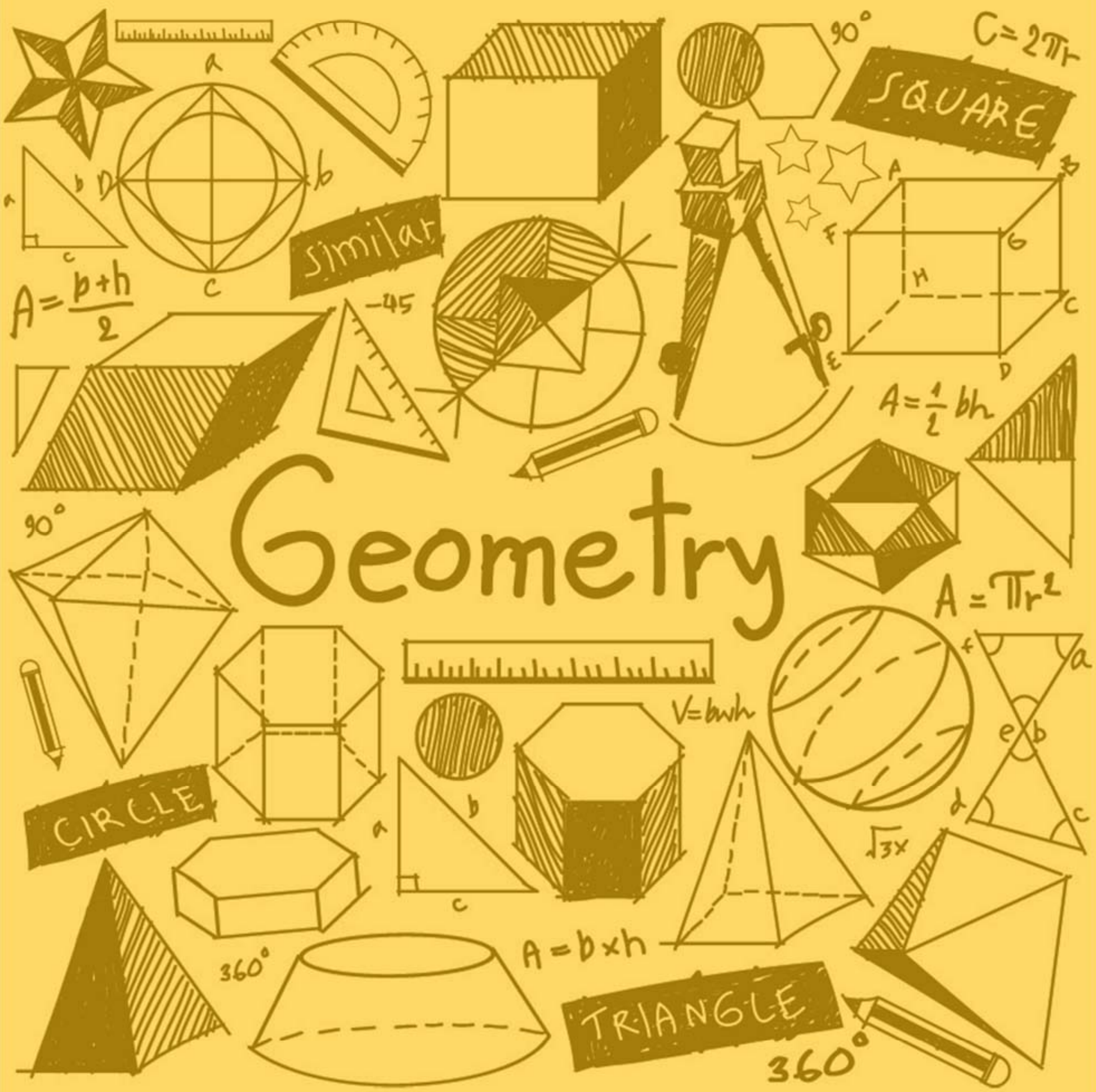
HOUSTON COUNTY
BOARD OF EDUCATION
HIGH-ACHIEVING TEACHERS
MATHEMATICS
DEPARTMENT



Student Workbook Pages

Geometry

Unit 1: Transformations on the Coordinate Plane

Concept 1: Geometry Definitions & Undefined Terms

Lesson A: Geometry Intro	(G.U1.C1.A.____.GeometryInfo)
Lesson B: Congruence, Midpoint, and Distance	(G.U1.C1.B. ____.≅MidpointDistance)

Concept 2: Experiment with Transformations & Sequences of Transformations on the Coordinate Plane

Lesson C: ≅, Rigid vs. Non-Rigid Transf.	(G.U1.C2.C.___.≅RigidVs.NonRigidTransformations)
Lesson D: Translations	(G.U1.C2.D.___.Translations)
Lesson E: Reflections	(G.U1.C2.E.___.Reflections)
Lesson F: Rotations	(G.U1.C2.F.___.Rotations)
Lesson G: Compositions of Transformations	(G.U1.C2.G.___.CompositionOfTransformations)

Unit 2A: Transformations on the Coordinate Plane

Concept 1: Transformations on the Coordinate Plane

Lesson A: Properties of Equalities	(G.U2A.C1.A.____.PropsOfEquality)
Lesson B: Logic, Conjectures, and Conditional Statements	(G.U2A.C1.B.____.Logic)
Lesson C: Definition and Properties of Congruence, Figure Marking, Postulates, and Theorems	G.U2A.C1.C.____.PropOfCongruence)

Unit 2B: Proof with Lines, Segments, and Angles Theorems

Concept 1: Theorems about Segments

Lesson A: Segment Theorems	(G.U2B.C1.A.____.SegmentTheorems)

Concept 2: Theorems about Angles

Lesson B: Angle Theorems	(G.U2B.C2.B.___.AngleTheorems)

Concept 3: Theorems about Parallel Lines

Lesson C: Parallel Theorems	(G.U2B.C3.C.___.ParallelTheorems)

Unit 2C: Proof With Triangles

Concept 1: Types of Triangles, Angles within Triangles

Lesson A: Types of Triangles, Angles within Triangle	(G.U2C.C1.A.____.TypesAndAnglesInTriangles)

Concept 2: Proving Triangles Congruent to One Another

Lesson B: Prove Triangles Congruent	(G.U2C.C2.B.___.TriangleCongruency)
Lesson C: Proofs with Triangles	(G.U2C.C2.C.___.TriangleProofs)

Concept 3: Special Segments within Triangles and Points of Concurrency

Lesson D: Parallel Theorems	(G.U2C.C3.D.___.ConstructionsSpecialSegs)

Unit 2D: Proof With Quadrilaterals

Concept 1: Definitions and Properties of Quadrilaterals

Lesson A: Types of Quadrilaterals, their Properties	(G.U2D.C1.A.____.DefineQuad)

Concept 2: The Resulting Theorems about Quadrilaterals

Lesson B: Prove Properties of Parallelograms	(G.U2D.C2.B.___.ParalPro)
Lesson C: Proving Properties of Rectangles, Rhombuses, and Squares	G.U2D.C2.C.___.RRS)

Unit 2E: Similarity and Dilation Transformations

Concept 1: Similar Polygons

Lesson A: Similar Polygons	(G.U2E.C1.A.____.SimilarPolygons)

Concept 2: Rigid vs. Non-Rigid Transformations: Dilations

Lesson B: Dilations	(G.U2E.C2.B.___.Dilations)

Unit 3: Right Triangle Trigonometry

Concept 1: Discover Trig Ratios with Similar Triangles

Lesson A: Intro to Trigonometry $S\frac{O}{H}, \ C\frac{A}{H}, \ T\frac{O}{A}$	(G.U3.C1.A.____.IntroToTrig)
Lesson B: Trig Relationships Between Angles, Compliments, and their Ratios	(G.U3.C1.B. ____.≅TrigRelationships)

Concept 2: Using Trig Functions to Solve Right Triangles + Applications

Lesson C: Solve for Unknown Sides and Angles	(G.U3.C2.C.___.SolveForSidesAngles)
Lesson D: Tangle of Elevation/Depression Problems	(G.U3.C2.D.___.ElevationDepression)

Unit 4: Circles and Volume

Concept 1: Angle Relationships in Circles

Lesson A: Circles Basics	(G.U4.C1.A.____.CircleBasics)
Lesson B: Central Angles, Inscribed Angles, and Inscribed Quadrilaterals and Theorems	(G.U4.C1.B. ____.CentralAndInscribed)
Lesson C: Arc and Angle Measures and Theorems	(G.U4.C1.C.____.ArcAngleMeasures)

Concept 2: Segment Relationships in Circles

Lesson D: Segment Lengths Formed within a Circle	(G.U4.C2.D.___.SegmentLengths)
Lesson E: Tangents	(G.U4.C2.E.___.Tangents)
Lesson F: Cords & Arcs	(G.U4.C2.F.___.ChordsAndArcs)

Concept 3: Arc Length & Sector Area of Circles

Lesson G: Arc Length and Sector Area	(G.U4.C3.G.___.ArcLengthSectorArea)

Concept 4: Volumes of All 3-D Shapes

Lesson H: Volume	(G.U4.C4.H.___.Volume)
Lesson I: Density	(G.U4.C4.I.___.Density)
Lesson J: Cavalierri's Principal	(G.U4.C5.J.___.CavalierisPrin)

Concept 5: Visualization and Design

Lesson K: Volumes of Rotation	(G.U4.C5.K.___.≅3DRotations)

Unit 5: Geometric and Algebraic Connections

Concept 1: Distance, Midpoint, and Partitioning a Directed Line Segment

Lesson A: Midpoint Distance, Revisited	(G.U5.C1.A.____.MidPointDistance)
Lesson B: Partitioning a Directed Line Segment	(G.U5.C1.B. ____.Partition)

Concept 2: Slope Criteria for Parallel and Perpendicular Lines

Lesson C: Perpendicular and Parallel Lines & Their Equations on the Coordinate Plane	(G.U5.C2.C.___.ParallelAndPerp)

Concept 3: Prove Geometric Theorems Algebraically

Lesson D: Coordinate Geometry Proofs (Triangles and Parallelograms	(G.U5.C3.D.___.ProvingShapes)

Concept 4: Equations of a Circle

Lesson E: Circles & Their Equations on the Coordinate Plane	(G.U5.C4.E.___.EquationOfCircle)

Unit 6: Application of Probability

Concept 1: Venn Diagrams, Intersection, Union, and Compliments of Sets

Lesson A: Using Venn Diagrams	(G.U6.C1.A.____.VennDiagrams)

Concept 2: Independent & Dependent Events Using $\times, +, -$, & Conditional Probability

Lesson B: Probability of Independent and Dependent Events & Conditional Probability	(G.U6.C2.B.___.IndepAndDepend)

Concept 3: Frequency Tables, Finding Probability Using Frequency Tables

Lesson C: Two-Way Tables	(G.U6.C3.C.___.TwoWayTables)

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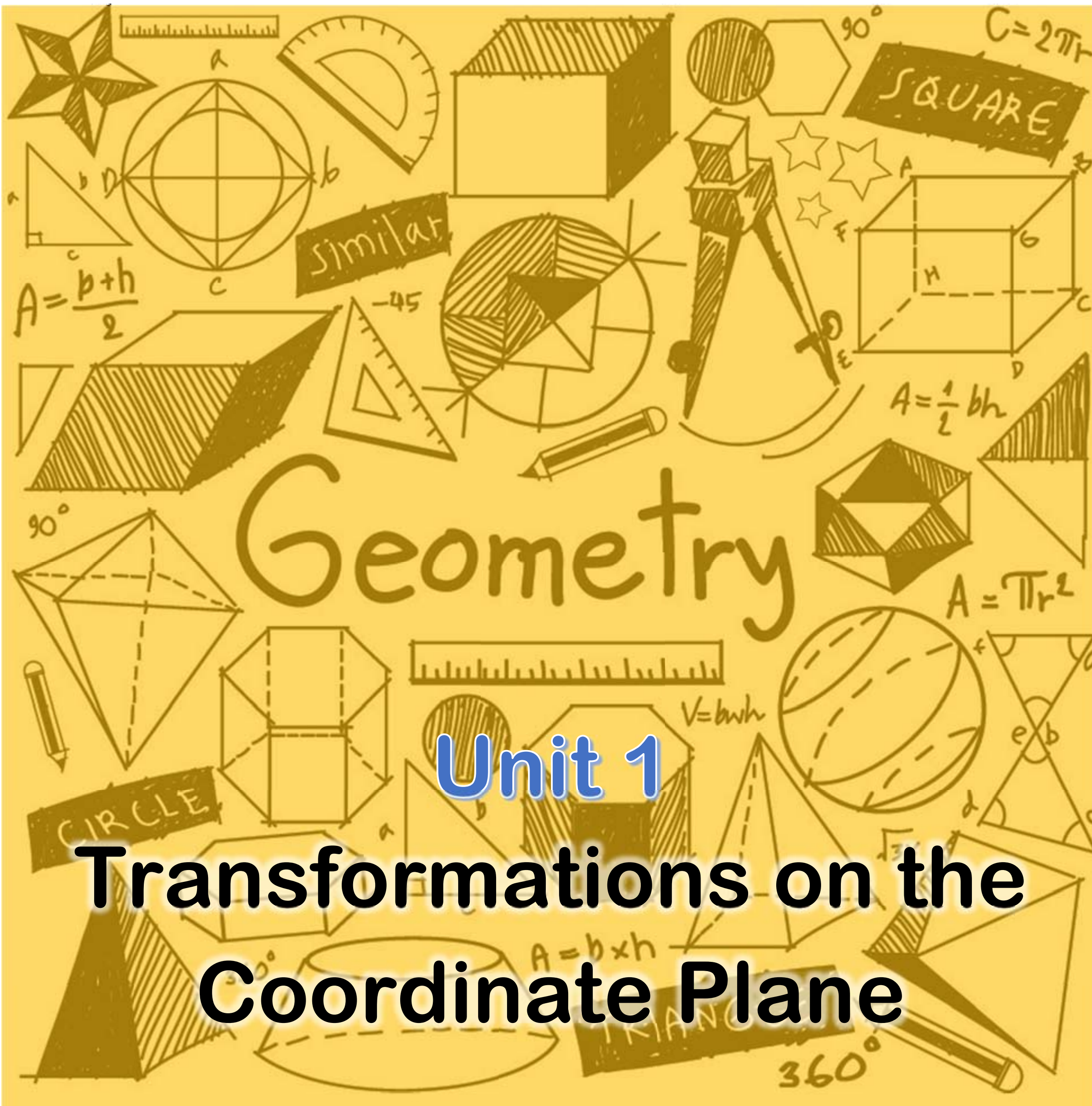
MATHEMATICS DEPARTMENT

Science Technology Engineering Mathematics



Unit 1

Transformations on the Coordinate Plane

HOUSTON COUNTY BOARD OF EDUCATION
HIGH-ACHIEVING TEACHERS

MATHEMATICS DEPARTMENT

Section	Term	Definition	Notation	Diagram/Visual
A	Between			
A	Circle			
A	Collinear			
A	Coplanar			
A	Intersection, Plane and Line			
A	Intersection, 2 Planes			
A	Line			
A	Non-Collinear			
A	Non-Coplanar			

Section	Term	Definition	Notation	Diagram/Visual
A	**Opposite Ray**			
A	**Parallel**			
A	**Perpendicular**			
A	**Plane**			
A	**Point**			
A	**Ray**			
A	**Segment**			
A	**Space**			

Section	Term	Definition	Notation	Diagram/Visual
A	Congruent			
B	Axiom			
B	Bisector			
B	Congruent Segment			
B	Distance Formula			
B	Equidistant			
B	Measure of a Segment			
B	Midpoint			
B	Midpoint Formula			

Geometry Unit # 1 Concept# Entire Unit

Lesson Name: VOCABULARY, POSTULATES, AND THEOREMS

Section	Term	Definition	Notation	Diagram/Visual
B	Midpoint Theorem			
B	Perpendicular Bisector Theorem			
B	Postulate			
B	Ruler Postulate			
B	Segment Addition Postulate			
B	Theorem			
C	Compass			
C	Construction			
C	Dilation			

Section	Term	Definition	Notation	Diagram/Visual
C	Image			
C	Isometry			
C	Mapping			
C	Mapping "onto"			
C	Onto			
C	Pre-Image			
C	Rigid Motion			
C	Straightedge			
C	Transformation			

Section	Term	Definition	Notation	Diagram/Visual
C,D	**Translation**			
C,E	**Reflection**			
C,F	**Rotation**			
C	**Dilation**			
D	**Directed Line Segment**			
D	***(enrichment)*** **Notation for Translations**			
E	**Line of Identity**			
E	**Line of Reflection**			
E	**Line of Symmetry**			

Section	Term	Definition	Notation	Diagram/Visual
E	*(enrichment)* **Notation for Reflection**			
E	**Reflectional Symmetry**			
C	**Symmetry**			
F	**Clockwise**			
F	**Counter-Clockwise**			
F	*(enrichment)* **Notation for Rotations**			
E	**Point of Rotation**			
G	**Composition (Sequence) of Transformations**			

Geometry Terms and Undefined Terms

Euclid

(pronounced "yoo-clid"), the 'Father of Geometry'.

Euclid did not really have a cell phone. However, if Euclid were there with you and asked you to define "cell phone," how would you respond? Telling him, "it's a phone that is cellular" would not really help him to understand, would it? This is an example of a "circular definition," because the definition uses the words that being defined. While the concept of "cell phone" is easy to describe or show, it is not very easy to define without using a circular definition.

Another type of circular definition: **oak** - "1: a tree that grows from an acorn." **acorn** - "1a: the fruit or nut of an oak tree"

Euclid realized that there are certain Geometry terms that are foundational to Geometry but that would yield circular logic in trying to define so he described them instead. These are called the ***UNDEFINED TERMS OF GEOMETRY**.

This first lesson includes many definitions (see vocab pages), and yet the four terms below are not included on those pages. Explain why.

GUIDED NOTES

Linear Term:	Description	Notation/Named by:	Diagram/Visual
POINT*	______________ ______________ ______________		
LINE*	______________ ______________ ______________		
PLANE*	______________ ______________ ______________		
SPACE*	______________ ______________ ______________		

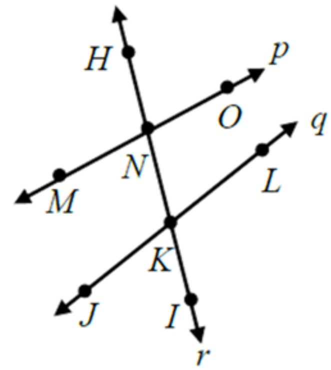
Name three real-life things that might be used to model each of the undefined terms above. Explain, also, why each of the "models" might fail the highly-technical geometric ideas of each, as discussed above.

Point		Line		Plane		Space	
Could be modeled by:	How it fails geometrically	Could be modeled by:	How it fails geometrically	Could be modeled by:	How it fails geometrically	Could be modeled by:	How it fails geometrically
1.		1.		1.		1.	
2.		2.		2.		2.	
3.		3.		3.		3.	

Now turn to your definitions for this section to continue guided notes as a part of this lesson.

SELF CHECK **Do you see how to name and define points, lines, planes, and do you understand their basic properties? Check your understanding below.**

1. Use the diagram to the right to name the following.



a) Four collinear points. ________________

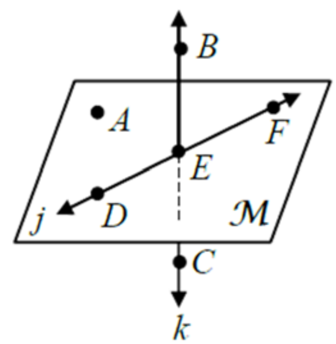
b) A line that contains point M. ________________

c) A line that contains points H and K. ________________

d) Another name for line q. ____________________

e) The intersection of lines p and r. ____________

2. Use the diagram to the right to name the following.



a) A line containing point F. _________

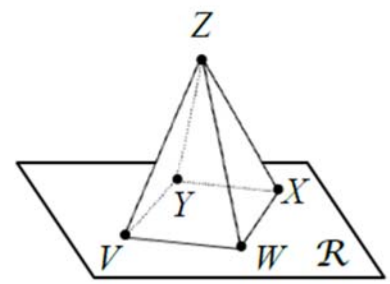
b) Another name for line k. _________

c) A plane containing point A. ________________

d) An example of three non-collinear points. _________

e) The intersection of plane $\mathcal{M}$ and line k. _________

3. Use the diagram to the right to name the following.



a) Three coplanar points. _______________

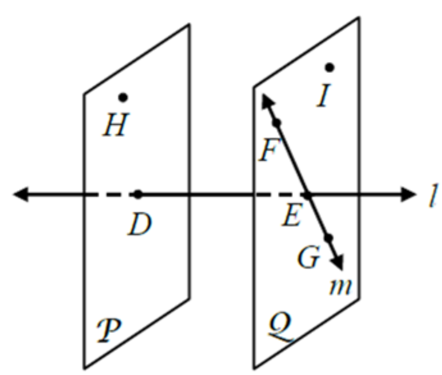
b) A plane containing point X. ____________________

c) The intersection of plane $\mathcal{R}$ and plane ZVY. _________

d) How many planes appear in the figure? _________

e) How many planes contain point W? _________

4. Use the diagram to the right to name the following.



a) The intersection of lines ℓ and m. _________

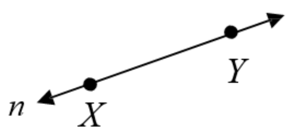
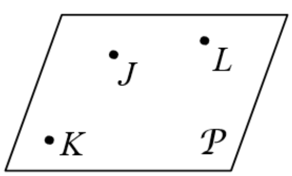
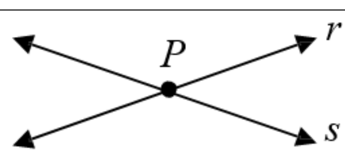
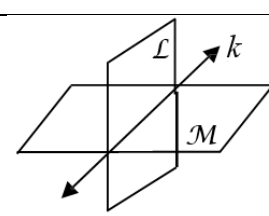
b) Another name for plane Q. ____________________

c) Are points D and E collinear or coplanar? ______________

d) How many times do planes P and Q intersect? ___________

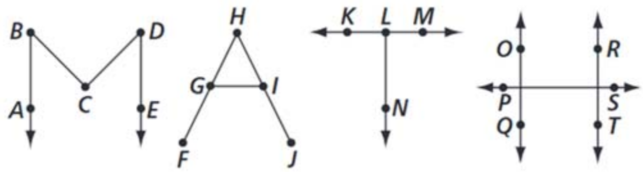
Re-Teaching and Independent Practice

Example! Fill in the blanks below with the correct answers that show that you understand the concepts of Point, Line, Plane, and Intersections.

Main Ideas/Questions:	Notes
POINT  	• By definition, a point is a ____________________. • A point is generally represented by ____________, when drawn on paper. • A point has no __________________ or __________________. • Always use a __________________ to name a point. **Example:____________________________**
LINE  	• A line is made up of ____________________. • A line is generally represented by ____________, when drawn on paper. • Any __________________ points form a line. • A line has no __________________ or __________________. • Name a line by ___________________________, or by __________________. **Example:____________________________.** **COLLINEAR POINTS** are points that lie on ________________________. **NON-COLLINEAR POINTS** are points that __________________________. (Must be at least ____ points!)
PLANE  	• A plane is a __________ _____________. made up of _______________. • A plane is generally represented by ____________, when drawn on paper. • Any __________________ points form a plane. • A plane extends _________________________in all directions. • Name a plane by ___________________________, or by __________________. **Example:____________________________.** **COLLINEAR POINTS** are points that lie on ________________________. **NON-COLLINEAR POINTS** are points that __________________________. **(Must be at least ____ points!)**
INTERSECTING LINES AND PLANES	 Two lines intersect at a________________.  Two planes intersect at a...________________.

Describing and Modeling with Linear Structures Task

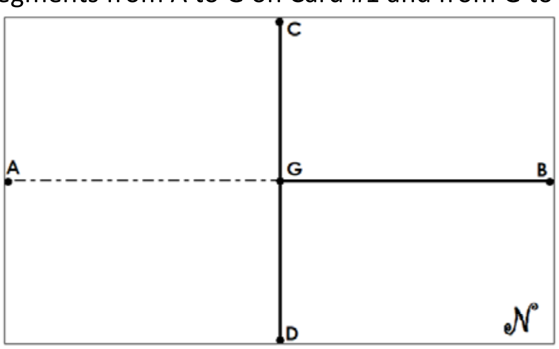
1. With geometric accuracy, name **all** of the distinct figures contained within each letter MATH, below.



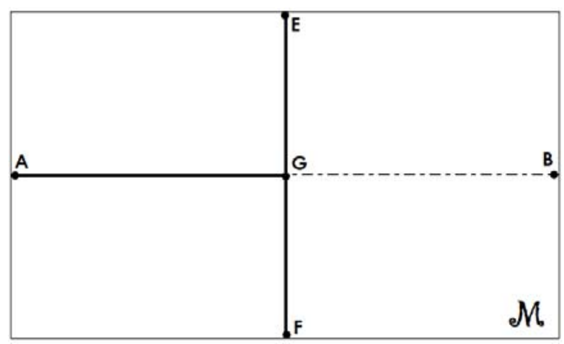
Bonus: name all of the structures you can find, and then identify which ones are actually redundant (describe the exact same structure).

M	
A	
T	
H	

2. You will need two index cards. On these cards, draw and label them as you see below. Then cut along the dotted line segments from A to G on Card #1 and from G to B on Card #2.

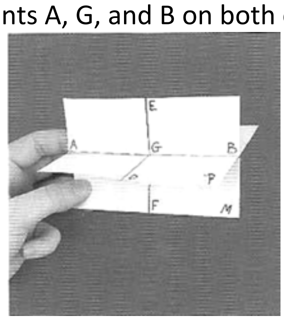


Card #1



Card #2

Slide the cards together as shown below, so points A, G, and B on both cards coincide (meet).



Then answer the following questions:

a) When the cards are NOT together, what is the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$?	
b) When the cards are NOT together, what is the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$?	
c) With the cards together, what is the intersection of $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$?	
d) What is the intersection of planes $\mathcal{M}$ and $\mathcal{P}$?	
e) Are $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ coplanar?	

Draw the following geometric structures exactly as described and answer questions, where there are questions.	
3. Line ℓ and $\overrightarrow{BC}$ intersect perpendicularly at point C.	
4. Line a and line b are parallel to one another, and $\overline{JK}$ intersects a at point K and intersects b at point J. $\overline{JK}$ is not parallel to a.	
5. Plane ABC intersects Plane XYZ at line q, and $\overleftrightarrow{AB}$ and $\overline{XY}$ are parallel to q.	
6. Planes Q and $\mathcal{R}$ are parallel, and both are perpendicular to plane $\mathcal{T}$.	

7. Plane N contains lines $\overleftrightarrow{XY}$ ad $\overline{WZ}$, which will never intersect.	
8. $\overrightarrow{BC}$ and $\overline{AB}$ are perpendicular.	
9. Line a and line b are coplanar. Line f is noncoplanar with line a but is coplanar with line b.	
10. Points A and B are collinear with C, but noncollinear with D, although C and D are collinear with E.	
11. THINKING TOWARD OUR NEXT LESSON: **Draw the figure described and answer the questions.** $\overline{GH}$ is cut into two segments of equal length, at point B, by line k. **Question A:** What are two segments of equal length called? **Question B:** How would you show the segments are of equal length in your drawing? **Question C:** The segments of equal length are distinct from one another, meaning they are two different segments. That means they are not "equal." How would you SAY they are of equal length so that it is clear that they are distinct segments of equal length?	

Homework #________

1. Use the diagram to the right to name the following.

a) How many points appear in the figure? _______

b) How many lines appear in the figure? _______

c) How many planes appear in the figure? _______

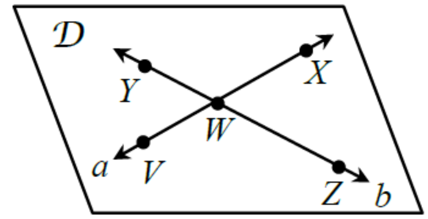
d) Name a line containing point V. ________________

e) Name the intersection of lines a and b. ________________

f) Give another name for line b. ________________

g) Name three non-collinear points. ____________________

h) Give another name for plane $\mathcal{D}$. __________________



2. Use the diagram to the right to name the following.

a) How many points appear in the figure? _______

b) How many lines appear in the figure? _______

c) How many planes appear in the figure? _______

d) Name three collinear points. ________________

e) Name four non-coplanar points. ________________

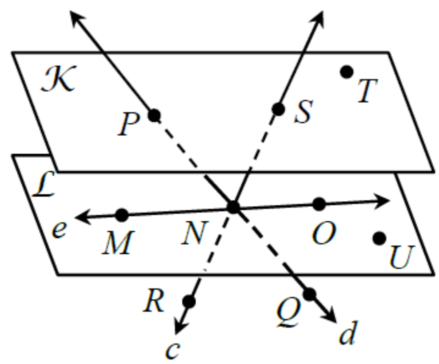
f) Give another name for line e. ________________

g) Name the intersection of $\overleftrightarrow{PQ}$ and $\overleftrightarrow{MO}$. ___________

h) Name the intersection of plane $\mathcal{K}$ and line c. ___________

i) Give another name for plane $\mathcal{L}$. ____________________

j) Give another name for $\overleftrightarrow{PQ}$. ____________



3. Use the diagram to the right to name the following.

a) How many points appear in the figure? _______

b) How many lines appear in the figure? __________

c) How many planes appear in the figure? __________

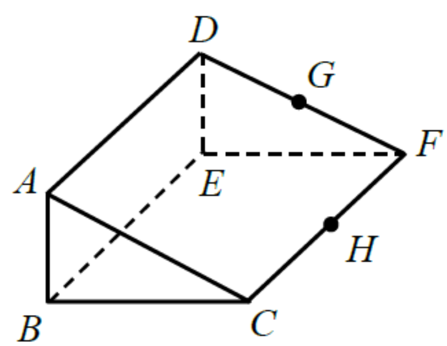
d) Name three collinear points. ________________

e) Name four coplanar points. ________________

f) Name the intersection of planes ABC and ABE. __________

g) Name the intersection of planes BCH and DEF. __________

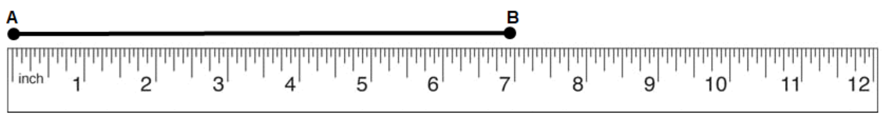
h) Name the intersection of $\overleftrightarrow{AD}$ and $\overleftrightarrow{DF}$. __________



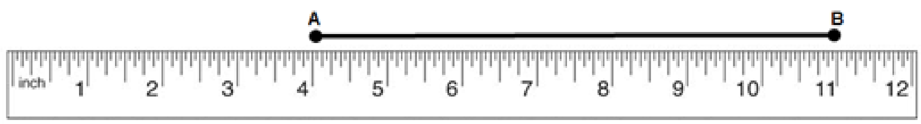
Ruler Postulate

In geometry ***postulates***, are statements that are accepted as true without proof in order to provide a starting point for deductive reasoning. Like *point, line,* and *plane*, ***distance along a line*** is an undefined term in geometry used to define other geometric terms. For example, the length of a line segment is the distance between its endpoints.

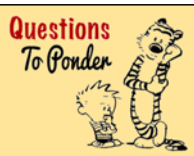
The ***ruler postulate*** states, essentially, that it is possible to measure a segment using a graduated measuring tool (like a ruler, which gives this postulate its name). We know when using a ruler to line up one point with zero so that the end point shows us the length of the object we are measuring.



However, the ruler postulate also states something else. If your segment happens NOT to be lined up with zero, finding the length is still possible, by subtraction. Fill in the blanks below with the correct mathematical sentence. Notice the **absolute value** symbols!



$AB = |$________ − ________$| = |$________$| =$ ________ **inches**



Is there another way you could have calculated AB?

$AB = |$________ − ________$| = |$________$| =$ ________ **Inches**

In calculating distance on a number line, what is the function of the absolute value symbols? What do they allow you to do that you could not do without them?

Postulate 1: Ruler Postulate a. To every pair of points there corresponds a unique positive number, called the distance between the points. **b.** The points on a line can be matched with the real numbers so that the distance between any two points is the absolute value of the difference of their associated numbers.

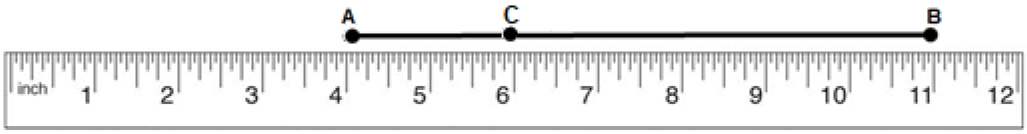
SELF CHECK

The number obtained as a measure of distance depends on the unit of length. The distance in inches will be a different number than the distance in centimeters. Use a ruler to determine the lengths of each segment below. Write as a full mathematical sentence.

Segment	Length, to nearest eighth of an inch	Length, to nearest tenth of a centimeter
$\overline{DE}$		
$\overline{EF}$		
$\overline{DF}$		

Segment Addition Postulate

Now – Let us look at what happens with collinear points on a segment. Given point C that lies on the line segment between A and B.

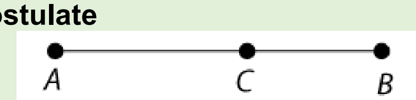


$AC =$ ______________, $CB =$ ____________________, $and\ AB =$ ______________________.

You can write a <u>numeric</u> mathematical sentence about what just happened:	…and you can write a <u>symbolic</u> mathematical sentence about what just happened, using the names of the segements:
__________**+**__________**=**_____________	__________**+**__________**=**_____________

The symbolic sentence you just wrote is called the **Segment Addition Postulate.**

Postulate 2: Segment Addition Postulate



Let A, B, and C be collinear points. If C is between A and B, then $AC + CB = AB$.

Example!

1. Three segment measures are given: $NQ = 17, NK = 6, and\ QK = 11$. The three points named are collinear. Determine which point is between the other two, and draw a sketch of what was described.

Solution:

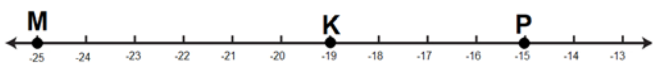
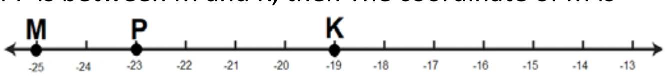
;
K has to be between N and Q.

SELF CHECK

Q, R, and S are collinear points, and R lies between Q and S. If $RS = 44cm$ and $QS = 68cm$, sketch the geometric situation described and find QR.

Example! **2. M, K, and P all lie on the same number line. $MK = 6, KP = 4$, and the coordinate of M is -25. What are the coordinates of all of these points?**

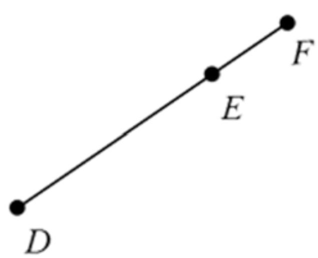
Solution: Read carefully for what questions say, and what they do not say. Without knowing for sure if K lies between M and P, or if P lies between M and K, there are two possible solutions.

Where are M, K, and P, relative to one another?	The coordinate of M:	The coordinate of K:	The coordinate of P:
If K is between M and P, then The coordinate of M is 	**-25**	**-19**	**-15**
If P is between M and K, then The coordinate of M is 	**-25**	**-19**	**-23**

…remember not to assume in Geometry!

SELF CHECK $Q, B, and\ A\ are\ all\ collinear.\ QB + BA = QA, AB = 10, QB = 8.$ The number line coordinate of B is 1. Find the coordinate of Q and A.

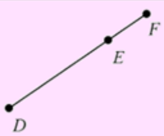
Example! **3. Suppose point E lies on $\overline{DF}$, between D and F.**
If $DE = 4x - 1, EF = 9$, and $DF = 9x - 22$, find the value of x.



Solution: For Alg/Geo mix problems, ALWAYS write out the true symbolic statement first. That way, you know how to set up the equation.

Statement	Reason
$DE + EF = DF$	Segment Addition Postulate
$(4x - 1) + (9) = (9x - 22)$	Substitution
$4x + 8 = 9x - 22$	Combining Like Terms
$-8 \quad -8$; $4x = 9x - 30$	Subtraction Property of Equality
$-9x \quad -9x$; $\frac{-5x}{-5} = \frac{-30}{-5}$	Subtraction Property of Equality
$x = 6$	Division Property of Equality

SELF CHECK

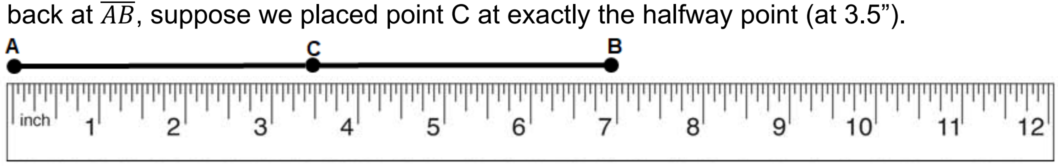


Suppose again that point E lies on $\overline{DF}$, between D and F. Solve the following problems (they are independent of one another, except for the diagram).

1) If $DF = 78$, $DE = 5x - 9$, and $EF = 2x + 10$, find EF. Show your work.	2) If $DE = 4x + 10$, $EF = 2x - 1$, and $DF = 9x - 15$, find DF. Show your work.

Midpoint of a segment

Now…looking back at $\overline{AB}$, suppose we placed point C at exactly the halfway point (at 3.5”).

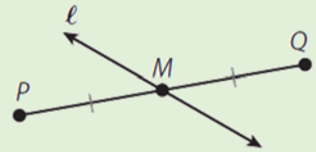


You can still write a numeric mathematical sentence about what just happened this time: __________+__________=____________	…and you can still write a symbolic mathematical sentence about what just happened, using the points that were superimposed on the image of the ruler. __________+__________=____________

However, NOW there is a third sentence, that can be written both numerically and symbolically, that is now true.

In a situation in which the cut happens at the midpoint, numerically this is true: ___________=____________	…and symbolically, this is true based on C now being at the center point (midpoint) of the ruler: ___________=____________ Also - ___________≅____________

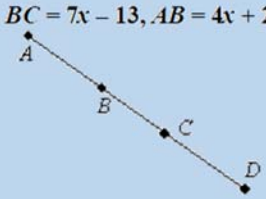
Midpoint, Bisector, and the Midpoint Theorem
The midpoint of a line segment is the point that divides the segment into two segments that have the same length. A line, ray, or other figure that passes through the midpoint of a segment is a segment bisector.
If M is the midpoint of PQ, and you know that P the figure, the tick marks show that $PM = MQ$. Therefore, M is the midpoint of $\overline{PQ}$ and line ℓ bisects $\overline{PQ}$.



Example!

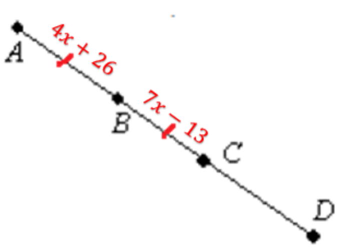
4. For Alg/Geo mix problems, ALWAYS write out the true symbolic statement first. That way, you know how to set up the equation.

$BC = 7x - 13$, $AB = 4x + 26$, B is the midpoint of $\overline{AC}$.



Find x.

Solution: If B is the midpoint of $\overline{AC}$, then $AB = BC$. (Who cares about $\overline{CD}$? No information was given about $\overline{CD}$, and no questions were asked, so we ignore that unused tail on the segment!)



Statement	Justification
B is the midpoint of AC	Given
$AB = BC$	Because B is the midpoint
$7x - 13 = 4x + 26$	Substitution
$-4x \quad\quad -4x$	
$3x - 13 = 26$	Subtraction property of equality
$+13 \quad\quad +13$	
$3x = 39$	Addition property of equality
$\frac{3x}{3} = \frac{39}{3}$	Division property of equality
$x = 13$	

SELF CHECK Try the following midpoint problems on your own or in a small group. Show your work!

1. If Q is the midpoint of $\overline{PR}$, $PQ = 7x - 16$, and $QR = 4x + 2$, sketch the situation, set up, and solve and equation to find the value of x, PQ, QR, and PR.

$x =$
$PQ =$
$QR =$
$PR =$

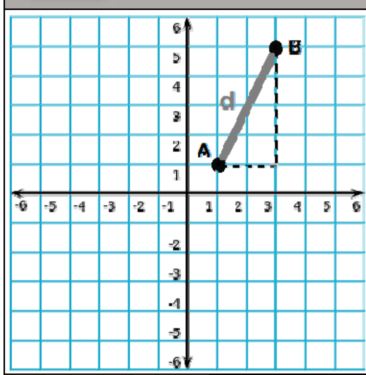
2. If H is the midpoint of $\overline{GI}$, $GH = 5x + 2$, and $HI = 9x - 10$, sketch the situation, set up, and solve and equation to find the value of x, GH, HI, and GI.

$x =$
$GH =$
$HI =$
$GI =$

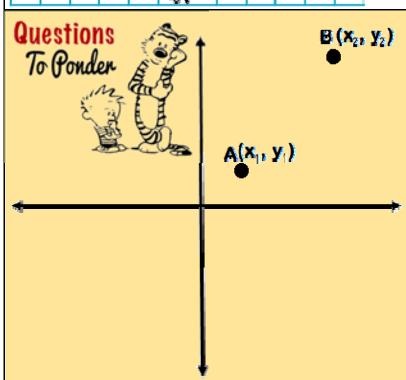
3. If X is the midpoint of $\overline{AB}$, $AX = 2n + 16$, and $AB = 7n + 1$, sketch the situation, set up, and solve and equation to find the value of n, AX, XB, and AB.

$n =$
$AX =$
$XB =$
$AB =$

Linear Formulas – Midpoint Formula and Distance Formula



Find the distance between A (you can see A is (1, 2) and B (3, 5) in the image to the left? (HINT: most math students do this in 7th or 8th grade, and it involved a different ancient mathematician).



Now – more generally, and without the aid of grid lines, retrace the same thoughts you had above. What would you do with (x_1, y_1) and (x_2, y_2) to find the distance between A and B? ***Don't let the subscripts scare you. They only indicate that there is a "first" point and a "second" point; that is, that you have two points. Whichever one you call "first" or "second" is up to you.***

Using the Distance Formula

DISTANCE FORMULA

____

____

____

WORKED EXAMPLE: What is the distance between (–2, 1) and (1, 5)?

$$d = \sqrt{(5-1)^2 + (1--2)^2}$$
$$d = \sqrt{(4)^2 + (3)^2}$$
$$d = \sqrt{16+9}$$
$$\boldsymbol{d = \sqrt{25} = 5}$$

SELF CHECK What is the distance between (–2, –3) and (–4, 4)?

MIDPOINT FORMULA

____

____

____

WORKED EXAMPLE: What is the midpoint of (–2, 1) and (1, 5)?

$$midpoint = \left(\frac{(-2+1)}{2}, \frac{(1+5)}{2}\right) = \left(\frac{(-1)}{2}, \frac{(6)}{2}\right) = (\boldsymbol{-0.5}, \ \boldsymbol{3})$$

SELF CHECK What is the midpoint of (–2, –3) and (–4, 4)?

SELF CHECK If a segment has an endpoint of (–21, 6) and a midpoint of (4, -1), what is the other endpoint?

Re-Teaching and Independent Practice

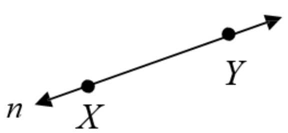
Example! Fill in the blanks below with the correct answers that show that you understand the concepts of Point, Line, Plane, and Intersections.

Main Ideas/Questions:	Notes
POINT	

- By definition, a point is a ____________________.
- A point is generally represented by ____________, when drawn on paper.
- A point has no __________________ or __________________.
- Always use a __________________ to name a point.

Example:____________________________

LINE

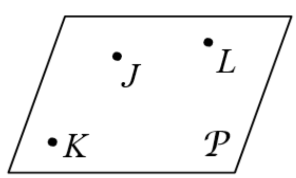


- A line is made up of ____________________.
- A line is generally represented by ____________, when drawn on paper.
- Any __________________ points form a line.
- A line has no __________________ or __________________.
- Name a line by ___________________________, or by __________________.

Example:____________________________.

COLLINEAR POINTS are points that lie on ________________________.
NON-COLLINEAR POINTS are points that __________________________.
(Must be at least ____ points!)

PLANE

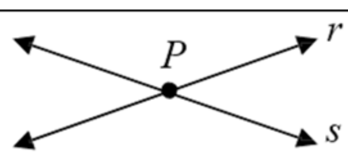


- A plane is a __________ _____________. made up of _______________.
- A plane is generally represented by ____________, when drawn on paper.
- Any __________________ points form a plane.
- A plane extends _________________________in all directions.
- Name a plane by ___________________________, or by __________________.

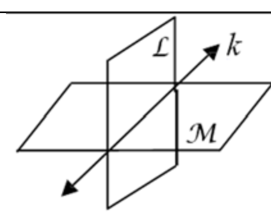
Example:____________________________.

COLLINEAR POINTS are points that lie on ________________________.
NON-COLLINEAR POINTS are points that __________________________.
(Must be at least ____ points!)

INTERSECTING LINES AND PLANES



Two lines intersect at a________________.

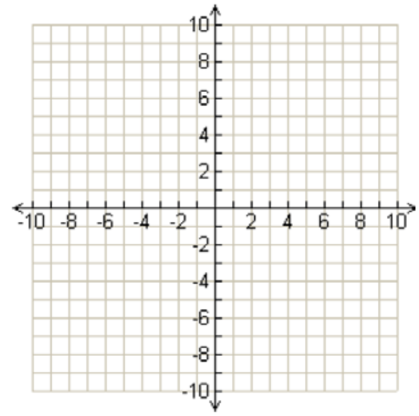


Two planes intersect at a…________________.

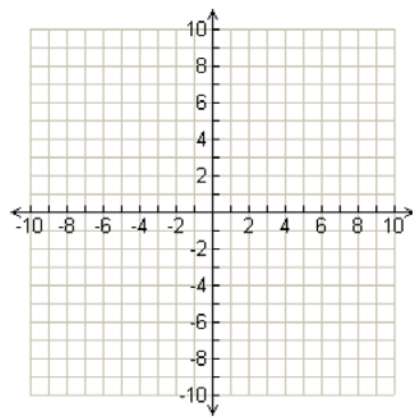
Midpoint and Distance Worksheet

Part 1: Graphing

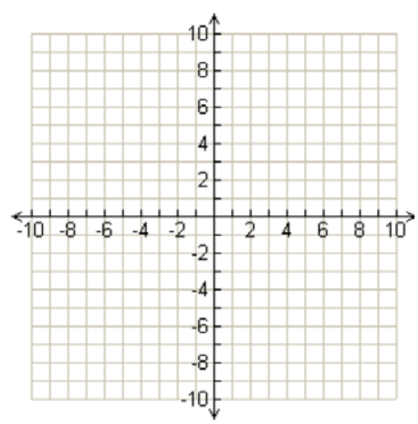
1) Graph the points A (1, 8) and B (9, 6). Find the midpoint of $\overline{AB}$. Find the length of $\overline{AB}$.



2) Graph the points C (2, -4) and D (6, 2). Find the midpoint of $\overline{CD}$. Find the length of $\overline{CD}$.



3) Graph the points E (-10, -9) and F (4, -3). Find the midpoint of $\overline{EF}$. Find the length of $\overline{EF}$.



Part 2: Midpoint Using Formula Only

Find the midpoint for each line segment using the formula (no graphing needed). Show the formula and all work.

4) G (6, 5) and H (9, 2)

5) I (1, 1) and J (-3, -3)

6) Given the midpoint of segment KL is M (1, -1) and L (8, -7). What are the coordinates of the other endpoint K?

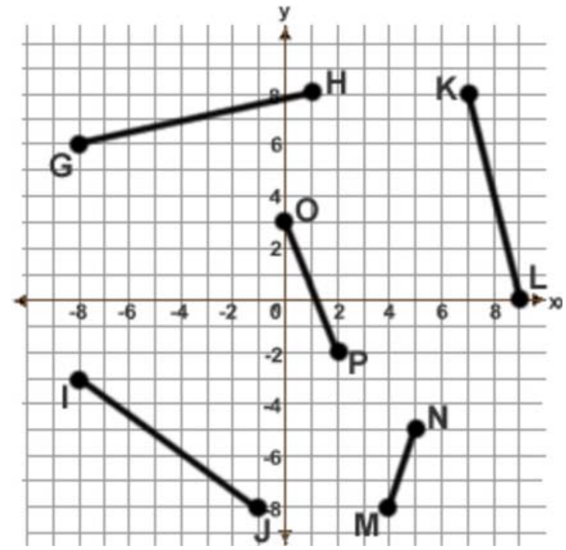
Part 3: Distance Using Formula Only

Find the distance between each set of points. Show the formula and all work.

7) (0, 0) and (4, 3)

8) (3, -3) and (2, 7)

9) Determine the coordinates of the points needed. Then find the length of each line segment.



a) GH G (,) H (,)

b) KL K (,) L (,)

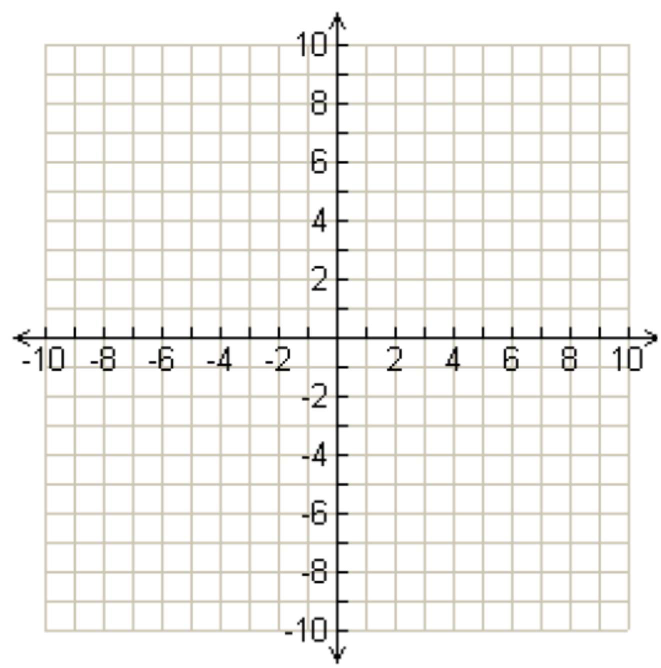
Part 4: Putting it All Together

10) Triangle ABC has coordinates A (3, 9), B (5,1) and C (9, 5). **D is the midpoint of AB** and **E is the midpoint of AC**.

a) Graph the points A, B, and C (make sure you label them). Find the coordinates of points D and E. Show all work.

D =

E =



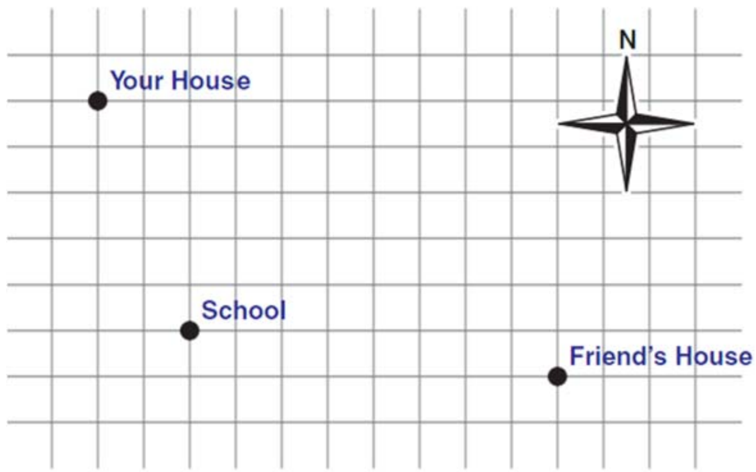
b) Plot points D and point E on the graph and label.

c) Find the length of BC. Show all work.

d) Find the **length of DE**. Show all work.

Task: As the Crow Flies

Suppose that the city in which you live has a system of evenly spaced perpendicular streets, forming square city blocks. The map below shows your school; your house, which is located two blocks west and five blocks north of the school; and your best friend's house, which is located eight blocks east and one block south of the school.



1. How many blocks would you have to drive to get from your house to your friend's house? Draw a path that you would drive, and calculate the distance.

2. What if you could use a helicopter to fly straight from your house to your friend's house? Draw the path that you would take. How could you find the distance "as the crow flies"?

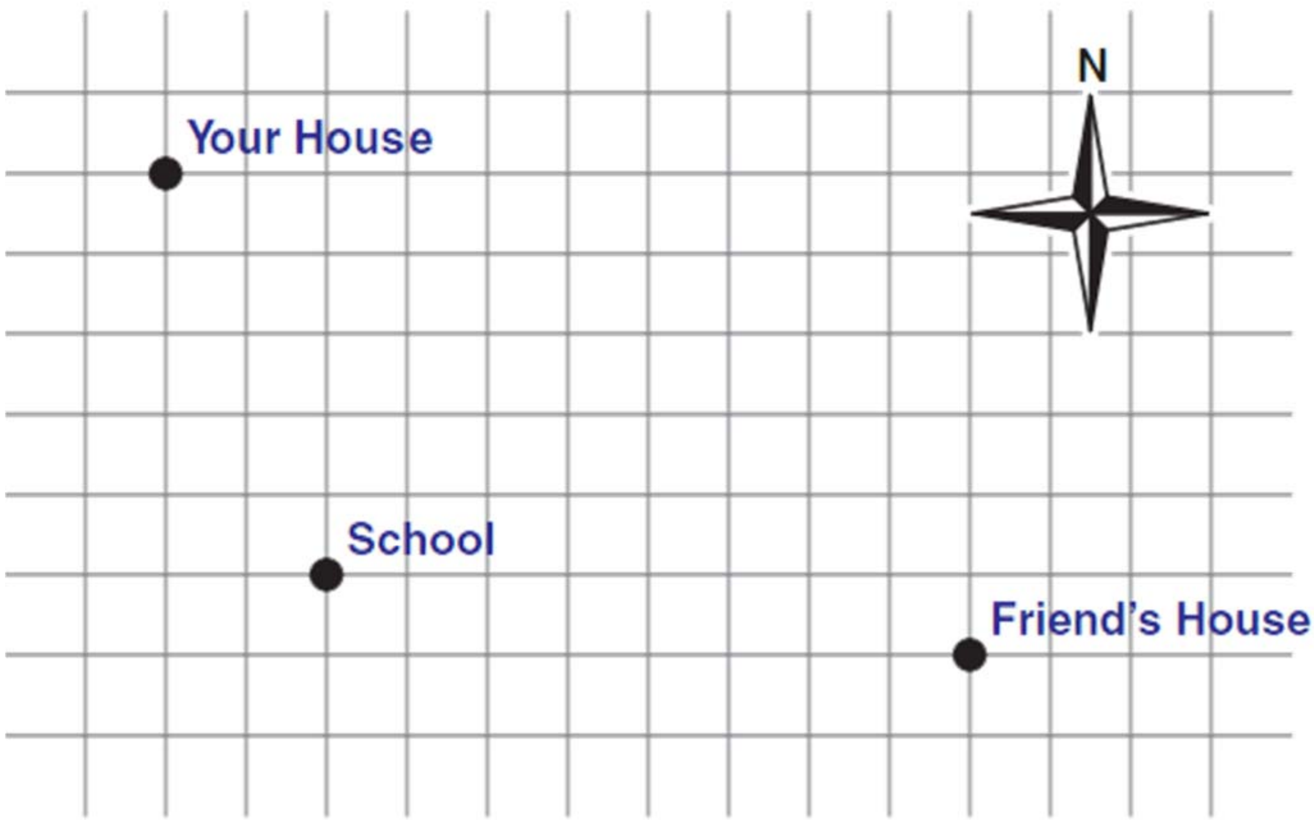
3. Establish a coordinate-axis system, using the school as the origin. What would the coordinates be for your house? For your friend's house?

4. How could you use the coordinates to calculate the distance "as the crow flies" from your house to your friend's house?

5. Suppose that your uncle lives two blocks east and one block south of the school and that you decide to stop by his house on the way home from your friend's house. Compute the round-trip distance from your house to your friend's house, to your uncle's house, and then back to your own house.

a. Using the school as the origin on the grid below, place your uncle's house on the plane and label it. What are the coordinates of your uncle's house?

b. Show how you could find the round-trip distance by using only coordinates.



Adapted from the National Council of Teachers of Mathematics, Inc. www.nctm.org.

1. MEASURING: Use a ruler to measure $\overline{AB}$ and $\overline{CD}$ below, as described in the table (both inches and centimeters)

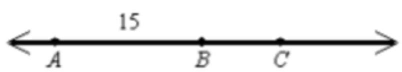
Segment	Length, to nearest eighth of an inch	Length, to nearest tenth of a centimeter
a. $\overline{AB}$		
b. $\overline{CD}$		

2. NUMBER LINE DISTANCES:

$KD, and\ E\ are\ all\ collinear.$ $KD + DE = KE, ED = 150, KD = 40.$ The number line coordinate of D is -121. Find the coordinate of K and E.

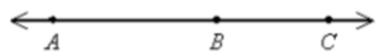
3. SEGMENT ADDITION POSTULATE WITH NUMBERS:

a.



If $AB = 15$ and $AC = 23$, find the length of $\overline{BC}$

b.



If $AB = 19$ and $AC = 32$, find the length of $\overline{BC}$.

4. SEGMENT ADDITION POSTULATE WITH ALGEBRA:

a. If $XY = 2n - 3$, $YZ = 7$, and $XZ = -3n + 24$, find the value of XZ.

b. If $XZ = 78, XY = 5a - 9$, and $YZ = 2a + 10$, find YZ.

c. If $XY = 4x + 10, YZ = 2x - 1,$ and $XZ = 9x - 15,$ find XZ.

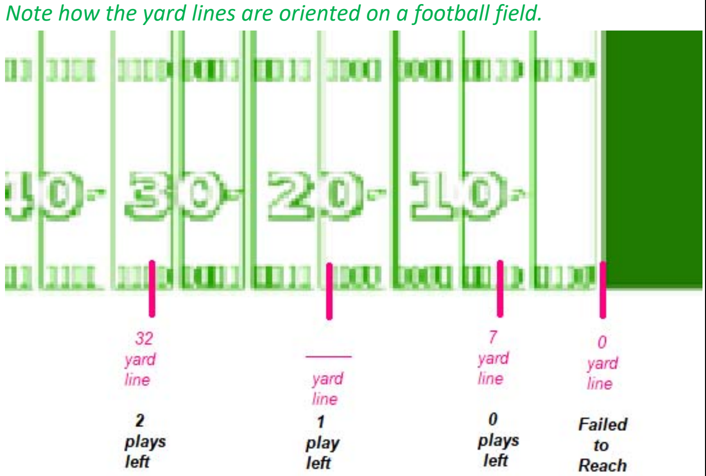
d. Let Y be between X and Z. Use the segment addition postulate to solve for v.
$XY = 3v - 30$
$YZ = 6v - 15$
$XZ = 27$

5. MIDPOINT WITH NUMBERS:

a. $K, D,$ and E are all collinear. D is the midpoint of $\overline{KE}$. $ED = 45$. The number line coordinate of K is -12. Find the coordinate of D and E.

b. With fifteen seconds and two plays left in the football game, the Bears needed to move the ball 32 more yards to get another touchdown. They ultimately failed to make a touchdown by 7 yards, but they advanced the exact same number of yards in both of their two attempts. From what yard line did they play their last down?

Note how the yard lines are oriented on a football field.



6. MIDPOINT WITH ALGEBRA:

a. If R is the midpoint of $\overline{QS}$, $QR = 8x - 51$ and $RS = 3x - 6$, find QS.

b. $R, T,$ and S are all collinear. T is the midpoint of $\overline{RS}$. $ST = 5x + 6$. $RS = 10x - 12$. Find the x and the le;ngths of $\overline{RT}, \overline{TS}, and\ \overline{ST}$.

7. MIDPOINT WITH COORDINATES Determine the coordinates of the midpoint for each segment. **The first one is done for you**.

a. . $\overline{PQ}$ has endpoints *P*(-3, -5) and *Q*(4, 1).

Midpoint:_____________________________/

b. . $\overline{RS}$ has endpoints $R(3, 2)$ and $S(9, 6)$.

Midpoint: _________________________

c. $\overline{XY}$ has endpoint $X(5, 12)$ and midpoint $M(7, 4)$.

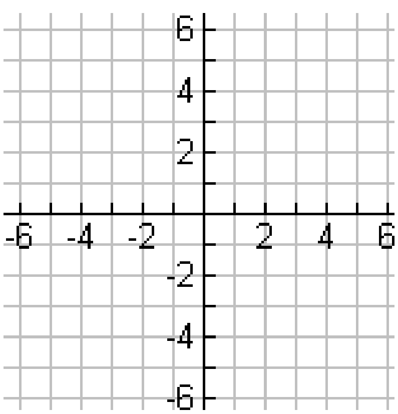
Endpoint Y:______________________

d. $\overline{PN}$ has endpoint $N(-3.5, 2)$ and midpoint $(-8, 17)$.

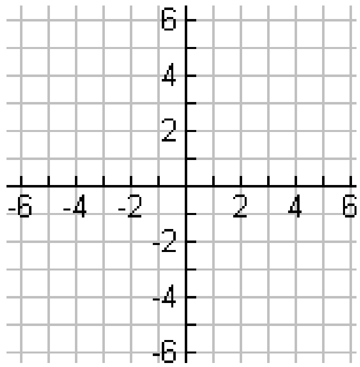
Endpoint P:______________________

8. **MIDPOINT WITH COORDINATE:S** Given triangle *ABC,* determine the coordinates of the vertices of a new triangle formed by the midpoints of each side of triangle *ABC*. *Graph all points to check on the axes provided.*

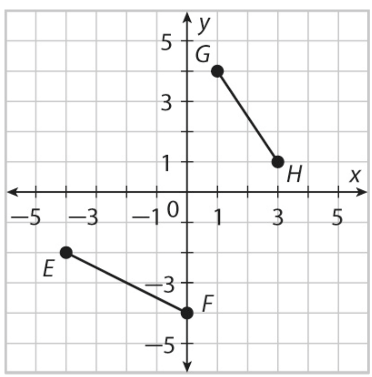
a. $A(-1, -3), B(4, -1), C(3, 4)$; $M_{\overline{AB}}$(______ , ________) ; M_{BC}(______ , ________) ; M_{CA}(______ , ________)



a) $A(4,-4), B(0,4), C(-3,-1)$; $M_{\overline{AB}}$(_______ , ________) ; M_{BC}(_______ , ________) ; M_{CA}(_______ , ________)



9. DISTANCE FORMULA: Use the distance formula to determine the length of each segment. **The first one is done for you.**



$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

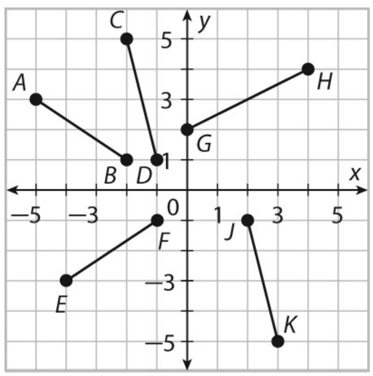
a.. $\overline{EF}$

$E(-4,-2),\ F(0,-4)$

$$EF = \sqrt{(0-(-4))^2 + (-4-(-2))^2} = 2\sqrt{5}$$

b. $\overline{GH}$

10: **DISTANCE FORMULA:** Answer the following questions about the lengths of the segments on the grid. Use the distance formula to show the length of each segment.



3. $\overline{CD}$ and _____ have the same length.

4. $\overline{EF}$ and _____ have the same length.

5. _____ has a different length than all of the other segments on the grid.

Lesson Name: G.U1.C1.B.05.**ExtraHW.≅MidpointDistance**

Find the distance, midpoint, and slope for the points given in the table below

Points	**Distance** $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	**Midpoint** $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$	**Slope** $m=\frac{y_2-y_1}{x_2-x_1}$	**Parallel** Same slope	**Perpendicular** opposite reciprocal
Example: Label your points as $(x_1,y_1),(x_2,y_2)$ A(-2,1), B(3,9)	$d=\sqrt{(9-1)^2+(3-(-2))^2}$ $d=\sqrt{(8)^2+(5)^2}$ $d=\sqrt{64+25}$ $d=\sqrt{89}$	$\left(\frac{-2+3}{2},\frac{1+9}{2}\right)$ $\left(\frac{1}{2},\frac{10}{2}\right)$ $\left(\frac{1}{2},5\right)$	$m=\frac{9-1}{3-(-2)}$ $m=\frac{8}{5}$ hint: keep it as improper fraction, do not simplify	$m=\frac{8}{5}$ hint: same number	$m=-\frac{5}{8}$ hint: change the sign and flip the fraction
C(0,0), D(6,8)					
E(3,0), F(6,8)					
G(3,2), H(10,8)					
I(-2,-4), J(4,5)					
K(2,3), L(5, -4)					
M(2,-6), N(2,7)					

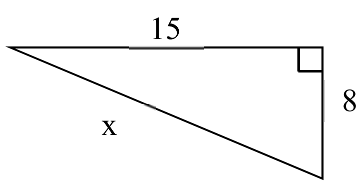
More hints on fractions: any number divided by 1 is the number itself,
any number divided by 0 is undefined,
0 divided by any number is 0.

Word Problems

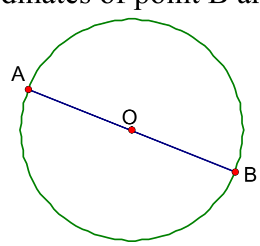
1) Find the midpoint of segment DE, with coordinates D(-4, -6), E(6, 10)

3) If point P has coordinates (-4, 2) and point Q has coordinates (2, 0), what is the distance from point P to point Q?

5) Find the value of x



7) Segment AB is the diameter of a circle whose center is point O. if the coordinates of point A are (1,5) and the coordinates of point B are (7, 3), find the coordinates of pint O.



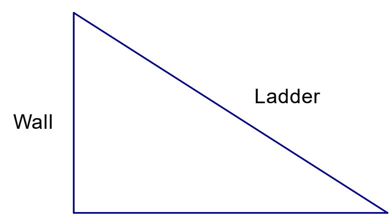
9) For the same picture above, find the length of diameter AB

2) Will is standing 30 yards due north of point P. Grace is standing 60 yards due west of point P. What is the shortest distance between Will and Grace?

4) What is the slope of a line m that passes through (0, -4) and (6, 25)? If line n is parallel l to line m, what is the slope of line n? if line p is perpendicular to line m, what is the slope of line p?

6) How long is a string reaching from the top of a 14-ft pole to a point 9 feet from the pole?

8) A ladder 15 feet long is placed against the wall of a building. The base of the ladder is 10 feet from the wall. How high up on the wall is the top of the ladder?



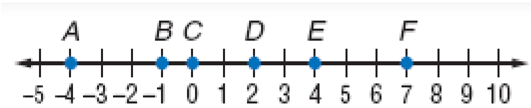
10) For the same picture above, if the ladder reaches 7 feet up the wall, and the base of the ladder is 24 feet from the wall. How long is the ladder?

Geometry Unit # 1 Concept# 1

Lesson Name: G.U1.C1.B.05.**ExtraExtraHW.≅MidpointDistance**

Distance and Midpoints

Use the number line to calculate each item:



1. AB = ______ 2. AD = _______ 3. CF = _______ 4. BE = ________

5. Midpoint of $\overline{AB}$ = ______ 6. Midpoint of $\overline{AD}$ = ______

7. Midpoint of $\overline{CF}$ = ______ 8. Midpoint of $\overline{FB}$ = ______

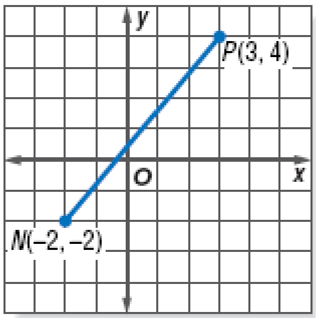
Use the distance formula to find the distance between each pair of points:

9. (7, 11) and (-1, 5) 10. (2, 0) and (8, 6)

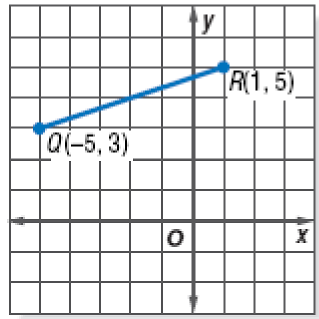
11. (-2, -1) and (3, 11) 12. (-2, -6) and (6, 9)

13. (-10, 2) and (-7, 6) 14. (-3, 2) and (6, 5)

15.



16.



Calculate the midpoint of each segment with the given endpoints:

17. A (9, 5) and B(17, 4)

18. C (8, 4) and D(12, 2)

19. E (-11, -4) and F(-9, -2)

20. A (4, 2) and B(8, -6)

21. A (-4, 3) and B(-1, 5)

22. A (2, 8) and B(-2, 2)

Point M is the midpoint of $\overline{AC}$, find the coordinates of the missing endpoint when you are given one endpoint and the coordinates of the midpoint.

23. M(0, 5.5) and C(-3, 6)

24. M(-1, 5) and A(-4, 3)

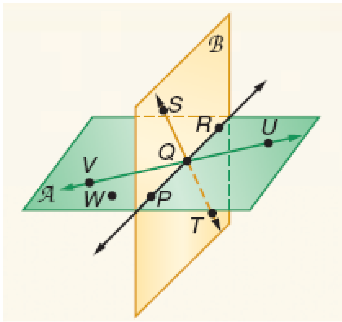
25. M(-2, 2) and A(2, 8)

26. M(4, 1) and A(5, -1)

Review:

27. Point A is between points M and C. If MC = 7x, AM = 2x+8, and AC = 3x+6, find the measure of each segment.

28.



a. Name the intersection of planes and B:

b. Name 4 coplanar points:

c. Name 3 collinear points:

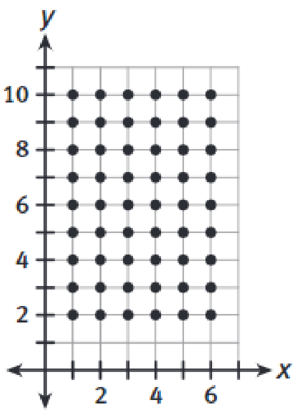
d. Name 3 non-collinear points:

e. Name 4 non-coplanar points:

f. Name the line with pt. V in 3 different ways:

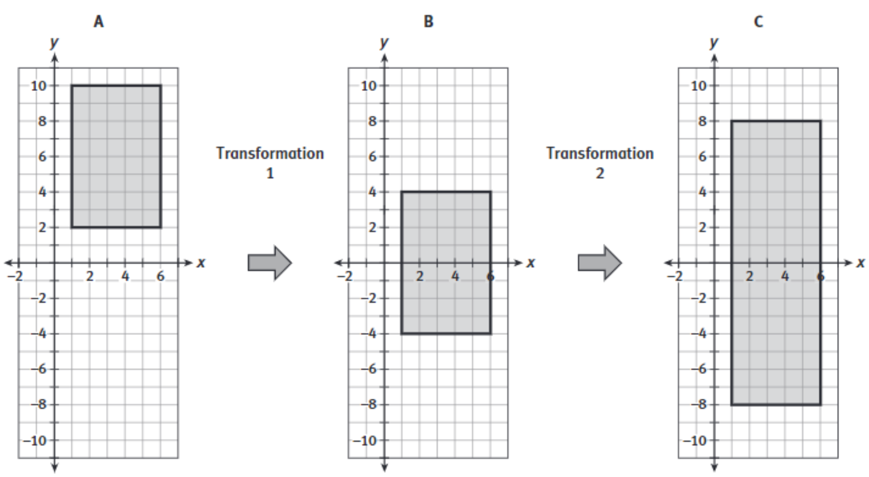
Transformations – Rigid vs. Non-Rigid

A translation slides all points of a figure the same distance in the same direction. Mr. Howell is the Band Director for the Marching Band at one Houston County high school in middle Georgia. He uses the coordinate plane to represent the football field. For the band's first show, he arranges the band in a rectangle that is 6 marchers wide and 9 marchers deep.



The band begins by marching down the grid in this formation. Then the marchers move apart from each other vertically, while keeping the same distance between marchers within the same row.

The diagrams below show the initial shape of the marchers, and the two ***transformations*** that they undergo. To describe and classify the transformations, compare the ***pre-image*** of a transformation to its **image**.



1. Use your own words to describe Transformation 1

2. Compare Transformation 1 and Transformation 2. How do the two transformations compare?

Notes

Notes

3. Model with Mathematics. Transformation 1 is an example of a ***rigid motion***. (Sometimes called an ***isometry***). A rigid motion keeps the same distance between the points that are transformed (in this situation, the marchers of the band); the shape and size of the pre-image and image are the same.

a. How does Transformation 1 affect the distance between any two marchers in the band?

b. How does Transformation 2 affect the distance between the marchers? Is Transformation 2 a rigid motion?

4. Review Transformation 1. Each point in the pre-image is mapped to a point in the image. For this reason, the transformation can be expressed as a function.

a. Complete the table to show the positions of the four corners of the rectangle when Figure A is mapped onto Figure B.

Figure A (pre-image)	Figure B (image)
(1, 10)	(1, 4)
(1, 2)	
(6, 10)	
(6, 2)	

b. For any given point, how does the transformation change the *x*-coordinate and *y*-coordinate?

c. You can use the notation (1, 10) → (1, 4) to show how a point is transformed. When you use this notation to show how a general point (x, y) is transformed, you are expressing the transformation as a function. Express Transformation 1 as a function.

5. Review Transformation 2.

a. Complete the table to show the positions of the four corners of the rectangle when Figure B is mapped onto Figure C.

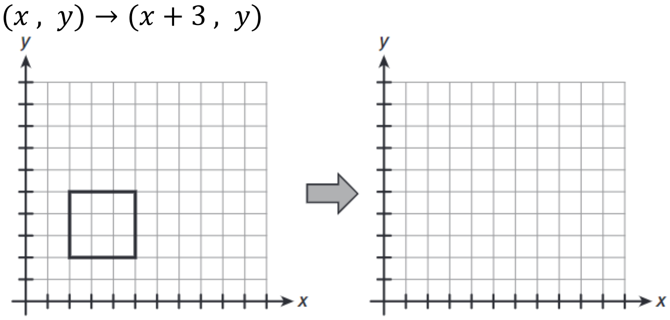
Figure B (pre-image)	Figure C (image)
(1, 4)	(1, 8)
(1, −4)	

b. For any given point, how does the transformation change the *x*-coordinate and *y*-coordinate?

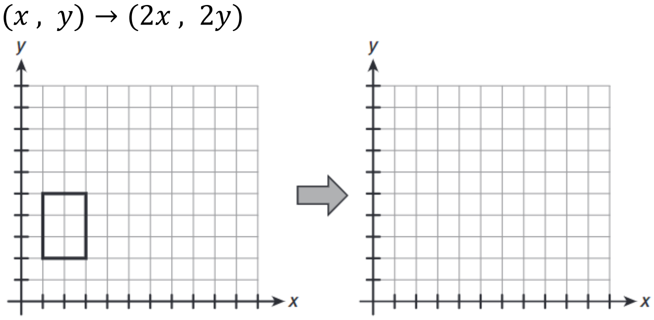
c. Can Transformation 2 also be expressed as a function? Explain why or why not. Write the function if it exists.

6. Draw each image on the graph to show how the pre-image is transformed by the function. Then classify the transformation as rigid or non-rigid.

a. $(x\,,\ y) \rightarrow (x + 3\,,\ y)$



b. $(x\,,\ y) \rightarrow (2x\,,\ 2y)$



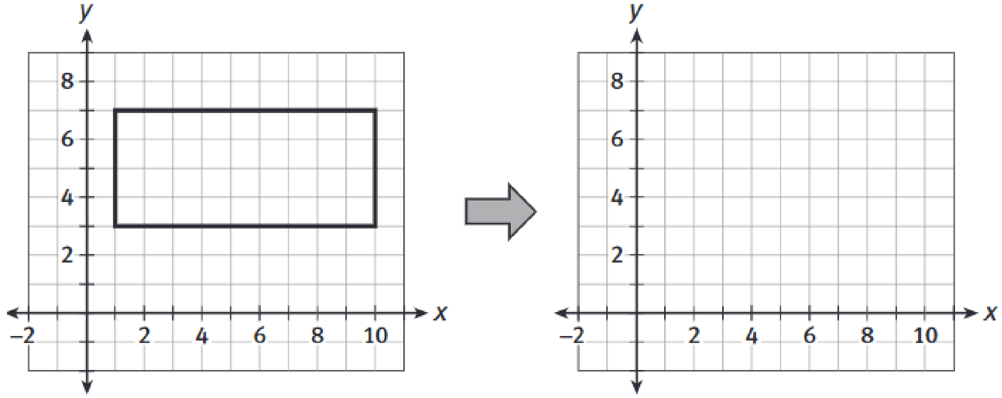
7. Write the letter "F" in the middle of each pre-image on Item 6. Describe how the letter should appear in each image.

Notes

Notes

SELF CHECK

Use the text and diagram to answer items 8 and 9.
The rectangle undergoes the transformation described by the function (x, y) → (x −2, y +1).

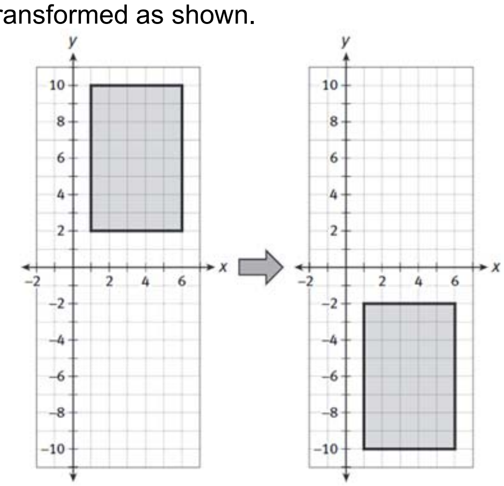


8. Complete the table to show the coordinates of the image and pre-image for the four corners of the rectangle.

Pre-image	Image
(1, 3)	____
(1, 7)	____
____	____
____	____

9. Graph the transformation of the figure. Is the transformation a rigid motion or non-rigid motion? Explain how you know.

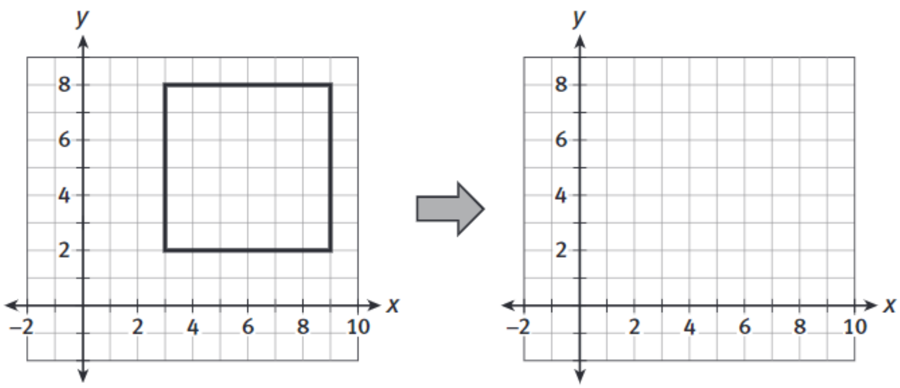
10. A rectangle is transformed as shown.



a. Which function describes the transformation?

b. Classify the transformation as rigid or non-rigid. Explain why you classified the transformation that way.

Example! **For Items 11 and 12, consider the following:** A rectangle undergoes the transformation described by the function $(x\,,y) \rightarrow \left(x\,,\frac{y}{2}\right)$.



11. Graph the transformation of the figure. Is the transformation a rigid motion? Explain.

12. Reason abstractly. Draw a multiplication sign $(\times)$ in the middle of the image. Describe how the transformation would change the multiplication sign.

13. Attend to precision. Use the graph of the rectangle to help you classify each of the following transformations.

a. Draw the image of the rectangle under the transformation described by the function $(x\,,y) \rightarrow \left(\frac{x}{2}\,,\ y\right)$. Classify the transformation as rigid or non-rigid.

b. Draw the image of the rectangle under the transformation described by the function $(x\,,y) \rightarrow (x\,.\ y + 2\,)$. Classify the transformation as rigid or non-rigid.

Notes

Introduction to Constructions

Constructions: The drawing of various shapes using only a pair of compasses and straightedge or ruler. No measurement of lengths or angles is allowed. The word construction in geometry has a very specific meaning: the drawing of geometric items such as lines and circles using only compasses and straightedge or ruler. Very importantly, you are not allowed to measure angles with a protractor, or measure lengths with a ruler.

Compasses

Compasses are a drawing instrument used for drawing circles and arcs. It has two legs, one with a point and the other with a pencil or lead. You can adjust the distance between the point and the pencil and that setting will remain until you change it.

This kind of compass has nothing to do with the kind used find the north direction when you are lost. A compass used to find the north direction is usually referred to in the singular - a compass. The kind we are talking about here is usually referred to in the plural - compasses. This plural reference is similar to the way we talk about scissors - with an 's' on the end.

Straightedge

A straightedge is simply a guide for the pencil when drawing straight lines. In most cases you will use a ruler for this, since it is the most likely to be available, but you must not use the markings on the ruler during constructions. If possible, turn the ruler over so you cannot see them.

Why we learn about constructions

The Greeks formulated much of what we think of as geometry over 2000 years ago. In particular, the mathematician Euclid documented it in his book titled "Elements", which is still regarded as an authoritative geometry reference. In that work, he uses these construction techniques extensively, and so they have become a part of the geometry field of study. They also provide insight into geometric concepts and give us tools to draw things when direct measurement is not appropriate.

Why did Euclid do it this way?

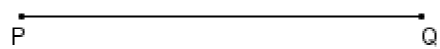
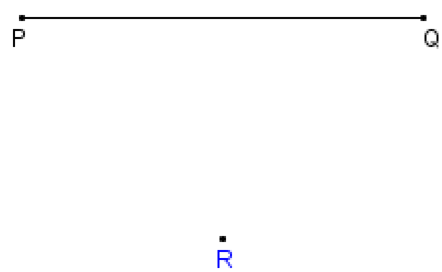
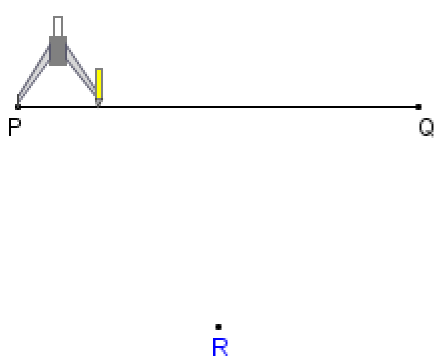
Why didn't Euclid just measure things with a ruler and calculate lengths? For example, one of the basic constructions is bisecting a line (dividing it into two equal parts). Why not just measure it with a ruler and divide by two?

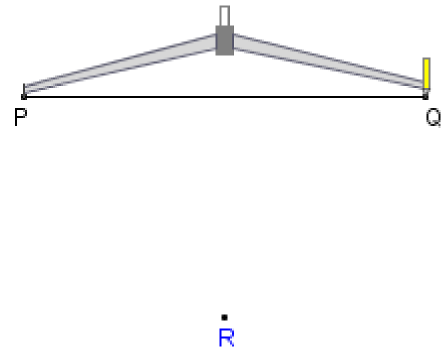
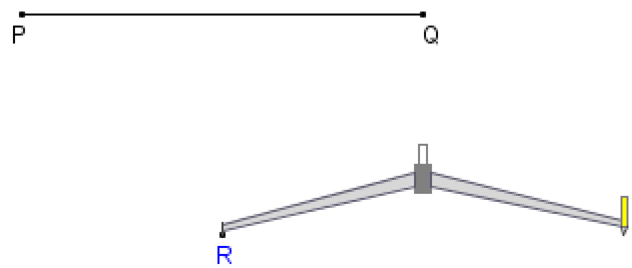
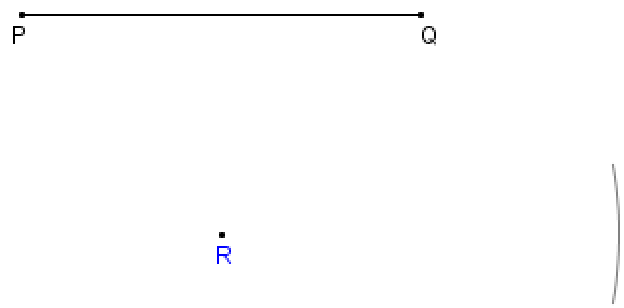
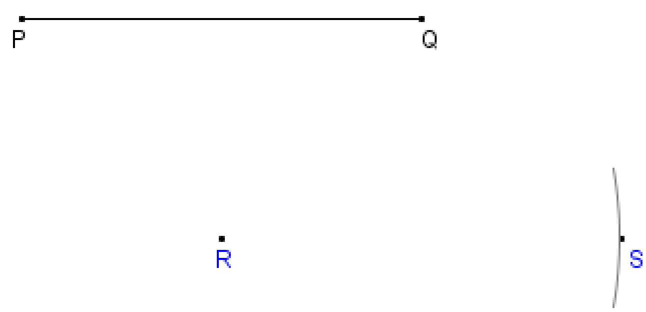
One theory is that the Greeks could not easily do arithmetic. They had only whole numbers, no zero, and no negative numbers. This meant they could not for example divide 5 by 2 and get 2.5, because 2.5 is not a whole number - the only kind they had. Also, their numbers did not use a positional system like ours, with units, tens , hundreds etc, but more like the Roman numerals. In short, it was quite difficult to do useful arithmetic.

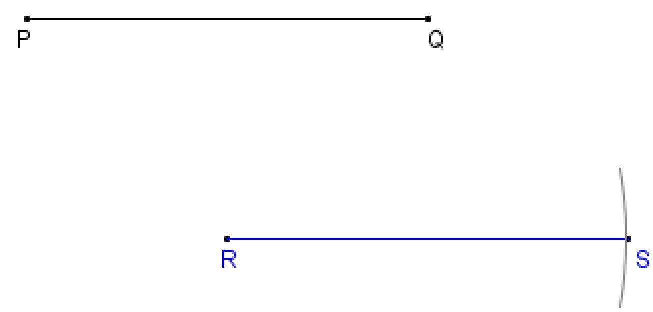
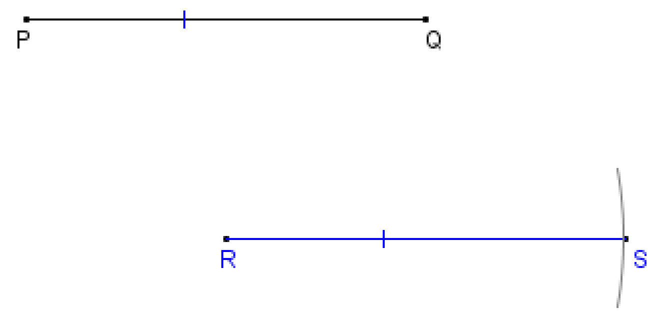
So, faced with the problem of finding the midpoint of a line, it was very difficult to do the obvious - measure it and divide by two. This led to the constructions using compass and straightedge or ruler. It is also why the straightedge has no markings. It is definitely not a graduated ruler, but simply a pencil guide for making straight lines. Euclid and the Greeks solved problems graphically, by drawing shapes instead of using arithmetic.

Construction #1: Copying a Line Segment

Using a compass and straightedge on your own paper, follow the instructions below to copy a segment. You may also use the QR code below to see an animation of the process.

	After doing this	Your work should look like this
	Start with a line segment PQ that we will copy.	
Step 1	Mark a point R that will be one endpoint of the new line segment.	
Step 2	Set the compasses' point on the point P of the line segment to be copied.	

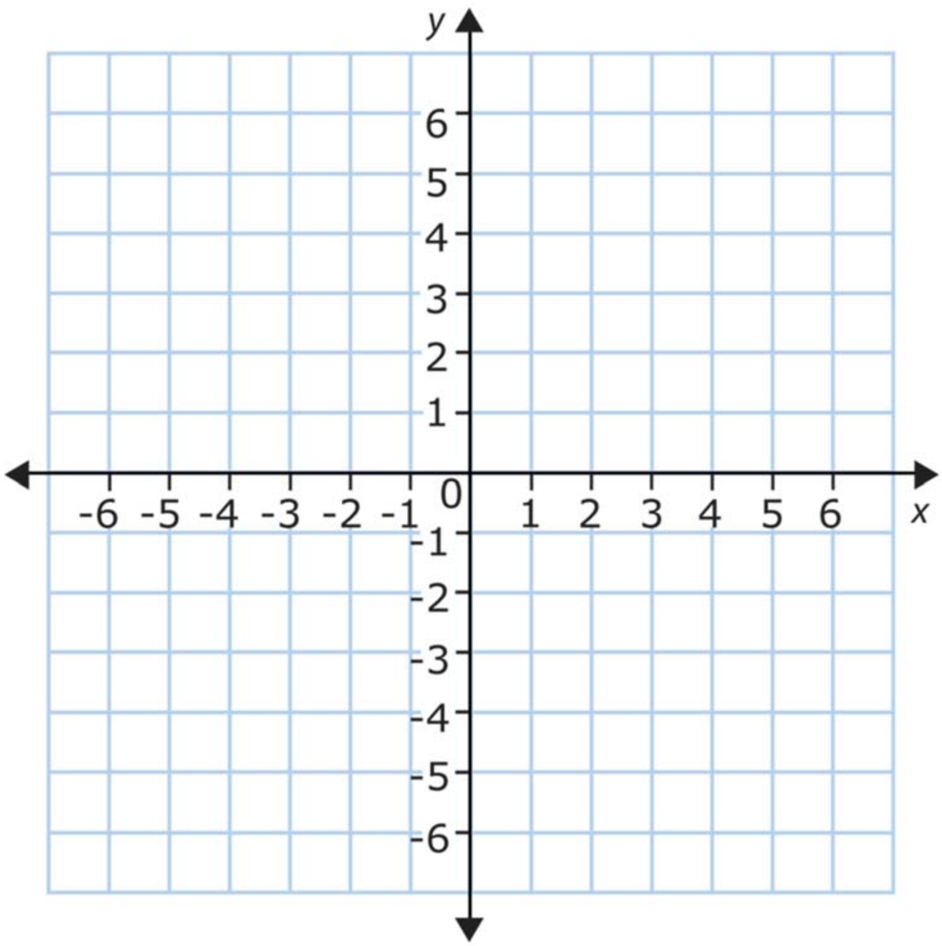
Step 3	Adjust the compasses' width to the point Q. The compasses' width is now equal to the length of the line segment PQ.	
Step 4	Without changing the compasses' width, place the compasses' point on the the point R on the line you drew in step 1	
Step 5	Without changing the compasses' width, Draw an arc roughly where the other endpoint will be.	
Step 6	Pick a point S on the arc that will be the other endpoint of the new line segment.	

Step 7	Draw a line from R to S.	
Step 8	Done. The line segment RS is equal in length (congruent to) the line segment PQ.	

Questions To Ponder

14. On the coordinate plane below, randomly plot a points P and Q so that they do not have the same x-coordinate or y-coordinate as each other. So that you have space, keep them fairly close together. Next, plot a point P' at another location on the coordinate plane.

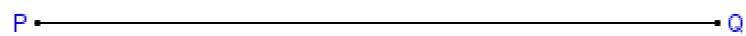
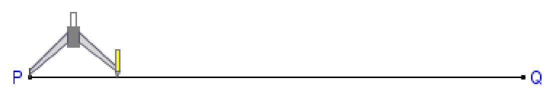
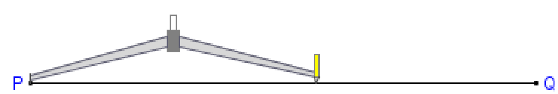
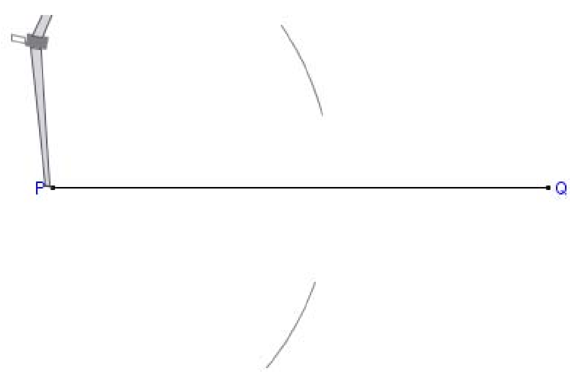
a. **What is the SLOPE of your $\overline{PQ}$?**
b. **Starting at P', use the slope of $\overline{PQ}$ (you may have to "jump" several times) to draw a new line containing point P' that is parallel to $\overline{PQ}$. Ensure that your new line containing P' extends across the entire coordinate plane.**

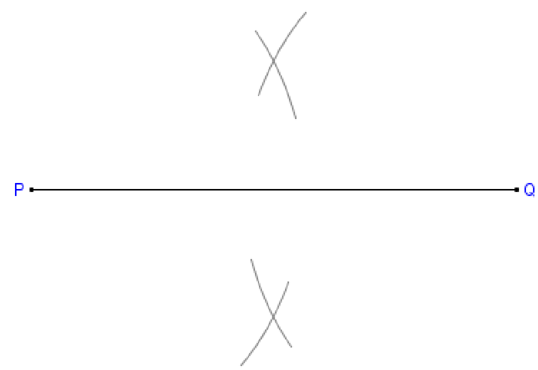
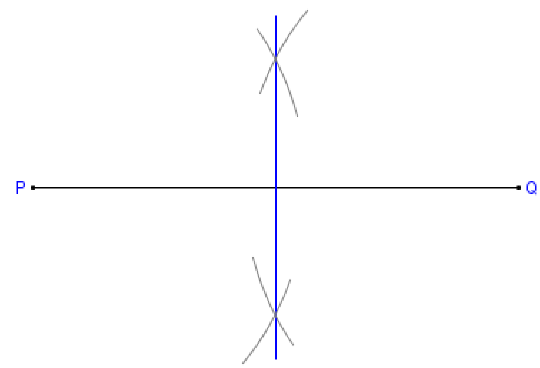
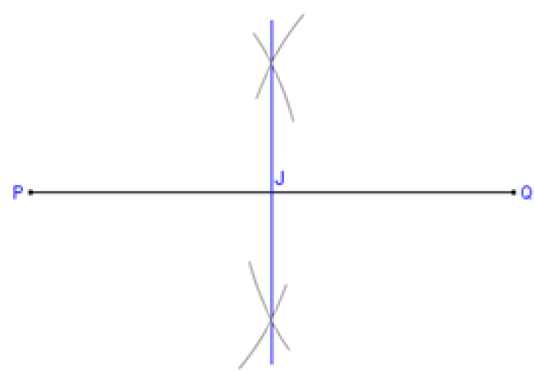


c. **Using a compass and straightedge, copy $\overline{PQ}$ to the line containing P' so that $\overline{P'Q'}$ is an exact copy of $\overline{PQ}$.**
d. **How can you verify that $\overline{PQ} \cong \overline{P'Q'}$?**

Construction #2: Bisection a Line Segment

Using a compass and straightedge on your own paper, follow the instructions below to bisect a segment. You may also use the QR code below to see an animation of the process.

	After doing this	Your work should look like this
	Start with a line segment PQ.	
1	Place the compasses on one end of the line segment.	
2	Set the compasses' width to a approximately two thirds the line length. The actual width does not matter.	
3	Without changing the compasses' width, draw an arc above and below the line.	

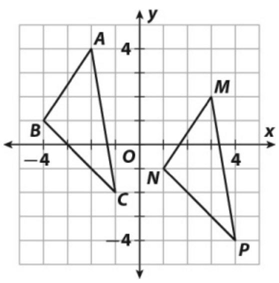
4	Again without changing the compasses' width, place the compasses' point on the the other end of the line. Draw an arc above and below the line so that the arcs cross the first two.	
5	Using a straightedge, draw a line between the points where the arcs intersect.	
6	Done. This line is perpendicular to the first line and bisects it (cuts it at the exact midpoint of the line).	

a. How should step #6, the final step of bisecting $\overline{PQ}$, be marked to show someone that it the segment has been bisected and also to show that is was bisected perpendicularly?
b. Mark the figure above so that it shows congruence and perpendicularity.
c. Go back to $\overline{P'Q'}$ that you copied on the coordinate plane before (on page 11). Find the coordinates of its midpoint, J. Then verify using the distance formula that $\overline{PJ} \cong \overline{JQ}$ below. Show your work.

RIGID TRANSFORMATION…OR NOT?

Determine whether △*ABC* and △*MNP* are congruent.
Explain your answer.

1.



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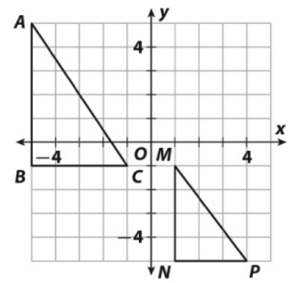
2.



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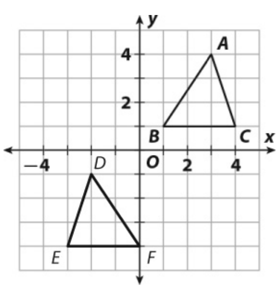
For each pair of congruent figures, specify a sequence of rigid motions that maps one figure onto the other.

3.



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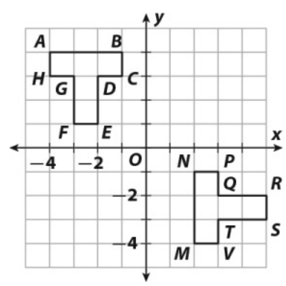
4.



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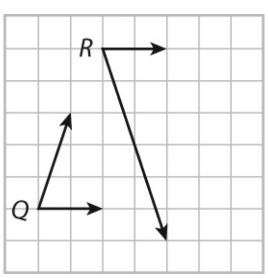
Decide if the angles or the segments in each pair are congruent.
Write Yes or No.

5. ∠*Q and* ∠*R*



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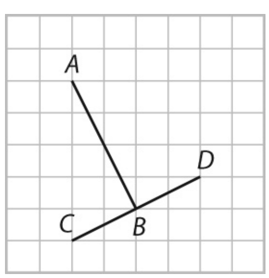
6. $\overline{AB}$ and $\overline{CD}$



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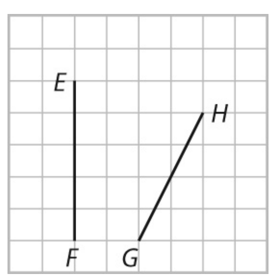
7. $\overline{EF}$ and $\overline{GH}$



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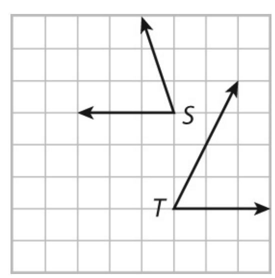
8. ∠*S* and ∠*T*



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Task: Coordinating Translations

Name___________________________________ **Date**___________________

Create any polygon you want on the coordinate plane, and then create polygons congruent to the one you designed using the three directions described below. Use the same coordinate plane for all transformations.

1. For each vertex of your original polygon in the form (x, y), create its image at the coordinates $(x + 4, y)$. Record your answers in the table below.

2. For each vertex of your original polygon in the form (x, y), create its image at the coordinates $(x, y - 3)$. Record your answers in the table below.

3. For each vertex of your original polygon in the form (x, y), create its image at the coordinates $(x - 4, y + 1)$. Record your answers in the table below.

Original polygon's vertices (x, y)	**#1** $(x + 4, y)$	**#2** $(x, y - 3)$	**#3** $(x - 4, y + 1)$

4. What kind of transformations are these?

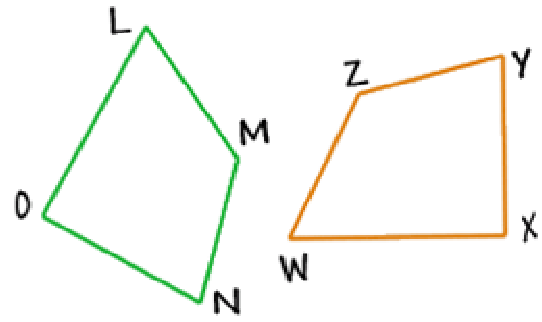
5. Can you create a translation $(x + 2, y + 2)$? Is it necessary that the same number is added or subtracted to the x and y coordinates of the polygon? Why or why not?

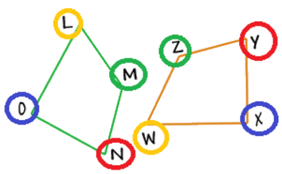
Provide a description of each of the following translations, where c represents a positive number.

6. $(x + c, y)$	**7.** $(x, y - c)$	**8.** $(x - c, y)$	**9.** $(x, y + c)$

Corresponding Parts of Congruent Figures Are Congruent

1. *By definition, two polygons are congruent if they are the same size and that is, if their corresponding angles and sides are equal. Suppose the quadrilaterals to the right are, in fact, congruent. Your teacher measured the sides and angles of the shapes to the right, you can see in colors below which ones correspond. Write a congruence statement that relates quadrilateral LMNO to quadrilateral XYZW.*





List all of the pairs of congruent angles and sides of the figures.

2. $\triangle KLM \cong \triangle GHI$

________ ≅ ________ ________ ≅ ________

________ ≅ ________ ________ ≅ ________

________ ≅ ________ ________ ≅ ________

3. Rhombus *WXYZ* ≅ rhombus *DEFG*

________ ≅ ________ ________ ≅ ______

________ ≅ ________ ________ ≅ ______

________ ≅ ________ ________ ≅ ______

________ ≅ ________ ________ ≅ ______

Quadrilateral *ABCD* ≅ quadrilateral *EFGH*. In quadrilateral *ABCD*, $AB = 16$, $BC = 5w + 7$, $m\angle C = (2z - 1)°$, and $m\angle D = 50°$. In quadrilateral *EFGH*, $EF = 3y + 1$, $FG = 8$, $m\angle G = 80°$, and $m\angle H = (2x)°$. Find the value of the indicated variable.

4. Find the value of *w*.

____

5. Find the value of *x*.

____

6. Find the value of *y*.

____

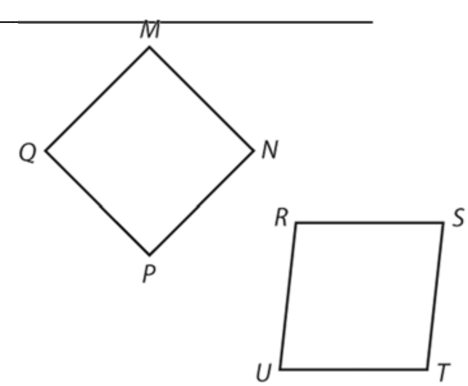
7. Find the value of *z*.

____

Write the proof.

8. Given: Quadrilateral *MNPQ* ≅ quadrilateral *RSTU*; $\overline{MN} \cong \overline{PQ}$

   Prove: $\overline{MN} \cong \overline{TU}$



CONGRUENCE AND RIGID TRANSFORMATIONS

1. An isometry is a transformation that always preserves (circle only the correct answers):

Length ~~Location~~ ~~Orientation~~ Area ~~Coordinates~~ Angle Perimeter

Fill in the blanks.

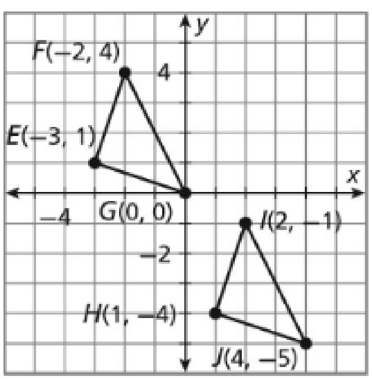
2. Because of these properties, isometries produce _____________ images.
3. A __________ transformation is another name for an isometry.
4. Dilations with scale factor $k \neq$ ______ are transformations that produce images that are not congruent to their preimages.

5. Fill in the chart below with each of the four types of transformations, and state whether or not it is an isometry, and whether the image is congruent to the preimage under that transformation.

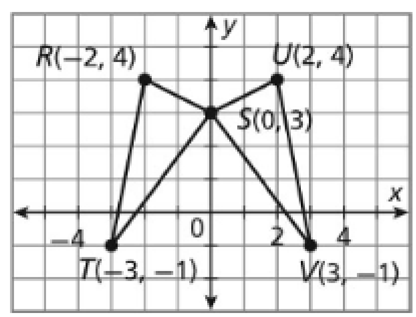
Transformation	Is it an Isometry?	Is the Image ≅ to the pre-image?

Determine below whether the polygons with the given vertices are congruent. If they are congruent, write congruence statements for the polygons, and state all pairs of congruent sides and/or angles.

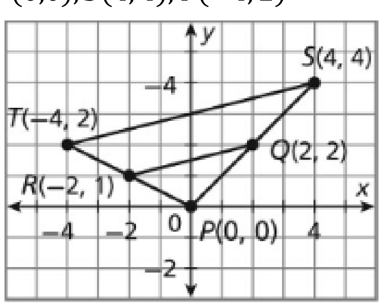
6. $E(-3,1), F(-2,4), G(0,0)$
$H(1,-4), I(2,-1), J(4,-5)$



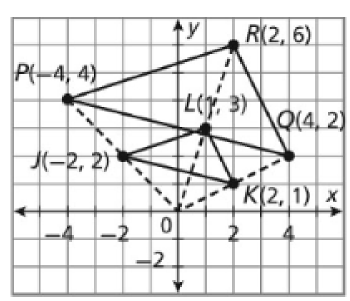
7. $R(-2,4), S(0,3), T(-3,-1)$
$U(2,4), S(0,3), V(3,-1)$



8. $P(0,0), Q(2,2), R(-2,1)$
$P(0,0), S(4,4), T(-4,2)$

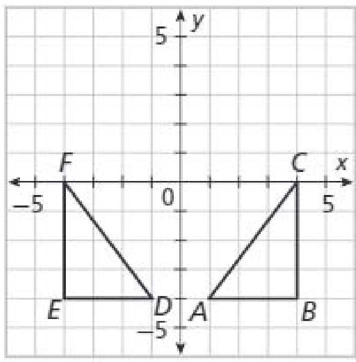


9. $J(-2,2), K(2,1), L(1,3)$
$P(-4,4), Q(4,2), R(2,6)$

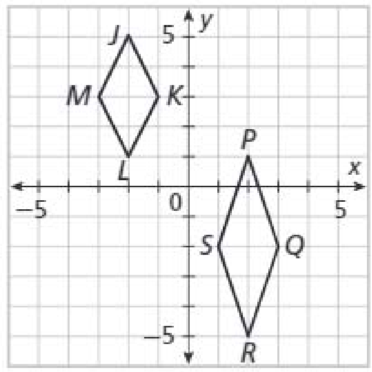


Use the definition of congruence in terms of rigid motions to determine whether the two figures below are congruent, and explain your answer.

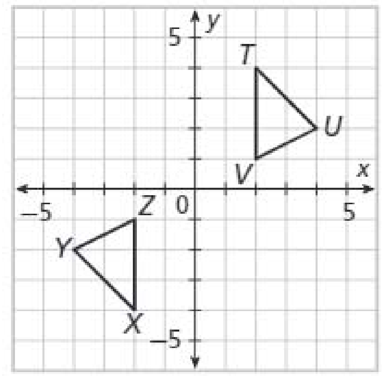
10.



11.



12.



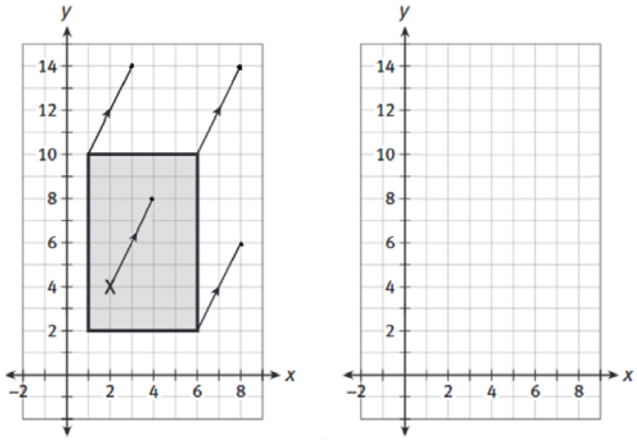
13. △ABC≅△DEF and △DEF ≅△GHJ. Can you conclude △ABC ≅△GHJ? Explain.

14. Given two congruent figures, is there a unique sequence of rigid motions that maps one figure to the other? Use one or more of the above examples to explain your answer.

15. Explain how you could use tracing paper to help you find a sequence of rigid motions that maps one figure to another congruent figure.

Transformations – Translations

Maria marches with the band. At the start of the halftime show, her position is (2, 4) on the coordinate plane. Then Mr. Howell tells everyone to move 2 yards to the right on the field and 4 yards up the field. The band's transformation is shown in the diagram, and Maria's position is marked with an X.



You may remember from 8th grade math class – this type of transformation is called a ***translation*** . On the coordinate plane, x can translate left or right and the y coordinate can translate up or down. In this situation, a translation is described by a function of the form $(x\,,y) \to (x+a\,,y+b)$in which a and b are positive or negative constants. In the example above, $a =\ 2$ and $b =\ 4$.

You can think of a translation as a figure sliding up or down, left or right, or diagonally. During the translation, every point of the figure moves the same distance and in the same direction. This distance and direction is called a ***directed line segment*** . In the diagram, the directed line segment of the translation is shown by each of the arrows.

1. Complete the table below to show how the translation affects four of the Marching Cougars.

Marcher	Pre-image	Image
Maria	(2, 4)	
Joe	(3, 7)	
Alfredo	(1, 8)	
LeJaya		(7, 11)

2. The band director uses ***directed line segments*** to show where/how the rectangular formation moves above. However, in this context we would not call those "rays." Make a conjecture about why these are directed line segments rather than rays.

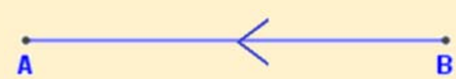
Notes

A note about directed line segments:
Generally, in geometry, talking about segment $\overline{AB}$ is the same thing as talking about $\overline{BA}$.

HOWEVER, $\overline{AB}$ **is not the same thing as** $\overline{BA}$ **if it is a directed line segment.** Translations, because they involve movement and therefore direction, often use directed line segments. Directed line segment $\overline{AB}$, below,

means you start at A and go in the direction of B.

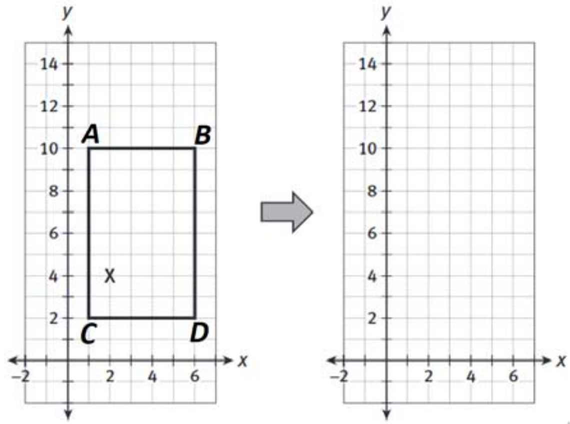
Directed line segment $\overline{BA}$, below



means you start at B and go towards A. You must be told that this is a directed line segment.

Notes

Mr. Howell arranges the band in a rectangle. Abby, Brian, Donny, and Callie are at each of the four corners. Then he directs the band member Brian at (6, 10) to move to (4, 13). The numbers on the coordinate plane show yards of the football field.



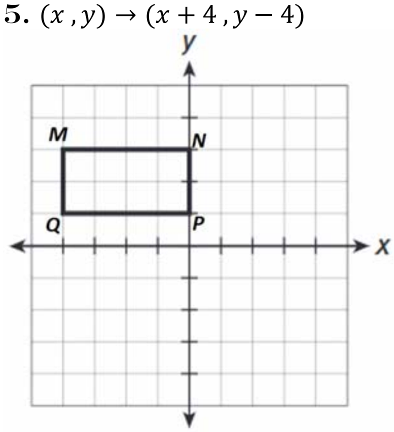
3. For the band to move in a translation, how should each band member move?

4. Critique the reasoning of others. Maria was positioned at point $(2, 4)$. Her friend tells her that Maria's new position is described by the function $(x + 3, y - 2)$ since her new position is $(7, 0)$. Is her friend correct? Explain.

SELF CHECK

Name each polygon below. Draw the image of the figure under the translation described, and name the polygon after translation. Lastly, identify each of the points' coordinates after the translations are completed.

5. $(x, y) \rightarrow (x + 4, y - 4)$

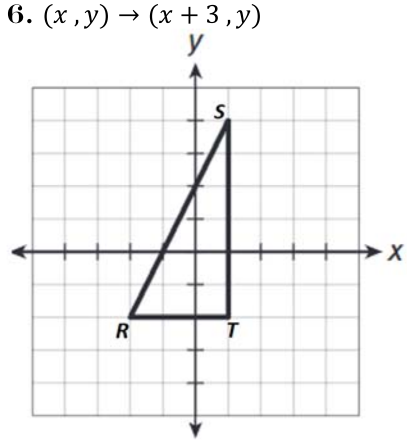


Name of Pre-Image:_______________

Q (,)	Q' (,)
P (,)	P' (,)
N (,)	N' (,)
M (,)	M' (,)

Name of translated image:_______________

6. $(x, y) \rightarrow (x + 3, y)$



Name of Pre-Image::_______________

S (,)	S' (,)
T (,)	T' (,)
R (,)	N' (,)

Name of translated image:_______________

Enrichment

Translations can also be defined without the coordinate plane. For the directed line segment $\overline{AB}$, a translation maps point P to point P' so that the following statements are true:

- $\overline{PP'}$ is parallel to $\overline{AB}$
- $PP' = AB$
- '$\overline{PP'}$ is in the same direction as $\overline{AB}$.

The expression $T_{\overline{AB}}$ describes the translation of a given point P by the directed line segment $\overline{AB}$. In the above example, $T_{\overline{AB}}(P) = P'$.

7. In $T_{\overline{AB}}(P) = P'$. T means _______________, the subscript means ________________, P means ________________, and P' means _____________________.

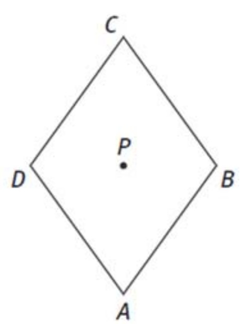
In regular words, $T_{\overline{AB}}(P) = P'$ means:

____

____

____

Items 8 through 10 refer to rhombus $ABCD$ shown below. Point P is in the center of the rhombus.



8. Draw the translation of the rhombus described by directed line segment $\overline{AB}'$. Include $P' = T_{\overline{AB}}(P)$.

9. Which part of the rhombus maps onto $\overline{BC}$?

10. Identify a translation of the rhombus that would map exactly one point of the rhombus onto another point of the rhombus.

11. Draw the following translations **on your own paper**. Show the pre-image and image, and label corresponding points in each.

a. Square $ABCD$, translated exactly 4 inches to the right.
b. Right triangle ABC, translated three inches up.
c. Right triangle ABC, translated by $T_{\overline{AB}}$.

Notes

← Can you complete the drawing of $\overline{PP'}$ below so that it fits the three criteria to the left (also listed below)?

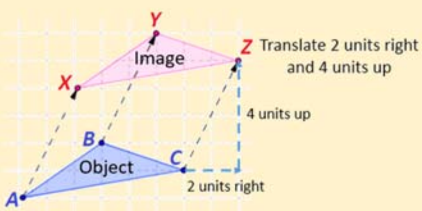
- $\overline{PP'}$ is parallel to $\overline{AB}$
- $PP' = AB$
- $\overline{PP'}$ is in the same direction as $\overline{AB}$.

Once you have completed your sketch above, both of the directed line segments $\overline{AB}$ and $\overline{PP'}$ will have arrows showing that they are going in the same direction, as the instructions say. What is another name for the relationship between these two directed line segments?

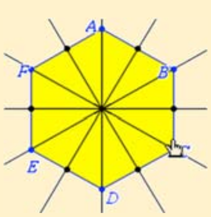
____

Notes

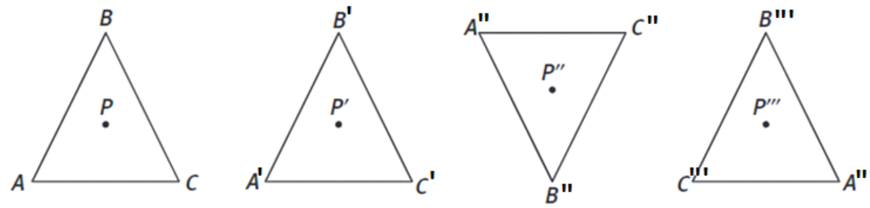
A note about "onto":
In Geometry, when a part of an image (sometimes, the entire image) transforms "onto" another image, it means that after transforming as directed, a part of the transformed image is on top of – literally exactly where- the image was. Below, if ΔABC is translated to ΔXYZ, $\overline{AB}$ translated onto $\overline{XY}$.



Another example of "onto" might be if the hexagon below were to rotate 60°clockwise. The whole hexagon would rotate "onto" itself, and A would rotate onto B, B would rotate onto C, etc.



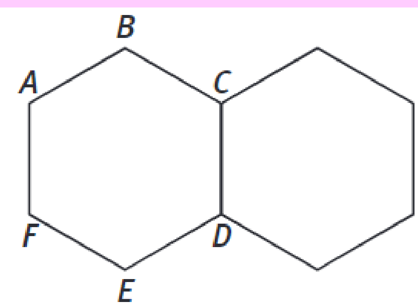
SELF CHECK **Mr. Howell arranges the band in the shape of triangle ABC, shown below. Point P is in the interior of the triangle. He plans three transformations of the triangle, in which point P is mapped onto P', P'', and P''', respectively.**



12. Which of the three transformations are translations? Explain your answer by applying the definition of a translation.

13. Maria stands at point P of triangle ABC. To move to point P', her instructions are to move eight steps to the right. Do these instructions apply to all the other band members? Explain.

SELF CHECK **The figure below shows hexagon $ABCDEF$ undergoing a translation to the right.**



14. Which part of the pre-image is translated onto $\overline{CD}$?

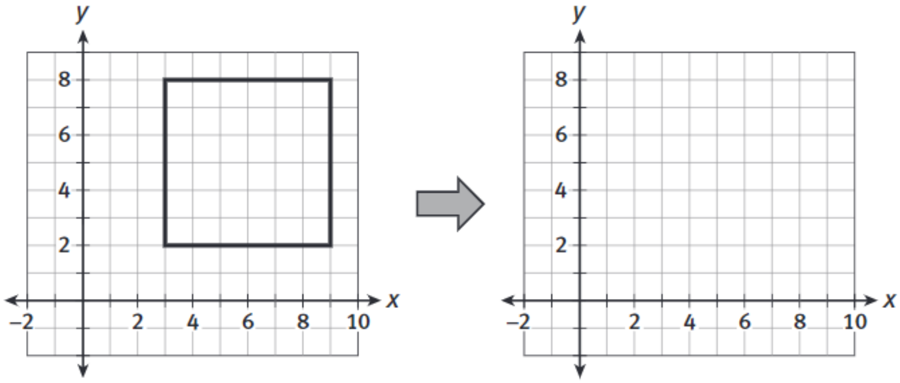
15. Express the translation as a function of two points of the hexagon.

16. **Make use of structure (enrichment).** By applying this translation many times, you could create a row of hexagons. What additional translation could be repeated to fill the page with hexagons? Is it $T_{\overline{BC}}, T_{\overline{BD}}, or\ T_{\overline{BE}}$?

17. **5-Minute ENRICHMENT SELF CHECK OF NOTATION:**
https://student.desmos.com/?prepopulateCode=agjyya
(You will need a larger screen than a phone).

Example! **For the next two Items, consider the following: A rectangle undergoes the transformation described by the function $(x, y) \rightarrow \left(x, \frac{y}{2}\right)$.**

Notes



18. Graph the transformation of the figure. Is the transformation a rigid motion? Explain.

19. **Reason abstractly**. Draw a multiplication sign $(\times)$ in the middle of the image. Describe how the transformation would change the multiplication sign.

20. **Attend to precision.** Use the graph of the rectangle to help you classify each of the following transformations.

a. Draw the image of the rectangle under the transformation described by the function $(x, y) \rightarrow \left(\frac{x}{2}, y\right)$. Classify the transformation as rigid or non-rigid.

b. Draw the image of the rectangle under the transformation described by the function $(x, y) \rightarrow (x . \ y + 2)$. Classify the transformation as rigid or non-rigid.

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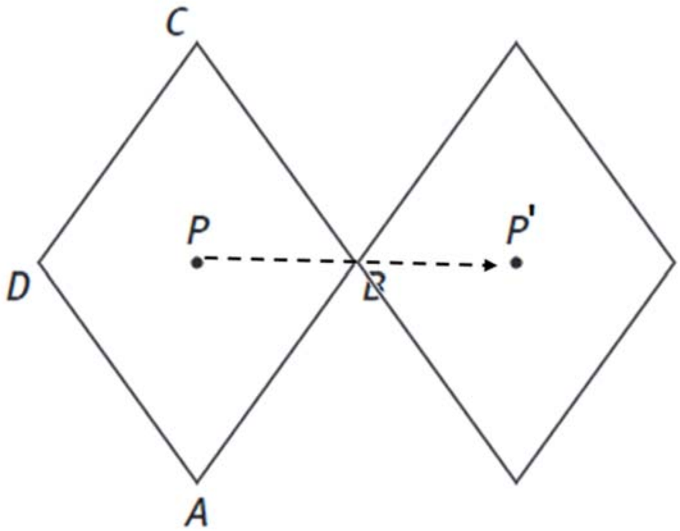

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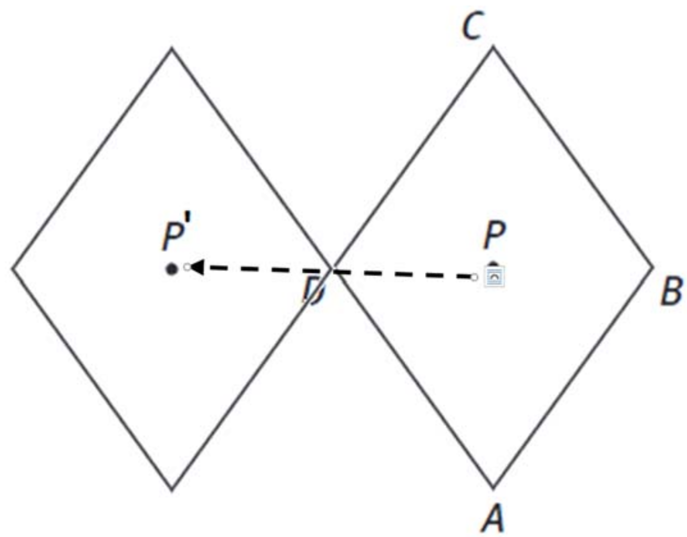

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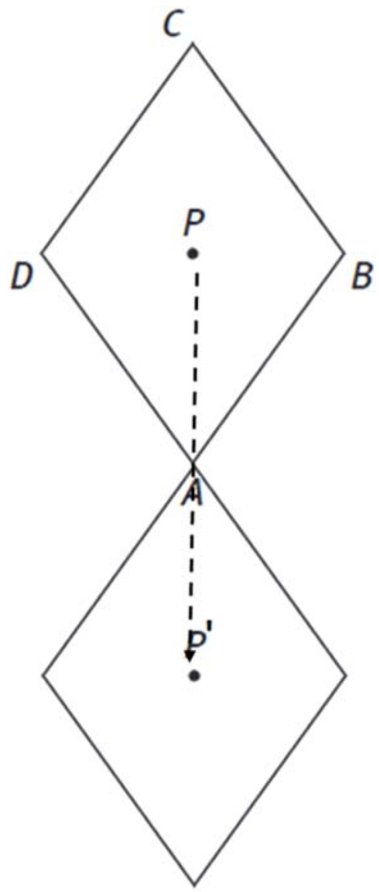

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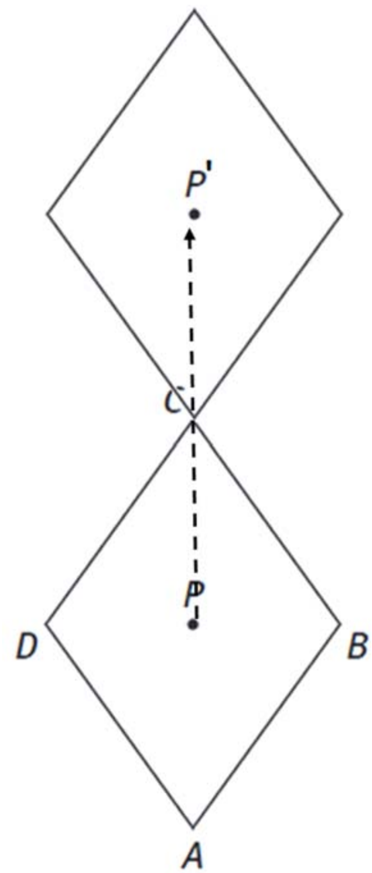
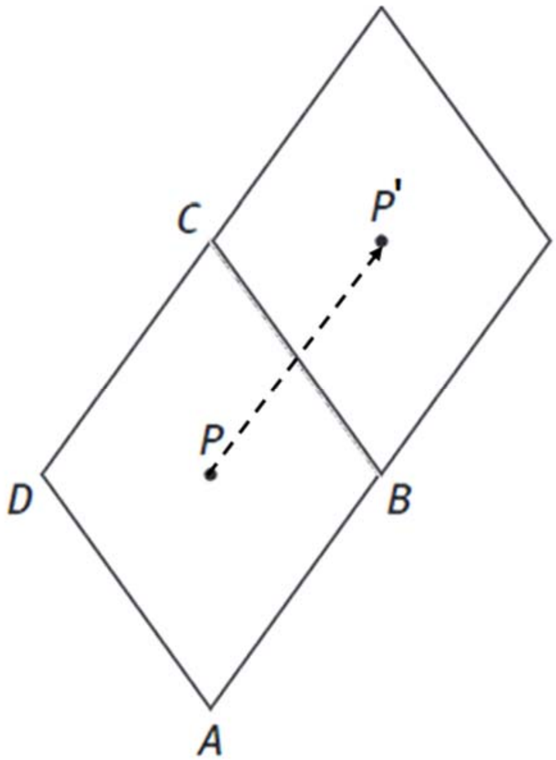
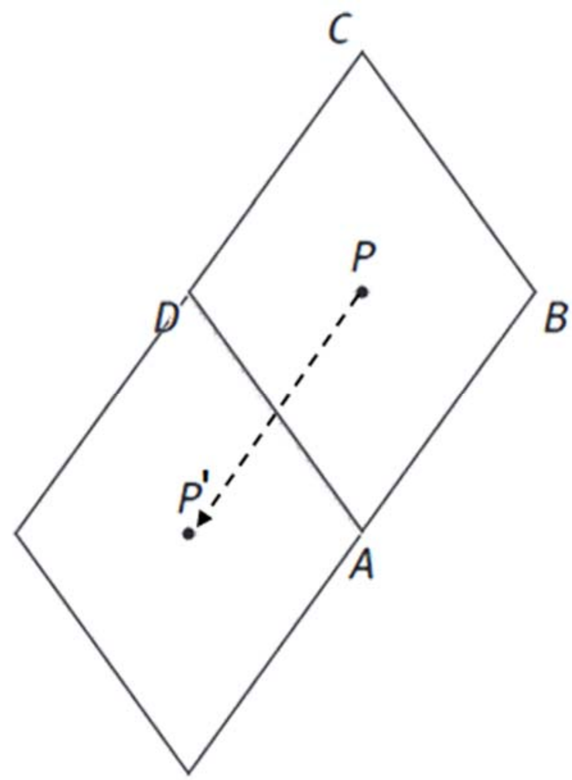
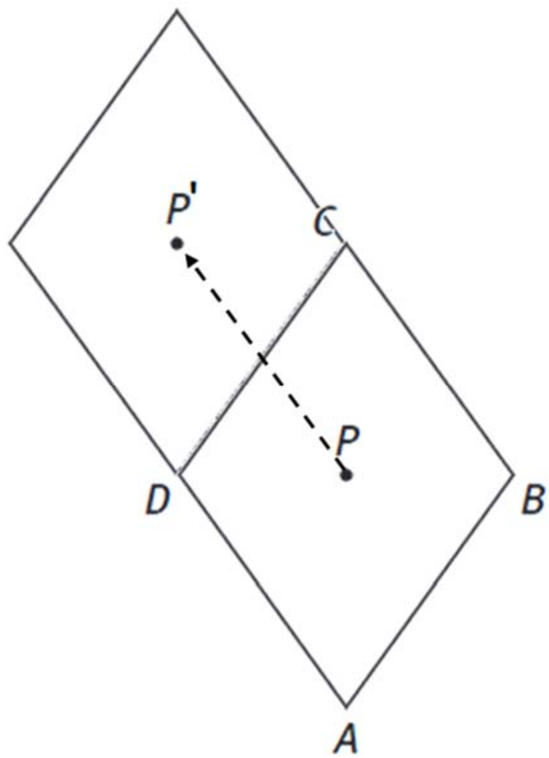
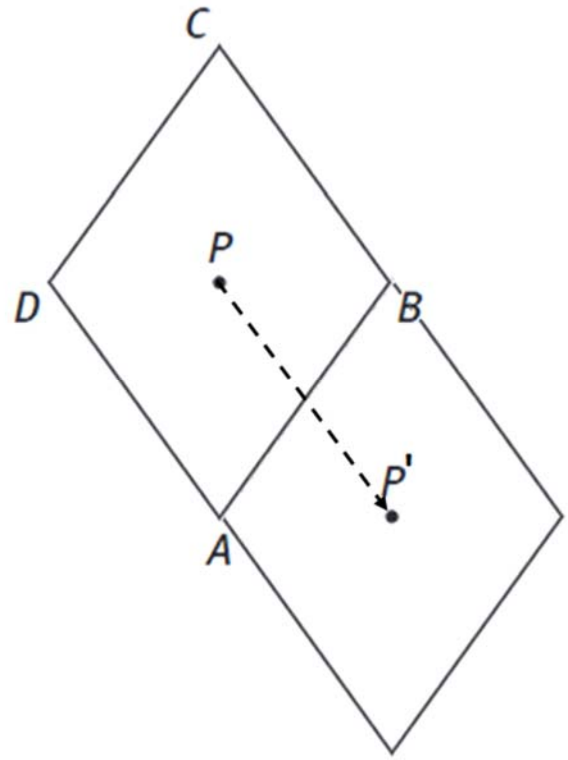


6	**8**
	

2	3
	

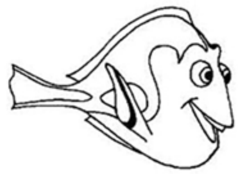
D $P' = T_{\overline{DB}}(P)$	F $P' = T_{\overline{BD}}(P)$
H $P' = T_{\overline{CA}}(P)$	K $P' = T_{\overline{AC}}(P)$
G $P' = T_{\overline{DC}}(P)$	L $P' = T_{\overline{AB}}(P)$
B $P' = T_{\overline{BA}}(P)$	A $P' = T_{\overline{CD}}(P)$

C $P' = T_{\overline{BC}}(P)$	E $P' = T_{\overline{AD}}(P)$
J $P' = T_{\overline{DA}}(P)$	M $P' = T_{\overline{CB}}(P)$

Translations Practice

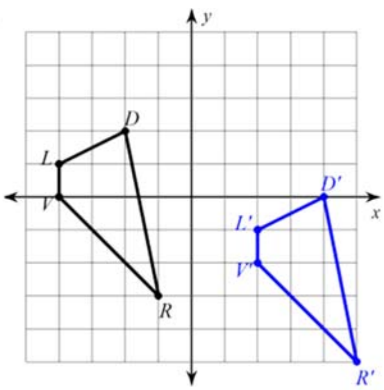
Define Translation in words:

Show what a translation would look like with the preimage below:

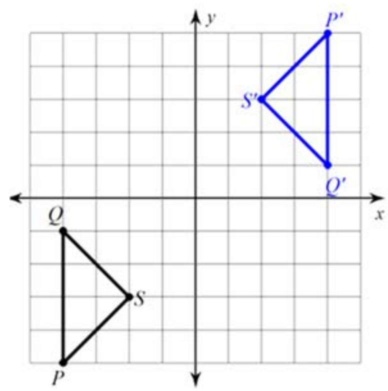


Are the following examples of translations? If not, explain why.

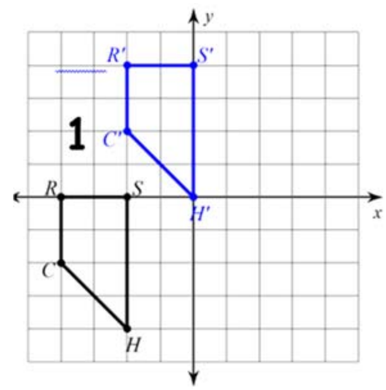
A.



B.



C.



THE MATHEMATICAL WAY TO DESCRIBE TRANSLATIONS…

Coordinate Notation
(x, y) → (x ± a, y ± b)
left/right up/down

Ex: left 3 units; up 7 units
$(x, y) \rightarrow (x-3, y+7)$

Rewrite the translations below mathematically.

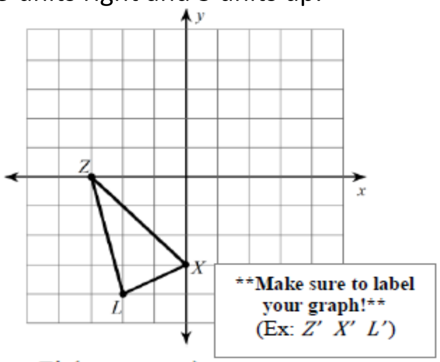
Left 15 units and up 24 units.	Right 8 units and down 4 units.
$(x\,,y) \rightarrow ($ $\quad$ $,$ $\quad$ $)$	$(x\,,y) \rightarrow ($ $\quad$ $,$ $\quad$ $)$

Find the coordinates of the image without graphing.

Point $G(6\,,-3)$ is translated 5 units to the left and 6 units up to form the point G'. What are the coordinates of G'? (,)	What is the image of point $H(-2\,,6)$ iafter the translation defined by $(x\,,y) \rightarrow (x+2\,,y-1)$? (,)
Use the translation $(x\,,y) \rightarrow (x-5\,,y+1)$ to find what point $(-2\,,-10)$ translates to: (,)	Use the translation $(x\,,y) \rightarrow (x-3\,,y-7)$ to find what point $(1\,,4)$ translates to: (,)

Graph the translation using the rule given. Then list the coordinates of the image.

5 units right and 3 units up.

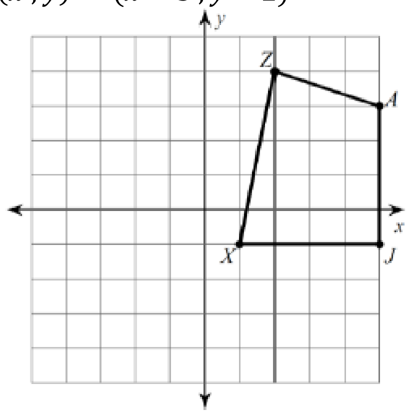


Z' (,)

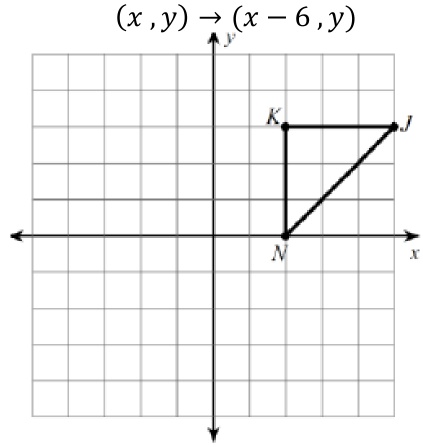
X' (,)

L' (,)

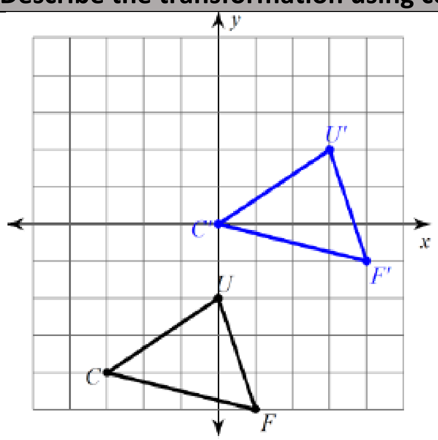
$(x, y) \rightarrow (x-3, y-2)$

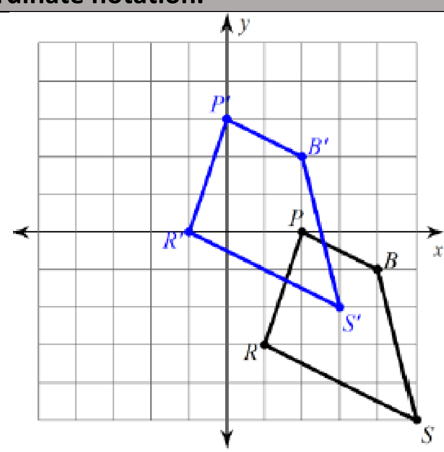


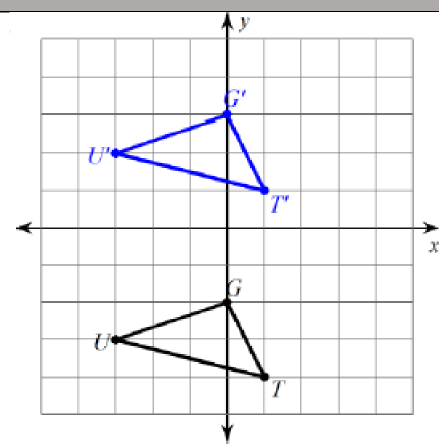
$(x, y) \rightarrow (x-6, y)$



Describe the transformation using coordinate notation.

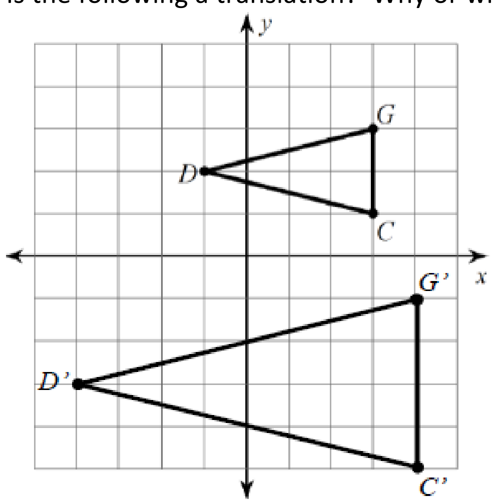




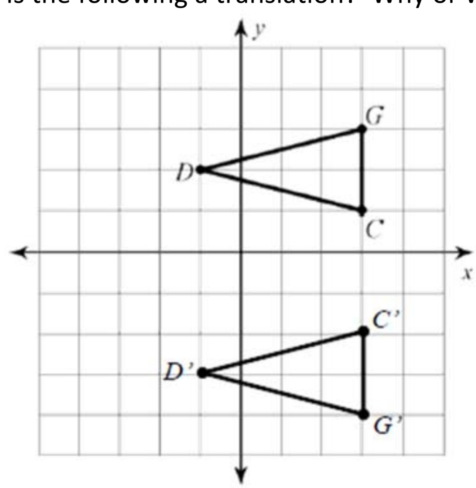


Critical Thinking: Partner up and Discuss!

Is the following a translation? Why or why not?

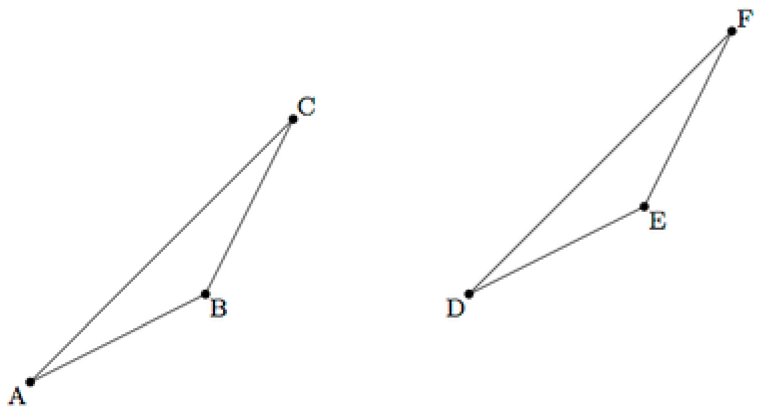


Is the following a translation? Why or why not?



IDENTIFYING TRANSLATIONS (ILLUSTRATIVE MATHEMATICS)

Suppose $\triangle DEF$ below can be obtained from $\triangle ABC$ by a translation:

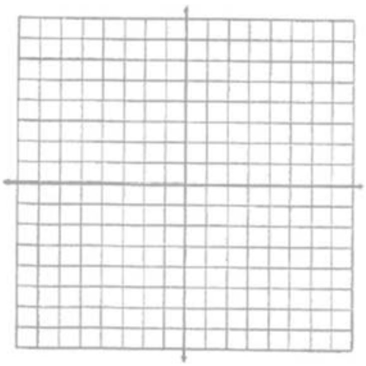


a. Explain why ABED, ACFD, and BCFE are parallelograms.

b. Explain why $|AD|=|BE|=|CF|$.

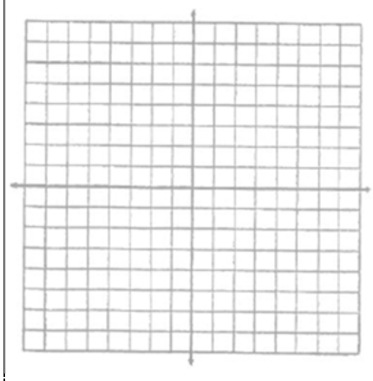
Practice: Graph and label each figure and its image under the given reflection. Give the new coordinates.

1. Rectangle $QRST$ with vertices $Q(-6,-1), R(-3,1)$, $S(1,-5)$, and $T(-2,-7)$: $(x,y) \to (x+5, y+7)$



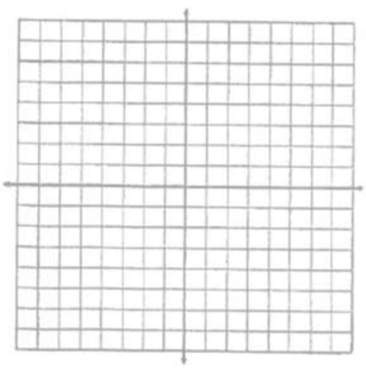
Q' (______ , ______)
R' (______ , ______)
S' (______ , ______)
T' (______ , ______)

2. Triangle CDE with vertices $C(2,-1), D(7,-4)$, and $E(4,-6)$: $(x,y) \to (x-3, y+8)$



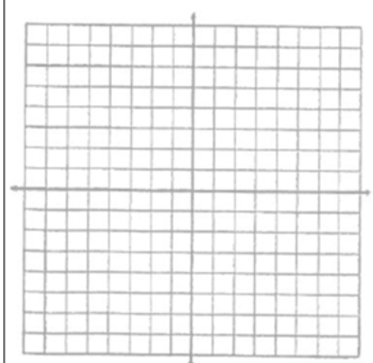
C' (______ , ______)
D' (______ , ______)
E' (______ , ______)

3. Rhombus $JKLM$ with vertices $J(-4,7), K(0,8)$, $L(-1,4)$, and $M(-5,3)$: $(x,y) \to (x+2, y-2)$



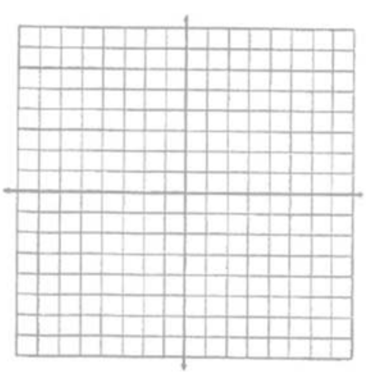
J' (______ , ______)
K' (______ , ______)
L' (______ , ______)
M' (______ , ______)

4. Square $WXYZ$ with vertices $W(1,7)$, $X(6,5)$, $Y(4,0)$, and $Z(-1,2)$: $(x,y) \to (x-7, y-6)$



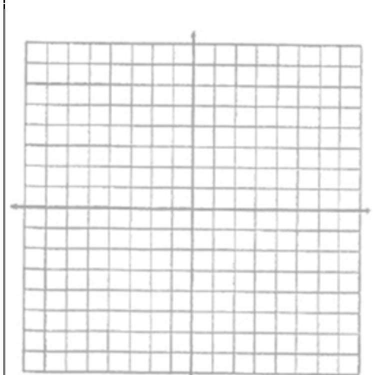
W' (______ , ______)
X' (______ , ______)
Y' (______ , ______)
Z' (______ , ______)

5. Triangle STU with vertices $S(-4,2)$, $T(5,-1)$, and $U(-2,-2)$: $(x,y) \to (x-1, y-5)$



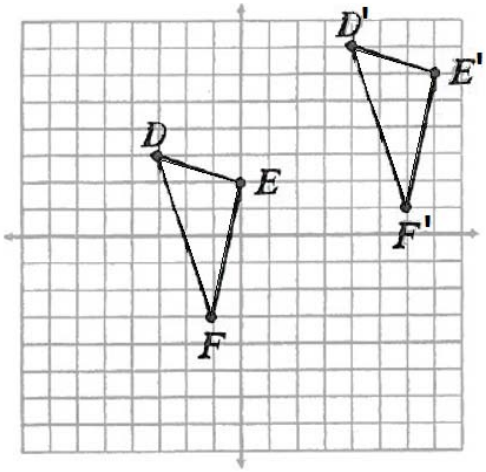
S' (______ , ______)
T' (______ , ______)
U' (______ , ______)

6. Trapezoid $ABCD$ with vertices $A(-3,6), B(0,7)$, $C(1,4)$, and $D(-5,2)$: $(x,y) \to (x+4, y-1)$

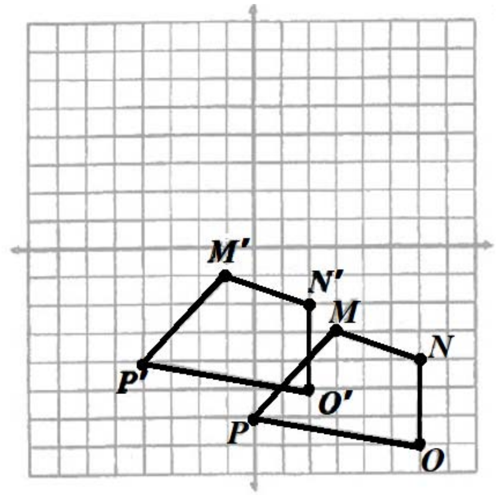


A' (______ , ______)
B' (______ , ______)
C' (______ , ______)
D' (______ , ______)

7. Write a rule describing the translation below:



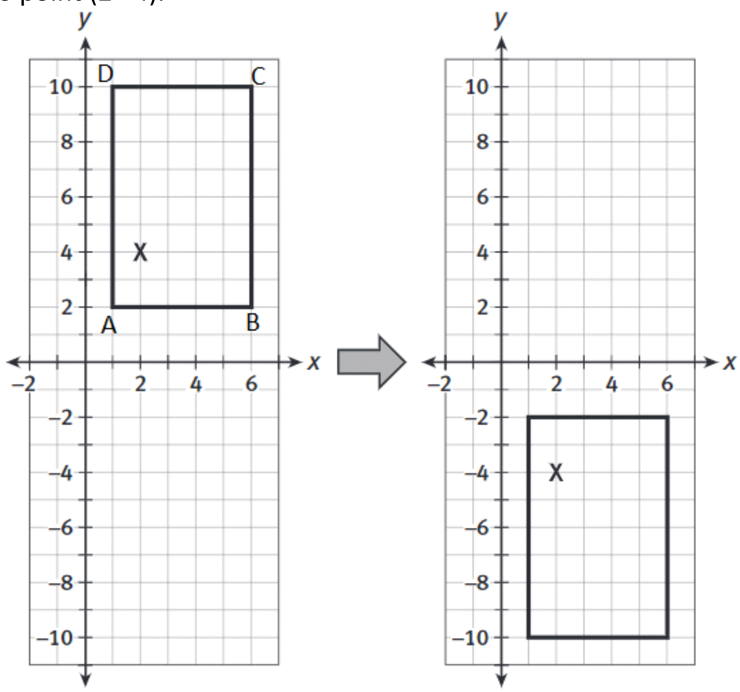
8. Write a rule describing the translation below:



THE BIG IDEA

Transformations – Reflections

Mr. Howell plans another transformation for the Marching Cougars. The sign of the y-coordinate of each marcher changes from positive to negative. Maria, whose position is shown by the X in the diagram, moves from point (2, 4) to point (2 −4).

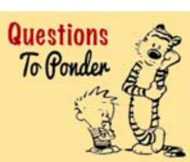


This type of transformation is called a ***reflection***. Reflections are sometimes called flips because the figure is flipped like a pancake. On the coordinate plane, examples of reflections are defined by the functions $(x , y) \to (-x , y)$, which is a reflection across the y-axis. The example shown above is described by $(x , y) \to (x , -y)$, which is a reflection across the x-axis. The function $(x , y) \to (y , x)$defines a reflection across the line $y = x$.

Every reflection has a ***line of reflection***, which is the line that the reflection maps to itself. In the above diagram, the line of reflection is the x-axis

1. Complete the table for the two reflections.

Pre-image	Image $(x, y) \to (x, -y)$	Image $(x, y) \to (-x, y)$
(1, 10)	(1, −10)	
(6, 10)		
(1, 2)		(−1, 2)
(6, 2)		



1. The line y=x is often called the "line of identity." Why do you think the line has this nickname?

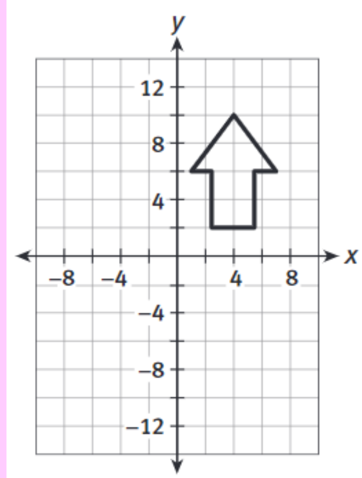
Notes

Graph the line y = x below. Draw your own axes.

Notes

SELF CHECK

The figure shows an arrow in the coordinate plane. The tip of the arrow is located at point $(4, 10)$.



2. Predict the direction of the arrow after these reflections:

a. across the x-axis
b. across the y-axis

3. Draw the two reflections. Were your predictions correct?

4. What reflection maps the arrow to a downward arrow that also has its tip at the point $(4, 10)$?

5. Could a reflection map the arrow so it points to the right or to the left (i.e., parallel to the x-axis)? If yes, describe the line of reflection.

Enrichment (notation)

The expression $r_{y-axis}(P)$ describes the reflection of a given point P over/across the y-axis. In the above example , $r_{y-axis}(P) = P'$.

6. In $r_{y-axis}(P) = P'$. r means ________________, the subscript (y-axis) means _________________, P means _________________, and P' means ______________________.

In regular words, $r_{y-axis}(P) = P'$ means:

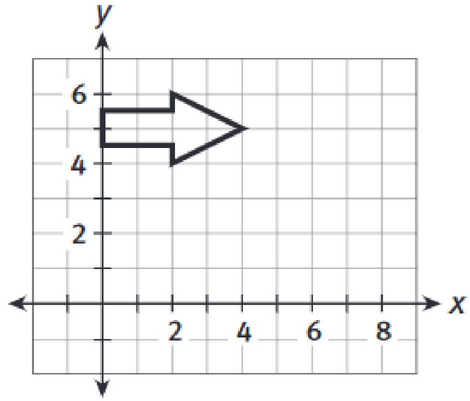
____
____
____
____
____

Mr. Howell arranges the tuba players in an arrow formation, shown below. Then the tuba players undergo a reflection that is described by the function $(x,\ y) \to (8-x\ ,y)$.

Questions To Ponder

$8-x$ may be easier to see if you write the expression a different way. Can you rewrite an equivalent expression to $8-x$ below? How might this be easier to use/consider?

____



7. Draw the reflection. Identify the line of reflection.

8. Which tuba players travel the longest distance during the reflection? Identify this distance.

9. Which tuba player does not travel any distance during the reflection?

10. Explain why the reflection does not change the distance between any given tuba player and the point $(4, 5)$.

11. **Use appropriate tools strategically.** Use geometry software, Mira, the graph to the right or patty paper to explore reflections. First, draw a pentagon. Then draw a line of reflection that passes through one side of the pentagon. Finally, reflect the pentagon over this line of reflection. Then answer the following questions:
a. What happens under this reflection to points that lie on the line of reflection?

b. How are points not on the line of reflection related to the image?

c. Measure the distance of a point to the line of reflection. Then measure the distance of the point's image to the line of reflection. What do you find?

d. Draw the segment that connects the point and its image. How is this segment related to the line of reflection?

Notes

REFLECTION HINT: Cut out paper shapes to model transformations. To show a reflection, flip the shape across the line of reflection.

Enrichment *(notation , 11a, 11b)*

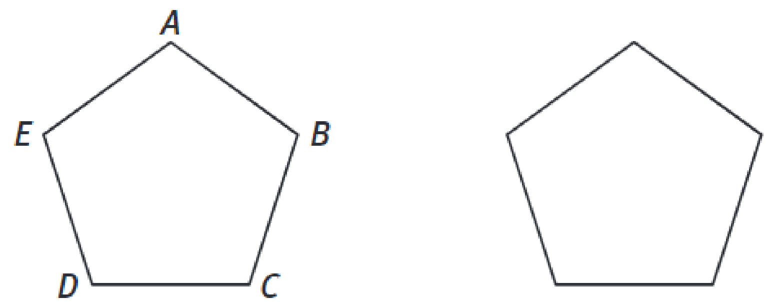
Notes

Like other transformations, reflections can be defined independently of the coordinate plane. A reflection is a transformation that maps P to P' across line such that:

- If P is not on ℓ , then ℓ is the perpendicular bisector of $\overline{PP'}$.
- If P is on ℓ , then $P = P'$.

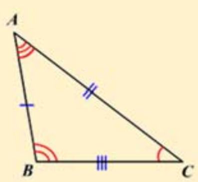
To describe reflections, we will use the notation $r_\ell(P) = P''$, in which r_ℓ is the function that maps point P to point P' across line ℓ , the line of reflection.

SELF CHECK **12.** The diagram below shows pentagon $ABCDE$ and the reflection r_ℓ.

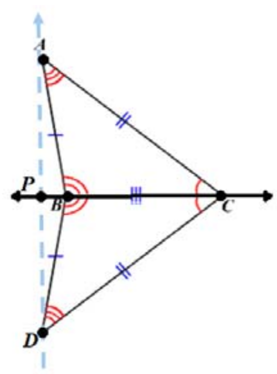


a. Draw line then ℓ .
b. Label the points $r_\ell(A) = A'$ and $r_\ell(B) = B'$, and so on for all of the five vertices.

A note about scalene:
In previous grades, you should have learned about scalene triangles. These are triangles with no equal sides.



SELF CHECK **13.** Quadrilateral $ABDC$ was constructed by drawing scalene triangle ABC, then drawing its reflection across line $\overleftrightarrow{BC}$. Point P is the intersection of $\overline{BC}$ and $\overline{AD}$.



a. How do you know that $\overline{BC}$ is perpendicular to $\overline{AD}$?

b. Explain why AB=BD.

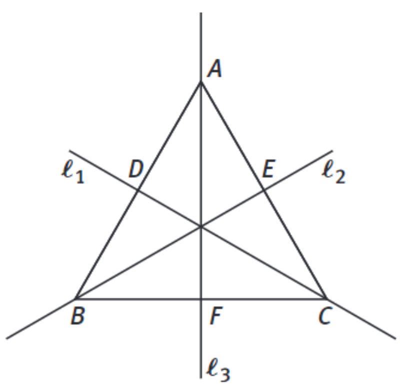
Item 13 below is related to the ***Perpendicular Bisector Theorem***.

> **Perpendicular Bisector Theorem.** if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. The converse (opposite) is also true – if a point equidistant from the endpoints of a segment, it is lying on the perpendicular bisector.

Look again at quadrilateral ABDC. It was constructed by a reflection of a figure, which means that it has ***reflectional symmetry*** . Line segment BC is the ***line of symmetry*** of the figure. A line of symmetry can be an actual line in the figure, or it may be imaginary. A figure may have any number of lines of symmetry, including an infinite number.

14. For each figure shown below, draw all of the lines of symmetry.

15. Triangle ABC has three lines of symmetry, labeled ℓ_1, ℓ_2, and ℓ_3 in the figure below. What can you conclude about the triangle? Explain.



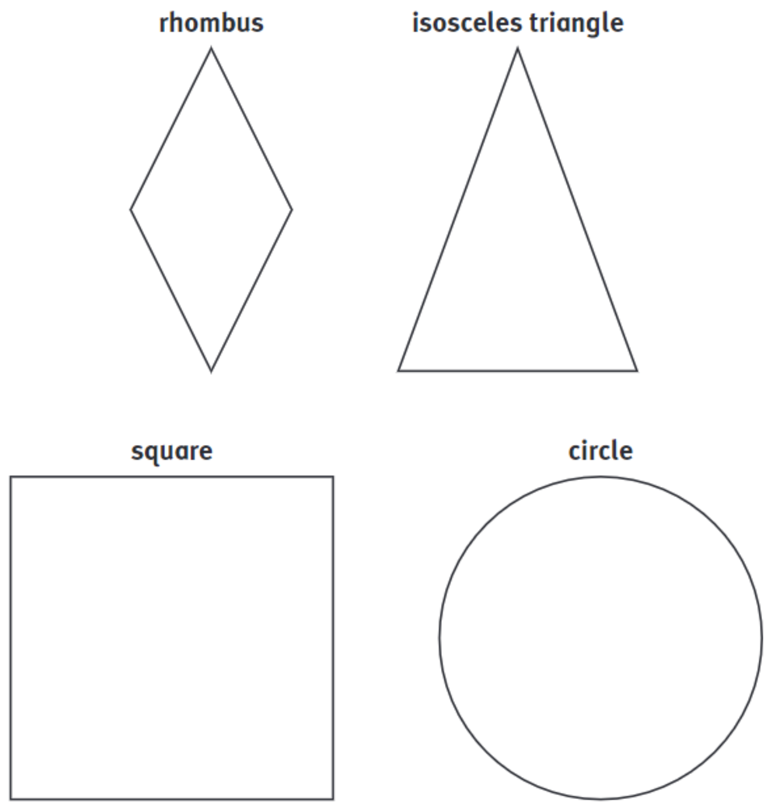
Notes

Notes

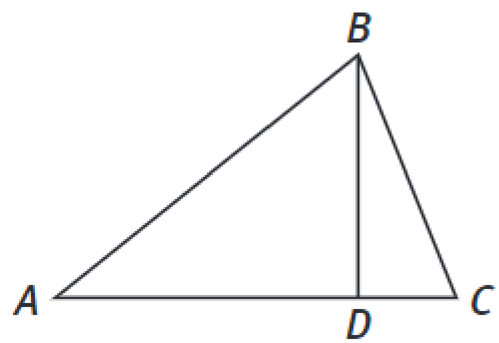
SELF CHECK

Answer the following questions:

16. How many lines of symmetry does each figure below have?



17. The figure shows scalene triangle ABC, which by definition has three unequal sides. Point D is on $\overline{AC}$, and $\overline{BD}$ is perpendicular to $\overline{AC}$.

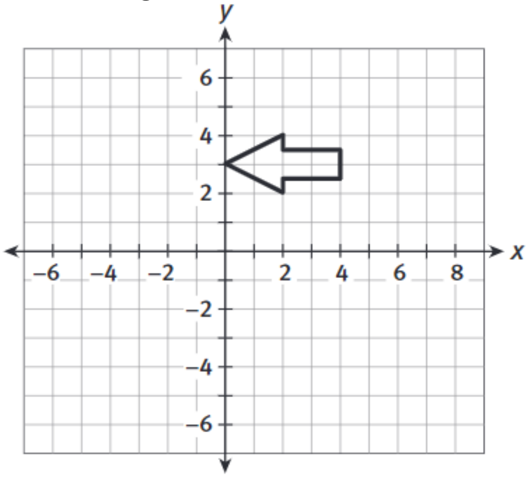


a. Explain why $\overline{BD}$ is not a line of symmetry for the triangle.

b. Does triangle ABC have any lines of symmetry? Explain.

SELF CHECK Reflections Practice.

Use the image below for 17 and 18.



18. Draw the reflection of the arrow described by each of these functions, and identify the line of reflection.

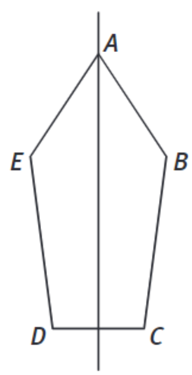
a. $(x\ ,\ y) \rightarrow (8 - x\ ,\ y)$

b. $(x\ ,\ y) \rightarrow (-2 - x\ ,\ y)$

c. $(x\ ,\ y) \rightarrow (x\ , -y)$

19. **Reason abstractly.** Describe a reflection that would map the arrow onto itself.

Irregular pentagon $ABCD$E has exactly one line of symmetry, which passes through point A.



20. What is the image of point C under a reflection across the line of symmetry?

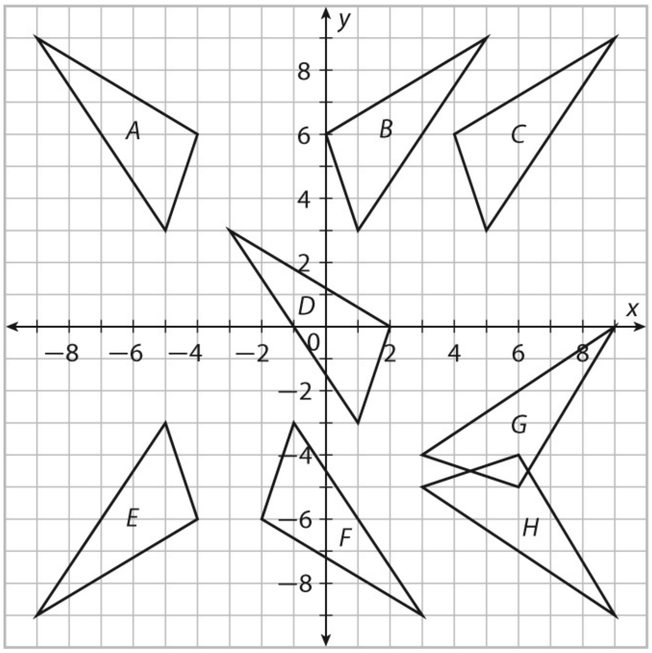
21. Does any point remain fixed under a reflection across the line of symmetry? Explain.

22. Which sides of the pentagon must be congruent?

Notes

REFLECTION HINT: Cut out paper shapes to model transformations. To show a reflection, flip the shape across the line of reflection.

Study the figures on the grid and answer the questions.

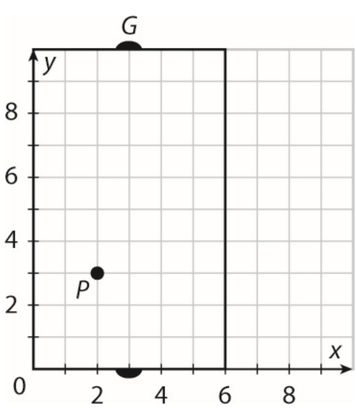


1. Which figure is the reflection of figure *A* over the *y*-axis? _____
2. Which two figures have $x = -3$ as their line of reflection? _____ and _____
3. Which figure is the reflection of figure *A* over the line $y = x$? _____
4. What is the equation of the line of reflection for figures *G* and *H*?

   ____

5. Which figures are **not** reflections of figure *A*? Name all. ________________

Use principles of reflections to determine where to place the puck.



Mike is playing air hockey and wants to bounce the puck off the wall and into the goal at *G*(3, 10).

6. If the puck is at *P* (2, 3), what point on the right wall ($x = 6$) should he aim for? Sketch and label a figure on the grid. Explain your answer.

   ____

   ____

   ____

7. If the puck is at (0, 4), what point on the wall should he aim for?

   (_____ , _____)

8. If the puck is at (3, 2), what point on the wall should he aim for?

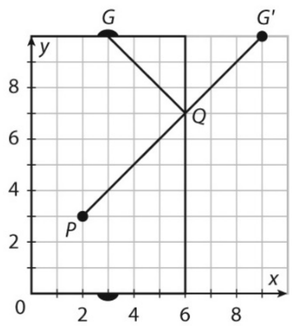
   (_____ , _____)

9. If the puck is at (3, 6), what point on the wall should he aim for?

   (_____ , _____)

Practice and Problem Solving

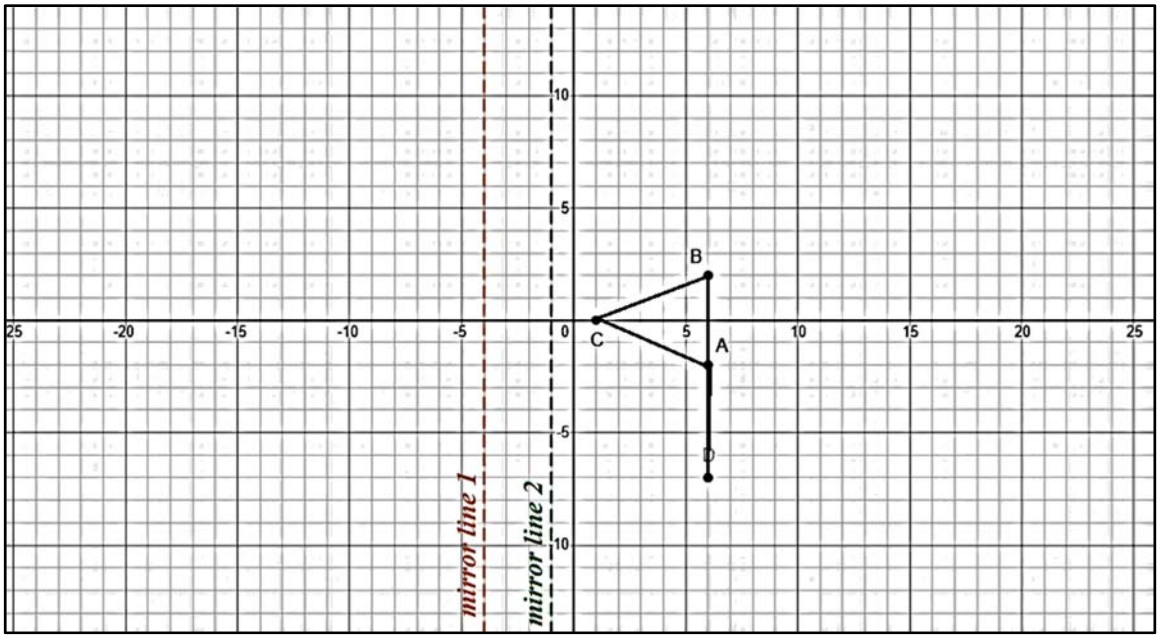
1. C
2. E and F
3. H
4. $y = -4.5$
5. D, F, G



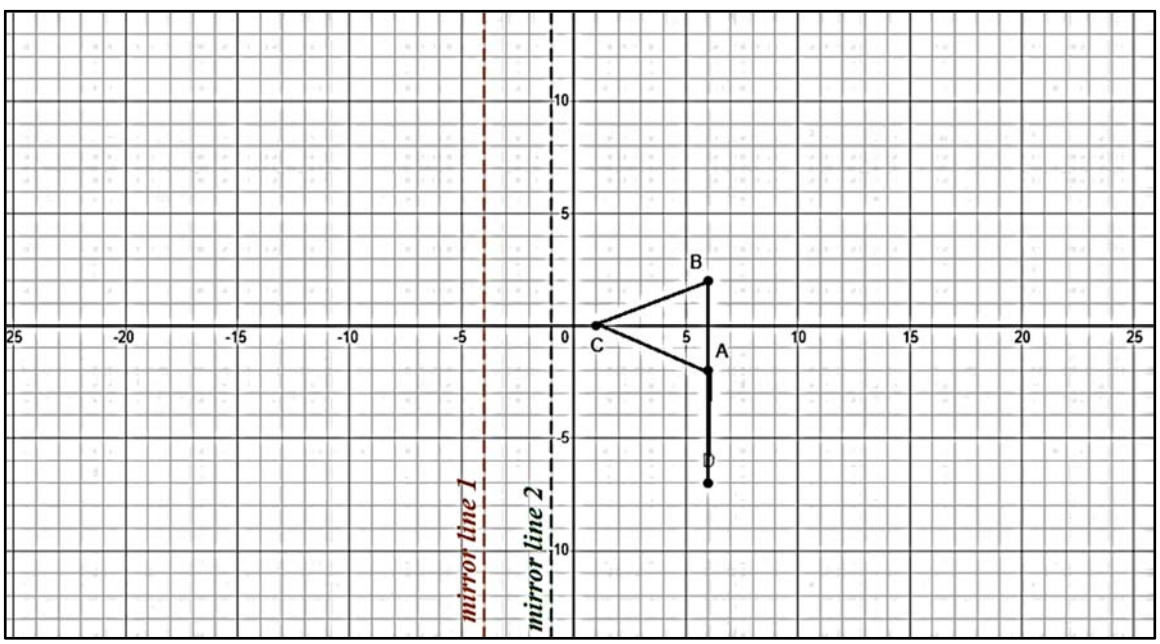
6. The reflection of $G(3, 10)$ over the line $x = 6$ is $G'(9, 10)$. A line from P to G' intersects the wall at $Q(6, 7)$. A puck that goes from P to Q bounces off the wall at the same angle that it hits and will land in the goal.
7. $\left(6, \frac{22}{7}\right)$
8. (6, 6)
9. $\left(6, \frac{29}{5}\right)$

MIRROR, MIRROR…DOES ORDER MATTER?

1. On the FIRST graph below, reflect the flag (made by connecting points A, B, C, and D) across mirror line 1, and then reflect the resulting image across mirror line 2. Label your points in the first and second reflections appropriately.



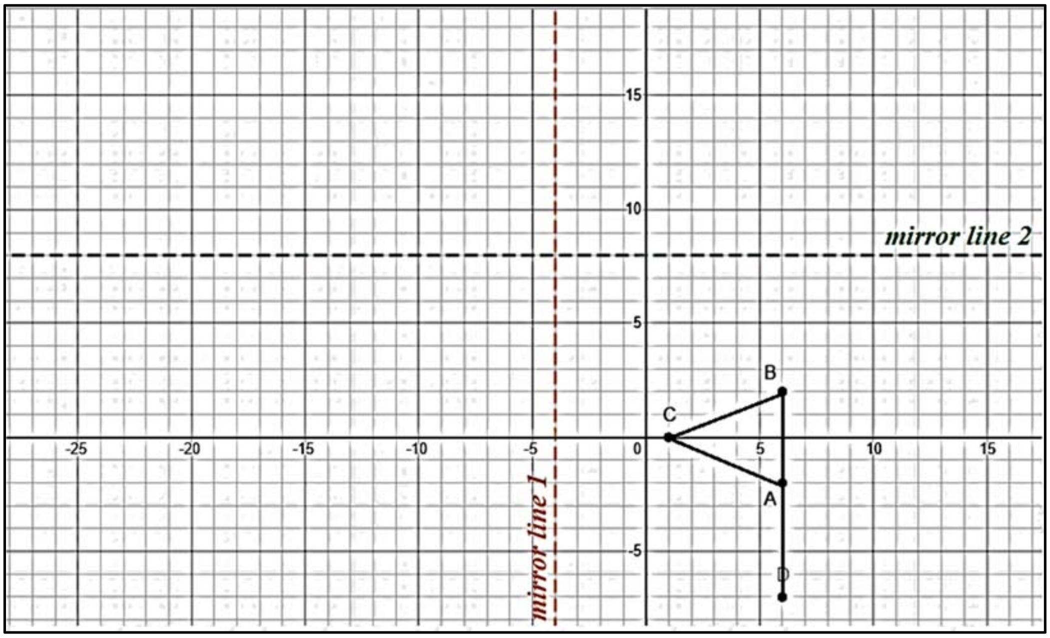
2. Then on the SECOND graph below, reflect the flag (made by connecting points A, B, C, and D) across mirror line 2, and then reflect the resulting image across mirror line 1. Label your points in the first and second reflections appropriately.

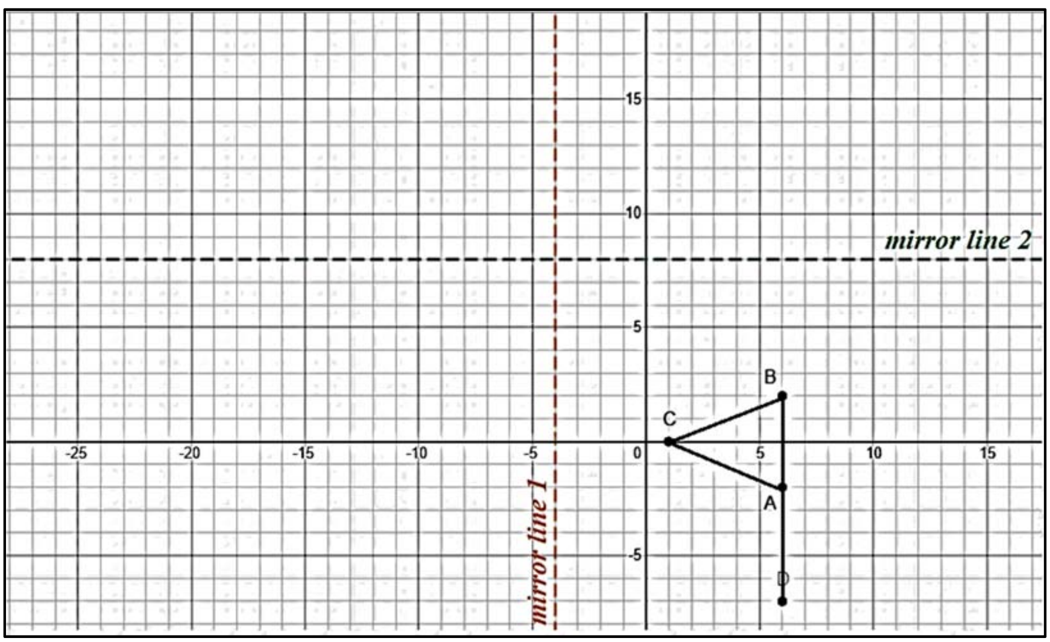


3. For each, an you describe the single transformation that maps the first flag to the last flag in #1 vs. #2?

4. Did it matter in which line you reflected first? Explain why or why not.

5. Now look at the new diagram below. Reflect the flag across mirror line 1, and then reflect the resulting image across mirror line 2, then reflect in the opposite order (line 2 then line 1) on the other graph. Identify the points after both reflections as well.

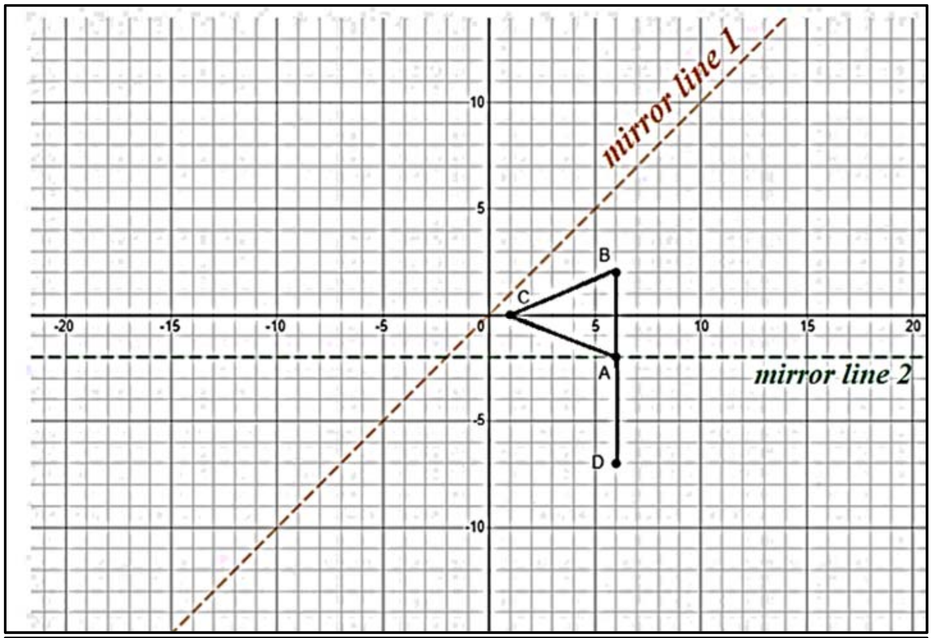


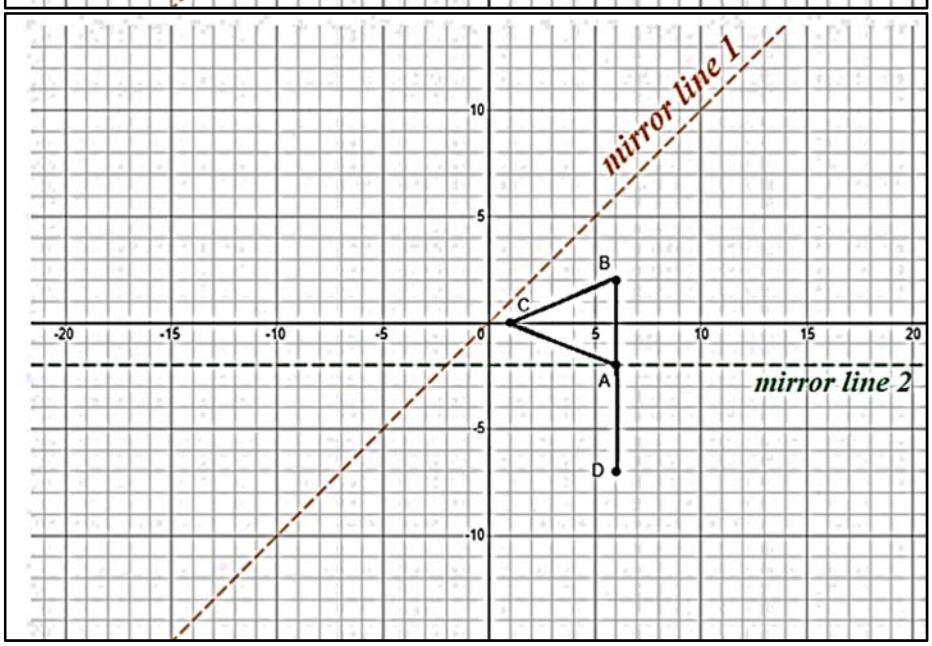


6. For each, can you describe the single transformation that takes the first flag to the last flag? Explain.

7. Does it matter over which line you reflect first? Explain why or why not.

8. Now try double reflecting with lines that meet at <u>45°</u> , below – reflect across line 1 then 2 on the first one, and in the opposite order on the second one.





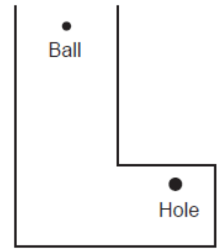
9. For each, can you describe the single transformation that takes the first flag to the last flag? Explain.

10. Does it matter over which line you reflect first? Explain why or why not.

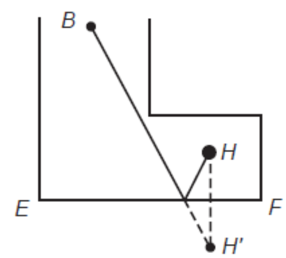
11. What is the equation and “nickname” of mirror line 1?

MINIATURE GOLF

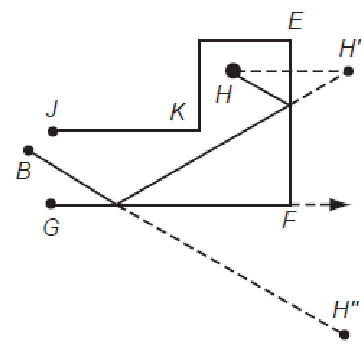
In miniature golf, the object of the game is to putt the golf ball into the hole in as few shots as possible. As in the diagram at the right, the hole is often placed so that a direct shot is impossible. If the ball does not have much spin, it will rebound off a wall in such a way that the two angles formed by the path of the ball and the wall will be congruent. Reflections can be used to help determine the direction that the ball should be struck in order to score a hole-in-one.



Example 1: Using wall $\overline{EF}$, find the path to use to score a hole-in-one.
Find the reflection image of the "hole" with respect to $\overline{EF}$ and label it H'. The intersection of $\overline{BH'}$ with wall $\overline{EF}$ is the point at which the shot should be directed.

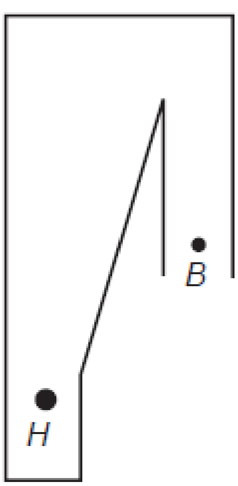


Example 2: For the hole at the right, find a path to score a hole-in-one.
Find the reflection image of H with respect to $\overline{EF}$ and label it H'. In this case, $\overline{BH'}$ intersects $\overline{JK}$ before intersecting $\overline{EF}$. Therefore, this path cannot be used.
To find a usable path, find the reflection image of H' over $\overrightarrow{GF}$ and label it H''.
Now, the intersection of $\overline{BH''}$ with wall $\overline{GF}$ is the point at which the shot should be directed. Notice how the path of the ball is generated using B, H'', H', and H.

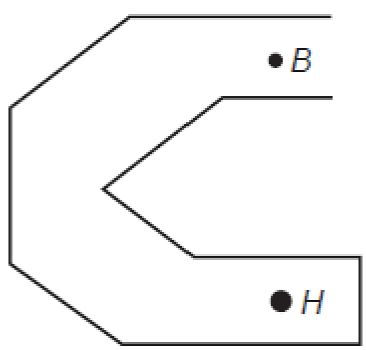


Use reflections to determine a possible path for a hole-in-one.

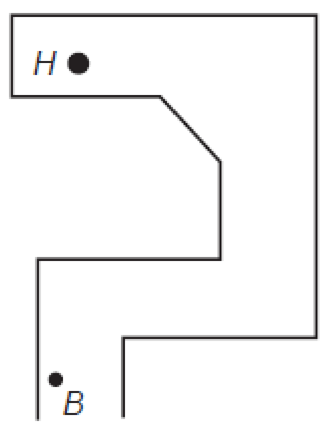
1.



2.



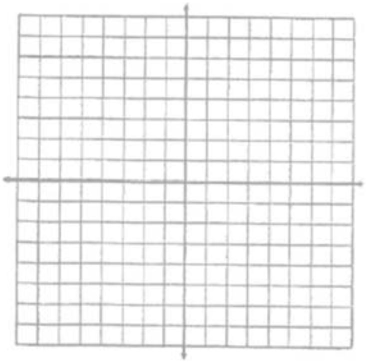
3.



G.U1.C2.E.05.**Homework.Reflections**

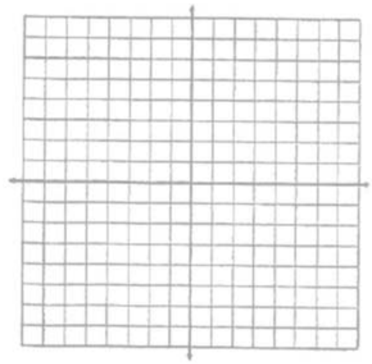
Practice: Graph and label each figure and its image under the given reflection. Give the new coordinates.

1. Triangle ABC with vertices $A(-4, 2)$, $B(4, 7)$, and $C(5, 1)$ reflected across the x-axis.



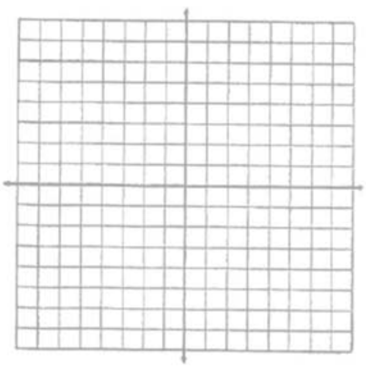
A' (______ , ______)
B' (______ , ______)
C' (______ , ______)

2. Rectangle $PQRS$ with vertices $P(1, 2)$, $Q(2, 5)$, $R(8, 3)$, and $S(7, 0)$ reflected across the y-axis.



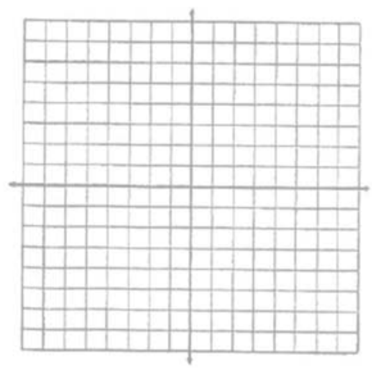
P' (______ , ______)
Q' (______ , ______)
R' (______ , ______)
S' (______ , ______)

3. Trapezoid $FGHI$ with vertices $F(-5, -2)$, $G(-2, -2)$ $H(0, -6)$, and $I(-8, -6)$ reflected across the y-axis.



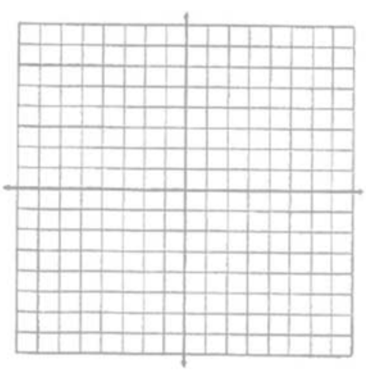
F' (______ , ______)
G' (______ , ______)
H' (______ , ______)
I' (______ , ______)

4. Rhombus $WXYZ$ with vertices $W(-2, -4)$, $X(1, -2)$, $Y(4, -4)$, and $Z(1, -6)$ reflected across the y-axis.



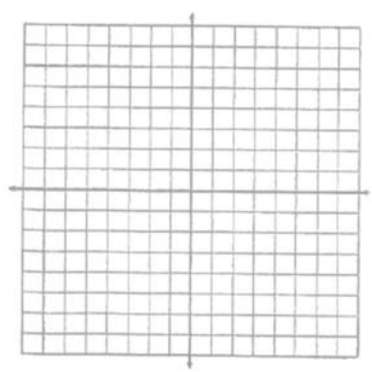
W' (______ , ______)
X' (______ , ______)
Y' (______ , ______)
Z' (______ , ______)

5. Triangle JKL with vertices $(1, -1)$, $K(2\ ,\ 3)$, and $L(3, -2)$ reflected across the line $x = 4$.



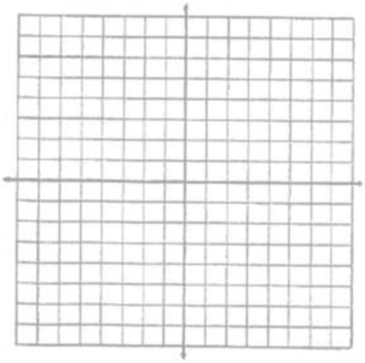
J' (______ , ______)
K' (______ , ______)
L' (______ , ______)

6. Square $RSTU$ with vertices $R(0, 3)$, $S(5, 4)$, $T(6, -1)$, and $U(1, -2)$ reflected across the line $x = -1$.



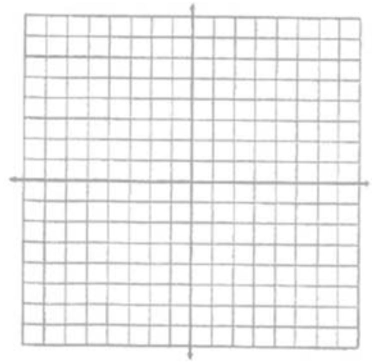
R' (______ , ______)
S' (______ , ______)
T' (______ , ______)
U' (______ , ______)

7. Parallelogram $CDEF$ with vertices $(2, -7)$, $D(1, -1)$, $E(2\ ,\ 3)$, and $F(3, -2)$ reflected across the line $y = 2$.



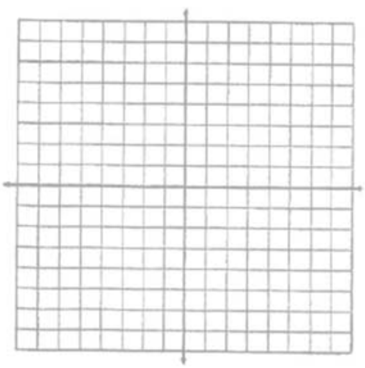
C' (______ , ______)
D' (______ , ______)
E' (______ , ______)
F' (______ , ______)

8. Triangle MNP with vertices $M(-6, -8)$, $N(-1, -6)$, and $P(-2, -8)$ reflected across the line $y = x$.



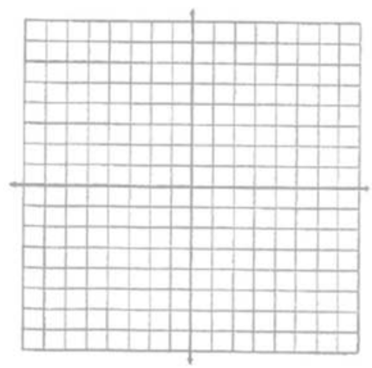
M' (______ , ______)
N' (______ , ______)
P' (______ , ______)

9. Triangle XYZ with vertices $(-5, -2)$, $Y(-3, 4)$, and $Z(-1, 1)$ reflected across the line $y = x$.



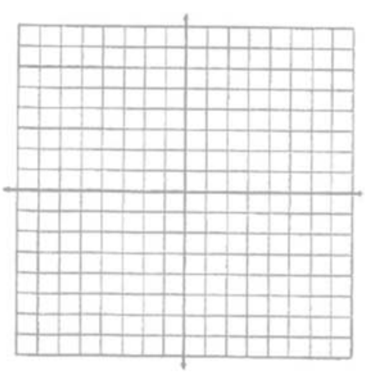
X' (______ , ______)
Y' (______ , ______)
Z' (______ , ______)

10. Rectangle $GHIJ$ with vertices $G(2, -1)$, $H(7, -1)$, $I(7, -4)$, and $J(2, -4)$ reflected across the line $y = -5$.



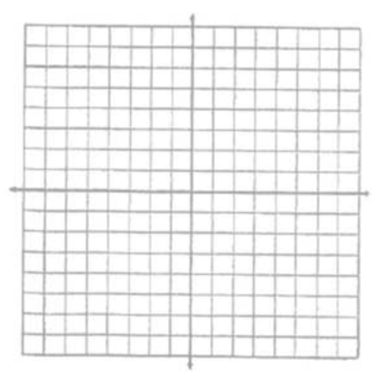
G' (______ , ______)
H' (______ , ______)
I' (______ , ______)
J' (______ , ______)

11. Square $ABCD$ with vertices $A(-1, 3)$, $B(0, 6)$, $C(3, 5)$, and $D(2, 2)$ reflected across the line $y = -x$.



A' (______ , ______)
B' (______ , ______)
C' (______ , ______)
D' (______ , ______)

12. Triangle STU with vertices $S(-1, -6)$, $T(0, -3)$, and $U(2, -4)$ reflected across the line $y = -x$.

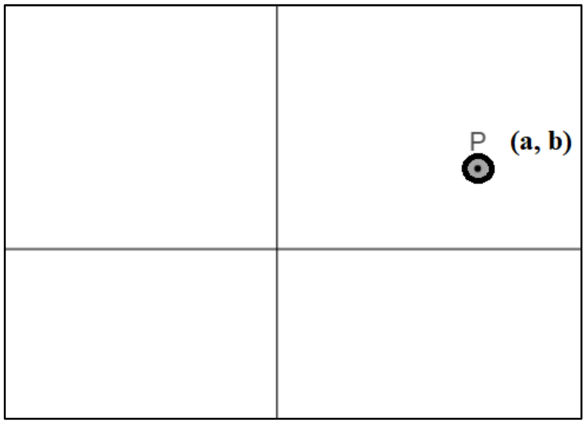


S' (______ , ______)
T' (______ , ______)
U' (______ , ______)

Transformations: Rotations

GUIDED NOTES

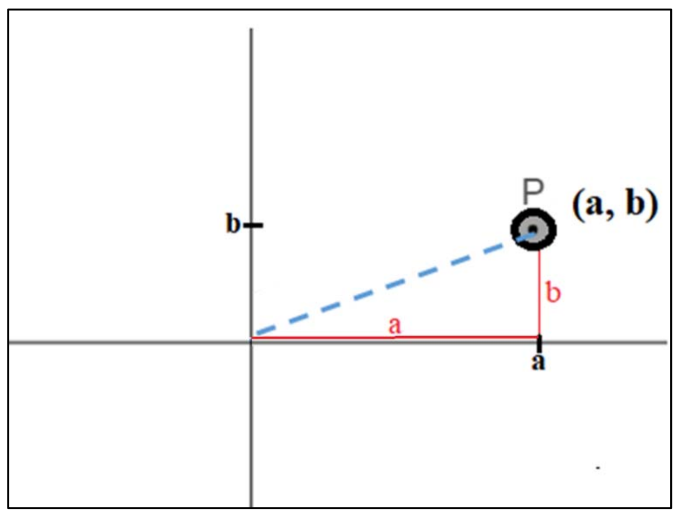
TRANSFORMATION THROWBACK TO 8th GRADE - ROTATIONS

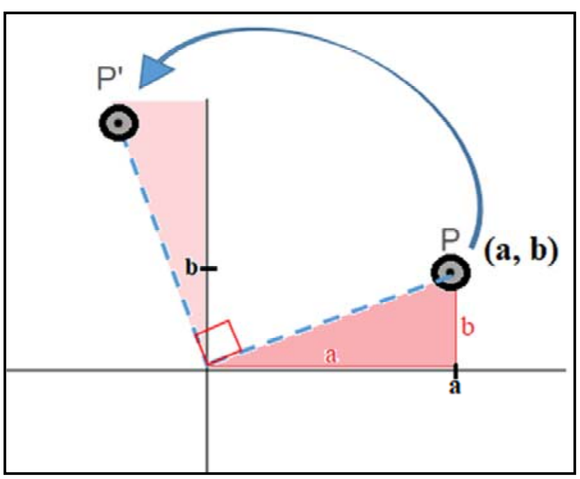


Recall that in 8th grade, you rotated points in a positive direction.

1) Positive rotations are always in a ___________ direction around the origin (the origin was called the **center of rotation**). Given point P, left, with coordinates (a, b). You were asked "What will the coordinates of P' be after you rotate it 90° about the origin?" You also may have been asked the same but for other multiples of 90 such as 180°, 270°, 360°. Negative rotations are new – but not difficult. This just means to rotate in a ______________direction!!!

You may recall that you rotated images, point-by-point, around the origin by imagining right triangles. You imagined the right triangle that would be formed by the horizontal length, a, the vertical length, b as the legs of a right triangle with the hypotenuse (the dotted blue line in the picture to the right) being the line you will be rotating 90°.
You connected the origin to the point you wanted to rotate to form the hypotenuse and that was the line you would swing counter-clockwise 90°.





Then you rotated point P, 90°. Where it lands is the new point, P', and the entire triangle followed. Note that the vertical leg b has now become horizontal, and the horizontal leg a has now become vertical. This changes the coordinate of P' from P in very specific ways.
In terms of a and b, what are the coordinates of P' after the rotation of 90°**?** Fill in the coordinate below then explain below it what you would do to any coordinate to rotate it 90°.

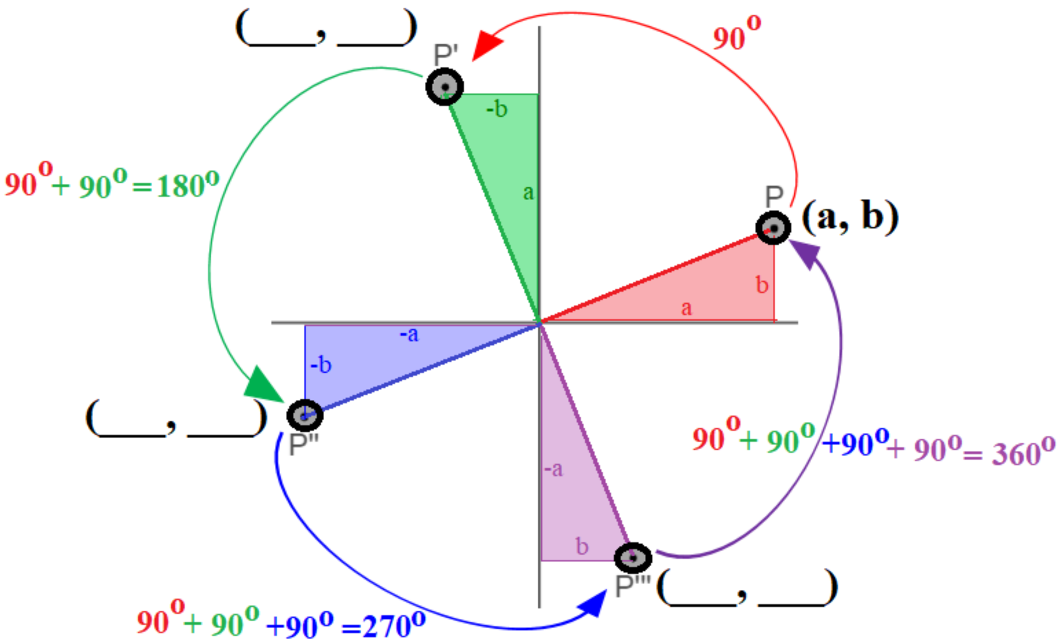
2. (_____ , _____)

3. To rotate 90°, you would negate the ____value, then switch it with ____.

How to rotate 90^o - Common Sense Is Better than Memorization!
Link 90^o + 90^o + 90^o…just keep negating y and exchanging it with x!

Link 90^0 rotations in a chain: If you can remember how a 90° rotation affects the coordinates of a point, you can just perform the same rotation twice to find the coordinates of a 180^0 rotation, do the same three times for a 270^o, and four times for 360^0! There is no need to memorize "rules" for rotations that are multiples of 90°!

4.



Fill in the coordinates of *P'*, *P''*, and *P'''* in terms of a and b on the picture above.

Fill in the charts below continuing the patterns you see filled in already!

POSITIVE COORDINATE ROTATION PATTERNS: Fill in the chart below to show how, based on the previous coordinates, you can find the coordinates of the next 90^o multiple's coordinates.

Given point P (a, b) (Counter-Clockwise)	
+90^o from previous	**Coordinates** (negate previous y, switch with x)
0^o	**(a, b)**
90^o	Negate b, switch with a: (-b, a)
180^o	Negate a, switch with -b: (-a, -b)
270^o	**5.**
360^o	**6.**

NEGATIVE COORDINATE ROTATION PATTERNS: Positive rotation angles travel counter-clockwise and negative rotation angles travel clockwise, but all negative rotations can be written as positive ones. Negative rotations can be changed into positive rotations by combining with 360^0. Fill in the chart below by subtracting and then filling in the correct coordinates.

Given point P (a, b) (Negative Angles - Clockwise)	
+360⁰ to convert to positive:	**Coordinates** (negate previous y, switch with x)
$0^0 = 360° - 0 = 360°$	**(a, b)**
$-90^0 = 360° - 90 = 270°$	**7.**
$-180^0 = 360° - 180 = 180°$	Negate a, switch with -b: (-a, -b)
$-270^0 = 360° - 270 = 90°$	Negate b, switch with a: (-b, a)
$-360^0 = 360° - 360 = 0°$	**8.**

SELF CHECK See if you understand negative angles, and how they related to positive ones, below!

9. -90^0 **has the same coordinates as the positive angle**___________^{0}.

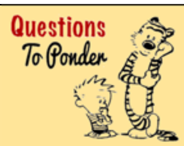
10. -180^0 **has the same coordinates as the positive angle** ___________ ^{0}.

11. -270^0 **has the same coordinates as the positive angle** ___________ ^{0}.

12. -360^0 **has the same coordinates as the angle** ___________ ^{0}**, and also as the positive angle**___________^{0}.

9. 270^0, **10.** 180^0, **11.** 90^0, **12.** 0^0 and 360^0.

Questions To Ponder

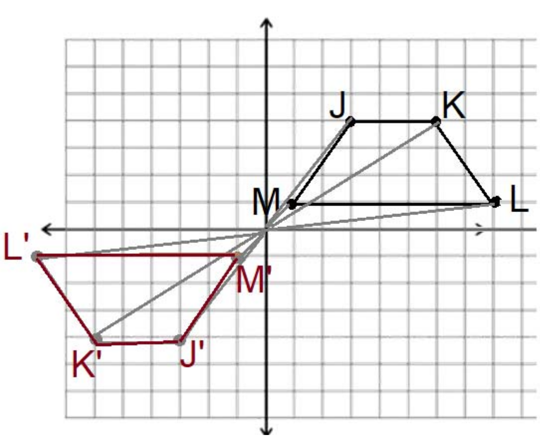


Why is it that $180°$ and $-180°$ have the same coordinate? Does negative vs. positive for $180°$ matter?

Example! Graph and label each figure and its image under the given rotation about the origin.

13. Trapezoid $JKLM$ with vertices $J(3,4), K(6,4), L(8,1)$ and $M(1,1)$:rotated $-180°$about the origin.

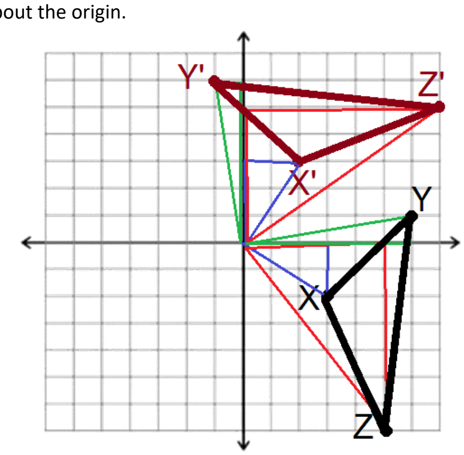
$J'(-3,-4)$
$K'(-6,-4)$
$L'(-8,-1)$
$M'(-1,-1)$



This image shows 180 degree straight angles tracking rotation of each point.

14. Triangle XYZ with vertices $X(3,-2), Y(6,1)$, and $Z(5,-7)$: rotated 90° about the origin.

$X'(2,3)$
$Y'(-1,6)$
$Z'(7,5)$



This image shows how drawing right triangles with x and y movement to the original points, then rotating the triangle (in this case, 90^0 rotations) is a good way of tracking rotations analytically on the coordinate plane.

Example! Graph and label each figure and its image under the given rotation about the origin.

Rotations are easy if you know how to negate y, then switch the current x and the negated y for every 90° rotation.

Original Point	(a, b)	(-5, 6)
Rotated 90°	(-b, a)	(-6, -5)
Rotated 90° more (180°)	(-a, -b)	(5, -6)
Rotated 90° more (270°)	(b, -a)	(6, 5)
Rotated 90° more (360°)	(a, b)	(-5, 6)

The result is below.

Original Point	(a, b)	(-5, 6)
Rotated 90°	(-b, a)	(-6, -5)
Rotated 90° more (180°)	(-a, -b)	(5, -6)
Rotated 90° more (270°)	(b, -a)	(6, 5)
Rotated 90° more (360°)	(a, b)	(-5, 6)

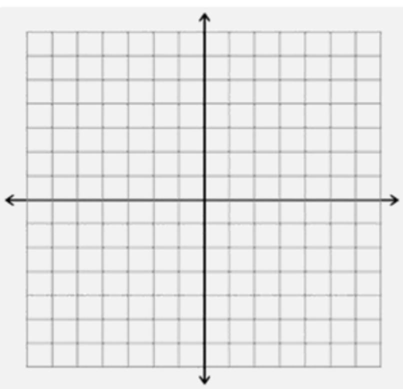
SELF CHECK For #15, Make sure you can rotate the points below, 90° at a time. Then apply this basic idea to the rotations on #16 and #17.

15. Can you show the correct coordinates after rotation?

Original Point	(-5, -4)	(3, 12)	(8, -2)
Rotated 90°			
Rotated 90° more (180°)			
Rotated 90° more (270°)			
Rotated 90° more (360°)			

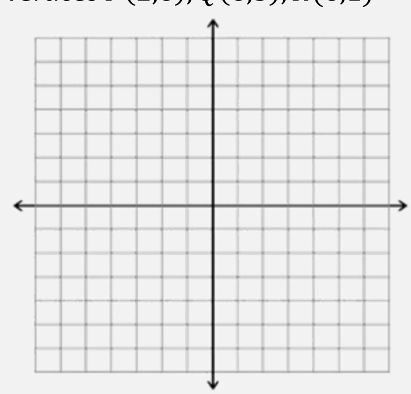
16. Triangle ABC with vertices $A(2,7), B(6,5)$, and $C(4,1)$: rotated $-90°$ about the origin.

A'(____ , ____)
B'(____ , ____)
C'(____ , ____)



17. Square PQRS with vertices $P(2,6), Q(6,5), R(6,1)$ and $S(1,2)$: rotated 180° about the origin.
18.

P'(____ , ____)
Q'(____ , ____)
R'(____ , ____)
S'(____ , ____)



ANSWERS:

Original Point	(-5, -4)	(3, 12)	(8, -2)
Rotated 90°	(4, -5)	(-12, 3)	(2, 8)
Rotated 90° more (180°)	(5, 4)	(-3, -12)	(-8, 2)
Rotated 90° more (270°)	(-4, 5)	(12, -3)	(-2, -8)
Rotated 90° more (360°)	(-5, -4)	(3, 12)	(8, -2)

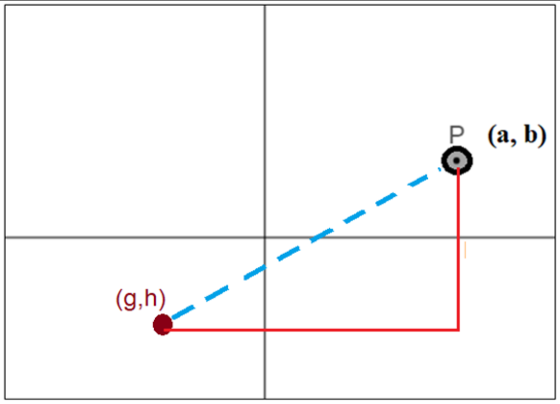
$A'(7,-2)$
$B'(5,-6)$
$C'(1,-4)$

$P'(-2,-6)$
$Q'(-6,-5)$
$R'(-6,-1)$
$S'(-1,-2)$

GUIDED NOTES

SO WHAT IS DIFFERENT NOW FROM 8TH GRADE?

You rotated **only around the origin in 8th grade**. But objects can be rotated over ANY point/center of rotation, not just the origin! Welcome to higher learning!



You can rotate point P around (g, h), even if it is not the origin!

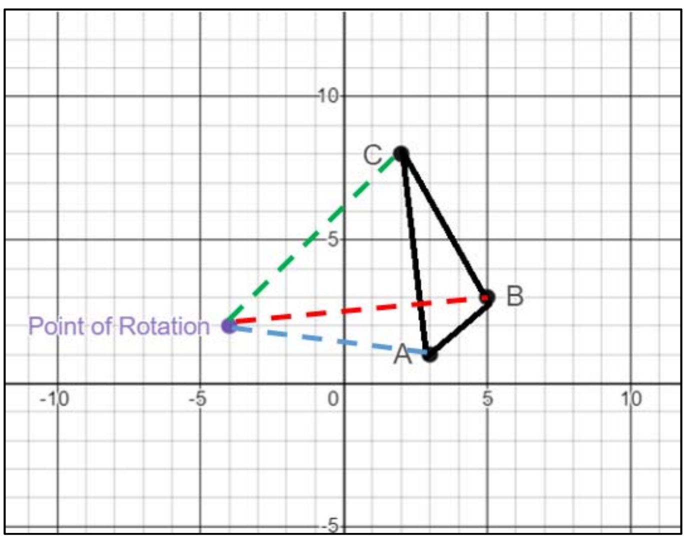
Sketch: Estimate a $90°$rotation of point P about the point (g, h) and sketch it in to the left.

18. Compare rotation around the origin with rotation around (g, h). What is different? What is the same?

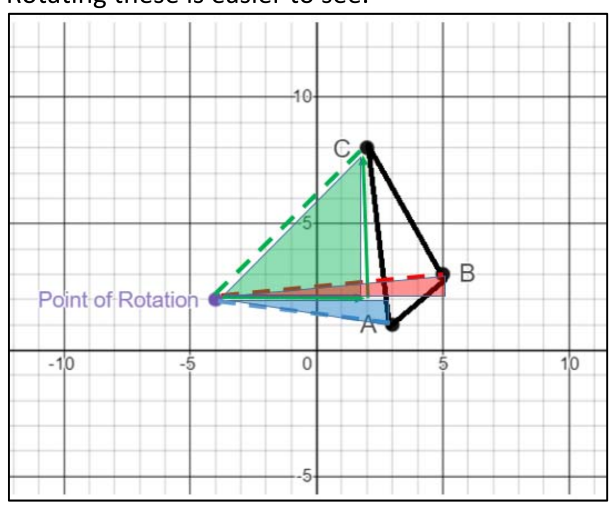
There are three ways to perform rotations around a point that is not the origin.

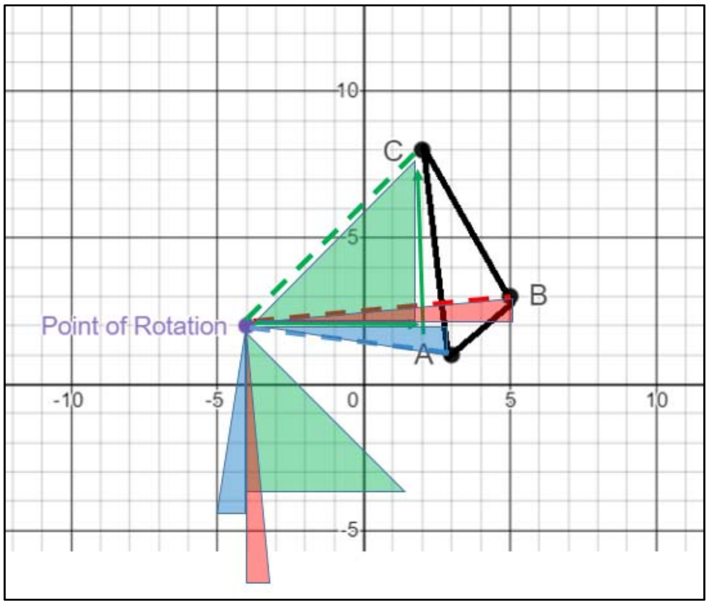
Method 1: Graphically Rotating. You can manually rotate the line connecting your point of rotation the number of degrees indicated on the axes (following "right triangles").

From the point of rotation, determine the paths to each of the vertices to be rotated.



Imagine (or actually draw) the right triangles whose hypotenuses would be the direct paths to those points. Rotating these is easier to see.



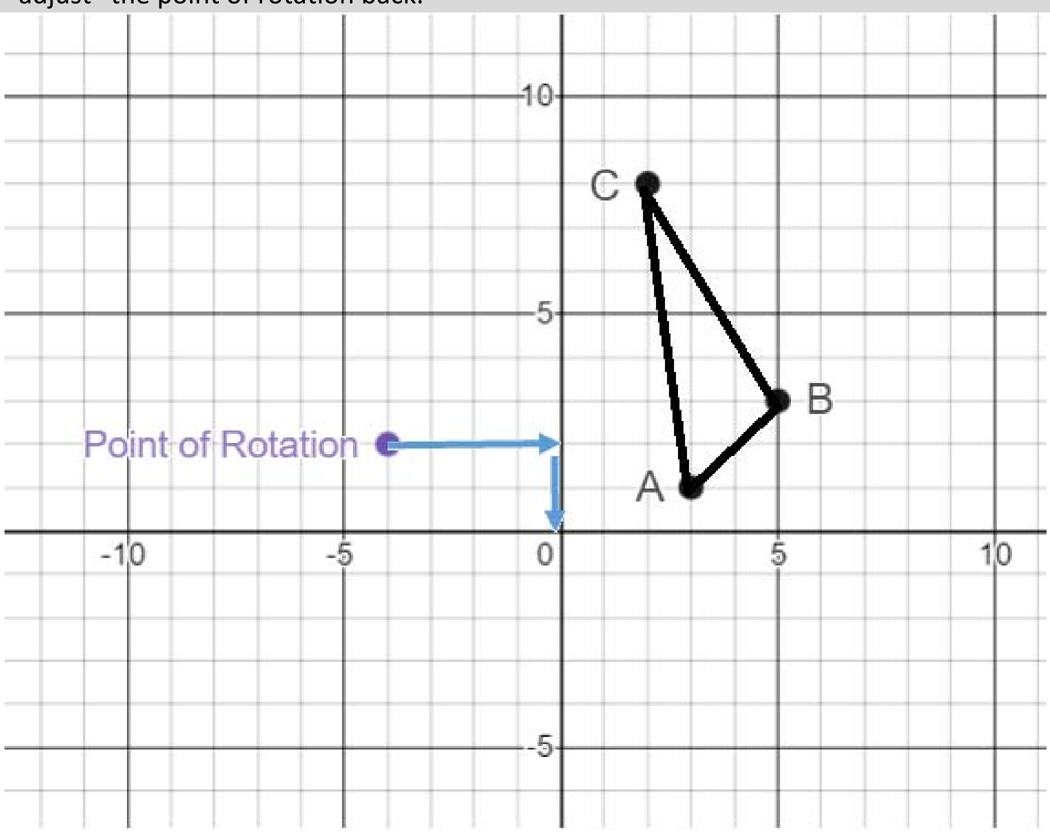


Rotate them each clockwise (which is -90°). Where the hypotenuses land is your new points.

HINT: to rotate, for instance, the red triangle, note that to reach B from the point of rotation originally, you had to hop forward 9 units and up 1 unit. That series of movements "rotates" to right 1 unit and down 9 units. From (-4, 2), that would be B'(-3, -7).
A'(-5, -5), B'(-3, -7) and C'(2, -4).

Upside to graphical rotations: for the visual learner, it is very clear what is happening.
Downside: If you accidentally count the squares wrong either while forming the triangles or when identifying the new points, you can easily get this wrong.

Method 2: Analytically Rotating: You can temporarily "adjust" the point of rotation to the origin, use your rotation rules, and then "adjust" the point of rotation back.



For the point of rotation to move back to the origin, you would have to add 4 units from the x values and subtract 2 units from the y-values. Making a chart is helpful. Remember too that -90° is the same as 270° in this situation.

The steps are to (1) adjust the point of rotation by doing the exact opposite of the point of rotation's coordinates to each point you are rotation, (2) use rotation rules as if you are rotating around the origin, and finally (3) adjust back the other way using the point of rotation coordinates to get the end coordinates.

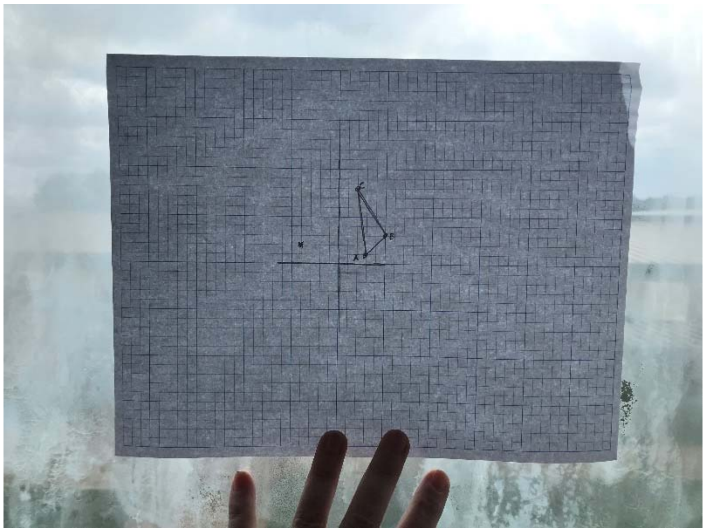
Pt Original Coordinate	Adjust point of rotation to origin (+4, -2)	Use rotation rules (negate b, switch a and b for every 90 degrees). $-90^o=270^o$ = (+b, -a)	Adjust point of rotation BACK to where it should be by doing the opposite (-4, +2)
A(3,1)	(7, -1)	(-1, -7)	$A'(-5,-5)$
B(5,3)	(9, 1)	(1, -9)	$B'(-3,7)$
C(2,8)	(6, 6)	(6, -6)	$C'(2,-4)$

Upsides to analytic rotation: very easy, minimal opportunities to make errors, and does not require graphing
Downside to analytic rotation: somewhat dependent upon memory of steps or ability to figure steps out once forgotten.

Method 3: Rotating Numerically/Hands-0n. Using constructions, geometry software, or patty paper, you can rotate points and segments 90° at a time until you have rotated them the correct number of degrees.

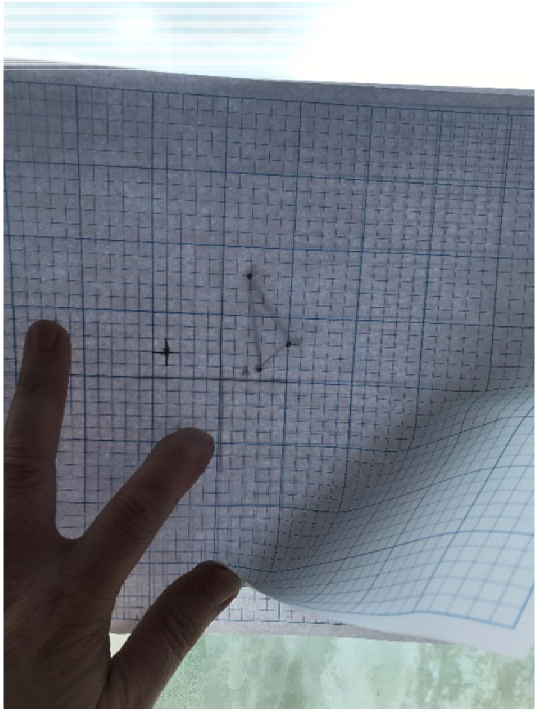
One of the most under-utilized tools (especially on standardized test that are taken on a computer) is the backlighting that a computer monitor provides. See below…in this example, a written problem taped to a window is used, but this would work similarly if using a computer screen. ***(Images are blurry because you are looking at an image THROUGH a piece of paper that is backlit through a window! Try it yourself!)***

1.



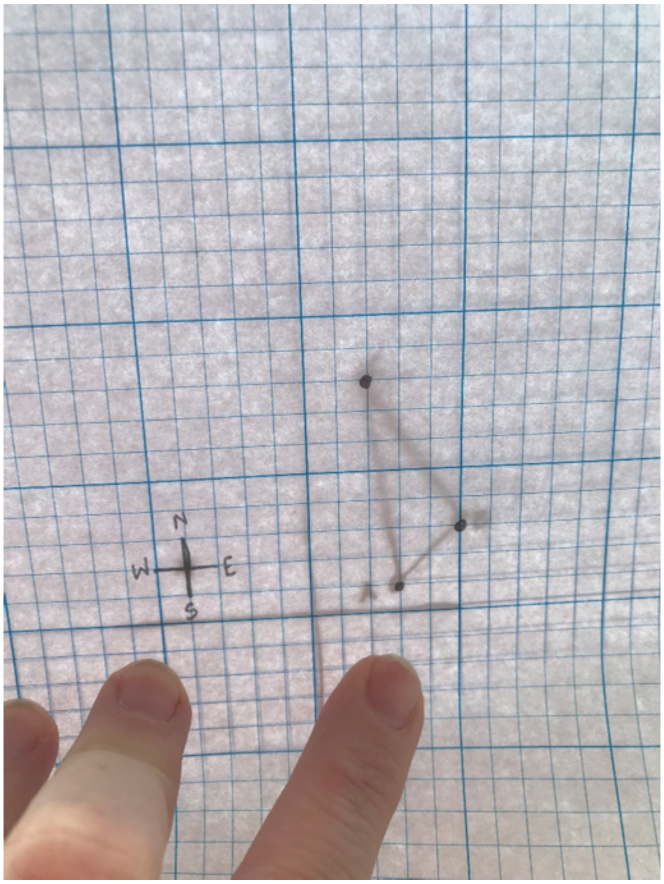
The same triangle is either projected or taped to a light source.

2.



On a separate piece of paper, the points are traced. Since the point of rotation is (-4, 2), small axes are drawn over that point, as well.

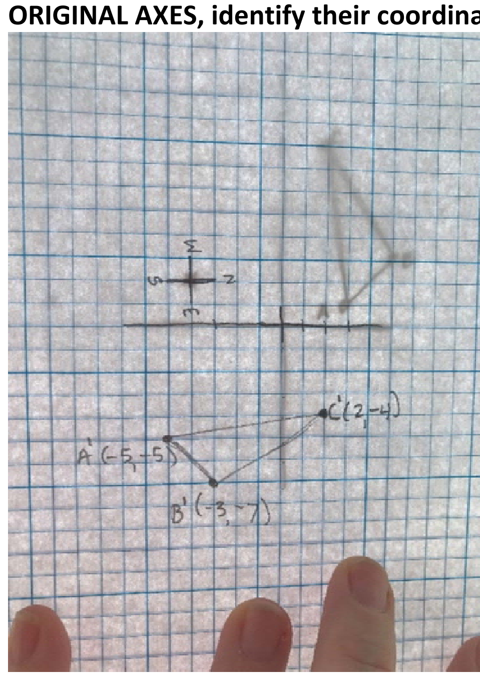
3.



Just for reference, the point of rotation's axes were tagged with North, South, East, and West.

4.

Placing the tip of your pencil or your finger on the point of rotation, rotate the tiny axes -90^{o} (you can see North is now on the right side…the background image is the same still). The A, B, and C points are now at their A', B', and C' locations. 5. USING THE ORIGINAL AXES, identify their coordinates.



Which method should you use? Whichever one you like best and that you can be the most accurate with.

SELF CHECK Perform the rotations using the method indicated. You will need to use your own paper.

19. Use Graphing
Given rhombus CDEF with vertices
C(-5,5)
D(-1,7)
E(-3,3)
F(-7,1)
Rotate -270^0 about the point (-2,1), and identify the new coordinates after the rotation.

20. Use Analytical
Given rectangle TUVW with vertices
T(-3,-1)
U(0,-2)
V(-2,-8)
W(-5,-7)
Rotate -180^0 about the point (-2,-3), and identify the new coordinates after the rotation.

21. Use a hands-on approach.
Given parallelogram MNOP with vertices
M(1,7)
N(8,5)
O(4,2)
P(-3,4)
Rotate 90^0 about he point (3, -5), and identify the new coordinates after the rotation.

C'(2,4); D'(4,0); E'(0,2); F'(0,5)

T'(-1,-5); U'(-4,-4); V'(-2,2); W'(1,1)

M'(-9,-7); N'(-7,0); O'(-4,-4); P'(-6,-11)

Example! Perform the rotations using whatever method works best for you. You will need to use your own paper.

22. Given triangle GHI with vertices
G(0,-2)
H(7,-6)
I(3,-8)

Rotate 270^0 about the point (4,1), and identify the new coordinates after the rotation.

23. Given square ABCD with vertices
A(-7,5)
B(-4,7)
C(-2,4)
D(-5,2)
Rotate 180^0 about the point (-1,-3), and identify the new coordinates after the rotation.

24. Given rectangle WXYZ with vertices
W(-3,-5)
X(1,-1)
Y(3,-3)
Z(-1,-7)
Rotate 90^0 about he point (-3, 2), and identify the new coordinates after the rotation.

Enrichment *(notation)*

The expression $R_{(2,-3),270°}(P)$ describes the rotation of a given point P about the point (2, -3) of 270°. In the above example , $R_{(2,-3),270°}(P) = P'$.

1. I$R_{(2,-3),270°}(P) = P'$,n R means ________________, the subscript (coordinate) means _________________, the subscript (degree measure) means _________________, P means _________________, and P' means _________________.

In regular words, $R_{(2,-3),270°}(P) = P'$ **means:**

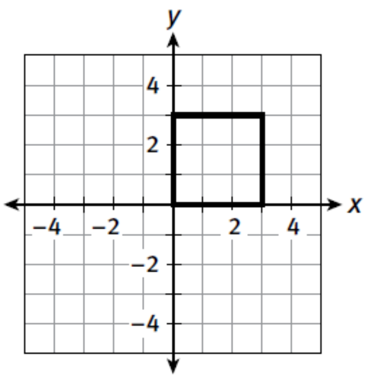
Unit 1 Concept 2 (Lessons C – F): Translations, Reflections, and Rotations

The Marching Cougars

PRACTICE Write your answers on notebook paper. Show your work.

Lesson C - Rigid or Not?

For Items 1–4, a square is drawn in the coordinate plane, with vertices as shown in the diagram. Then the square undergoes a rigid motion.



1. The function that describes the rigid motion could be $(x, y) \rightarrow$

A. $(2x\ ,\ y)$.	B. $(x\ -\ 3\ ,\ y)$.
C. $(\ y\ ,\ -2x)$.	D. $(x\ ,\ 3)$.

2. If the point (0, 3) is mapped to (0, 0), what could be the image of (3, 0)?

A. $(0\ ,\ 0)$.	B. $(0\ ,3)$.
C. $(3\ ,\ -3)$.	D. $(-3\ ,\ 3)$.

3. The length of the diagonal of the square is $3\sqrt{2}$. Can you determine the length of the diagonal of the image of the square? Explain.

4. Draw the transformations of the square described by these functions. Classify each as rigid or non-rigid.

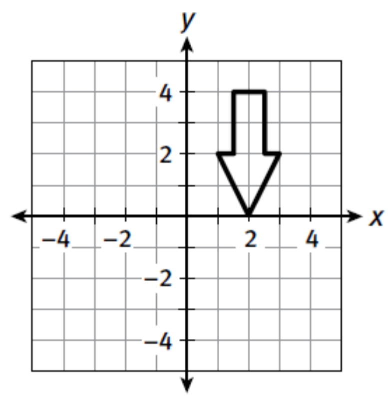
a. $(x\ ,\ y\ + 2)$

b. $(x\ + 3\ ,\ y\ - 3)$

c. $\left(2x\ ,\ \frac{1}{2}y\right)$

Lesson D - Translations

For Items 5 and 6, a down arrow is drawn with its tip at $(2, 0)$. Then it undergoes a translation described by the directed line segment $(-3, 1)$.



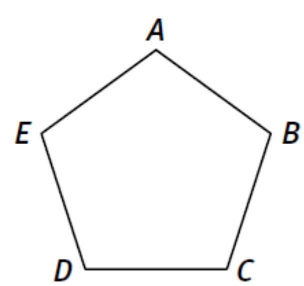
5. The image of the tip of the arrow is at point

A. $(2\ ,\ 1)$.	B. $(-\ 1\ ,1)$.
C. $(\ 2\ ,\ -3)$.	D. $(-3\ ,\ 2)$.

6. The image of the arrow points in which direction?

A. Down	B. Left
C. Right	D. Diagonal

7. The diagram shows regular pentagon ABCDE.



a. Draw the translation $T_{\overline{EB}}$. Label the images of each vertex.

b. Is it possible for a translation to map more than one point of the pentagon onto another point of the pentagon? Explain.

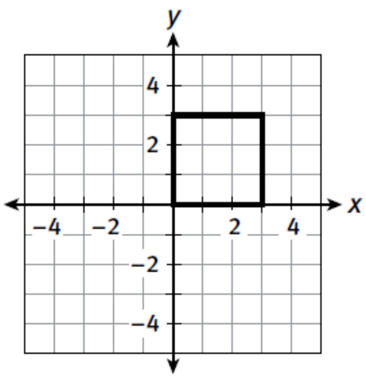
Unit 1 Concept 2 (Lessons C – F): Translations, Reflections, and Rotations

The Marching Cougars

Continued

Lesson E - Reflections

For Items 8–10, a square is drawn in the coordinate plane, with vertices as shown in the diagram. Then the square is reflected across the x-axis.



8. The function that describes the reflection Is $(x, y) \rightarrow$

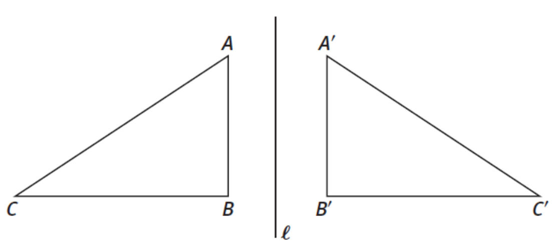
A. $(x\ ,\ y-3)$.	B. $(x\ ,\ y-6)$.
C. $(-x\ ,\ y)$.	D. $(x\ ,-y)$.

9. An up-pointing arrow is drawn inside the square. In the image, the arrow points in which direction?

A. Up	B. Down
C. Left	D. Right

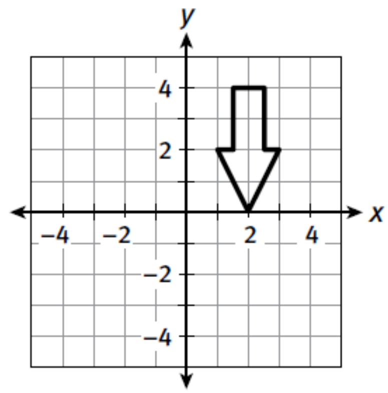
10. Describe a reflection of the square that would map it onto itself.

11. Right triangle ABC is reflected across line ℓ, which is parallel to but distinct from $\overline{AB}$, one of the legs of the triangle. Prove that points B, C, B' and C are collinear.



Lesson F - Rotations

For Items 12 and 13, a down arrow is drawn with its tip at (2, 0). Then it undergoes a clockwise rotation of 90°.



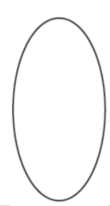
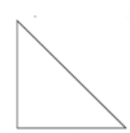
12. If the tip of the arrow moves to (0, 2), what is the center of rotation?

A. $(2\ 2)$.	B. $(2\ ,4)$.
C. $(\ 0\ ,\ 2)$.	D. $(0\ ,\ 0)$.

13. How many possible centers of rotation will produce an image of a left-pointing arrow?

A. zero	B. one
C. two	D. infinite

14. Describe the rotational and reflectional symmetry of these shapes.

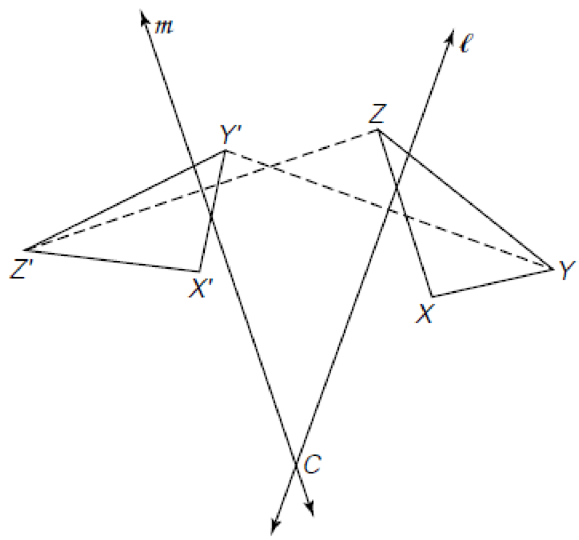
A. Ellipse 	B. right isosceles triangle 
B. letter S 	C. plus sign 

15. Figure A is a square that is centered at the origin O. A student claims that the reflection $r_{(y\,=\,0)}$ and the rotation $R_{O,180^{\circ}}$ transform the square in the same way. Critique this claim.

Finding the Center of Rotation

Suppose you are told that $\Delta X'Y'Z'$ is the rotation image of ΔXYZ, but you are not told where the center of rotation is nor the measure of the angle of rotation. Can you find them? Yes, you can. Connect two pairs of corresponding vertices with segments. In the figure, the segments $\overline{YY'}$ and $\overline{ZZ'}$ are used. Draw the perpendicular bisectors, ℓ and m, of these segments.

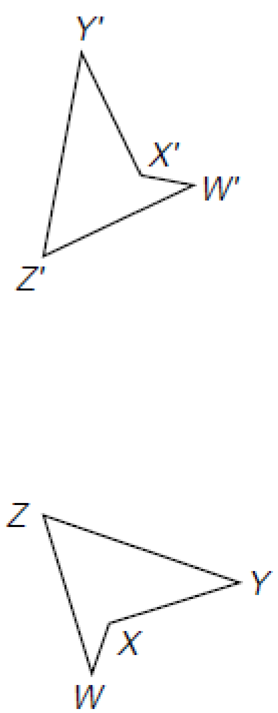
The point C where ℓ and m intersect is the center of rotation.



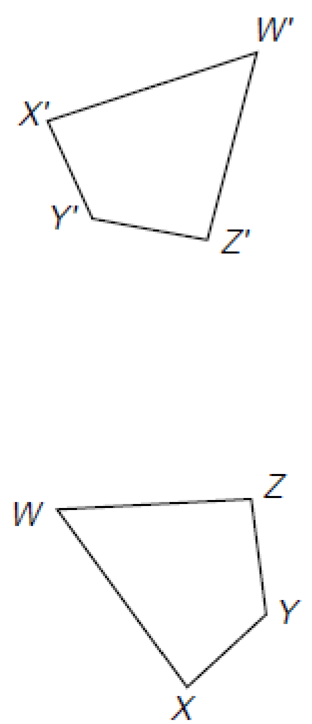
1. How can you find the measure of the angle of rotation in the figure above?

Locate the center of rotation for the rotation that maps $WXYZ$ onto $W'X'Y'Z'$. Then find the measure of the angle of rotation.

2.



3.

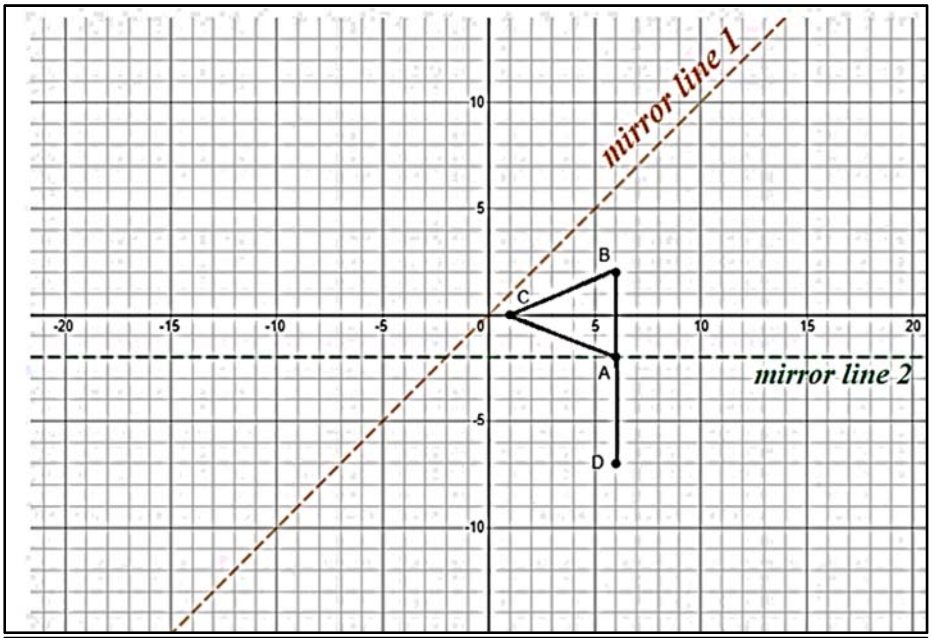


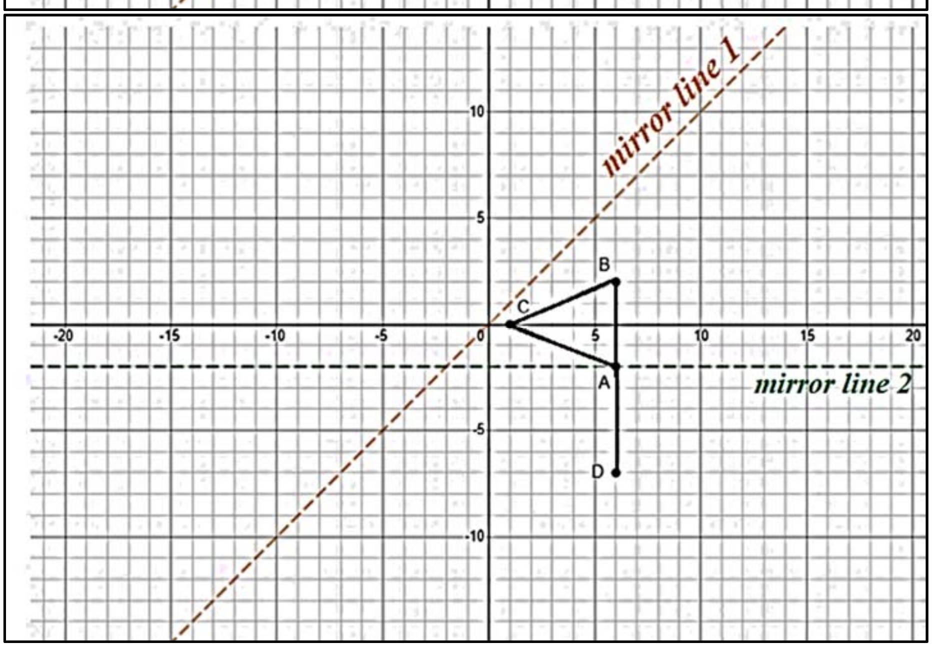
G.U1.C2.**F.04.Task2.Rotations**

ON THE WALL…DOES ORDER MATTER?

This is a follow-up task to "Mirror, Mirror" from last lesson.

1. Now try double reflecting with lines that meet at <u>45^o</u> , below – reflect across line 1 then 2 on the first one, and in the opposite order on the second one.





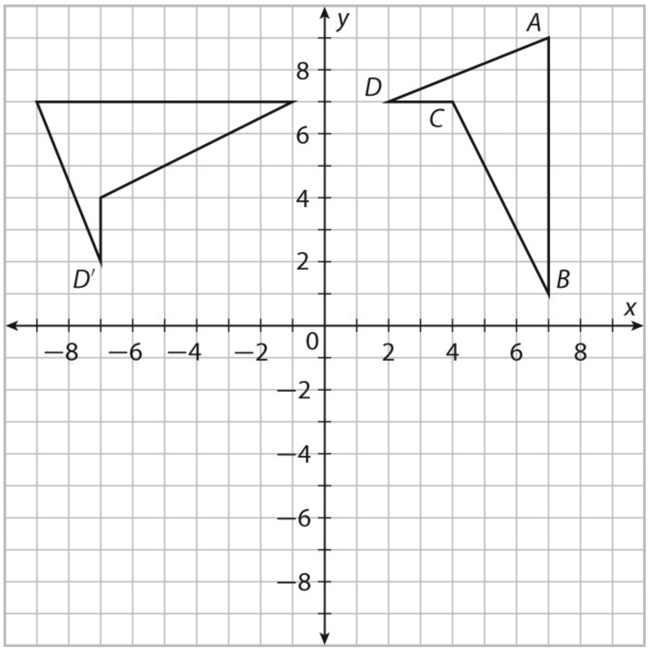
2. For each, can you describe the single transformation that takes the first flag to the last flag? Explain.

3. Does it matter over which line you reflect first? Explain why or why not.

4. What is the equation and "nickname" of mirror line 1?

Rotations Homework

Follow the directions for Problems 1–5 to analyze rotations.



1. Draw a line from the origin, *O*, to point *D* and from *O* to *D′*. Measure the angle formed by $\overline{OD}$ and $\overline{OD'}$. How many degrees was figure

   ABCD rotated? ___ degrees

2. Find the coordinates of points on *ABCD* and corresponding points on its image. Label *A′*, *B′*, and *C′*.

   A(___, ___) *A′*(___, ___)

   B(___, ___) *B′*(___, ___)

   C(___, ___) *C′*(___, ___)

   D(___, ___) *D′*(___, ___)

   P(*x*, *y*) *P′*(_______ , _______)

3. If you rotate *A′B′C′D′* counterclockwise 90°, what is the sign of the

   x-coordinates of the new image? ___ Of the *y*-coordinates? ___

   In what quadrant is the new image? _________

4. Draw and label *A″B″C″D″*, the image of *A′B′C′D′* after being rotated 90° counterclockwise.

5. If (*x*, *y*) is a point on *ABCD,* what is its image on *A″B″C″D″*? (_______, _______)

Use principles of rotations to answer Problems 6–8.

6. What clockwise rotation produces the same image as a counterclockwise rotation of 220°? _______° clockwise

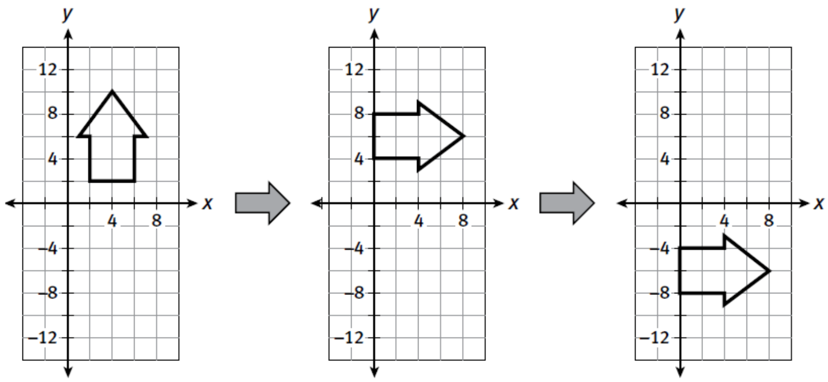
7. Tony Hawk was the first skateboarder to do a “900,” a rotation of 900°. How many times did he rotate on the skateboard? _______ times

8. Each arm of this pinwheel is the image of another arm rotated around the center. What is the angle of rotation between one arm and the next? _______°

Transformations – Composition of Transformations

Consider the series of transformations shown in the figures above. The first is $R_{(4,6),\,-90°}$, or a clockwise rotation of 90° about the point (4, 6). The second transformation is $r_{y=0}$, or a reflection across the *x*-axis. Together they are a ***composition of transformations,*** which is two or more transformations performed in sequence.

The notation for a composition of transformations is similar to the way you express the composition of functions. The composition pictured above is described as $r_{y=0}\,(R_{(4,6),\,-90°})$. If a reflection across the *x*-axis were added as the third transformation of the sequence, then the notation would be $r_{x=0}(r_{y=0}(R_{(4,6),\,-90°}))$. Notice that the transformation that occurs first in the series is in the interior of the notation, and subsequent transformations are written outside of it.



Notes

Learning Targets:

- Find the image of a figure under a composition of rigid motions.
- Find the pre-image of a figure under a composition of rigid motions.

SELF CHECK

1. **Attend to precision.** Write the notations for these compositions of transformations. Use the points *A*(0, 0), *B*(1, 1), and *C*(0, −1).

a. a clockwise rotation of 60° about the origin, followed by a translation by directed line segment *AB.*

b. a reflection about the line $x = 1$, followed by a reflection about the line $x = 2$

c. three translations, each of directed line segment $\overline{AC}$.

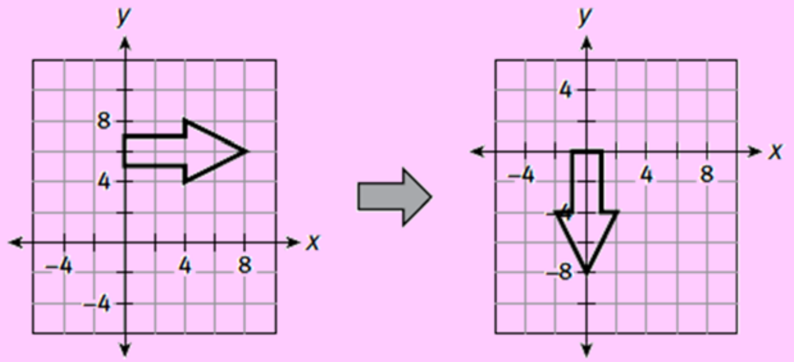
Questions To Ponder

A **composition of transformations** is a series of two or more transformations performed on a figure one after another. Would order matter if you perform several transformations? Give an example in your explanation.

Notes

SELF CHECK

Use the figure for Items 2–4.



2. Identify a composition of transformations that could map the arrow on the left to the image of the arrow on the right.

3. Consider the composition you identified in Item 2 but with the transformations in reverse order. Does it still map the arrow to the same image?

4. Identify a composition that undoes the mapping, meaning it maps the image of the arrow on the right to the pre-image on the left.

You can also find compositions of transformations away from the coordinate plane.

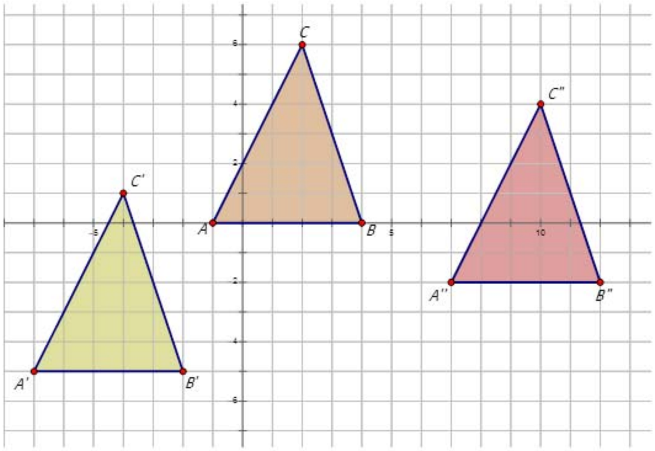
5. Points A, B, C, and D are points on the right-pointing arrow shown here. Predict the direction of the arrow after it is mapped by these compositions.

a. $T_{\overline{DB}}\,(T_{\overline{AC}})$

b. $r_{\overline{D'B'}}\,(R_{D,90^\circ})$

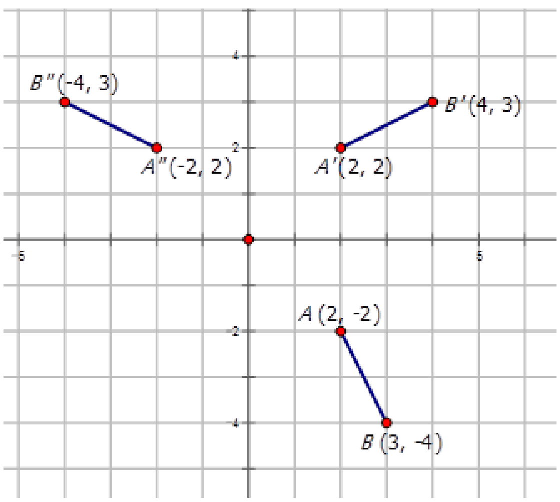
c. $R_{A,180^\circ}(r_{\overline{AC}})$

6. Look at the following diagram. It involves two translations. Identify the two translations of triangle $\boldsymbol{ABC}$.

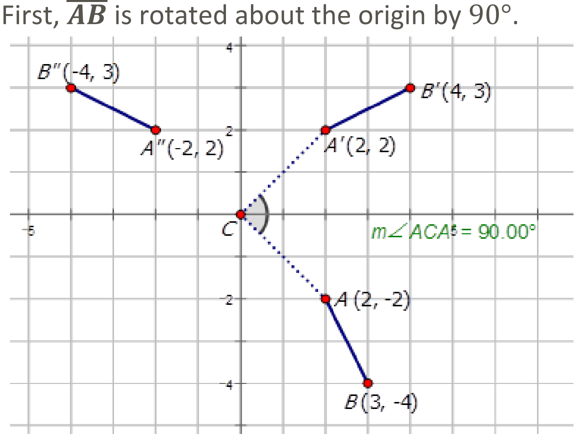


Example! **Let's describe the transformations in the following diagram:**

7. The transformations involve a reflection and a rotation.



First, $\overline{\boldsymbol{AB}}$ is rotated about the origin by 90°.



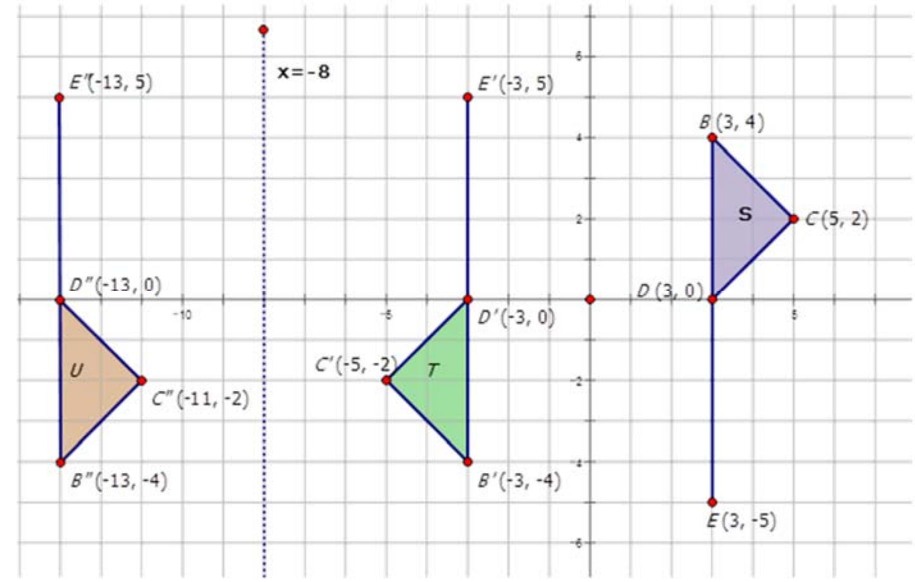
Then, the line $\boldsymbol{A'B'}$ is reflected about the y-axis to produce line $\boldsymbol{A''B''}$.

Notes

Notes

Example! See the image below. How did S transform into U through a series of transformations?

8.



The flag in diagram S is rotated about the origin $180°$ to produce flag T. You know this because if you look at one point you notice that both $\boldsymbol{x}-$ and $\boldsymbol{y}-$coordinate points is multiplied by -1 which is consistent with a $180°$ rotation about the origin. Flag T is then reflected about the line $\boldsymbol{x}=-8$ to produce flag U.

SELF CHECK Now let's draw the diagram described by the following transformations:

9. Triangle ABC where the vertices of ΔABC are $A(-1,-3)$, $B(-4,-1)$, and $C(-6,-4)$ undergoes a composition of transformations described as:

a. $T_{(10,0)}$, then
b. r_{x-axis}

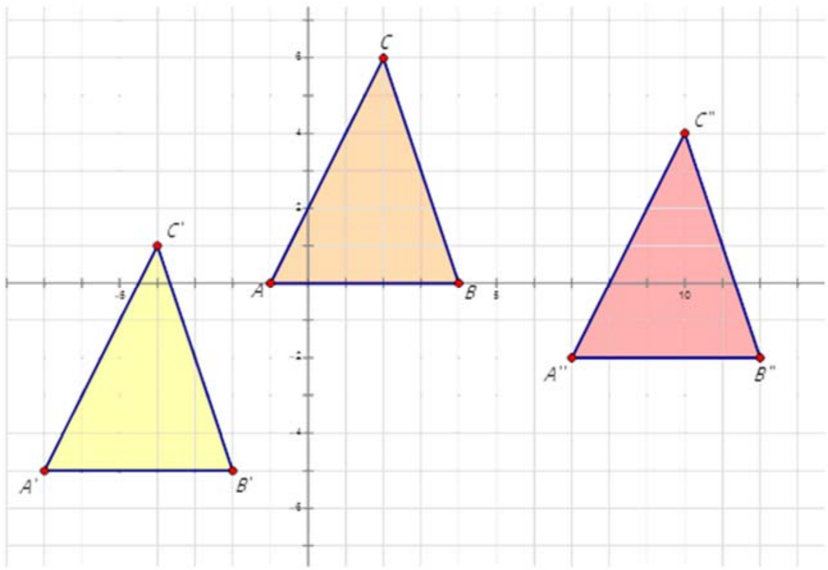
Draw the diagram to represent this composition of transformations. What are the vertices of the triangle after both transformations are applied? Also, how would you write this composition of transformations as a single expression?

Example!

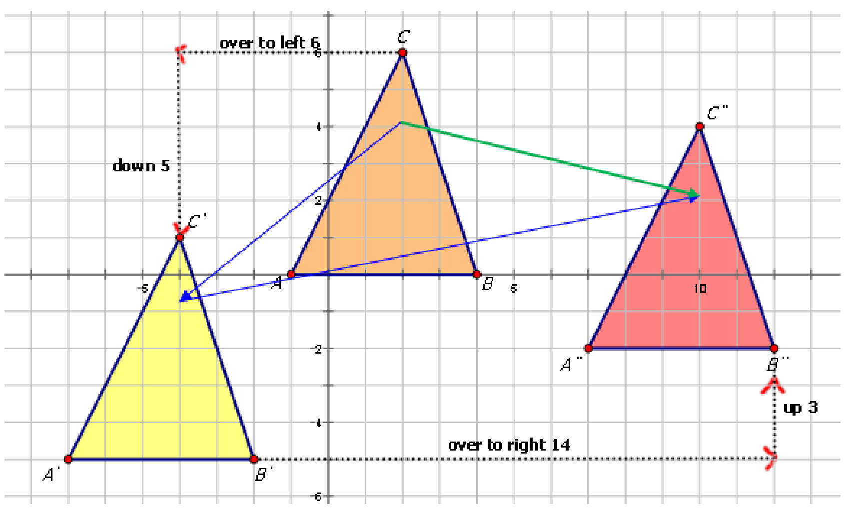
Notes

10. Example 1

Earlier, you were asked to identify the two translations of triangle ABC that are shown below:



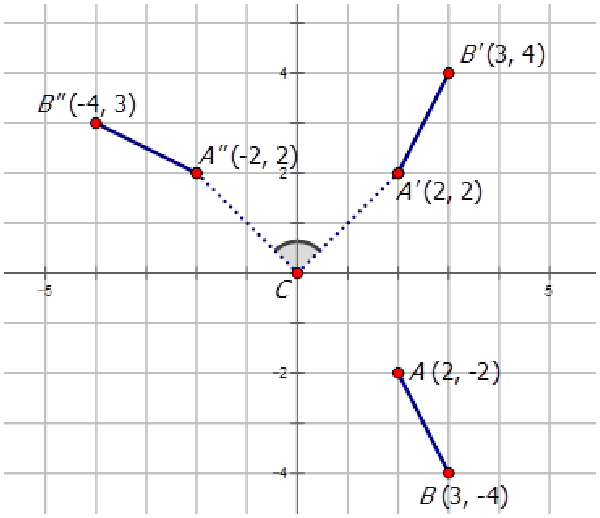
ΔABC moves over 6 to the left and down 5 to produce $\Delta A'B'C'$.
Then $\Delta A'B'C'$ moves over 14 to the right and up 3 to produce $\Delta A''B''C''$.
These translations are represented by the blue arrows in the diagram.



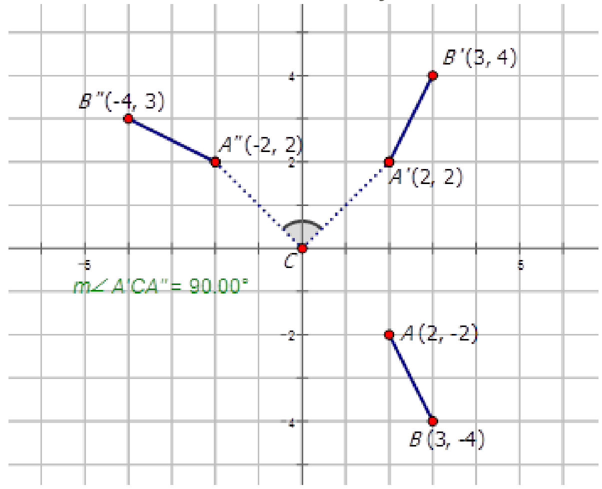
All together, ΔABC moves over 8 to the right and down 2 to produce $\Delta A''B''C''$. The green arrows show the total translations (the combination of both).

11. Example 2

Describe the transformations in the diagram below.
The transformations involve a rotation and a reflection.



The transformations involve a reflection and a rotation.
First $\overline{AB}$ is reflected about the $y-axis$ to produce $\overline{A'B'}$.



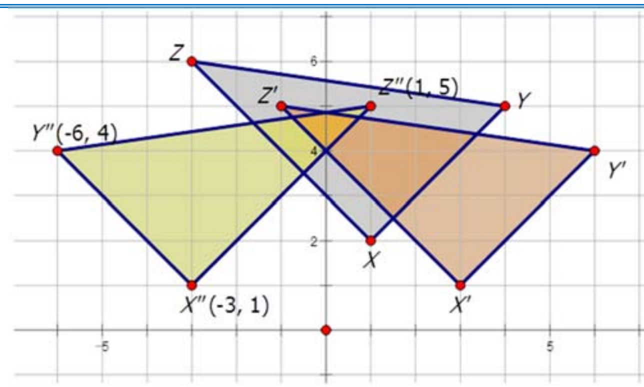
Then $\overline{A'B'}$ is rotated about the origin by 90^0 to produce $\overline{A''B''}$.

12. Example 3 Triangle XYZ has coordinates $X(1,2)$, $Y(-3,6)$ and $Z(4,5)$.

The triangle undergoes a rotation of 2 units to the right and 1 unit down to form triangle $X'Y'Z'$.

Triangle $X'Y'Z'$ is then reflected about the y-axis to form triangle $X''Y''Z''$ **.**

Draw the diagram of this composite transformation and determine the vertices for triangle $''Y''Z''$.



13. Example 4

The coordinates of the vertices of ΔJAK are $J(1,6)$, $B(2,9)$, and $C(7,10)$.

a. Draw and label ΔJAK.
b. ΔJAK is reflected over the line $y = x$. Graph and state the coordinates of $\Delta J'A'K'$.
c. $\Delta J'A'K'$ is then reflected about the x-axis. Graph and state the coordinates of $\Delta J''A''K''$.
d. $\Delta J''A''K''$undergoes a translation of 5 units to the left and 3 units up. Graph and state the coordinates of $\Delta J'''A'''K'''$.

14. A point $\boldsymbol{X}$ has coordinates $(-\mathbf{1},-\mathbf{8})$. $\boldsymbol{X''}=\boldsymbol{T}_{(4,6)}\left(\boldsymbol{r}_{y-axis}(\boldsymbol{X})\right)$. What are the coordinates of $\boldsymbol{X'}$ ***and*** $\boldsymbol{X''}$?

15. A point $\boldsymbol{A}$ has coordinates $(\mathbf{2},-\mathbf{3})$. $\boldsymbol{A''}=\boldsymbol{r}_{x-axis}\left(\boldsymbol{T}_{(-3,5)}(\boldsymbol{A})\right)$. What are the coordinates of $\boldsymbol{A'}$ ***and*** $\boldsymbol{A''}$?

16. A point $\boldsymbol{P}$ has coordinates $(\mathbf{5},-\mathbf{6})$. $\boldsymbol{P''}=\boldsymbol{R}_{(-3,2),-90^\circ}\left(\boldsymbol{r}_{y=x}(\boldsymbol{P})\right)$. What are the coordinates of $\boldsymbol{P'}$ ***and*** $\boldsymbol{P''}$?

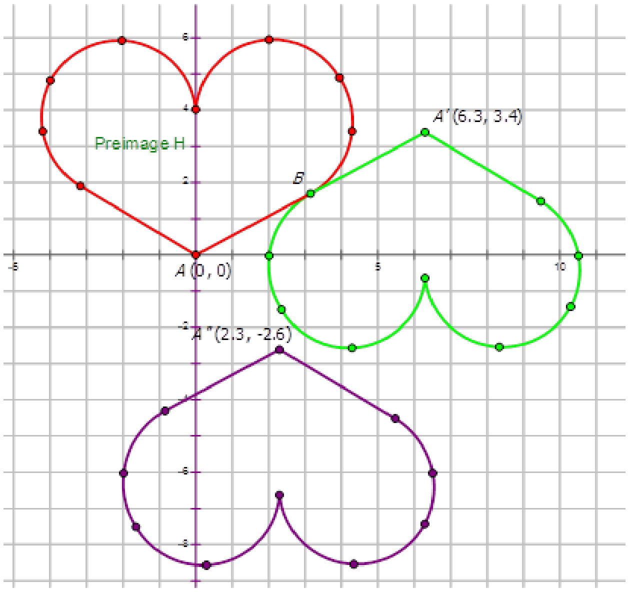
17. $\overline{\boldsymbol{JT}}$ has coordinates $\boldsymbol{J}(-\mathbf{2},-\mathbf{5})$ and $\boldsymbol{T}(\mathbf{2},\mathbf{3})$ $\overline{\boldsymbol{J''T''}}=\boldsymbol{T}_{(6,-3)}\left(\boldsymbol{R}_{(2,-2),-90^\circ}(\overline{\boldsymbol{JT}})\right)$. What are the coordinates of $\overline{\boldsymbol{J'T'}}$ and $\overline{\boldsymbol{J''T''}}$?

18. $\overline{\boldsymbol{SK}}$ has coordinates $\boldsymbol{S}(-\mathbf{1},-\mathbf{8})$ and $\boldsymbol{K}(\mathbf{1},\mathbf{2})$. The segment is translated over 3 to the right and up 3 to form $\overline{\boldsymbol{S'K'}}$. Then $\overline{\boldsymbol{S'K'}}$ is rotated 90° counter-clockwise about the point $(-1,-3)$. Write this composition of transformations as one transformation using proper composite transformation notation (as in 1, 2, and 3, above). What are the coordinates of $\overline{\boldsymbol{S'K'}}$ and $\overline{\boldsymbol{S''K''}}$?

19. A point $\boldsymbol{K}$ has coordinates $(-\mathbf{1},\mathbf{4})$. The point is reflected across the line $\boldsymbol{y}=-\boldsymbol{x}$ to form $\boldsymbol{K'}$. $\boldsymbol{K'}$ is rotated $\mathbf{270}^\circ$ about the point (1, 5) to form $\boldsymbol{K''}$. Write this composition of transformations as one transformation using proper composite transformation notation (as in 1, 2, and 3, above). What are the coordinates of K′ and K′′?

Describe the following composite transformations:

20.



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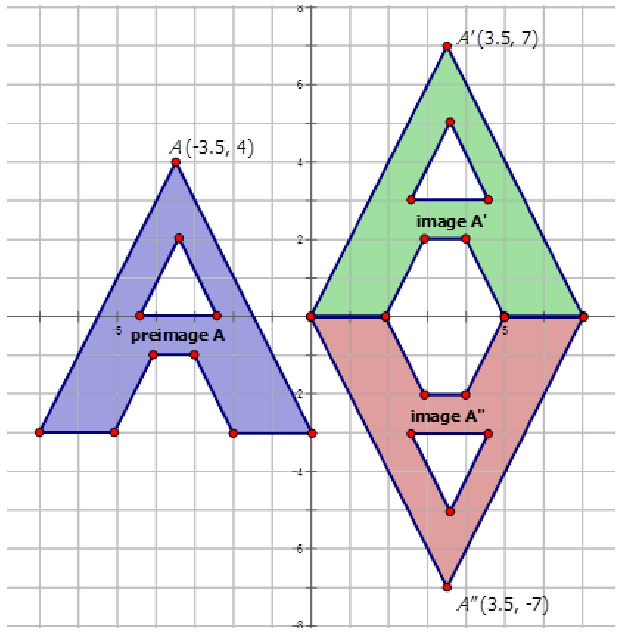

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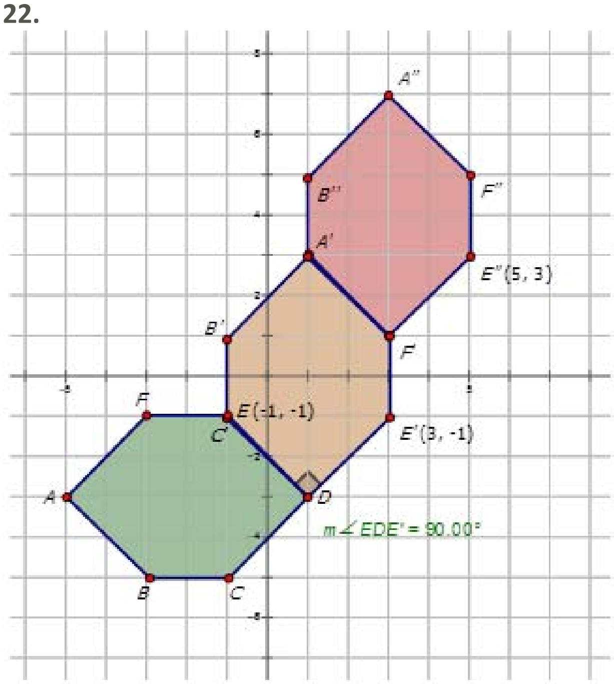

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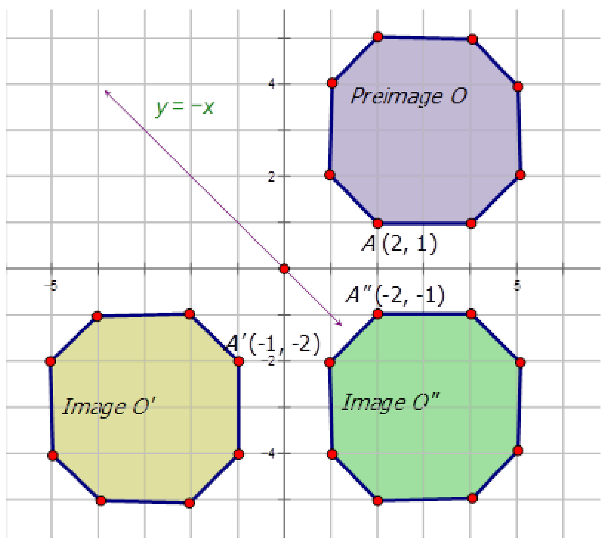

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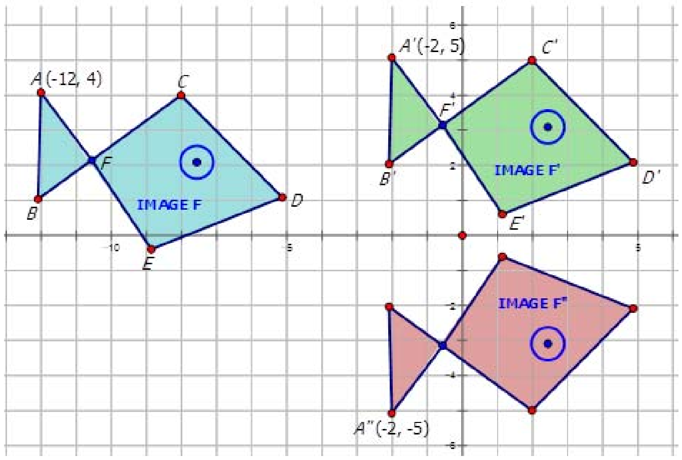


25. Explore what happens when you reflect a shape twice, over a pair of parallel lines. What one transformation could have been performed to achieve the same result?

26. Explore what happens when you reflect a shape twice, over a pair of intersecting lines. What one transformation could have been performed to achieve the same result?

27. Explore what happens when you reflect a shape over the x-axis and then the y-axis. What one transformation could have been performed to achieve the same result?

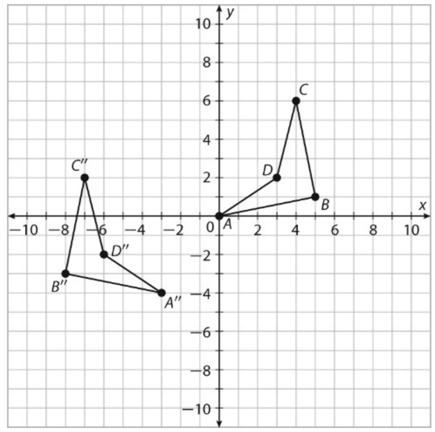
28. A composition of a reflection and a translation is often called a glide reflection. Make up an example of a glide reflection. Why do you think it's called a **glide** reflection?

A figure can be transformed more than once to create an image figure. When *rigid* transformations are used, the figures are the same shape and size. When *nonrigid* transformations are used, the figures could be different shapes or sizes.

Look at this sequence of transformations.

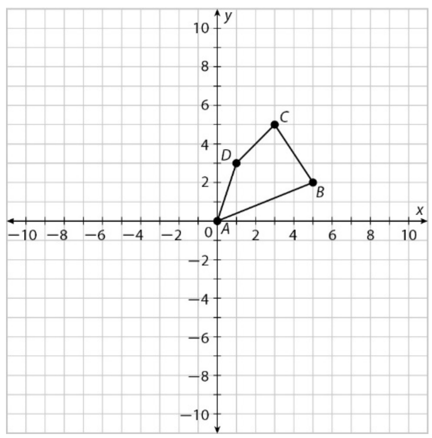
$(x, y) \rightarrow (x + 3, y - 4) \rightarrow (-x, y)$

First, figure *ABCD* was moved 3 units right and 4 units down. Then, the image was reflected over the *y*-axis to form figure *A″B″C″D″*.

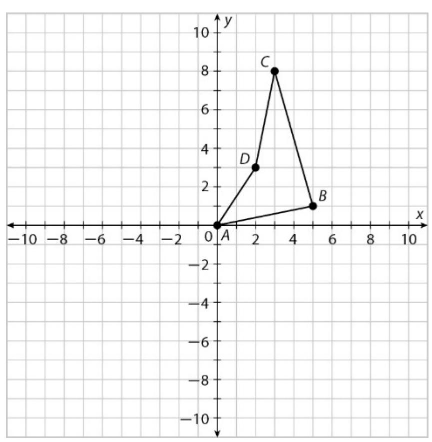


For each figure, draw the image after the given transformations.

1. $(x, y) \rightarrow (x + 2, y) \rightarrow (x, -y)$

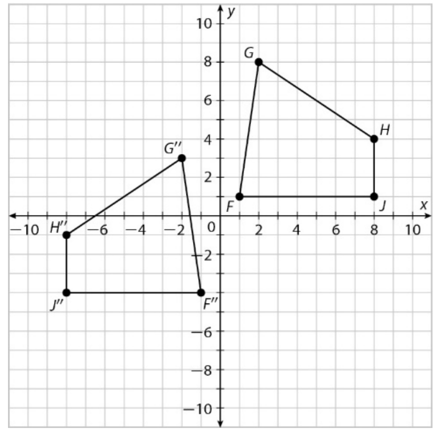


2. $(x, y) \rightarrow (x, y - 8) \rightarrow (2x, y)$



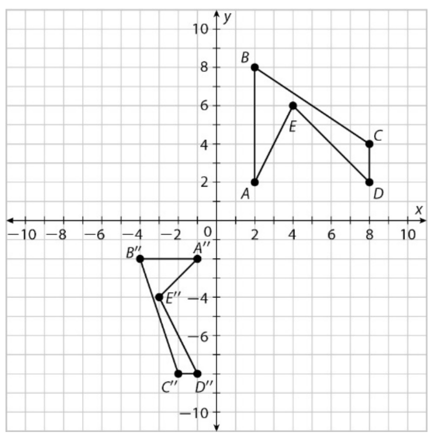
Two figures are congruent if they are exactly the same shape and size. Congruent figures are formed by using rigid transformations (translations, reflections, and rotations).

Figure *FGHJ* was reflected and translated to form figure *F″G″H″J″*, so figure *FGHJ* and figure *F″G″H″J″* are congruent.

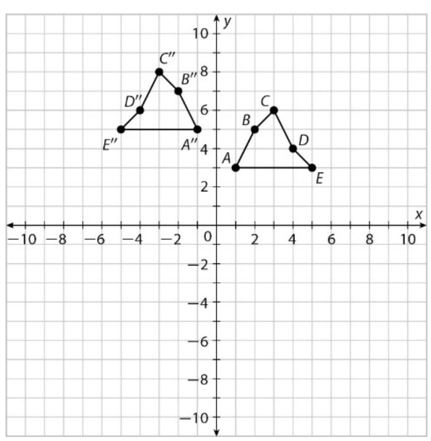


Determine whether each pair of figures is congruent. If so, name the transformations used to form the image figure.

3.



4.

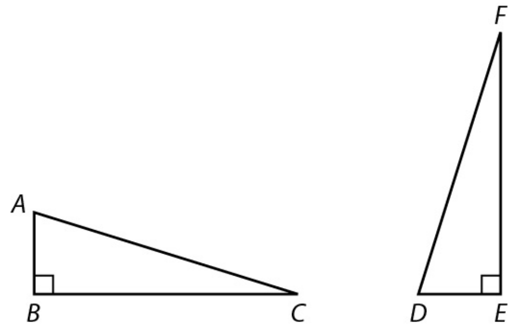


When two figures are congruent, the corresponding sides and corresponding angles are congruent.

Triangles *ABC* and *DEF* are congruent.

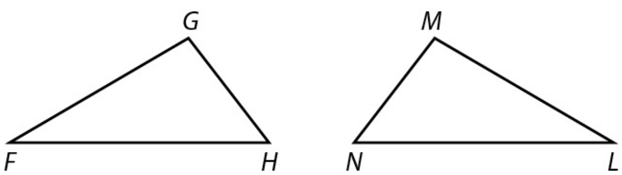
$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{CA} \cong \overline{FD}$

$\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$



Each of these pairs of figures is congruent. Complete the congruence statements.

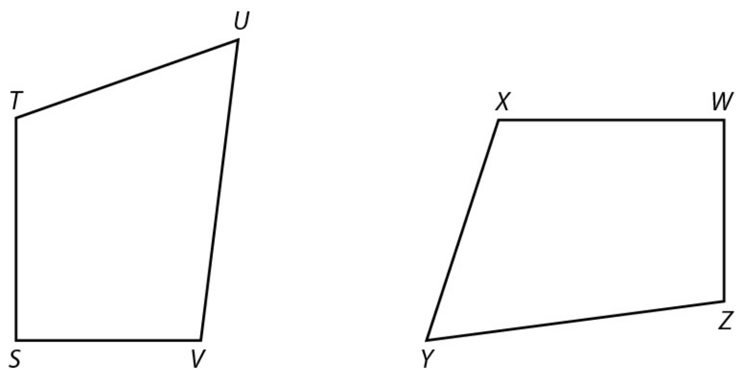
5. Triangle *FGH* is congruent to triangle *LMN.*



$\overline{FG} \cong$ ___, $\overline{GH} \cong$ ___, and ___ $\cong \overline{NL}$

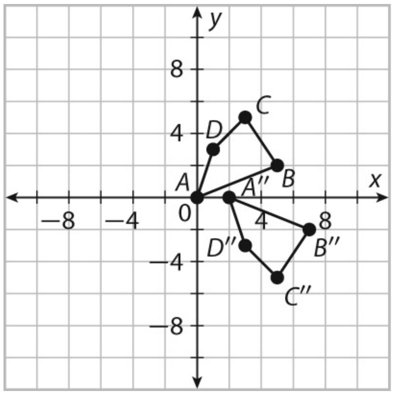
$\angle F \cong$ ___, ___ $\cong \angle M$, and $\angle H \cong$ ___

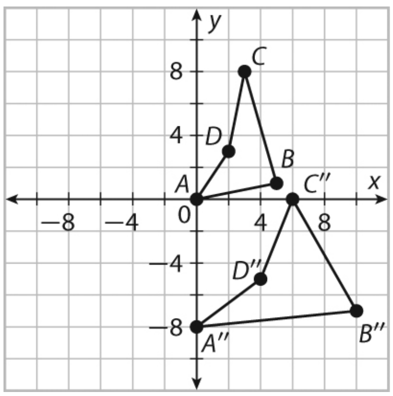
6. Figure *STUV* is congruent to figure *WXYZ.*



$\overline{ST} \cong$ ___, $\overline{TU} \cong$ ___, ___ $\cong \overline{YZ}$, and ___ $\cong \overline{ZW}$

$\angle S \cong$ ___, $\angle T \cong$ ___, ___ $\cong \angle Y$, and ___ $\cong \angle Z$

1. 

2. 

3. 1. not congruent

4. 2. congruent (reflection across *y*-axis and translation 2 units up)

5. 1. $\overline{FG} \cong \overline{LM}$, $\overline{GH} \cong \overline{MN}$, and $\overline{HF} \cong \overline{NL}$
$\angle F \cong \angle L$, $\angle G \cong \angle M$, and $\angle H \cong \angle N$

6. 2. $\overline{ST} \cong \overline{WX}$, $\overline{TU} \cong \overline{XY}$, $\overline{UV} \cong \overline{YZ}$,
and $\overline{VS} \cong \overline{ZW}$; $\angle S \cong \angle W$,
$\angle T \cong \angle X$, $\angle U \cong \angle Y$, and $\angle V \cong \angle Z$

7.

Transformations Task

desmos

Hey, students!

Go to student.desmos.com
and type in:

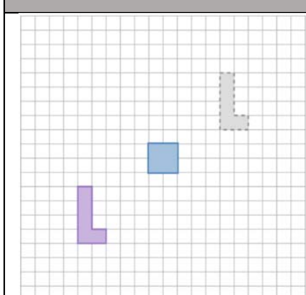
4UG U7P

OR…you can just use the QR code below.

Use the following pages as recording sheets that you can hand in.

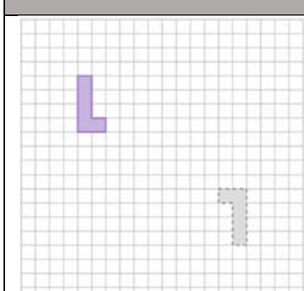
In this activity, you'll use your understanding of translations, reflections, and rotations to complete a round of transformation golf. For each challenge, your task is the same: Use one or more of those transformations to transform the pre-image onto the image. Good luck!

Page 2 – Challenge #1



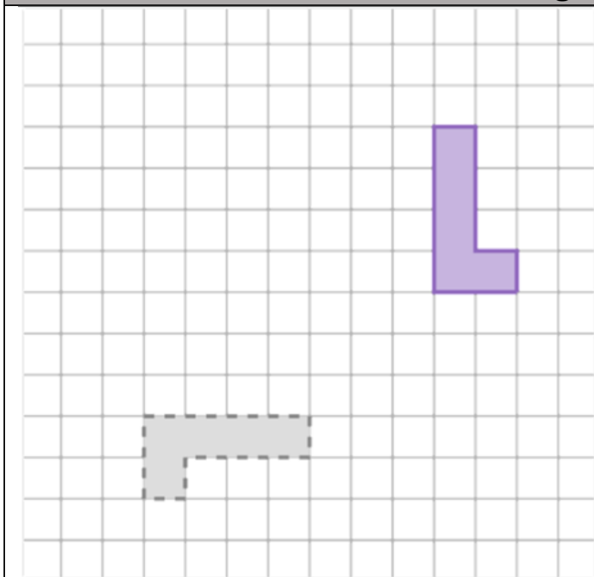
List the transformation(s) you used to map the purple L onto the pink one.

Page 3 – Challenge #2



List the transformation(s) you used to map the purple L onto the pink one.

Page 4 – Settle the Dispute

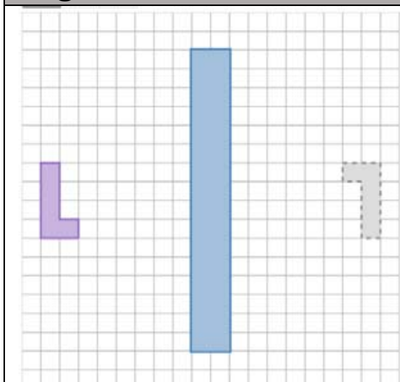


Who is correct – Anya or Braden?

Show what you tried to the left.

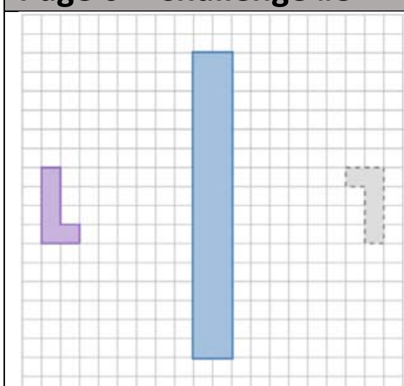
In Desmos, what explanations made the most sense to you?

Page 5 – Describe Your Strategy



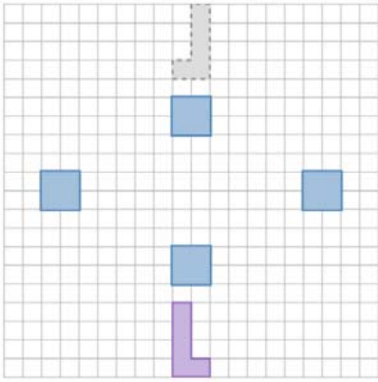
Describe the sequence of transformations you'll use to transform the pre-image onto the image.

Page 6 – Challenge #3



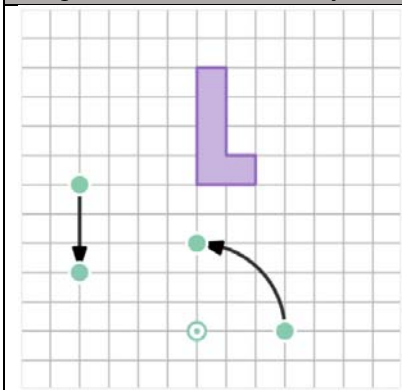
List the transformation(s) you used:

Page 7 – Challenge #4



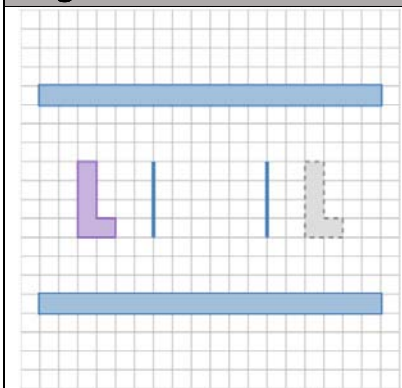
List the transformation(s) you used:

Page 8 – Settle a Dispute (part 2)



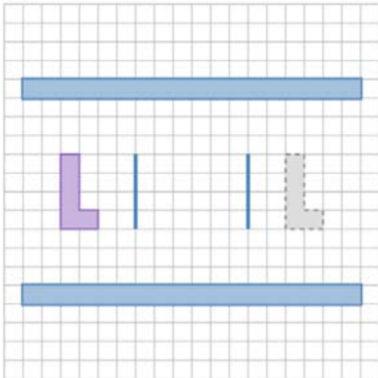
Braden thinks that the order they used to perform their transformations doesn't matter. Is he right? Explain/show your thinking.

Page 9 – Describe Your Strategy



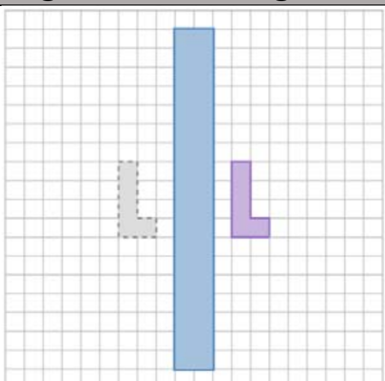
List the transformation(s) you used:

Page 10 – Challenge #5



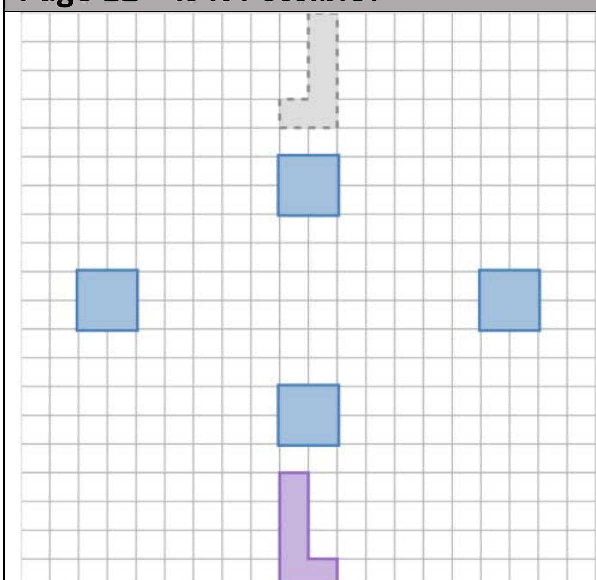
List the transformation(s) you used:

Page 11 – Challenge #6



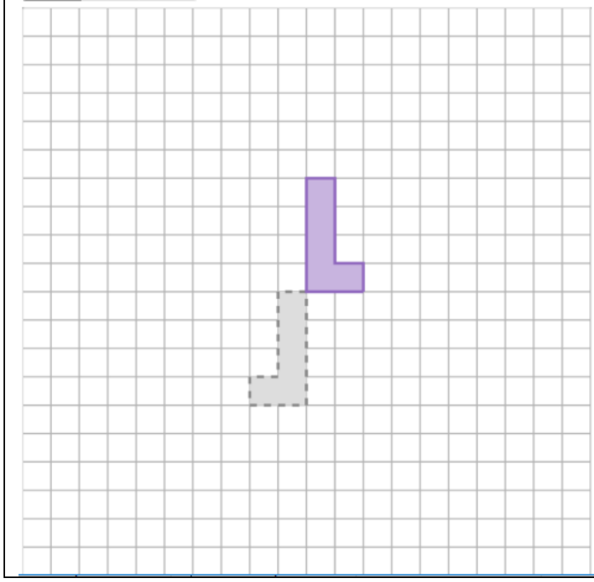
List the transformation(s) you used:

Page 12 – Is It Possible?



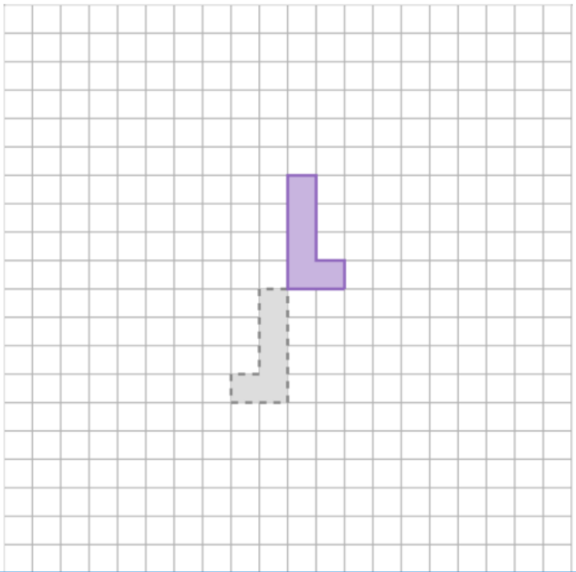
Describe the transformation(s) you used and show to the left what you would do!

Page 13 – Describe Your Strategy



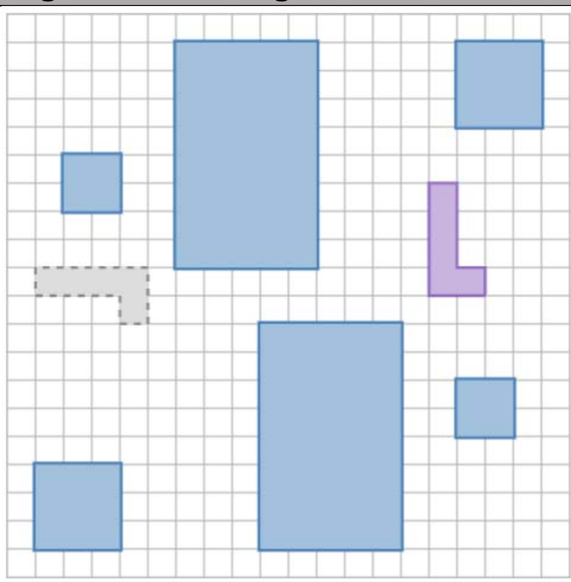
Describe the transformation(s) you used in words.

Page 14 – Challenge #2



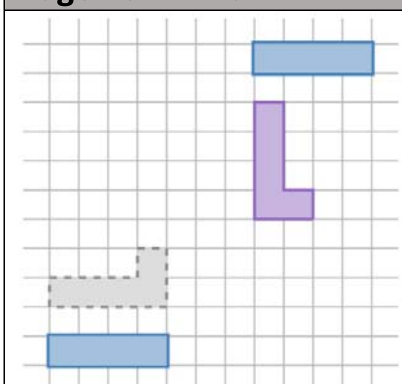
List the transformation(s) you used:

Page 15 – Challenge #8



List/show the transformation(s) you used:

Page 16 – What will NEVER change?



In a sequence of translations, reflections, and rotations, what will NEVER change from the pre-image to the image?

Select all that apply.)

— The lengths of the line segments.
— The position of the object.
— The measures of the angles.
— The orientation of the object.

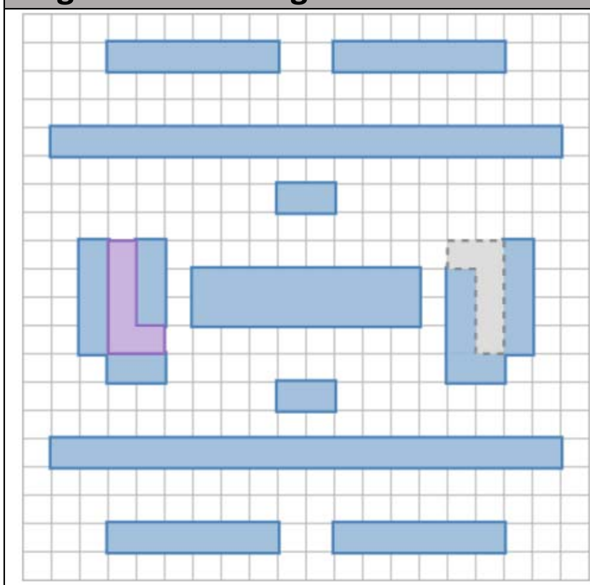
Page 17 – True or False

Which of these statements is true?

(Select all that apply.)

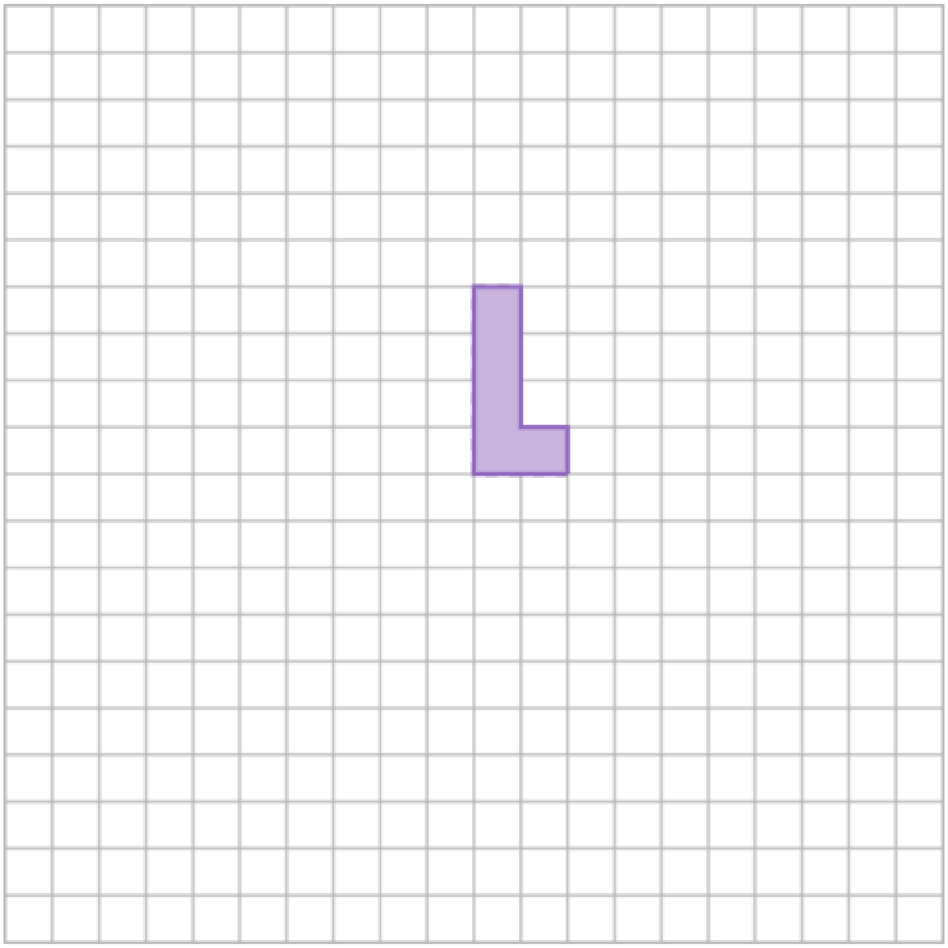
— Any translation can be replaced by two reflections.
— Any translation can be replaced by two rotations.
— Any rotation can be replaced by a reflection.
— Any reflection can be replaced by a rotation followed by a translation.

Page 18 – Challenge #9



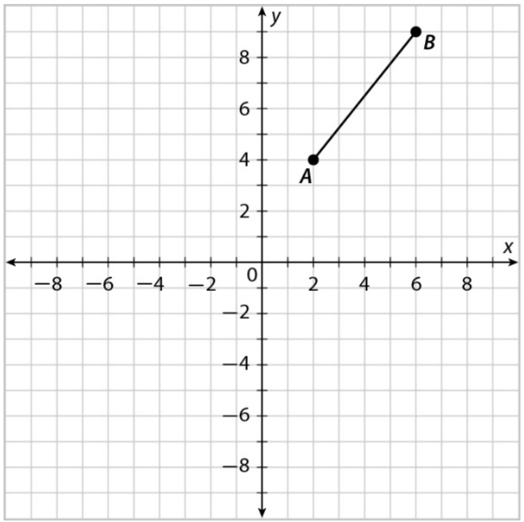
List the transformation(s) you used:

Page 19– Explore



List the transformation(s) you used:

Use the figure to answer questions 1–3.



1. Line segment *AB* will be reflected across the *x*-axis. The image that is formed will then be reflected across the *y*-axis to form line segment *A″B″*. What are the coordinates of points *A″* and *B″*?

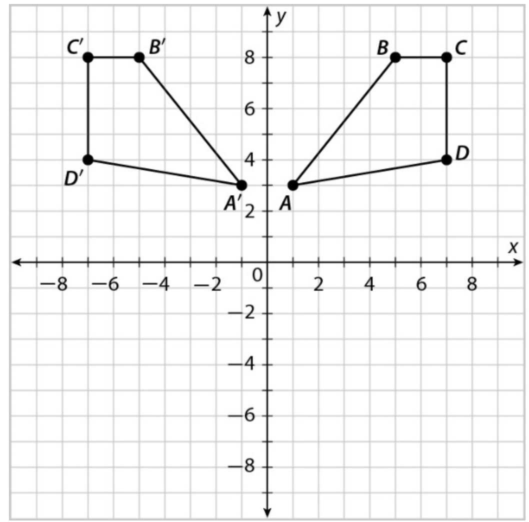
____

2. Name another set of reflections that will produce the same image line segment *A″B″* that was found in question 1.

____

3. Which other transformation would produce the same image line segment *A″B″* that was found in question 1?
   - A rotation 180° about the origin
   - B rotation 270° clockwise about the origin
   - C translation 2 units left and 4 units down
   - D translation 4 units left and 8 units down

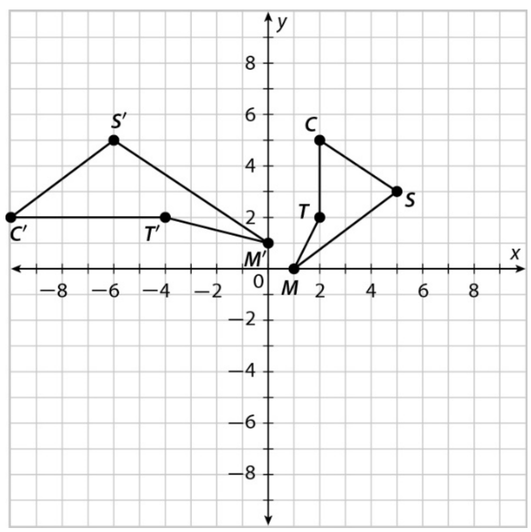
4. Look at the graph below.



Are quadrilaterals *ABCD* and *A′B′C′D′* congruent? Explain why or why not.

____

5. Look at the graph below.



Are quadrilaterals *MTCS* and *M′T′C′S′* congruent? Explain why or why not.

____

6. Triangle *PQR* has vertices *P*(0, 0), *Q*(3, 4), and *R*(3, 0). If triangle *PQR* is rotated 180° about the origin, what is the length of side *P′R′*?

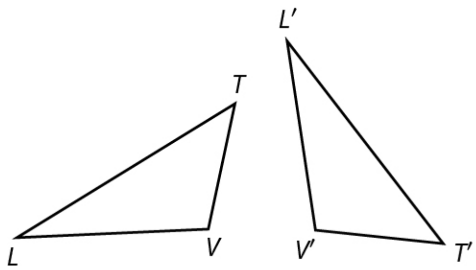
   A 3 units

   B 4 units

   C 5 units

   D 6 units

7. Triangles *LVT* and *L′V′T′* are congruent.



   Which angle must be congruent to $\angle T$?

   A $\angle L$

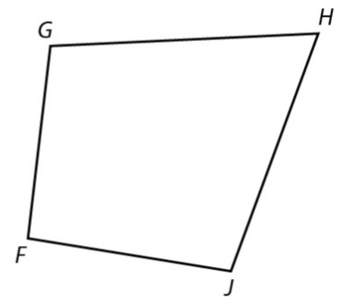
   B $\angle L'$

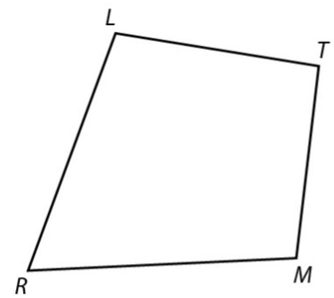
   C $\angle V'$

   D $\angle T'$

Use the figures below to answer questions 8 and 9.

Figures *FGHJ* and *TMRL* are congruent.





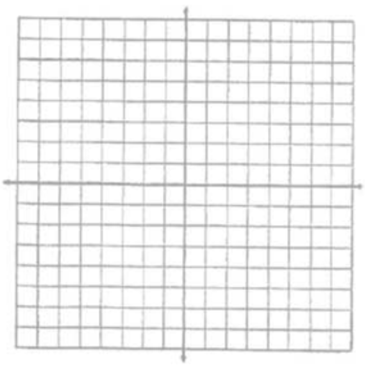
8. Write congruence statements listing all of the congruent sides.

   ____

9. Write congruence statements listing all of the congruent angles.

Directions: Graph and label each figure and its image under the given transformations. Give the new coordinates.

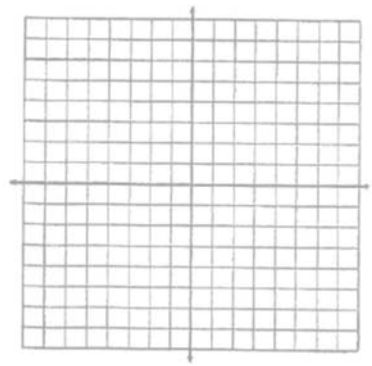
10. Triangle XYZ with vertices $X(-3,7)$, $Y(-2,1)$, and $Z(-5,2)$:
    (a) Reflection: across the x-axis
    (b) Translation: $(x, y) \rightarrow (x+9, y+2)$



X' (______ , ______)
Y' (______ , ______)
Z' (______ , ______)

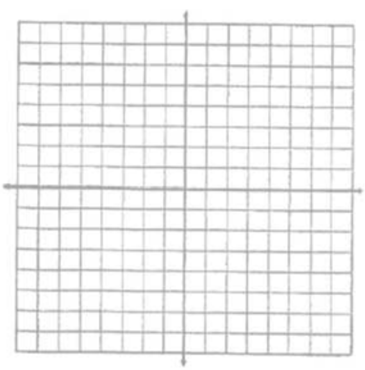
X'' (______ , ______)
Y'' (______ , ______)
Z'' (______ , ______)

11. Parallelogram $QRST$ with vertices $Q(2,-1)$, $R(7,-1)$, $S(7,-4)$, and $T(2,-4)$:
    (a) Reflection: across the line of identity
    (b) Translation: $(x, y) \rightarrow (x-3, y-7)$



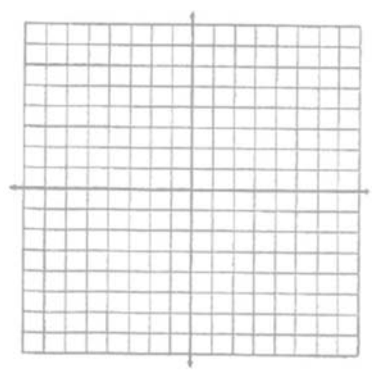
Q' (______ , ______)
R' (______ , ______)
S' (______ , ______)
T' (______ , ______)
R'' (______ , ______)
S'' (______ , ______)
T'' (______ , ______)

12. Rectangle $LMNP$ with vertices $L(-2,7)$, $M(2,3)$, $N(0,1)$, and $P(-4,5)$:
    (a) Translation: $(x, y) \rightarrow (x+6, y-8)$
    (b) Reflection: across the y-axis



L' (______ , ______)
M' (______ , ______)
N' (______ , ______)
P' (______ , ______)
L'' (______ , ______)
M'' (______ , ______)
N'' (______ , ______)
P'' (______ , ______)

13. Triangle GHI with vertices $G(-1,6)$, $H(-1,3)$, and $I(-6,6)$:
    (a) Translation: $(x, y) \rightarrow (x+7, y-5)$
    (b) Reflection: across the line $y=-3$



G' (______ , ______)
H' (______ , ______)
I' (______ , ______)
G'' (______ , ______)
H'' (______ , ______)
I'' (______ , ______)

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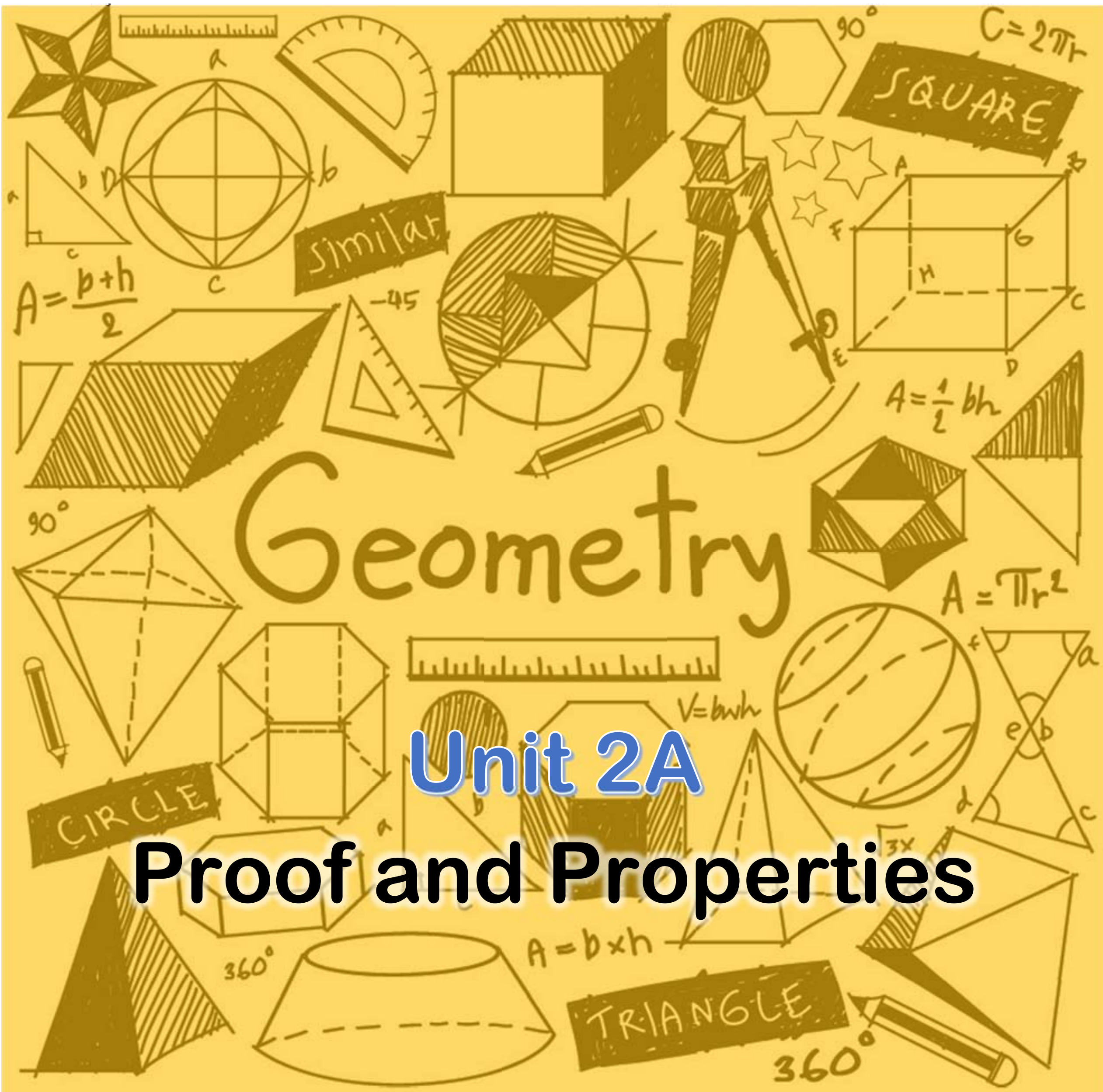


Unit 2A

Proof and Properties

HOUSTON COUNTY BOARD OF EDUCATION

HIGH-ACHIEVING TEACHERS

MATHEMATICS DEPARTMENT

Science Technology Engineering Mathematics

	Diagram/Visual		
Axiom			
Corollary			
Equivalent Statement			
Point			
Postulate			
Proof			
Propositions			
Theorem			

Properties of Equality

GUIDED NOTES

PROPERTIES OF EQUALITY ("∈ ℝ" is the mathematical expression which means "is an element of the set of all Real Numbers.")	
Addition Property of Equality	→ IF a=b THEN a+c=b+c
Additive Identity Property	→ IF a ∈ ℝ, , THEN a+0=a
Additive Inverse Property	→ IF a ∈ ℝ, a+(-a)=0
Division Property of Equality	→ IF a, b, c ∈ ℝ, c≠0, and a=b, THEN $\frac{a}{c}=\frac{b}{c}$
Multiplication Property of Equality	→ IF a=b THEN ac=bc
Subtraction Property of Equality	→ IF a=b THEN a-c=b-c
Transitive Property of Equality	→ IF a=b and b=c THEN a=c
Reflexive Property of Equality	→ IF a ∈ ℝ, , THEN a = a
Symmetric Property of Equality	→ IF a,b ∈ ℝ and a = b, THEN b = a
Transitive Property of Equality	→ IF a, b, c ∈ ℝ and a = b and b = c, THEN a = c
PROPERTIES OF REAL NUMBER OPERATIONS (below, a,b,c ∈ ℝ)	
Associative Property of Addition	→ (a+b)+c = a+(b+c)
Associative Property of Multiplication	→ (ab)c = a(bc)
Commutative Property of Addition	→ a+b = b+a
Commutative Property of Multiplication	→ ab = ba
Distributive Property	→ a(b + c) = ab + ac AND a(b - c) = ab - ac
Multiplicative Identity Property	→ 1*a = a
Multiplicative Inverse Property	→ a*$\frac{1}{a}$ = 1 (a≠0)
Substitution Property	→ IF a = b THEN either may be substituted for the other
Zero Product Property	→ IF a*b = 0 THEN a = 0, or b = 0 or both a,b = 0

Use the properties to solve the algebraic proof.

Given: $\frac{a}{-6}+2=5$	*Prove: a = -18*

SELF CHECK

Use the properties to solve the algebraic proof.

Given: -9(2x – 3) = 63	Prove: x = -2

Questions To Ponder

Why do you think we go over this again even though you should already know it?

1.Given: $2x + 30 = -4(5x - 2)$

Prove: $x = -1$

Statements	Reasons

2.

Given: $18x - 2(3x + 1) = 5x - 16$

Prove: $x = -2$

Statements	Reasons

3.

Given: $7 = \frac{5y-1}{2}$

Prove: y = 3

Statements	Reasons

4.

Given: $\frac{m}{-3} + 10 = -1$

Prove: m = 33

Statements	Reasons

Introductory Activity

Introductory Activity (Spotlight Task)

Part 1

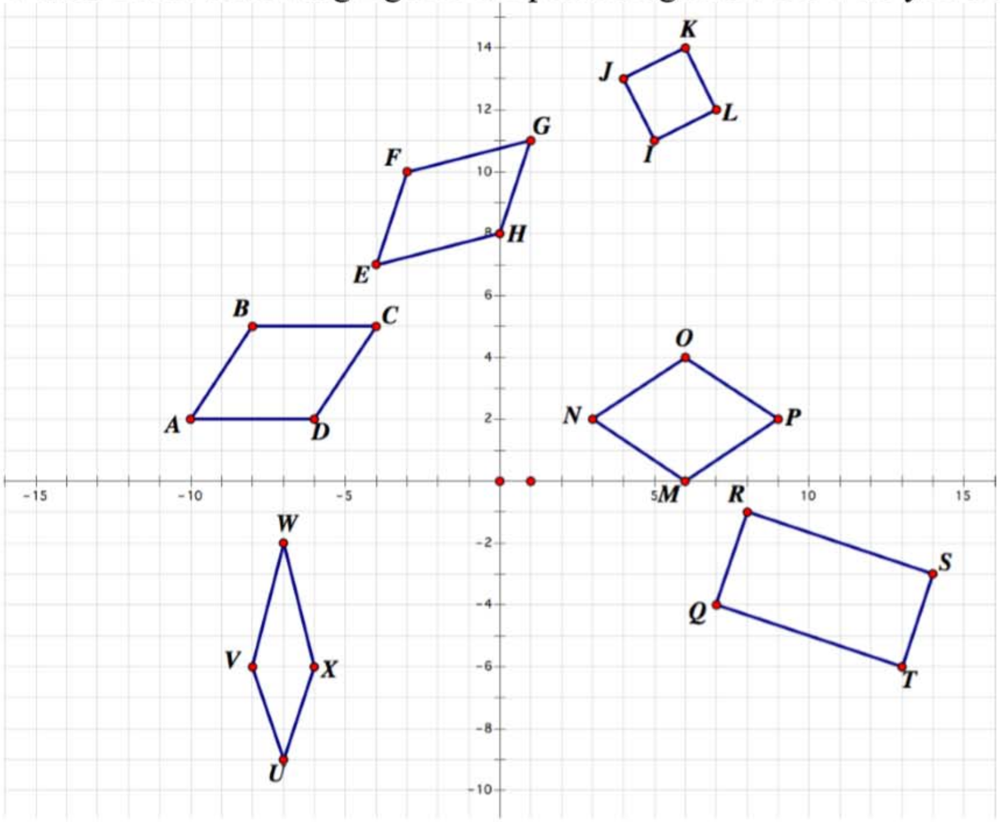
1. Brainstorm: What do you know about the following figures? List definitions and properties of each.

Square	Rhombus	Trapezoid
Rectangle	Parallelogram	Kite
Isosceles triangle	Right triangle	Equilateral triangle

Part 2

2. Brian thinks that every square is also a rectangle. He says this is so because a rectangle is a quadrilateral with four right angles and a square fits this definition. Is Brian right? Why or why not? Did Brian make a convincing argument?

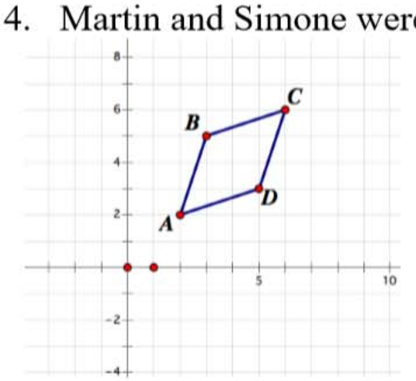
3. Consider the following set of figures on a coordinate plane.
   a. Which of the following figures are parallelograms? How do you know?



   b. Can you identify all of the parallelograms? Write an argument that would convince a skeptic that you have found all of the parallelograms in this figure.

   c. Could you classify any of the parallelograms as another type of mathematical shape? If so, which ones? If not, why not?

4. Martin and Simone were given quadrilateral ABCD on a coordinate plane:



Martin said: "Quadrilateral ABCD is a rhombus because AB||DC and AD||BC and it doesn't have any right angles."

Simone said: "Quadrilateral ABCD is a rhombus because it has two pairs of parallel sides and AB=BC=CD=DA."

Whose argument is better? Why? Can you write a more precise mathematical argument than Martin and Simone?

Directions: Name the property of equality that justifies each statement.

_______ **1.** If $a = 2b$, then $a - c = 2b - c$

_______ **2.** $x = x$

_______ **3.** $3(p - 7) = 3p - 21$

_______ **4.** If $-7k = -42$, then $k = 6$

_______ **5.** If $m + n = 15$ and $n = 2$, then $m + 2 = 15$

_______ **6.** If $\frac{x}{4} = -5$, $x = -20$

_______ **7.** If $w^2 = 2x$ and $2x = y$, then $w^2 = y$

_______ **8.** If $c - 9 = -1$, then $c = 8$

_______ **9.** If $n = -3$, then $-3 = n$

A. Addition Property of Equality
B. Subtraction Property of Equality
C. Multiplication Property of Equality
D. Division Property of Equality
E. Distributive Property
F. Substitution Property
G. Reflexive Property
H. Symmetric Property
I. Transitive Property

10.

Given: $-16 = \frac{m}{5} - 18$

Prove: $m = 10$

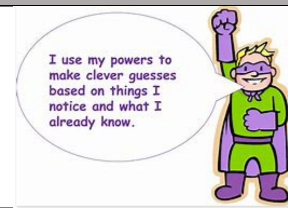
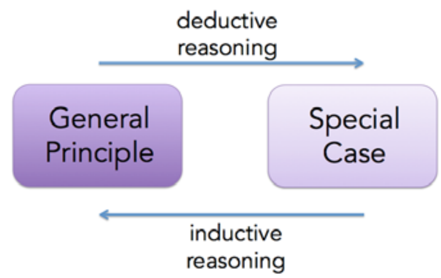
Statements	Reasons

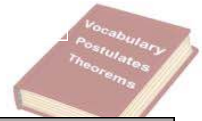
11.

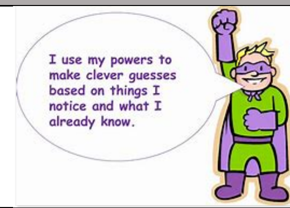
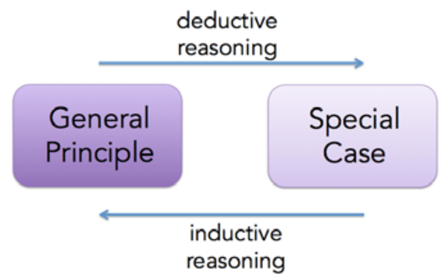
Given: -5 = $\frac{2y-1}{-3}$

Prove: y = 8

Statements	Reasons

Term	Definition	Notation	Diagram/Visual
Conjecture			
Inductive Reasoning			
Deductive Reasoning			
Conditional Statement			
Hypothesis			If you pet the cat, then it will purr. 
Conclusion			
If-then Form			



Term	Definition	Notation	Diagram/Visual
Conjecture	An unproven statement based on observations.		
Inductive Reasoning	The process of finding a pattern for specific cases and then writing a conjecture for the general case.		
Deductive Reasoning	Uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.		
Conditional Statement	A logical statement that has two parts, a hypothesis and conclusion.		
Hypothesis	The "if" part of a conditional statement; usually the subject of the sentence.		If you pet the cat, then it will purr. 
Conclusion	The "then" part of a conditional statement; usually the predicate of the sentence.		
If-then Form	A form of a conditional statement in which the "if" part contains the hypothesis and the "then" part contains the conclusion.		

THE BIG IDEA

Logic

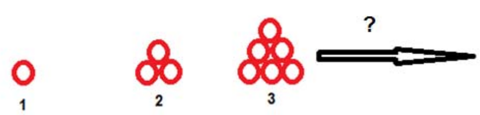
GUIDED NOTES

Conjectures and Inductive Reasoning

If we look at data over the precipitation in a city for 29 out of 30 days and see that it has been raining every single day, can you conclude that it will be raining the 30th day as well?

A **conjecture** is an educated guess that is based on known information.

Example!



1. Use Inductive Reasoning to make a conjecture about the next two images in the sequence above. ____

2. **Statement:** Geometry is a 10th grade course.
   Use Inductive Reasoning to write a conjecture about the age of students enrolled in Geometry:____

3. A **counterexample** is an example that disproves a conjecture. Write a counterexample to disprove your conjecture above. ____

SELF CHECK

A car salesman sold 5 used cars to five different couples. He noticed that each couple was under 30 years old. The following day, he sold a new, luxury car to a couple in their 60's.

Conjecture: The salesman determined that only younger couples by used cars.

Write a counterexample to show the conjecture is false.____

Example!

Given: noncollinear points A, B, C, and D
Conjecture: A, B, C, and D are coplanar.

Based on the given information, is the conjecture TRUE or FALSE?

Explain your answer. Give a counterexample for a FALSE conjecture.

Given: $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ intersect at X.

Write a conjecture based on the given information. If needed, draw a figure to illustrate your conjecture.

SELF CHECK

Given: Point C between H and V.
Conjecture: $\angle HCV$ is a straight angle.

Based on the given information, is the conjecture TRUE or FALSE?

Deductive Reasoning and Conditional Statements

GUIDED NOTES

Conditional Statement

- A statement that can be written in _____-________ form.

Symbolic Form: _________________ ← Read as "if p, then q" or, "p ______________ q".

- The ___________________ is the phrase immediately following the word _____.
- The ___________________ is the phrase immediately following the word __________.

Inverse	Formed by __________________ the hypothesis and conclusion. **Symbolic form:** ________________
Converse	Formed by __________________ the hypothesis and conclusion. **Symbolic form:** ________________
Contrapositive	Formed by __________________ and __________________ the hypothesis and conclusion. **Symbolic form:** ________________

Biconditional

_______________________________ if and only if _________________________________

Symbolic Form:__________________________

***Only true when both the conditional and the converse are true.

Example!

1. Consider the statement: Invertebrates have no backbone.

Rewrite the conditional statement in if-then form. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion. Underline the hypothesis and circle the conclusion, then rewrite below.

2.
Conditional: If you live in Warner Robins, then you live in Georgia.

*a) Inverse*____
____
______________________________________*Truth Value*______________

*b) Converse*____
____
______________________________________*Truth Value*______________

*c) Contrapositive*____
____
____*Truth Value*______________

*d) Biconditional*____
____
____*Truth Value*______________

SELF CHECK

1. Consider the statement: A rhombus is a quadrilateral with 4 congruent sides.

Rewrite the conditional statement in if-then form. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion. Underline the hypothesis and circle the conclusion, then rewrite below.

2.
Conditional: If you have a library card, then you can check out books.

*a) Inverse*____
____
______________________________________*Truth Value*______________

*b) Converse*____
____
______________________________________*Truth Value*______________

*c) Contrapositive*____
____
____*Truth Value*______________

d) *Biconditional*____
____
____*Truth Value*______________

Law of Detachment and Syllogism

GUIDED NOTES

Law of Detachment

Given a conditional statement, if the ____________________ is __________, then the ____________________ is __________.

Symbolic Map:

Law of Syllogism

Allows you to draw a conclusion from ______ conditional statements in which the__________________ of the first statement is the __________________ of the second statement.

Symbolic Map:

Example!

Rewrite the following statements to make a valid conclusion (Law of Syllogism) provided the hypothesis of the statement is true (Law of Datachment).

First: Identify the hypothesis of the first statement and the conclusion of the second statement by undelining these phrases. **Second:** Identify the the conclusion of the first statement and the hypothesis of the second statement by drawing a circle around them. If the two inner statements agree, then combine the first hypothesis with the last conclusion for a new valid statement.

Statement: If you live in Atlanta, then you live in Georgia**.** If you live in Georgia, then you live in the United States.

If ____, then ____.

Rewrite the following statements to make a valid conclusion (Law of Syllogism) provided the hypothesis of the statements are true (Law of Datachment). If a valid conclusion does not follow, write <u>no conclusion</u>.

1. If Rachel lives in Tampa, then she lives in Florida.
   If Rachel lives in Florida, then she lives in the United States.

2. If Spot is a dog, then he has four legs.
   Spot has four legs.

3. If Jim is a Texan, then he is an American.
   Jim is a Texan.

SELF CHECK

Use the Law of Detachment to make a valid conclusion. If not possible, write no valid conclusion.

1. If a polynomial is prime, then it cannot be factored.
5x + 13y is prime.
Conclusion:______________________________________

2. If you visit Paris, then you see the Eiffel Tower.
You did not see the Eiffel Tower.
Conclusion:______________________________________

Use the Law of Syllogism to make a valid conclusion. If not possible, write no valid conclusion.

3. If Nicole is tardy to class again, then she will get a detention.
If Nicole gets detention, then her mom will take away her phone.
Conclusion:____
____

4. If it is sunny this weekend, then you will go boating.
If it is sunny this weekend, then you will wear shorts.
Conclusion:____
____

Why is logic important to learn in Geometry?

1. Identify the hypothesis and conclusion

If the product of two numbers is 0, then at least one of the numbers must be 0.

Hypothesis:_____________________________________

Conclusion:_____________________________________

2. Write the following statement in if-then form.

Those that finish the marathon will get a medal.

____

____

3. *Conditional: If it snows, then school will be cancelled.*

a) *Inverse*____
____
_______________________________*Truth Value*_____________

b) *Converse*____
____
_________________________________*Truth Value*_____________

c) *Contrapositive*____
____
_____________________________________*Truth Value*_____________

d) *Biconditional*____
____
____________________________________*Truth Value*_____________

Use the Law of Detachment to make a valid conclusion. If not possible, write no valid conclusion.

4. If you are 18 years old, then you can register to vote.

Olivia is not 18 years old.

Conclusion:_____________________________________

5. If the sum of the measures of two angles is 90°, then they are complementary.

m∠J = 58° and m∠K = 32°

Conclusion:_____________________________________

Use the Law of Syllogism to make a valid conclusion. If not possible, write no valid conclusion.

6. If it is Saturday, then Jake has a baseball tournament.

If Jake has a baseball tournament, then he will need to pack his lunch.

Conclusion:____
____

7. If a quadrilateral is a square, then it is a rectangle.

If a quadrilateral is a rectangle, then it has four right angles.

Conclusion:____
____

Conditional Statements in Advertising

Assignment:

- Find 2 different advertisements in magazines or newspapers.
- Cut the ads out and NEATLY mount them on a piece of paper. If you do not own the magazine you are using, a photocopy of the advertisement would be acceptable.
- For each of the 2 advertisements, complete the following questions:
    1. Write an "If … then …" conditional statement for the advertisement.
    2. Underline each hypothesis.
    3. Circle each conclusion.
    4. Write the converse, inverse, and contrapositive for each advertisement.
    5. Write one or two sentences describing whether the advertisement is valid in the condition implied or stated, or if the advertisement is false and misleading. If the advertisement is false, then describe how you would change it to a valid conditional statement.
    6. If possible, write a counterexample for the advertisement. **This will be worth 5 extra credit points.**

Monty Python

Monty Python and the Holy Grail
The Witch Scene and Reasoning about Proofs

***Watch the video from Monty Python and the Holy Grail – this is the transcript.

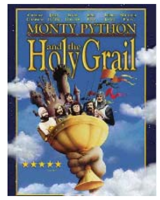
Transcript of the Scene

VILLAGER #1: We have found a witch, might we burn her?
CROWD: Burn her! Burn!
BEDEVERE: How do you know she is a witch?
VILLAGER #2: She looks like one.
BEDEVERE: Bring her forward.
WITCH: I'm not a witch. I'm not a witch.
BEDEVERE: But you are dressed as one.
WITCH: They dressed me up like this.
CROWD: No, we didn't -- no.
WITCH: And this isn't my nose, it's a false one.
BEDEVERE: Well?
VILLAGER #1: Well, we did do the nose.
BEDEVERE: The nose?
VILLAGER #1: And the hat -- but she is a witch!
CROWD: Burn her! Witch! Witch! Burn her!

BEDEVERE: Did you dress her up like this?

CROWD: No, no... no... yes. Yes, yes, a bit, a bit.
VILLAGER #1: She has got a wart.
BEDEVERE: What makes you think she is a witch?
VILLAGER #3: Well, she turned me into a newt.
BEDEVERE: A newt?
VILLAGER #3: I got better.
VILLAGER #2: Burn her anyway!
CROWD: Burn! Burn her!
BEDEVERE: Quiet! quiet! Quiet! There are ways of telling whether she is a witch.
CROWD: Are there? What are they?
VILLAGER #2: Do they hurt?
BEDEVERE: Tell me, what do you do with witches?
VILLAGER #2: Burn!
CROWD: Burn, burn them up!
BEDEVERE: And what do you burn apart from witches?
VILLAGER #1: More witches!
VILLAGER #2: Wood!
BEDEVERE: So, why do witches burn?

[pause]
VILLAGER #3: B--... 'cause they're made of wood?
BEDEVERE: Good!
CROWD: Oh yeah, yeah.
BEDEVERE: So, how do we tell whether she is made of wood?
VILLAGER #1: Build a bridge out of her.
BEDEVERE: Aah, but can you not also make bridges out of stone?
VILLAGER #2: Oh, yeah.
BEDEVERE: Does wood sink in water?
VILLAGER #1: No, no.
VILLAGER #2: It floats! It floats!
VILLAGER #1: Throw her into the pond!
CROWD: The pond!
BEDEVERE: What also floats in water?
VILLAGER #1: Bread!
VILLAGER #2: Apples!
VILLAGER #3: Very small rocks!
VILLAGER #1: Cider!
VILLAGER #2: Uhhh, gravy!
VILLAGER #1: Cherries!
VILLAGER #2: Mud!
VILLAGER #3: Churches -- churches!
VILLAGER #2: Lead -- lead!
ARTHUR: A duck.
CROWD: Oooh.
BEDEVERE: Exactly! So, logically...
VILLAGER #1: If... she... weighs the same as a duck.. she's made of wood.
BEDEVERE: And therefore?
VILLAGER #1: A witch!
CROWD: A witch! A witch! A witch!
BEDEVERE: We shall use my largest scales!
[yelling]
BEDEVERE: Right, remove the supports!
[whop] [creak]
CROWD: A witch! A witch!
WITCH: It's a fair cop.
CROWD: Burn her! Burn her!
[yelling]
BEDEVERE: Who are you who are so wise in the ways of science?
ARTHUR: I am Arthur, King of the Britons.
BEDEVERE: My liege!
ARTHUR: Good Sir knight, will you come with me to Camelot, and join us at the Round Table?
BEDEVERE: My liege! I would be honored.
ARTHUR: What is your name?
BEDEVERE: Bedevere, my leige.
ARTHUR: Then I dub you Sir Bedevere, Knight of the Round Table.



STUDENTS: Statements of fact can almost always be written in conditional ("If-Then") form. For instance, "Cats are felines" can be written as "If an animal is a cat, then it is a feline."

Instructions:

1. Identify as many implied "If-Then" statements as you can from the MP&HG scene. *To make it easier, you may state the if-then statement as if you are a caveman. (For instance, "if cat, then feline" would be sufficient for the first column.*
2. State the converse, inverse, and contrapositive in the appropriate columns.
3. Tell whether each statement is true or false.
4. If it is false, provide a counterexample.
5. Look for a True or False pattern. If the conditional statement is true, which statement is also always true? If the converse is true, which statement is also true? Be prepared to discuss.

Conditional Statement If p, then q. $(p \rightarrow q)$	**T or F?**	**Converse** If q, then p. $(q \rightarrow p)$	**T or F?**	**Inverse** If not p, then not q. $(\sim p \rightarrow \sim q)$	**T or F?**	**Contrapositive** If not q, then not p. $(\sim q \rightarrow \sim p)$	**T or F?**
If cat, then feline.	**T**	**If feline, then cat.**	**T**	**If not cat, then not feline.**	**T**	**If not feline, then not cat.**	**T**
If duck, then floats.	**T**	**If floats, then duck.** **Counterexample: A boat.**	**F**	**If not duck, then doesn't float.** **Counterexample: Oil**	**F**	**If doesn't float, then not duck.**	**T**

Conditional Statement If p, then q. $(p \rightarrow q)$	**T or F?**	**Converse** If q, then p. $(q \rightarrow p)$	**T or F?**	**Inverse** If not p, then not q. $(\sim p \rightarrow \sim q)$	**T or F?**	**Contrapositive** If not q, then not p. $(\sim q \rightarrow \sim p)$	**T or F?**

Conditional Statement If p, then q. $(p \rightarrow q)$	T or F?	**Converse** If q, then p. $(q \rightarrow p)$	T or F?	**Inverse** If not p, then not q. $(\sim p \rightarrow \sim q)$	T or F?	**Contrapositive** If not q, then not p. $(\sim q \rightarrow \sim p)$	T or F?

1. Identify the hypothesis and conclusion

If you plan to attend prom, then you must purchase a ticket.

Hypothesis:_____________________________________

Conclusion:_____________________________________

2. Write the following statement in if-then form.

All freshmen are required to attend orientation.

____

____

3. *Conditional: If a number is divisible by 2, then it is also divisible by 4.*

*a) Inverse*____
____
________________________________*Truth Value*_____________

*b) Converse*____
____
__________________________________*Truth Value*_____________

*c) Contrapositive*____
____
______________________________________*Truth Value*_____________

*d) Biconditional*____
____
_____________________________________*Truth Value*_____________

Use the Law of Detachment to make a valid conclusion. If not possible, write no valid conclusion.

4. If you plan to attend prom, then you must purchase a ticket.

Sarah purchases a ticket.

Conclusion:_____________________________________

5. If Mark saves $50, then he can purchase a video game.

Mark saved $50.

Conclusion:_____________________________________

Use the Law of Syllogism to make a valid conclusion. If not possible, write no valid conclusion.

6. If you shop at Target, then you will use your Target Red Card.

If you do not use your Target Red Card, then you will not save 5%.

Conclusion:____
____

7. If Sally goes to the airport, then she will need to pay for parking.

If Sally pays for parking, then her car will not get towed.

Conclusion:____
____

Term	Definition	Notation	Diagram/Visual
Corresponding parts			
Reflexive			
Symmetric			
Transitive			
Congruent			
Equal			
Protractor postulate			
Ruler postulate			
Properties of equality			

Properties of Congruence

GUIDED NOTES

What's the story on this notation between things that are congruent and those that are equal?

The notation used in geometry can often be confusing. The major problems seem to develop when working with segments and angles. Let's see if we can clarify "what" gets used "when".

Basic knowledge:	$\overline{AB}$ with the bar on top, means the actual segment itself.	$\measuredangle ABC$ means the actual angle itself.
	AB without the bar on top, means the length of the segment labeled A and B.	$m\measuredangle ABC$ means the measure of the angle labeled A, B and C.

When to use congruent:	When to use equal:
Figures are congruent. Segments are congruent. Angles are congruent. Triangles are congruent. This refers to the actual diagrams themselves. $\overline{AB} \cong \overline{CD}$ $\measuredangle ABC \cong \measuredangle DEF$	**Numerical values are equal.** When an expression represents a length or measure, equal should be used. $AB = CD$ $m\measuredangle ABC = m\measuredangle DEF$

Notation at Work:

Let's see what we should write if we wish to add the lengths of two sets of segments known to be congruent:

$\overline{AB} \cong \overline{CD}$ $\overline{PQ} \cong \overline{RS}$	Before we try to add these statements, we need to change these congruent entities to numerical values. After all, we add numbers, not sets of points.
$AB = CD$ $PQ = RS$	Congruent segments are segments of equal measure.

Now that we have these numerical values, we are ready to add:

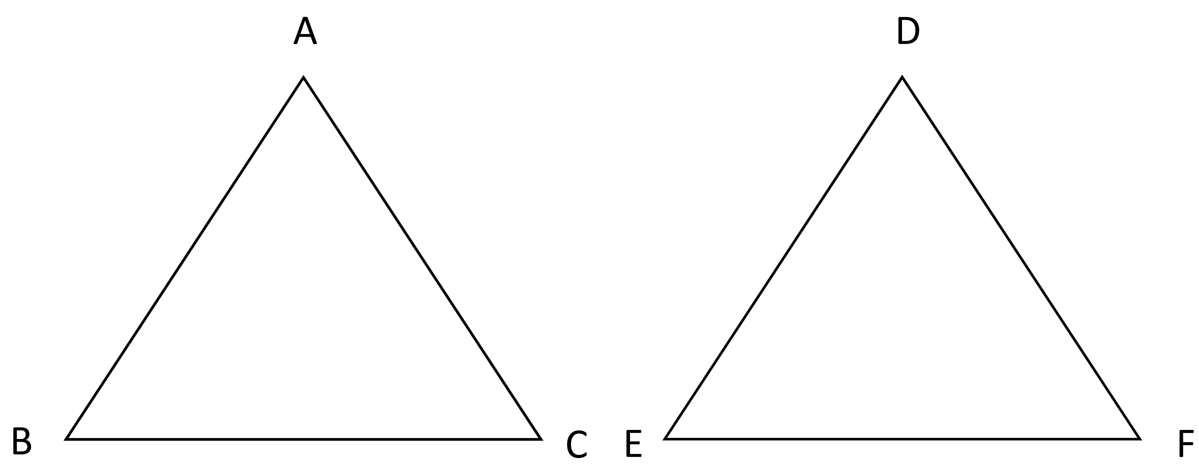
$$AB + PQ = CD + RS$$

Things you should and should not write:

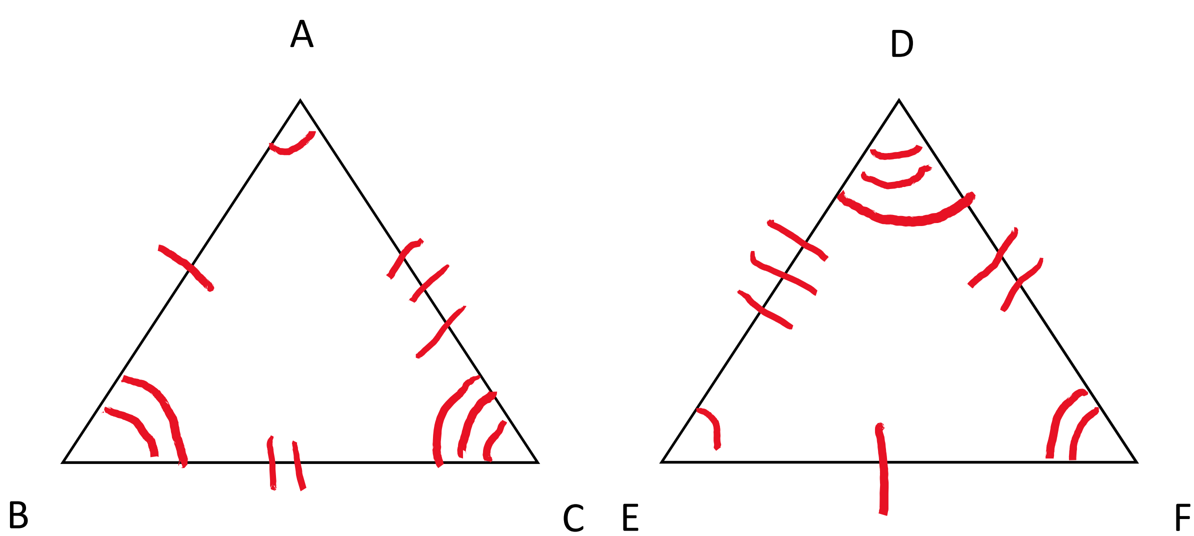
Statement	
$\overline{AB} + \overline{PQ} = \overline{CD} + \overline{RS}$	NO
$AB + PQ = CD + RS$	YES
$\measuredangle ABC + \measuredangle DEF = \measuredangle PQR$	NO
$m\measuredangle ABC + m\measuredangle DEF = m\measuredangle PQR$	YES
$\overline{AB} = \overline{CD}$	NO
$AB = CD$ and $\overline{AB} \cong \overline{CD}$	YES
$AB \cong CD$	NO
$\overline{AB} \cong \overline{CD}$	YES
$\measuredangle ABC = \measuredangle DEF$	NO
$m\measuredangle ABC = m\measuredangle DEF$	YES
$\measuredangle ABC \cong \measuredangle DEF$	YES

Marking Shapes for Congruence

$\Delta ABC \cong \Delta DEF$



Writing congruence statements from marked shapes.



Sides	Angles
________ ≅ ________	________ ≅ ________
________ ≅ ________	________ ≅ ________
________ ≅ ________	________ ≅ ________

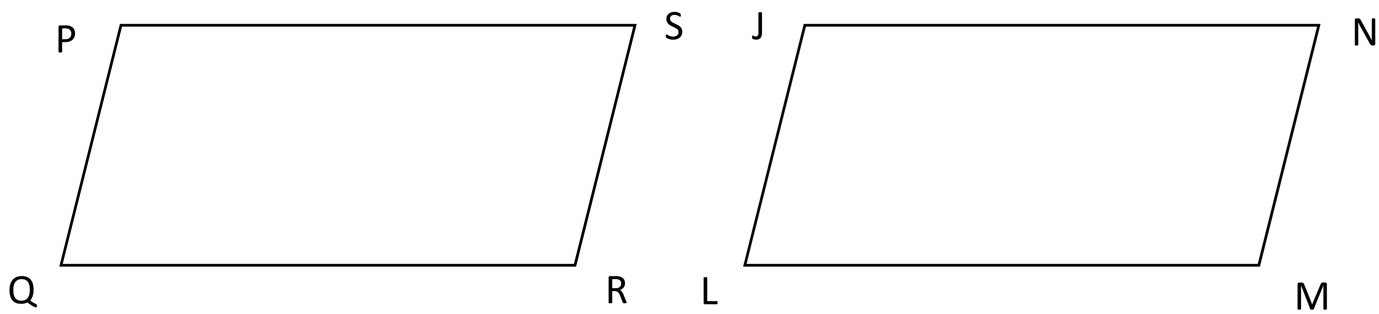
Congruence Statement

__________ ≅ __________

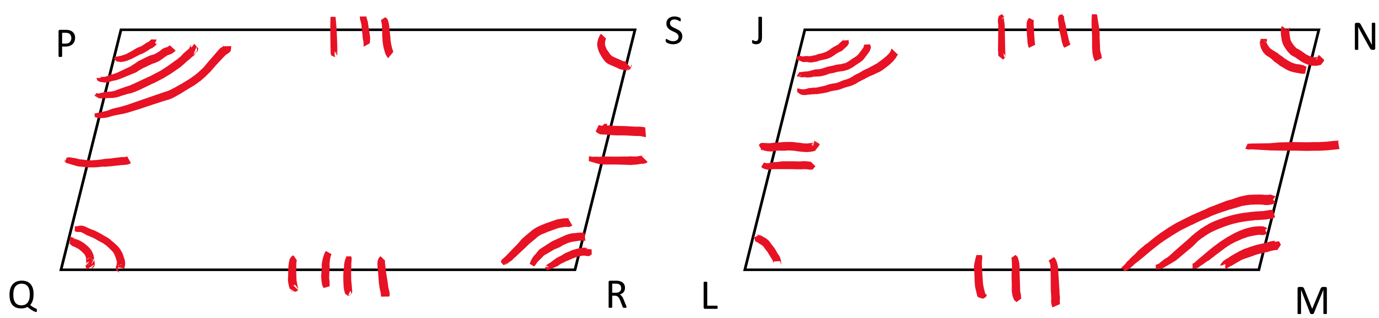
Example!

1. ▱ PQRS ≅ ▱ JLMN

Mark the congruent parts of the parallelograms.



2. Use the marks to identify the congruent angles, sides and write the congruence statement about the parallelograms.



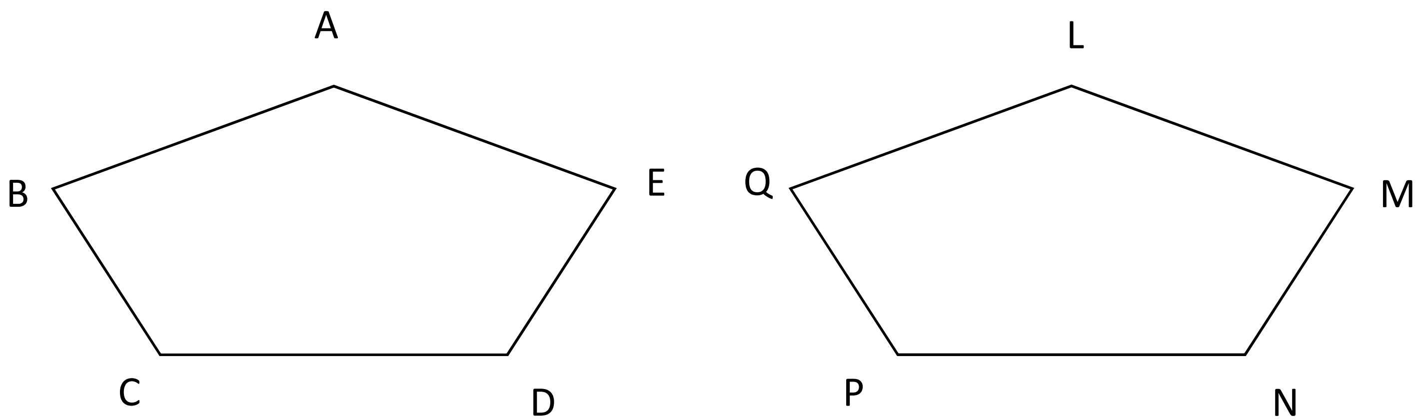
Sides	Angles
___________≅___________	___________≅___________
___________≅___________	___________≅___________
___________≅___________	___________≅___________
___________≅___________	___________≅___________

Congruence Statement
______________≅______________

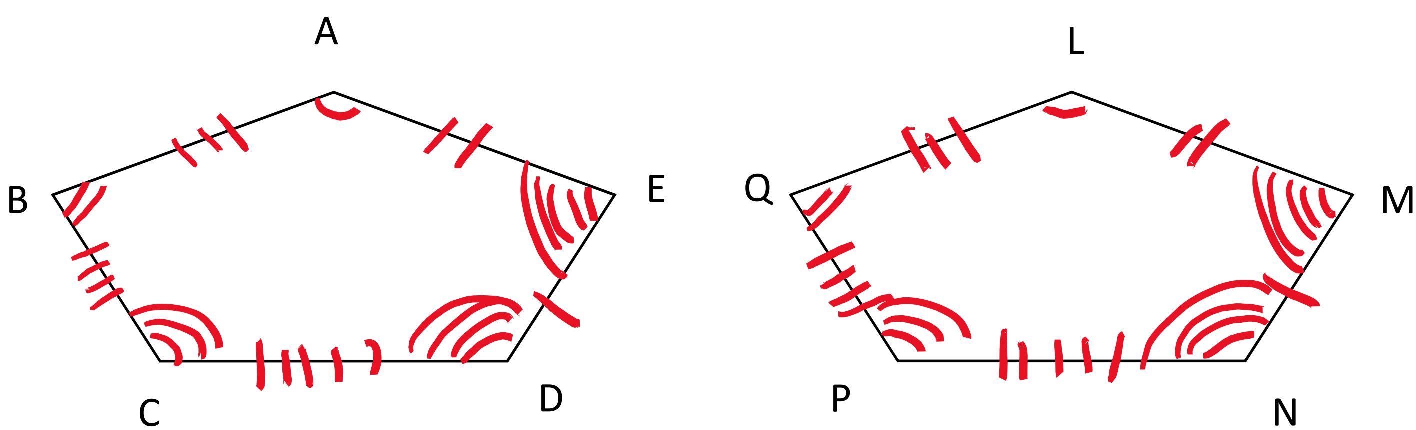
SELF CHECK

1. Pentagon ABCDE ≅ Pentagon LMNPQ

Mark the congruent parts of the pentagon



2. Use the marks to identify the congruent angles, sides and write the congruence statement about the pentagons.



Sides	Angles
____________≅____________	____________≅____________
____________≅____________	____________≅____________
____________≅____________	____________≅____________
____________≅____________	____________≅____________
____________≅____________	____________≅____________

Congruence Statement

_______________≅_______________

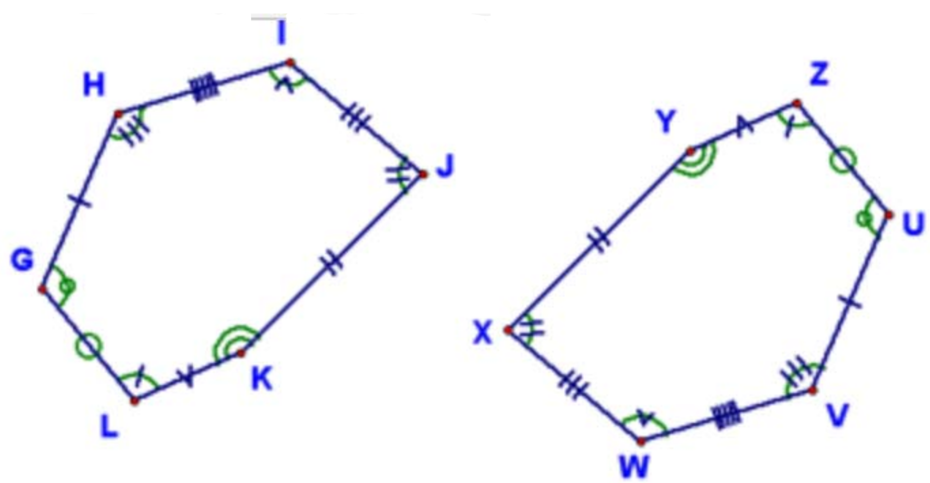
Do you need a diagram when trying to figure out the congruent parts, or will a congruent statement be enough? Why?

Draw a picture and mark the congruent parts.

1. $\Delta EMS \cong \Delta CHR$

2. Quadrilateral MATH ≅ Quadrilateral GEOM

3. List all the congruent parts and the write the congruence statement.

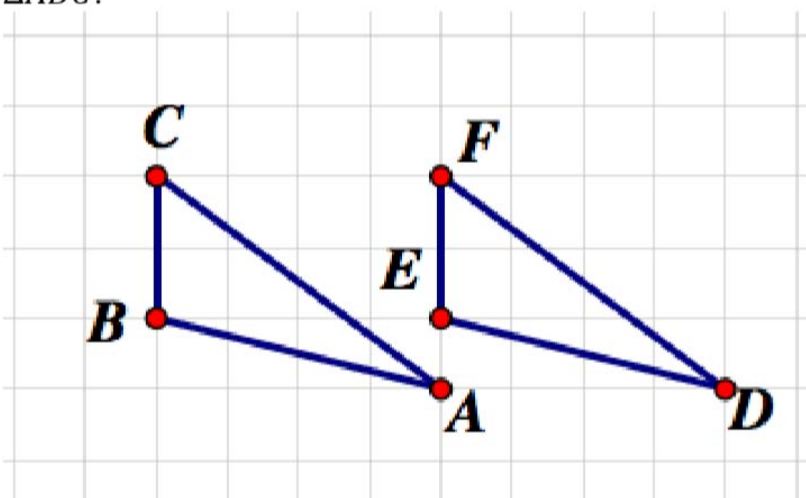


Introducing Congruence

Definition: Two triangles are congruent if and only if one can be obtained from the other by a sequence of rigid motions (translations, reflections, rotations)

From Coordinate Algebra, how can we tell if a figure is a translation, reflection, or rotation of another figure?

Triangle DEF is congruent to triangle ABC ($\Delta DEF \cong \Delta ABC$) because ΔDEF is a translation of ΔABC.

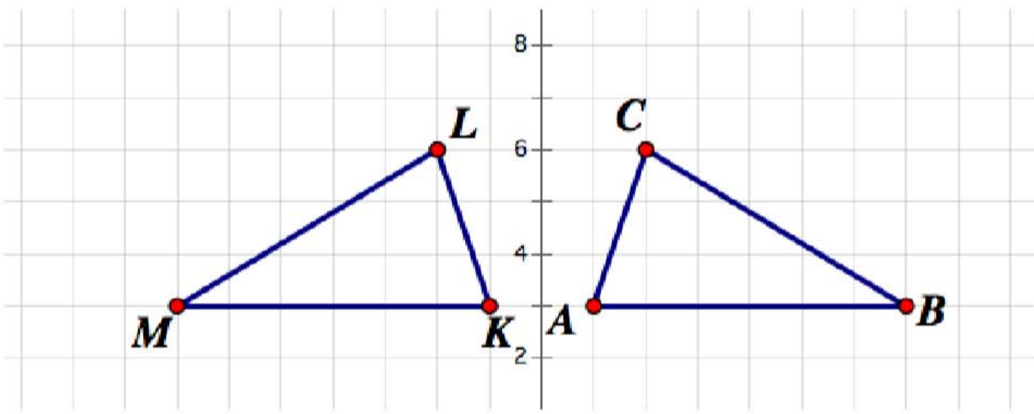


If ΔDEF is a translation of ΔABC, then what is true about the various sides and angles of the triangles? Specifically, what parts of these two triangles are related, and how are they related?

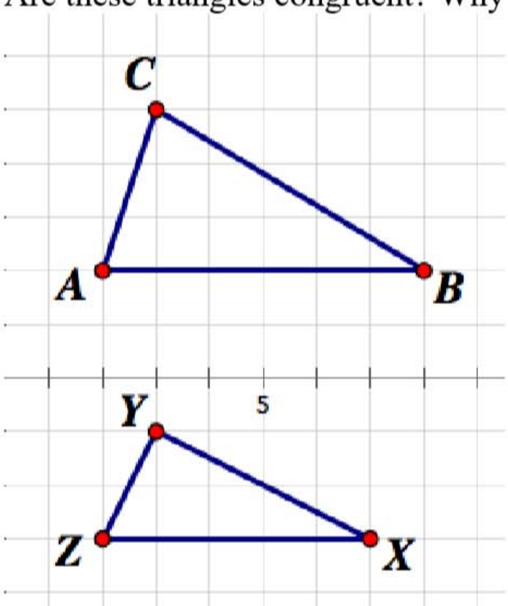
In general, what can you say about sides and angles of figures that are translations, reflections, or rotations of other figures?

So then, if two triangles are congruent (and congruence depends on translations, reflections, and rotations), then what must be true about their sides and angles?

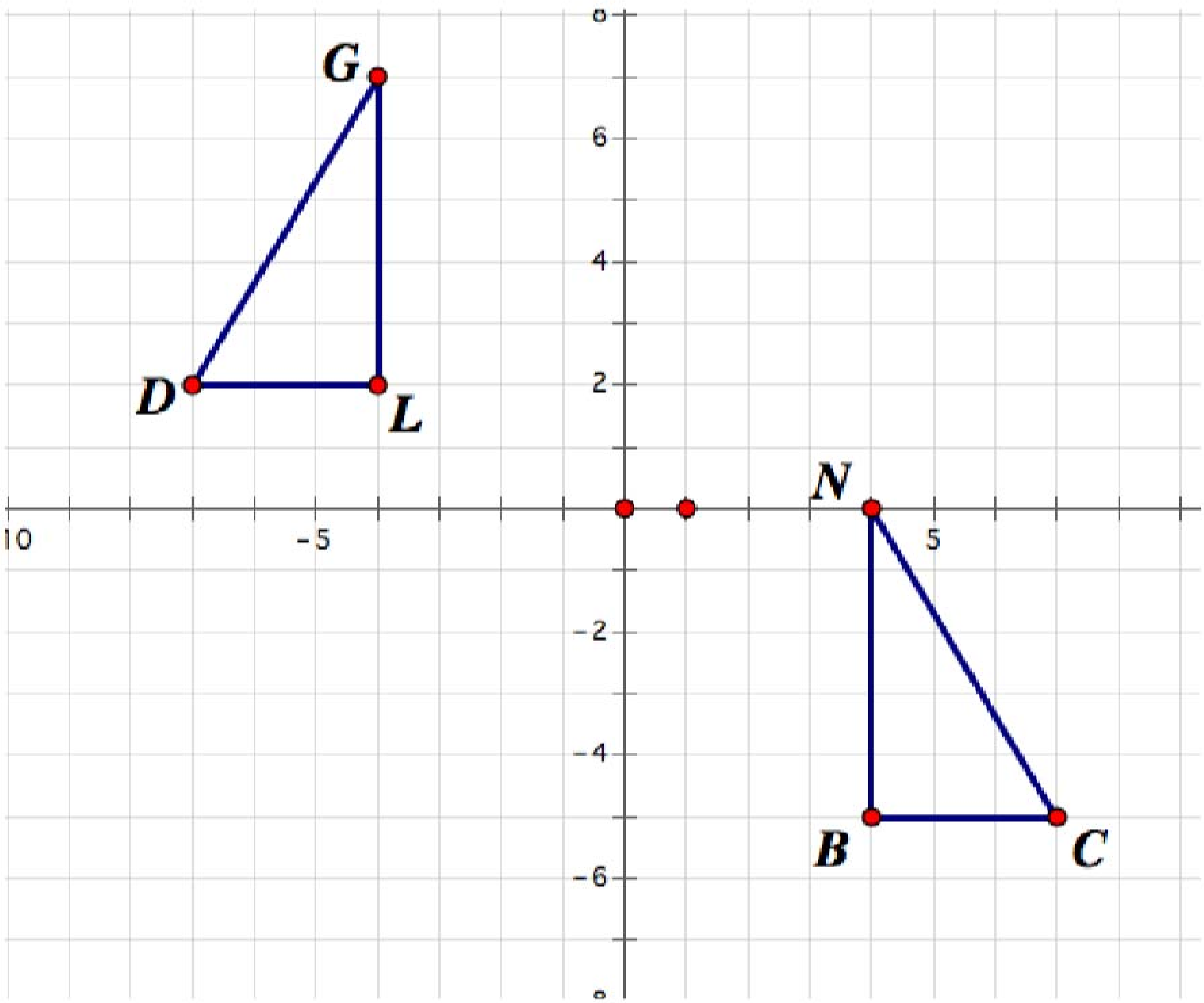
Are these triangles congruent? Why or why not?



Are these triangles congruent? Why or why not?



What sequence of rigid motions maps ΔDGL to ΔCNB?



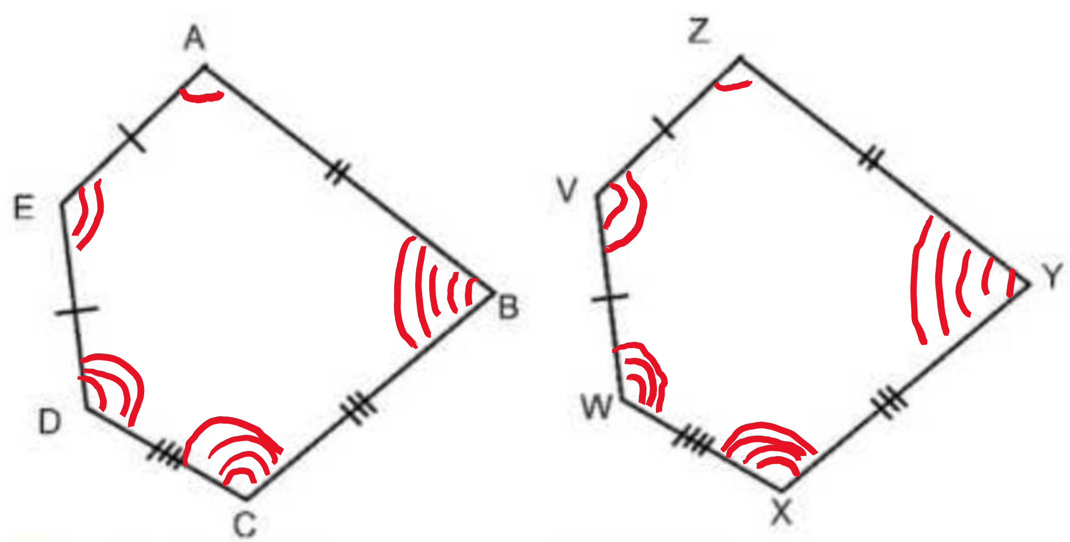
Which parts of these triangles must be congruent?

Draw a picture and mark the congruent parts.

1. $\Delta LMN \cong \Delta PQR$

2. Quadrilateral DEFG ≅ Quadrilateral WXYZ

3. List all the congruent parts and the write the congruence statement.



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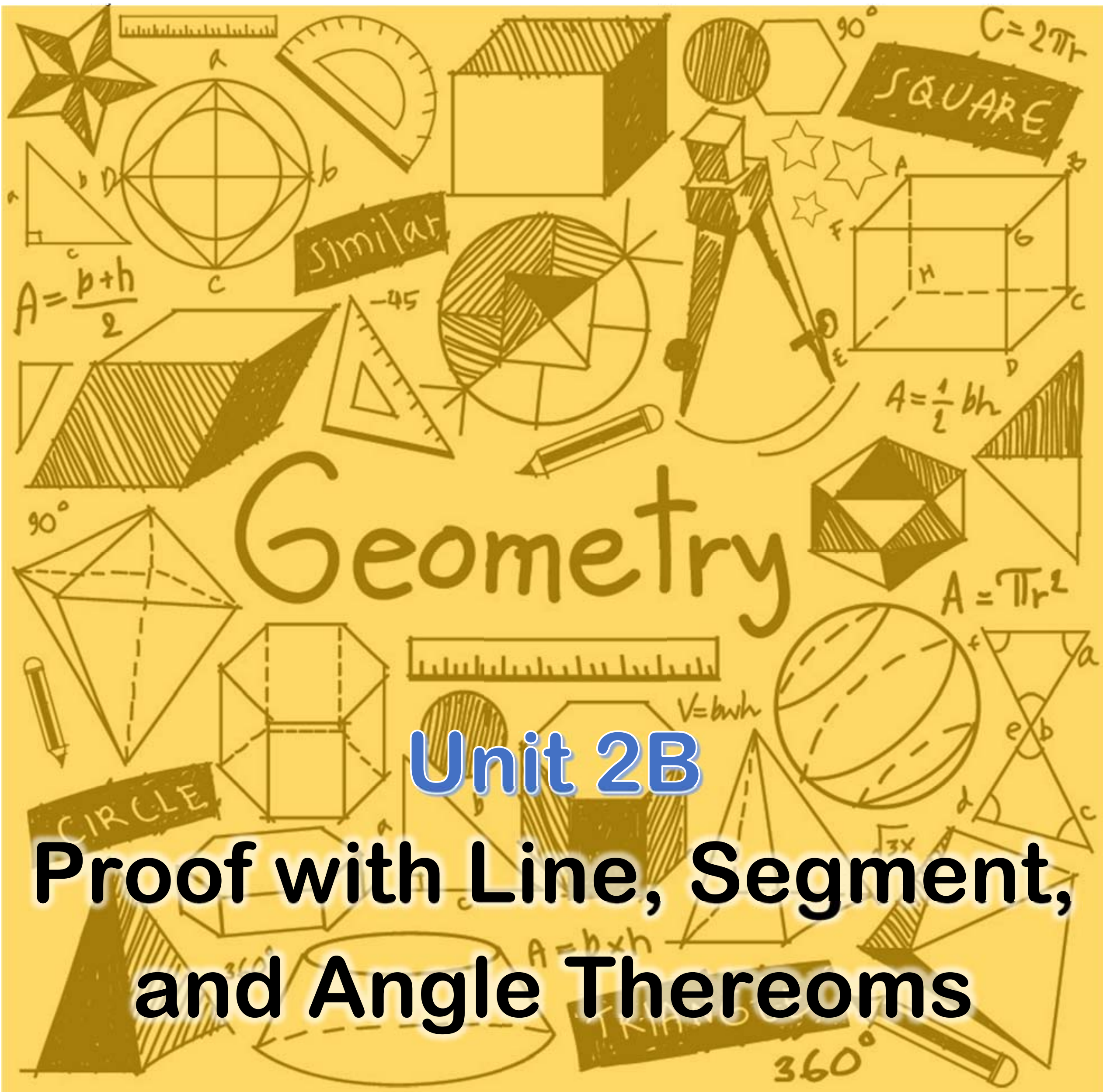

Unit 2B

Proof with Line, Segment, and Angle Thereoms

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MATHEMATICS DEPARTMENT

Diagram/Visual			
Segment			
Segment Addition Postulate			
Geometric Constructions			
Midpoint Theorem			
Ruler Postulate			

Segment Theorems

GUIDED NOTES

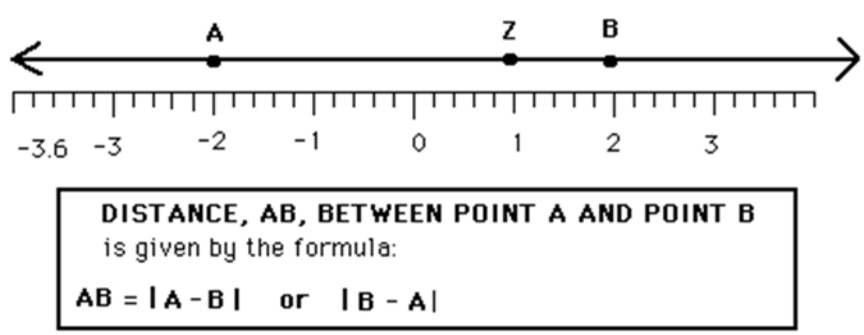
Ruler Postulate

1. On a number line, every point can be paired with a number and every number can be paired with a point.
2. The length of a line segment is a positive number.
3. The distance between two points on a number line can be found by taking the absolute value of the difference of their coordinates.

The Ruler Postulate (in simplified terms)

Suppose we have the points R and S on a number line where the coordinate of R=r and the coordinate of S=s. Then the Ruler Postulate tells us how to find the length RS. We subtract the coordinates (in any order) and take the absolute value.

$RS = |r - s|$ or $|s - r|$



Ruler Postulate:

- The points on a line can be placed in a one-to-one correspondence with real numbers so that
  - 1. for every point on the number line, there is exactly one real number.
  - 2. for every real number, there is exactly one point on the line.
  - 3. the distance between any two points is the absolute value of the difference of the corresponding real numbers.

Ruler Placement Postulate

- Given two points, A and B on a line, the number line can be chosen so that A is at zero and B is a positive number.

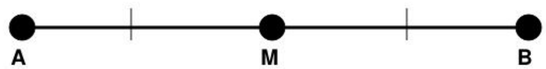
Midpoint Theorem

MIDPOINT

Recall

Definition of Midpoint: The point that divides a segment into 2 congruent segments.

*If M is the midpoint of $\overline{AB}$, then AM = MB



MIDPOINT THEOREM

Theorem 2-1 Midpoint Theorem:

If M is the midpoint of AB,
then AM = ½ AB and MB = ½ AB.

Segment Addition Postulate

Segment Addition Postulate

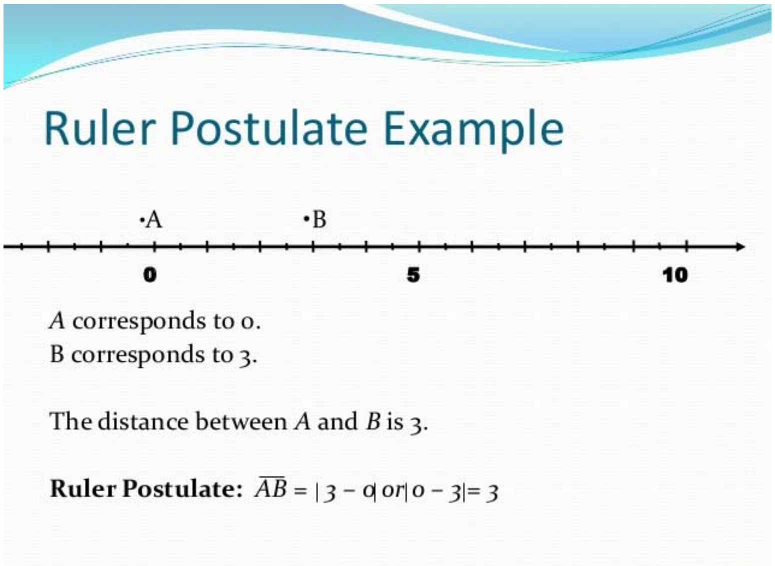
- If three points (A, B, and C) are collinear and B is between A and C, then AB + BC = AC.
  - In other words: If three points are on a line, then the length of the 2 segments created can be added together to get the length of the entire segment!

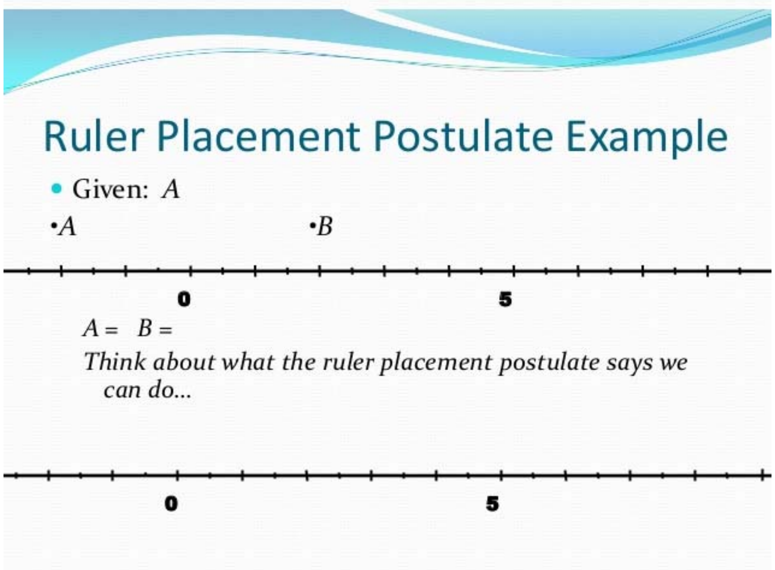
Segment Addition Postulate

- If B is between A and C, then $\overline{AB} + \overline{BC} = \overline{AC}$.

Example!

Ruler Postulate

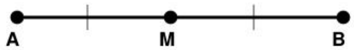




Midpoint Theorem

EXAMPLES

M is the midpoint of AB. If AM = 3x + 5 and AB = 46, find x.



Method 1

AM = ½ AB

3x + 5 = ½ (**46**)

3x + 5 = 23

3x = 18

x = 6

Method 2

AM = MB

So, AM + MB = AB

(**3x + 5**) + (**3x + 5**) = **46**

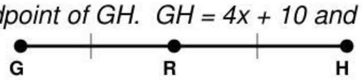
6x + 10 = 46

6x = 36

x = 6

EXAMPLES

R is the midpoint of GH. GH = 4x + 10 and GR = 3x. Find x.



Method 1

GR = ½ GH

3x = ½ (**4x + 10**)

3x = 2x + 5

x = 5

Method 2

GR = RH

GR + RH = GH

3x + **3x** = **4x + 10**

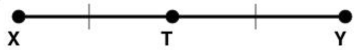
6x = 4x + 10

2x = 10

X = 5

EXAMPLES

T is the midpoint of XY. XT = 6x - 20 and XY = 2x. Find x.



Method 1

XT = ½ XY

6x – 20 = ½ (**2x**)

6x – 20 = x

6x = x + 20

5x = 20

x = 4

Method 2

XT = TY

XT + TY = XY

(**6x – 20**) + (**6x – 20**) = **2x**

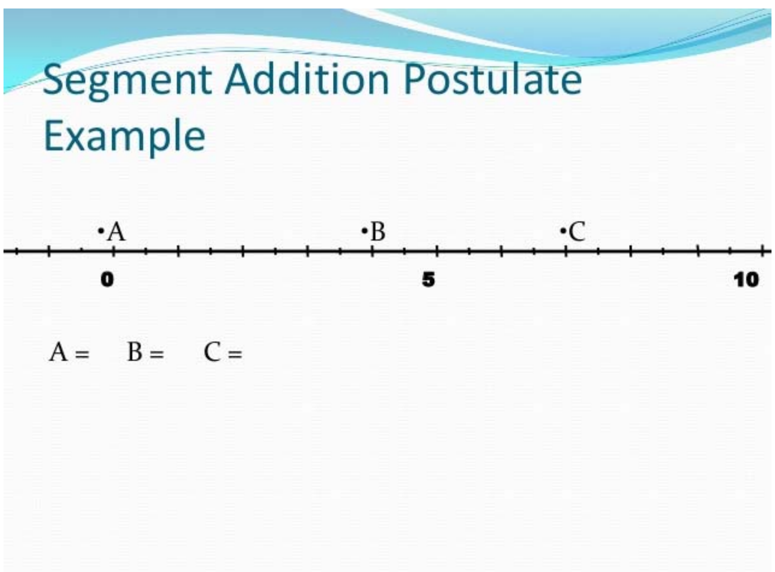
12x – 40 = 2x

12x = 2x + 40

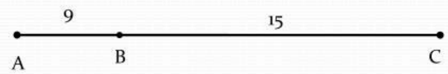
10x = 40

x = 4

Segment Addition Postulate



Example



If AB=9 and BC=15, find AC.

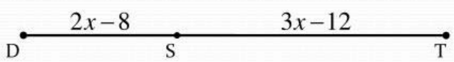
AC=9+15=24

If AB=8 and BC=3, find AC.

AC=8+3=11

Example

- Example 1: If DT = 60, find the value of x, then find DS and ST.



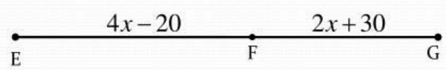
$DS + ST = DT$
$2x - 8 + 3x - 12 = 60$
$5x - 20 = 60$
$5x = 80$
$x = 16$

$DS = 2x - 8$
$DS = 2(16) - 8$
$DS = 32 - 8$
$DS = 24$

$ST = 3x - 12$
$ST = 3(16) - 12$
$ST = 48 - 12$
$ST = 36$

Example:

- EG =100. Find x, then find EF and FG.



$$EF + FG = EG$$
$$4x - 20 + 2x + 30 = 100$$
$$6x + 10 = 100$$
$$6x = 90$$
$$x = 15$$

$$EF = 4x - 20$$
$$EF = 4(15) - 20$$
$$EF = 60 - 20$$
$$EF = 40$$

$$FG = 2x + 30$$
$$FG = 2(15) + 30$$
$$FG = 30 + 30$$
$$FG = 60$$

1. Let GH = 45 in, HT = 15 in. Find GT.

2. Let GT = 37 cm, HT = 17 cm. Find GH.

Write the Segment Addition Postulate equation for the points described. Draw a picture.

J is between S and H.

SELF CHECK

3. Let HT = 14 ft, GH = 36 ft. Find GT.

4. Let TG = 100 in, GH = 60 in. Find HT.

5. Let GT = 34 m, HT = 20 m. Find GH.

Write the Segment Addition Postulate equation for the points described. Draw a picture.

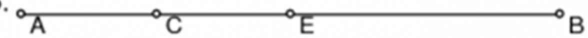
S is between D and P.

Points A, B, C, D, and E are collinear and in that order. Find AC if $AE = x + 50$ and $CE = x + 32$.

Geometry A
Segment Addition

Name__________________

Use the figure to the right to answer questions 1 – 5.
For each problem, write the Segment Addition relationship that applies before substituting. For example, you might write AC + CB = AB



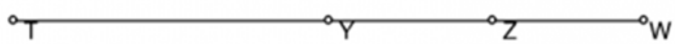
1. If AC = 5 and CB = 12, find AB.

2. If AC = 4, CE = 6, and AB = 18, find EB

3. If E is the midpoint of AB, AC = 6, and EB = 9, how long is CE?

4. If E is the midpoint of AB, C is the midpoint of AE, and AB = 28, how long is CE?

<u>Using algebra to solve segment problems…</u>



For questions 5 – 10, use the figure to the right
Write an equation to solve each problem, then solve.

5. If TZ = 2x, ZW = 3, and TW = 15 Find x.

6. If TY = 4x + 5, YW = 6x, and TW = 20, Find x.

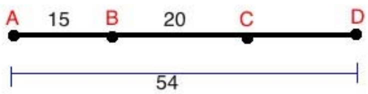
7. If Y is the midpoint of TW, TY = 3x and TW = 30, find x.

8. If Y is the midpoint of TW, TY = 4x – 4 and YW = 2x + 8, find x.

9. If Y is the midpoint of TW, Z is the midpoint of YW, ZW = 2x, and TW = 40, find x.

10. If Y is the midpoint of TW, Z is the midpoint of YW, YZ = 2x – 1, and TW = 20, find x.

1. Find the value of CD given the information about line segment AD in the image.

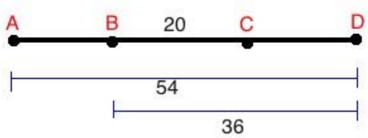


- 19
- 21
- 20
- 18

2. Find AC if AB= 12 and BC=8.

- 24
- 20
- 18
- 22

3. Find the value of line segment AC given the values of the other segments in this image.



- 41
- 40
- 38
- 39

Segment Theorems

The study of Geometry was born in Ancient Greece, where mathematics was thought to be embedded in everything from music to art to the governing of the universe. Plato, an ancient philosopher and teacher, had the statement, "Let no man ignorant of geometry enter here," placed at the entrance of his school. This illustrates the importance of the study of shapes and logic during that era. Everyone who learned geometry was challenged to construct geometric objects using two simple tools, known as Euclidean tools:

- A straight edge without any markings
- A compass

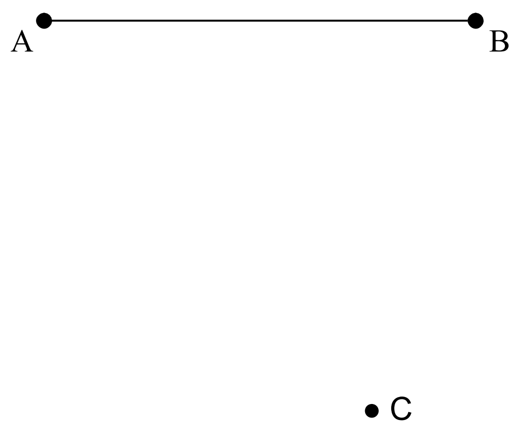
The straight edge could be used to construct lines, the compass to construct circles. As geometry grew in popularity, math students and mathematicians would challenge each other to create constructions using only these two tools. Some constructions were fairly easy (Can you construct a square?), some more challenging, (Can you construct a regular pentagon?), and some impossible even for the greatest geometers (Can you trisect an angle? In other words, can you divide an angle into three equal angles?). Archimedes (287-212 B.C.E.) came close to solving the trisection problem, but his solution used a marked straight edge. What constructions can you create?

Your First Challenge: Can you copy a line segment?

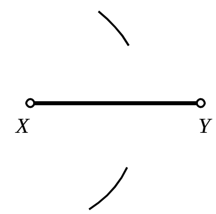
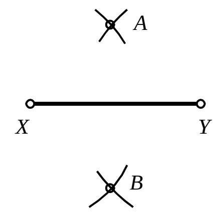
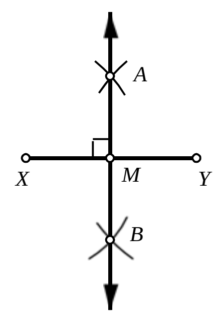
Step 1	Construct a circle with a compass on a sheet of paper.
Step 2	Mark the center of the circle and label it point A.
Step 3	Mark a point on the circle and label it point B.
Step 4	Draw $\overline{AB}$

Your Second Challenge: Can you copy any line segment?

Below is a line segment $\overline{AB}$. Using only an unmarked straight edge and compass, can you construct another line segment the same length beginning at point C? Write instructions that explain the steps you used to complete the construction. (*Hint: An ancient geometer would require you to "cut off from the greater of two lines" a line segment equal to a given segment.*)



Your Fourth Challenge: Can you bisect a segment?

1. Begin with line segment *XY*.	
2. Place the compass at point *X*. Adjust the compass radius so that it is more than (½)*XY*. Draw two arcs as shown here.	
3. Without changing the compass radius, place the compass on point *Y*. Draw two arcs intersecting the previously drawn arcs. Label the intersection points *A* and *B*.	
4. Using the straightedge, draw line *AB*. Label the intersection point *M*. Point *M* is the midpoint of line segment *XY*, and line *AB* is perpendicular to line segment *XY*.	

Construct the perpendicular bisector of the segments. Mark congruent segments and right angles. Check your work with a protractor.

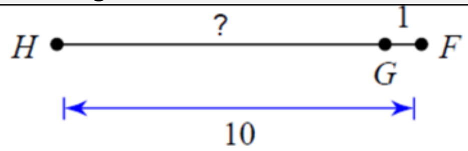
1.

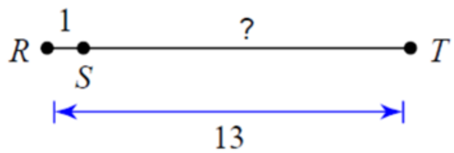
2.

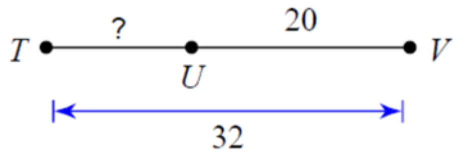
3.

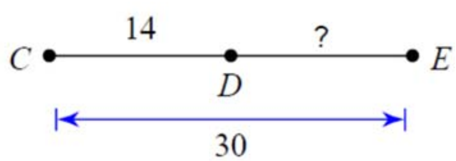
Segment Addition Postulate

Find the length indicated.

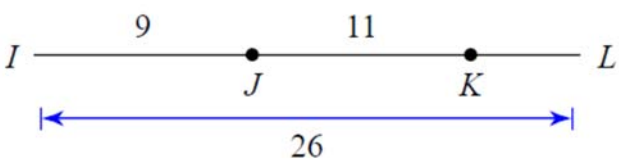
1) 

2) 

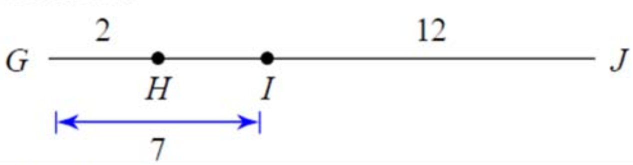
3) 

4) 

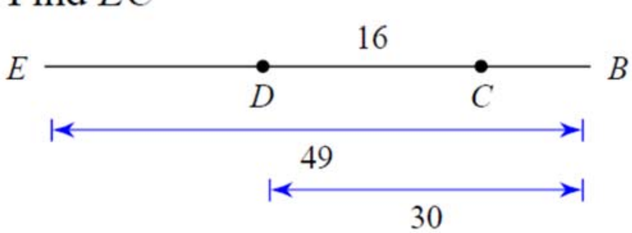
5) Find KL



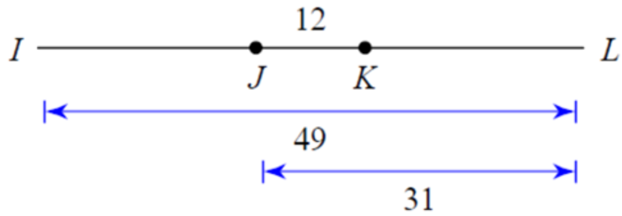
6) Find HJ



7) Find EC



8) Find IK



Points A, B, and C are collinear. Point B is between A and C. Find the length indicated.

9) Find AC if $AB = 16$ and $BC = 12$.

10) Find AC if $AB = 13$ and $BC = 9$.

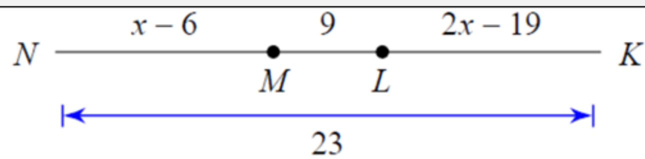
Points A, B, and C are collinear. Point B is between A and C. Solve for *x*.

11) $AC = 3x + 3$, $AB = -1 + 2x$, and $BC = 11$. Find x.

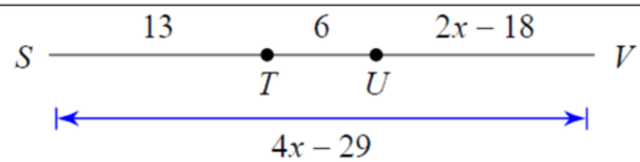
12) $AC = 22$, $BC = x + 14$, and $AB = x + 10$. Find x.

Solve for *x*.

13)

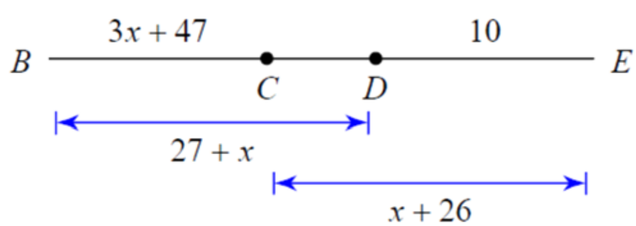


14)

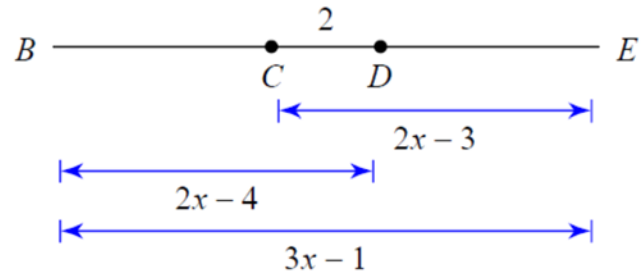


Find the length indicated.

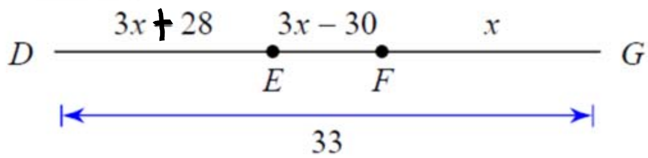
15) Find CE



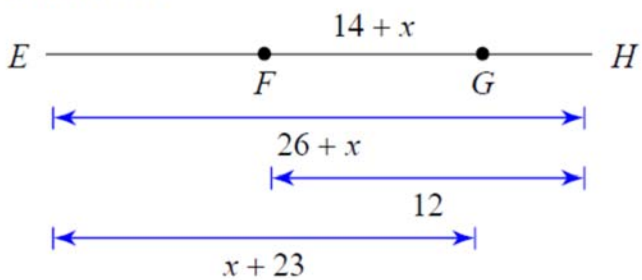
16) Find BD



17) Find DE



18) Find EG



Term	Definition	Notation	Diagram/Visual
Protractor Postulate			
Angle Addition Postulate			
Vertical Angles			
Supplemental Angles			
Complementary Angles			
Vertical Pairs			

Angle Theorems

GUIDED NOTES

Protractor Postulate

The protractor postulate states that the measurement of an angle between two rays can be designated as a unique number, and this number would be between 0 and 180 degrees. This postulate enables the use of a protractor to measure angles.

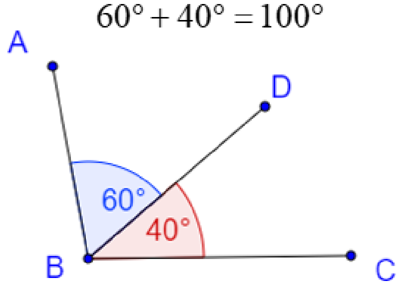
Angle Addition Postulate

Angle Addition Postulate

The angle addition postulate states that if D is in the interior of ABC then

$$\angle ABD + \angle CBD = \angle ABC$$

$$60° + 40° = 100°$$



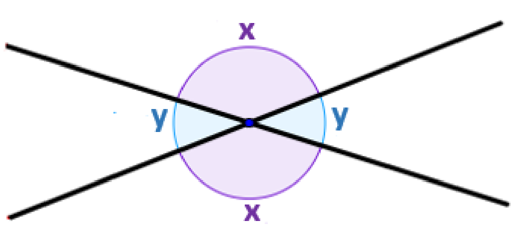
Vertical Angles

Vertical Angles

Vertical Angles are pairs of opposite angles made by intersecting lines.

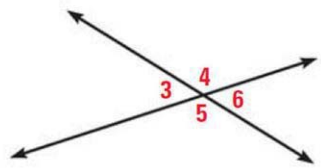
Vertical Angle Theorem

If 2 angles are vertical then they are congruent.



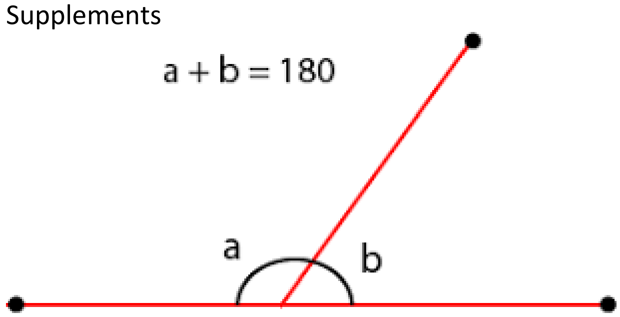
Vertical Angles

Vertical angles are any pair of non-adjacent angles formed by two intersecting lines.



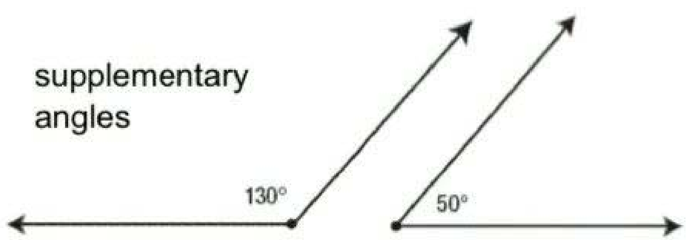
∠3 and ∠6 are vertical angles.
∠4 and ∠5 are vertical angles.

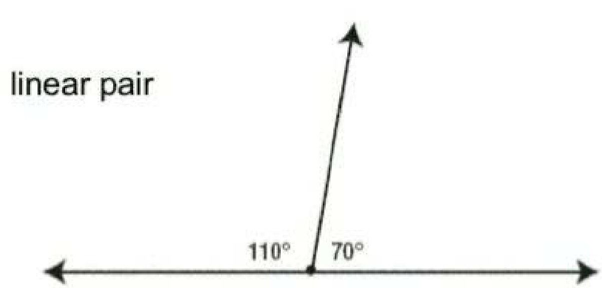
Supplements

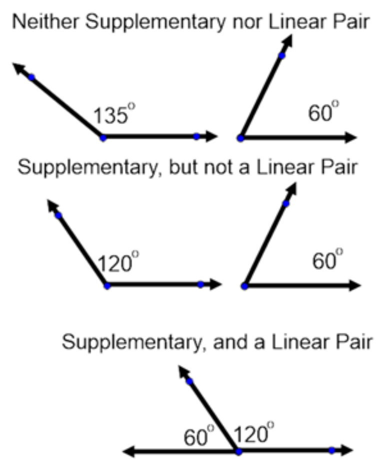


Supplementary angle

Complements

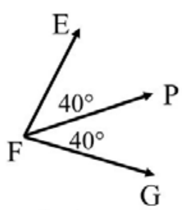






Angle Bisector Theorem

<u>*Def:* Angle Bisector:</u> The ***<u>bisector</u>*** of an angle is a ray that divides the angle into two <u>congruent</u> angles.



How do we ***<u>bisect</u>*** $\angle EFG$?

<u>Ans.:</u> Make a ray from the vertex that divides it in half.

<u>Conclusions:</u>

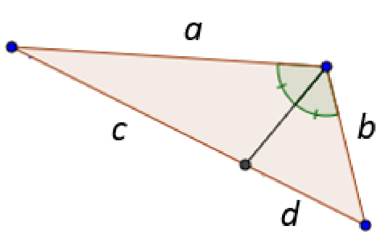
If $\overline{FP}$ bisects $\angle EFG$, then:

A) $\angle EFP \cong \angle PFG$ Or $m\angle EFP = m\angle PFG$

B) $m\angle EFP = \frac{1}{2} m\angle EFG$ C) $m\angle PFG = \frac{1}{2} m\angle EFG$

Angle Bisector Theorem

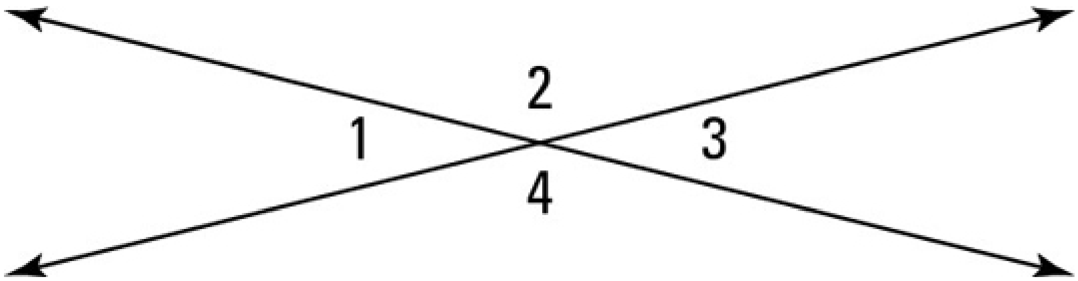
If a ray bisects an angle of a triangle, then it divides the opposite side of the triangle into segments that are proportional to the other two sides.



$$\frac{a}{c} = \frac{b}{d}$$

Proving Vertical Angles are Congruent

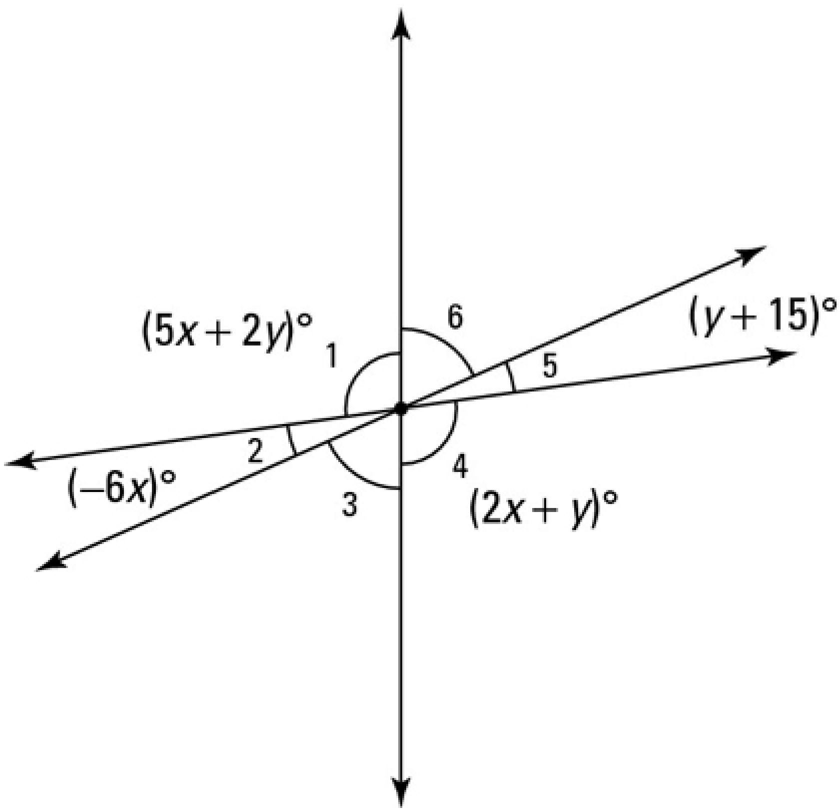
When two lines intersect to make an X, angles on opposite sides of the X are called vertical angles. These angles are equal, and here's the official theorem that tells you so.



Vertical angles are congruent: If two angles are vertical angles, then they're congruent (see the above figure).

Vertical angles are one of the most frequently used things in proofs and other types of geometry problems, and they're one of the easiest things to spot in a diagram. Don't neglect to check for them!

Here's an algebraic geometry problem that illustrates this simple concept: Determine the measure of the six angles in the following figure.



Vertical angles are congruent, so

$\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 5$;

and thus you can set their measures equal to each other:

$\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 5$

$5x + 2y = 2x + y$ $\quad -6x = y + 15$

Now you have a system of two equations and two unknowns. To solve the system, first solve each equation for *y*:

$y = -3x$

$y = -6x - 15$

Next, because both equations are solved for *y*, you can set the two *x*-expressions equal to each other and solve for *x*:

$-3x = -6x - 15$

$3x = -15$

$x = -5$

To get *y*, plug in –5 for *x* in the first simplified equation:

$y = -3x$

$y = -3(-5)$

$y = 15$

Now plug –5 and 15 into the angle expressions to get four of the six angles:

$\angle 4 \cong \angle 1 = 5x + 2y = 5(-5) + 2(15) = 5^\circ$

$\angle 5 \cong \angle 2 = -6x = -6(-5) = 30^\circ$

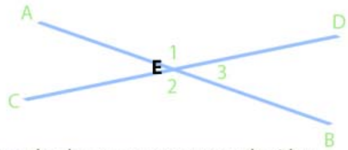
To get angle 3, note that angles 1, 2, and 3 make a straight line, so they must sum to 180°:

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$

$5^\circ + 30^\circ + \angle 3 = 180^\circ$

$\angle 3 = 145^\circ$

Finally, angle 3 and angle 6 are congruent vertical angles, so angle 6 must be 145° as well. Did you notice that the angles in the figure are absurdly out of scale? Don't forget that you can't assume anything about the relative sizes of angles or segments in a diagram.



To prove the vertical angles theorem, you must work with a diagram.

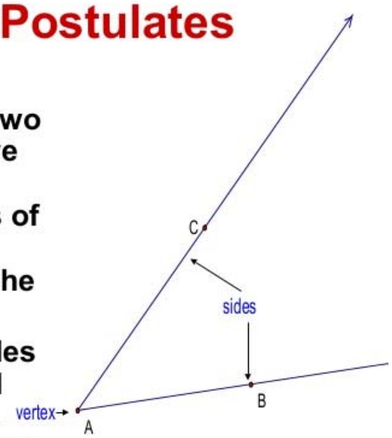
Statements	Reasons
1. AB intersects CD at E	1. Given
2. $m\angle 1 + m\angle 3 = 180$ $m\angle 2 + m\angle 3 = 180$	2. Angle Add. Post.
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	3. Substituion Prop (=)
4. $m\angle 1 = m\angle 2$	4. Subtration Prop (=)
5. $\angle 1 \cong \angle 2$	5. Definition of Congruence

Vertical Angles Theorem

Vertical angles are congruent. In the diagram above angle 1 is congruent to angle 2 because they are vertical angles.

Using Angle Postulates

- **An angle consists of two different rays that have the same initial point. The rays are the sides of the angle. The initial point is the vertex of the angle.**
- **The angle that has sides AB and AC is denoted by ∠BAC, ∠CAB, ∠A. The point A is the vertex of the angle.**



Ex.1: Naming Angles

- **Name the angles in the figure:**

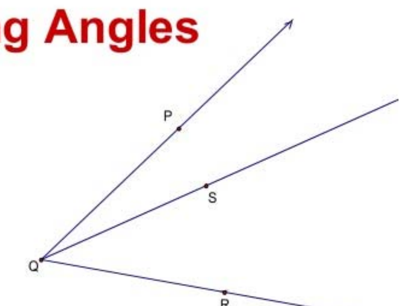
SOLUTION:

There are three different angles.

∀ ∠PQS or ∠SQP

∀ ∠SQR or ∠RQS

∀ ∠PQR or ∠RQP



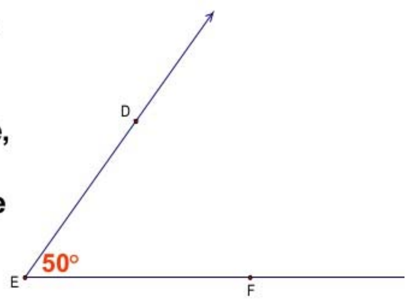
You should not name any of these angles as ∠Q because all three angles have Q as their vertex. The name ∠Q would not distinguish one angle from the others.

Note:

- **The measure of ∠A is denoted by m∠A. The measure of an angle can be approximated using a protractor, using units called degrees(°). For instance, ∠BAC has a measure of 50°, which can be written as**
  m∠BAC = 50°.

more . . .

- **Angles that have the same measure are called congruent angles. For instance, ∠BAC and ∠DEF each have a measure of 50°, so they are congruent.**



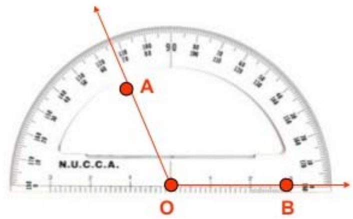
Note – Geometry doesn't use equal signs like Algebra

MEASURES ARE EQUAL	ANGLES ARE CONGRUENT
m∠BAC = m∠DEF	∠BAC ≅ ∠DEF
↑	↑
"is equal to"	"is congruent to"

Note that there is an m in front when you say equal to; whereas the congruency symbol ≅ ; you would say congruent to. (no m's in front of the angle symbols).

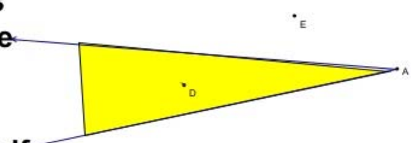
Postulate 3: Protractor Postulate

- Consider a point A on one side of OB. The rays of the form OA can be matched one to one with the real numbers from 1-180.
- The measure of ∠AOB is equal to the absolute value of the difference between the real numbers for OA and OB.



Interior/Exterior

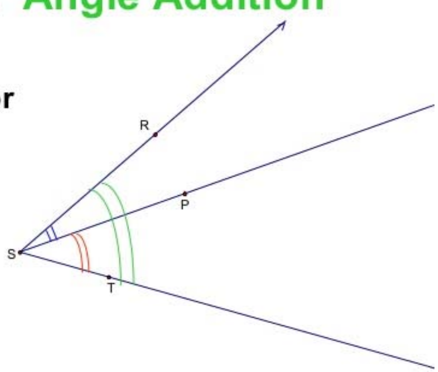
- A point is in the interior of an angle if it is between points that lie on each side of the angle.
- A point is in the exterior of an angle if it is not on the angle or in its interior.



Postulate 4: Angle Addition Postulate

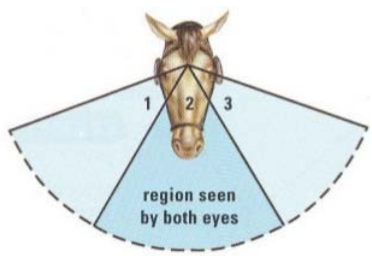
- If P is in the interior of ∠RST, then

$m\angle RSP + m\angle PST = m\angle RST$



Ex. 2: Calculating Angle Measures

- VISION. Each eye of a horse wearing blinkers has an angle of vision that measures 100°. The angle of vision that is seen by both eyes measures 60°.
- Find the angle of vision seen by the left eye alone.



Solution:

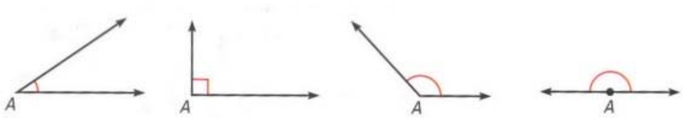
You can use the Angle Addition Postulate.

$m\angle 2 + m\angle 3 = 100°$	Total vision for left eye is 100°.
$m\angle 3 = 100° - m\angle 2$	Subtract $m\angle 2$ from each side.
$m\angle 3 = 100° - 60°$	Substitute 60° for $m\angle 2$.
$m\angle 3 = 40°$	Subtract.

So, the vision for the left eye alone measures 40°.

Classifying Angles

- Angles are classified as acute, right, obtuse, and straight, according to their measures. Angles have measures greater than 0° and less than or equal to 180°.



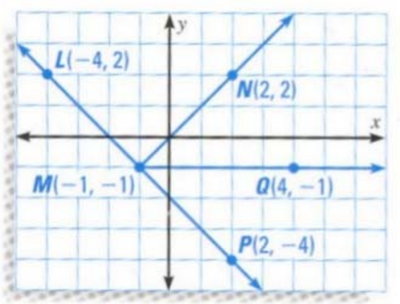
Ex. 3: Classifying Angles in a Coordinate Plane

- **Plot the points L(-4,2), M(-1,-1), N(2,2), Q(4,-1), and P(2,-4). Then measure and classify the following angles as acute, right, obtuse, or straight.**

α. ∠LMN
β. ∠LMP
χ. ∠NMQ
δ. ∠LMQ

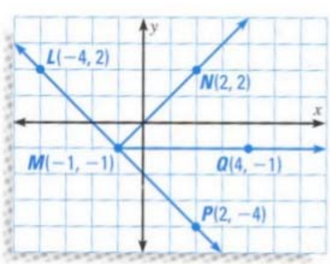
Solution:

- **Begin by plotting the points. Then use a protractor to measure each angle.**



Solution:

- **Begin by plotting the points. Then use a protractor to measure each angle.**



	MEASURE	CLASSIFICATION
a.	$m\angle LMN = 90°$	right angle
b.	$m\angle LMP = 180°$	straight angle
c.	$m\angle NMQ = 45°$	acute angle
d.	$m\angle LMQ = 135°$	obtuse angle

Two angles are adjacent angles if they share a common vertex and side, but have no common interior points.

Ex. 4: Drawing Adjacent Angles

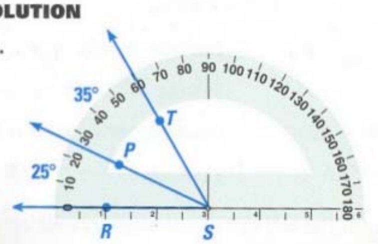
- **Use a protractor to draw two adjacent acute angles ∠RSP and ∠PST so that ∠RST is (a) acute and (b) obtuse.**

Ex. 4: Drawing Adjacent Angles

- **Use a protractor to draw two adjacent acute angles ∠RSP and ∠PST so that ∠RST is (a) acute and (b) obtuse.**

SOLUTION

a.

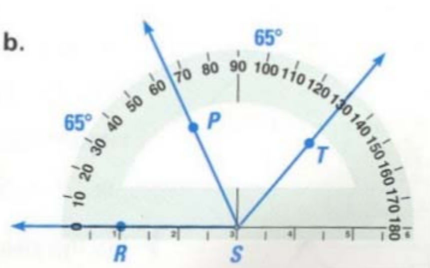


Ex. 4: Drawing Adjacent Angles

- **Use a protractor to draw two adjacent acute angles ∠RSP and ∠PST so that ∠RST is (a) acute and (b) obtuse.**

Solution:

b.

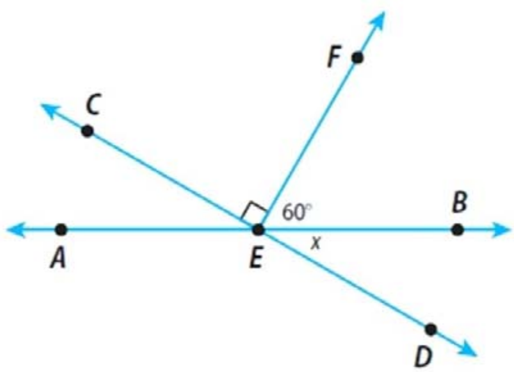


Closure Question:

- **Describe how angles are classified.**

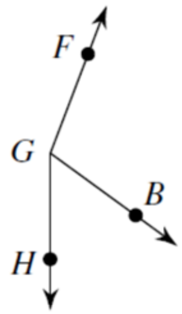
Angles are classified according to their measure. Those measuring less than 90° are acute. Those measuring 90° are right. Those measuring between 90° and 180° are obtuse, and those measuring exactly 180° are straight angles.

Example!

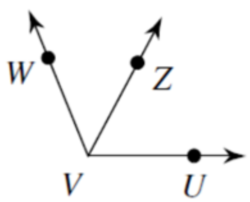


- Name a pair of vertical angles.
- Name a pair of adjacent angles.
- Name a pair of complementary angles.
- Name a pair of supplementary angles.

Find $m\angle FGH$ if $m\angle FGB = 105°$ and $m\angle BGH = 54°$.

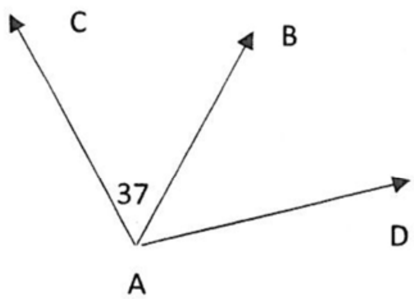


Find $m\angle WVU$ if $m\angle ZVU = 62°$ and $m\angle WVZ = 50°$.

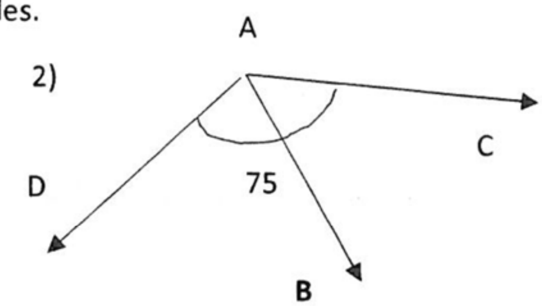


The angle is bisected by AB. Find the missing two angles.

1)

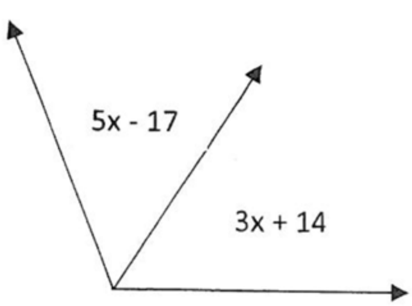


2)

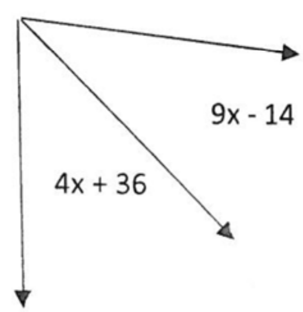


Solve for x.

3)

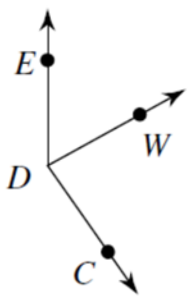
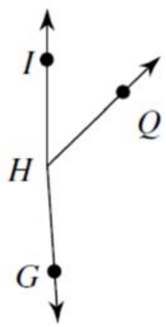


4)

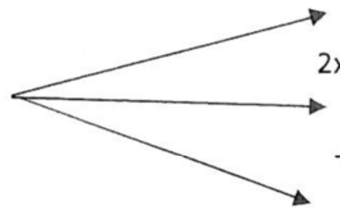


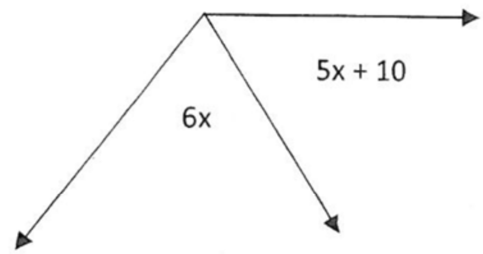
Let O be in the interior of ∠PQR. Use the angle addition postulate to solve for x. Find the measure of each angle.	
$m\angle PQO = (x + 4)°$ $m\angle OQR = (2x - 2)°$ $m\angle PQR = 26°$	

SELF CHECK

Find $m\angle WDC$ if $m\angle EDC = 145°$ and $m\angle EDW = 61°$. 	
Find $m\angle IHQ$ if $m\angle IHG = 176°$ and $m\angle QHG = 130°$. 	

The angle is bisected by AB. Find the missing two angles.





7) Use the diagram and the following information to answer the questions.

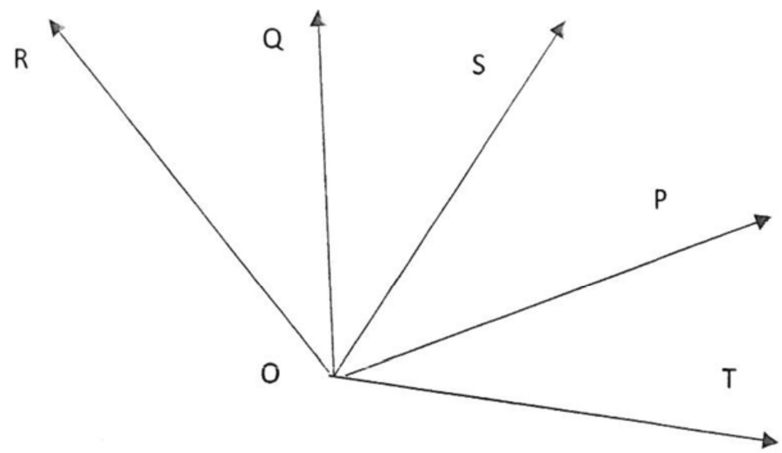
m∠ROT = 127 m∠SOT = 71 m∠ROQ = m∠QOS = m∠POT

a) m∠QOP

b) m∠QOT

c) m∠ROQ

d) m∠SOP

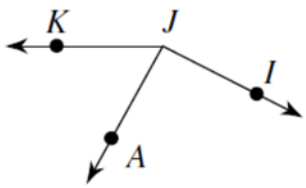


Let O be in the interior of ∠PQR. Use the angle addition postulate to solve for x. Find the measure of each angle.

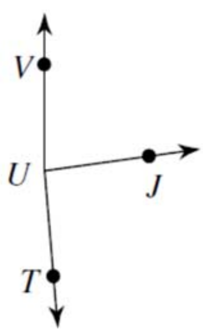
m∠PQO = (3x + 7)° m∠QOR = (5x - 2)° m∠PQR = 61°	
m∠PQO = $(\frac{1}{3}x + \frac{1}{3})°$ m∠OQR = $(2x + \frac{4}{3})°$ m∠PQR = (5x - 1)°	

If you wanted to predict the location of the moon in the sky at 9:15pm, how might you go about doing that?

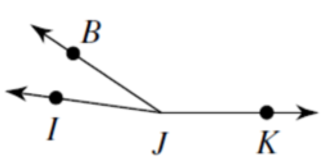
Find $m\angle IJA$ if $m\angle AJK = 61°$ and $m\angle IJK = 153°$.



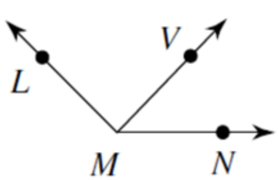
$m\angle VUT = 175°$, $m\angle VUJ = 17x - 3$, and $m\angle JUT = 17x + 8$. Find x.

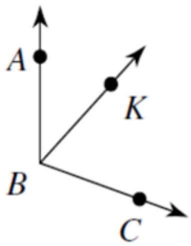
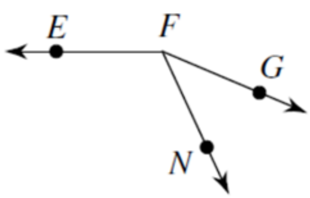


Find x if $m\angle BJK = 146 + 2x$, $m\angle IJK = 172°$, and $m\angle IJB = 2x + 26$.



Find x if $m\angle LMN = 135°$, $m\angle LMV = -1 + 45x$, and $m\angle VMN = 23x$.



$m\angle ABC = 17x + 8$, $m\angle ABK = 42°$, and $m\angle KBC = 12x - 4$. Find $m\angle ABC$. 	
$m\angle GFN = 4x + 10$, $m\angle NFE = 14x + 3$, and $m\angle GFE = 157°$. Find $m\angle NFE$. 	

Angle Theorems

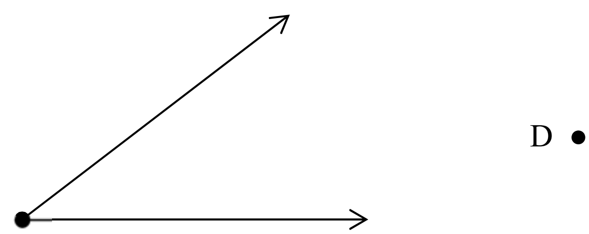
The study of Geometry was born in Ancient Greece, where mathematics was thought to be embedded in everything from music to art to the governing of the universe. Plato, an ancient philosopher and teacher, had the statement, "Let no man ignorant of geometry enter here," placed at the entrance of his school. This illustrates the importance of the study of shapes and logic during that era. Everyone who learned geometry was challenged to construct geometric objects using two simple tools, known as Euclidean tools:

- A straight edge without any markings
- A compass

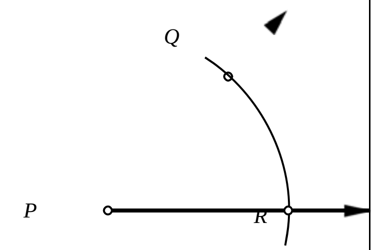
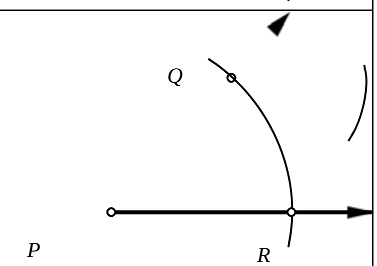
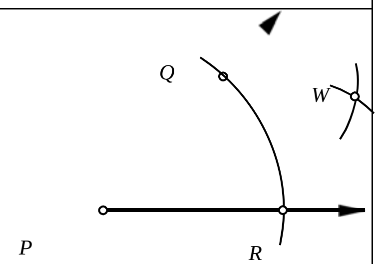
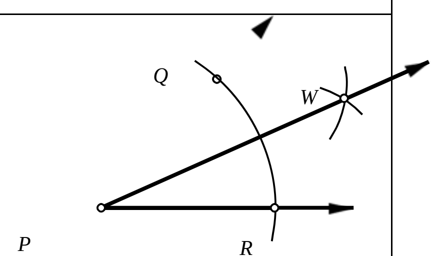
The straight edge could be used to construct lines, the compass to construct circles. As geometry grew in popularity, math students and mathematicians would challenge each other to create constructions using only these two tools. Some constructions were fairly easy (Can you construct a square?), some more challenging, (Can you construct a regular pentagon?), and some impossible even for the greatest geometers (Can you trisect an angle? In other words, can you divide an angle into three equal angles?). Archimedes (287-212 B.C.E.) came close to solving the trisection problem, but his solution used a marked straight edge. What constructions can you create?

Your Third Challenge: Can you copy an angle?

Now that you know how to copy a segment, copying an angle is easy. How would you construct a copy of an angle at a new point? Discuss this with a partner and come up with a strategy. Think about what congruent triangles are imbedded in your construction and use them to justify why your construction works. Be prepared to share your ideas with the class.

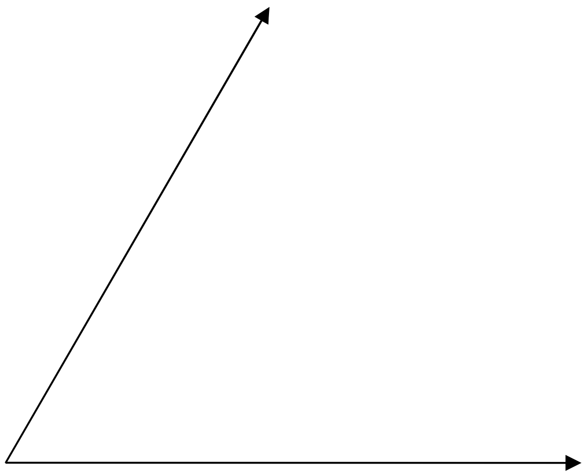


Your Fifth Challenge: Can you bisect an angle?

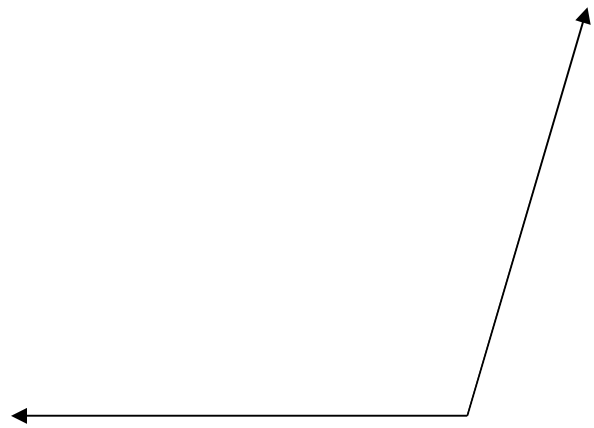
1. Let point *P* be the vertex of the angle. Place the compass on point *P* and draw an arc across both sides of the angle. Label the intersection points *Q* and *R*.	
2. Place the compass on point *Q* and draw an arc across the interior of the angle.	
3. Without changing the radius of the compass, place it on point *R* and draw an arc intersecting the one drawn in the previous step. Label the intersection point *W*.	
4. Using the straightedge, draw ray *PW*. This is the bisector of $\angle QPR$.	

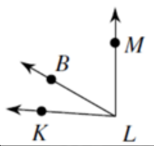
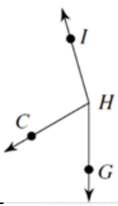
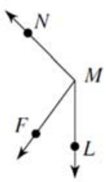
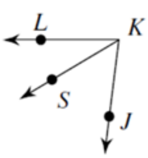
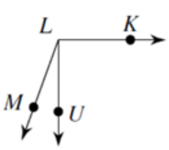
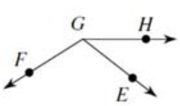
Construct the angle bisector. Mark congruent angles. Check your construction by measuring with a protractor.

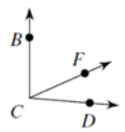
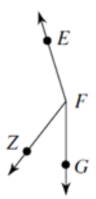
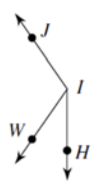
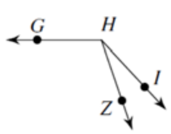
1.



2.



Find $m\angle KLM$ if $m\angle KLB = 26°$ and $m\angle BLM = 60°$.  	
$m\angle GHC = 60°$ and $m\angle CHI = 104°$ Find $m\angle GHI$.  	
$m\angle FMN = 99°$ and $m\angle LMF = 36°$. Find $m\angle LMN$.  	
Find $m\angle JKL$ if $m\angle SKL = 31°$ and $m\angle JKS = 52°$.  	
Find $m\angle KLU$ if $m\angle ULM = 20°$ and $m\angle KLM = 110°$.  	
$m\angle HGF = 16x + 4$, $m\angle EGF = 110°$, and $m\angle HGE = 3x + 11$. Find x.  	

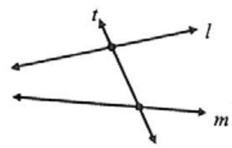
$m\angle FCD = x + 41$, $m\angle BCF = x + 78$, and $m\angle BCD = 95°$. Find x. 	
$m\angle GFZ = 38°$, $m\angle ZFE = 2x + 125$, and $m\angle GFE = x + 163$. Find x. 	
Find $m\angle HIW$ if $m\angle WIJ = 10x$, $m\angle HIJ = 145°$, and $m\angle HIW = 2x + 13$. 	
$m\angle ZHG = 11x - 1$, $m\angle IHZ = 24°$, and $m\angle IHG = 12x + 13$. Find $m\angle IHG$. 	

Term	Definition	Notation	Diagram/Visual
Parallel Lines			
Corresponding Angles Postulate			
Transversal			
Alternate Interior Angles Theorem			
Consecutive (same side) Interior Angles Theorem			
Alternate Exterior Angles Theorem			
Consecutive (same side) Exterior Angles Theorem			

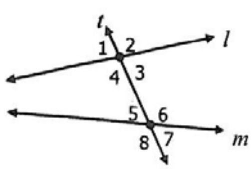
THE BIG IDEA

Parallel Lines Theorems

GUIDED NOTES

TRANSVERSAL		• A line that intersects two or more lines. • **Example:** ____________________

ANGLES *formed by* TRANSVERSALS

Diagram	Angle Pairs	Examples
	**Corresponding Angles** (Angles on the same side of the transversal and in the same position.)	________, ________, ________, ________
	Alternate Interior Angles (Interior angles, non-adjacent, and on opposite sides of the transversal.)	________, ________
	Alternate Exterior Angles (Exterior angles, non-adjacent, and on opposite sides of the transversal.)	________, ________
	Consecutive Interior Angles (Interior angles that are on the same side of the transversal.)	________, ________

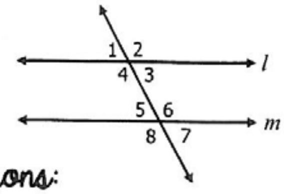
Parallel Lines & Transversals

If two PARALLEL lines are cut by a transversal, then...

- Each pair of **corresponding angles** is congruent
- Each pair of **alternate interior angles** is congruent.
- Each pair of **alternate exterior angles** is congruent.
- Each pair of **consecutive interior angles** is supplementary.

And recall from Unit 1, **vertical angles** are always congruent and a **linear pair is** always supplementary.

Proving Lines Parallel



You can prove lines are parallel by the following reasons:

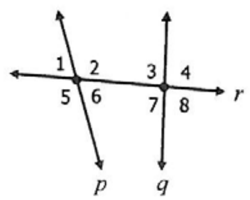
Corresponding Angles Converse	If two lines are cut by a transversal so that corresponding angles are congruent, then the lines are parallel. **Example:** ____________
Alternate Interior Angles Converse	If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel. **Example:** ____________
Alternate Exterior Angles Converse	If two lines are cut by a transversal so that alternate exterior angles are congruent, then the lines are parallel. **Example:** ____________
Consecutive Interior Angles Converse	If two lines are cut by a transversal so that consecutive interior angles are supplementary, then the lines are parallel. **Example:** ____________

Corresponding Angles	If two **parallel** lines are cut by a transversal, then the pairs of corresponding angles are congruent.
Corresponding Angles Converse	If two lines are cut by a transversal and the corresponding angles are congruent, the lines are **parallel**.
Alternate Interior Angles	If two **parallel** lines are cut by a transversal, then the alternate interior angles are congruent.
Alternate Exterior Angles	If two **parallel** lines are cut by a transversal, then the alternate exterior angles are congruent.
Interiors on Same Side	If two **parallel** lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary.
Alternate Interior Angles Converse	If two lines are cut by a transversal and the alternate interior angles are congruent, the lines are **parallel**.
Alternate Exterior Angles Converse	If two lines are cut by a transversal and the alternate exterior angles are congruent, the lines are **parallel.**
Interiors on Same Side Converse	If two lines are cut by a transversal and the interior angles on the same side of the transversal are supplementary, the lines are **parallel.**

Example!

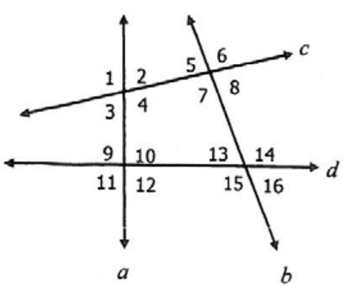
Examples! Name the type of angle relationship. If no relationship, write "none."

1



a. ∠1 and ∠8	
b. ∠2 and ∠3	
c. ∠5 and ∠7	
d. ∠2 and ∠7	
e. ∠1 and ∠3	
f. ∠6 and ∠7	

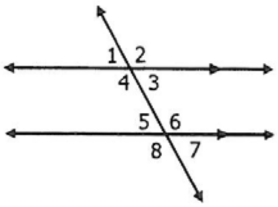
2



Important!
Angles must belong to the SAME transversal to be an angle pair.

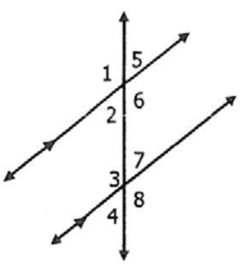
a. ∠5 and ∠13	
b. ∠7 and ∠14	
c. ∠3 and ∠6	
d. ∠9 and ∠16	
e. ∠4 and ∠7	
f. ∠2 and ∠10	
g. ∠8 and ∠14	
h. ∠6 and ∠11	
i. ∠4 and ∠13	
j. ∠4 and ∠9	

Example 1 Given m∠1 = 65°, find the measure of each missing angle. Give your reasoning.



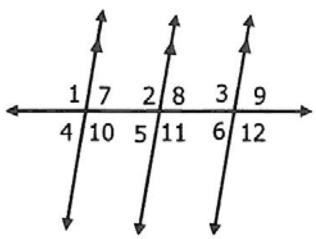
a. $m\angle 2$ =	
b. $m\angle 3$ =	
c. $m\angle 4$ =	
d. $m\angle 5$ =	
e. $m\angle 6$ =	
f. $m\angle 7$ =	
g. $m\angle 8$ =	

Example 2 Given m∠6 = 142°, find the measure of each missing angle. Give your reasoning.



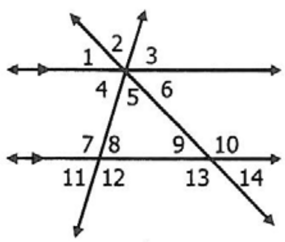
a. $m\angle 1$ =	
b. $m\angle 2$ =	
c. $m\angle 3$ =	
d. $m\angle 4$ =	
e. $m\angle 5$ =	
f. $m\angle 7$ =	
g. $m\angle 8$ =	

Example 3 Given m∠5 = 82°, find the measure of each missing angle.



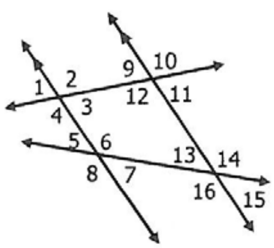
a. $m\angle 1 =$	**e.** $m\angle 6 =$	**i.** $m\angle 10 =$
b. $m\angle 2 =$	**f.** $m\angle 7 =$	**j.** $m\angle 11 =$
c. $m\angle 3 =$	**g.** $m\angle 8 =$	**k.** $m\angle 12 =$
d. $m\angle 4 =$	**h.** $m\angle 9 =$	

Example 4 Given m∠12 = 121° and m∠6 = 75°, find the measure of each missing angle.



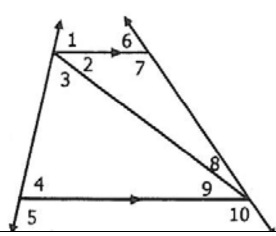
a. $m\angle 1 =$	**e.** $m\angle 5 =$	**i.** $m\angle 10 =$
b. $m\angle 2 =$	**f.** $m\angle 7 =$	**j.** $m\angle 11 =$
c. $m\angle 3 =$	**g.** $m\angle 8 =$	**k.** $m\angle 13 =$
d. $m\angle 4 =$	**h.** $m\angle 9 =$	**l.** $m\angle 14 =$

Example 5 Given m∠7 = 38° and m∠10 = 102°, find the measure of each missing angle.



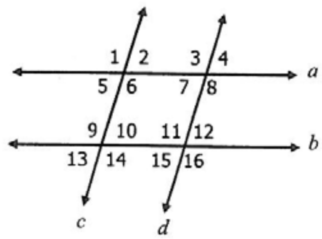
a. $m\angle 1 =$	**f.** $m\angle 6 =$	**k.** $m\angle 13 =$
b. $m\angle 2 =$	**g.** $m\angle 8 =$	**l.** $m\angle 14 =$
c. $m\angle 3 =$	**h.** $m\angle 9 =$	**m.** $m\angle 15 =$
d. $m\angle 4 =$	**i.** $m\angle 11 =$	**n.** $m\angle 16 =$
e. $m\angle 5 =$	**j.** $m\angle 12 =$	

Example 6 Given m∠2 = 41°, m∠5 = 94°, and m∠10 = 109°, find the measure of each missing angle.



a. $m\angle 1 =$	**d.** $m\angle 6 =$	**g.** $m\angle 9 =$
b. $m\angle 3 =$	**e.** $m\angle 7 =$	
c. $m\angle 4 =$	**f.** $m\angle 8 =$	

Determine which lines, if any, are parallel. Use converse theorems to justify your answers.

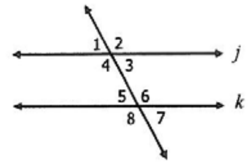


1

Given	Parallel Lines	Converse
a. $\angle 2 \cong \angle 4$		
b. $\angle 5 \cong \angle 10$		
c. $m\angle 6 + m\angle 10 = 180°$		
d. $\angle 1 \cong \angle 14$		
e. $m\angle 14 + m\angle 15 = 180°$		
f. $\angle 11 \cong \angle 16$		
g. $\angle 4 \cong \angle 15$		
h. $\angle 10 \cong \angle 12$		
i. $m\angle 9 + m\angle 13 = 180°$		
j. $\angle 2 \cong \angle 7$		
k. $\angle 6 \cong \angle 11$		

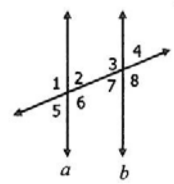
Proofs: Complete the proof below by filling in the missing reasons.

③ **Given:** ∠4 and ∠5 are supplementary
Prove: $j \parallel k$



Statements	Reasons
1. ∠ 4 and ∠5 are supplementary	**1.**
2. $m\angle 4 + m\angle 5 = 180°$	**2.**
3. $j \parallel k$	**3.**

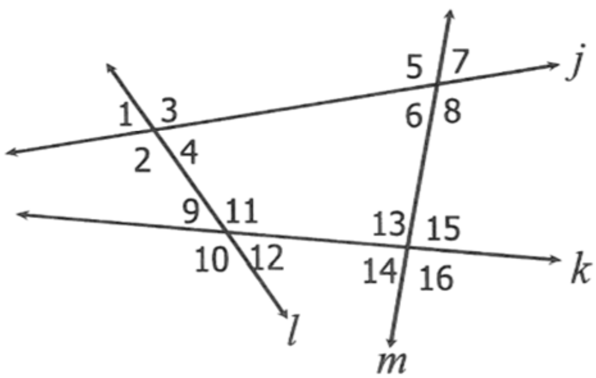
④ **Given:** ∠1 and ∠2 form a linear pair; ∠1 and ∠4 are supplementary
Prove: $a \parallel b$



Statements	Reasons
1. ∠1 and ∠2 form a linear pair	**1.**
2. ∠1 and ∠2 are supplementary	**2.**
3. ∠1 and ∠4 are supplementary	**3.**
4. ∠2 ≅ ∠4	**4.**
5. $a \parallel b$	**5.**

SELF CHECK

ANGLES *formed by* TRANSVERSALS



Name that Transversal!

$\angle 1$ and $\angle 12$: ______________

$\angle 3$ and $\angle 6$: ______________

$\angle 7$ and $\angle 15$: ______________

$\angle 12$ and $\angle 14$: ______________

Corresponding Angles

- ____________________
- ____________________
- ____________________
- ____________________
- ____________________

Alternate Interior Angles

- ____________________
- ____________________
- ____________________
- ____________________
- ____________________

Alternate Exterior Angles

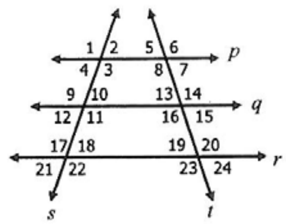
- ____________________
- ____________________
- ____________________
- ____________________
- ____________________

Consecutive Interior Angles

- ____________________
- ____________________
- ____________________
- ____________________
- ____________________

Non-Examples: ______________________________ ______________________________ ______

Determine which lines, if any, are parallel. Use converse theorems to justify your answers.



2

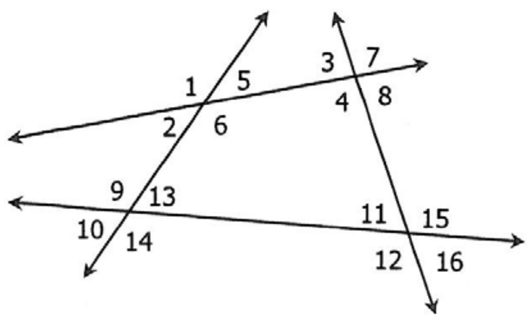
Given	Parallel Lines	Converse
a. $\angle 9 \cong \angle 22$		
b. $m\angle 8 + m\angle 13 = 180°$		
c. $\angle 1 \cong \angle 17$		
d. $\angle 16 \cong \angle 20$		
e. $\angle 5 \cong \angle 24$		
f. $m\angle 4 + m\angle 17 = 180°$		
g. $\angle 10 \cong \angle 13$		
h. $\angle 3 \cong \angle 22$		
i. $\angle 5 \cong \angle 15$		
j. $m\angle 11 + m\angle 16 = 180°$		

If two lines that intersect are vertical angles and one angle measure is given, do you have enough information to know all of the angle measures? Why or why not?

Independent Practice

Name that Angle Pair!

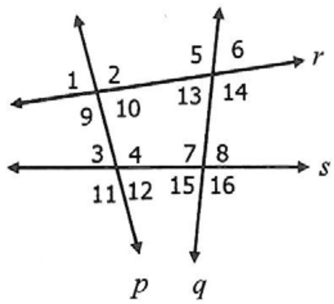
Directions: Given the diagram below, determine whether the angle pairs are corresponding, alternate interior, alternate exterior, consecutive interior, or no relationship. Color the boxes using the color codes below.



Corresponding Angles: Pink
Alternate Interior Angles: Light Blue
Alternate Exterior Angles: Yellow
Consecutive Interior Angles: Light Green
No Relationship: Uncolored

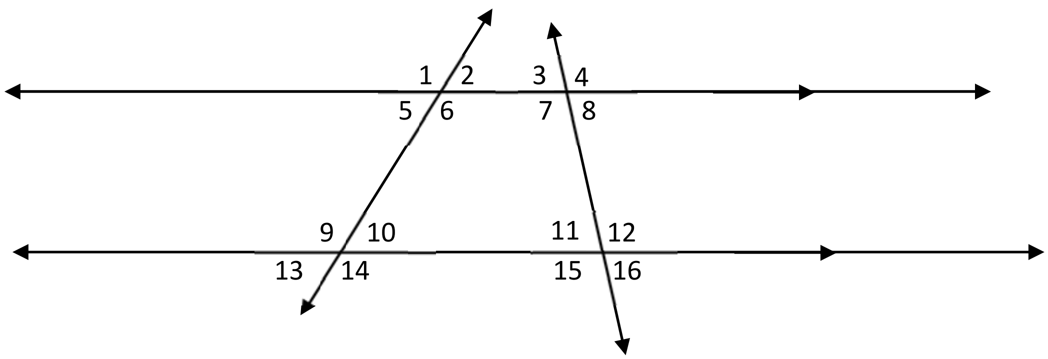
∠2 and ∠13	∠4 and ∠11	∠1 and ∠16	∠8 and ∠11
∠3 and ∠5	∠14 and ∠16	∠1 and ∠3	∠5 and ∠10
∠3 and ∠15	∠4 and ∠5	∠6 and ∠13	∠4 and ∠13
∠2 and ∠7	∠4 and ∠12	∠3 and ∠16	∠12 and ∠14
∠7 and ∠15	∠6 and ∠16	∠11 and ∠14	∠1 and ∠3
∠1 and ∠11	∠8 and ∠15	∠7 and ∠13	∠1 and ∠14

5. Using the diagram below, classify the angle pairs as corresponding, alternate interior, alternate exterior, consecutive interior, or none.



a. ∠4 and ∠7	**b.** ∠2 and ∠11
c. ∠12 and ∠16	**d.** ∠8 and ∠13
e. ∠11 and ∠15	**f.** ∠7 and ∠10
g. ∠1 and ∠14	**h.** ∠12 and ∠15
i. ∠6 and ∠7	**j.** ∠1 and ∠3
k. ∠14 and ∠16	**l.** ∠6 and ∠15
m. ∠5 and ∠10	**n.** ∠8 and ∠14

Parallel Lines and Transversals



Let m∠1 = 115° and m∠12 = 110°

1. m∠9 = ______	5. m∠4 = ______
2. m∠10 = ______	6. m∠11 = ______
3. m∠8 = ______	7. m∠5 = ______
4. m∠3 = ______	8. m∠14 = ______

Refer to the above figure and identify the special angle pair name.

9) ∠7 and ∠2 ____

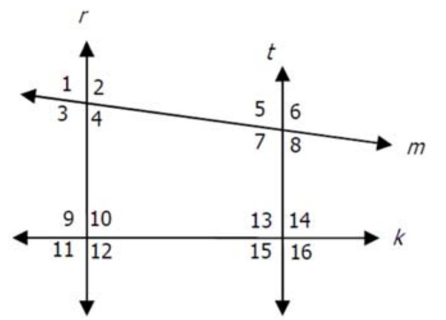
10) ∠6 and ∠14 ____

11) ∠13 and ∠12 ____

12) ∠7 and ∠11 ____

13) ∠4 and ∠8 ____

Using the figure below, state the transversal that forms each pair of angles. Then identify the special name for the angle pair.



$\angle 1$ and $\angle 12$ transversal = ______ special name = ________________

$\angle 2$ and $\angle 10$ transversal = ______ special name = ________________

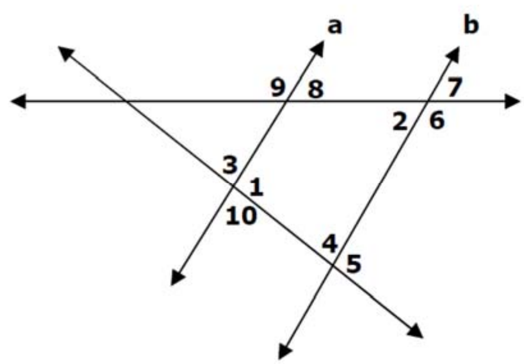
$\angle 4$ and $\angle 9$ transversal = ______ special name = ________________

$\angle 6$ and $\angle 3$ transversal = ______ special name = ________________

$\angle 14$ and $\angle 10$ transversal = ______ special name = ________________

$\angle 7$ and $\angle 13$ transversal = ______ special name = ________________

$a \parallel b$, $m\angle 1 = 78°$, and $m\angle 2 = 47°$.
Find measure of each angle.



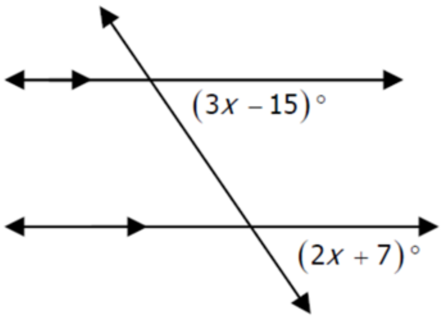
$\angle 3$ $\angle 4$

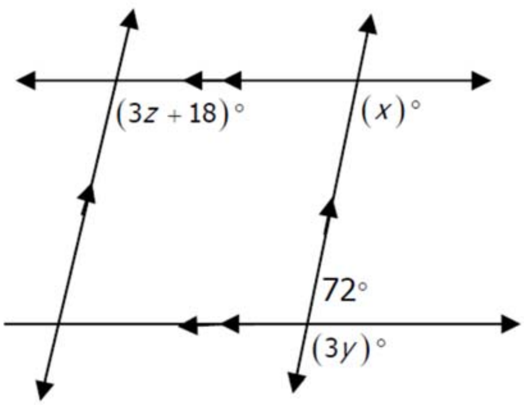
$\angle 5$ $\angle 6$

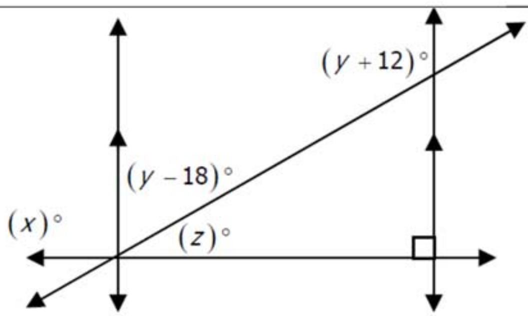
$\angle 7$ $\angle 8$

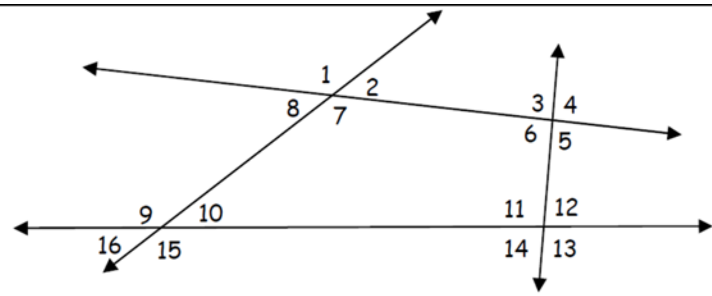
$\angle 9$ $\angle 10$

Find the missing values of x and y.









Use the picture above to identify the special name for the angle pairs.

43) $\angle 2$ and $\angle 6$ ____________________

44) $\angle 1$ and $\angle 9$ ____________________

45) $\angle 9$ and $\angle 6$ ____________________

46) $\angle 9$ and $\angle 13$ ____________________

47) $\angle 14$ and $\angle 16$ ____________________

48) $\angle 10$ and $\angle 16$ ____________________

49) $\angle 2$ and $\angle 1$ ____________________

50) $\angle 10$ and $\angle 14$ ____________________

51) $\angle 11$ and $\angle 6$ ____________________

52) $\angle 15$ and $\angle 11$ ____________________

53) $\angle 4$ and $\angle 13$ ____________________

54) $\angle 3$ and $\angle 11$ ____________________

If $m\angle 2 = 58°$ and $m\angle 13 = 111°$, then find the missing angle measures. x ll m, z ll y

$m\angle 1 =$ _______

$m\angle 2 =$ _______

$m\angle 3 =$ _______

$m\angle 4 =$ _______

$m\angle 5 =$ _______

$m\angle 6 =$ _______

$m\angle 7 =$ _______

$m\angle 8 =$ _______

$m\angle 9 =$ _______

$m\angle 10 =$ _______

$m\angle 11 =$ _______

$m\angle 12 =$ _______

$m\angle 13 =$ _______

$m\angle 14 =$ _______

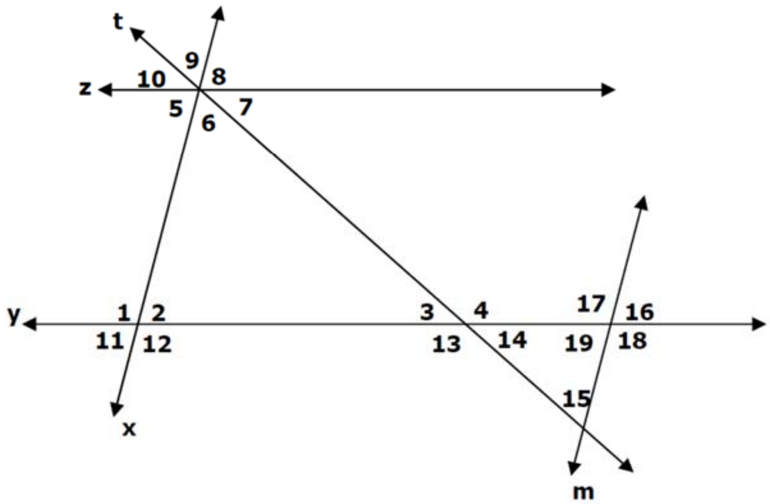
$m\angle 15 =$ _______

$m\angle 16 =$ _______ (16-19 look at line x and m)

$m\angle 17 =$ _______

$m\angle 18 =$ _______

$m\angle 19 =$ _______



Parallel Lines Theorems Tasks

Lunch Lines

Mathematical Goals

- Prove vertical angles are congruent.
- Understand when a transversal is drawn through parallel lines, special angles relationships occur.
- Prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angels are congruent.

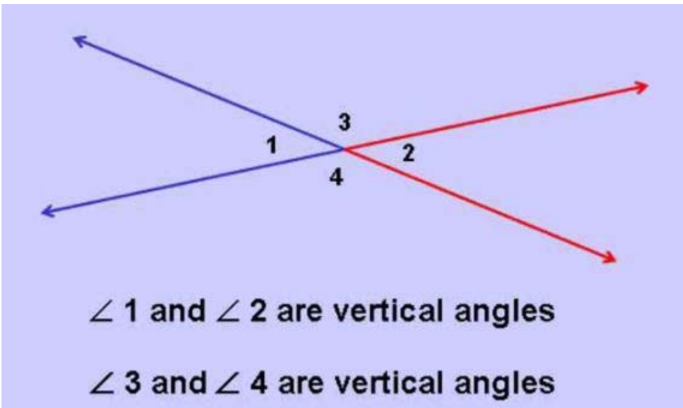
STANDARDS ADDRESSED IN THIS TASK

MGSE9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them** by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
3. **Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. **Model with mathematics** by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.
5. **Use appropriate tools strategically** by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.
6. **Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

Two angles are <u>vertical angles</u> if their sides form two pairs of opposite rays.



How do you know that vertical angles are congruent?

$m\angle 1 + m\angle 3 = 180°$ because the Linear Pair postulate $m\angle 2 + m\angle 3 = 180°$ because the Linear Pair postulate

Set the two equations equal to each other since they both equal 180 degrees.

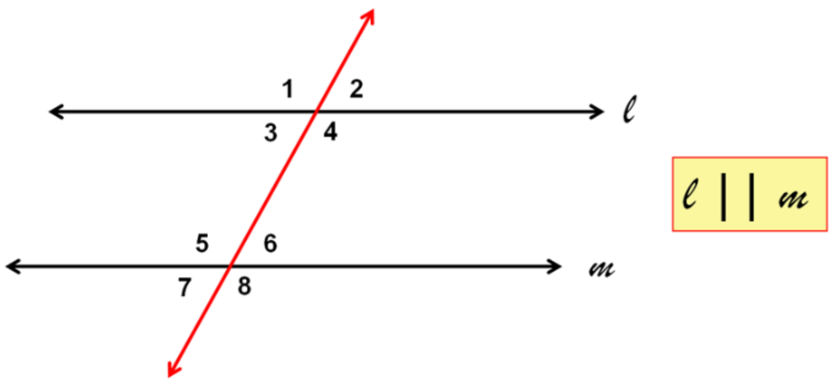
$m\angle 2 + m\angle 3 = m\angle 1 + m\angle 3$
$-m\angle 3 \qquad -m\angle 3$ $m\angle 2 = m\angle 1$

Therefore: $\angle 2 \cong \angle 1$

Prove that $\angle 3 \cong \angle 4$ using a similar method.

When a transversal crosses parallel lines, there are several pairs of special angles. Let's look at a few together.

Corresponding Angle Postulate: If two parallel lines are cut by a transversal, then corresponding angles are congruent.



Using this postulate, name a pair of congruent angles. How do

we know that $\angle 3 \cong \angle 6$?

Alternate Interior Angle Theorem: If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

Prove this theorem using the figure above.

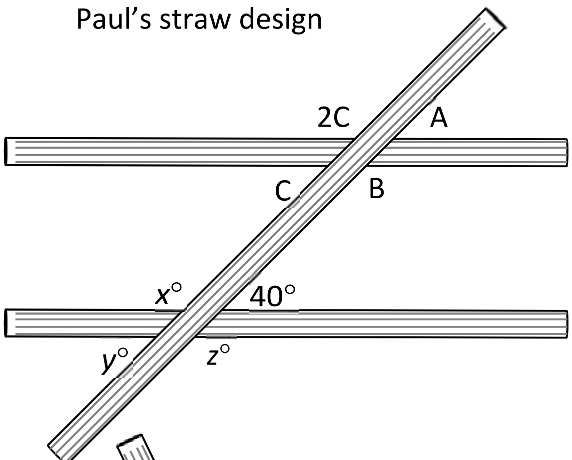
How do we know that $\angle 3 \cong \angle 5$ are supplementary?

Same-Side Interior Angle Theorem: If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.

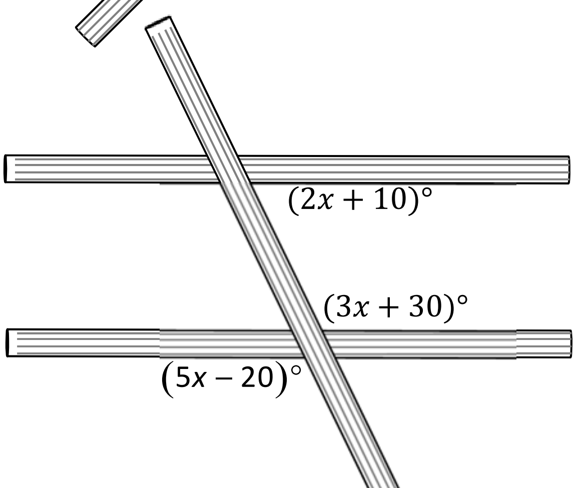
Prove this theorem using the figure above.

Paul, Jane, Justin, and Opal were finished with lunch and began playing with drink straws. Each one was making a line design using either 3 or 4 straws.
They had just come from math class where they had been studying special angles.
Paul pulled his pencil out of his book bag and labeled some of the angles and lines. He then challenged himself and the others to find all the labeled angle measurements in Paul and Justin's straw designs and to determine whether the lines that appear to be parallel really are parallel.

Paul's straw design

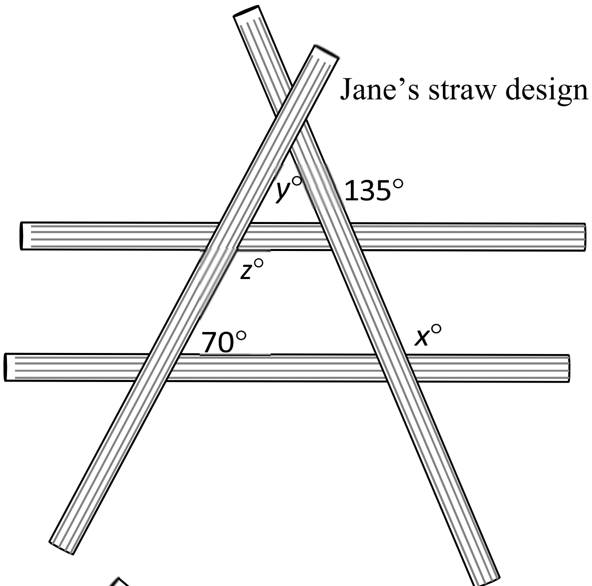


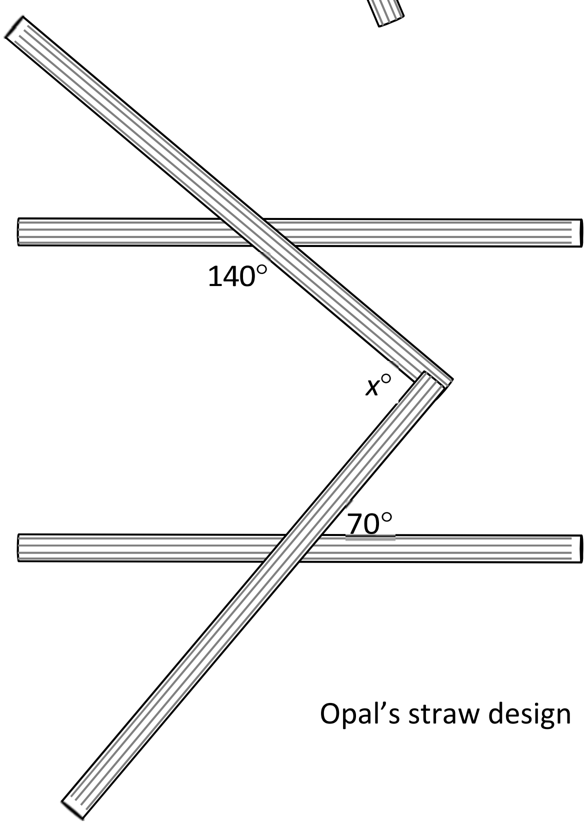
- Find all of the labeled angle measurements.
- Determine whether the lines that appear to be parallel really are parallel.
- Explain the reasoning for your results.



Justin's straw design

Paul then challenged himself and the others to find all the labeled angle measurements in Jane and Opal's straw designs knowing that the lines created by the straws in their designs were parallel.





- Find all of the labeled angle measurements (knowing that the lines created by the straws are parallel).
- Explain the reasoning for your results

Parallel Lines Theorems Tasks

Lunch Lines

Mathematical Goals

- Prove vertical angles are congruent.
- Understand when a transversal is drawn through parallel lines, special angles relationships occur.
- Prove when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angels are congruent.

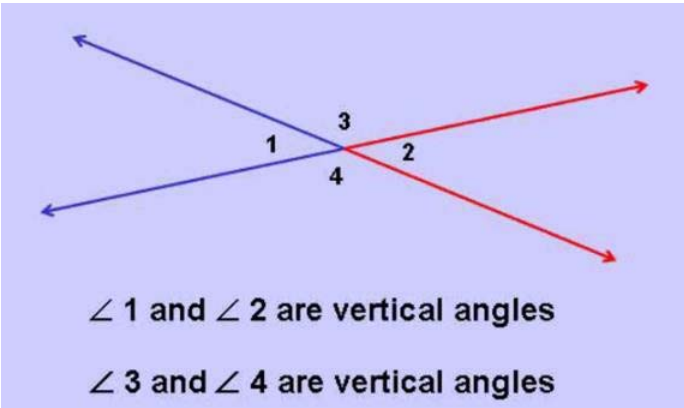
STANDARDS ADDRESSED IN THIS TASK

MGSE9-12.G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them** by requiring students to interpret and make meaning of a problem and find a logical starting point, and to monitor their progress and change their approach to solving the problem, if necessary.
2. **Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.
3. **Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. **Model with mathematics** by expecting students to apply the mathematics concepts they know in order to solve problems arising in everyday situations, and reflect on whether the results are sensible for the given scenario.
5. **Use appropriate tools strategically** by expecting students to consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a compass, a calculator, software, etc.
6. **Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

Two angles are <u>vertical angles</u> if their sides form two pairs of opposite rays.



How do you know that vertical angles are congruent?

$m\angle 1 + m\angle 3 = 180°$ because the Linear Pair postulate $m\angle 2 +$
$m\angle 3 = 180°$ because the Linear Pair postulate

Set the two equations equal to each other since they both equal 180 degrees.

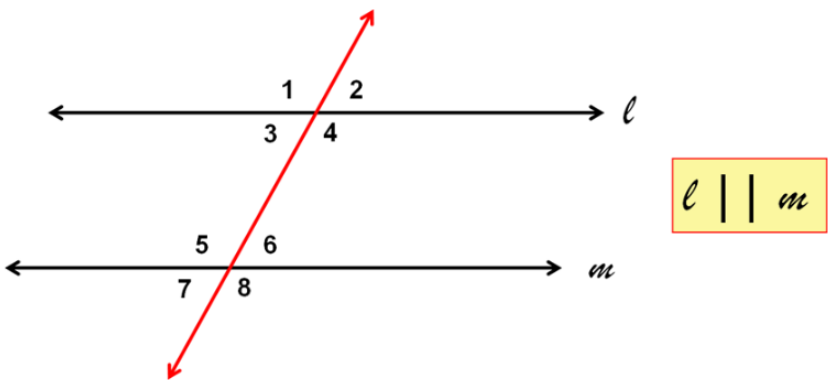
$m\angle 2 + m\angle 3 = m\angle 1 + m\angle 3$
$-m\angle 3$ $-m\angle 3$ $m\angle 2 =$
$m\angle 1$

Therefore: $\angle 2 \cong \angle 1$

Prove that $\angle 3 \cong \angle 4$ using a similar method.

When a transversal crosses parallel lines, there are several pairs of special angles. Let's look at a few together.

Corresponding Angle Postulate: If two parallel lines are cut by a transversal, then corresponding angles are congruent.



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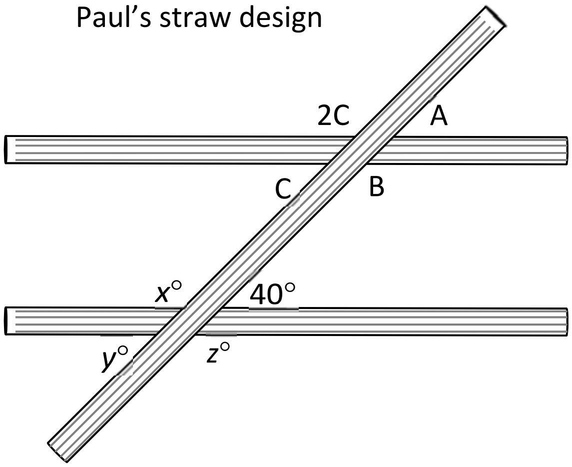
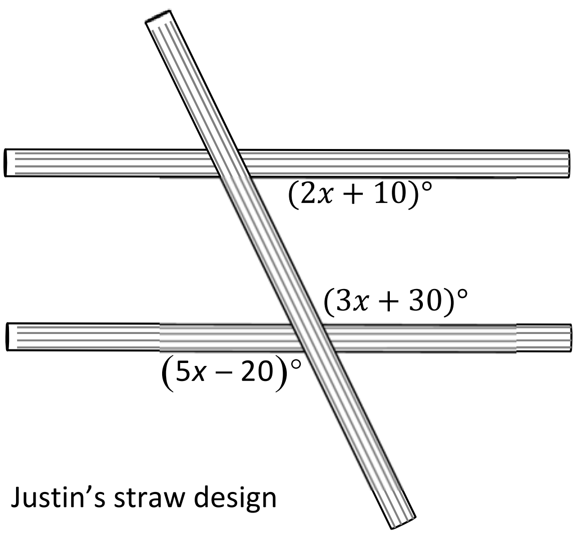
Paul's straw design



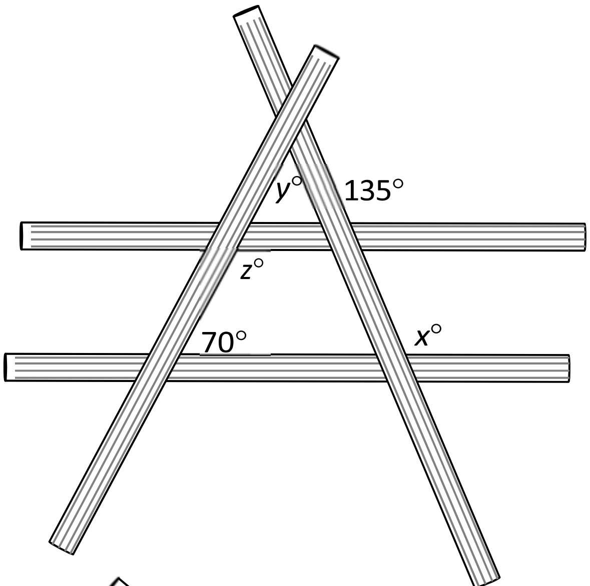
- Find all of the labeled angle measurements.
- Determine whether the lines that appear to be parallel really are parallel.
- Explain the reasoning for your results.



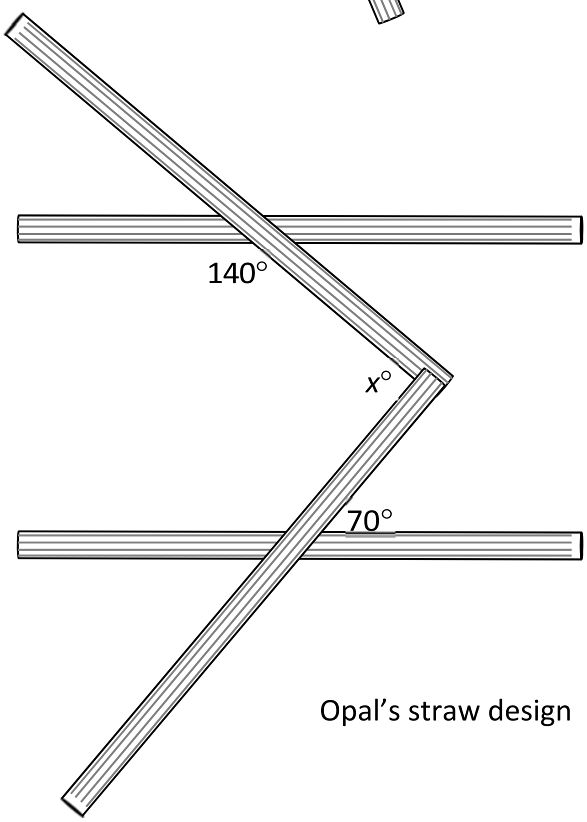
Justin's straw design

Paul then challenged himself and the others to find all the labeled angle measurements in Jane and Opal's straw designs knowing that the lines created by the straws in their designs were parallel.

Jane's straw design



- Find all of the labeled angle measurements (knowing that the lines created by the straws are parallel).
- Explain the reasoning for your results



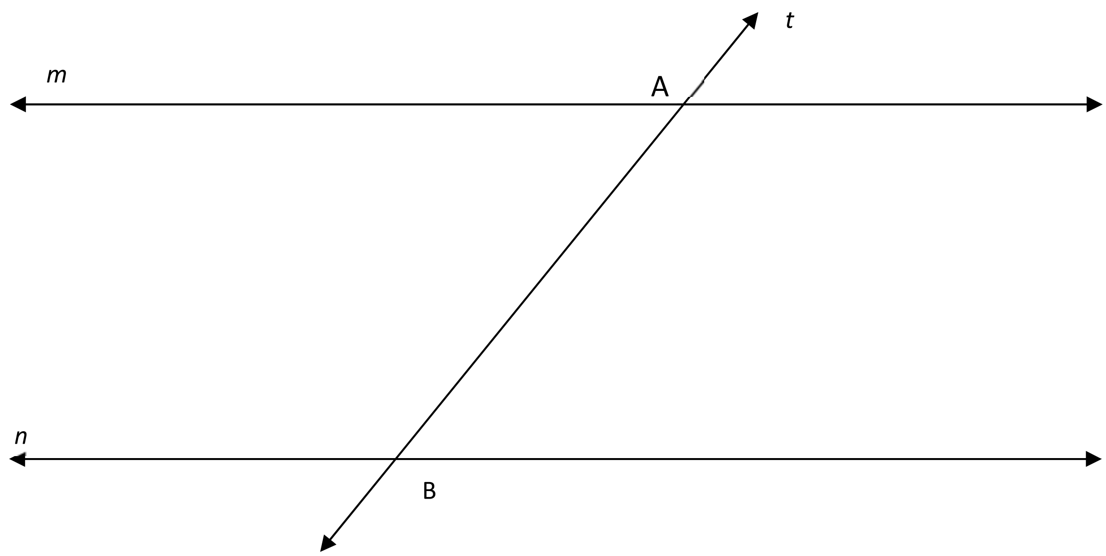
Opal's straw design

Constructing Parallel and Perpendicular Lines

Let's start by exploring features of parallel lines.

In the figure below, lines *m* and *n* are parallel and the line *t* intersects both.

- Label a new point *C* anywhere you choose on the line *m*. Connect *B* and *C* to form ΔABC.
- Construct a point *D* on line *n* so that points D and C are on opposite sides of line *t* and $AC = BD$.



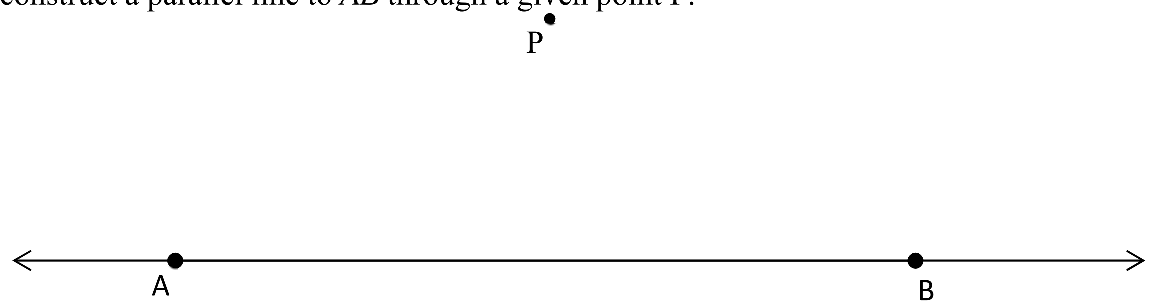
- Verify that ΔABC is congruent to ΔABD.

1. Name all corresponding and congruent parts of this construction.

2. What can you conclude about ∠CAB and ∠DBA? Will this always be true, regardless of where you choose *C* to be? Does it matter how line *t* is drawn? (*In other words could line t be perpendicular to both lines? Or slanted the other way*?)

3. What type of quadrilateral is *CADB*? Why do you think this is true?

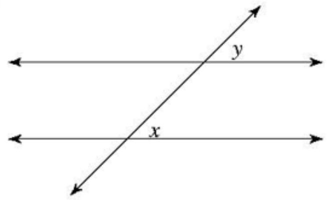
Drawing a line that intersects two parallel lines creates two sets of four congruent angles. Use this observation to construct a parallel line to $\overline{AB}$ through a given point P.



4. Construct a perpendicular line to $\overline{AB}$ that passes through *P.* Label the intersection with line *m* as Q.

Identify each pair of angles as corresponding, alternate interior, alternate exterior, or consecutive interior.

1)



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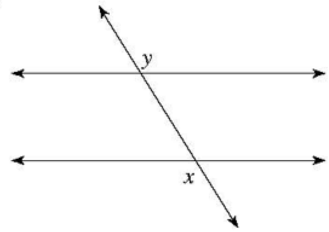

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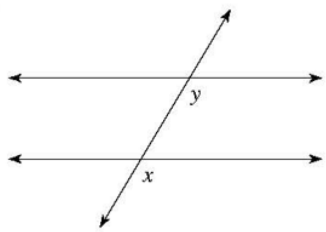

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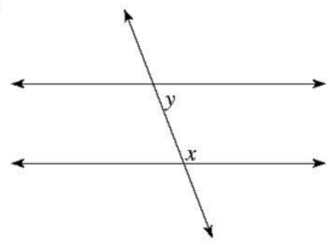

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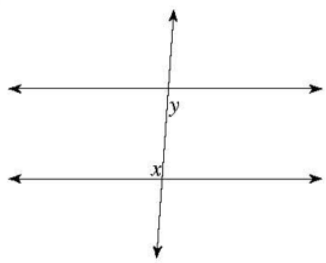

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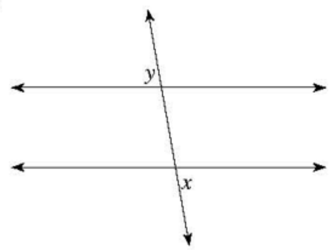

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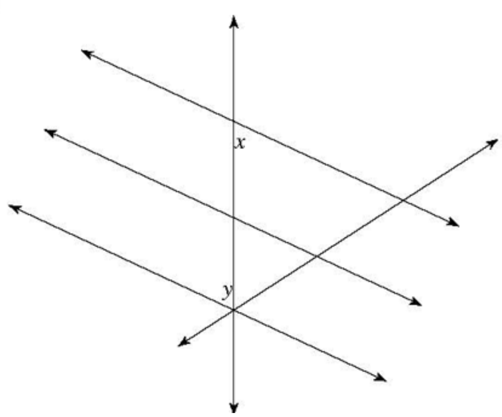

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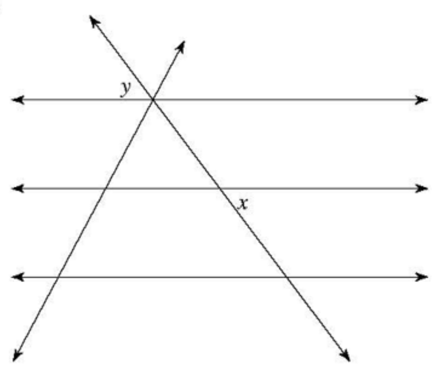

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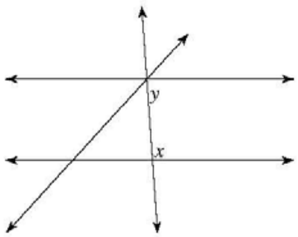

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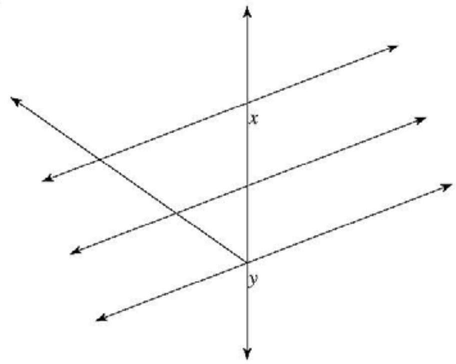
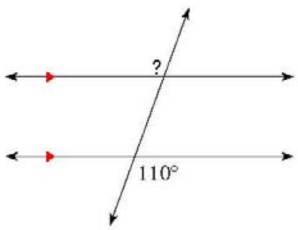


Find the measure of each angle indicated.

11)



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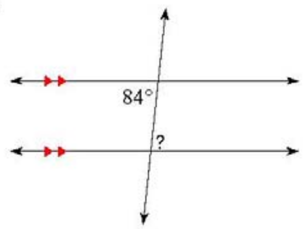

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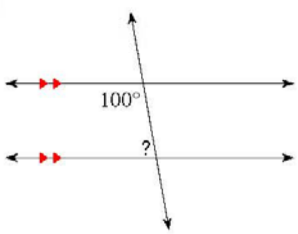

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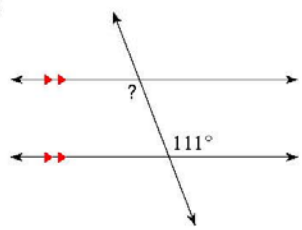

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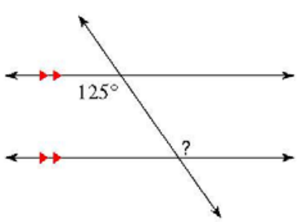

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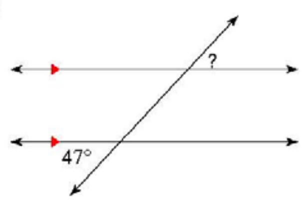

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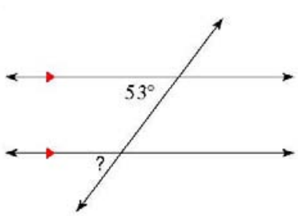

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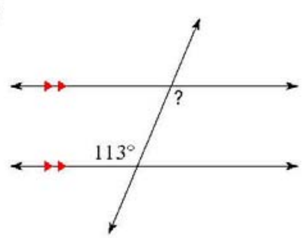
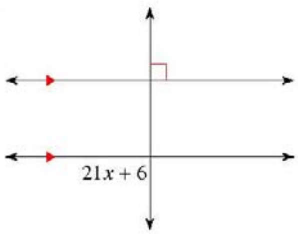


Solve for *x*.

19)



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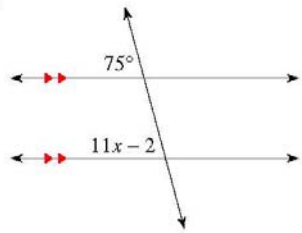

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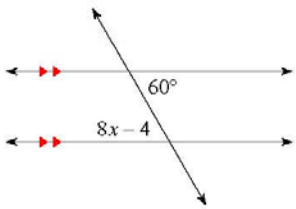

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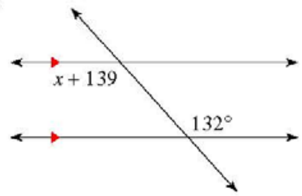

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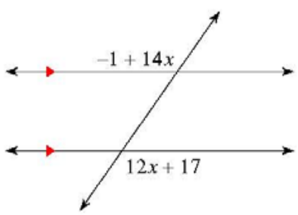

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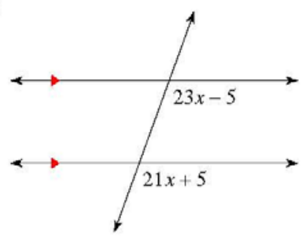
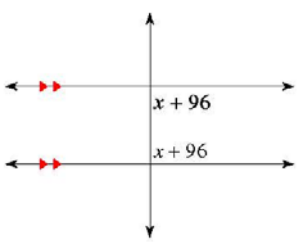


Find the measure of the angle indicated in bold.

25)



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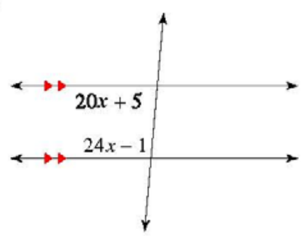

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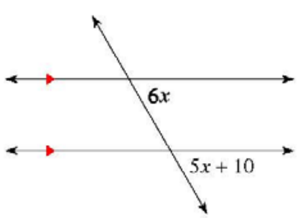

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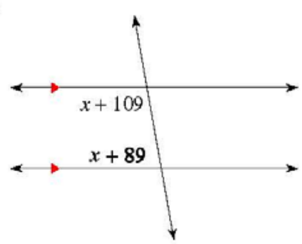
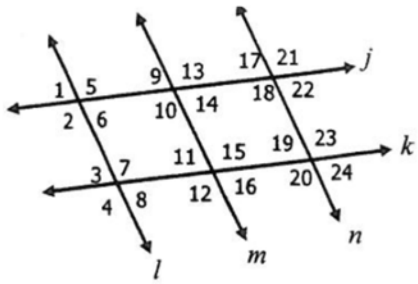


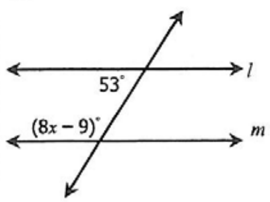
Given the following information, determine which lines, if any, are parallel. State the converse which justifies you answer.



Given	Parallel Lines	Converse
a. $\angle 10 \cong \angle 15$		
b. $m\angle 14 + m\angle 18 = 180°$		
c. $\angle 4 \cong \angle 20$		
d. $\angle 3 \cong \angle 16$		
e. $\angle 10 \cong \angle 12$		
f. $m\angle 7 + m\angle 19 = 180°$		
g. $\angle 6 \cong \angle 17$		
h. $\angle 9 \cong \angle 24$		
i. $\angle 2 \cong \angle 21$		
j. $m\angle 3 + m\angle 7 = 180°$		
k. $\angle 6 \cong \angle 11$		
l. $\angle 1 \cong \angle 3$		
m. $\angle 12 \cong \angle 15$		
n. $m\angle 10 + m\angle 11 = 180°$		
o. $\angle 15 \cong \angle 18$		

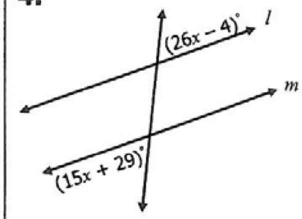
Directions: Find x so that $l \parallel m$. State the converse used

3.



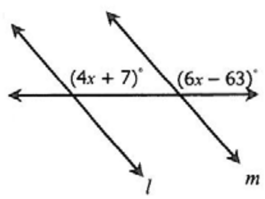
Converse: ______________________________

4.



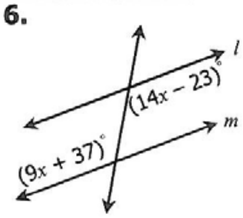
Converse: ______________________________

5.



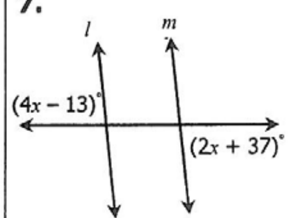
Converse: ______________________________

6.



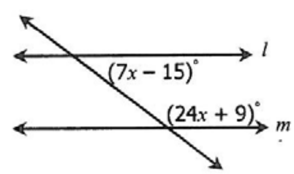
Converse: ______________________________

7.



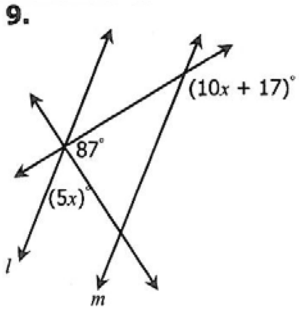
Converse: ______________________________

8.



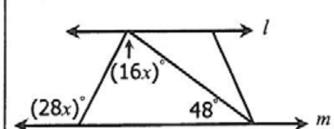
Converse: ______________________________

9.



Converse: ______________________________

10.



Converse: ______________________________

HOUSTON COUNTY
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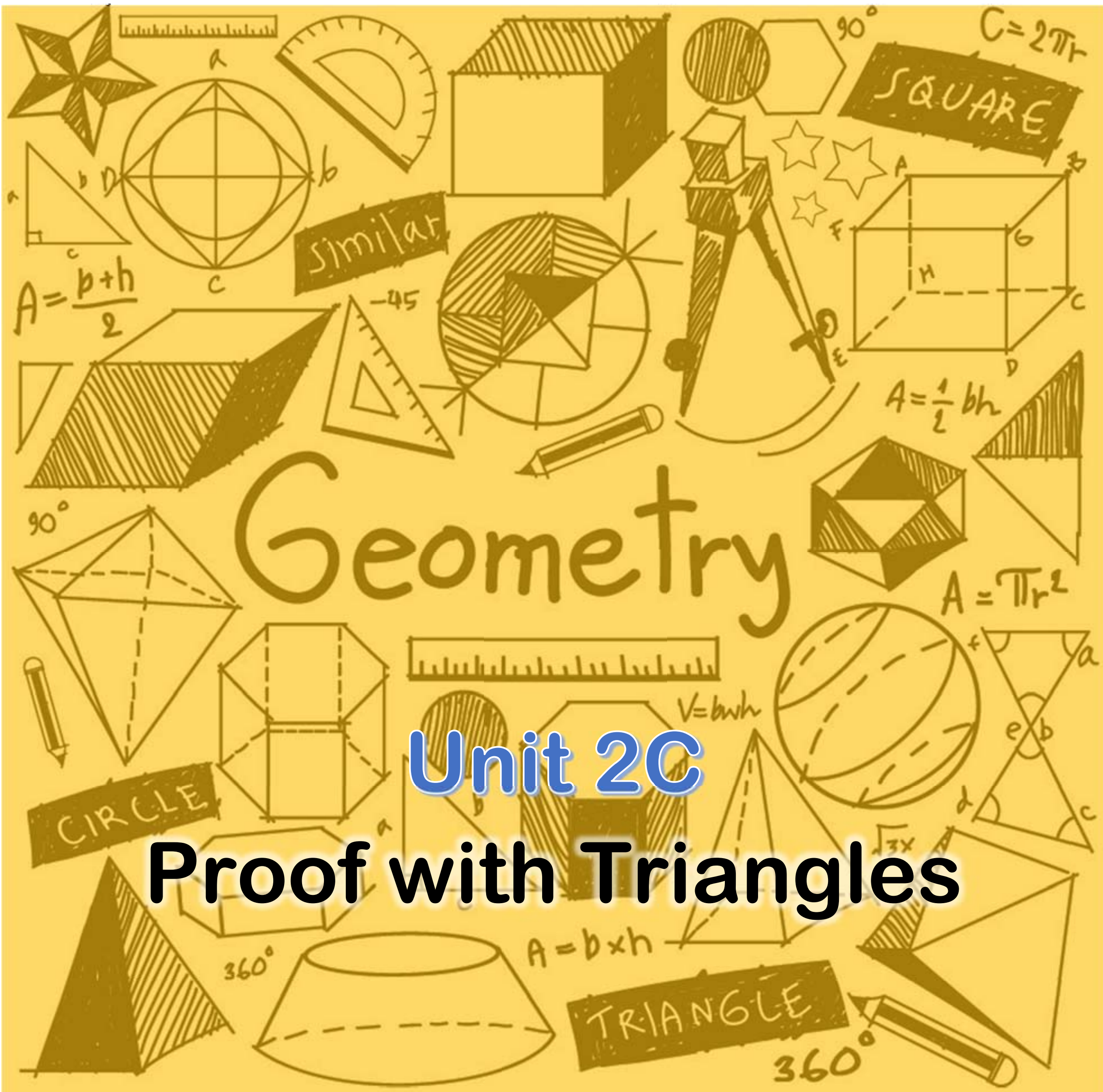
MATHEMATICS
DEPARTMENT



Unit 2C

Proof with Triangles

HOUSTON COUNTY
BOARD OF EDUCATION
HIGH-ACHIEVING TEACHERS

MATHEMATICS DEPARTMENT

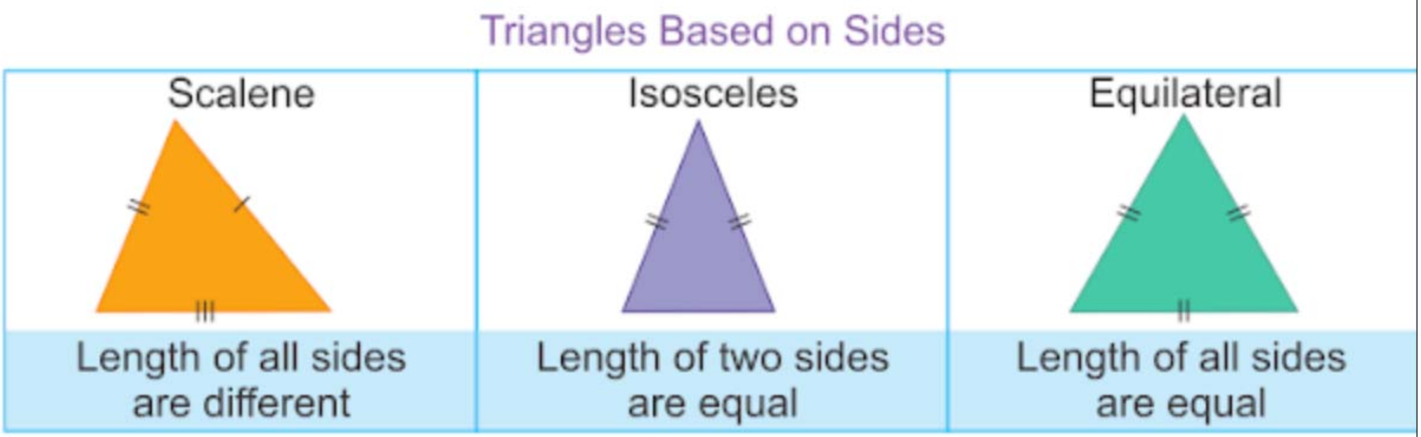
Science Technology Engineering Mathematics

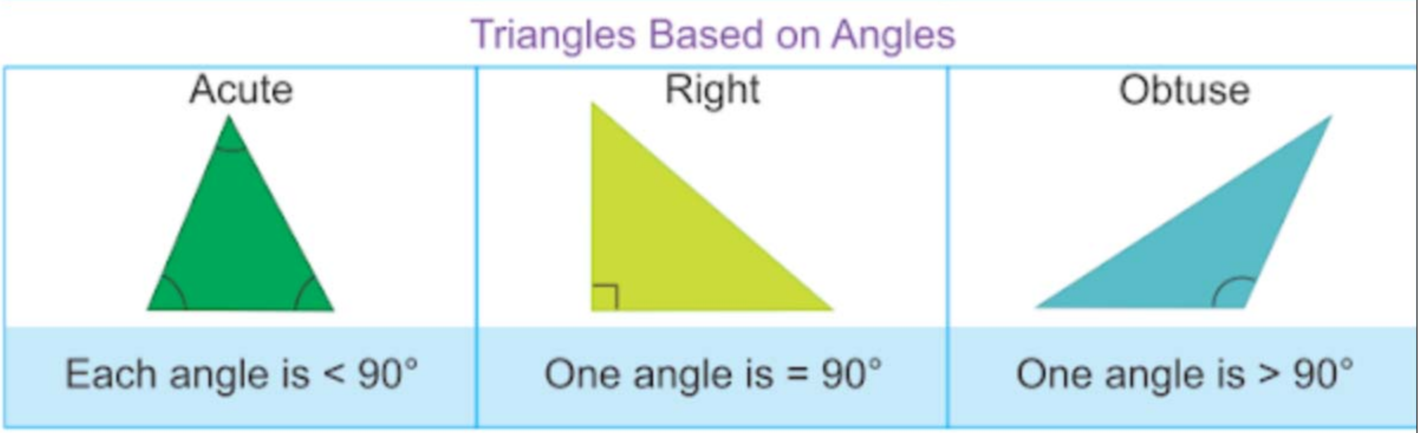
Term	Definition	Notation	Diagram/Visual
Acute Triangle			
Angle Sum Theorem			
Equiangular Triangle			
Equilateral Triangle			
Exterior Angles			
Exterior Angle Theorem			
Hypotenuse			
Interior Angles			
Isosceles Triangle			

Triangle Legs			
Obtuse Triangle			
Remote Interior Angles			
Right Triangle			
Scalene Triangle			
Sides			
Vertices			

Types of Triangles

GUIDED NOTES

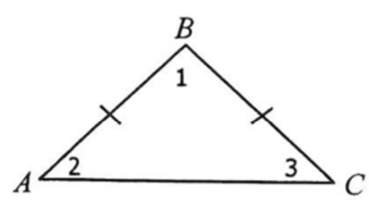




ISOSCELES TRIANGLES

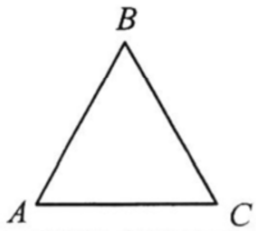
Parts of an Isosceles Triangle:

- The two congruent sides are called **legs**.
- The angle where the legs intersect is called the **vertex angle.**
- The side opposite the vertex angle is called the **base**.
- The angles along the base are called **base angles**.



Isosceles Triangle Theorem	***If two sides of a triangle are congruent, then the angles opposite those sides are congruent.*** **Example:** ____________________
Converse of Isosceles Triangle Theorem	***If two angles of a triangle are congruent, then the sides opposite those angles are congruent.*** **Example:** ____________________

EQUILATERAL TRIANGLES



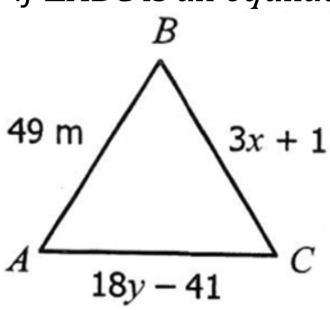
A triangle is equilateral if and only if it is equiangular!

- If $m\angle A = m\angle B = m\angle C$, then ______________________________
- If $AB = BC = AC$, then ______________________________

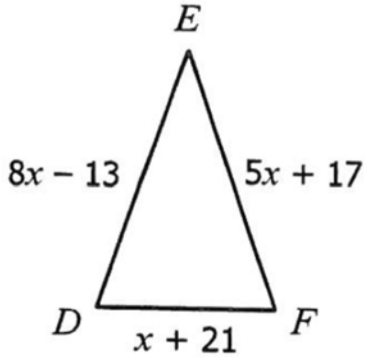
Example!

1. *Find the measures of the sides of ΔJKL then classify it by its sides.*
J(-7, -7), K(-9, 1), L(-1, -1)

2. *If ΔABC is an equilateral triangle, solve for both x and y.*



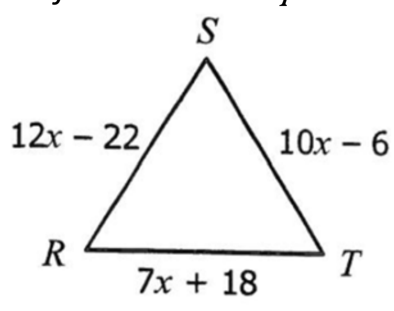
3. *If ΔDEF is an isosceles triangle with DE ≅ EF, find x and the measure of all sides.*



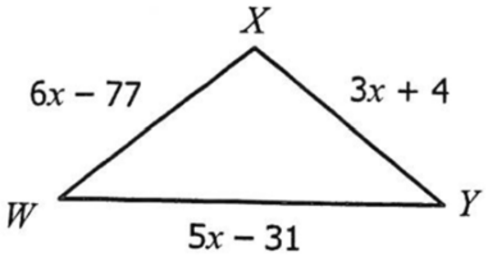
SELF CHECK

1. *Find the measures of the sides of ΔJKL then classify it by its sides.*
J(-3, 2), K(2, 6), L(8, -1)

2. *If ΔRST is an equilateral triangle, find x and the measure of all sides.*



3. *If ΔWXY is an isosceles triangle with $WX \cong XY$, find x and the measure of all sides.*



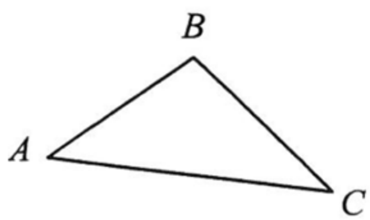
Angles within Triangles

GUIDED NOTES

Triangle Sum	The sum of the interior angles of a triangle is 180°.
Exterior Angle	The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles.
Base Angle Theorem (Isosceles Triangle)	If two sides of a triangle are congruent, the angles opposite these sides are congruent.
Base Angle Converse (Isosceles Triangle)	If two angles of a triangle are congruent, the sides opposite these angles are congruent.

Triangle Angle Sum Theorem

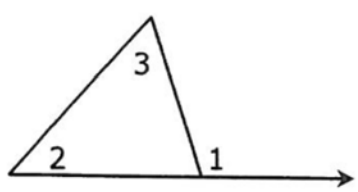
The sum of the measures of the angles of a triangle is 180°.



$m\angle A + m\angle B + m\angle C =$

Exterior Angle Theorem

An exterior angle is formed by extending any one side of the triangle.

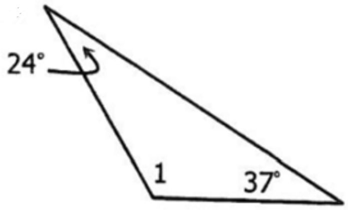


An exterior is always equal to the sum of the two non-adjacent interior angles.

$m\angle 1 =$

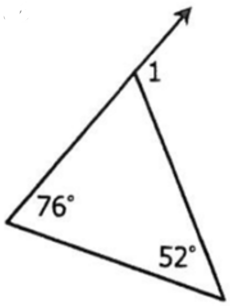
Example!

1.



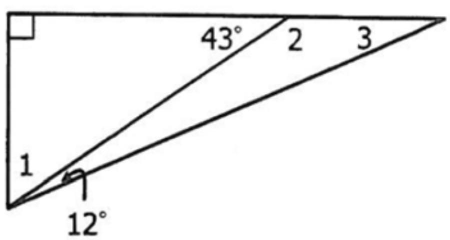
$m\angle 1 =$ ______

2.



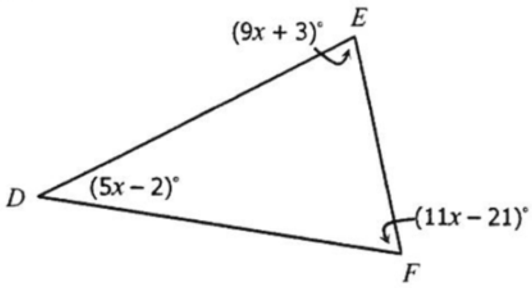
$m\angle 1 =$ ______

3.

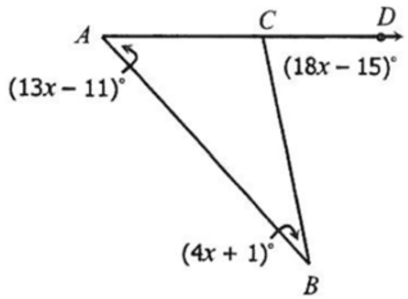


$m\angle 1 =$ ______
$m\angle 2 =$ ______
$m\angle 3 =$ ______

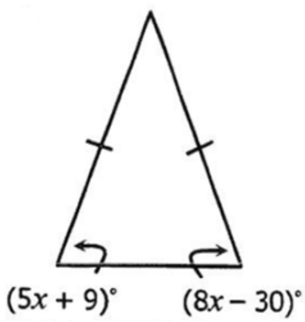
For 4-7, Solve for x.
4.



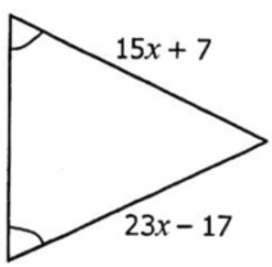
5.



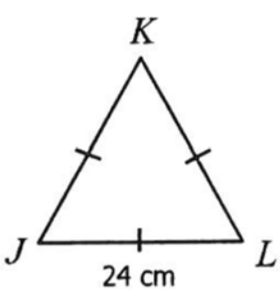
6.



7.



8.



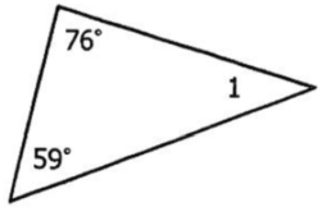
$m\angle J =$ ________

$m\angle K =$ ________

$m\angle L =$ ________

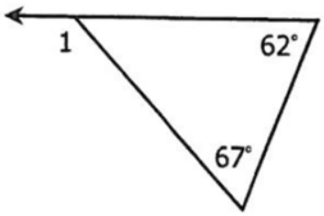
SELF CHECK

1.



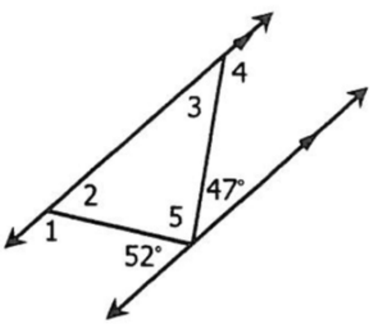
$m\angle 1 =$ ________

2.



$m\angle 1 =$ ________

3.



$m\angle 1 =$ ________

$m\angle 2 =$ ________

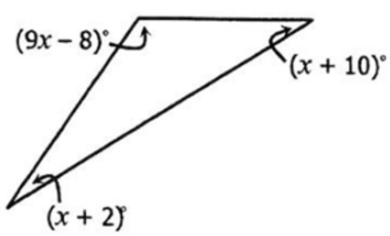
$m\angle 3 =$ ________

$m\angle 4 =$ ________

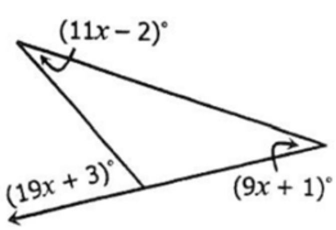
$m\angle 5 =$ ________

For 4-7, Solve for x.

4.



5.



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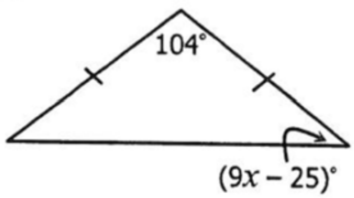

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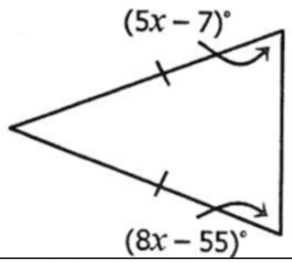
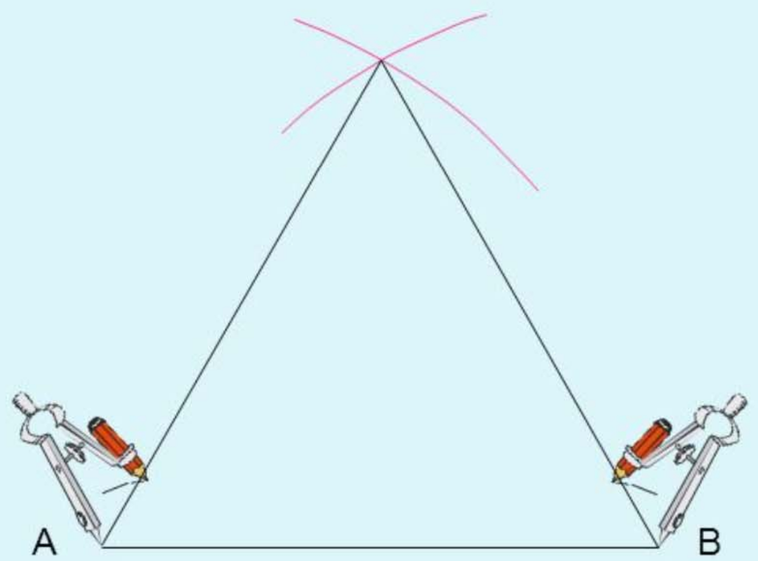


Constructions

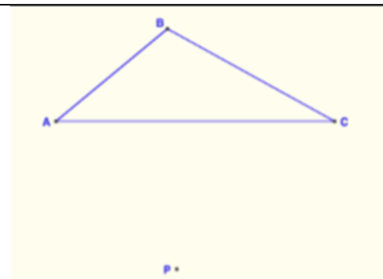
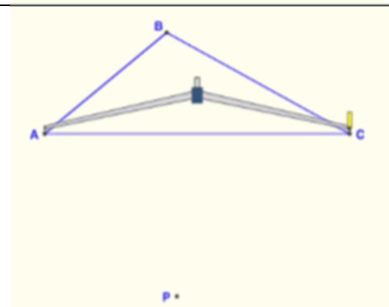
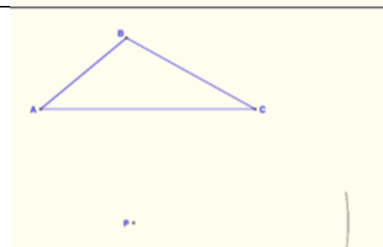
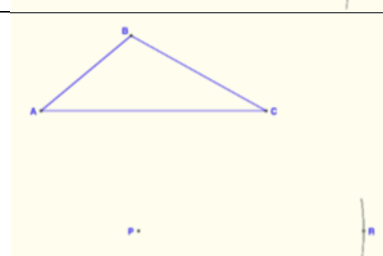
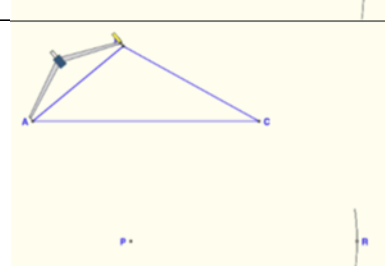
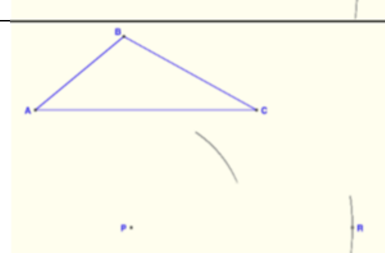
GUIDED NOTES

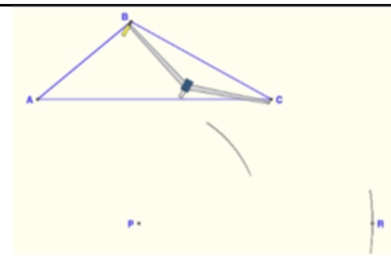
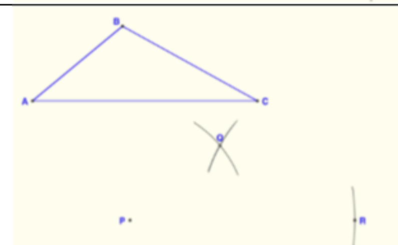
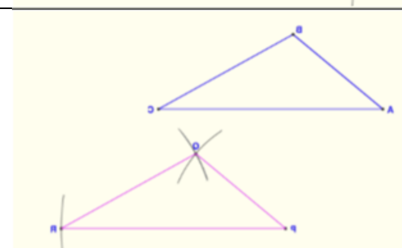
Equilateral Triangle

1. Draw base line AB of any length.
2. Place compass at A, set to distance AB and draw arc.
3. Place compass at B, with same distance set, draw an arc to intersect first one.
4. Join intersection point to A and B to form an equilateral triangle.

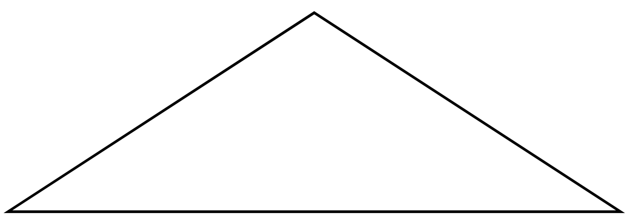


Copy a Triangle

1. Mark a point P that will become one vertex of the triangle	
2. Set the compass width to the desired length AC.	
3. From P, draw an arc.	
4. Mark a point R on this arc.	
5. Set the compass width to the distance AB.	
6. From P, draw an arc roughly where the third vertex will be.	

7. Set the compass width to the distance BC.	
8. From R, draw an arc across the first, creating point Q.	
9. Draw the lines PQ, QR, PR The triangle PQR is congruent to ABC	

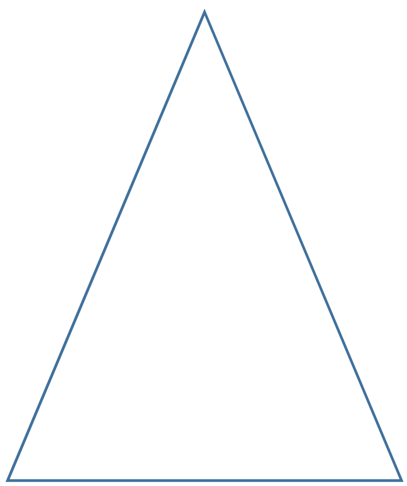
1. Copy the triangle.



SELF CHECK

1. Using a compass, draw an equilateral triangle.

2. Copy the following triangle.



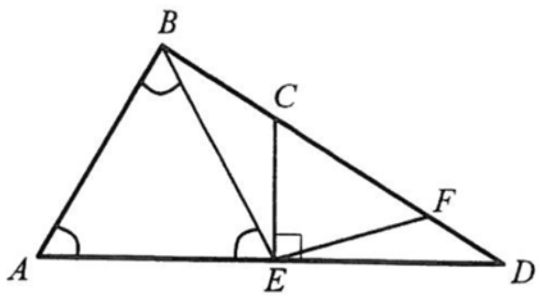
1. Can there be more than one right angle in a triangle? Why or why not?

2. Can there be more than one obtuse angle in a triangle? Why or why not?

3. In a right triangle, what should the two acute angles add to?

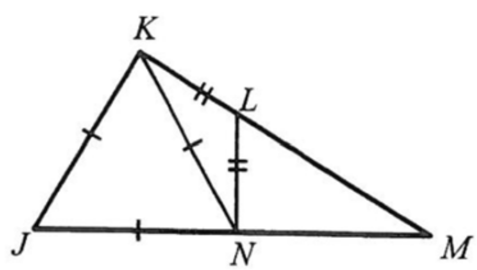
4. You have two triangles. If two angles in the first triangle are congruent to two angles in the second triangle, what do you know about the third angle in both triangles? (HINT: Try sketching it out.)

Use the diagram to classify each triangle by its angles.



1. *ΔCDE:________________________*
2. *ΔBCE:________________________*
3. *ΔABE:________________________*
4. *ΔBDE:________________________*
5. *ΔCFE:________________________*

Use the diagram to classify each triangle by its sides.



6. *ΔJKM:________________________*
7. *ΔKLN:________________________*
8. *ΔJKN:________________________*

9. *Find the measures of the sides of ΔJKL then classify it by its sides.*

J(7, -2), K(-4, 9), L(-3, -1)

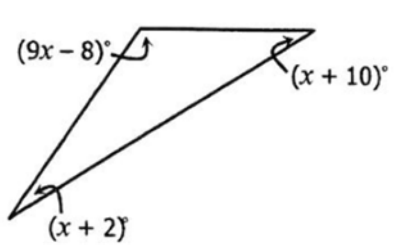
10.

ΔSTU is an equilateral triangle. If ST is one less than twice x, SU is 37 less than five times x, and TU is 11 more than x, find x and the measure of each side.

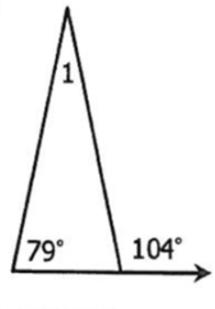
11.

ΔABC is an isosceles triangle with $\overline{AB} \cong \overline{BC}$. If $AB = 3x + 41$, $BC = 8x - 54$, and $AC = 12x - 103$, find x and the measure of each side.

12.

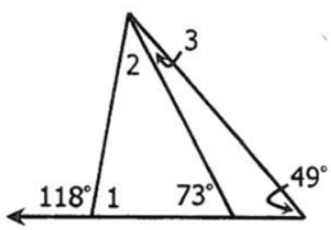


13.



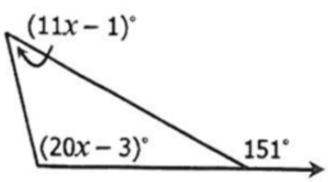
$m\angle 1 =$ ______

14.



$m\angle 1 =$ ______
$m\angle 2 =$ ______
$m\angle 3 =$ ______

15.

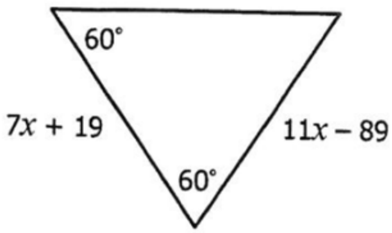


16.

In ΔWXY, if $\overline{WX} \cong \overline{WY}$, $m\angle W = x + 6$, $m\angle X = 5x - 12$, and $m\angle Y = 7x - 48$, find x and the measure of each angle.

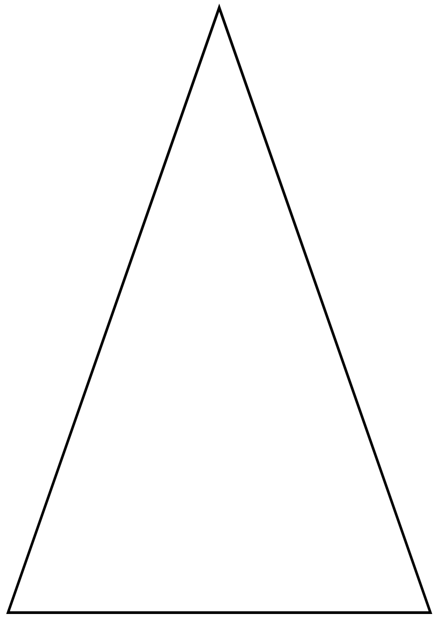
$x =$ ______
$m\angle W =$ ______
$m\angle X =$ ______
$m\angle Y =$ ______

17.



18. Use a compass to draw an equilateral triangle.

19. Use a compass to copy the following triangle.



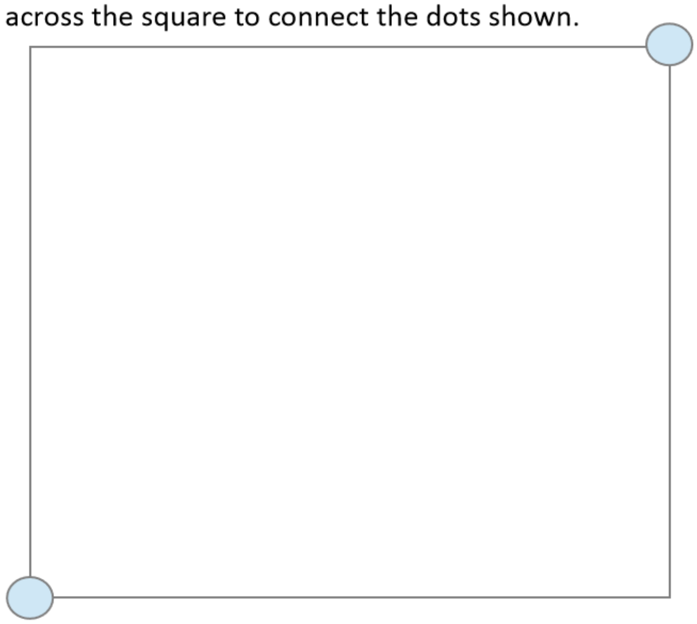
Triangles Activity

1. Briefly explain each type of triangle:
    - scalene:
    - isosceles:
    - equilateral:

2. What is special about a right triangle? ____

Use the image below to complete the next questions.

3. Draw a diagonal line across the square to connect the dots shown.



4. Circle the kind of triangle that is now shown above.
    - isosceles triangle
    - equilateral triangle
    - equiangular triangle

5. If the length of the bottom line of the triangle is 2 inches, then what is the length of the opposite side? ______________________________________

6. If the length of the bottom line of the triangle is 2 inches, then will the length of the hypotenuse be greater or smaller than 2 inches? ______________________________________

7. If every angle between the sides of a square is 90 degrees, then what is the angle formed by drawing a line half-way through the 90-degree angle? ______________________________________

HOMEWORK

Directions: If $\overline{PR}$ bisects $\angle SRT$ and U is the midpoint of $\overline{RT}$, classify each triangle by its <u>angles</u> and <u>sides</u>.

ΔUQR: ______________________________

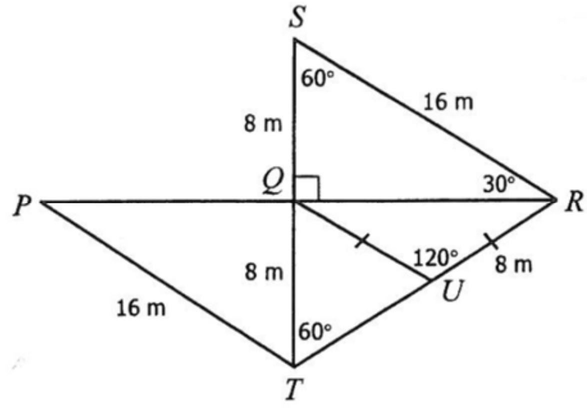
ΔRST: ______________________________

ΔSRQ: ______________________________

ΔPRT: ______________________________

ΔTQU: ______________________________

ΔPQT: ______________________________



1. Find the measures of the sides of ΔJKL then classify it by its sides.

J(1, -13), K(3, 3), L(10, -6)

2.

If ΔPQR is an equilateral triangle, $PQ = 18x + 1$, $QR = 24x - 17$, and $PR = 15x + 10$, find x and the measure of each side.

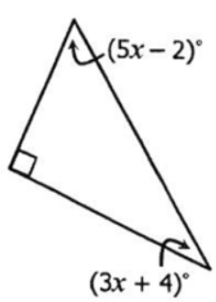
3.

ΔMKL is an isosceles triangle with $\overline{MK} \cong \overline{ML}$. If $MK = 7x - 15$, $KL = 4x - 5$, and $ML = 10x - 42$, find x and the measure of each side.

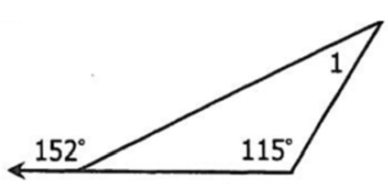
4.

ΔGHI is an isosceles triangle with $\overline{GI} \cong \overline{HI}$. If GH is three more than x, HI is 17 less than four times x, and GI is 45 less than six times x, find x and the measure of each side.

5.

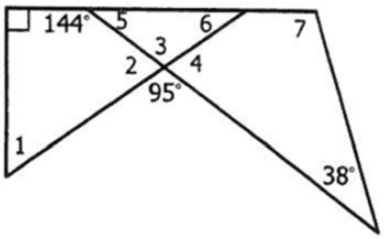


6.



$m\angle 1 =$ ________

7.



$m\angle 1 =$ ________

$m\angle 2 =$ ________

$m\angle 3 =$ ________

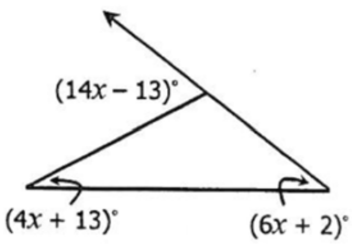
$m\angle 4 =$ ________

$m\angle 5 =$ ________

$m\angle 6 =$ ________

$m\angle 7 =$ ________

8.



9.

In ΔRST, if $\overline{RT} \cong \overline{ST}$, $m\angle R = 9x + 2$, $m\angle S = 13x - 18$, and $m\angle T = 17x + 1$, find x and the measure of each angle.

$x =$ ______

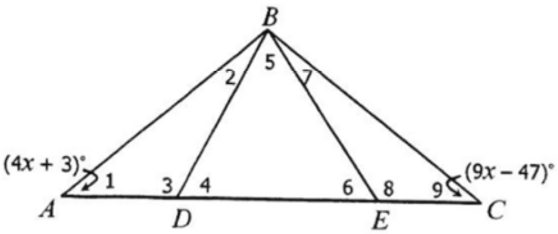
$m\angle R =$ ______

$m\angle S =$ ______

$m\angle T =$ ______

10.

If ΔABC is an isosceles triangle and ΔDBE is an equilateral triangle, find each missing measure.



$m\angle 1 =$ ______ $m\angle 6 =$ ______

$m\angle 2 =$ ______ $m\angle 7 =$ ______

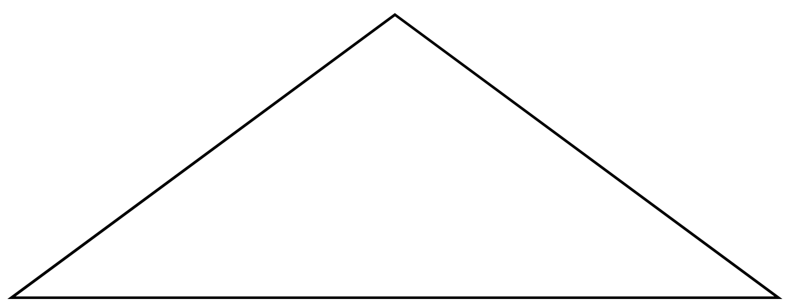
$m\angle 3 =$ ______ $m\angle 8 =$ ______

$m\angle 4 =$ ______ $m\angle 9 =$ ______

$m\angle 5 =$ ______

11. *Use a compass to draw an equilateral triangle.*

12. *Use a compass to copy the following triangle.*



Term	Definition	Notation	Diagram/Visual
Angle-Angle - Side			
Angle-Side - Angle			
Corresponding Parts of Congruent Triangles are Congruent			
Hypotenuse - Leg			
Isosceles Triangle Theorem			
Adjacent			
Side – Angle - Side			
Side – Side – Side			

Congruent Triangles

GUIDED NOTES

Congruent Triangles

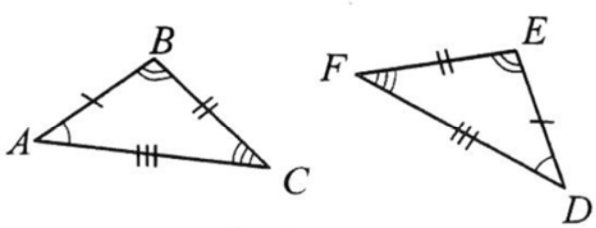
Triangles with the same ____________ and __________.

This means all **corresponding parts**

(____________ and ____________)are congruent.

Congruency Statements

When triangles are congruent, we can write a **congruency statement**.



$\Delta ABC \cong \Delta DEF$

A *valid congruency statement* must match all corresponding angles and sides.

CPCTC

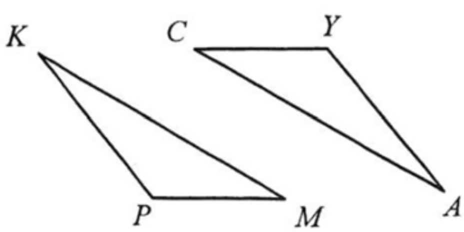
_____________________ ____________ of ______________

_______________ are _______________.

If we know two triangles are congruent, then we know that every pair of corresponding parts is also congruent.

Example!

1. *Given ΔKPM ≅ ΔAYC, complete the following statements.*

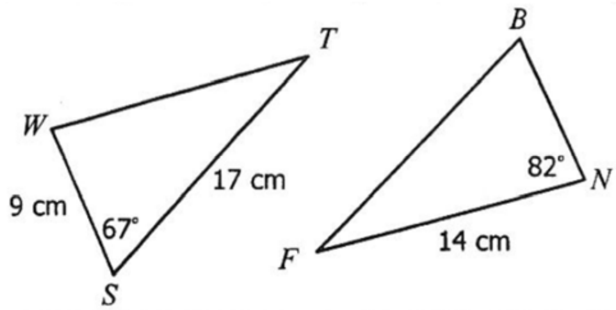


a) $\overline{KM} \cong$ _____ **d)** $\angle Y \cong$ _____ **g)** $\Delta MPK \cong$ ________

b) $\overline{CY} \cong$ _____ **e)** $\angle K \cong$ _____ **h)** $\Delta YAC \cong$ ________

c) $\overline{PK} \cong$ _____ **f)** $\angle ACY \cong$ _______

2. Given ΔSTW ≅ ΔBFN, find each missing measure.



a) $BN =$ ______

b) $TW =$ ______

c) $BF =$ ______

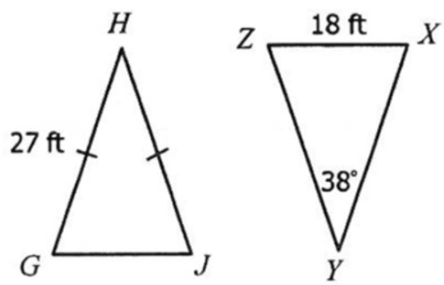
d) $m\angle W =$ ______

e) $m\angle B =$ ______

f) $m\angle F =$ ______

SELF CHECK

1. Given ΔGHJ ≅ ΔXYZ, find each missing measure.



a) $GJ =$ ______

b) $XY =$ ______

c) $ZY =$ ______

d) $m\angle H =$ ______

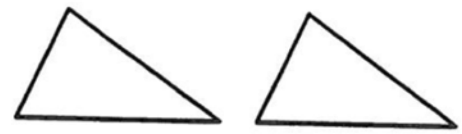
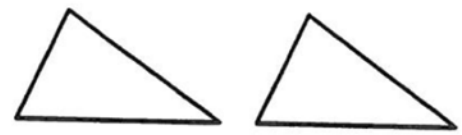
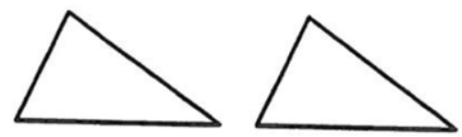
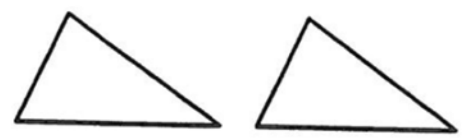
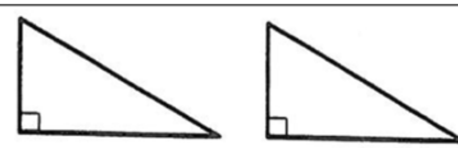
e) $m\angle Z =$ ______

f) $m\angle J =$ ______

Reasons Triangles are Congruent

GUIDED NOTES

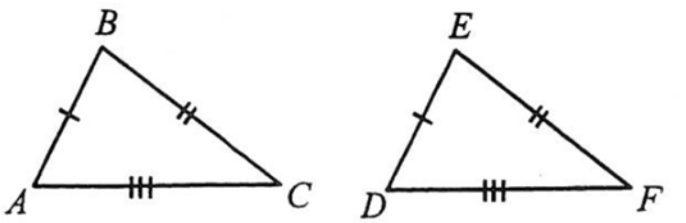
Reference Page

SSS (Side-Side-Side)	Three pairs of Congruent Sides	
SAS (Side-Angle-Side)	Two Sides and an Included Angle	
ASA (Angle-Side-Angle)	Two Angles and an Included Side	
AAS (Angle-Angle-Side)	Two Angles and a Side Opposite Them	
HL (Hypotenuse-Leg)	The Hypotenuse and any one Leg of a Right Triangle	

Reasons Sides are Congruent	• Its Given • Definition of Midpoint (A midpoint will create two congruent sides) • Reflexive Property (A side is congruent to itself)
Reasons Angles are Congruent	• Its Given • Vertical Angles • Alternate Interior Angles (must have parallel lines) • Alternate Exterior Angles (must have parallel lines) • Corresponding Angles (must have parallel lines) • Definition of an Angle Bisector (bisector will create two congruent angles)

Side-Side-Side (SSS)

If three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent

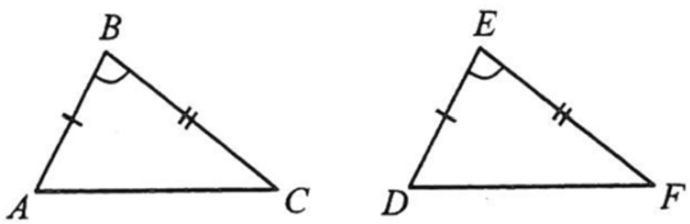


If ______________________(**S**ide)
______________________(**S**ide)
______________________(**S**ide)

then, ___________________________

Side-Angle-Side (SAS)

If two sides and the included angle of one triangle is congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

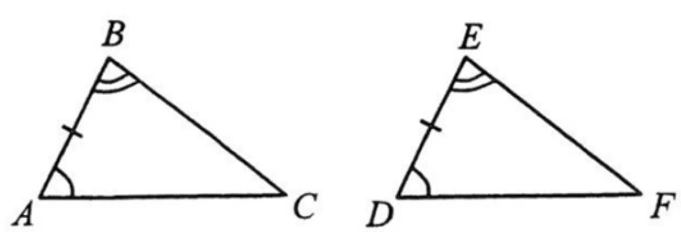


If ______________________(**S**ide)
______________________(**A**ngle)
______________________(**S**ide)

then, ___________________________

Angle-Side-Angle (ASA)

If two angles and the included side of one triangle is congruent to two angles and the included side of another triangle, then the two triangles are congruent.



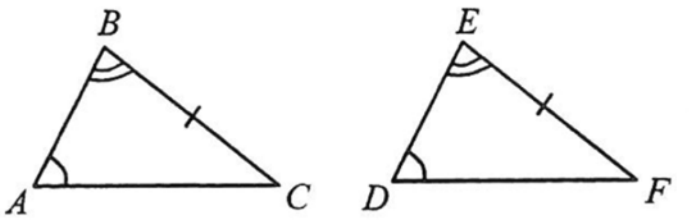
If ______________________(**A**ngle)
______________________(**S**ide)
______________________(**A**ngle)

then, ___________________________

Angle-Angle-Side (AAS)

If two angles and the non-included side of one triangle is congruent to two angles and the non-included side of another triangle, then the two triangles are congruent.

***Non-included means the side opposite



If ______________________(**A**ngle)
______________________(**A**ngle)
______________________(**S**ide)

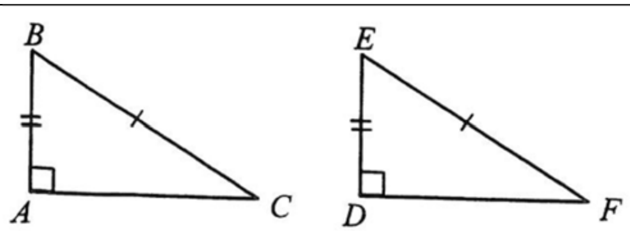
then, ___________________________

Hypotenuse-Leg (HL)

If the hypotenuse and leg of one right triangle is congruent to the hypotenuse and leg of another right triangle, then the two right triangles are congruent.

***The HYPOTENUSE the side opposite the right angle

***A LEG is a side adjacent to the right angle



If ______________________(**H**ypotenuse)
______________________(**L**eg)

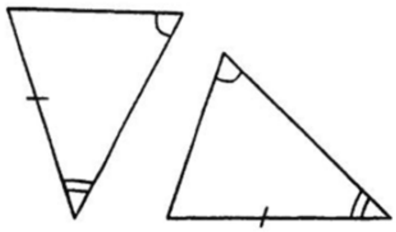
then, ___________________________

Example!

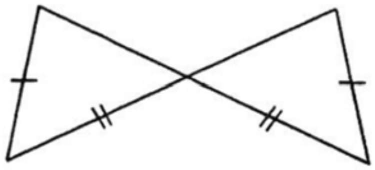
Compare the triangles and determine whether they can be proven congruent, if possible, by SSS, SAS, ASA, AAS, or HL. If not, write not possible.

1.

2.



3.



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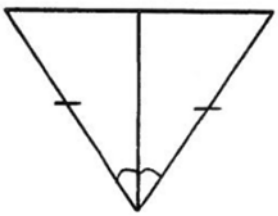
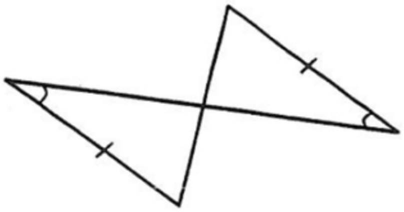


SELF CHECK

Compare the triangles and determine whether they can be proven congruent, if possible, by SSS, SAS, ASA, AAS, or HL. If not, write not possible.

1.



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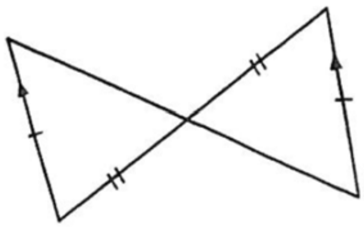

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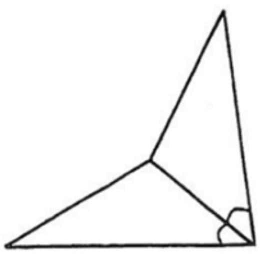

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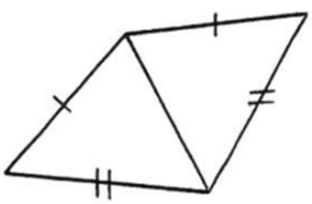
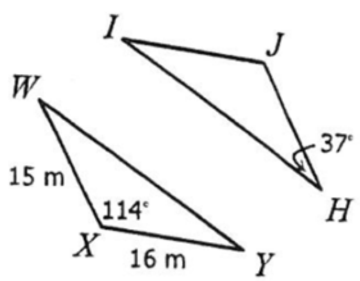


1. What does "included side" and "included angle" mean? Sketch a picture as part of your response.

If ΔKPL ≅ ΔACM, complete the following statements.

1. $\overline{KL}$ ≅ __________
2. $\overline{AC}$ ≅ __________
3. $\overline{PL}$ ≅ __________
4. ∠P ≅ __________
5. ∠K ≅ __________
6. ∠M ≅ __________

If ΔWXY ≅ ΔHJI, complete each part.



7. JI = ______

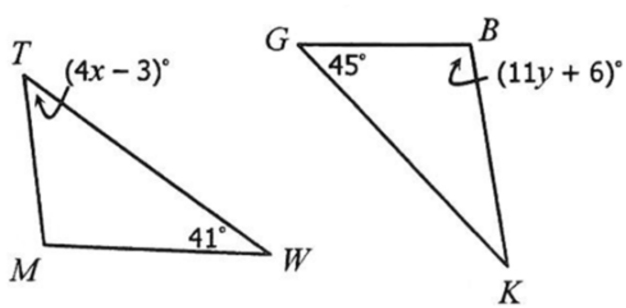
8. JH = ______

9. m∠W = ______

10. m∠J = ______

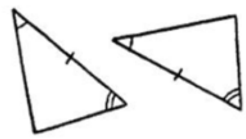
11. m∠I = ______

12. Given ΔMTW ≅ ΔBGK, find the values of x and y.



Compare the triangles and determine whether they can be proven congruent, if possible, by SSS, SAS, ASA, AAS, or HL. If not, write not possible.

13.



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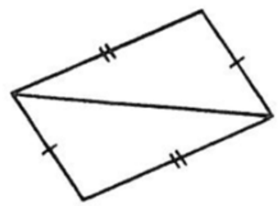

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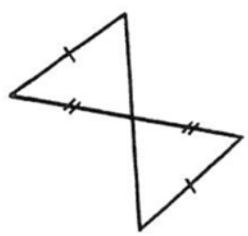

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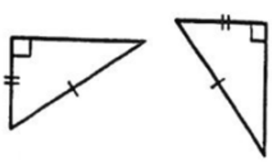


Formalizing Triangle Congruence

Part 1:

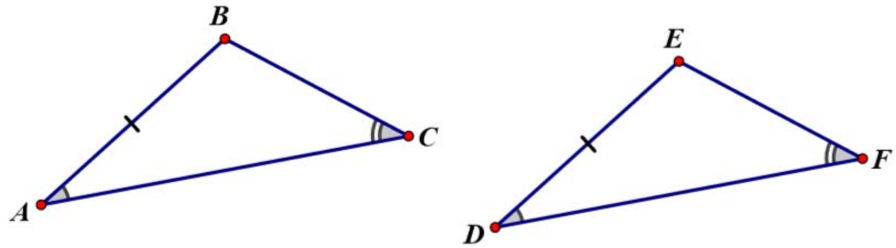
Challenge 1: You have two triangles, and you know that two sides of one triangle are congruent to two sides of the other triangle. You also know that the angle between the two known sides in the first triangle is congruent to the angle between the two known sides in the second triangle. Sketch the triangles and label the known information. Can you show that the two triangles are congruent? Hint: You should use what you know about congruence and rigid motions.

Challenge 2: You have two triangles, and you know that two angles of one triangle are congruent to two angles of the other triangle. You also know that the side between the two known angles in the first triangle is congruent to the side between the two known angles in the second triangle. Sketch the triangles and label the known information. Can you show that the two triangles are congruent? Hint: You should use what you know about congruence and rigid motions.

Challenge 3: You have two triangles, and you know that three sides of one triangle are congruent to three sides of the other triangle. Sketch the triangles and label the known information. Can you show that the two triangles are congruent? Hint: You should use what you know about congruence and rigid motions.

Part 2:

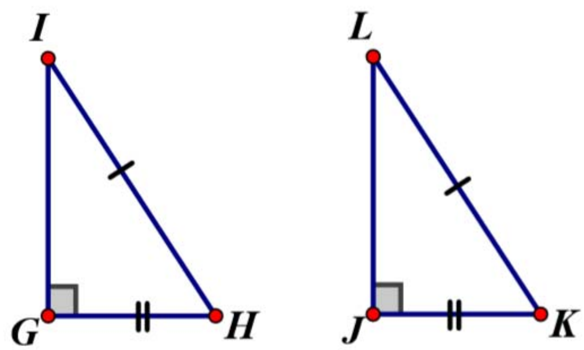
Aiden and Noah were given the following diagram of two triangles with the congruent parts marked.



Aiden says that you cannot tell whether the two triangles are congruent because the side is not between the two given angles.
Noah says that the triangles are congruent because he knows that the third angles in the triangles have to be congruent, and then you have a side between two angles.

Are the triangles congruent? Can you tell? Do you agree with Aiden or Noah? Revise the argument with which you agree to make it more convincing.

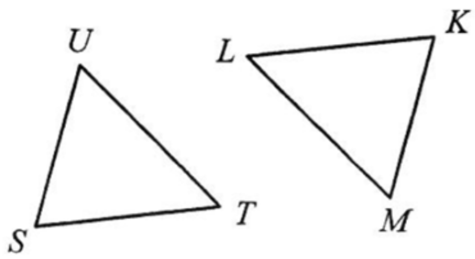
Tillie and Payton were given the following diagram of two right triangles with congruent parts marked.



Tillie says this is a situation where you have two sides without the angle between them, so you can't tell if the triangles are congruent.
Payton says that the triangles are congruent because she can find the length of the third side in a right triangle and then use SSS, since all three corresponding sides must be congruent.

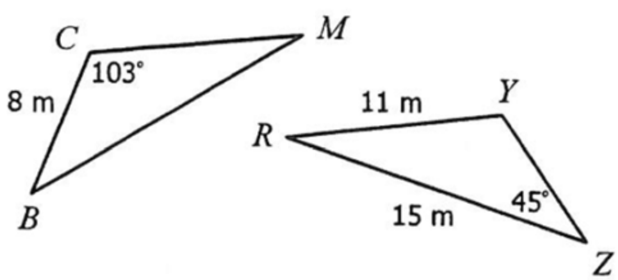
Are the triangles congruent? Can you tell? Do you agree with Tillie or Payton? Revise the argument with which you agree to make it more convincing.

If ΔSTU ≅ ΔKLM, complete the following statements.



1. $\overline{TU}$ ≅ __________
2. $\overline{KM}$ ≅ __________
3. $\overline{LK}$ ≅ __________
4. ∠M ≅ __________
5. ∠T ≅ __________
6. ∠UST ≅ __________
7. *ΔUST* ≅ __________
8. *ΔTUS* ≅ __________

If ΔBCM ≅ ΔZYR, complete each part.



9. CM = ______

10. BM = ______

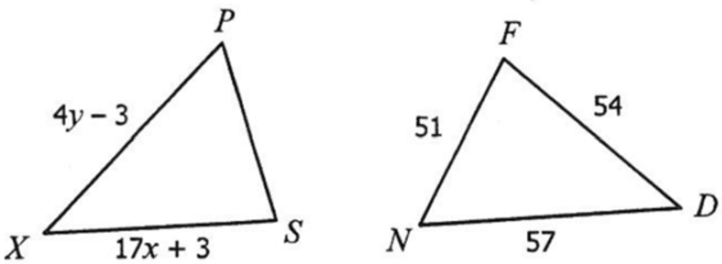
11. YZ = ______

12. m∠B = ______

13. m∠M = ______

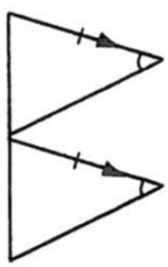
14. m∠Y = ______

15. Given ΔXPS ≅ ΔDNF, find the values of x and y.



Compare the triangles and determine whether they can be proven congruent, if possible, by SSS, SAS, ASA, AAS, or HL. If not, write not possible.

16.



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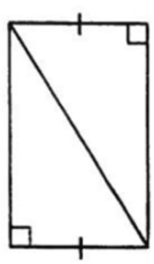

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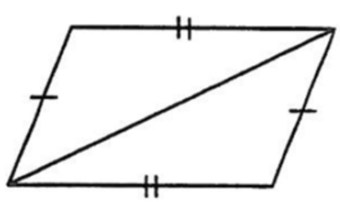

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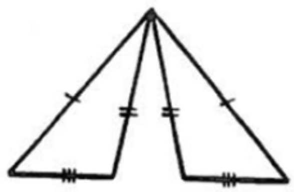

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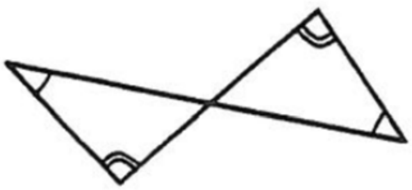

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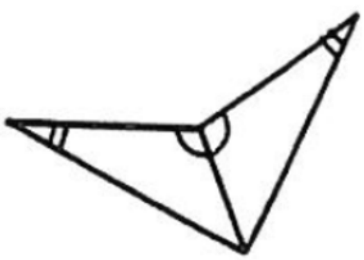


Triangle Proofs

GUIDED NOTES

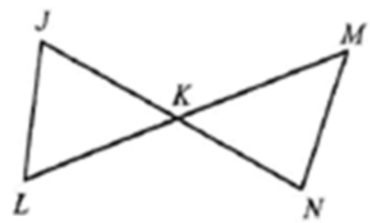
Side-Side-Side (SSS) Congruence	If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.
Side-Angle-Side (SAS) Congruence	If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.
Angle-Side-Angle (ASA) Congruence	If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.
Angle-Angle-Side (AAS) Congruence	If two angles and the non-included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.
Hypotenuse-Leg (HL) Congruence (right triangle)	If the hypotenuse and leg of one right triangle are congruent to the corresponding parts of another right triangle, the two right triangles are congruent.
CPCTC	Corresponding parts of congruent triangles are congruent.

Example!

1.

Given: $\overline{JL} \cong \overline{NM}$, K is the midpoint of $\overline{JN}$ and $\overline{LM}$

Prove : $\Delta JKL \cong \Delta NKM$

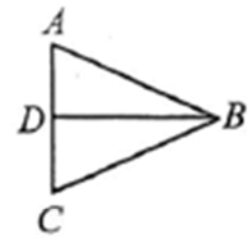


Statements	Reasons
1. $\overline{JL} \cong \overline{NM}$	1.
2. K is the midpoint of $\overline{JN}$ and $\overline{LM}$	2.
3. $\overline{JK} \cong \overline{NK}$	3.
4. $\overline{LK} \cong \overline{MK}$	4.
5. $\Delta JKL \cong \Delta NKM$	5.

2.

Given: $\overline{AB} \cong \overline{CB}$, $\overline{BD}$ bisects $\angle ABC$

Prove : $\Delta ABD \cong \Delta CBD$

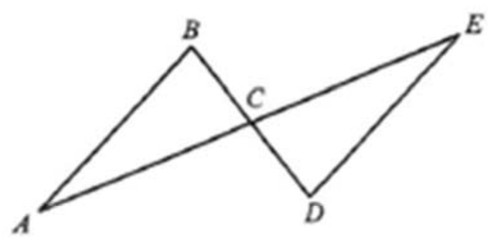


Statements	Reasons
1. $\overline{AB} \cong \overline{CB}$	1.
2. $\overline{BD}$ bisects $\angle ABC$	2.
3. $\angle ABD \cong \angle CBD$	3.
4. $\overline{BD} \cong \overline{BD}$	4.
5. $\Delta ABD \cong \Delta CBD$	5.

3.

Given: $\angle BAC \cong \angle DEC$, C is the midpoint of $\overline{AE}$

Prove : $\Delta ABC \cong \Delta EDC$

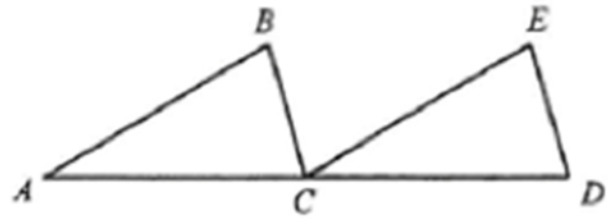


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

4.

Given: $\angle ABC \cong \angle CED$, $\overline{AB} \parallel \overline{CE}$
C is the midpoint of $\overline{AD}$

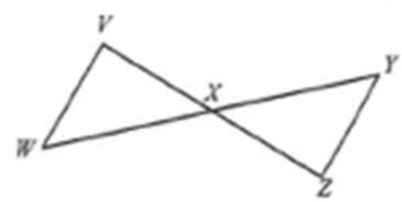
Prove : $\Delta ABC \cong \Delta CED$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

5.

Given: $\triangle$WVX and $\triangle$YZX are right triangles, $\overline{WV} \cong \overline{YZ}$
X is the midpoint of $\overline{WY}$
Prove: $\triangle$ LMP $\cong$ $\triangle$NMP

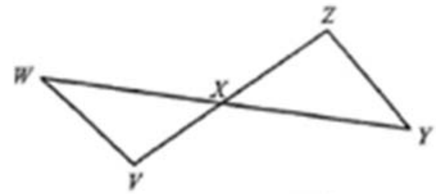


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

6.

Given: X is the midpoint of $\overline{WY}$ *and* $\overline{YZ}$

Prove : $\angle XWV \cong \angle XYZ$



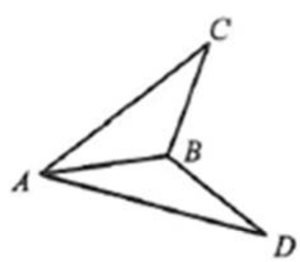
Statements	Reasons
1. X is the midpoint of $\overline{WY}$ and $\overline{YZ}$	1.
2. $\overline{WX} \cong \overline{YX}$	2.
3. $\overline{VX} \cong \overline{ZX}$	3.
4. $\angle WXV \cong \angle YXZ$	4.
5. $\Delta WXV \cong \Delta YXZ$	5.
6. $\angle XWV \cong \angle XYZ$	6.

SELF CHECK

1.

Given: $\overline{AC} \cong \overline{AD}, \overline{CB} \cong \overline{DB}$

Prove : $\Delta ABC \cong \Delta ABD$

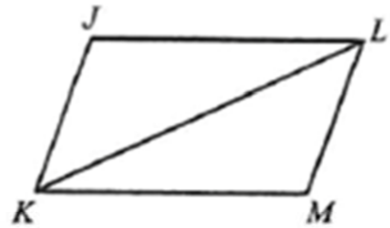


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

2.

Given: $\overline{JK} \parallel \overline{LM}, \overline{JL} \parallel \overline{KM}$

Prove : $\Delta JKL \cong \Delta MLK$

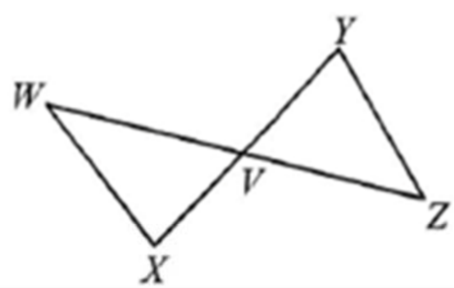


Statements	Reasons
1. $\overline{JK} \parallel \overline{LM}, \overline{JL} \parallel \overline{KM}$	1.
2. $\angle JKL \cong \angle MLK$	2.
3. $\angle JLK \cong \angle MKL$	3.
4. $\overline{KL} \cong \overline{LK}$	4.
5. $\Delta JKL \cong \Delta MLK$	5.

3.

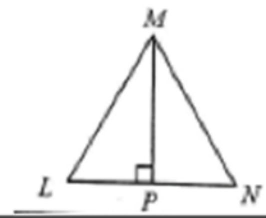
Given: V is the midpoint of $\overline{WZ}$ and $\overline{XY}$

Prove : $\Delta WXV \cong \Delta ZYV$



Statements	Reasons
1. V is the midpoint of $\overline{WZ}$ and $\overline{XY}$	1.
2. $\overline{WV} \cong \overline{ZV}$	2.
3. $\angle WVX \cong \angle ZVY$	3.
4. $\overline{XV} \cong \overline{YV}$	4.
5. $\Delta WXV \cong \Delta ZYV$	5.

4.



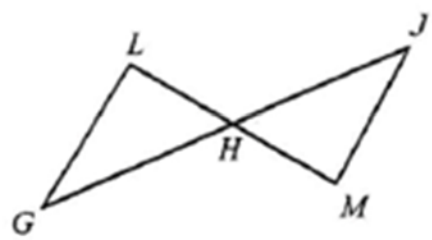
Given: △LMP and △MNP are right triangles, $\overline{ML} \cong \overline{MN}$
Prove: △ LMP ≅ △NMP

Statements	Reasons
1. △LMP and △MNP are right triangles	
2. $\overline{ML} \cong \overline{MN}$	
3. $\overline{MP} \cong \overline{MP}$	
4. △ LMP ≅ △NMP	

5.

Given: $\overline{LG} \parallel \overline{JM}$, H is the midpoint of $\overline{LM}$

Prove : $\Delta LGH \cong \Delta MJH$

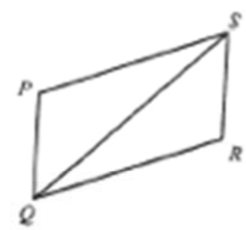


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

6.

Given: $\overline{PS} \parallel \overline{QR}$, $\angle QPS = \angle SRQ$

Prove : $\overline{PQ} \cong \overline{RS}$

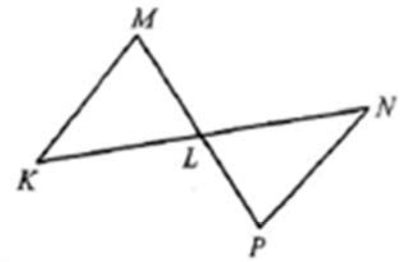


Statements	**Reasons**
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

1.

Given: L is the midpoint of $\overline{KN}$ and $\overline{MP}$

Prove : $\Delta MKL \cong \Delta PNL$

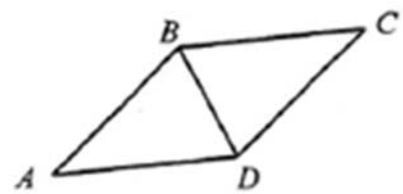


Statements	Reasons
1.	1.
2. $\overline{KL} \cong \overline{NL}$	2.
3.	3. Definition of Midpoint
4. $\angle MLK \cong \angle PLN$	4.
5.	5.

2.

Given: $\overline{BD}$ bisects $\angle ABC$, $\angle BAD \cong \angle BCD$

Prove : $\Delta ABD \cong \Delta CBD$

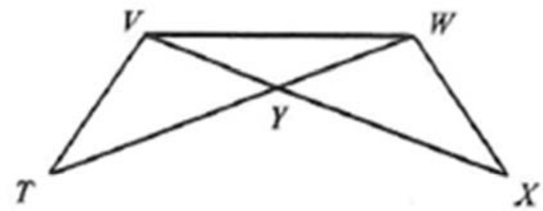


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

3.

Given: $\overline{TY} \cong \overline{XY}$, $\angle TVY \cong \angle XWY$

Prove : $\Delta TVY \cong \Delta XWY$

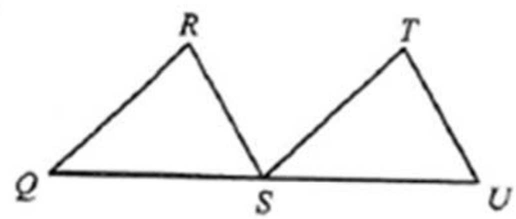


Statements	Reasons
1. $\overline{TY} \cong \overline{XY}$	1.
2. $\angle TVY \cong \angle XWY$	2.
3. $\angle TYV \cong \angle XYW$	3.
4. $\Delta TVY \cong \Delta XWY$	4.

4.

Given: S is the midpoint of $\overline{QU}$, $\overline{QR} \cong \overline{ST}$, $\overline{RS} \cong \overline{TU}$

Prove : $\Delta QRS \cong \Delta STU$

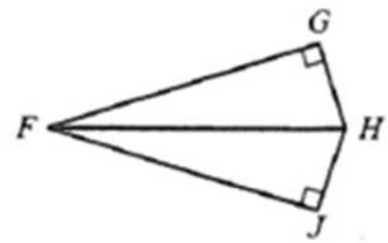


Statements	Reasons
1.	1.Given
2.	2. Definition of Midpoint
3.	3.
4.	4.
5. $\Delta QRS \cong \Delta STU$	5.

5.

Given: ΔFGH and ΔFJH are right triangles, $\overline{GH} \cong \overline{JH}$

Prove : $\Delta FGH \cong \Delta FJH$

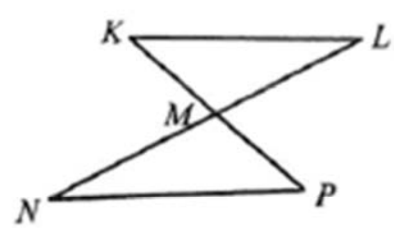


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.

6.

Given: $\overline{KL} \parallel \overline{NP}$, $\overline{KL} \cong \overline{PN}$

Prove : $\Delta KML \cong \Delta PMN$

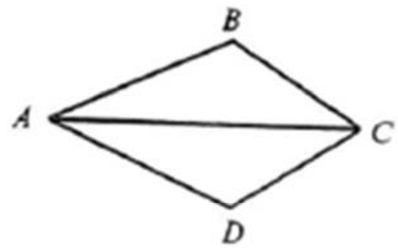


Statements	Reasons
1. $\overline{KL} \parallel \overline{NP}$	1.
2. $\overline{KL} \cong \overline{PN}$	2.
3. $\angle L \cong \angle N$	3.
4. $\angle NMP \cong \angle LMK$	4.
5. $\Delta KML \cong \Delta PMN$	5.

7.

Given: $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{DC}$

Prove : $\angle BCA \cong \angle DCA$

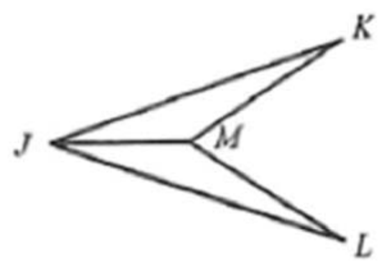


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

8.

Given: $\overline{JM}$ bisects $\angle KJL$, $\angle JMK \cong \angle JML$

Prove : $\overline{JK} \cong \overline{JL}$



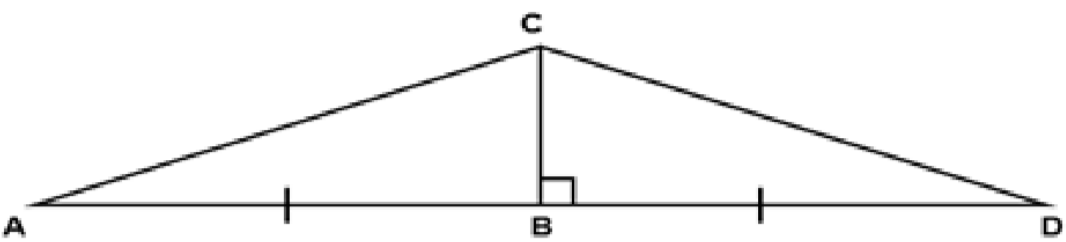
Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Triangle Proofs

Corresponding
Parts of
Congruent
Triangles are
Congruent

Remember the definition of congruent figures?
If two geometric figures are congruent, then their corresponding parts are congruent.

Example: In the figure below, how do we know that $\Delta ABC \cong \Delta DBC$?



Statements	Reason
1. $\overline{AB} \cong \overline{BD}$	1. Given
2. $\angle ABC \cong \angle DBC$	2. By Euclid's Perpendicular Postulate
3. $\overline{CB} \cong \overline{CB}$	3. Reflexive Property of Congruence
4. $\Delta ABC \cong \Delta DBC$	4. SAS Postulate

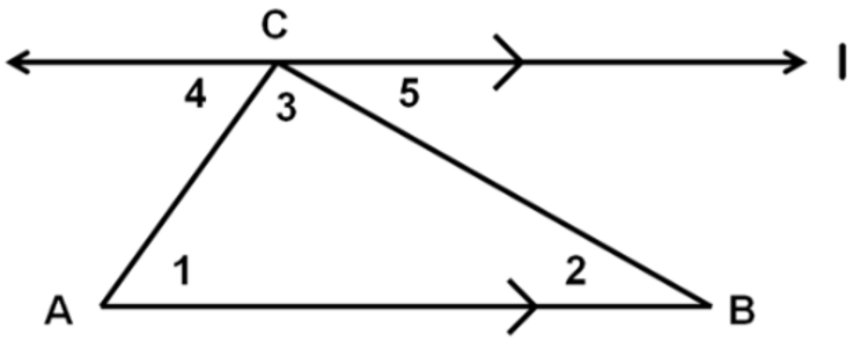
. . . And now that we know that the two triangles are congruent then by CPCTC all the other corresponding parts are congruent as well.

$\angle A \cong \angle D$
$\angle ACB \cong \angle DCB$
$\overline{AC} \cong \overline{DC}$

Let's start proving theorems about triangles using two column proofs. Fill in the missing statements and reasons in the proof below.

Theorem: The sum of the measure of the angles of any triangle is 180°.

Proof:
Given: The top line (that touches the top of the triangle) is parallel to the base of the triangle.
Prove: $m\angle 1 + m\angle 3 + m\angle 2 = 180°$



Statements	Reason
1. $m\angle 4 = m\angle 1$	1.
2. $m\angle 5 = m\angle 2$	2.
3.	3. Three angles form one side of the straight line
4. $m\angle 1 + m\angle 3 + m\angle 2 = 180°$	4.

Isosceles Triangles

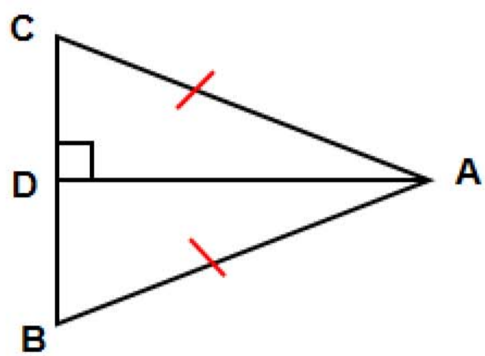
Theorem: If two sides of a triangle are congruent, than the angles opposite those sides are congruent.

Fill in the following proof using postulates, theorems, and properties that you have learned.

Proof:

Given: $\overline{AC} \cong \overline{AB}$

Prove: $\angle C \cong \angle B$

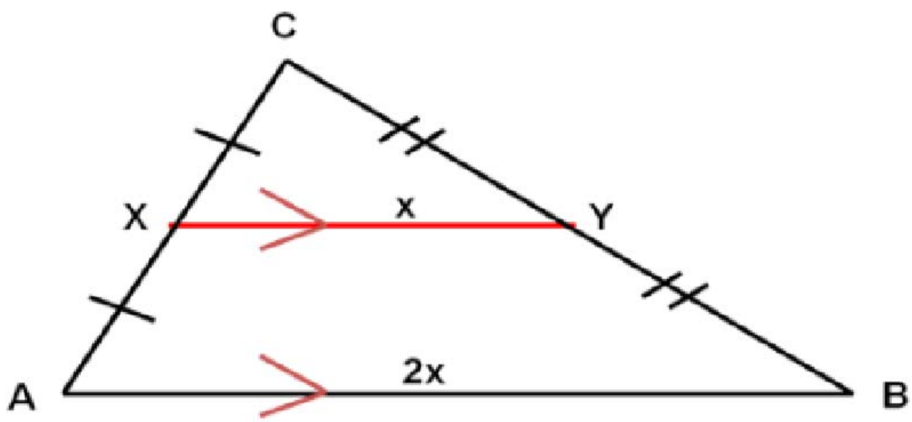


Statements	Reason
1.	1. Given
2. Draw $\overline{AD} \perp \overline{CB}$	2.
3.	3. Reflexive Property of Congruence
4. $\Delta CDA \cong \Delta BDA$	4.
5.	5.

Definition: A line segment whose endpoints are the midpoint of two sides of a triangle is called a midsegment of the triangle.

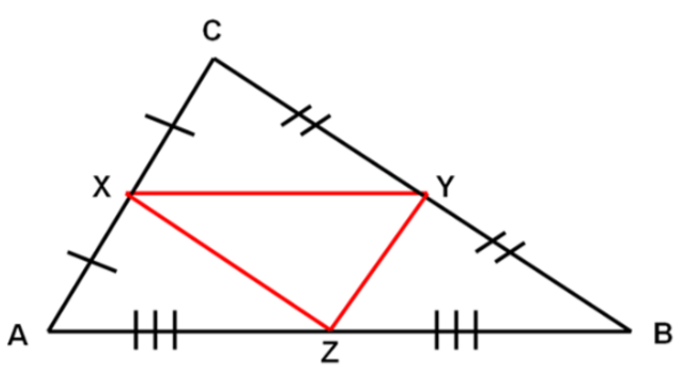
Theorem: The segment connecting the midpoints of two sides of the triangle is parallel to the third side and half the length of the third side.

In the figure below $\overline{XY} \parallel \overline{AB}$ and XY = ½ (AB) or AB = 2(XY)



Let's prove this theorem using a sheet of patty paper.

1) Draw Δ ABC on a sheet of patty paper.
2) Fold and pinch to locate the three midpoints of the triangle.
3) Draw and label the three midpoints X, Y, Z.
4) Draw segments $\overline{XY}$, $\overline{YZ}$, and $\overline{XZ}$.



Using your construction, verify:

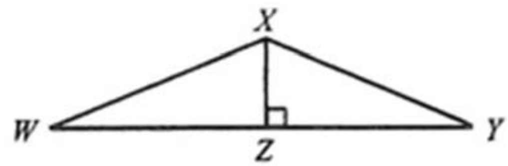
$\overline{XY} \parallel \overline{AB}$, $\overline{YZ} \parallel \overline{XY}$, $\overline{CA}$, and $\overline{XZ} \parallel \overline{CB}$

Complete the following triangle proofs. Some may include CPCTC. Some charts may also include more lines than needed.

1

Given: ΔWZX and ΔYZX are right triangles,
$\overline{WX} \cong \overline{YX}$

Prove : $\angle WXZ \cong \angle YXZ$

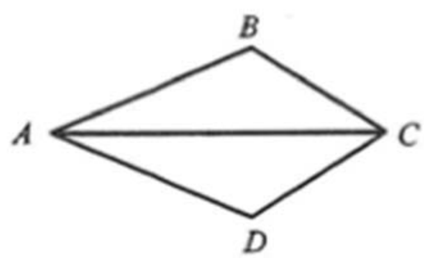


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

2.

Given: $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$

Prove : $\Delta ABC \cong \Delta ADC$

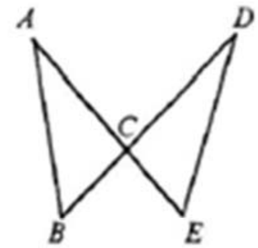


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

3.

Given: $\angle BAC \cong \angle EDC$, $\overline{BC} \cong \overline{EC}$

Prove : $\angle ABC \cong \Delta DEC$

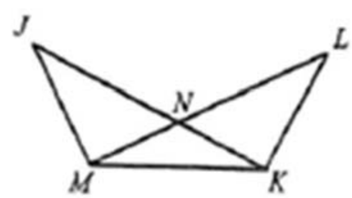


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

4.

Given: $\angle JKM \cong \angle LMK$, $\overline{JK} \cong \overline{LM}$

Prove : $\Delta JMK \cong \Delta LKM$

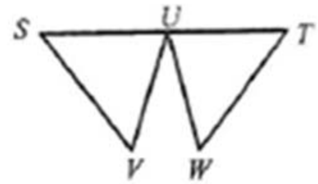


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

5.

Given: U is the midpoint of $\overline{ST}$,
$\overline{SV} \cong \overline{TW}$, $\overline{VU} \cong \overline{WU}$

Prove : $\angle SVU \cong \angle TWU$

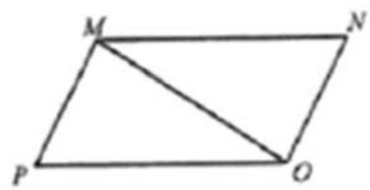


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

6.

Given: $\overline{MN} \parallel \overline{PO}$, $\overline{MP} \parallel \overline{NO}$

Prove : $\overline{MP} \cong \overline{ON}$

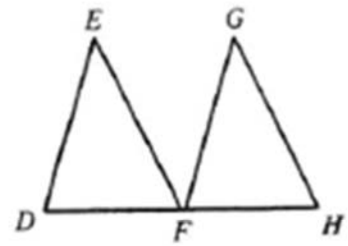


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

7.

Given: $\overline{DE} \parallel \overline{FG}, \overline{DE} \cong \overline{FG}, \angle DEF \cong \angle FGH$

Prove : $\angle DFE \cong \angle FHG$

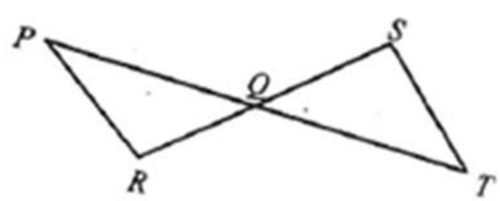


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

8.

Given: Q is the midpoint of $\overline{PT}$ and $\overline{RS}$

Prove : $\Delta PQR \cong \Delta TQS$

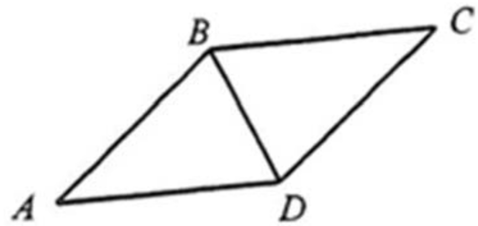


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

9.

Given: $\overline{BC} \parallel \overline{AD}$, $\angle BAD \cong \angle DCB$

Prove : $\overline{AB} \cong \overline{CD}$

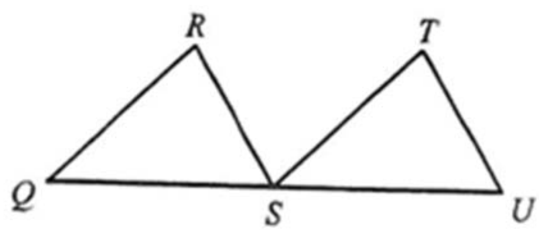


Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

10.

Given: $\overline{QR} \parallel \overline{ST}$, $\overline{RS} \parallel \overline{TU}$, $\overline{QS} \cong \overline{SU}$

Prove : $\angle QRS \cong \angle STU$



Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

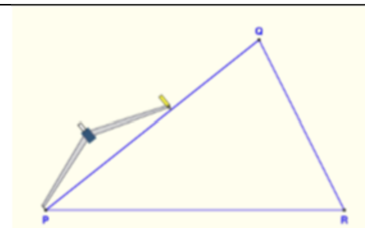
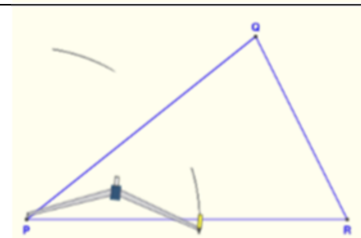
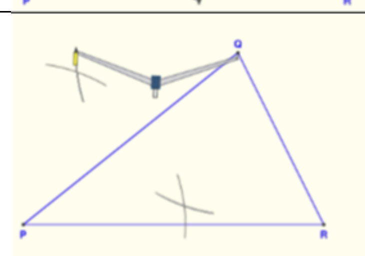
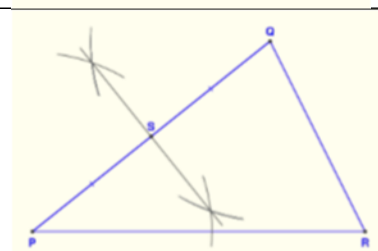
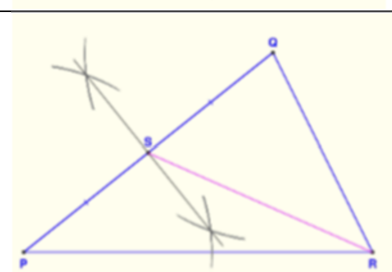
Term	Definition	Notation	Diagram/Visual
Constructions			
Altitude (of a Triangle)			
Angle Bisector (of a Triangle)			
Centroid (of a Triangle)			
Circumcenter (of a Triangle)			
Incenter (of a Triangle)			
Median (of a Triangle)			
Orthocenter (of a Triangle)			

Constructions – Special Segments Within Triangles and Points of Concurency

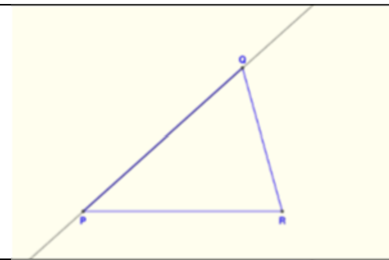
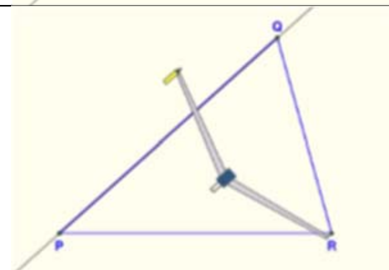
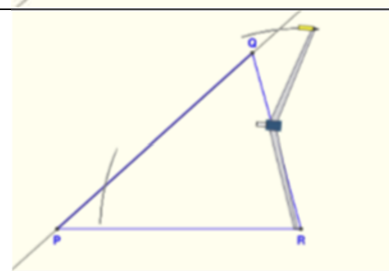
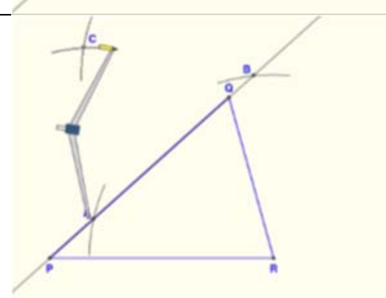
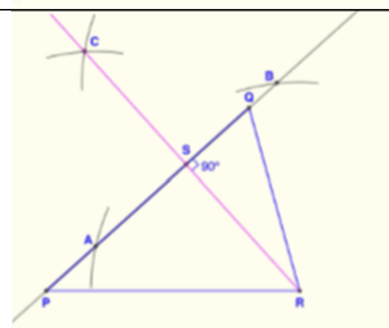
GUIDED NOTES

All virtual diagrams can be found on the website Math Open Reference.

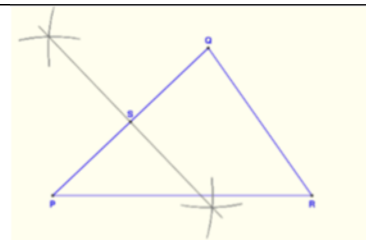
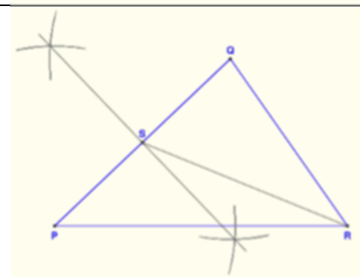
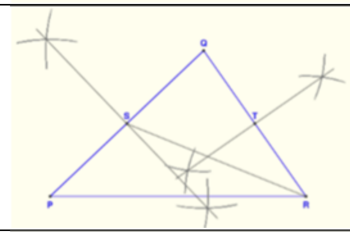
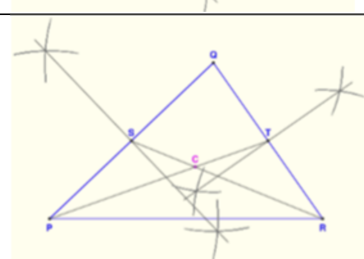
Constructing the Median of a Triangle

1. With the compass on P, set it to any medium width.	
2. Draw an arc on each side of the Line PQ.	
3. Without changing the compass width, repeat for the other end Q of the line.	
4. Draw a straight line between the two arc intersections.	
5. Draw a straight line from S to the opposite vertex of the triangle.	

Constructing the Altitude of a Triangle

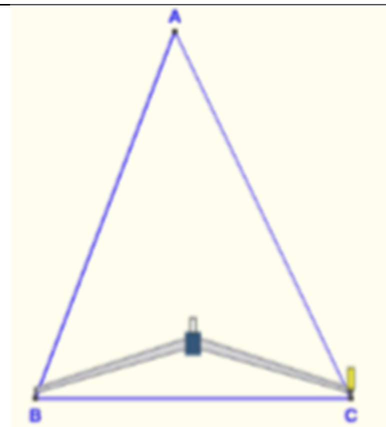
1. Extend a side of the triangle at both ends.	
2. From R, set the compass width beyond PQ	
3. Draw two arcs across the line PQ, creating points A,B	
4. Draw arcs from A and B, creating point C.	
5. Draw a line from R to C, crossing PQ at S.	

Constructing Centroid of a Triangle

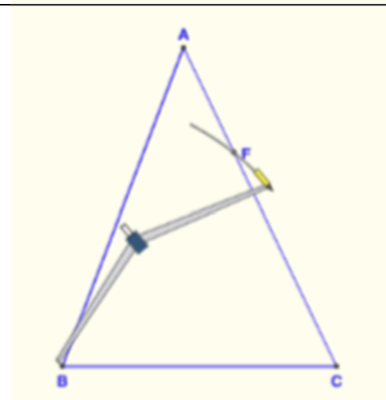
1. Construct the perpendicular bisector of PQ to find its midpoint S.	
2. Draw the first median from the midpoint S to the opposite vertex.	
3. Find the midpoint T of another side of the triangle.	
4. Draw a second median from the midpoint T to the opposite vertex. The centroid is C where the two medians intersect.	

Constructing Orthocenter of a Triangle

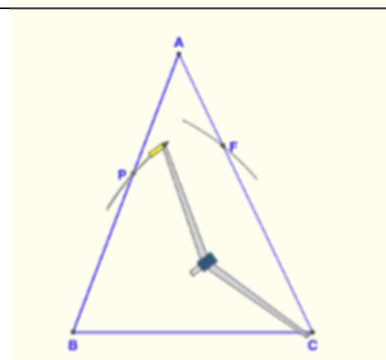
1. Set the compass width to the length of a side of the triangle.



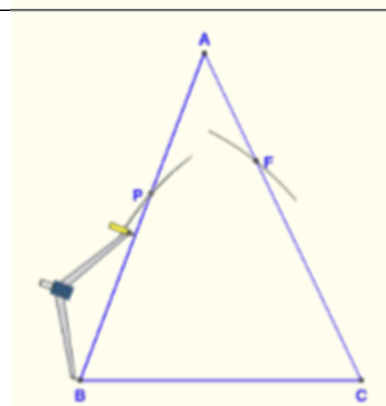
2. From B draw an arc across AC creating point F

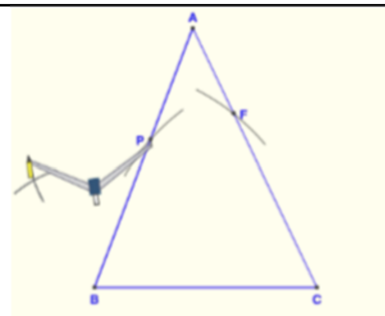
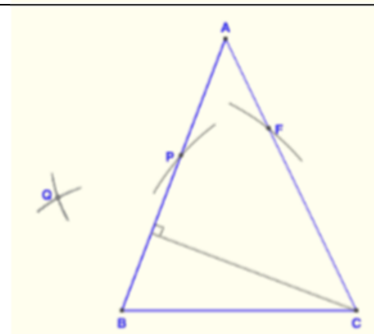
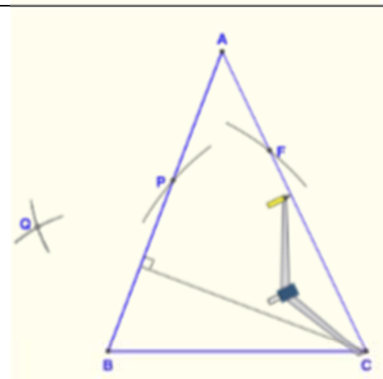
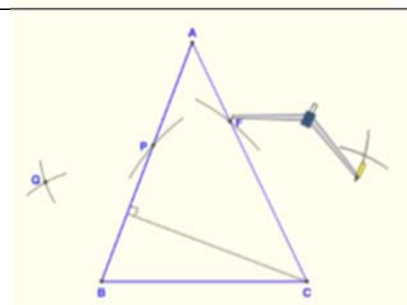
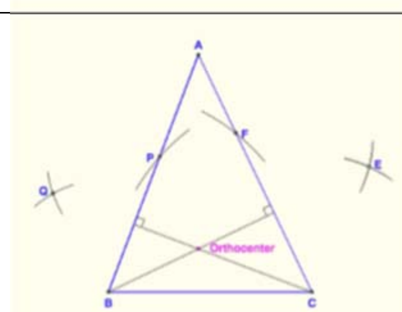


3. From C draw an arc across BA creating point P

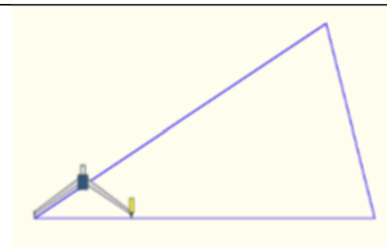
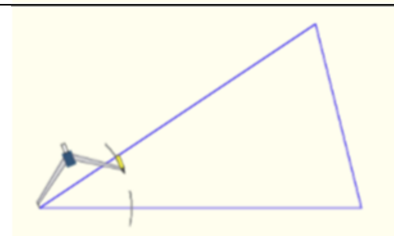
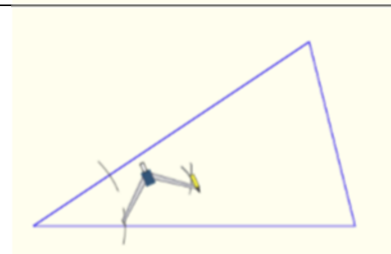
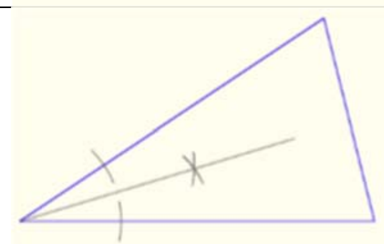
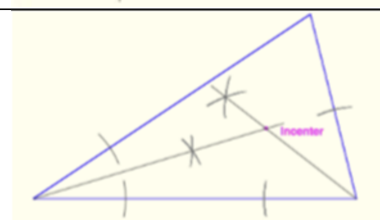


4. Set the compass width to more than half the distance BP.



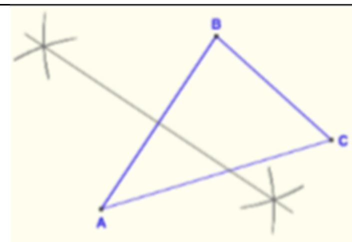
5. From B and P, draw arcs that intersect at point Q.	
6. With a straightedge on C and Q, draw an altitude of the triangle.	
7. Set the compass width to more than half the distance CF.	
8. From C and F, draw arcs that intersect at point E.	
9. With a straightedge on B and E, draw another altitude of the triangle The intersection of the altitudes is the orthocenter of the triangle.	

Constructing the Incenter of a Triangle

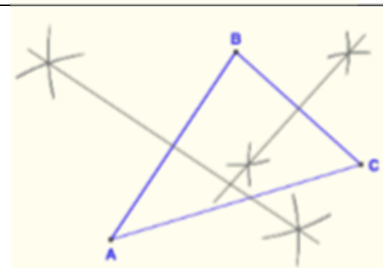
1. With the compass on a vertex, adjust its width to a medium setting.	
2. Draw an arc across each leg.	
3. Draw two intersecting arcs, one from each previous arc	
4. Draw the angle bisector line.	
5. Repeat for one of the other vertices. The two bisectors intersect at the Incenter of the triangle.	

Constructing the Circumcenter of a Triangle

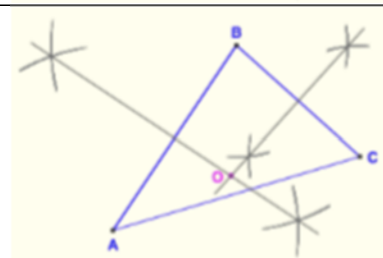
1. Draw the perpendicular bisector of side AB.



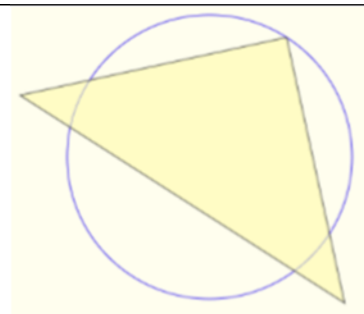
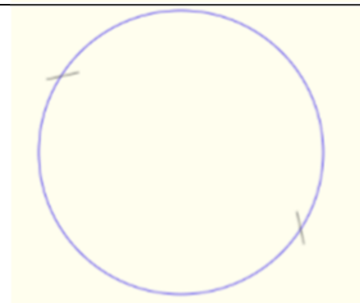
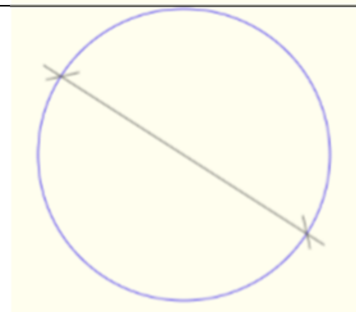
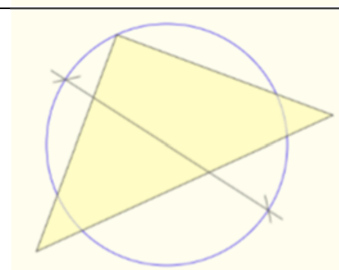
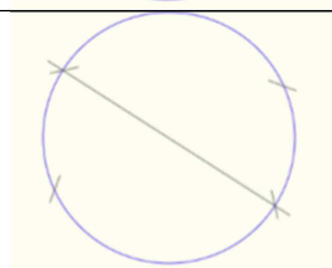
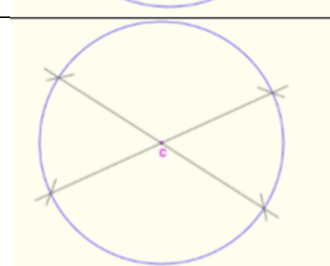
2. Draw the perpendicular bisector of the side BC.



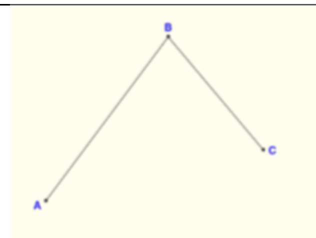
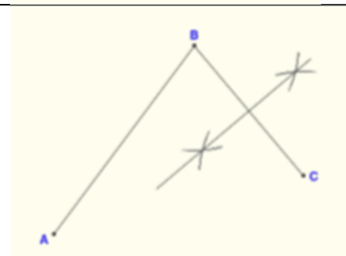
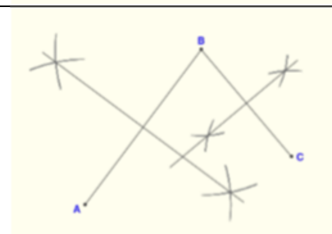
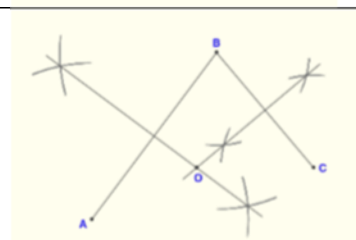
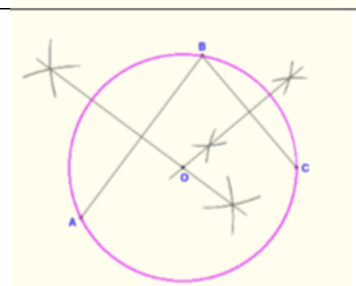
3. Make the points where the bisectors intersect as "O"
   The point "O" is the circumcenter of the triangle ABC.



Constructing – Finding the center of a circle

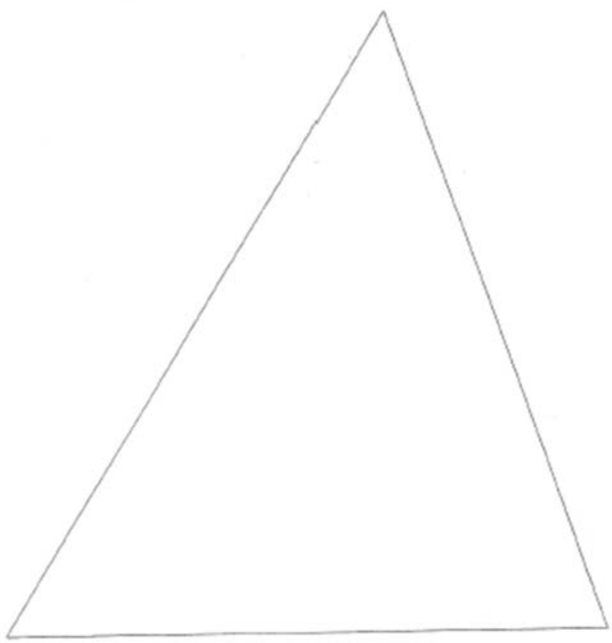
1. Place the vertex of a right angled object at any point on the circle	
2. Make two marks where the sides cross the circle.	
3. Draw a line between these points – the first diameter.	
4. Place the vertex of a right angled object at another point on the circle.	
5. Make two marks were the sides cross the circle.	
6. Draw a line between these points – the second diameter. The center of the circle is at C where the two diameters intersect.	

Constructing a Circle through three points

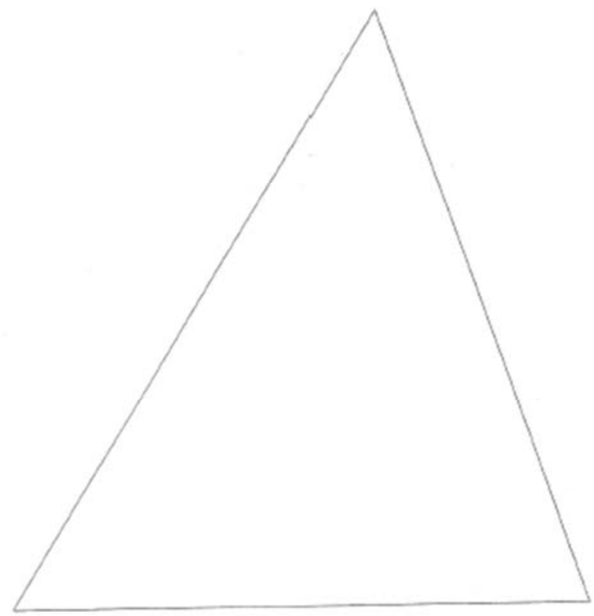
1. (Optional) Draw a line between any two point pairs. (here AB and BC)	
2. Construct the perpendicular bisector of BC.	
3. Repeat for the other line AB.	
4. The two perpendicular bisectors intersect at O, the circle center.	
5. From O, set the compass width to any point A,B or C and draw a circle. The circle is the only one that will pass through the three points.	

Example!

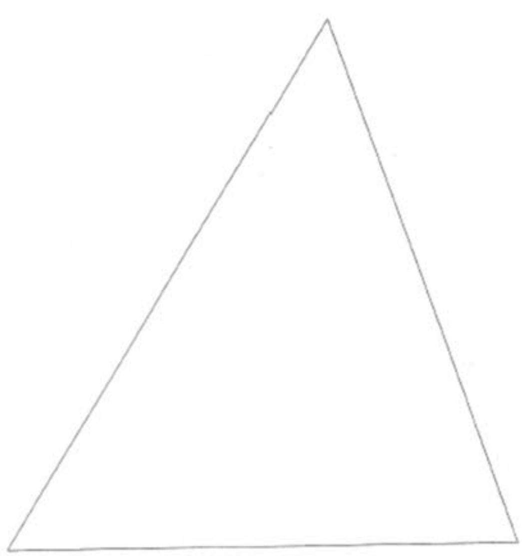
1. *Find the Mediean of the Triangle*



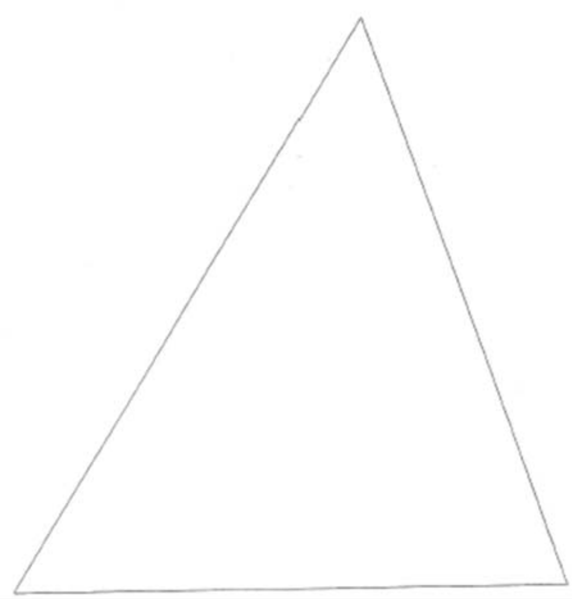
2. *Find the Altitude of the Triangle*



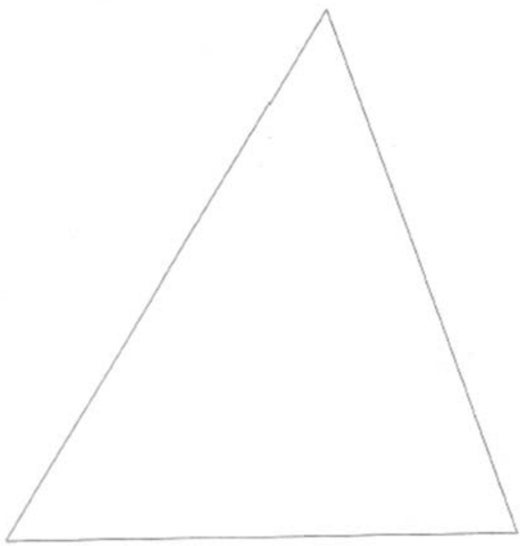
3. *Construct the CENTROID of the triangle*



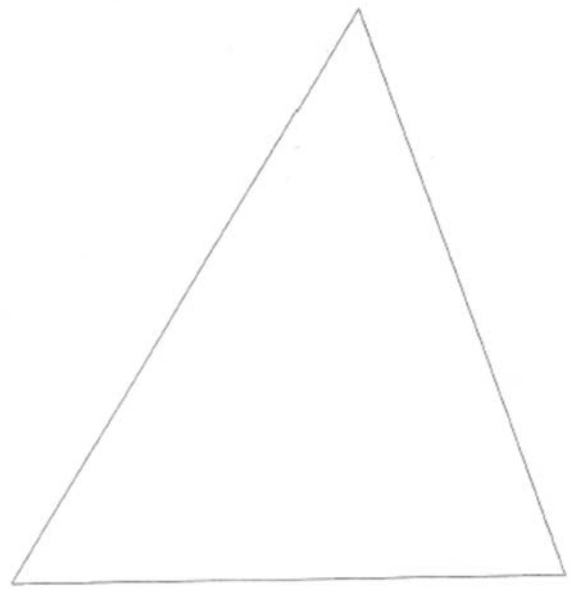
4. *Construct the ORTHOCENTER of the triangle*



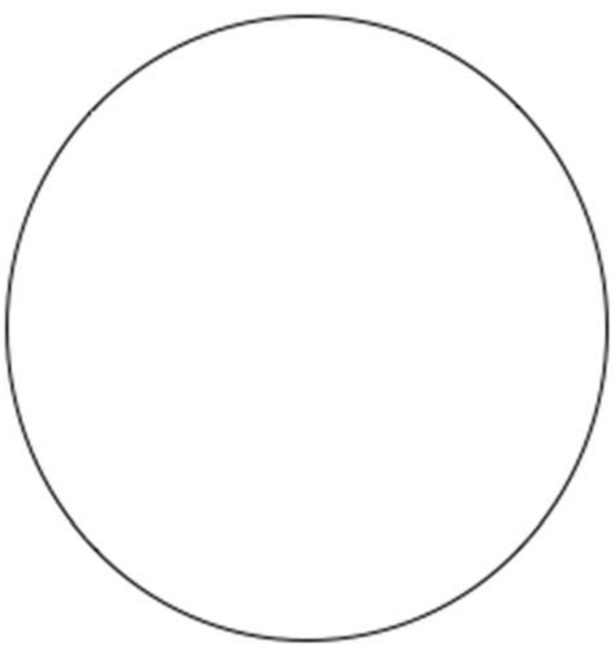
5. Construct the INCENTER of the triangle



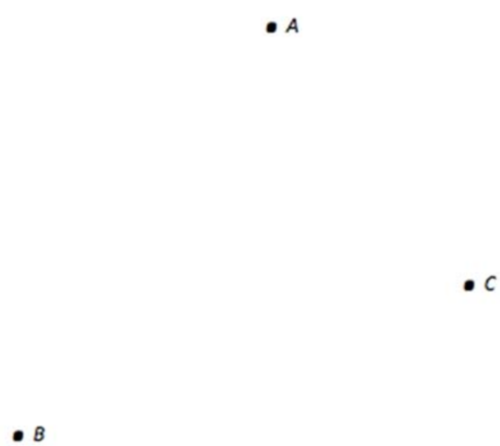
6. Construct the CIRCUMCENTER of the triangle



7. Find the center of a circle

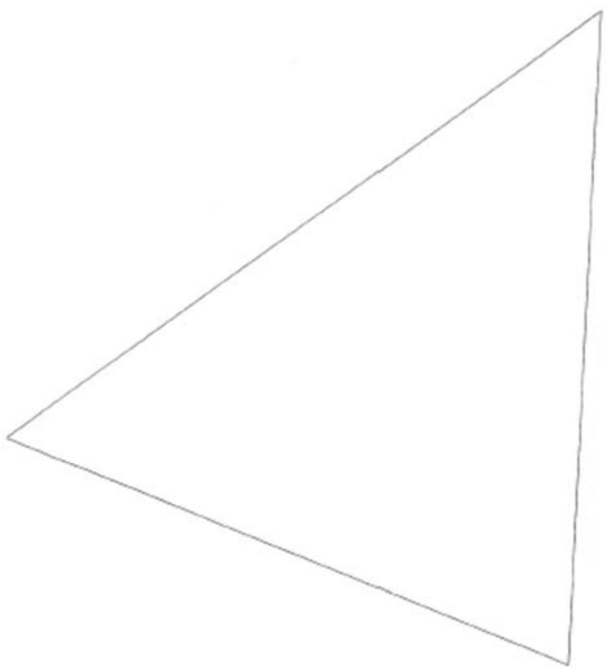


8. Construct a circle through three points

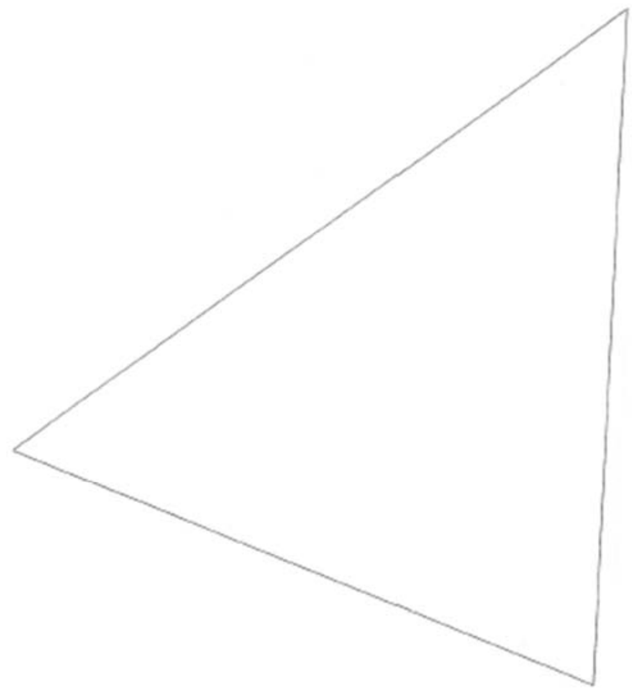


SELF CHECK

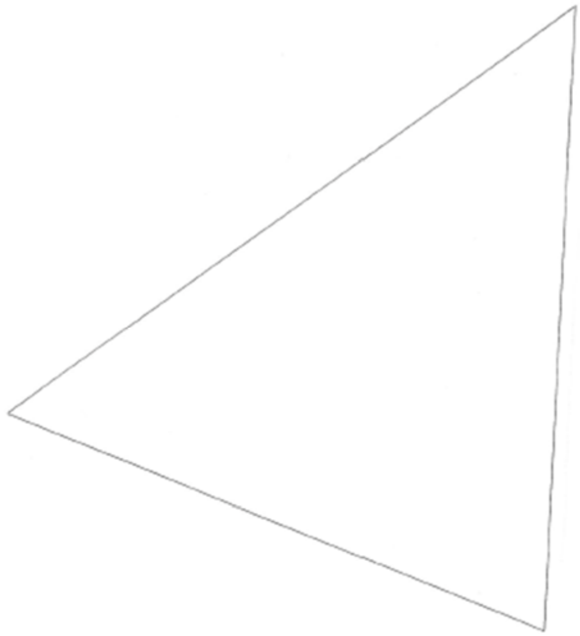
1. *Find the MEDIAN of the Triangle*



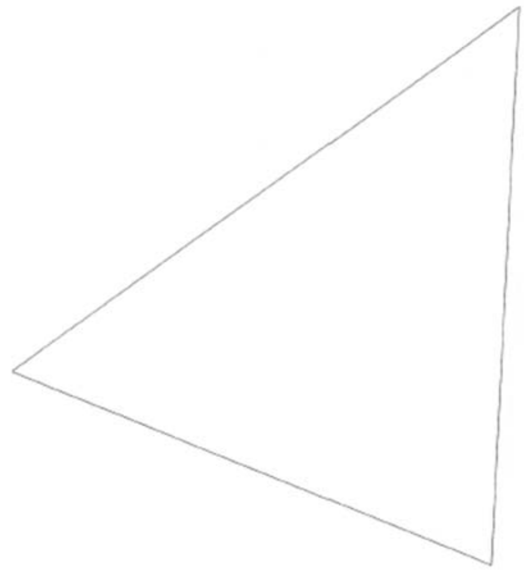
2. *Find the ALTITUDE of the Triangle*



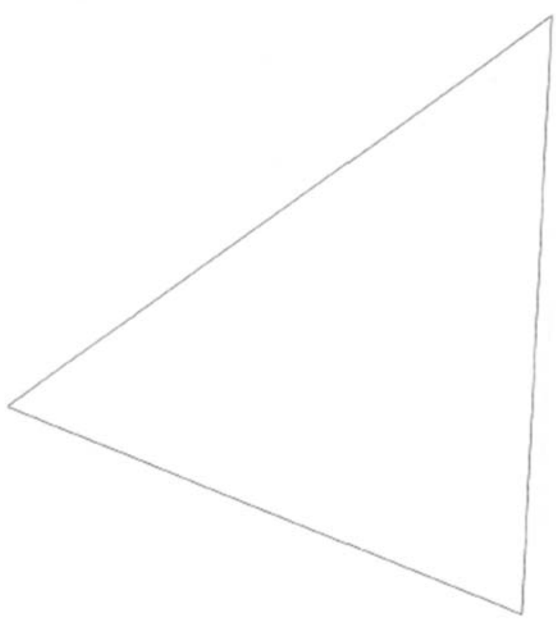
3. Construct the CENTROID of the triangle



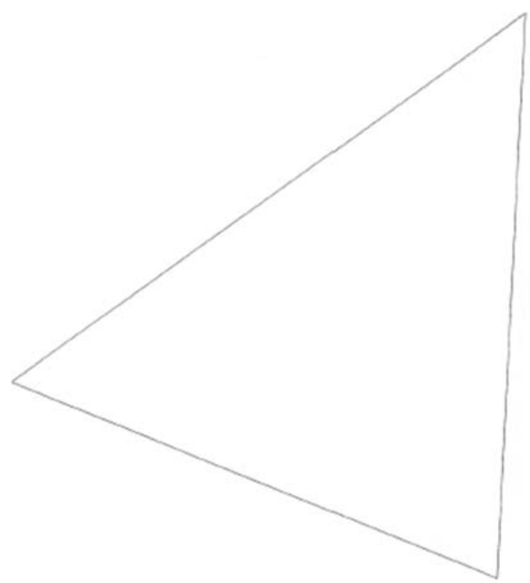
4. Construct the ORTHOCENTER of the triangle



5. Construct the INCENTER of the triangle

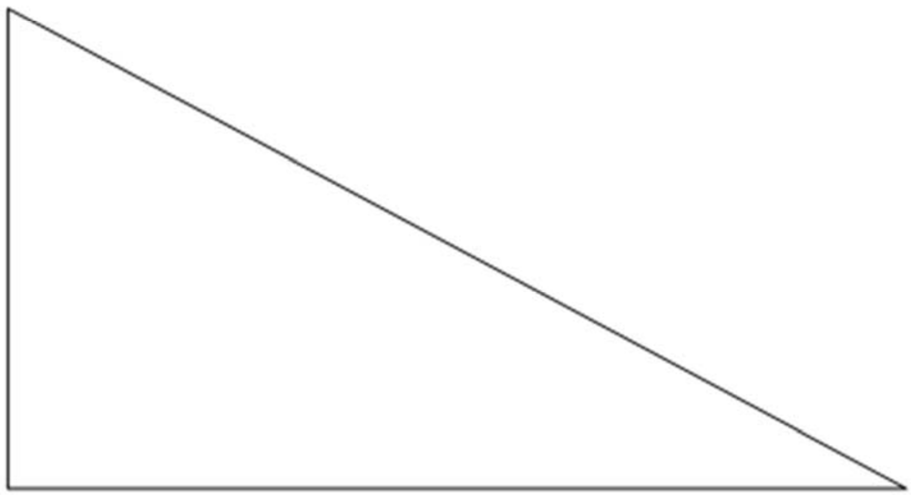


6. Construct the CIRCUMCENTER of the triangle



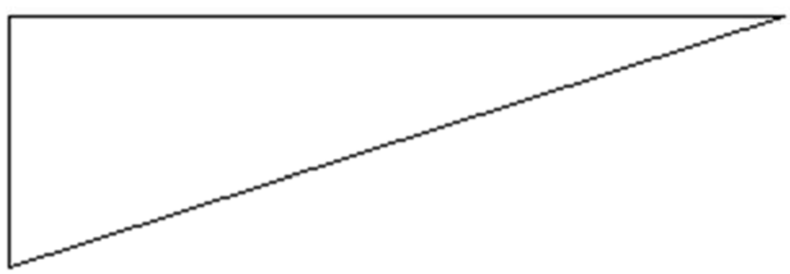
1.

Construct the centroid of the triangle below.



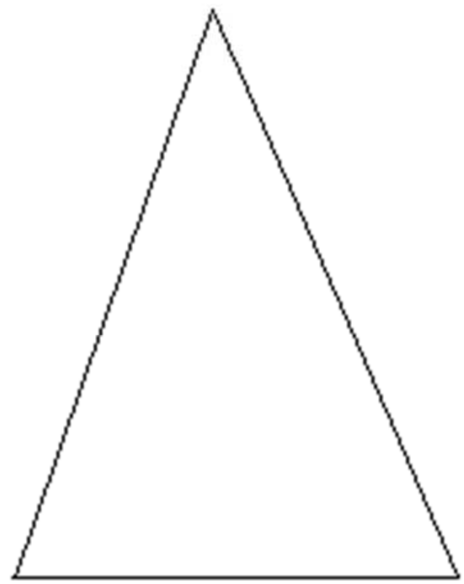
2.

Construct the incenter of the triangle below.



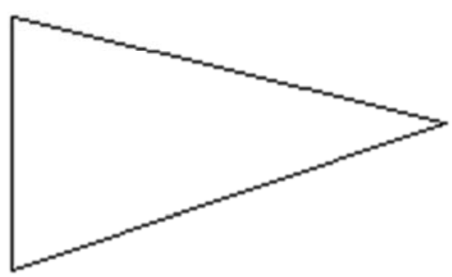
3.

Construct the orthocenter of the triangle below



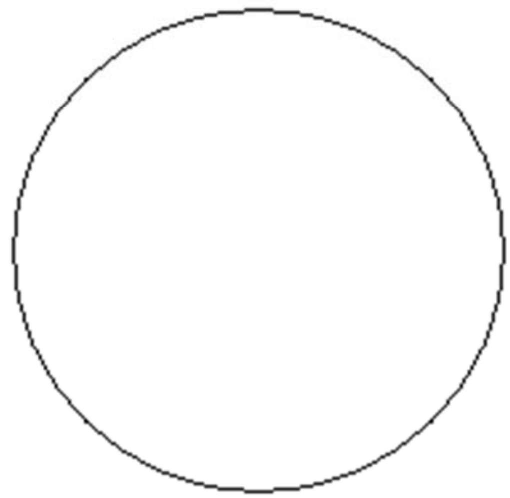
4.

. Construct the circumcenter of the triangle below.



5.

Find the center of the circle below.



6.

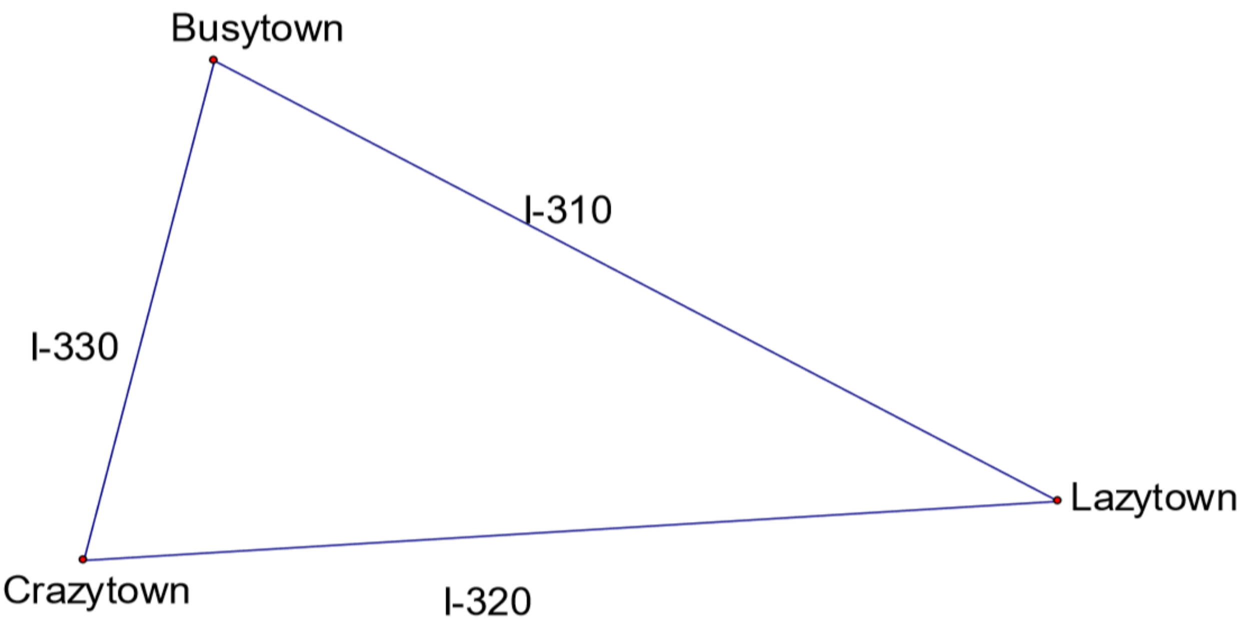
Draw the only circle that passes through the three points below.

Real Mathematics:
Application

Centers of Triangles

Triangle Center:	Point of Concurrency of:	Significance of:
Incenter	Angle bisectors	Center of inscribed circle Equidistant from the sides of the triangle
Circumcenter	Perpendicular bisectors	Center of the circumscribing circle Equidistant from the vertices of the
Orthocenter	Altitudes	
Centroid	Medians	Center of balance or gravity The distance from a vertex to the centroid is twice the distance from the centroid to the opposite side.

A developer plans to build an amusement park but wants to locate it within easy access of the three largest towns in the area as shown on the map below. The developer has to decide on the best location and is working with the ABC Construction Company to minimize costs wherever possible. No matter where the amusement park is located, roads will have to be built for access directly to the towns or to the existing highways.



1. Just by looking at the map, choose the location that you think will be best for building the amusement park. Explain your thinking.

2. Now you will use some mathematical concepts to help you choose a location for the tower. Investigate the problem above by constructing the following:
   a) all 3 medians of the triangle

   b) all 3 altitudes of the triangle
   c) all 3 angle bisectors of the triangle
   d) all 3 perpendicular bisectors of the triangle

   You have four different kinds of tools at your disposal- patty paper, MIRA, compass and straight edge, and Geometer's Sketch Pad. Use a different tool for each of your constructions.

3. Choose a location for the amusement park based on the work you did in part 2. Explain why you chose this point.

4. How close is the point you chose in part 3, based on mathematics, to the point you chose by observation?

You have now discovered that each set of segments resulting from the constructions above always has a point of intersection. These four points of intersection are called the ***points of concurrency*** of a triangle.

The intersection point of the medians is called the ***centroid*** of the triangle.
The intersection point of the angle bisectors is called the ***incenter*** of the triangle.
The intersection point of the perpendicular bisectors is called the ***circumcenter*** of the triangle.
The intersection point of the altitudes is called the ***orthocenter*** of the triangle.

5. Can you give a reasonable guess as to why the specific names were given to each point of concurrency?

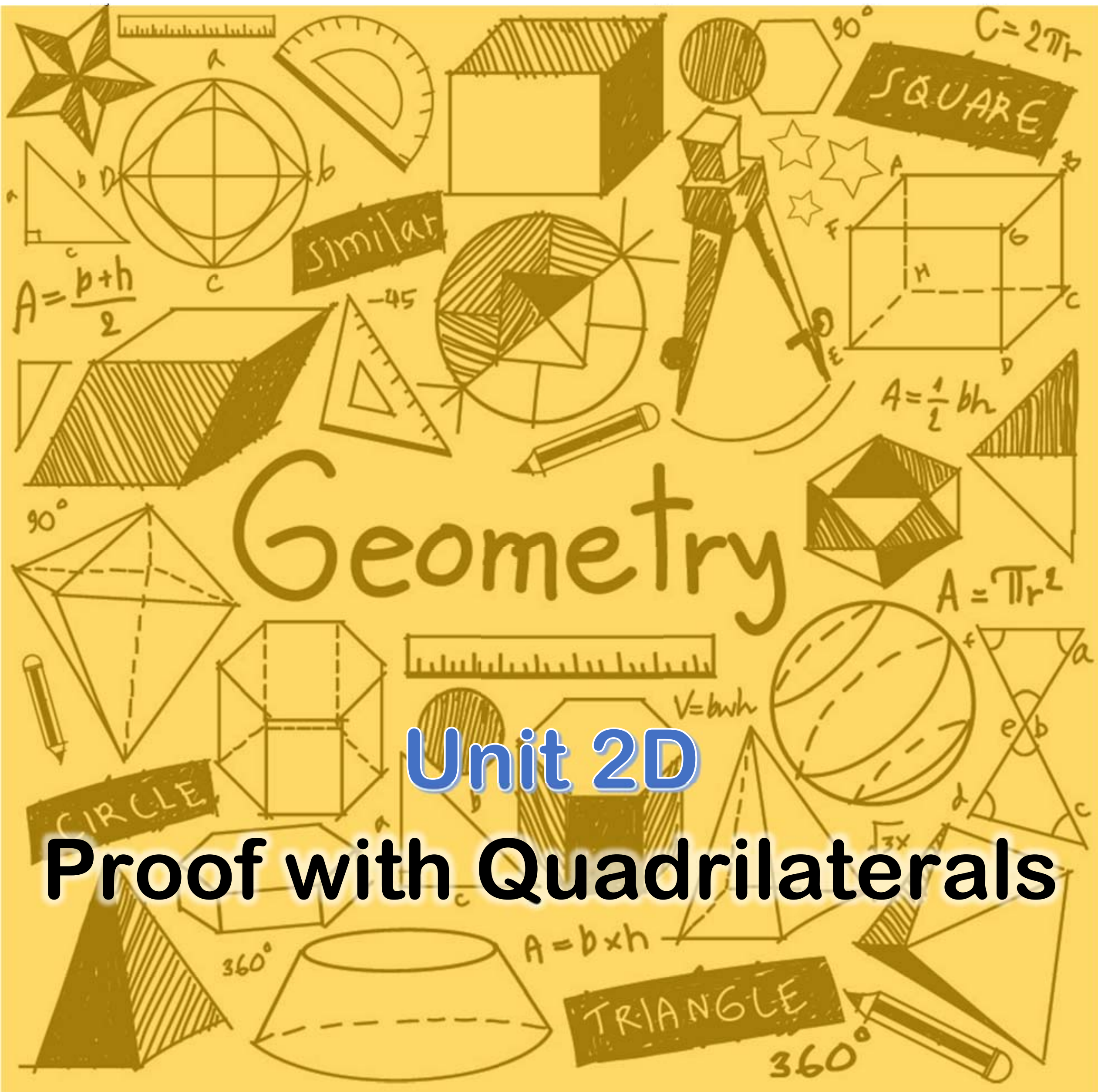
6. Which triangle center did you recommend for the location of the amusement park?

7. The president of the company building the park is concerned about the cost of building roads from the towns to the park. What recommendation would you give him? Write a memo to the president explaining your recommendation.

HOUSTON COUNTY BOARD OF EDUCATION

HIGH-ACHIEVING TEACHERS

MATHEMATICS DEPARTMENT



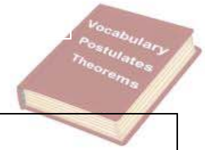
Unit 2D

Proof with Quadrilaterals

HOUSTON COUNTY
BOARD OF EDUCATION
HIGH-ACHIEVING TEACHERS

MATHEMATICS
DEPARTMENT

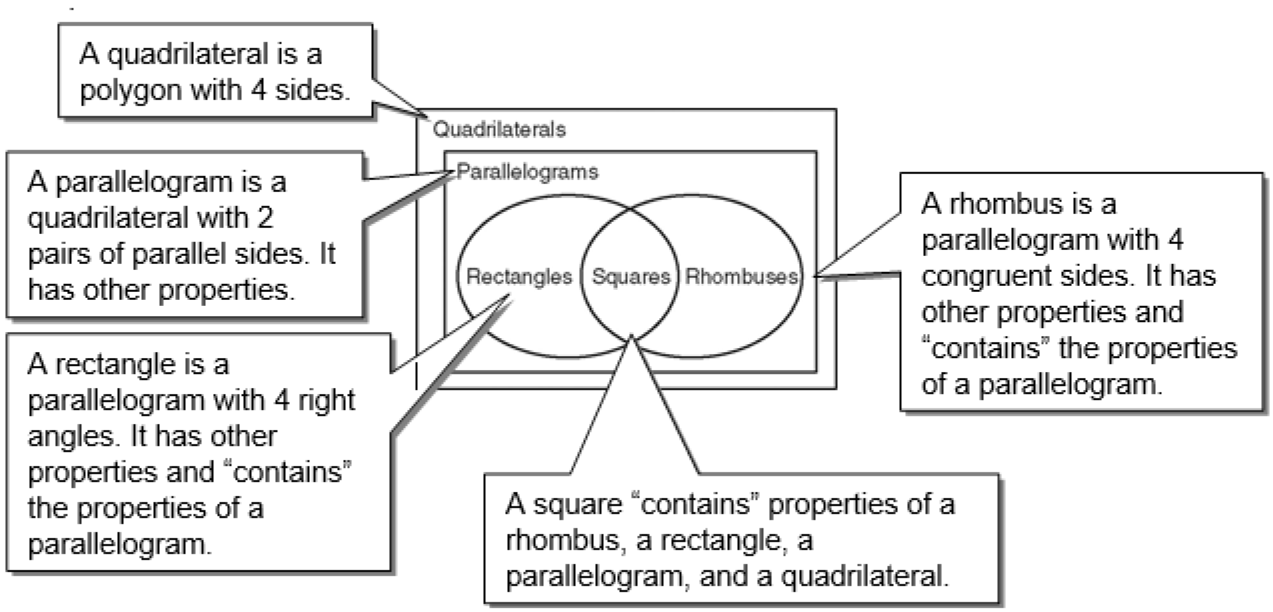
Science Technology Engineering Mathematics



	Diagram/Visual		
Parallelogram			
Rectangle			
Rhombus			
Square			
Trapezoid			
Diagonal			
Bisect			
Perpendicular			

Definitions and Properties of Quadrilaterals

A **quadrilateral** is a polygon with four sides. A **parallelogram** is a quadrilateral that has two pairs of parallel sides. A **rectangle** is a parallelogram with four right angles. A **rhombus** is a parallelogram with four congruent sides. A **square** is a parallelogram with four sides congruent and four right angles.



GUIDED NOTES

Explain why each conditional statement is true.

A. If a quadrilateral is a square, then it is a parallelogram. By definition, a square is a quadrilateral with four congruent sides.

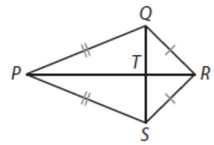
   Any quadrilateral with both pairs of opposite sides congruent is a parallelogram, so a square is a parallelogram.

B. If a quadrilateral is a square, then it is a rectangle.
   *By definition, a square is a quadrilateral with four*__________________ .
   By definition, a rectangle is also a quadrilateral with four ______________ .
   Therefore, a square is a rectangle.

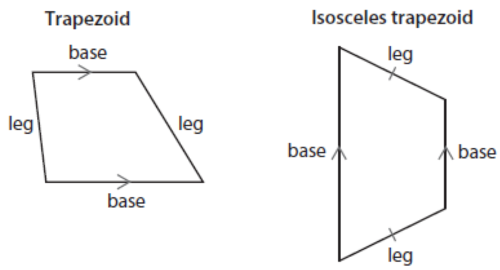
SELF CHECK

Explain why this conditional statement is true: If a quadrilateral is a square, then it is a rhombus.

A **kite** is a quadrilateral with two distinct pairs of congruent consecutive sides.

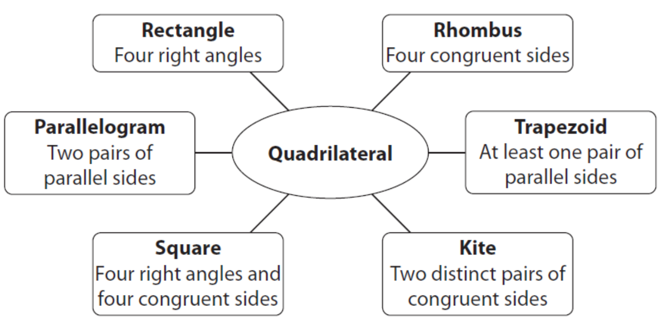


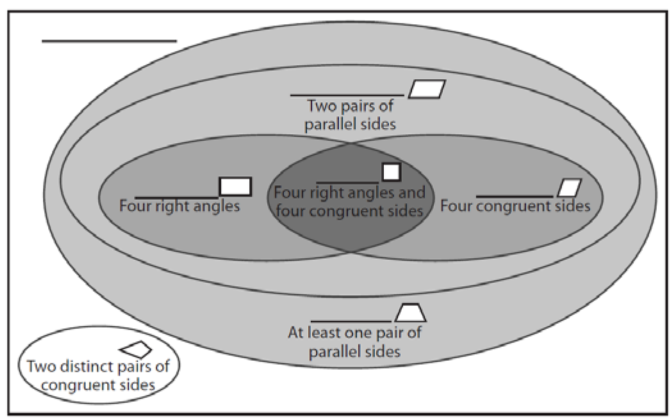
A **trapezoid** is a quadrilateral with at least one pair of parallel sides. The pair of parallel sides of the trapezoid (or either pair of parallel sides if the trapezoid is a parallelogram) are called the bases of the trapezoid. The other two sides are called the legs of the trapezoid. A trapezoid has two pairs of base angles: each pair consists of the two angles adjacent to one of the bases. An **isosceles trapezoid** is one in which the legs are congruent but not parallel.



GUIDED NOTES

Use the information in the graphic organizer to complete the Venn diagram.





What can you conclude about all parallelograms?

SELF CHECK

Determine whether each of the following describes a kite or a trapezoid.

A. Has two distinct pairs of congruent consecutive sides

B. Has diagonals that are perpendicular

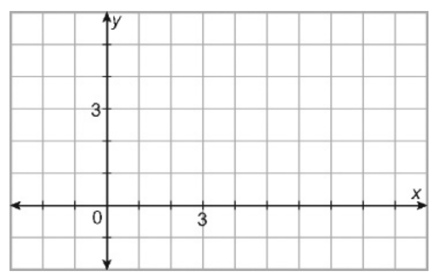
C. Has at least one pair of parallel sides

D. Has exactly one pair of opposite angles that are congruent

E. Has two pairs of base angles

GUIDED NOTES

Plot *G*(0, 0), *H*(2, 3), *I*(8,4) and *J*(6, 1).



Find the rise (difference in the *y*-coordinates) from *G* to *H*. ________

Find the run (difference in the *x*-coordinates) from *G* to *H*. _______

What is the slope of $\overline{GH}$? _________

Repeat the process to find the slope of $\overline{IJ}$. __________________

What do the slopes tell you about $\overline{GH}$ and $\overline{IJ}$? ________________________________

Repeat the process to find the slopes of $\overline{HI}$ and $\overline{JG}$. What do you know about $\overline{HI}$ and $\overline{JG}$?

____

Quadrilateral $GHIJ$ is a __________________________. The definition of a ____________________

is ____.

You can use the definitions of quadrilaterals and the coordinate plane to show quadrilaterals are special quadrilaterals, such as the following:

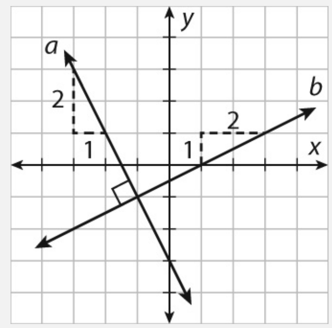
- To prove a quadrilateral is a parallelogram, show that the opposite sides are parallel using the slope formula.
- To prove a quadrilateral is a rectangle, show that the opposite sides are parallel and the consecutive sides are perpendicular using the slope formula.
- To prove a quadrilateral is a rhombus, show that all four sides are congruent using the distance formula.
- To prove a quadrilateral is a square, show that all four sides are congruent and consecutive sides are perpendicular using the distance and slope formulas.

Example!

Two non-vertical lines are perpendicular to each other if the product of their slopes is –1.

The slope of line *a* is –2. The slope of line *b* is $\frac{1}{2}$.

Since $-2 \times \frac{1}{2} = -1$, the lines are perpendicular.



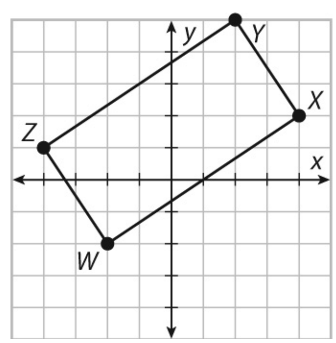
If a parallelogram on the coordinate plane has four right angles, that parallelogram is a rectangle. To prove this, show the consecutive sides of the parallelogram are perpendicular. So, show the product of the slopes is –1.

Questions To Ponder

Is there another way to describe the relationship between the slopes of two perpendicular lines?

GUIDED NOTES

Show that Quadrilateral *WXYZ* is a rectangle.



1. Name the coordinates of *W*, *X*, *Y*, and *Z*.

   W __________ *X* __________ *Y* __________ *Z* __________

2. Calculate the slopes of each side of the parallelogram.

   $\overline{WX}$ = __________ $\overline{XY}$ = __________

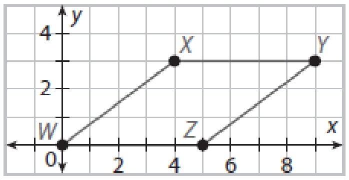
   $\overline{YZ}$ = __________ $\overline{ZW}$ = __________

3. Is *WXYZ* a rectangle? Why or why not?

   ____

Example!

The Distance Formula can also be used to determine if a figure is a special quadrilateral. Use slope and/or the distance formula to show that the figure is a rhombus.

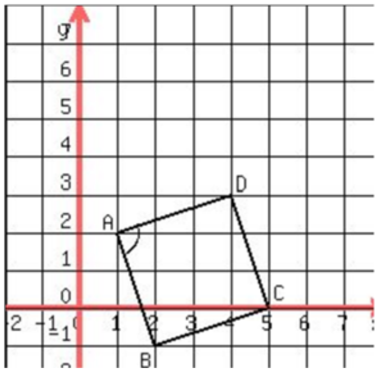


$$WX = \sqrt{(4-0)^2 + (3-0)^2} = \sqrt{25} = 5$$
$$XY = \sqrt{(9-4)^2 + (3-3)^2} = \sqrt{25} = 5$$
$$YZ = \sqrt{(5-9)^2 + (0-3)^2} = \sqrt{25} = 5$$
$$ZW = \sqrt{(0-5)^2 + (0-0)^2} = \sqrt{25} = 5$$

Since all four sides have the same length, *WXYZ* is a rhombus.

GUIDED NOTES

Show that Quadrilateral *ABCD* is a square.



1. Name the coordinates of *A, B, C, and D.*

   *A*____________ *B*____________ C ____________ *D* ____________

2. Calculate the slopes of each side of the parallelogram.

   $\overline{AB}$____________ $\overline{BC}$ ____________

   $\overline{CD}$____________ $\overline{DA}$____________

3. Calculate the length of each side of the parallelogram.

   AB___________ BC____________

   CD____________ DA ____________

Is *ABCD* a square? Why or why not?

SELF CHECK

Name each of the figures below using as many names as possible and state as many properties as you can about each figure.

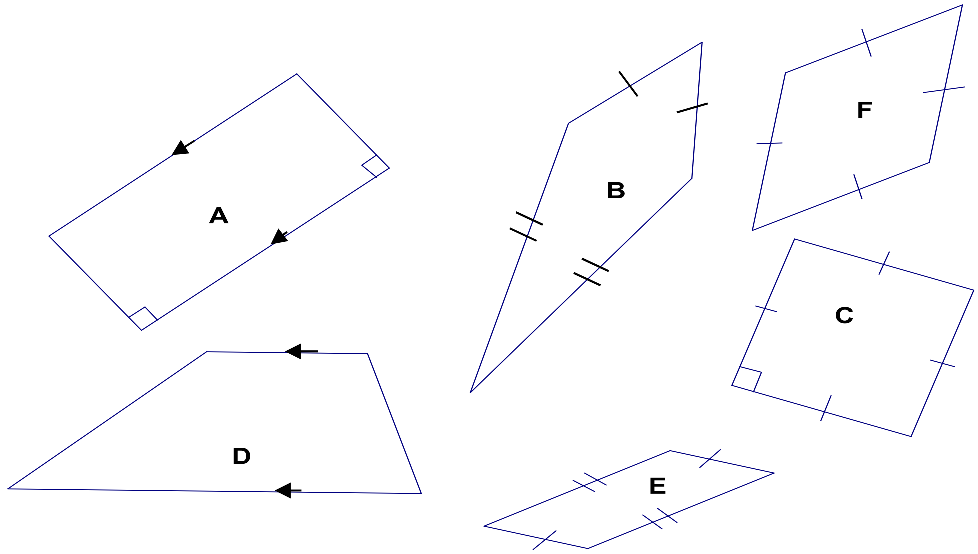


Figure	Names	Properties
A		
B		
C		
D		
E		
F		

	Exactly one pair of parallel sides	Trapezoid
	Two pairs of parallel sides	Parallelogram
	Parallelogram with congruent sides	Rhombus
	Parallelogram with right angles	Rectangle
	Rectangle with congruent sides	Square

Complete the statements below with the word ALL, SOME or NO to make the statements about quadrilaterals correct. Give reasons for your choice. Your reasons can include diagrams.

1. _____________ rectangles are squares.
   Reasons:

2. ____________ rhombuses are parallelograms.

   Reasons:

3. ____________ trapezoids are rectangles.

   Reasons:

4. ___________ kites are rhombuses.

   Reasons:

5. Which of the following quadrilaterals must have at least one pair of parallel sides?

Rectangle	Square	Trapezoid	Parallelogram	Kite	Rhombus

Explain your answer:

Determine whether the figure below is a parallelogram. Determine if yes, determine if it is also a rectangle, rhombus, or square.

6. $A(-7.4), B(1,2), C(9,-8), D(1,-6)$

7. $S(1,5), T(10,7), U(14,1), V(-3,1)$

8. $K(2,7), L(6,12), M(13,13), N(9,8)$

Desmos Polygraph – Basic Quadrilaterals

https://teacher.desmos.com/polygraph-basic-quad

Eric says that he can tell all quadrilaterals apart by asking these three questions:

- Is it concave?
- Does it have any parallel sides?
- Does it have a right angle?

Do you agree with Eric? Why or why not?

Proving Properties of Parallelograms

GUIDED NOTES

In the previous lesson, we defined special quadrilaterals. Today, we will explore additional theorems of quadrilaterals and prove these relationships.

A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Given parallelogram $KATY$ as shown.

a. Which angles are consecutive to $\angle K$?

b. Use what you know about parallel lines to complete the theorem.

Consecutive angles of a parallelogram are _______________.
We will prove this later in the lesson.

Use three index cards and a protractor to draw three different parallelograms. Then, cut out each parallelogram and draw a **diagonal**. Cut along the diagonal to form two triangles. What do you notice about each pair of triangles?

Based upon your exploration, complete the conjecture. Each diagonal of a parallelogram divides that parallelogram into ____.

Let's use your conjecture you made above can used to prove additional theorems about parallelograms.

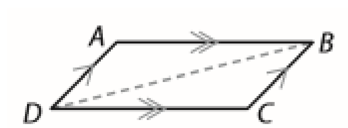
Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

Example! Prove the opposite sides of a parallelogram are congruent.

Given: $ABCD$ is a parallelogram

Prove: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$



Statements	Reasons
1. *ABCD* is a parallelogram.	1. Given
2. Draw $\overline{DB}$.	2. Through any two points, there is exactly one line.
3. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$	3. Definition of parallelogram
4. $\angle ADB \cong \angle CBD$ $\angle ABD \cong \angle CDB$	4. Alternate Interior Angles Theorem
5. $\overline{DB} \cong \overline{DB}$	5. Reflexive Property of Congruence
6. $\triangle ABD \cong \triangle CDB$	6. ASA Triangle Congruence Theorem
7. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$	7. CPCTC

Additionally, the theorem below can also be proven.

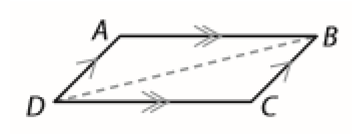
Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

SELF CHECK Prove the opposite angles of a parallelogram are congruent.

Given: $ABCD$ is a parallelogram

Prove: $\angle A \cong \angle C$ (A similar proof shows $\angle B \cong \angle D$.)



Statements	Reasons
1. *ABCD* is a parallelogram.	1.
2. Draw $\overline{DB}$.	2.
3. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$	3.
4.	4. Alternate Interior Angles Theorem
5.	5. Reflexive Property of Congruence
6.	6. ASA Triangle Congruence Theorem
7.	7.

Questions To Ponder

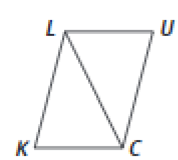
At the beginning of the lesson, you noticed that the consecutive angles of a parallelogram are supplementary. This can be stated as theorem as well.

If a quadrilateral is a parallelogram, then it's consecutive angles are supplementary.

Explain why this theorem is true.

SELF CHECK Given Parallelogram $LUCK$, use the figure and the theorems above to find the following.

Find $m\angle K$, if $m\angle KCU = 10x - 15$ and $m\angle K = 6x + 3$



Solve for x and y if $KL = 2x + y, LU = 7, UC = 14$ and $KC = 5y - 4x$.

GUIDED NOTES

The diagonals of a parallelogram bisect each other can also be proven and stated as a theorem.

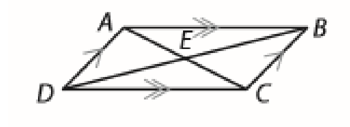
Theorem

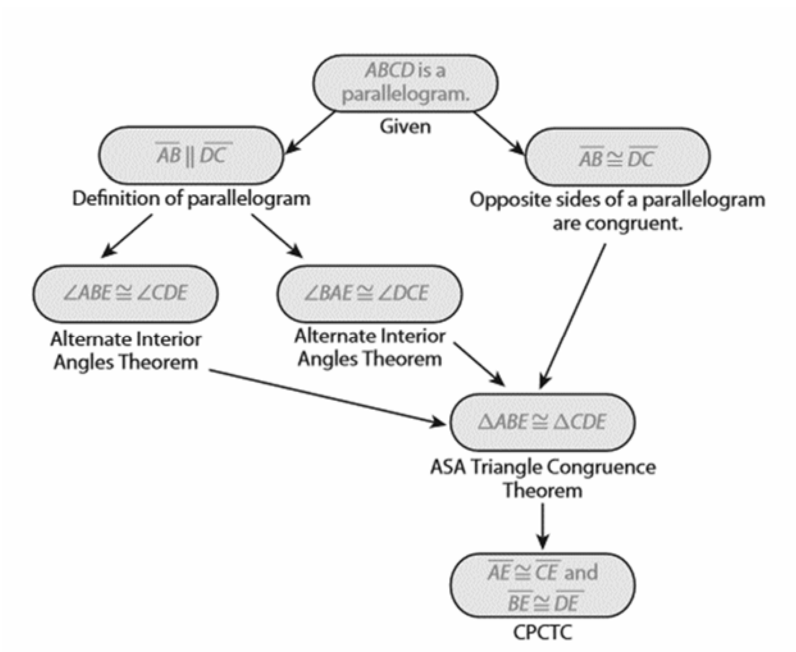
If a quadrilateral is a parallelogram, then its diagonals bisect each other.

One proof is shown below.

Given: $ABCD$ is a parallelogram

Prove: $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$





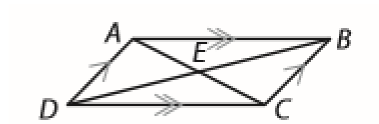
Is it possible to prove the theorem using a different triangle congruence theorem? Explain.

SELF CHECK

Write the proof that the diagonals of a parallelogram bisect each other as a two-column proof.

Given: $ABCD$ is a parallelogram

Prove: $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$

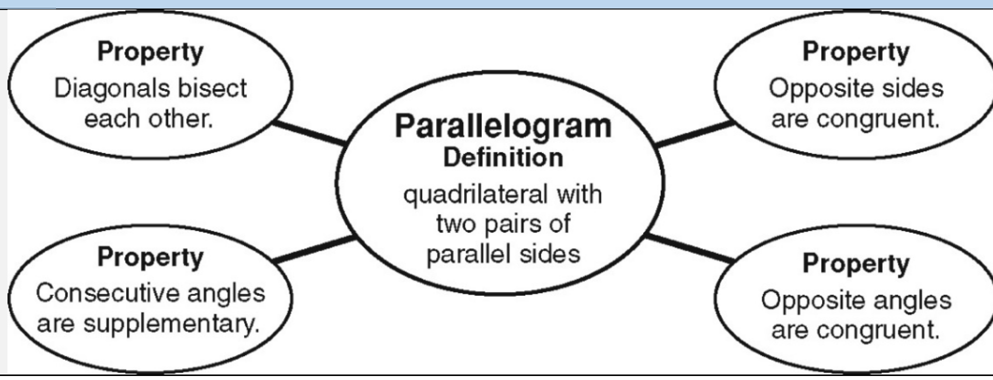


Statements	Reasons
1.	1.

Questions To Ponder

Why do you think this theorem was introduced after the theorems about the sides and angles of a parallelogram?

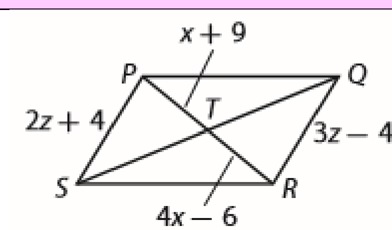
Example! Theorems about Parallelograms



SELF CHECK $PQRS$ is a parallelogram. Find each measure.

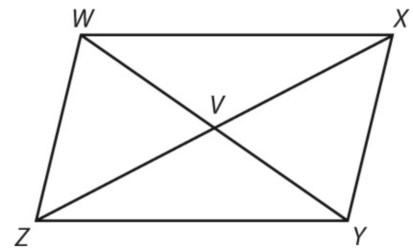
1. QR

2. PR



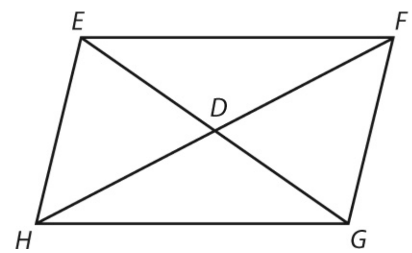
A parallelogram is a quadrilateral (four-sided figure) with the following properties:

The opposite sides are congruent.
The opposite angles are congruent.
The diagonals bisect each other.



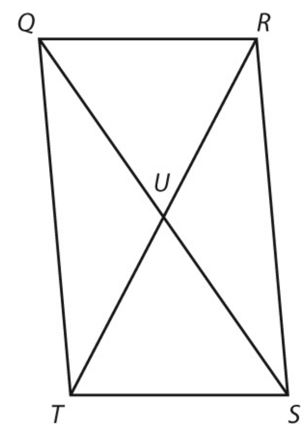
In the figure, ▱$WXYZ$ is a parallelogram.
$\overline{WZ} \cong \overline{XY}$ and $\overline{WX} \cong \overline{ZY}$
$\angle ZWX \cong \angle XYZ$ and $\angle WZY \cong \angle WXY$
$\overline{WY}$ and $\overline{XZ}$ bisect each other.

In the figure, ▱*EFGH* is a parallelogram. Complete the following statements.



1. $\angle HEF \cong$ _____.
2. $\overline{ED} \cong$ _____.
3. $\overline{HG} \cong$ _____.
4. $\overline{HF}$ bisects _____.

In the figure, ▱*QRST* is a parallelogram. Complete the following statements.

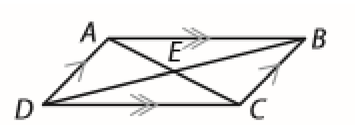


5. $QR = 16$, $TS =$ ______
6. $m\angle QTS = 95°$, $m\angle SRQ =$ ______
7. $QU = 4$, $SU =$ ______
8. $TR = 20$, $TU =$ ______

9. Write the proof that the opposite angles of a parallelogram are congruent as a paragraph proof.

Given: $ABCD$ is a parallelogram

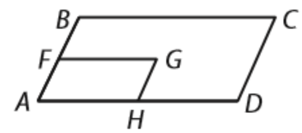
Prove: $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$



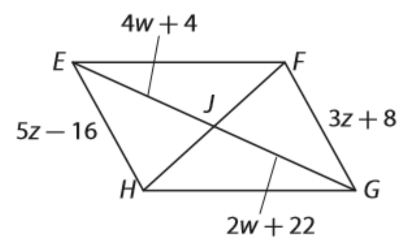
9. Write a two column, paragraph, or flow-chat proof.

Given: $ABCD$ and $AFGH$ are parallelograms

Prove: $\angle C \cong \angle G$



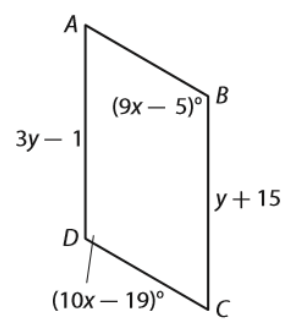
$EFGH$ is a parallelogram. Find each measure.



10. FG

11. EG

$ABCD$ is a parallelogram. Find each measure.



12. $m\angle B$

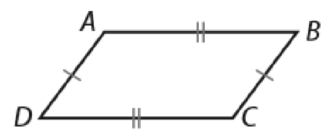
13. AD

Conditions of Parallelograms

You can prove that a quadrilateral is a parallelogram by using the definition and properties of a parallelogram.

For example, if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Complete the proof of the theorem.



Given: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$

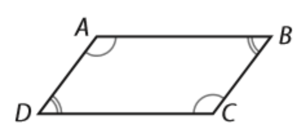
Prove: $ABCD$ is a parallelogram.

First, draw diagonal $\overline{DB}$

Use triangle congruence theorems and corresponding parts to complete the proof that the opposite sides are parallel so the quadrilateral is a parallelogram.

Statements	Reasons
1. Draw $\overline{DB}$.	1. Through any two points, there is exactly one line.
2. $\overline{DB} \cong \overline{DB}$	2.
3. $\overline{AB} \cong \overline{CD}$; $\overline{AD} \cong \overline{CB}$	3.
4. $\triangle ABD \cong \triangle CDB$	4.
5. $\angle ABD \cong \angle CDB$; $\angle ADB \cong \angle CBD$	5.
6. $\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$	6.
7. $ABCD$ is a parallelogram.	7.

Try another to prove that if both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



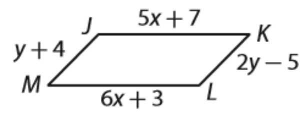
Given: $\angle A \cong \angle C$ and $\angle B \cong \angle D$

Prove: $ABCD$ is a parallelogram.

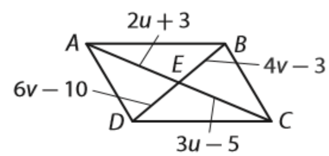
Statements	Reasons
1.	1.

Show that each quadrilateral is a parallelogram for the given values of the variables.

5. $x = 4$ and $y = 9$

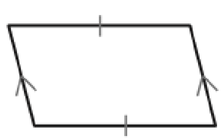


6. $u = 8$ and $v = 3.5$



Determine if each quadrilateral must be a parallelogram. Justify your answer.

7.



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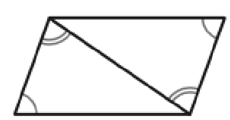
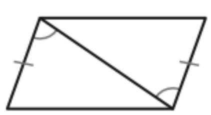
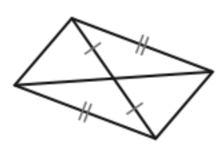


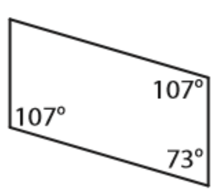
9.



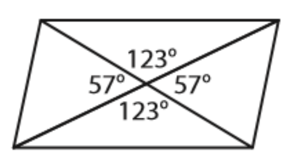
10.



11.



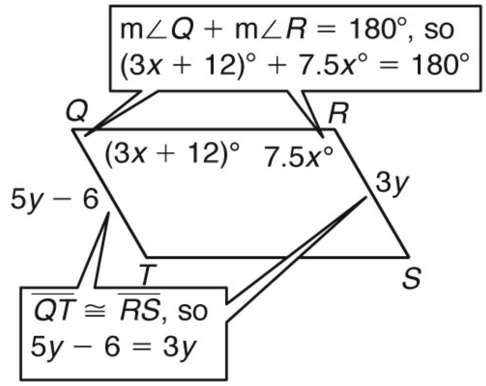
12.



PROVING PROPERTIES OF PARALLELOGRAMS

QRST is a parallelogram.

Parallelograms:
Opposite sides are congruent.
Consecutive angles are supplementary.



Find *QT*.

$5y - 6 = 3y$	Set the given lengt
$-3y \quad -3y$	Subtract 3*y* from both sides.
$2y - 6 = 0$	Simplify.
$+6 \quad +6$	Add 6 to both sides.
$2y = 6$	Simplify.
$\frac{2y}{2} = \frac{6}{2}$	Divide both sides by 2.
$y = 3$	Simplify.
$QT = 5y - 6 = 5(3) - 6 = 9$	Substitute the value of *y* and simplify.

Find $m\angle TQR$.

$7.5x + 3x + 12 = 180$	Set the measures of the consecutive angles equal to 180.
$10.5x + 12 = 180$	Simplify.
$-12 \quad -12$	Subtract 12 from both sides.
$10.5x = 168$	Simplify.
$\frac{10.5x}{10.5} = \frac{168}{10.5}$	Divide both sides by 10.5.
$x = 16$	Simplify.
$m\angle TQR = (3x + 12)° = [3(16) + 12]° = 60°$	Substitute the value of *x* and simplify.

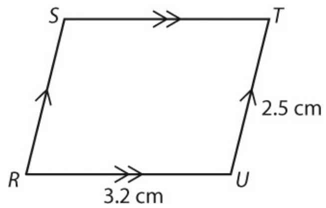
Fill in the blanks with words from the Word Bank to complete each definition or theorem. The first one is done for you.

1. If a quadrilateral is a parallelogram, then its consecutive angles are **supplementary**.
2. If a quadrilateral is a parallelogram, then its opposite sides are
   ____________________.
3. If a quadrilateral is a parallelogram, then its diagonals
   ____________________ each other.
4. If a quadrilateral is a parallelogram, then its opposite angles are
   ____________________.

Word Bank
bisect
congruent
parallel
supplementary

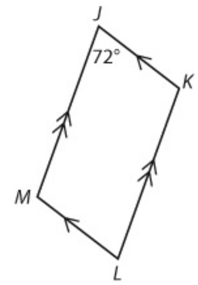
Find each measure..

5. *RS*



2.5 cm

6. m∠*K*

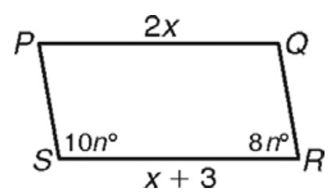


***PQRS* is a parallelogram. Find each measure.**

7. *RS* __________

8. m∠*S* __________

9. m∠*R* __________



Determine whether each statement is always, sometimes, or never true. Explain your reasoning.

10. If quadrilateral $RSTU$ is a parallelogram, then $\overline{RS} \cong \overline{ST}$.

11. If a parallelogram has a $30°$ angle, then it also has a $150°$ angle.

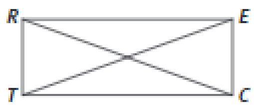
12. If quadrilateral $GHJK$ is a parallelogram, then $\overline{GH}$ is congruent to $\overline{JK}$.

Proving Properties of Rectangles, Rhombuses, and Squares

GUIDED NOTES

A rectangle is a parallelogram with four right angles.

Given quadrilateral $RECT$ is a rectangle. List all right triangles in the figure. Explain how you know the triangles are congruent.



Complete the theorem.
The diagonals of a rectangle are ______________________.

Explain how you know the theorem is true.

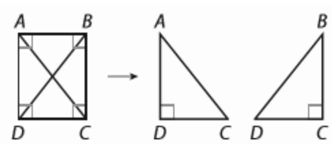
Example! You can use the definition of a rectangle to prove the following theorems.

Properties of Rectangles

If a quadrilateral is a rectangle, then it is a parallelogram.
If a parallelogram is a rectangle, then its diagonals are congruent.

Given: $ABCD$ is a rectangle.

Prove: $\overline{AC} \cong \overline{BD}$



Statements	Reasons
1. *ABCD* is a rectangle.	1. Given
2. $\angle A$ and $\angle C$ are right angles.	2. Definition of rectangle
3. $\angle A \cong \angle C$	3. All right angles are congruent.
4. $\angle B$ and $\angle D$ are right angles.	4. Definition of rectangle
5. $\angle B \cong \angle D$	5. All right angles are congruent.
6. *ABCD* is a parallelogram.	6. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
7. $\overline{AD} \cong \overline{CB}$	7. If a quadrilateral is a parallelogram, then its opposite sides are congruent.
8. $\overline{DC} \cong \overline{DC}$	8. Reflexive Property of Congruence
9. $\angle D$ and $\angle C$ are right angles.	9. Definition of rectangle
10. $\angle D \cong \angle C$	10. All right angles are congruent.
11. $\triangle ADC \cong \triangle BCD$	11. SAS Triangle Congruence Theorem
12. $\overline{AC} \cong \overline{BD}$	12. CPCTC

Questions To Ponder

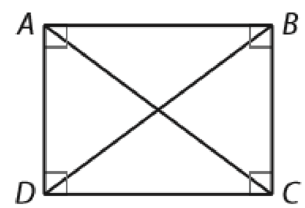
A student says you can also prove the diagonals are congruent by using the SSS Triangle Congruence Theorem to show that $\triangle ADC \cong \triangle BCD$. Do you agree? Explain

SELF CHECK

Find each measure.

$AD = 7.5$ cm and $DC = 10$ cm. Find DB

$AB = 17$ cm and $BC = 12.75$ cm. Find DB



Construct two segments of different length that are perpendicular bisectors of each other. Connect the four end points to form a quadrilateral. Measure each side length of the quadrilateral. Use those measurements to name the shape.

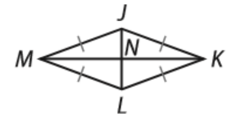
A rhombus is a quadrilateral with four congruent sides.

Properties of Rhombuses

If a quadrilateral is a rhombus, then it is a parallelogram.
If a parallelogram is a rhombus, then its diagonals are perpendicular.
If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

SELF CHECK Prove that the diagonals of a rhombus are perpendicular.

Given: $JKLM$ is a rhombus
Prove: $\overline{JL} \perp \overline{MK}$

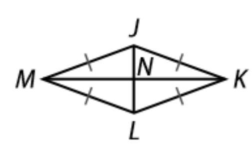


Statements	Reasons
1. $\overline{JM} \cong \overline{JK}$	1. Definition of rhombus
2. $\overline{MN} \cong \overline{KN}$	2.
3. $\overline{JN} \cong \overline{JN}$	3. Reflexive Property of Congruence
4.	4. SSS Triangle Congruence Theorem
5. $\angle JNM \cong \angle JNK$	5.
6. $\angle JNM$ and $\angle JNK$ are supplementary.	6.
7.	7. Definition of supplementary
8. $\angle JNM = \angle JNK$	8. Definition of congruence
9. ______ $+ \angle JNK = 180°$	9. Substitution Property of Equality
10. $2m\angle JNK = 180°$	10. Addition
11. $m\angle JNK = 90°$	11. Division Property of Equality
12.	12. Definition of perpendicular lines

What property about the diagonals of a rhombus is the same as a property of all parallelograms?
What special property do the diagonals of a rhombus have?

SELF CHECK Prove that if a parallologram is a rhombus, then each diagonal bisects a pair of opposite angles.

Given: $JKLM$ is a rhombus
Prove: $\overline{MK}$ bisects $\angle JML$ and $\angle JKL$; $\overline{JL}$ bisects $\angle MJK$ and $\angle MKL$



Example! Use rhombus $VWXY$ to find each measure.

Find XY.

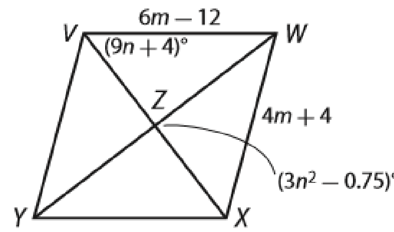
All sides of a rhombus are congruent, so $\overline{VW} \cong \overline{WX}$ and $VW = WX$.

Substitute values for VW and WX. $6m - 12 = 4m + 4$

Solve for m. $m = 8$

Sustitute the value of m to find VW. $VW = 6(8) - 12 = 36$

Because all sides of the rhombus are congruent, then $\overline{VW} \cong \overline{XY}$, and $XY = 36$.



SELF CHECK Find $\angle YVW$

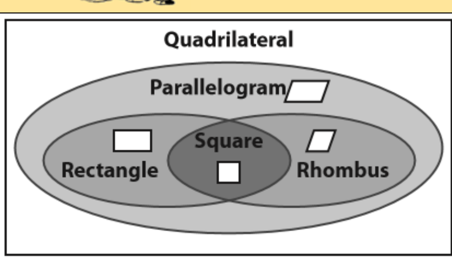
GUIDED NOTES

A **square** is a quadrilateral with four sides congruent and four right angles.

If a quadrilateral is a square, then it is a parallelogram.
By definition, a square is a quadrilateral with four congruent sides. Any quadrilateral with both pairs of opposite sides congruent is a parallelogram, so a square is a parallelogram.

If a quadrilateral is a square, then it is a rectangle.
By definition, a square is a quadrilateral with four ________________. By definition, a rectangle is also a quadrilateral with four ________________. Therefore, a square is a rectangle.

Questions To Ponder The Venn diagram shows how quadrilaterals, parallelograms, rectangles, rhombuses, and squares are related to each other. From this lesson, what do you notice about the definitions and theorems regarding these figures?



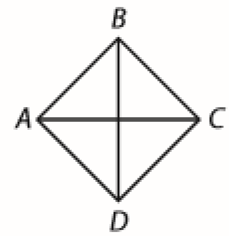
SELF CHECK List the properties that a square "inherits" because it is each of the following quadrilaterals.

a. a parallelogram

b. a rectangle

c. a rhombus

Example! Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.



Given: $\overline{AB} \cong \overline{CD}$; $\overline{BC} \cong \overline{DA}$; $\overline{AD} \perp \overline{DC}$; $\overline{AC} \perp \overline{BD}$

Conclusion: *ABCD* is a square.

To prove that a given quadrilateral is a square, it is sufficient to show that the figure is both a rectangle and a rhombus.

Step 1: Determine if *ABCD* is a parallelogram.

$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$ are given. Since a quadrilateral with opposite sides congruent is a parallelogram, we know that *ABCD* is a parallelogram.

Step 2: Determine if *ABCD* is a rectangle.

Since $\overline{AD} \perp \overline{DC}$, by definition of perpendicular lines, $\angle ADC$ is a right angle. A parallelogram with one right angle is a rectangle, so *ABCD* is a rectangle.

Step 3: Determine if *ABCD* is a rhombus.

$\overline{AC} \perp \overline{BD}$. A parallelogram with perpendicular diagonals is a rhombus. So *ABCD* is a rhombus.

Step 4: Determine if *ABCD* is a square.

Since *ABCD* is a rectangle and a rhombus, it has four right angles and four congruent sides. So *ABCD* is a square by definition.

So, the conclusion is valid.

GUIDED NOTES

Let's review conditions for rectangles, rhombuses, and squares.

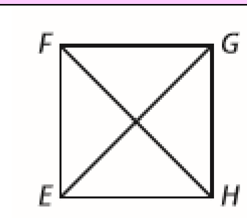
If a parallelogram,

- has one right angle, it is a _____________.
- has congruent diagonals, it is a rectangle.
- has congruent consecutive sides, it is a rhombus.
- has perpendicular diagonals, it is a ______________.
- is a rectangle and a rhombus, it is a square.

SELF CHECK $EFGH$ is a parallelogram. In $EFGH, \overline{EG} \cong \overline{FH}$. Which conclusion is incorrect?

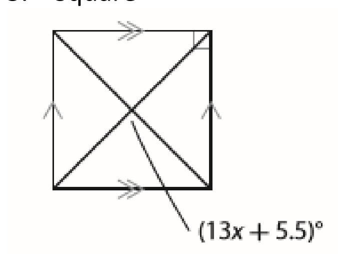
1. $EFGH$ is a rectangle

2. $EFGH$ is a square.

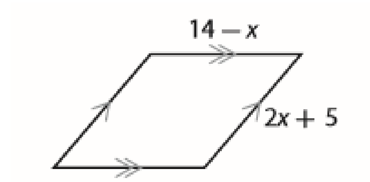


SELF CHECK Find the value that makes each parallelogram the given type.

3. Square

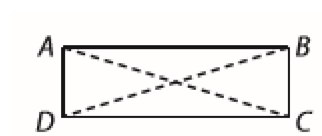


4. Rhombus



SELF CHECK Proove that if the diagonals of a parallelolgram are congruent, then the parallelogram is a rectangle.

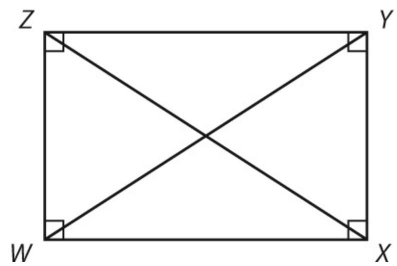
Given: $ABCD$ is a parallelogram, $\overline{AC} \cong \overline{BD}$.
Prove: $ABCD$ is a rectangle.



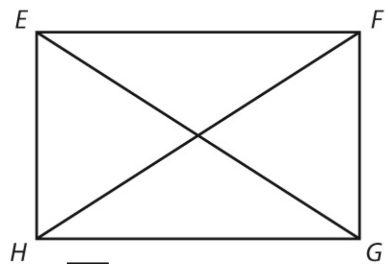
A rectangle is a parallelogram that contains four right angles. The diagonals of a rectangle are congruent.

Remember, all of the properties of parallelograms are true for rectangles. Opposite sides are congruent and parallel.

In the figure, if *WXYZ* is a rectangle, then: $\angle ZWX$, $\angle WXY$, $\angle XYZ$, and $\angle YZW$ are right angles, and $\overline{WY} \cong \overline{XZ}$.

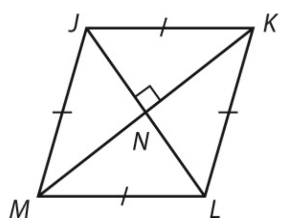


***EFGH* is a rectangle. Complete the statements that must be true about *EFGH*.**

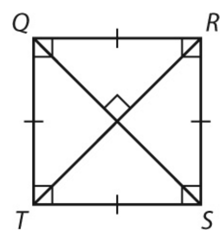


1. $\overline{EG} \cong$ ______
2. $m\angle EHG =$ ______
3. $\overline{EH} \parallel =$ ______

A rhombus is a parallelogram with four congruent sides. A rhombus has perpendicular diagonals.



A square is a rhombus with four congruent sides and four right angles. A square is, therefore, also a parallelogram and a rectangle.



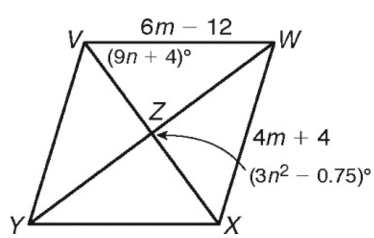
***JKLM* is a rhombus. Fill in the missing information. Use the figure shown above.**

***QRST* is a square. Fill in the missing information. Use the figure shown above.**

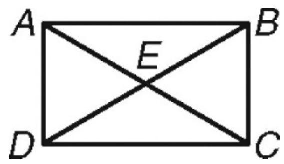
4. If $ML = 32$, $LK =$ _____
5. $m\angle MNL =$ _____
6. $\overline{QT} \cong$ ____ $\cong$ _____ $\cong$ _____

***VWXY* is a rhombus. Find each measure.**

7. $XY =$ ______________________
8. $m\angle YVW =$ ______________________
9. $m\angle VYX =$ ______________________
10. $m\angle XYZ =$ ______________________



Write a paragraph proof.



11. **Given:** *ABCD* is a rectangle.
Prove: $\angle EDC \cong \angle ECD$

Fill in the blanks to complete each theorem.

12. If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a ______________________.

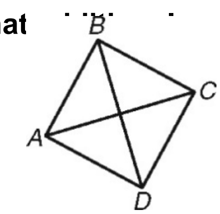
13. If the diagonals of a parallelogram are ______________________, then the parallelogram is a rhombus.

14. If the ______________________ of a parallelogram are congruent, then the parallelogram is a rectangle.

15. If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a ______________________.

16. If one angle of a parallelogram is a right angle, then the parallelogram is a ______________________.

Use the figure. Determine whether each conclusion is valid. If not, tell what information is needed to make it valid.



17. **Given:** $\overline{AC}$ and $\overline{BD}$ bisect each other. $\overline{AC} \cong \overline{BD}$
Conclusion: *ABCD* is a square.

18. **Given:** $\overline{AC} \perp \overline{BD}, \overline{AB} \cong \overline{BC}$
Conclusion: *ABCD* is a rhombus.

19. The vertices of square *JKLM* are *J*(–2, 4), *K*(–3, –1), *L*(2, –2), and *M*(3, 3). Find each of the following to show that the diagonals of square *JKLM* are congruent perpendicular bisectors of each other.

$JL =$ _________ $KM =$ _________

slope of $\overline{JL} =$ _________ slope of $\overline{KM} =$ _________

midpoint of $\overline{JL} =$ (______, ______) midpoint of $\overline{KM} =$ (______, ______)

Properties of Special Quadrilaterals

1. Complete the chart below by identifying the quadrilateral(s) for which the given condition is necessary.

Conditions	Quadrilateral(s)	Explain your reasoning
Diagonals are perpendicular.		
Diagonals are congruent and intersect but are not perpendicular		
Diagonals bisect each other.		
Diagonals are perpendicular and bisect each other.		
Diagonals are congruent and bisect each other.		
Diagonals are congruent, perpendicular and bisect each other.		

2. Identify the properties that are always true for the given quadrilateral by placing an X in the appropriate box.

Property	Parallelogram	Rectangle	Rhombus	Square
Opposite sides are parallel.				
Opposite sides are congruent.				
Opposite angles are congruent.				
Each diagonal forms 2 ≅ triangles.				
Diagonals bisect each other.				
Diagonals are perpendicular.				
Diagonals are congruent.				
Diagonals bisect vertex angles.				
All $\angle$s are right $\angle$s.				
All sides are congruent.				

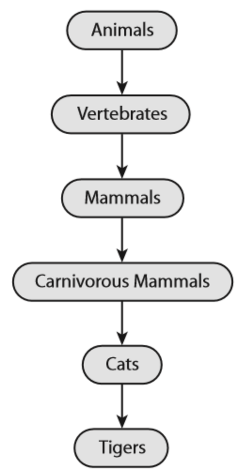
3. Using the properties in the table above, list the **minimum** conditions necessary to prove that a quadrilateral is:
   a. a parallelogram

   b. a rectangle

   c. a rhombus

   d. a square

4. The diagram shows the organizational ladder of groups to which tigers belong.



   a. Use the terms below to create a similar ladder in which each term is a subset of the term above it.

Parallelogram   Geometric figures   Squares   Quadrilaterals   Rhombuses

   b. Decide which of the following statements is true. Then write two more statements like it, using terms from the list in part (a).

   If a figure is a rhombus, then it is a parallelogram.

   If a figure is a parallelogram, then it is a rhombus.

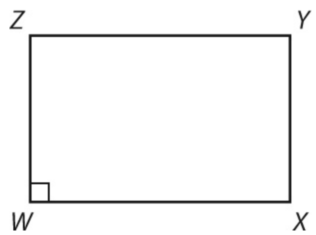
   c. Explain how you can use the ladder you created above to write if-then statements involving the terms on the list.

5. Suppose Anna draws two line segments, $\overline{AB}$ and $\overline{CD}$ that intersect at point E. She draws them in such a way that $\overline{AB} \cong \overline{CD}$, $\overline{AB} \perp \overline{CD}$, and $\overline{AB}$ and $\overline{CD}$ bisect each other. What is the best name to describe $ABCD$? Explain.

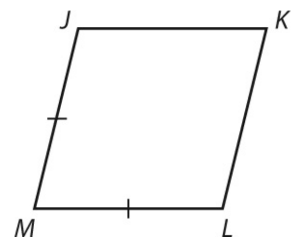
PROVING PROPERTIES OF RECTANGLES. RHOMBUSES, AND SQUARE

Certain conditions of a parallelogram are enough to prove that a parallelogram is a rectangle, a rhombus, or a square.

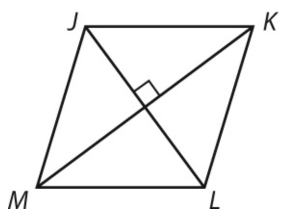
If one angle of a parallelogram is a right angle, the parallelogram is a rectangle.



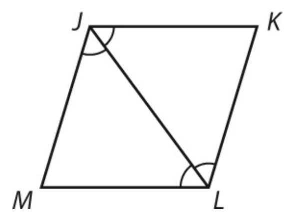
If two consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.



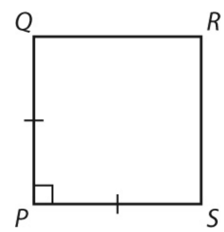
If the diagonals of a parallelogram are perpendicular, it is a rhombus.



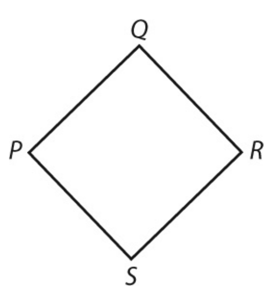
If one diagonal of a parallelogram bisects a pair of opposite angles, it is a rhombus.



If a parallelogram can be proven to be a rectangle and a rhombus, it is a square.



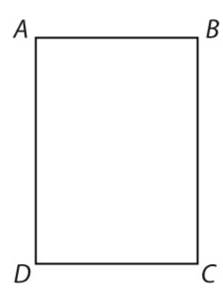
State whether the figure is a rectangle, rhombus, or square. Explain your reasoning. There may be more than 1 answer.



1. $\overline{PQ} \cong \overline{QR}$

____

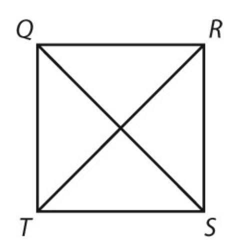
____



2. $m\angle D = 90^\circ$

____

____



3. $\overline{QS} \perp \overline{TR}$; $\overline{TQ} \cong \overline{QR}$

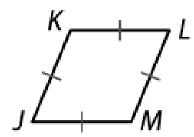
____

____

Fill in the blanks to complete each theorem. The first one is done for you.

4. If a parallelogram is a rhombus, then its diagonals are **perpendicular**.
5. If a parallelogram is a rectangle, then its diagonals are ______________________.
6. If a quadrilateral is a rectangle, then it is a ______________________.
7. If a parallelogram is a rhombus, then each diagonal ______________________ a pair of opposite angles.

8. Find and explain the error in this paragraph proof. Then describe a way to correct the proof.



Given: $JKLM$ is a rhombus.

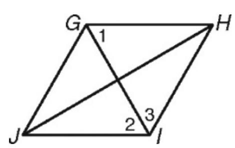
Prove: $JKLM$ is a parallelogram.

Proof: It is given that $JKLM$ is a rhombus. So, by definition of a rhombus, $\overline{JK} \cong \overline{LM}$, and $\overline{KL} \cong \overline{MJ}$. If a quadrilateral is a parallelogram, then it's opposite sides are congruent. So $JKLM$ is a parallelogram.

Use the phrases and theorems from the Word Bank to complete this two-column proof. The first step is done for you.

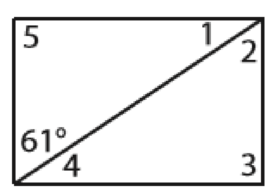
Alternate Interior ∠s Thm.
Trans. Prop. of ≅
∠2 ≅ ∠3
Given
parallel

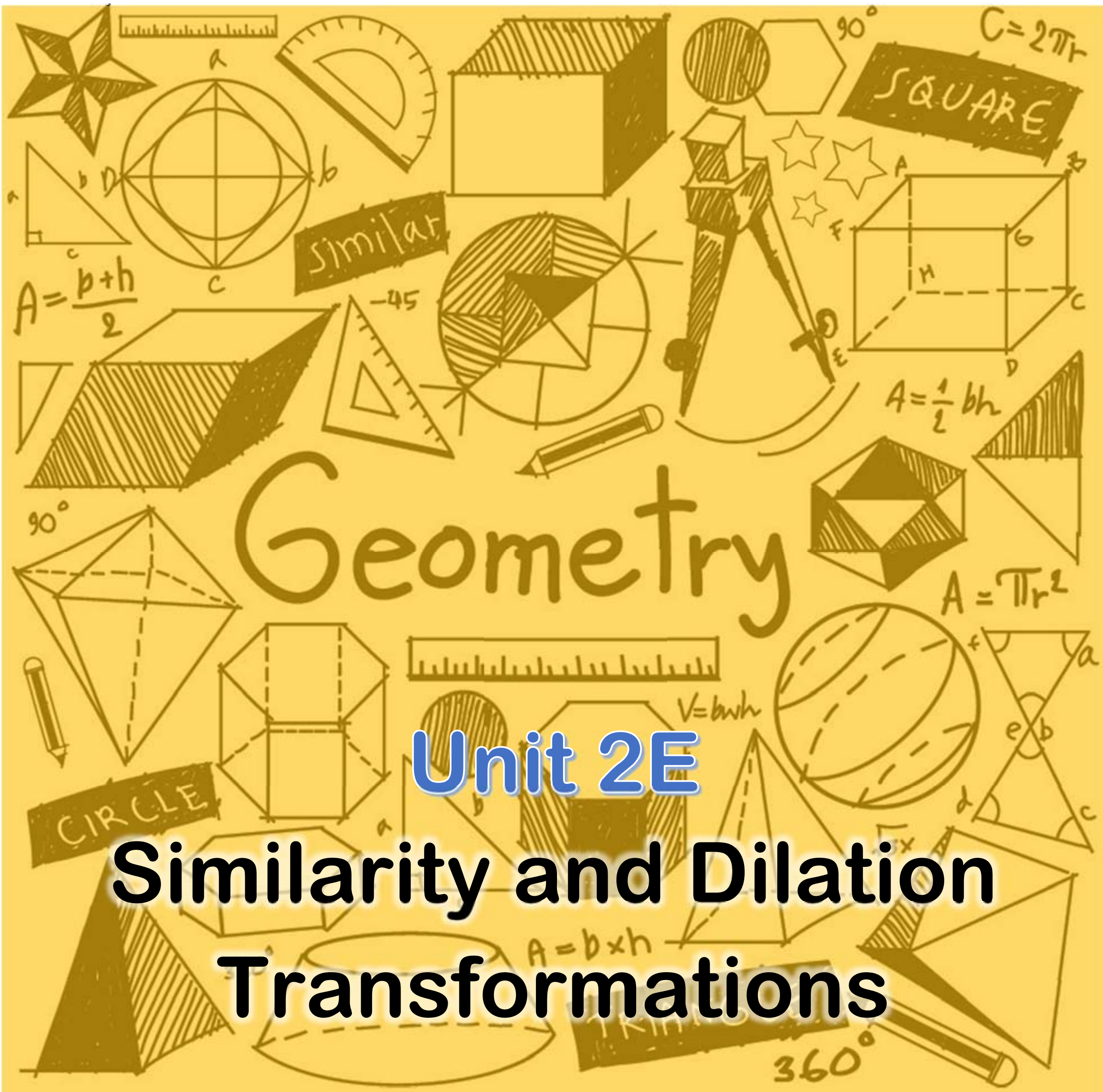
9. **Given:** *GHIJ* is a rhombus.
   Prove: ∠1 ≅ ∠3



Statements	Reasons
1. *GHIJ* is a rhombus.	1. **a.** Given
2. *GHIJ* is a parallelogram.	2. rhomb. → parallelogram
3. $\overline{GH} \parallel \overline{JI}$	3. parallelogram opp. sides are **b.** ________
4. ∠1 ≅ ∠2	4. **c.** ______________________
5. d. ____________________________	5. rhomb. → each diag. bisects opp. ∠s
6. ∠1 ≅ ∠3	6. e. ____________________________

10. Find the measure of each angle in the rectangle.





Unit 2E

Similarity and Dilation Transformations

HOUSTON COUNTY
BOARD OF EDUCATION
HIGH-ACHIEVING TEACHERS

MATHEMATICS
DEPARTMENT

Lesson Name: G.U2E.A.C1.A.01.**Vocab.SimilarPolygons**

Diagram/Visual			
Congruent Figures			
Corresponding Angles			
Corresponding Sides			
Proportion			
Ratio			
Reflection			
Reflection Line			
Remote Interior Angles of a Triangle			
Rotation			
Scale Factor			

Similar Figures			

Ratios

GUIDED NOTES

What is a ratio?

- A ________________________ of ___________________ quantities.
- Ways to represent a ratio:___________;___________;__________
- Ratios can be ______________________!

Extended Ratios

- A ________________________ of ___________________ quantities.
- Extended Ratios are written as:______________________

Example!

1. A music store has 40 trumpets, 39 clarinets, 24 violins, 51 flutes, and 16 trombones in stock. Give each as a simplified ratio:

a) Trumpets to violins__________

b) Flutes to clarinets__________

c) Trombones to trumpets__________

d) Violins to total instruments__________

In 2-5, use ratios to find angles and sides.

2. The ratio of two complementary angles is 3:7. Find the measures of both angles.

3. The ratio of two supplementary angles is 4:1. Find the measures of both angles.

4. The ratio of measures of the angles in a triangle are 4:7:9. Find the measures of the angles.

5. The ratio of measures of the sides of a triangle are 10:15:6. If the perimeter of the triangle is 217 inches, find the length of the shortest side.

SELF CHECK

1. The ratio of measures of the angles in a triangle are 11:2:5. Find the measure of the largest angle.

5. The ratio of measures of the sides of a triangle are 2:8:9. If the perimeter of the triangle is 76 inches, find the length of each side.

Proportions

GUIDED NOTES

What is a proportion?

- An_________________________ that states two ____________________ are equal.
- A proportion is written as:_______________________
- **Cross Product Property:** For any proportion, ________________________

Example!

Use the cross product property to solve.

1.

$$\frac{4}{x} = \frac{2}{7}$$

2.

$$\frac{x-1}{6} = \frac{13}{19}$$

3.

$$\frac{x-20}{3} = \frac{x-11}{18}$$

SELF CHECK

Use the cross product property to solve.

1.

$$\frac{19}{10}=\frac{x}{12}$$

2.

$$\frac{5}{17}=\frac{19}{x+4}$$

3.

$$\frac{6}{x+16}=\frac{7}{3x+3}$$

Similar Polygons

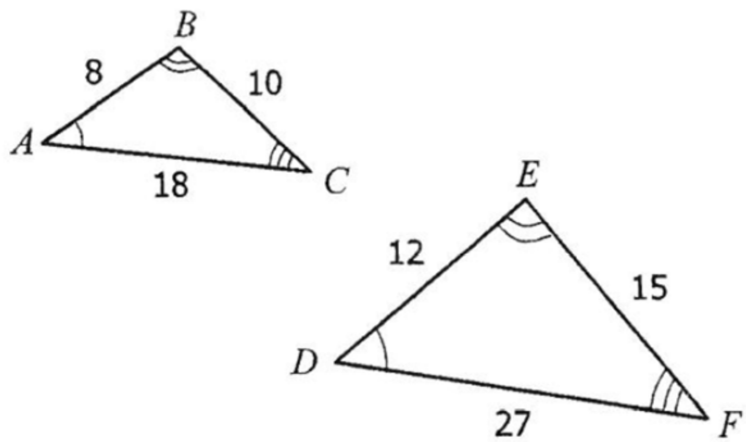
GUIDED NOTES

Similar Polygons

- Polygons with the same ____________________ but different _____________
- Polygons are similar if:
  (1)____
  (2)____
- The ratio of the corresponding sides is called the ____________ ___________
- If polygons are similar, then their __________________ are also proportional.

Scale Factor
(ORDER MATTERS!!!)

What is the scale factor of ΔABC to ΔDEF?	
What is the scale factor of ΔDEF to ΔABC?	
What is the ratio of the perimeter of ΔDEF to ΔABC?	



Similarity Statements
Symbol for Similar:_____
A **valid similarity statement** must match corresponding angles and sides!

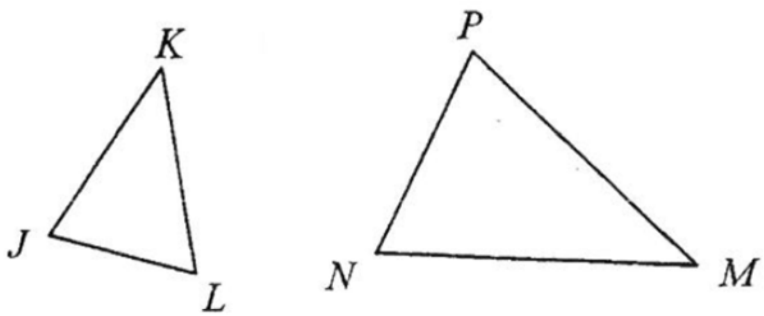
Write a similarity statement for the triangles above.

____

Example!

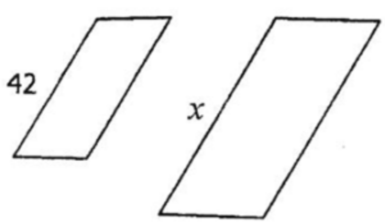
1. List all congruent angles and write a proportion that relates the corresponding sides.

$\Delta JKL \sim \Delta PMN$

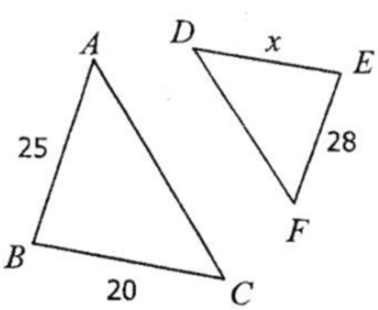


Angles	Sides

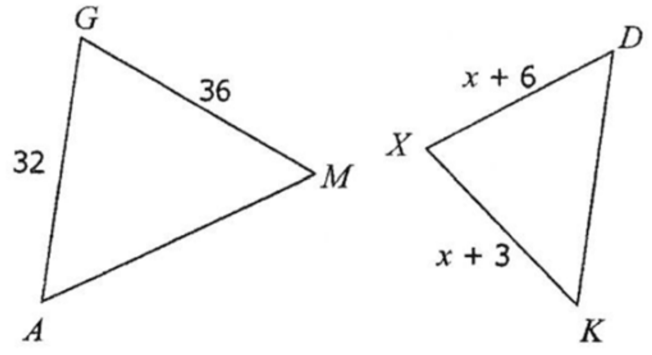
2. If the figures below are similar with a scale factor of 2:3, find the value of x.



3. If ΔABC ~ ΔDEF, find the value of x.



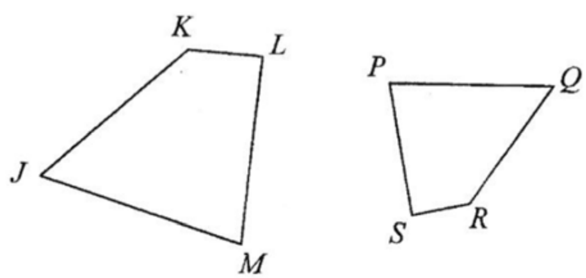
4. If ΔAGM ~ ΔKXD, find the value of x.



SELF CHECK

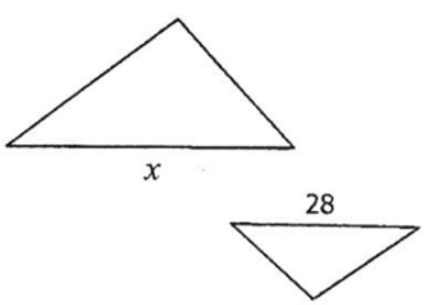
1. List all congruent angles and write a proportion that relates the corresponding sides.

JKLM ~ *QRSP*

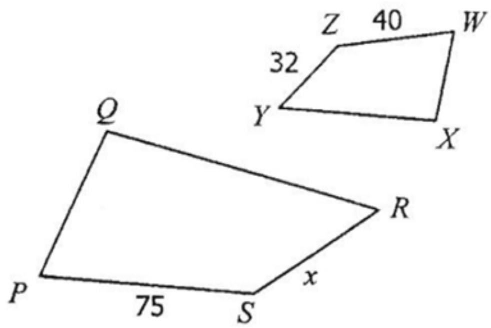


Angles	Sides

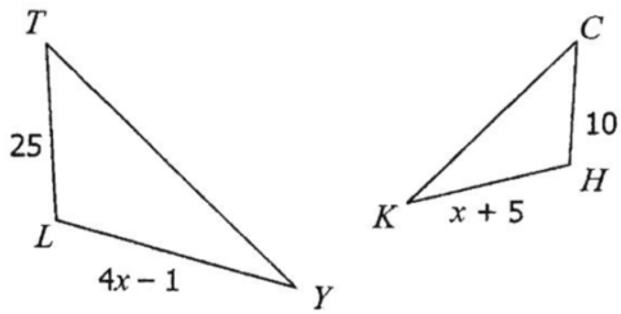
2. If the figures below are similar with a scale factor of 6:5, find the value of x.



3. If PQRS ~ WXYZ, find the value of x.



4. If ΔTLY ~ ΔCHK, find the value of x.



Triangle Similarity

GUIDED NOTES

Triangle Similarity

<u>AA~ (Angle-Angle Similarity)</u>: If two corresponding angles are congruent, then the triangles are similar.

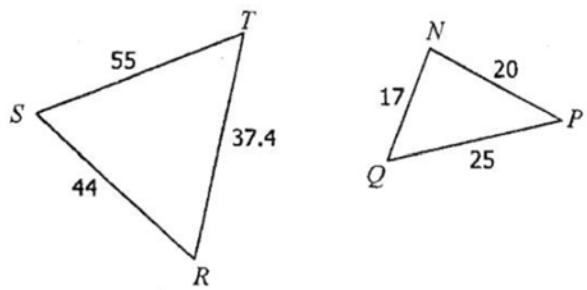
<u>SSS~ (Side-Side-Side Similarity)</u>: If all corresponding sides are proportional, then the triangles are similar.

<u>SAS~ (Side-Angle-Side Similarity)</u>: If two corresponding sides are proportional and the included angles are congruent, then the triangles are similar.

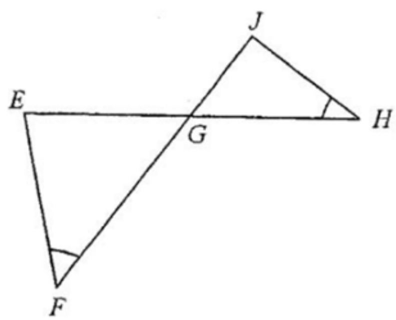
Example!

Determine whether the triangles are similar by AA~, SSS~, SAS~, or not similar.

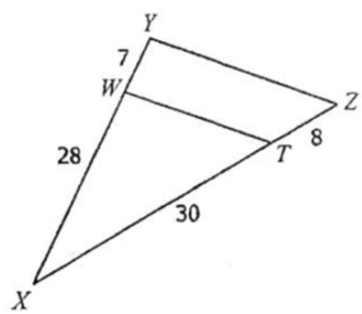
1.



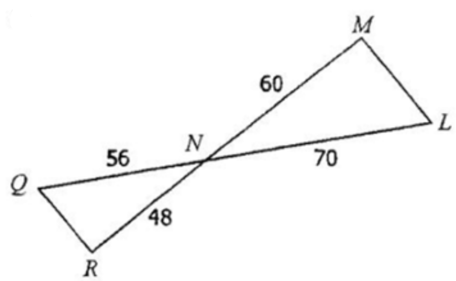
2.



3.



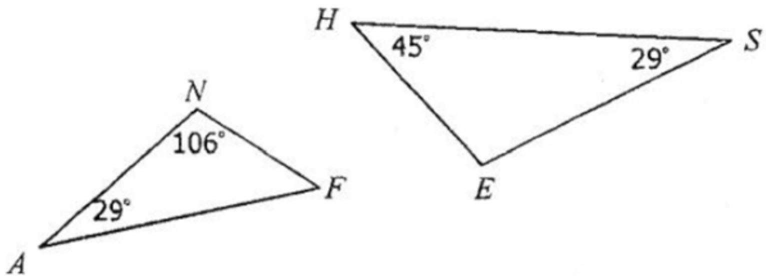
4.



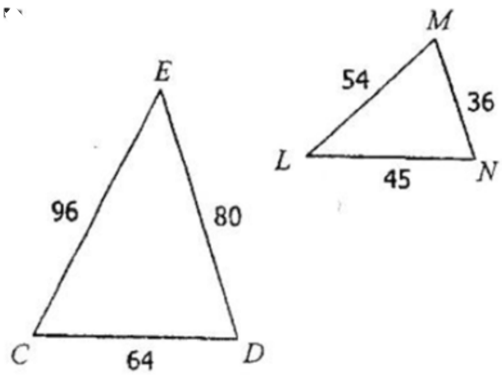
SELF CHECK

Determine whether the triangles are similar by AA~, SSS~, SAS~, or not similar.

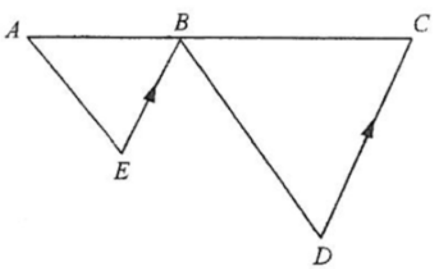
1.



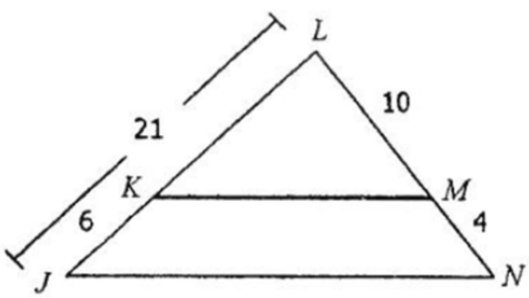
2.



3.



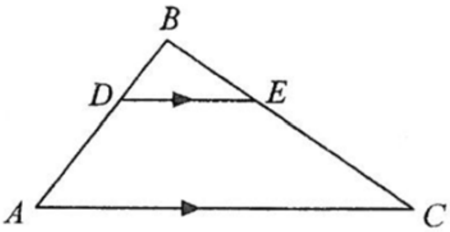
4.



Triangle Proportionality Theorem

GUIDED NOTES

Triangle Proportionality Theorem
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.



If _______________________, then ___________________________________

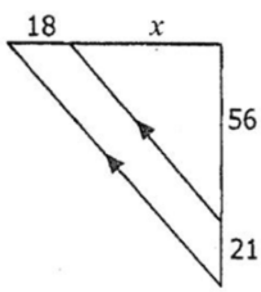
Converse of the Triangle Proportionality Theorem:

If _______________________, then ___________________________________

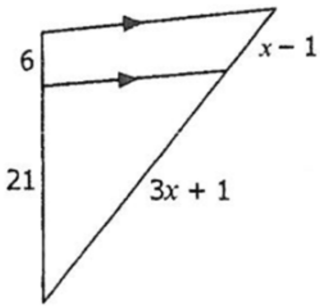
Example!

Find the value of x.

1.



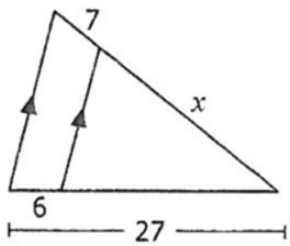
2.



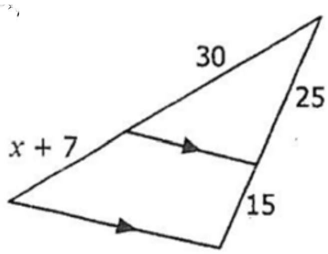
SELF CHECK

Find the value of x.

1.



2.

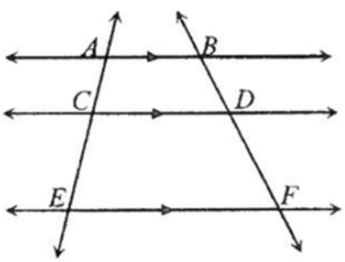


Proportional Parts and Parallel Lines

GUIDED NOTES

Proportional Parts and Parallel Lines

If three or more parallel lines intersect two transversals, then they cut of the transversals proportionally.

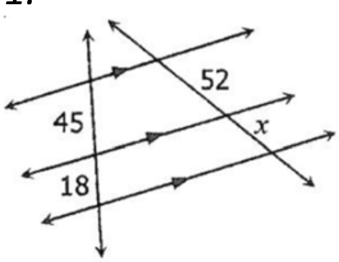


If ______________________, then ________________________________

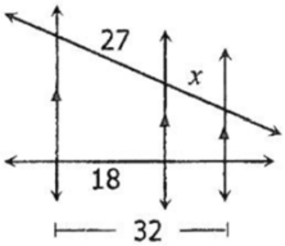
Example!

Find the value of x.

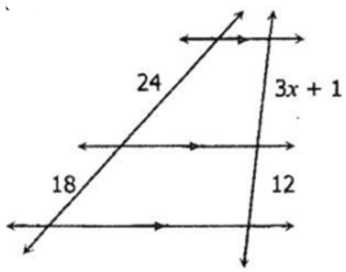
1.



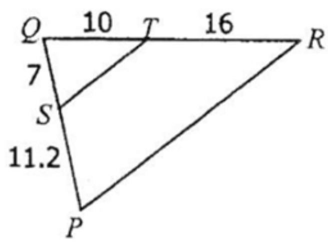
2.



3.



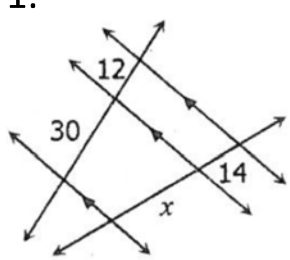
4. Determine is $\overline{ST}$ *is parallel to* $\overline{PR}$*.*



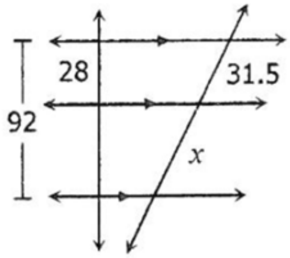
SELF CHECK

Find the value of x.

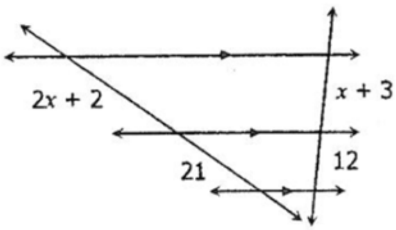
1.



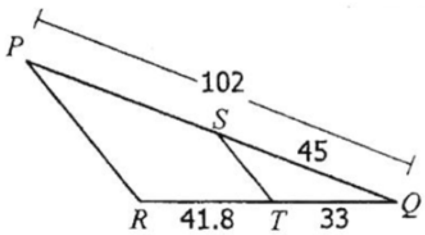
2.



3.



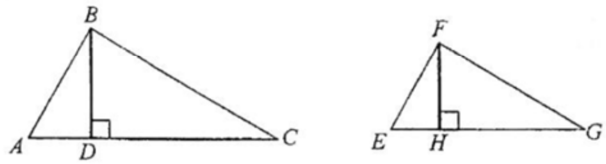
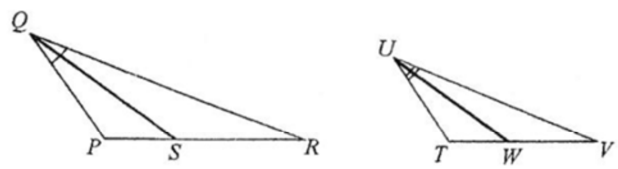
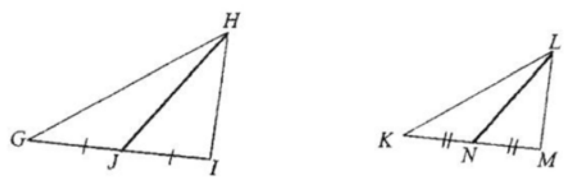
4. Determine is $\overline{ST}$ is parallel to $\overline{PR}$.



Parts of Similar Triangles

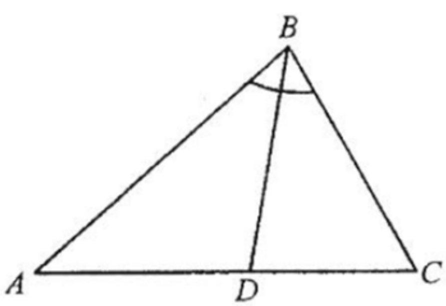
GUIDED NOTES

Parts of Similar Triangles

If two triangles are similar, then the following corresponding parts are also proportional to the corresponding sides:		
______________		
______________ ______________		
______________		

Triangle Angle Bisector Theorem

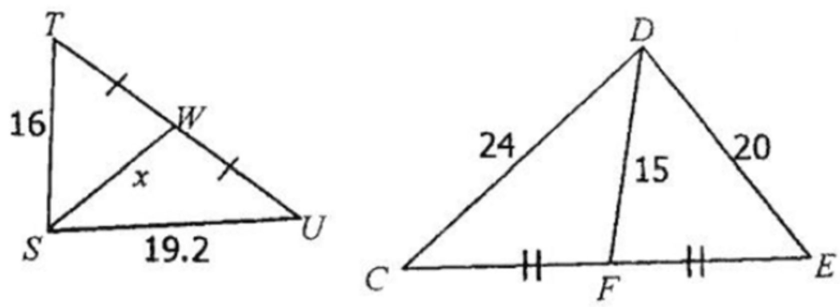
An angle bisector in a triangle separates the opposite sides into two segments that are proportional to the lengths of the other two sides.



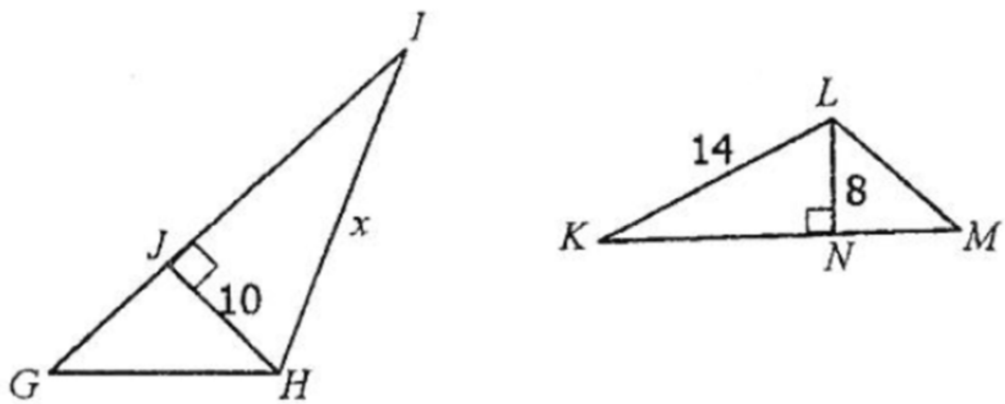
Example!

Given the similar triangles, solve for x.

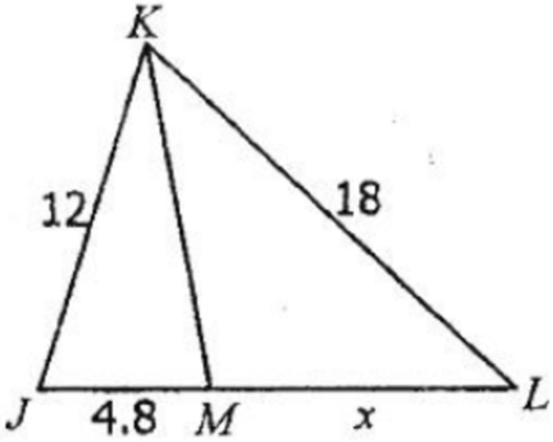
1. *ΔSTU ~ ΔDEC*



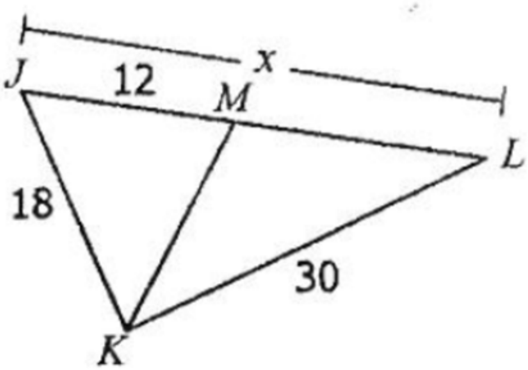
2. *ΔGHI ~ ΔMLK*



3. *If $\overline{KM}$ represents the angle bisector, solve for x.*



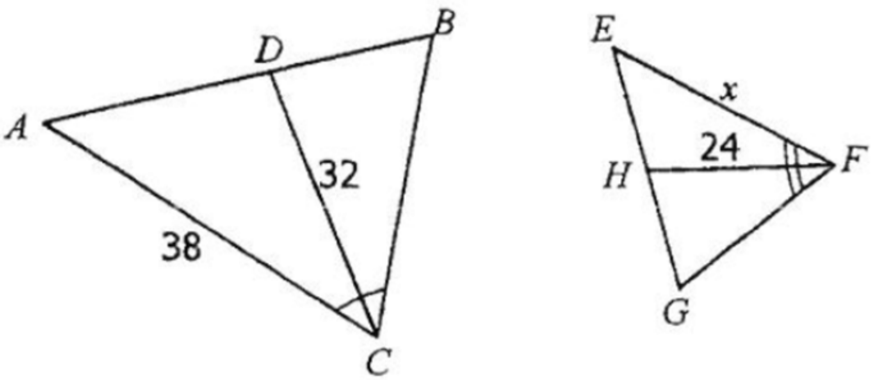
4. *If $\overline{KM}$ represents the angle bisector, solve for x.*



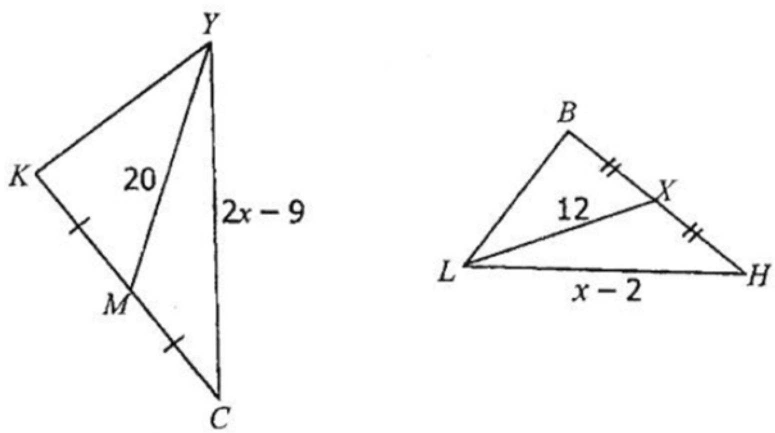
SELF CHECK

Given the similar triangles, solve for x.

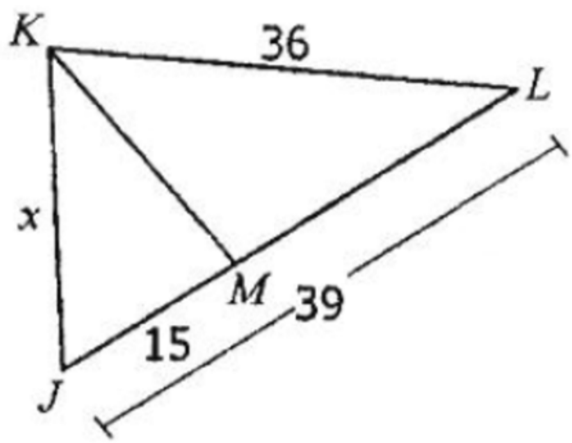
1. $\Delta ABC \sim \Delta EGF$



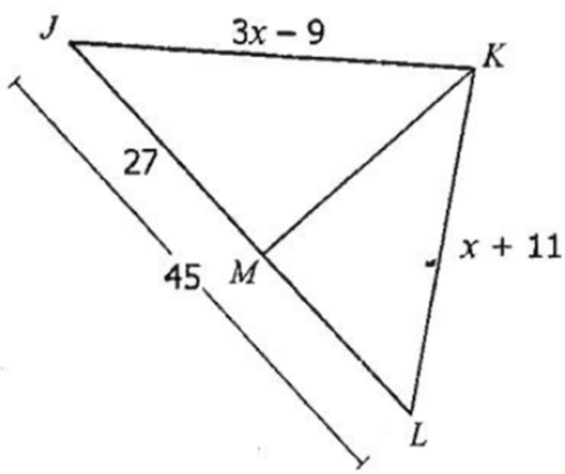
2. $\Delta KYC \sim \Delta BLH$



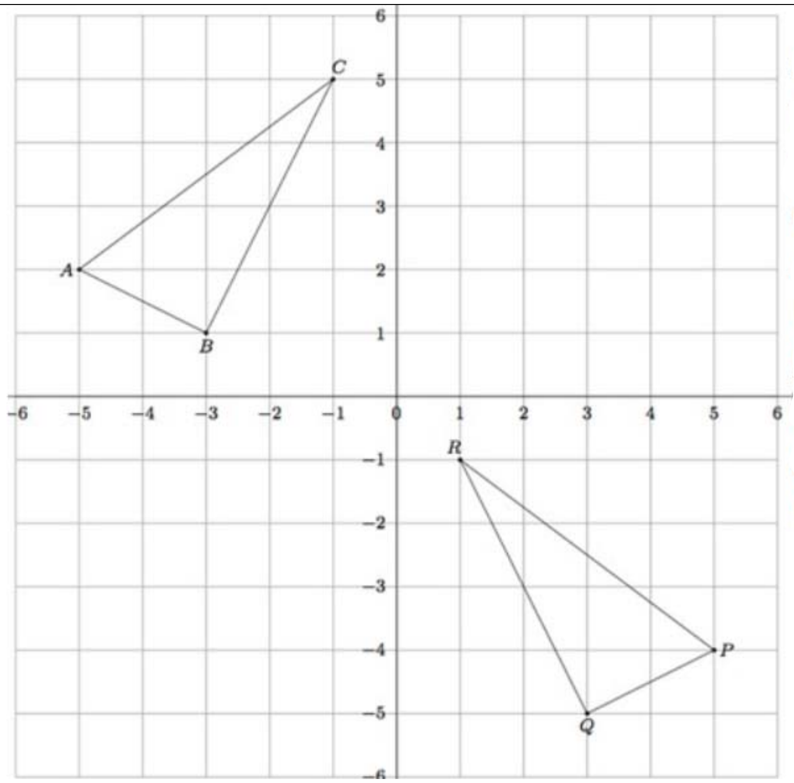
3. If $\overline{KM}$ represents the angle bisector, solve for x.



4. *If $\overline{KM}$ represents the angle bisector, solve for x.*



5. Are these two triangles congruent? (HINT: Use the distance formula)



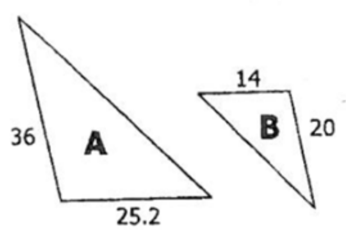
AB =
BC =
AC =
RP =
QP =
RQ =

Think back to when we proved triangles with congruence. Explain why we can have AA~ as a proof of similarity, but not AAA as a proof of congruence.

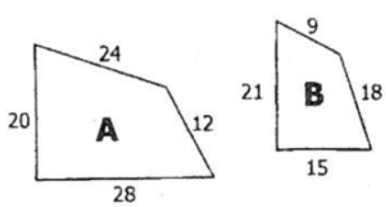
1. The ratio of the measures of the angles in a triangle is 8:3:4. Find the measures of the angles.

2. The ratio of the measures of the sides of a triangle is 9:12:5. If the perimeter of the triangle is 130 feet, find the measures of the sides.

3. Find the scale factor of Figure A to Figure B.

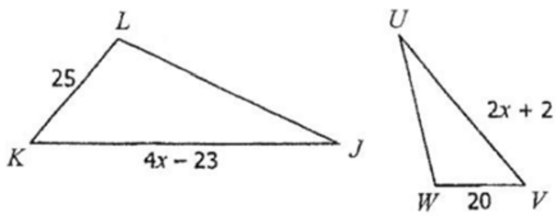


4. Find the scale factor of Figure B to Figure A.



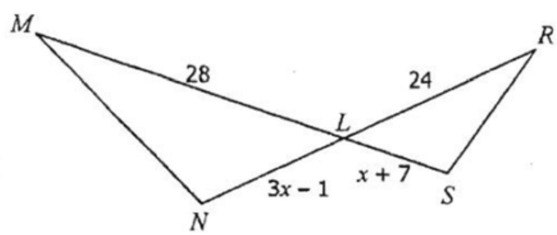
5.

If $\Delta KLJ \sim \Delta VWU$, find the value of x.



6.

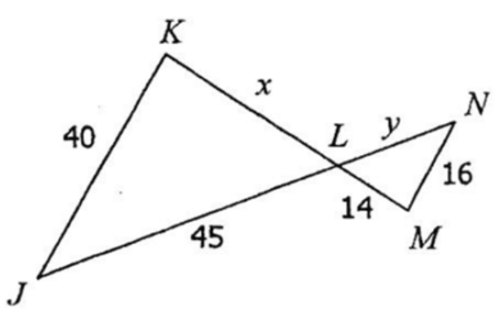
If $\Delta NML \sim \Delta SRL$ find the value of x.



For 7- 16, given the similar polygons, use a proportion to find the value of each variable

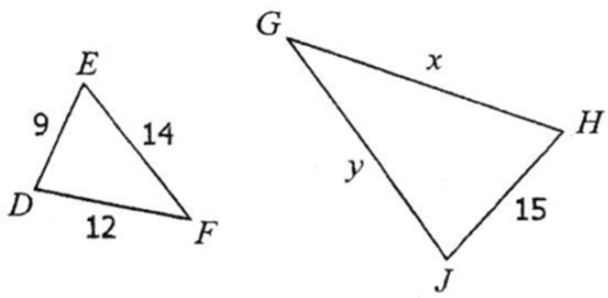
7.

$\Delta JKL \sim \Delta NML$



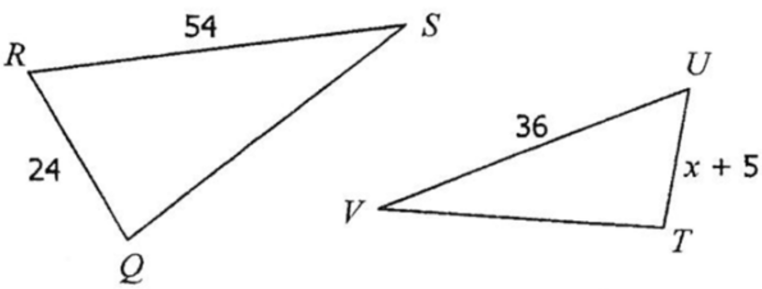
8.

$\Delta DEF \sim \Delta HJG$



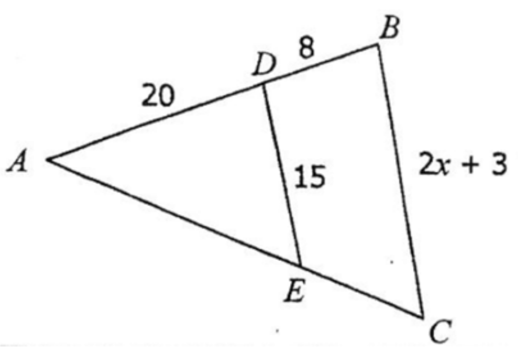
9.

$\Delta QRS \sim \Delta TUV$



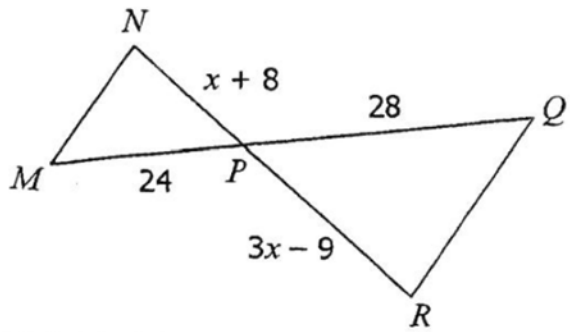
10.

$\Delta ABC \sim \Delta ADE$



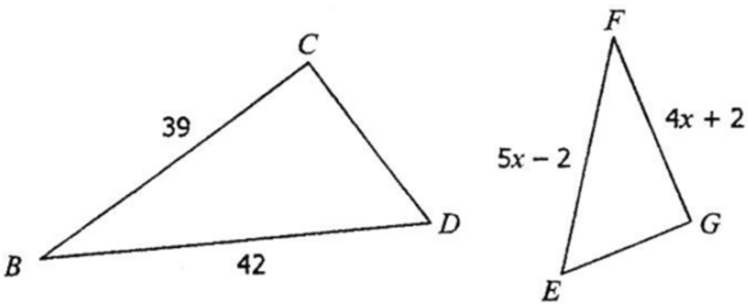
11.

$\Delta MNP \sim \Delta QRP$



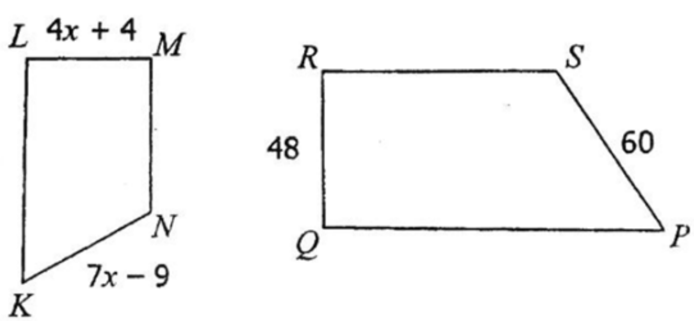
12.

$\Delta BCD \sim \Delta FGE$



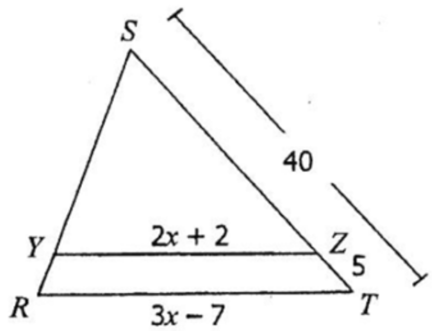
13.

$KLMN \sim PQRS$



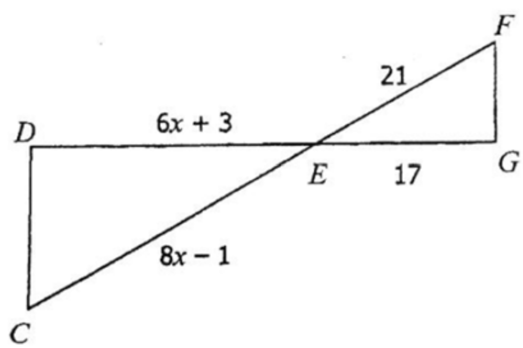
14.

$\Delta RST \sim \Delta YSZ$



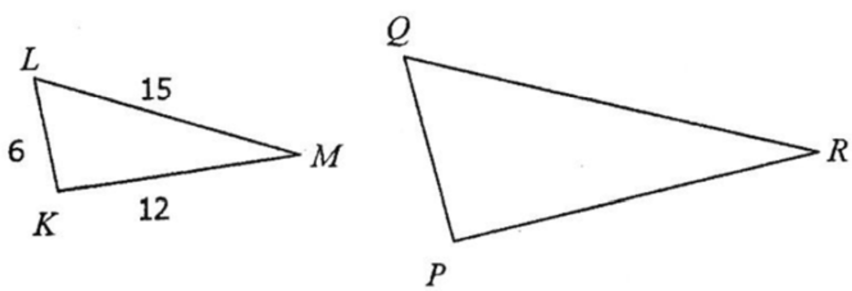
15.

$\Delta CDE \sim \Delta FGE$



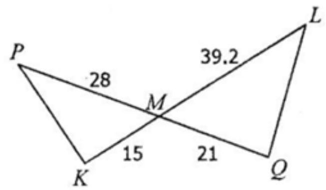
16.

If $\Delta KLM \sim \Delta PQR$ with a scale factor of 3:5, find the perimeter of ΔPQR.



For 17-22, determine if the triangles are similar by AA~, SSS~, SAS~, or not similar.

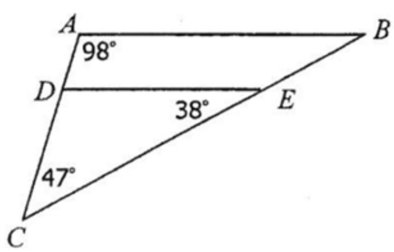
17.



Similar By: ______________

$\Delta PKM \sim$ ______________

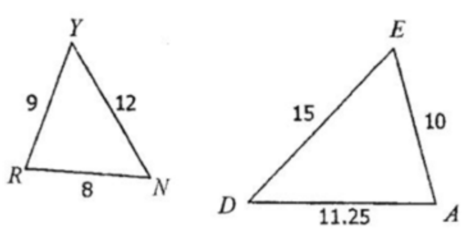
18.



Similar By: ______________

$\Delta CAB \sim$ ______________

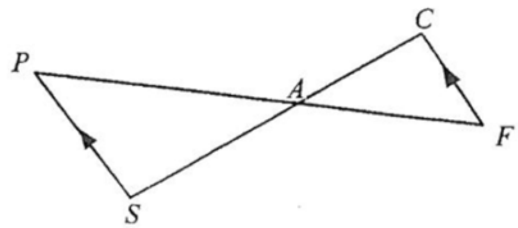
19.



Similar By: ______________

$\Delta RYN \sim$ ______________

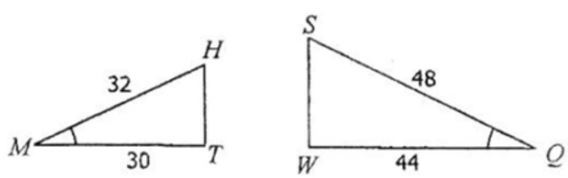
20.



Similar By: ______________

$\Delta PAS \sim$ ______________

21.



Similar By: ______________

$\Delta MTH \sim$ ______________

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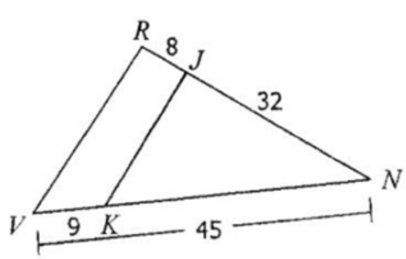
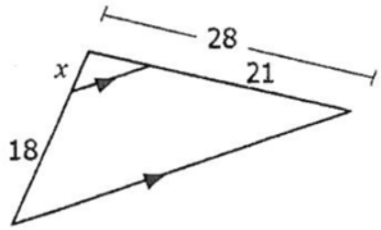


Similar By: ______________

$\Delta JKN \sim$ ______________

For 23-28, solve for x.

23.



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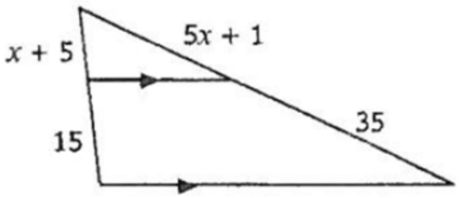

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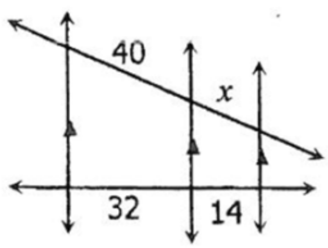

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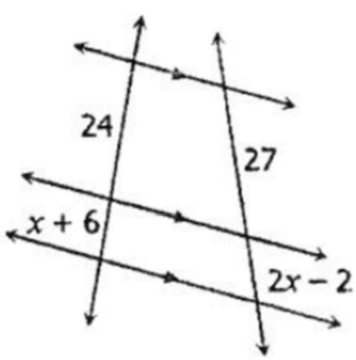
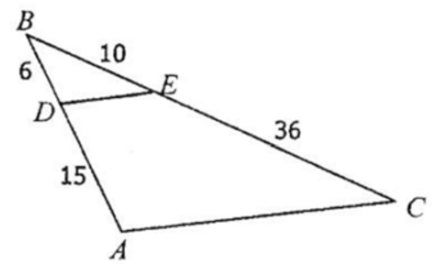


For 27-28, determine if $\overline{DE}$ is parallel to $\overline{AC}$.

27.



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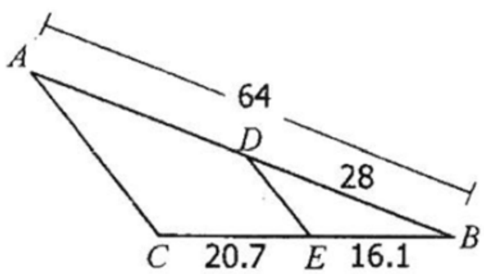
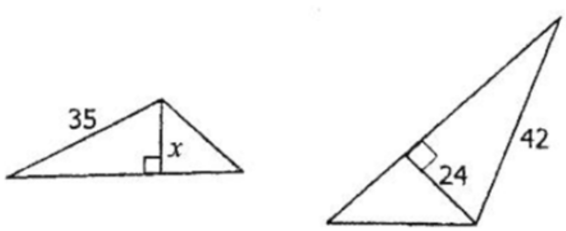


For 29-33, the triangles are similar. Solve for x.

29.



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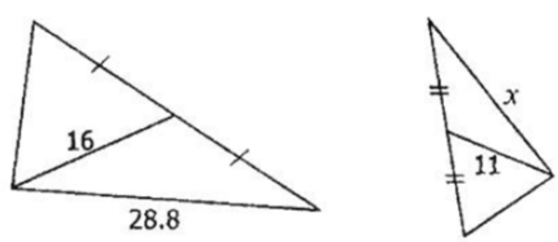

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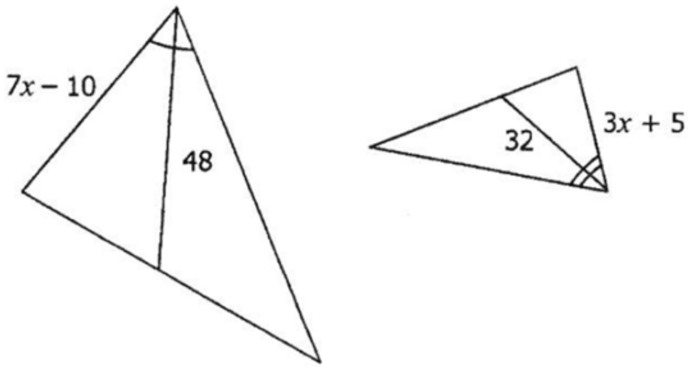

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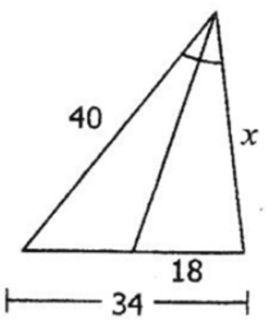

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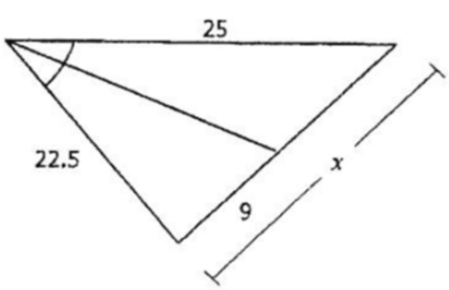


Triangle Proportionality Theorem

Draw two parallel lines that contain points A and B. (Refer to Figure 1.)

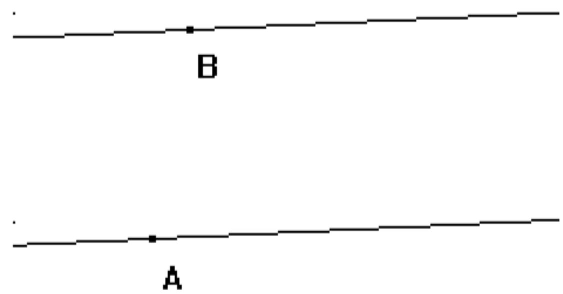


Figure 1

Draw line $\overline{AB}$. Create a new point C on $\overline{AB}$ and draw another transversal. Label the intersections of this line with the parallel lines as points D and E as shown in Figure 2.

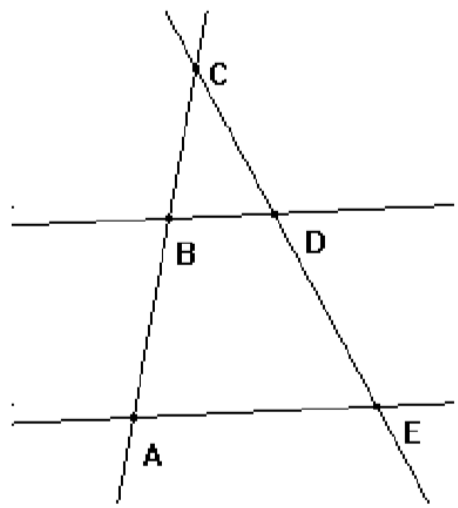


Figure 2

	Side 1	Side 2	Side 3	Perimeter
ΔCBD	$CB =$	$CD =$	$BD =$	$Perim_{\Delta CBD} =$
ΔCAE	$CA =$	$CE =$	$AE =$	$Perim_{\Delta CAE} =$
Ratio	$\frac{CB}{CA} =$	$\frac{CD}{CE} =$	$\frac{BD}{AE} =$	$\frac{Perim_{\Delta CBD}}{Perim_{\Delta CAE}} =$

Answer the following questions:

1. Explain why $\overline{AB}$ (illustrated in figure 2) is a transversal.

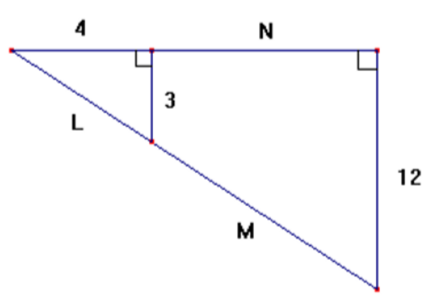
2. Explain why segments $\overline{CB}$ and $\overline{CA}$ are called corresponding segments.

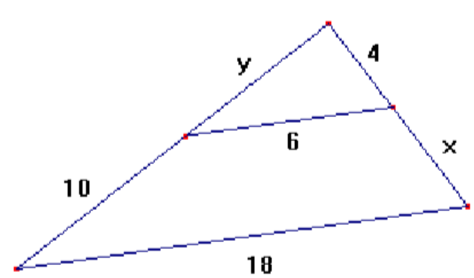
3. In view of the last row of results in the table, what appears to be true about the ratio of lengths defined by two transversals intersecting parallel lines?

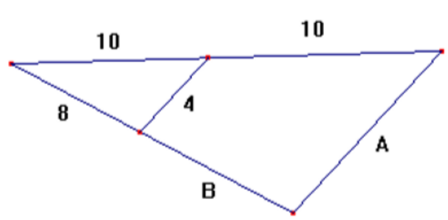
4. Grab different points and lines in the construction and move them around, if possible. While all of the measurements will change, one relationship will continue to hold no matter how the construction is changed. What is that relationship?

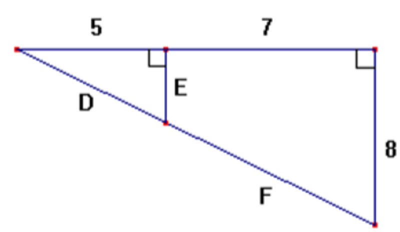
5. Compare your findings from question 4 to those of a classmate. Did everyone discover the same relationship?

6. Use the relationship that you have observed to solve for the unknown quantities in each of the following figures. You may assume that lines which look parallel in each figure are parallel.



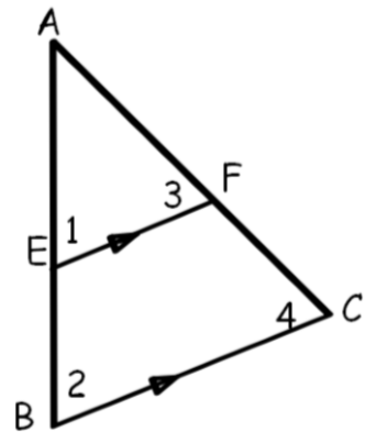






Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.



Proof #1:

Given: $\overleftrightarrow{EF} \parallel \overline{BC}$

Prove: $\frac{AE}{EB} = \frac{AF}{FC}$

Complete the proof:

Show that ΔAEF ~ ΔABC
Since $\overleftrightarrow{EF} \parallel \overline{BC}$, you can conclude that $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ by __________

So ΔAEF ~ ΔABC by ____________

Use the fact that corresponding sides of similar triangles are proportional to complete the proof

$\frac{AB}{AE} =$____________ Corresponding sides are proportional

$\frac{AE+EB}{AE} =$____________ Segment Addition Postulate

$1 + \frac{EB}{AE} =$_______________ Use the property that $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{EB}{AE} =$_______________ Subtract 1 from both sides.

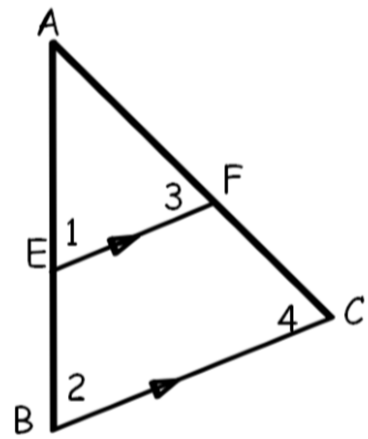
$\frac{AE}{EB} =$________________ Take the reciprocal of both sides.

Converse of the Triangle Proportionality Theorem
If a line divides two sides of a triangle proportionally, then it is parallel to the third side

Proof #2

Given: $\frac{AE}{EB} = \frac{AF}{FC}$

Prove: $\overleftrightarrow{EF} \parallel \overline{BC}$



Complete the proof. Show that ΔAEF ~ ΔABC
It is given that $\frac{AE}{EB} = \frac{AF}{FC}$ and taking the reciprocal of both sides show that ____________________.

Now add 1 to both sides by adding $\frac{AE}{AE}$ to the left side and $\frac{AF}{AF}$ to the right side.

____

Adding and using the Segment Addition Postulate gives _________________________.

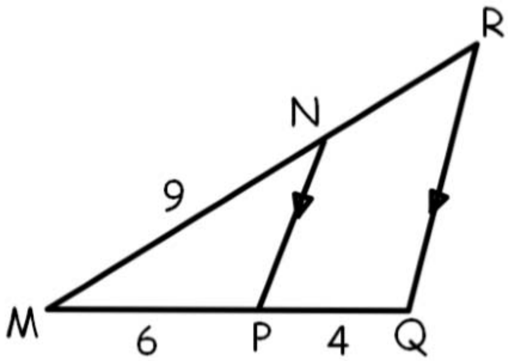
Since $\angle A \cong \angle A, \Delta AEF \sim \Delta ABC$ by ______________________

As corresponding angles of similar triangles, ∠AEF ≅ _________________

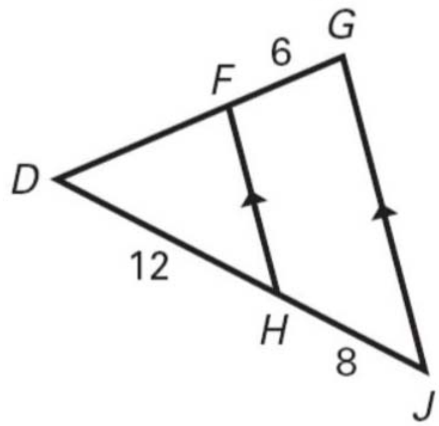
So, $\overleftrightarrow{EF} \parallel \overline{BC}$ by __________________

Let's practice finding the length of a segment since you know how to prove the Triangle Proportionality Theorem and its converse.

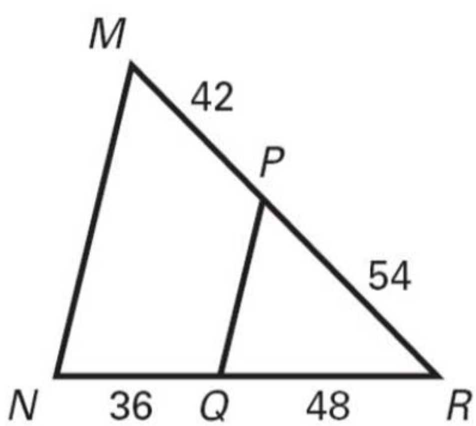
1. What is the length of NR?



2. What is the length of DF?



3. Given the diagram, determine whether $\overline{MN}$ is parallel to $\overline{PQ}$.



Proving Similar Triangles

Introduction
This task identifies the three ways to prove that two triangles are similar by showing that they satisfy the two requirements for similar polygons. Examples and practice problems are provided.

You can always prove that two triangles are similar by showing that they satisfy the two requirements for similar polygons.

1.) Show that corresponding angles are congruent AND
2.) Show that corresponding sides are proportional.

However, there are 3 simpler methods.

Angle-Angle Similarity Postulate (AA~) If two angles of one triangle are congruent to two angles of another triangle then the triangles are similar.

Examples of AA~

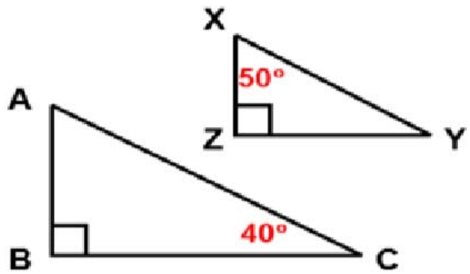
The ΔABC ~ ΔXZY are similar by AA~ because

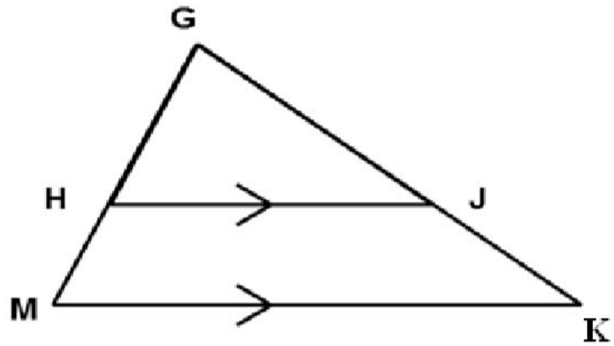
1) They are both right triangles; therefore they both have a 90 degree angle.

2) All triangles add up to 180 degrees, since angle C is 40 degrees in ΔABC angle A will be 50 degrees. Therefore, ∠ A and ∠ X are congruent.

The ΔGHJ ~ ΔGMK are similar by AA~ because

1) ∠ H and ∠ M are congruent by Corresponding Angles Postulate.
2) ∠ HGJ and ∠ MGK are congruent since they are the same angle.





Side-Side-Side Similarity (SSS~): If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar.

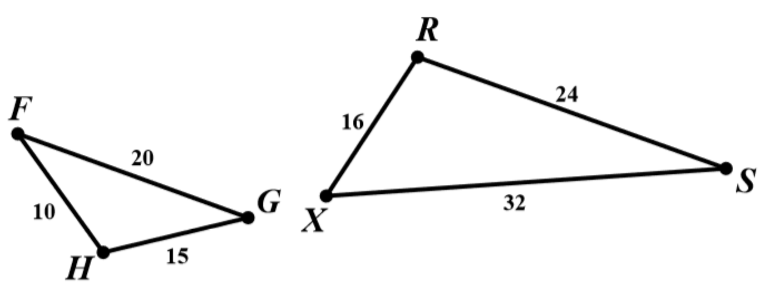
Example of SSS~

ΔFHG ~ ΔXRS because three sides of one triangle are proportional to three sides of another triangle.

$$\frac{FH}{XR} = \frac{10}{16} \equiv \frac{5}{8}$$

$$\frac{HG}{RS} = \frac{15}{24} = \frac{5}{8}$$

$$\frac{FG}{XS} = \frac{20}{32} = \frac{5}{8}$$

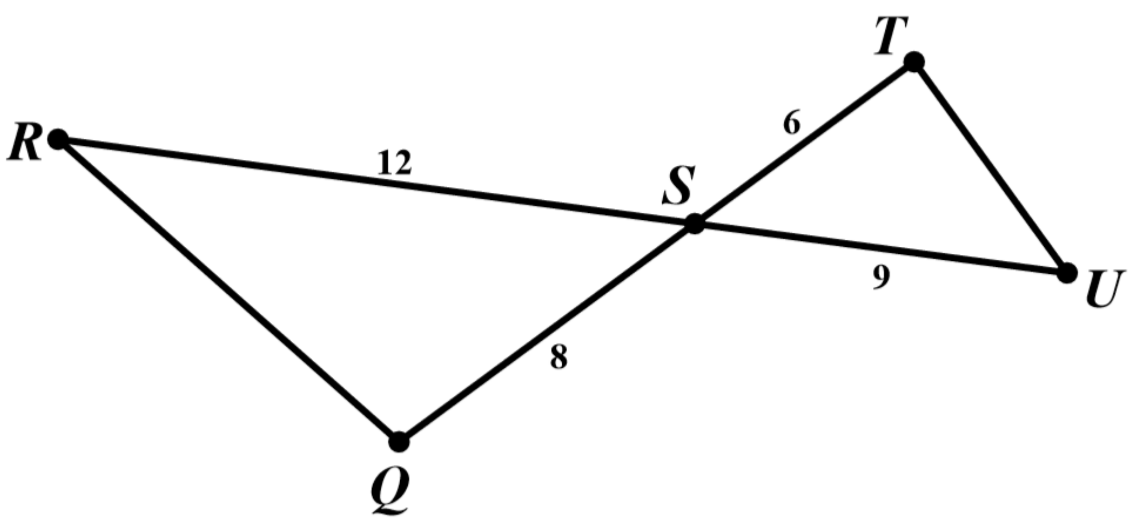


Side-Angle-Side Similarity (SAS~): If two sides of one triangle are proportional to two sides of another triangle and the included angles of these sides are congruent, then the two triangles are similar.

Example of SAS~

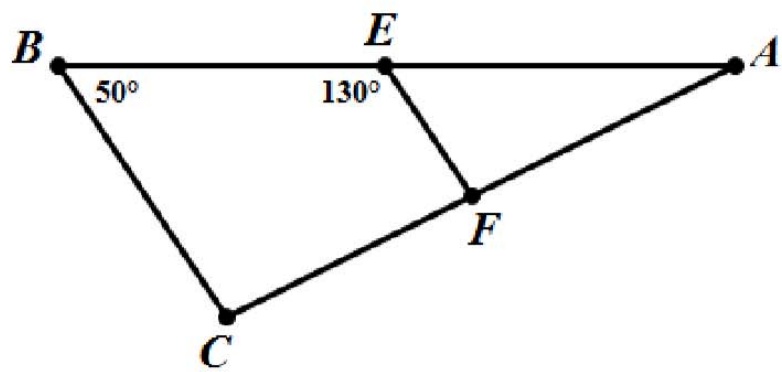
ΔRSQ ~ ΔUST because

1) $\angle$ RSQ $\cong$ $\angle$ UST since Vertical Angles are Congruent
2) $\frac{\text{RS}}{\text{US}} = \frac{12}{9} = \frac{4}{3}$, Two sides of one triangle are proportional to two sides of another triangle.
$\frac{\text{SQ}}{\text{ST}} = \frac{8}{6} = \frac{4}{3}$

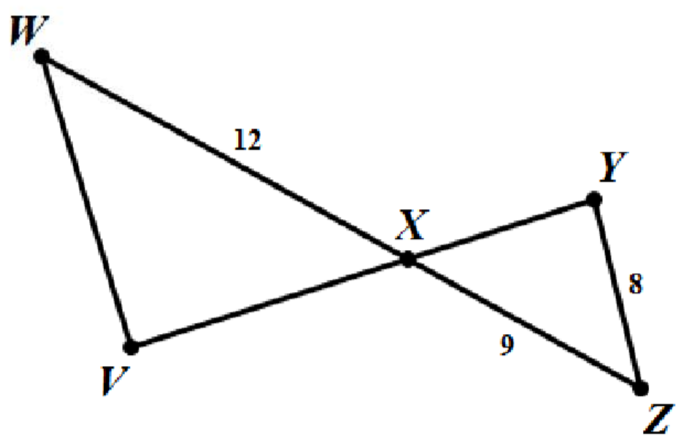


Can the two triangles shown be proved similar? If so, write a similarity statement and tell which method you used.

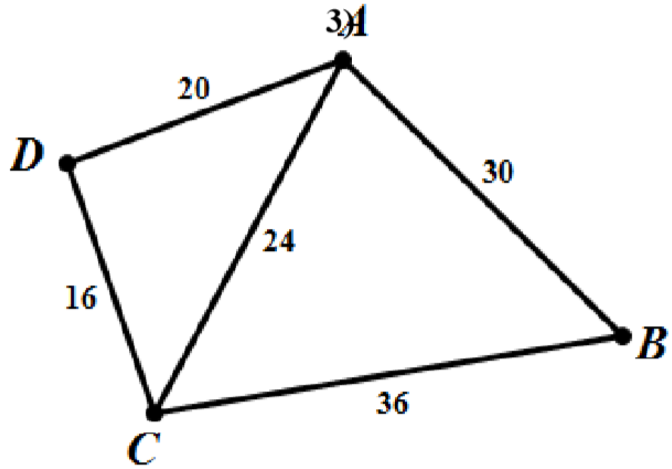
1)



2)



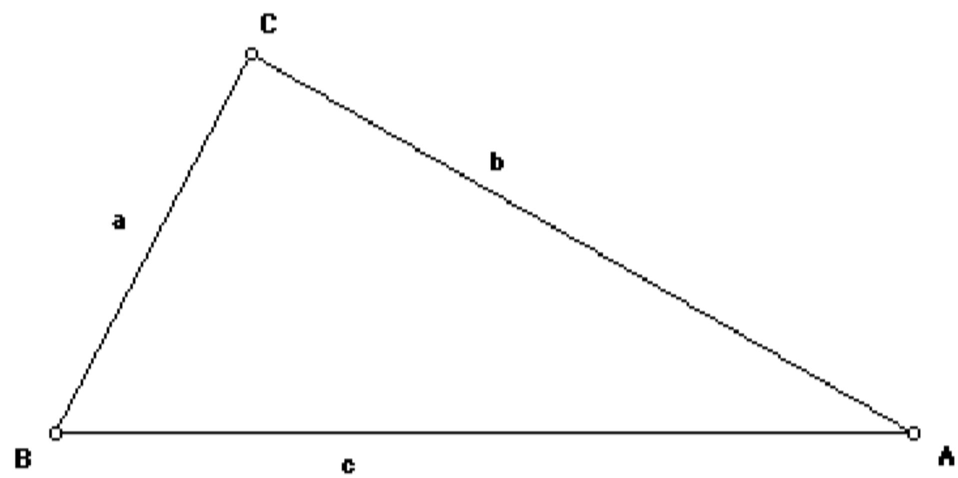
3)



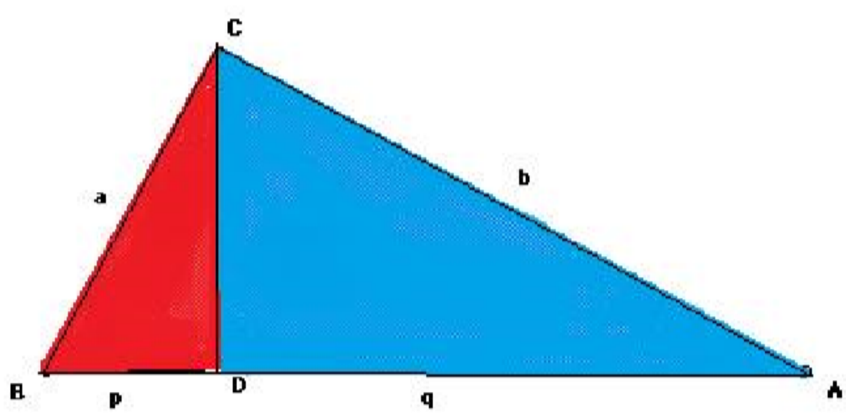
Pythagorean Theorem Using Triangle Similarity

Use cardboard cutouts to illustrate the following investigation:

In the next figure, draw triangle ABC as a right triangle. Its right angle is angle C.



Next, draw CD perpendicular to AB as shown in the next figure.

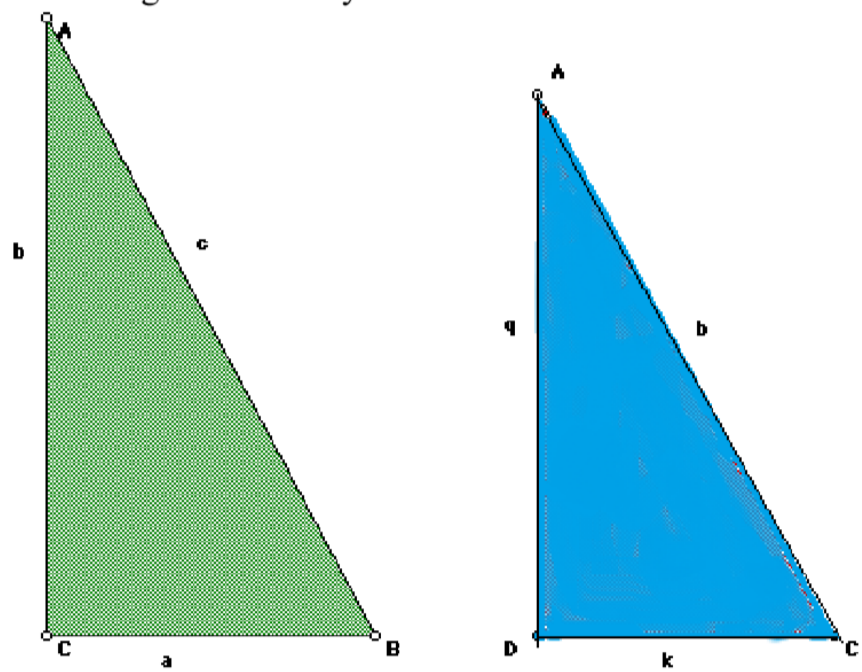


How many triangles do you see in the figure?

Why are the three triangles similar to each other?

Compare triangles 1 and 3:

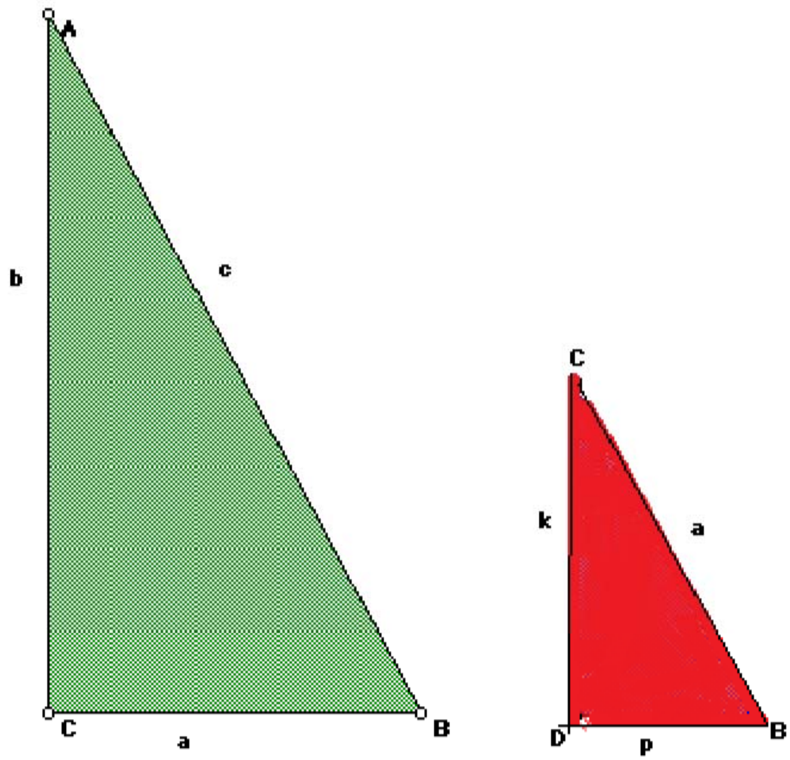
Triangle 1 (green) is the right triangle that we began with prior to constructing CD. Triangle 3 (blue) is one of the two triangles formed by the construction of CD.



By comparing these two triangles, we can see that $\frac{c}{b} = \frac{b}{q}$ and $b^2 = cq$

Compare triangles 1 and 2:

Triangle 1 (green) is the same as above. Triangle 2 (red) is the other triangle formed by constructing CD. Its right angle is angle D.



By comparing these two triangles, we see that $\frac{c}{a} = \frac{a}{p}$ and $a^2 = cp$

By adding the two equations:

$$a^2 + b^2 = cp + cq$$
$$a^2 + b^2 = c(p + q)$$

CD, we have that (p + q) = c. By substitution, we get

$$a^2 + b^2 = c^2$$

For 1-3, write the ratio in the simplest terms.

1. 180 red marbles to 145 blue marbles.

2. The hockey team played 82 regular season games last year. If they won 44 games, what is the ratio of wins to losses.

3. In the word FLASHLIGHT, what is the ratio of vowels to total letters.

For 4-7, use the given ratios to solve each problem.

4. The ratio of the measure of two complementary angles is 7:8. What is the measure of the smaller angle?

5. The ratio of the measure of the three angles in a triangle is 2:9:4. Find the measures of the angles.

6. The ratio of the vertex angle to the base angle of an isosceles triangle is 8:5. Find the measure of the vertex angle.

7. The ratio of the measures of the sides of the triangle is 4:7:5. If the perimeter of the triangle is 128 yards, find the length of the longest side.

For 8-11, solve each proportion.

8.

$$\frac{9}{16} = \frac{x}{12}$$

9.

$$\frac{3x-4}{14} = \frac{9}{10}$$

10.

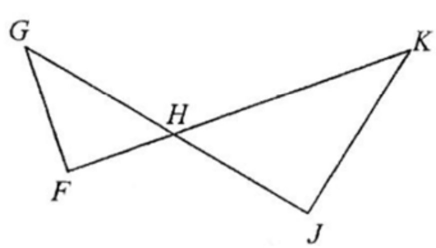
$$\frac{x-16}{x+6} = \frac{3}{5}$$

11.

$$\frac{x-9}{15} = \frac{2x-9}{10}$$

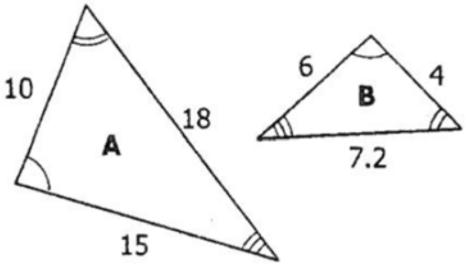
12. List all the congruent angles and write a proportion that relates the corresponding sides.

$\Delta FGH \sim \Delta JKH$

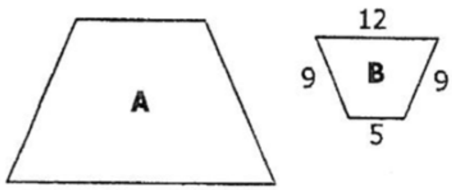


Angles	Sides

13. The pairs of polygons are similar. Give the scale factor of Figure A to Figure B.

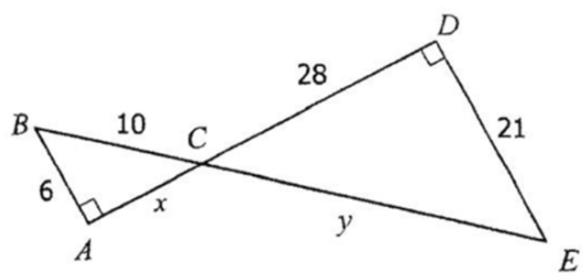


14. If the scale factor of Figure A to Figure B is 7:2, find the perimeter of Firgure A.



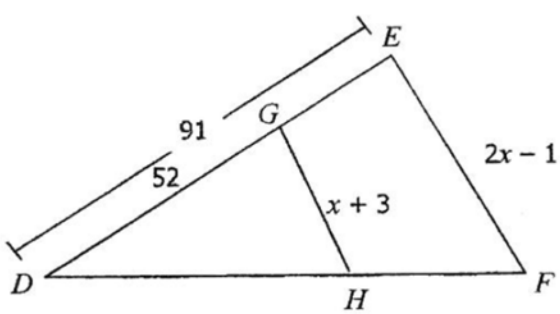
15.

If $\Delta ABC \sim \Delta DEC$, find the value of x and y.



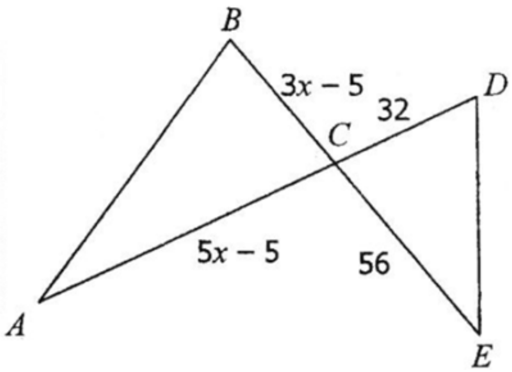
16.

If $\Delta DGH \sim \Delta DEF$, find the value of x.



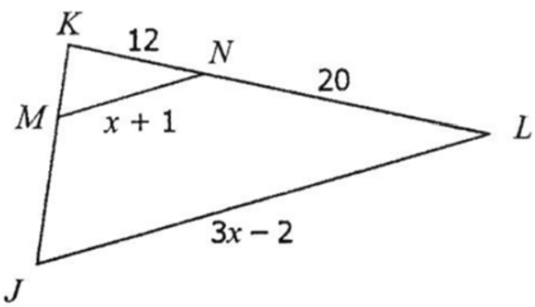
17.

If $\Delta ABC \sim \Delta EDC$, find the value of x.



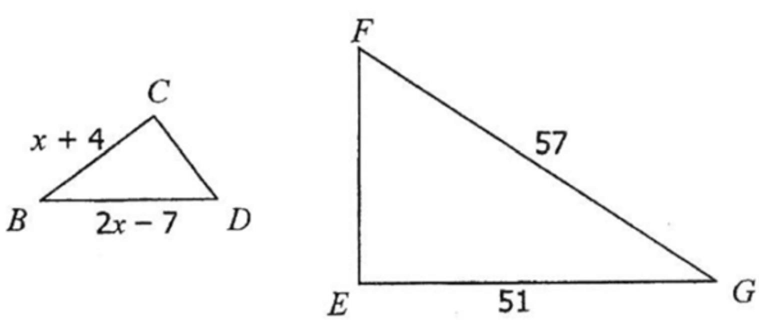
18.

If $\Delta JKL \sim \Delta MKN$, find the value of x.



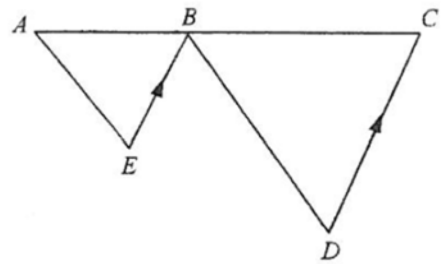
19.

If $\Delta BCD \sim \Delta GEF$, find the value of x.



For 20-25, determine if the triangles are similar by AA~, SSS~, SAS~, or not similar.

20.



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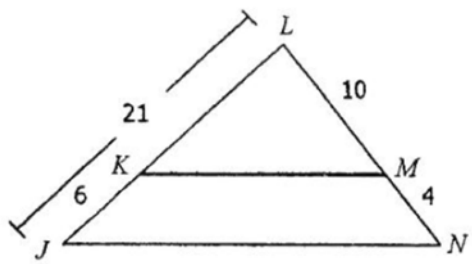

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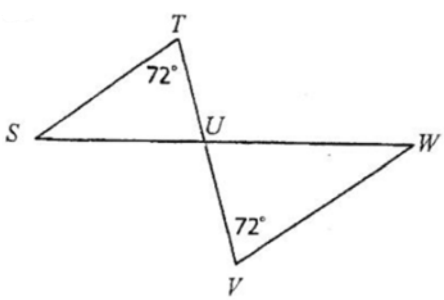

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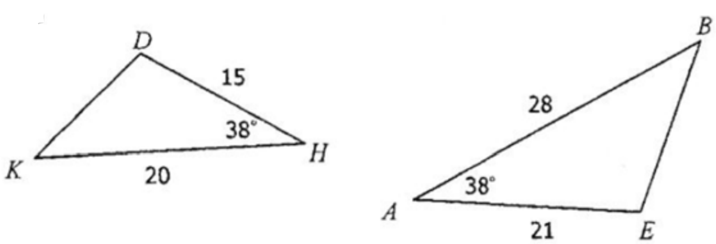
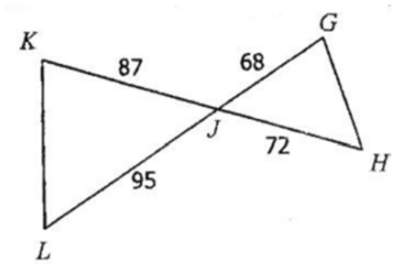


24.



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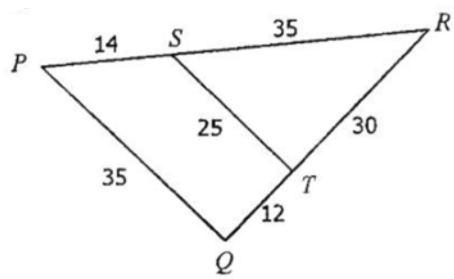
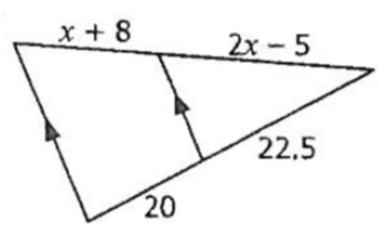


For 26-31, solve for x.

26.



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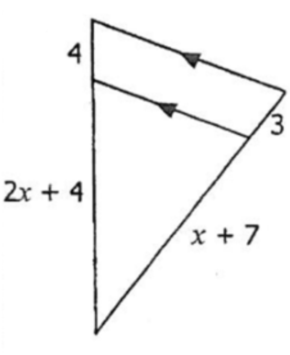

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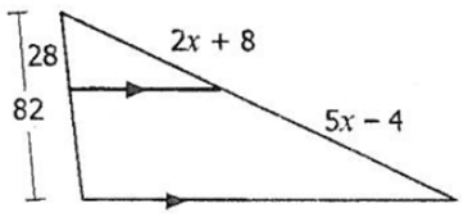
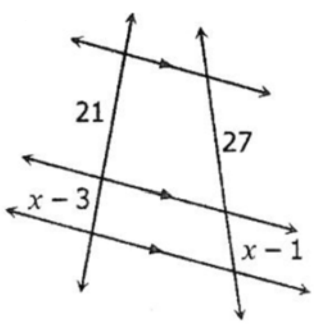
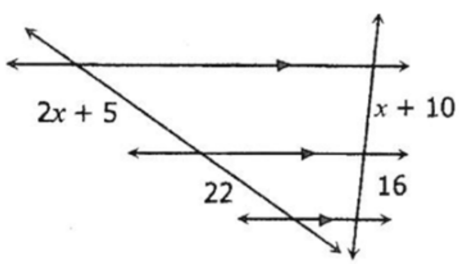


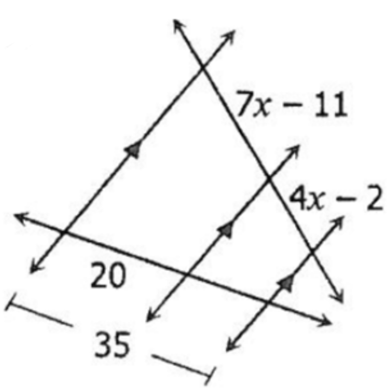
29.



30.



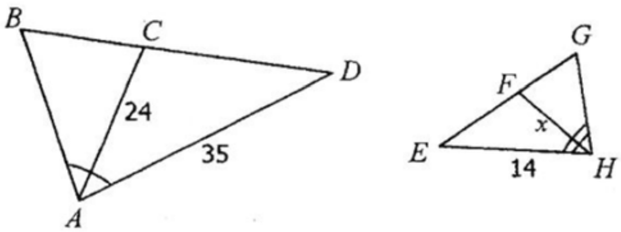
31.



For 32-34, given each pair of similar triangles, find the missing value.

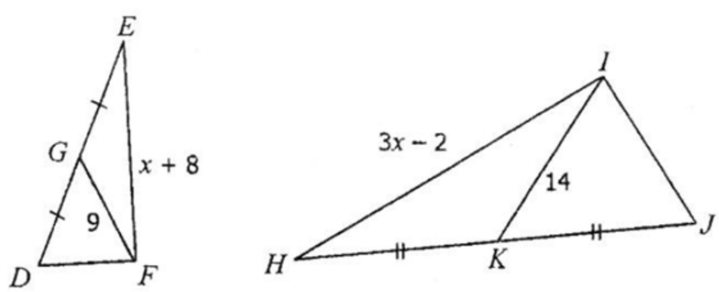
32.

$\Delta ABD \sim \Delta HGE$; Find x.



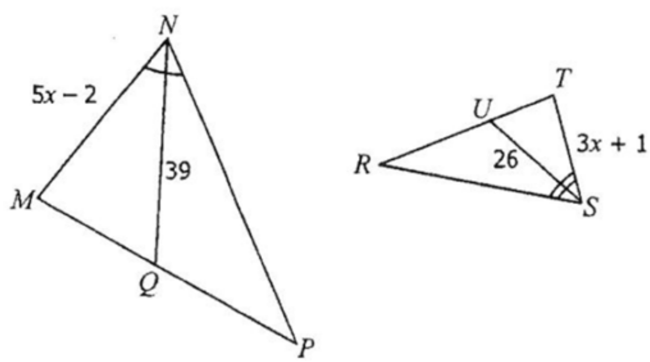
33.

$\Delta DEF \sim \Delta JHI$; Find EF.



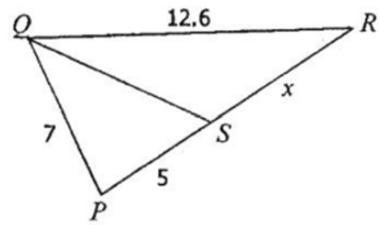
34.

$\Delta MNP \sim \Delta TSR$; Find TS.

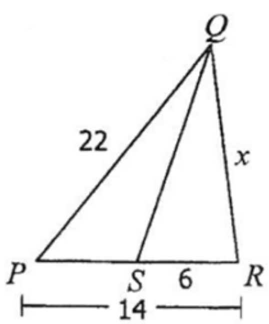


For 35-40, if $\overline{QS}$ represents an angle bisector, solve for x.

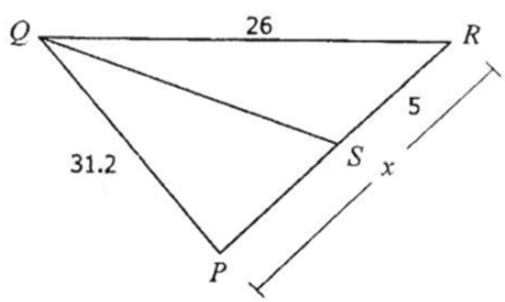
35.



36.



37.



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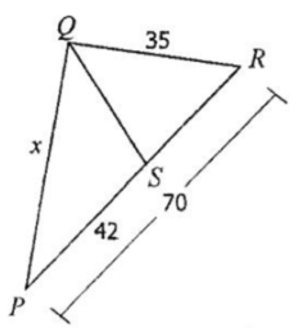

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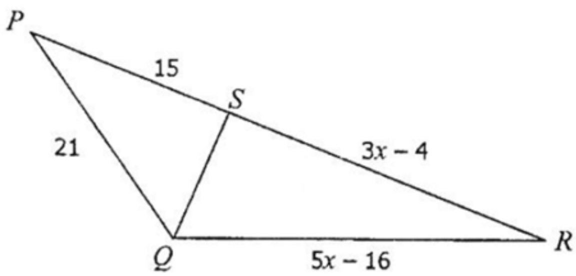
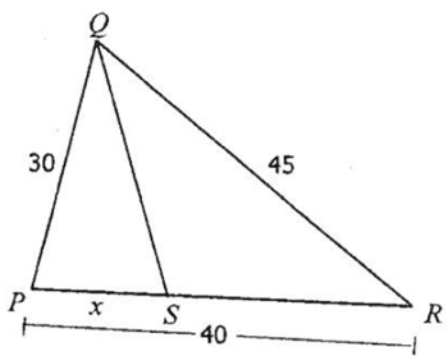


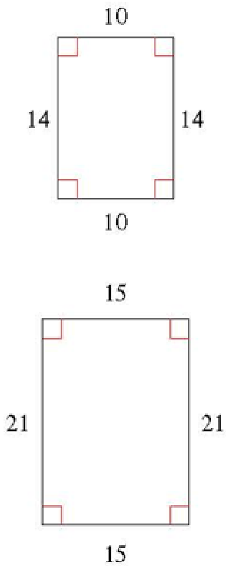
40.



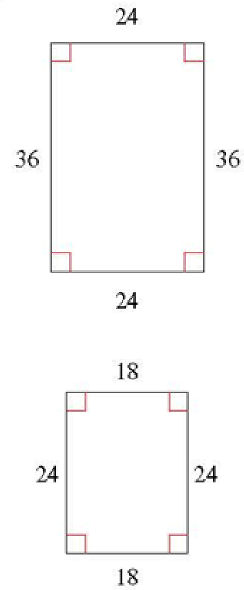
Similar Polygons

State if the polygons are similar.

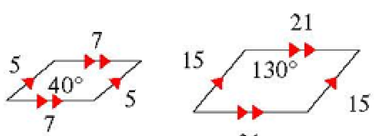
1)



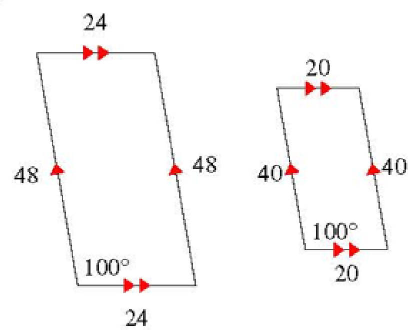
2)



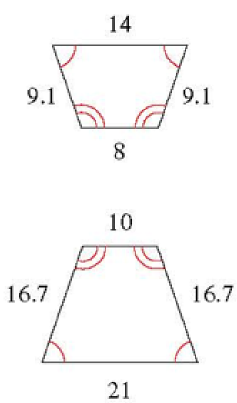
3)



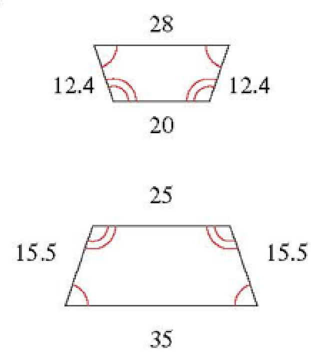
4)



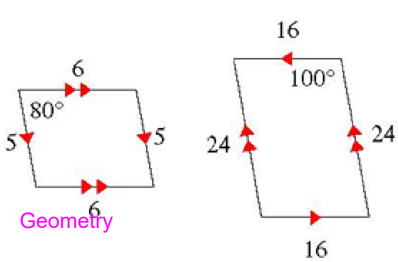
5)



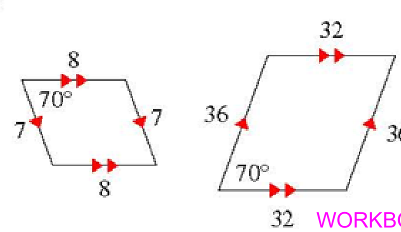
6)



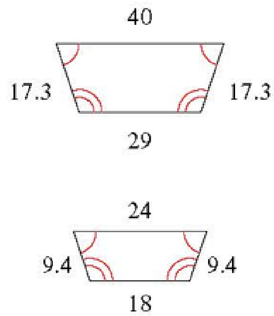
7)



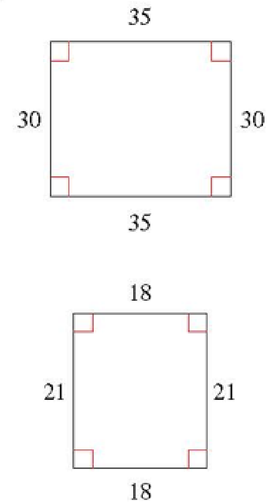
8)



9)

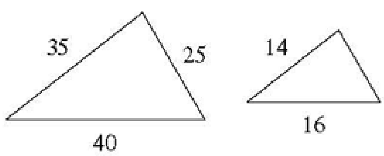


10)

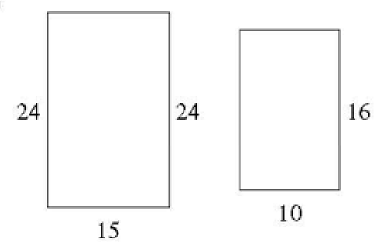


The polygons in each pair are similar. Find the scale factor of the smaller figure to the larger figure.

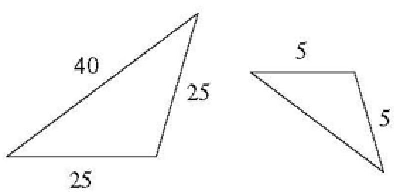
11)



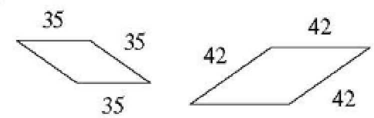
12)



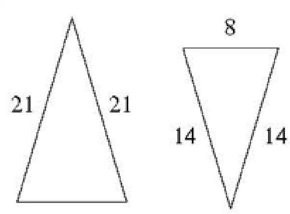
13)



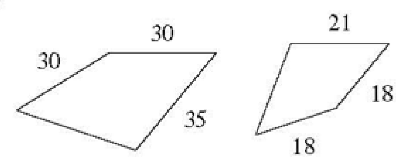
14)



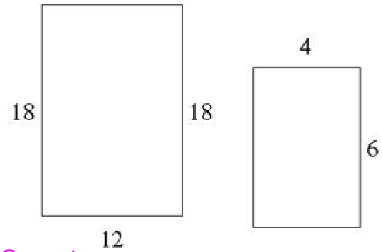
15)



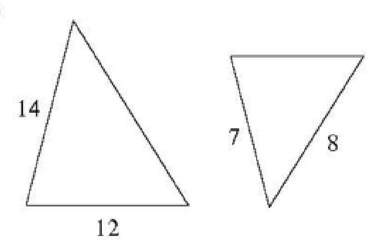
16)



17)



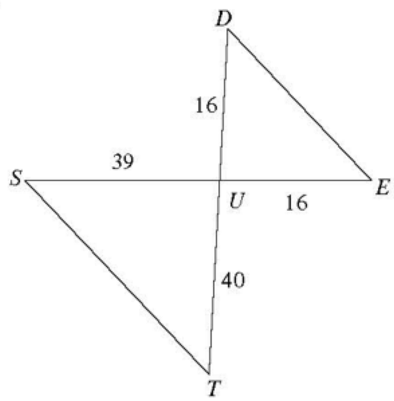
18)



Similar Triangles

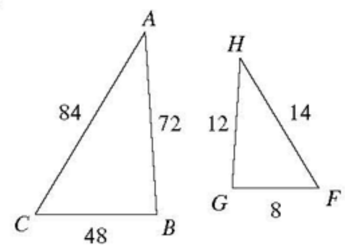
State if the triangles in each pair are similar. If so, state how you know they are similar and complete the similarity statement.

1)



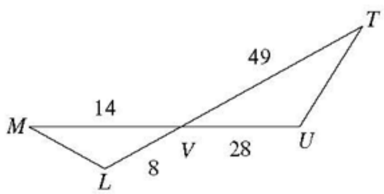
$\Delta UTS \sim$ _______

2)



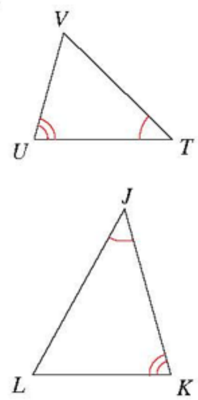
$\Delta CBA \sim$ _______

3)



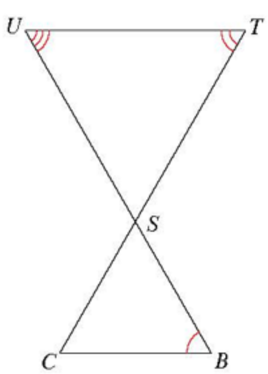
$\Delta VUT \sim$ _______

4)



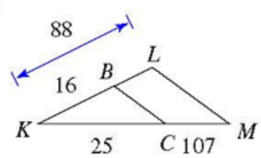
$\Delta JKL \sim$ _______

5)



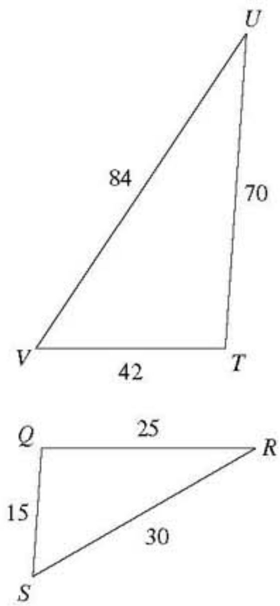
$\Delta STU \sim$ _______

6)



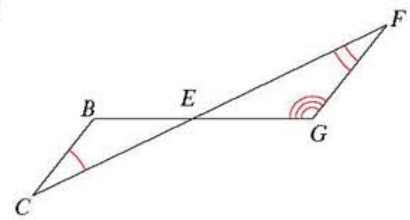
$\Delta KLM \sim$ _______

7)



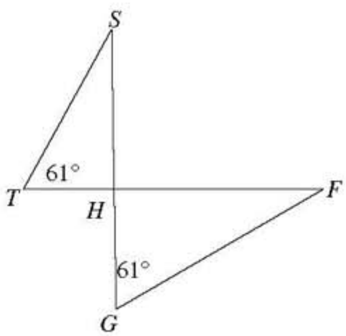
$\Delta TUV \sim$ ______

8)



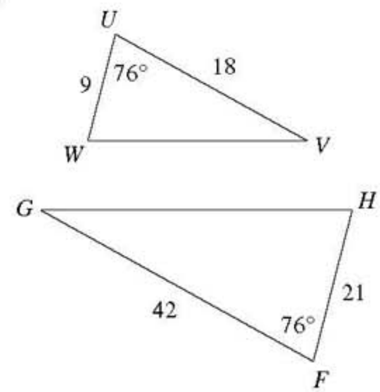
$\Delta EFG \sim$ ______

9)



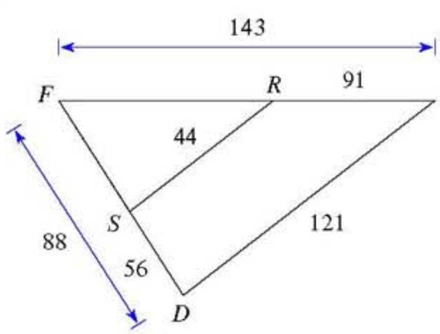
$\Delta HGF \sim$ ______

10)



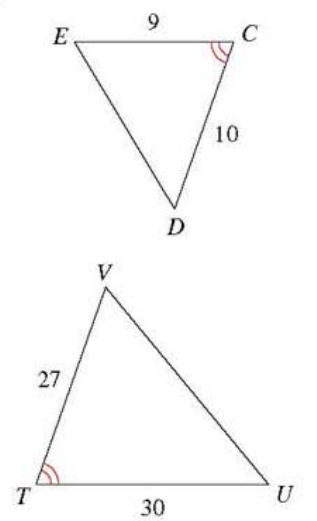
$\Delta FGH \sim$ ______

11)



$\Delta FED \sim$ ______

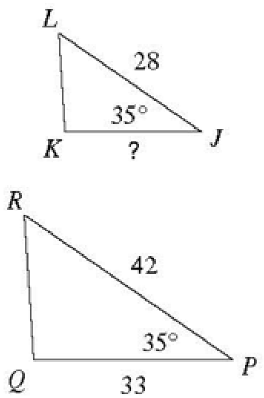
12)



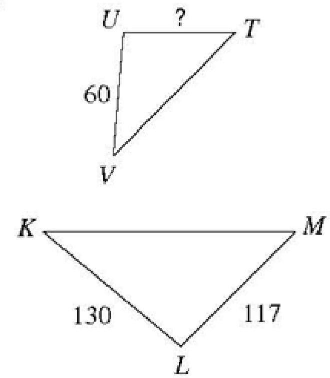
$\Delta TUV \sim$ ______

Find the missing length. The triangles in each pair are similar.

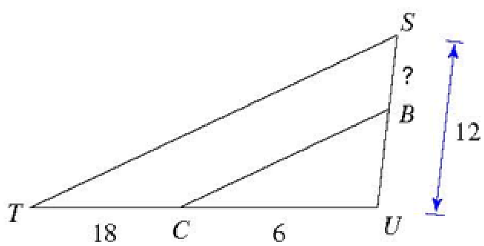
13)



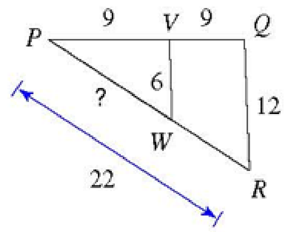
14)



15)

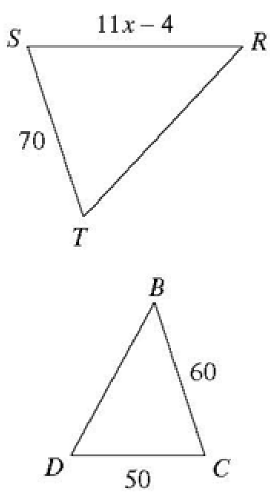


16)

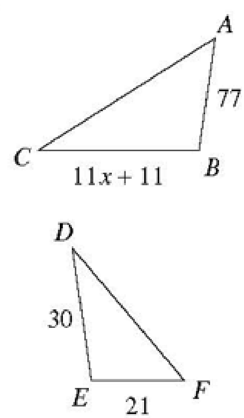


Solve for x. The triangles in each pair are similar.

17)



18)



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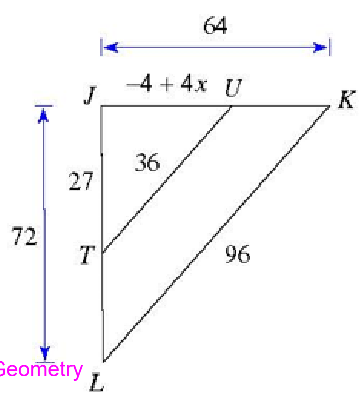
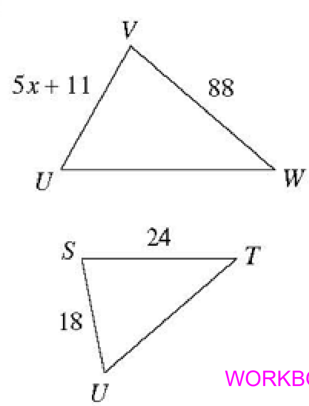


20)



Solving Proportions

Solve each proportion. Leave your answer as a fraction in simplest form.

1) $\frac{6}{2} = \frac{4}{p}$

2) $\frac{4}{k} = \frac{8}{2}$

3) $\frac{n}{4} = \frac{8}{7}$

4) $\frac{5}{3} = \frac{x}{4}$

5) $\frac{m}{5} = \frac{7}{2}$

6) $\frac{7}{4} = \frac{r}{5}$

7) $\frac{7}{6} = \frac{5}{x}$

8) $\frac{6}{5} = \frac{2}{5n}$

Solve each proportion. Round your answers to the nearest hundredth.

9) $\frac{7.7}{3.6} = \frac{2.3}{b}$

10) $\frac{v}{4.9} = \frac{5.4}{6.1}$

11) $\frac{6.3}{x} = \frac{2.56}{9.3}$

12) $\frac{3.4}{x} = \frac{2.17}{7.7}$

Solve each proportion. Leave your answer as a fraction in simplest form.

13) $\frac{9}{8} = \frac{k+6}{6}$

14) $\frac{2}{10} = \frac{4}{a-3}$

15) $\frac{10}{p+2} = \frac{4}{3}$

16) $\frac{4}{6} = \frac{8}{x-1}$

17) $\frac{m}{8} = \frac{m+7}{9}$

18) $\frac{n}{n+1} = \frac{3}{5}$

19) $\frac{9}{4} = \frac{r-10}{r}$

20) $\frac{x+6}{x} = \frac{10}{7}$

21) $\frac{n-9}{n+5} = \frac{7}{4}$

22) $\frac{6}{b+9} = \frac{4}{b+5}$

23) $\frac{8}{3} = \frac{v-9}{7v+4}$

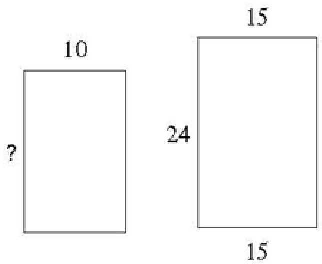
24) $\frac{8}{5x-4} = \frac{6}{x+5}$

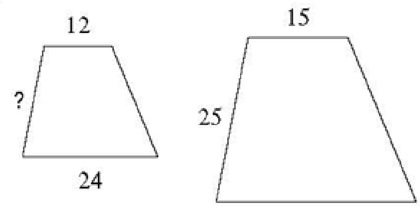
Critical thinking questions:

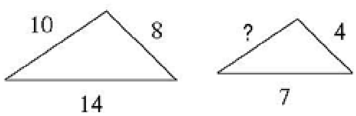
25) Do you think that a person's age and the amount they eat each day are basically in proportion?

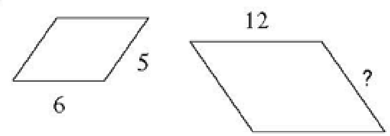
Using Similar Polygons

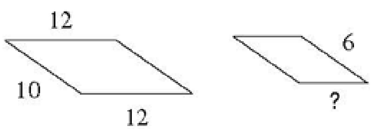
The polygons in each pair are similar. Find the missing side length.

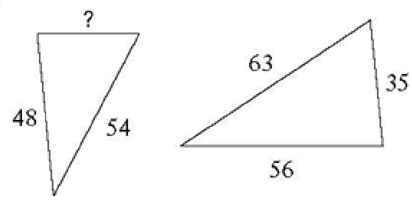
1)


2)


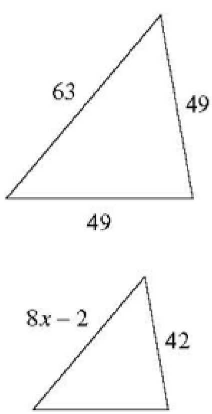
3)


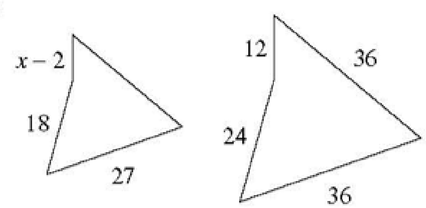
4)


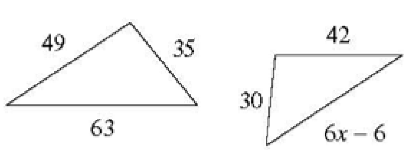
5)


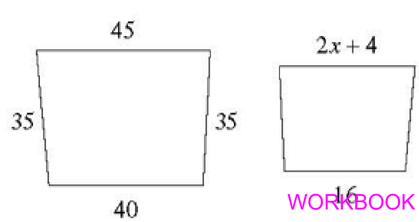
6)


Solve for *x*. The polygons in each pair are similar.

15)


16)


17)


18)


Lesson Name: G.U2E.B.C2.01.**Vocab.Dilations**

Term	Definition	Notation	Diagram/Visual
Rigid			
Non-Rigid			
Dilations			
Scale Factor			
Center of a Dilation			

Dilations

GUIDED NOTES

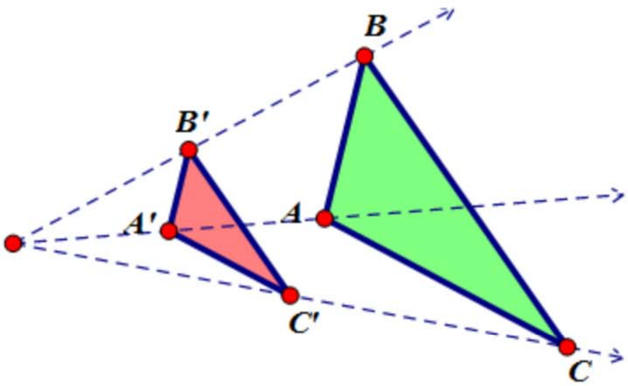
The ____________________________________ or ___________________________________ of a figure.

The ________________ __________________ indicates how much a figure will enlarge or reduce.

Variable for scale factor: _____________

When ____________, the dilation is an ______________________

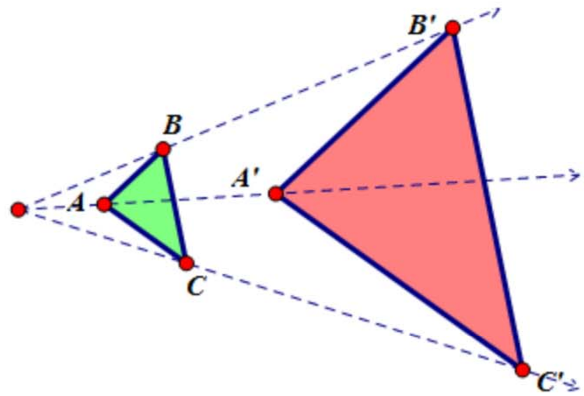
When ____________, the dilation is a ______________________



B'C' < BC and $\frac{B'C'}{BC} = k < 1$

A'C' < AC and $\frac{A'C'}{AC} = k < 1$

A'B' < AB and $\frac{A'B'}{AB} = k < 1$



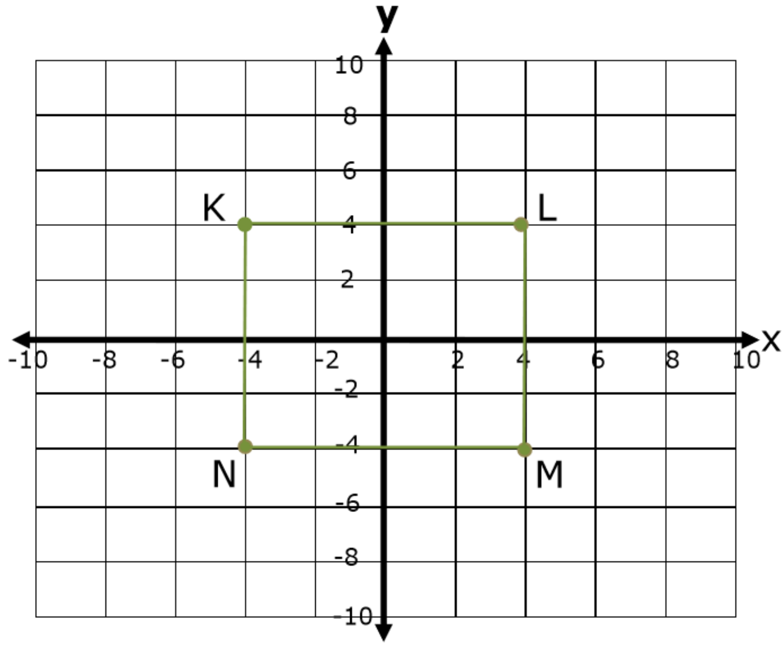
B'C' > BC and $\frac{B'C'}{BC} = k > 1$

A'C' > AC and $\frac{A'C'}{AC} = k > 1$

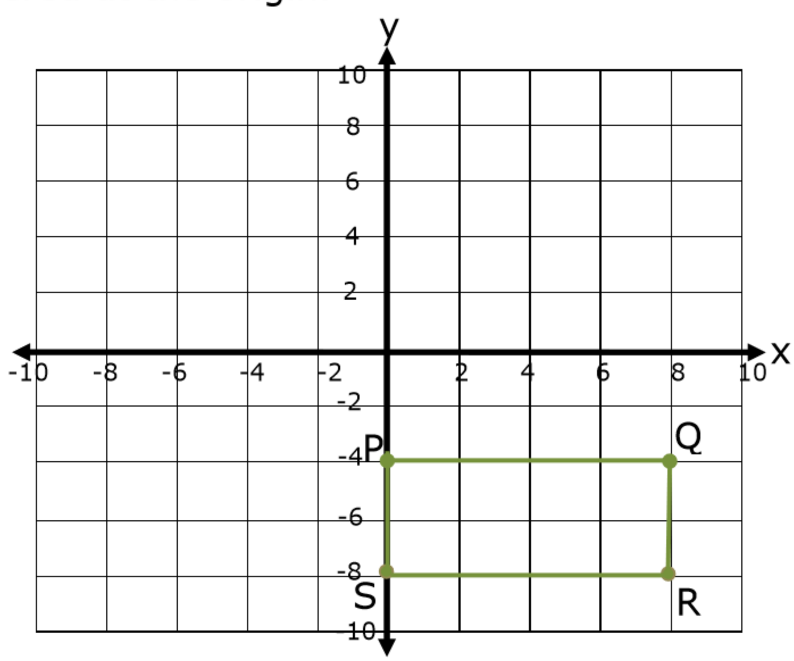
A'B' > AB and $\frac{A'B'}{AB} = k > 1$

Example!

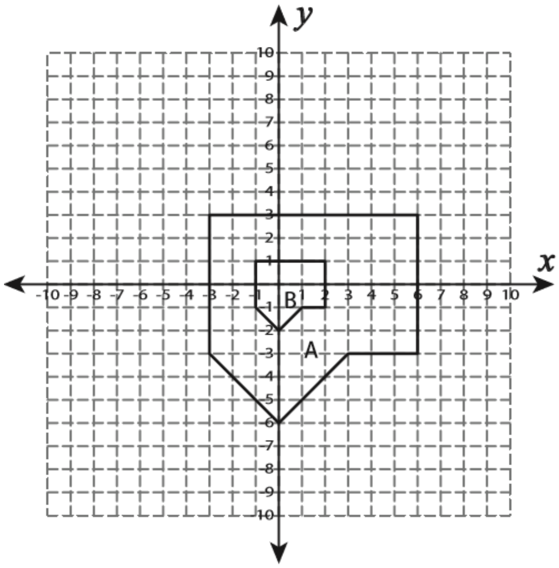
1. Graph the image of rectangle KLMN after dilation with a scale factor of 2, centered at the origin.



2. Graph the image of rectangle PQRS after a dilation with a scale factor of 1/4, centered at the origin.



3.



Scale factor = ______________

For 4 and 5 state whether the dilation is a reduction or an enlargement.

4.

Scale factor = 2

____

5.

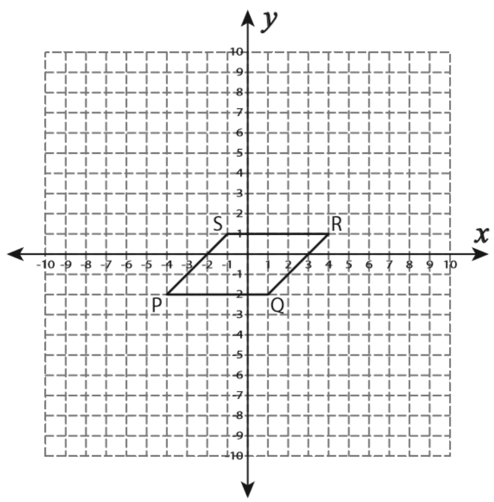
Scale factor = $\frac{1}{8}$

____

For 6 and 7, find the dilated coordinates with the given scale factor. Draw the dilated image.

6.

Scale factor = 2

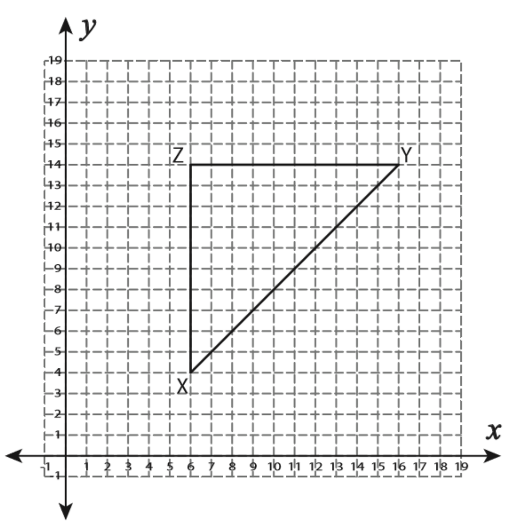


P′ : ______________ , Q′ : ______________

R′ : ______________ , S′ : ______________

7.

Scale factor = 0.5



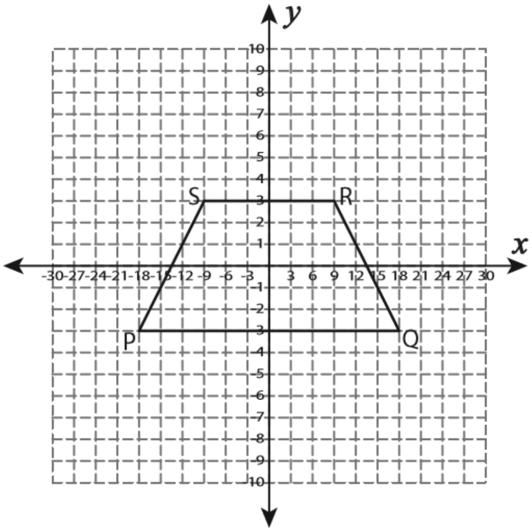
X′ : ______________ , Y′ : ______________

Z′ : ______________

For 8 and 9, draw the dilated image with the given scale factor.

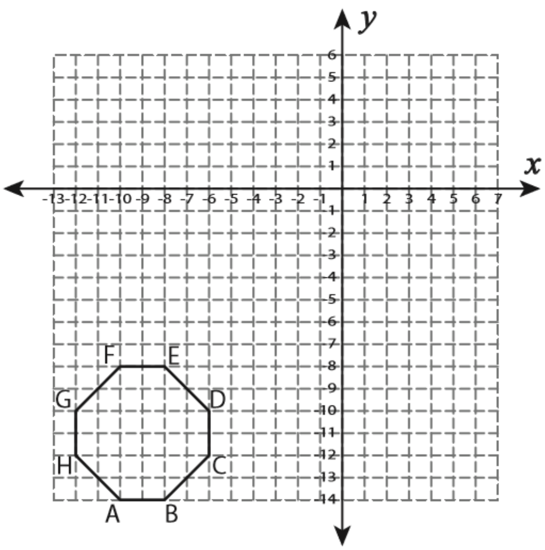
8.

Scale factor = $\frac{4}{3}$



9.

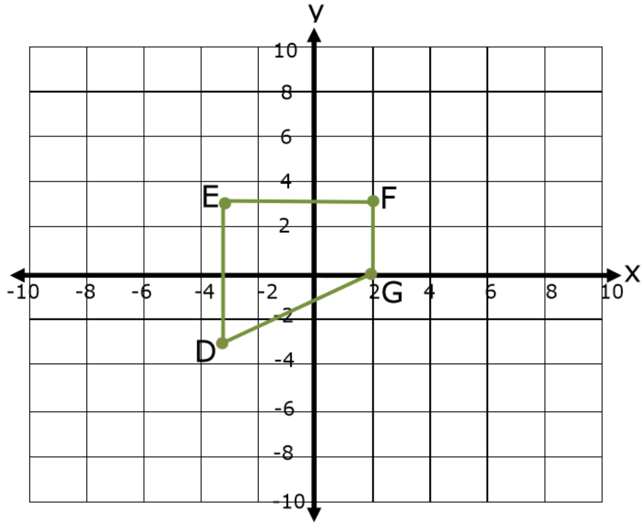
Scale factor = 0.5



SELF CHECK

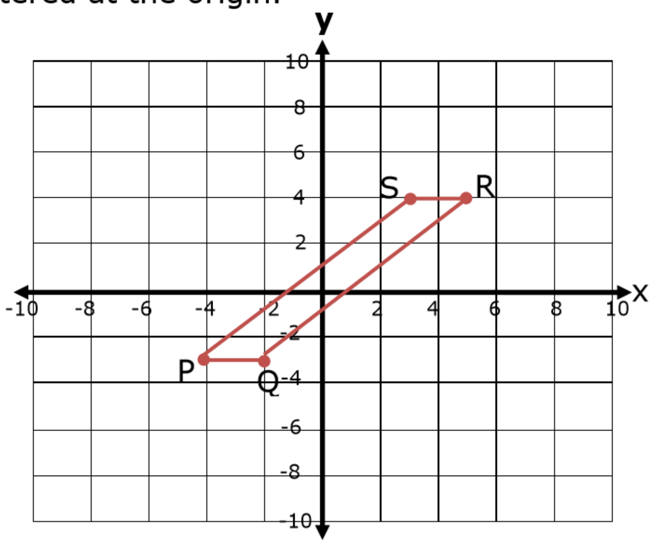
1.

Graph the image of quadrilateral EFGD after a dilation with a scale factor of 3, centered at the origin.

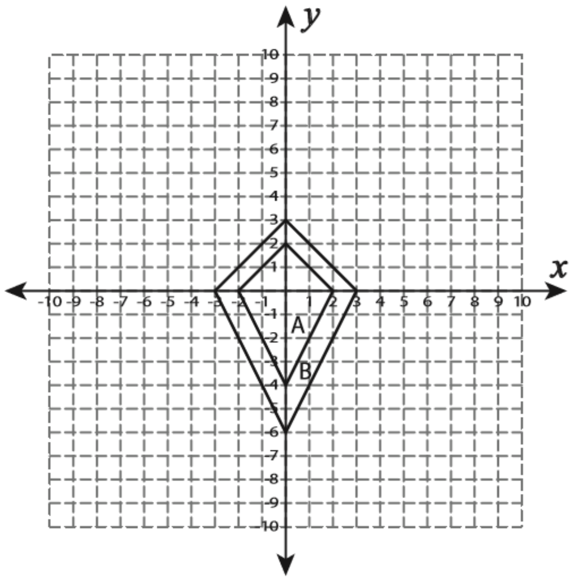


2.

Graph the image of quadrilateral PQRS after a dilation with a scale factor of 2, centered at the origin.



3.



Scale factor = ____________

For 4 and 5 state whether the dilation is a reduction or an enlargement.

4.

Scale factor = $\frac{5}{4}$

____

5.

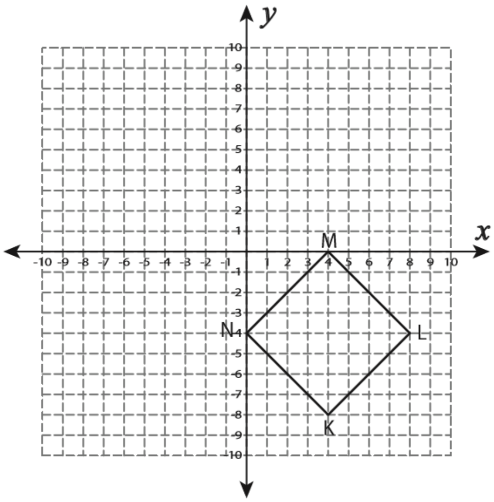
Scale factor = $\frac{2}{3}$

____

For 6 and 7, find the dilated coordinates with the given scale factor. Draw the dilated image.

6.

Scale factor = $\frac{1}{4}$

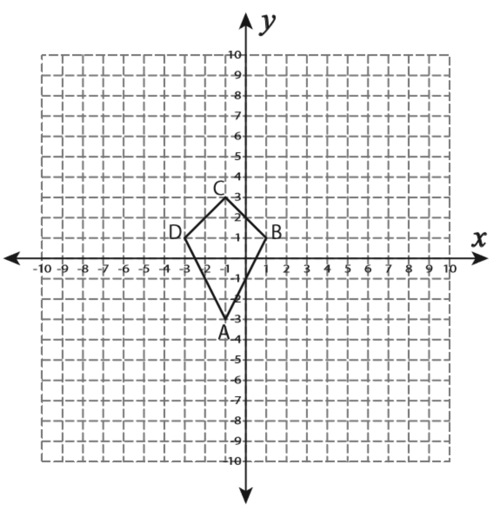


K′ : ______________ , L′ : ______________

M′ : ______________ , N′ : ______________

7.

Scale factor = 3



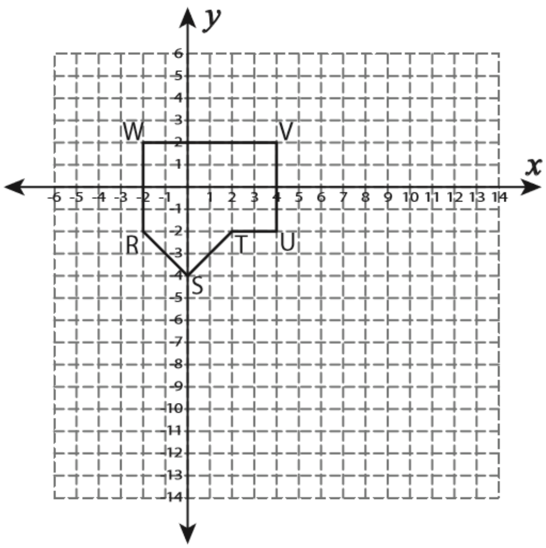
A′ : ______________ , B′ : ______________

C′ : ______________ , D′ : ______________

For 8 and 9, draw the dilated image wit the given scale factor.

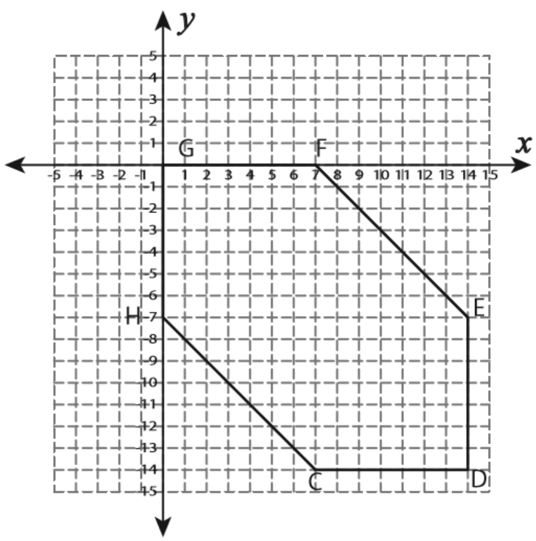
8.

Scale factor = 2.5



9.

Scale factor = $\frac{3}{7}$

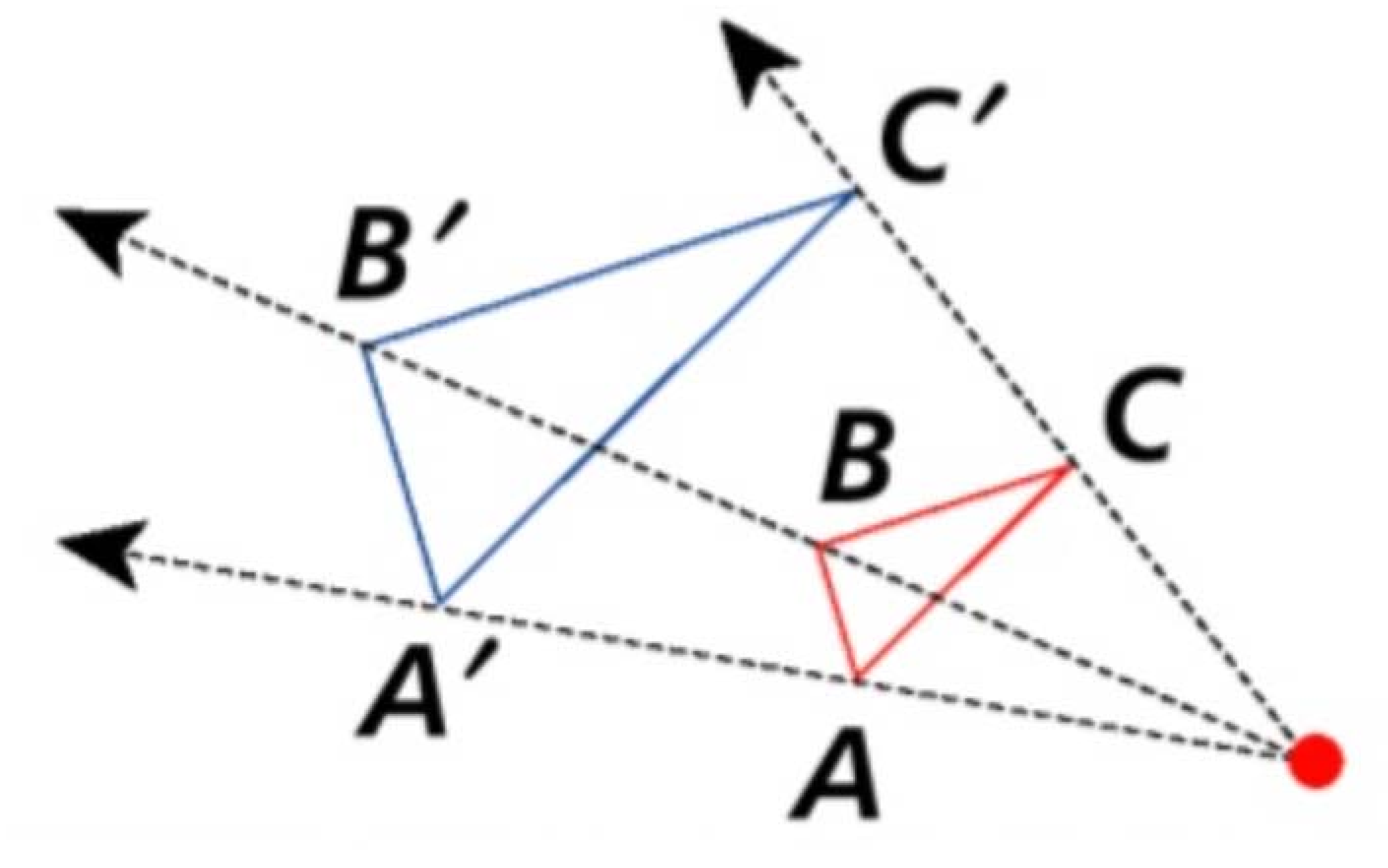


Center of a Dilation

GUIDED NOTES

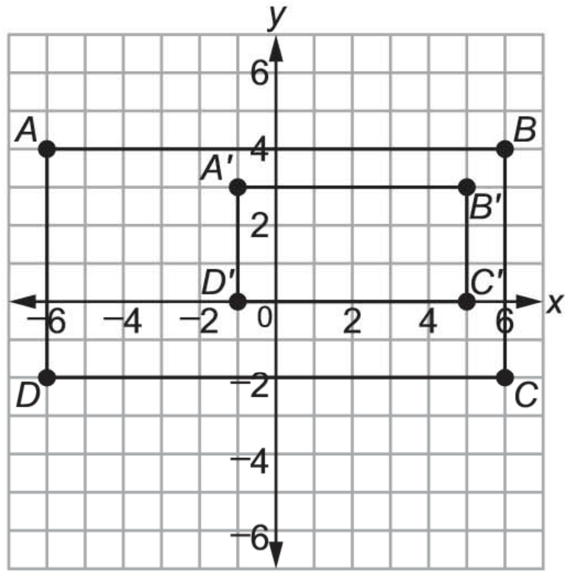
Facts about the Center of a Dilation:

- On a Coordinate plane, the center of dilation can be any point, but the origin is commonly used.
- The ratio of the distance from the center of dilation to any point on the image compared to the distance from the center of dilation to the corresponding point on the pre-image will result in the scale factor, k.
- Lines drawn through each point on the pre-image and its corresponding image point will intersect at the center of the dilation.
- When the origin is the center of the dilation, each point on the pre-image (x, y) corresponds to (kx, ky) on the image.



How to find the center when it is not the origin

Figure $A'B'C'D'$ is a dilation of figure $ABCD$.



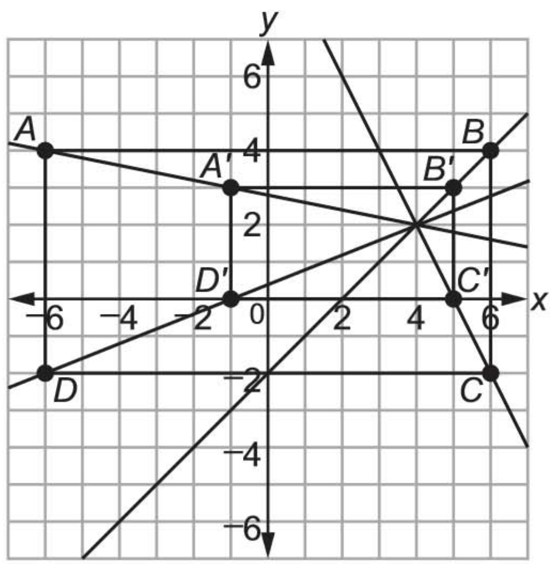
a. Determine the center of dilation.

b. Determine the scale factor of the dilation.

c. What is the relationship between the sides of the pre-image and the corresponding sides of the image?

Solution:

a. To find the center of dilation, draw lines connecting each corresponding vertex from the pre-imag to the image. The lines meet at the center of dilation.



The center of dilation is (4, 2).

b. Find the ratios of the lengths of the corresponding sides.

$$\frac{A'B'}{AB} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{B'C'}{BC} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{C'D'}{CD} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{A'D'}{AD} = \frac{3}{6} = \frac{1}{2}$$

The ratio for each pair of corresponding sides is $\frac{1}{2}$, so the scale factor is $\frac{1}{2}$.

c. Each side of the image is parallel to the corresponding side of its pre-image and is $\frac{1}{2}$ the length.

Note: Lines connecting corresponding points pass through the center of dilation.

Writing the rule for translating a dilation when the center is not the origin

This can be accomplished by first translating the center of the dilation and figure, so the origin becomes the center, and then translating it back.

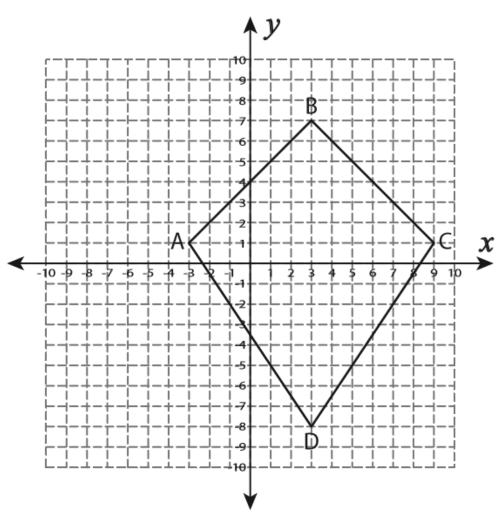
Example: Find a coordinate rule for the dilation with center (5, -3) and scale factor 2.

Solution: If (x, y) is a point on a figure to be dilated, we first translate left 5 and up 3. This gives us the point (x – 5, y + 3) and the origin becomes the center of the dilation. Distribute the scale factor (2) out to the new point and you will get (2x – 10, 2y + 6). The we translate back – right 5 and down 3, which gives us (2x – 10 + 5, 2y + 6 – 3). So the coordinate rule is: (x, y) → (2x – 5, 2y + 3)

Example!

1.

center : (–6, 1) , $k = \frac{1}{3}$

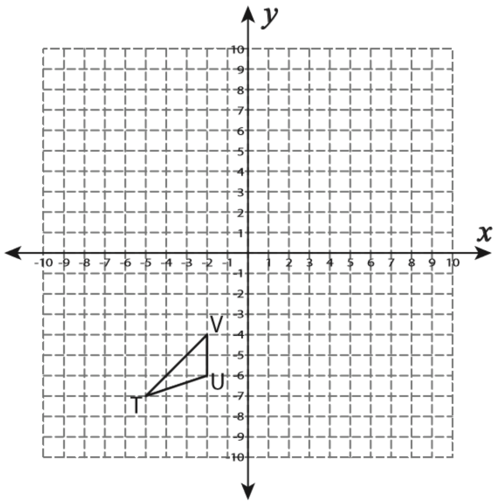


A′ : ______________ , B′ : ______________

C′ : ______________ , D′ : ______________

2.

center : (–2, –7) , k = 2



T′ : ______________ , U′ : ______________

V′ : ______________

3.

P(–1, 2), Q(3, 8), R(5, –10), S(–5, –4)

center : (–7, 2) , $k = \frac{1}{2}$

P′ : ______________ , Q′ : ______________

R′ : ______________ , S′ : ______________

4.

C(1, 2), D(2, 2), E(2, 0)

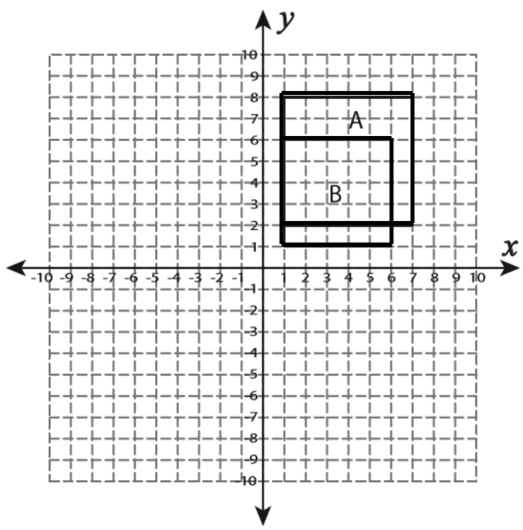
center : (0, 2) , k = 3

C′ : ______________ , D′ : ______________

E′ : ______________

5.

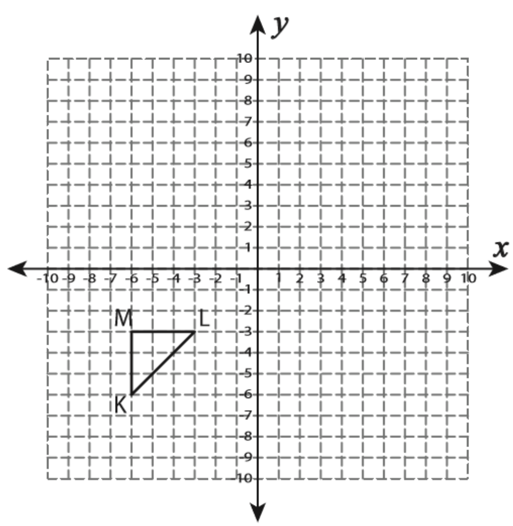
k = 1.2



Center = ____________________

6.

center : (−3, 0) , k = $\frac{4}{3}$



For 7 and 8, write the coordinate rule with the given center and scale factor.

7.

center : (6, 2) , k = 3.5

____

8.

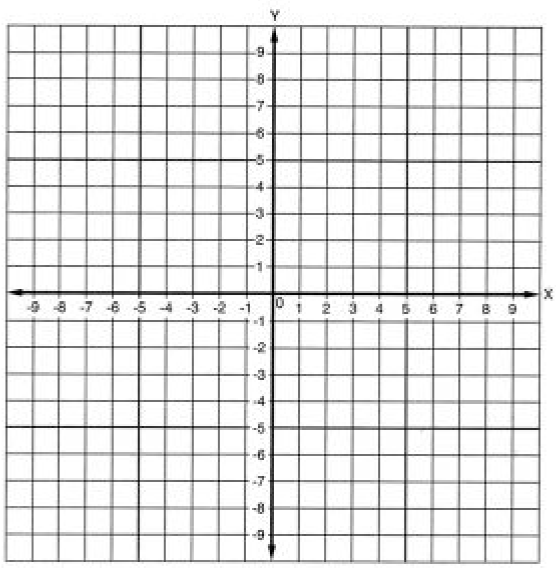
center : (5, −8) , k = 4

____

9. Find the center of the dilation.

Q(10, −3), R(2, −7), S(6, 1) are dilated to Q′(1, 3), R′(−1, 2), S′(0, 4), $k = \frac{1}{4}$

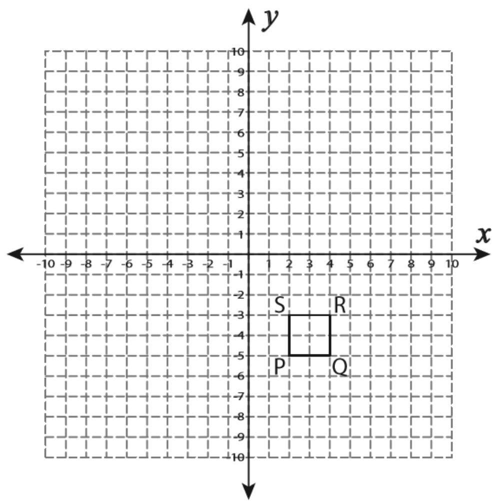
Center = ______________________



SELF CHECK

1.

center : (3, −5) , k = 6

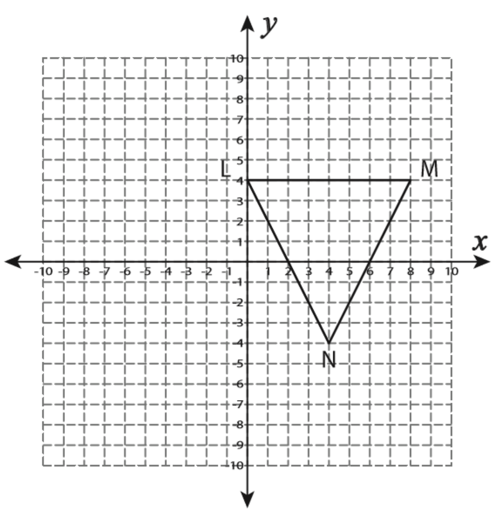


P′ : ______________ , Q′ : ______________

R′ : ______________ , S′ : ______________

2.

center : (4, 2) , k = 0.5

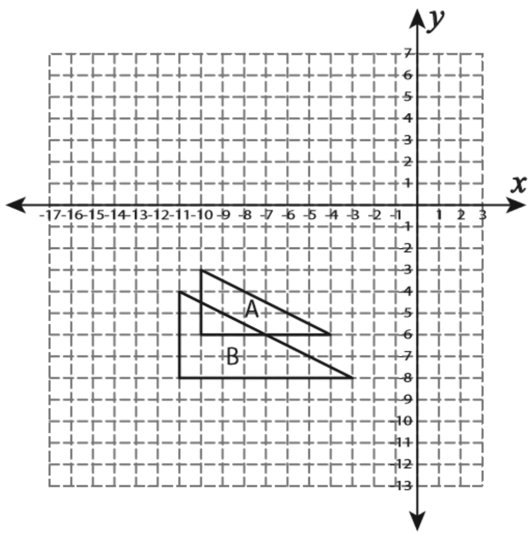


L′ : ______________ , M′ : ______________

N′ : ______________

3.

$k = \frac{3}{4}$



Center = ____________________

4.

LMN is dilated to L'M'N' with a scale factor of $\frac{3}{4}$ and the center of dilation is (–3, 0). The coordinates of the original image are given by L(–7, –8), M(–3, –4) and N(–11, 12). Find the dilated coordinates.

5.

L(–4, –3), M(5, –1), N(2, 1)

center : (–3, –1) , k = 4

L' : ____________ , M' : ____________

N' : ____________

6.

T(4, 1), U(4, –8), V(–5, –4), W(–4, –1)

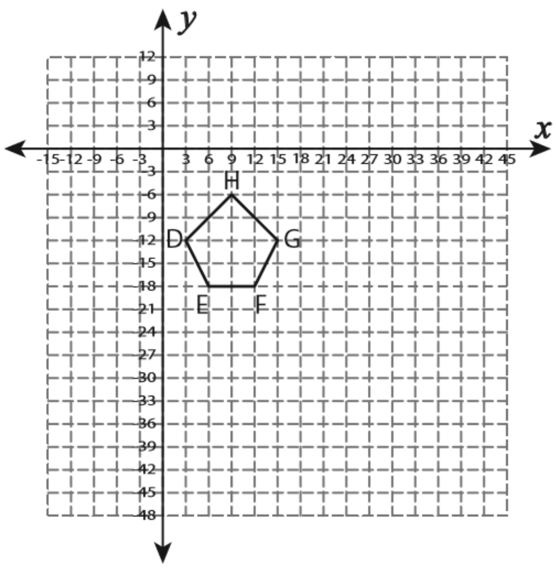
center : (–6, –4) , k = 0.9

T′ : ______________ , U′ : ______________

V′ : ______________ , W′ : ______________

7.

center : (9, –12) , k = 4



For 8 and 9, write the coordinate rule with the given center and scale factor.

8.

center : (–7, –3) , k = 2

____

9.

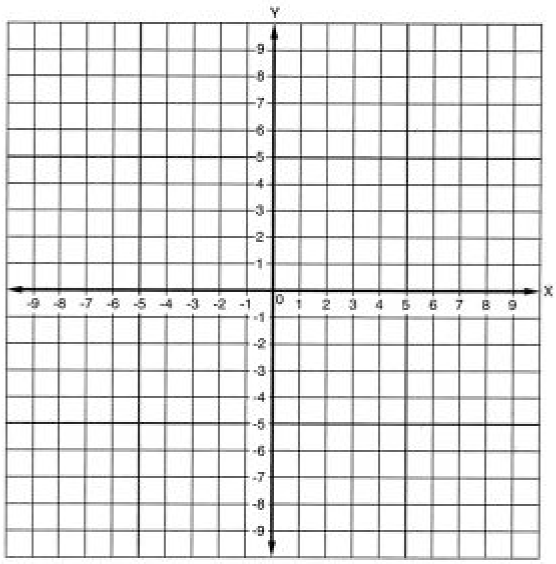
center : $\left(\frac{3}{2}, -\frac{9}{2}\right), k = \frac{1}{3}$

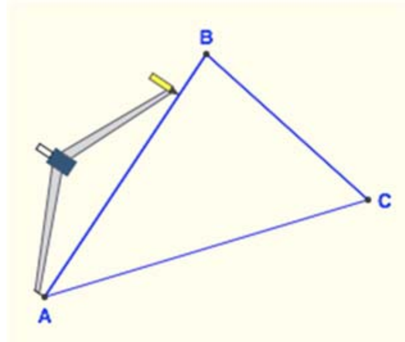
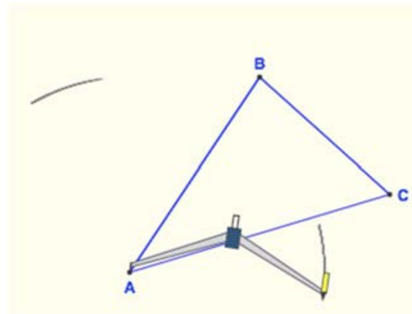
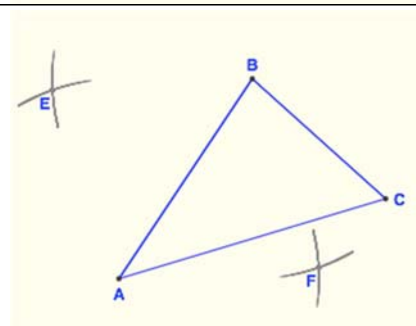
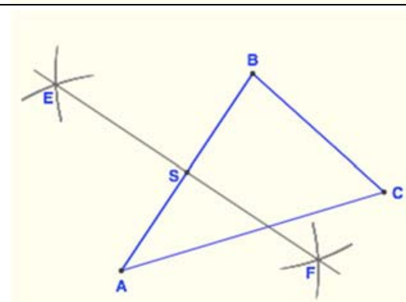
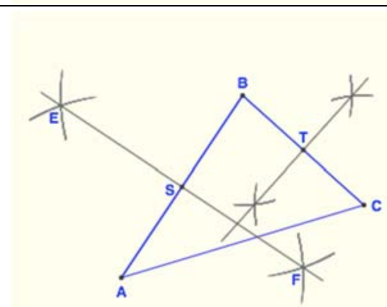
____

10. Find the center of the dilation.

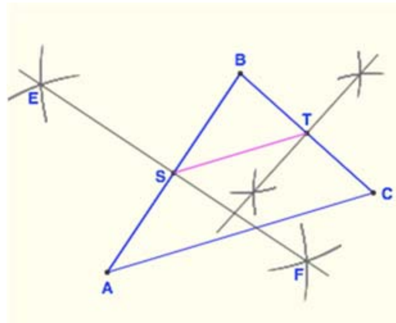
A(−3, 5), B(1, 4), C(0, −1), D(−2, −2) are dilated to A′(−31, 12), B′(1, 4), C′(−7, −36), D′(−23, −44), k = 8

Center = ______________________



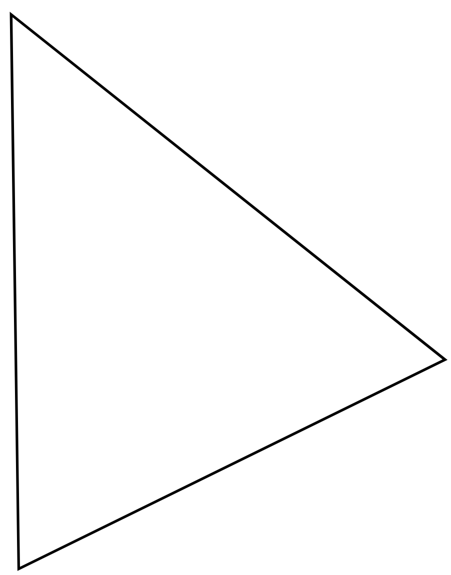
Construct the Midsegment of a Triangle	
1. Set the compass to medium width.	
2. From A, scribe an arc on each side of AB.	
3. From B, scribe an arc on each side of AB creating points E and F.	
4. Draw the line EF, creating the midpoint S of AB.	
5. Repeat for the line BC, creating its midpoint T.	

6. Draw the line ST.
   The segments ST is a mid-segment of the triangle ABC.



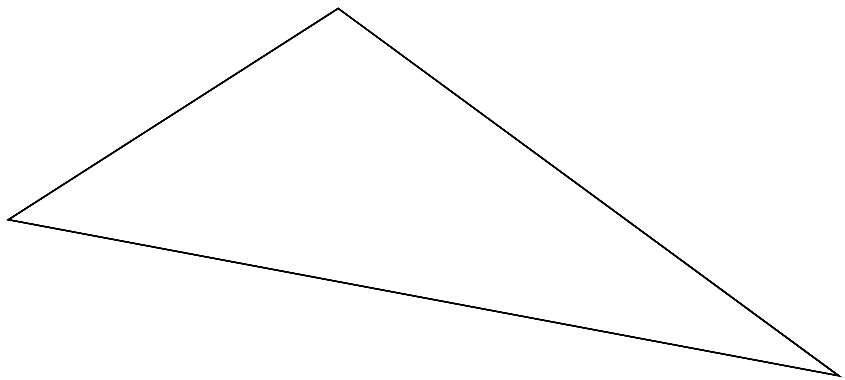
Example!

Construct the midsegment of a triangle



SELF CHECK

Construct the midsegment of a triangle.

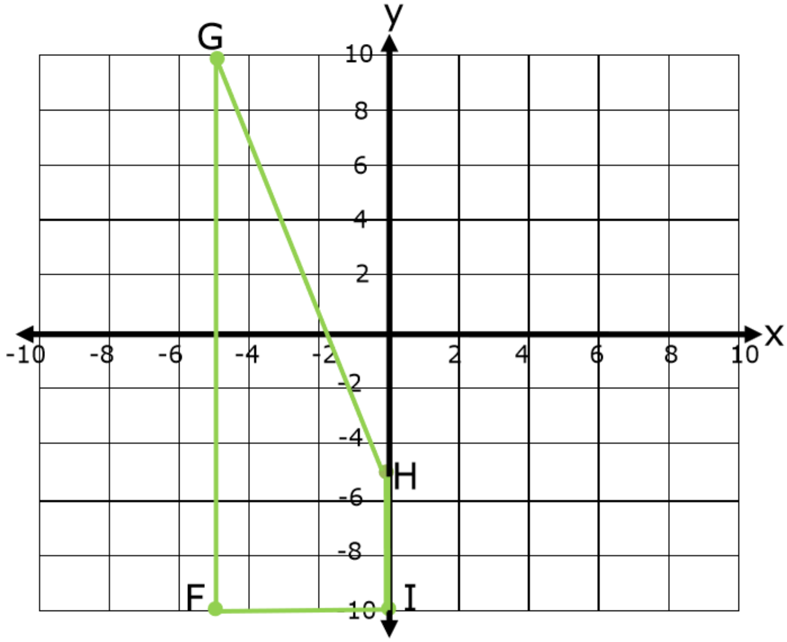


Questions To Ponder

Why does where the center of the dilation is matter?

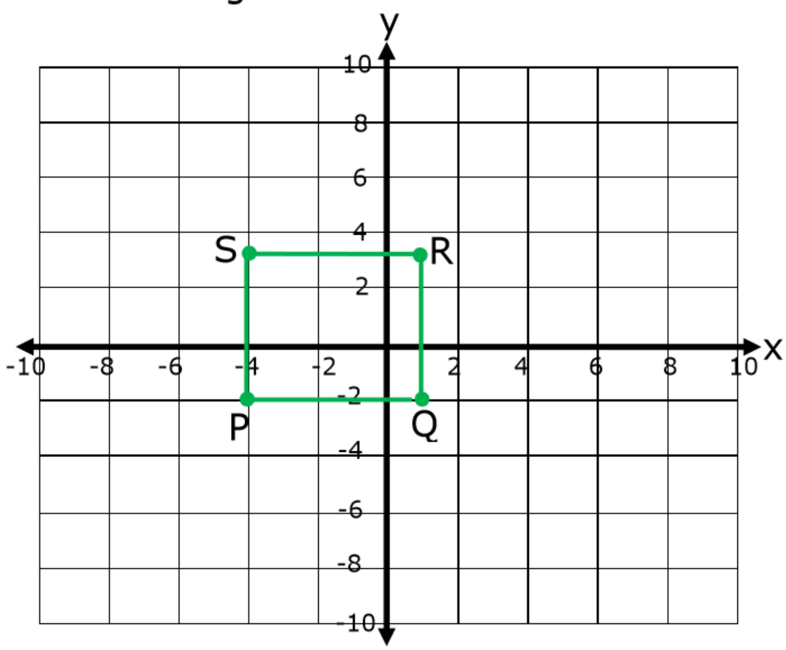
1.

5. Graph the image of quadrilateral FGHI after a dilation with a scale factor of 1/5, centered at the origin.

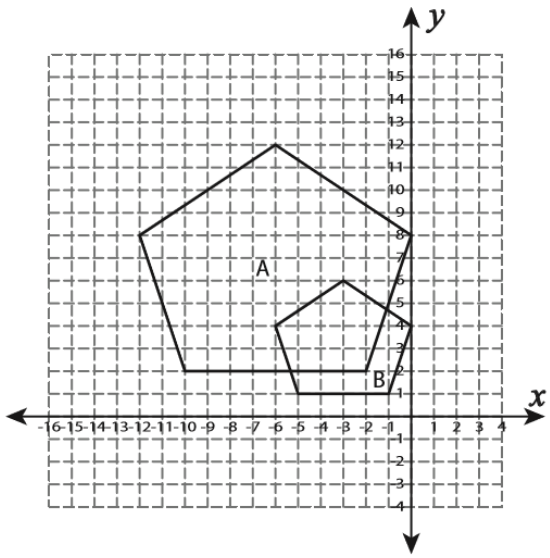


2.

6. Graph the image of rectangle PQRS after a dilation with a scale factor of 2, centered at the origin.



3.



Scale factor = ______________

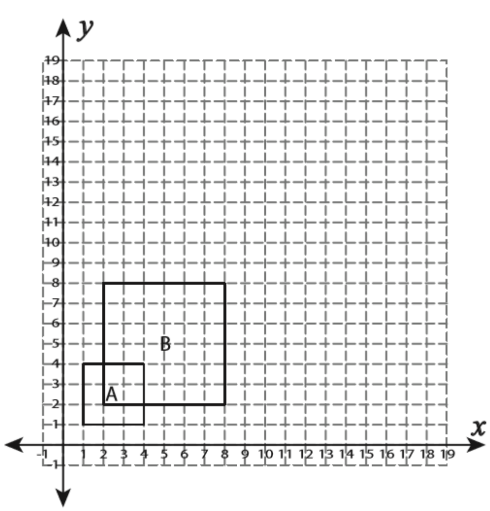
For 4 and 5 state whether the scale factor is a reduction or an enlargement.

4.

Scale factor = 3

____

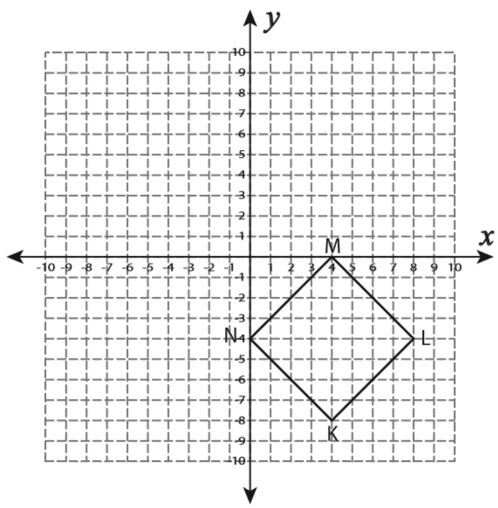
5.



____

6. Find the dilated coordinates with the given scale factor. Draw the dilated image.

Scale factor = $\frac{1}{4}$

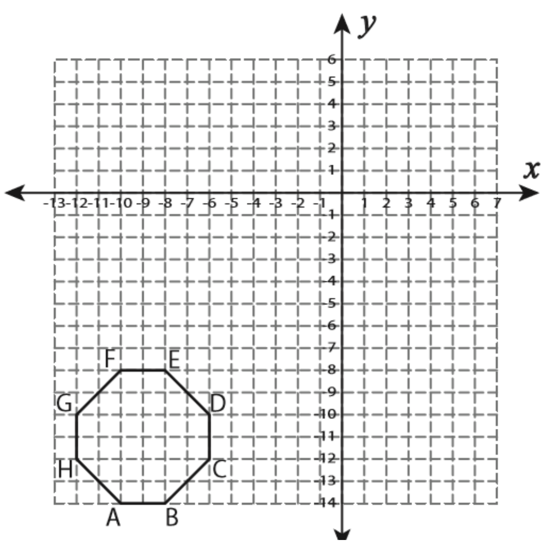


K′ : ______________ , L′ : ______________

M′ : ______________ , N′ : ______________

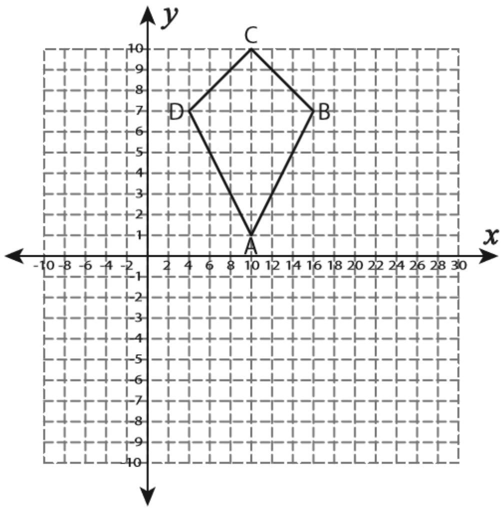
7. Draw the dilated image with the given scale factor.

Scale factor = 0.5



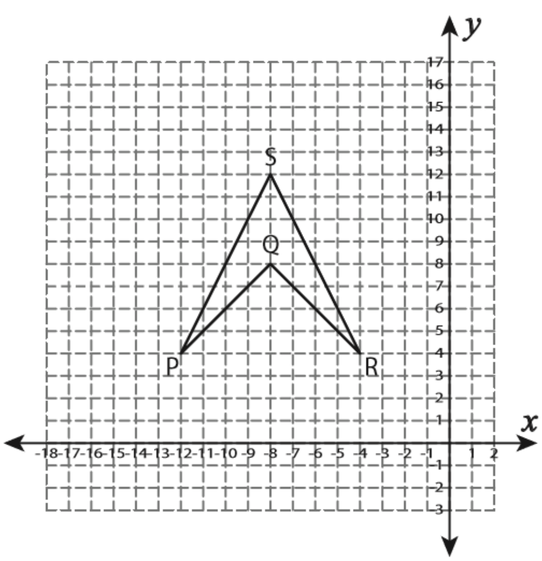
8. Draw the dilated image with the following center and scale factor.

center : (10, 4) , $k = \frac{2}{3}$



9. Draw the dilated image with the following center and scale factor.

center : (–4, 8) , k = 1.25



For 10 and 11, find the new coordinates given the center and scale factor.

10.

W(−6, 2), X(−1, 3), Y(1, 7), Z(−4, 5)

center : (2, 5) , k = 1.5

W′ : ______________ , X′ : ______________

Y′ : ______________ , Z′ : ______________

11.

U(5, 5), V(10, 0), W(0, 10), X(−1, −10)

center : (0, −5) , $k = \frac{9}{5}$

U′ : ______________ , V′ : ______________

W′ : ______________ , X′ : ______________

For 12 and 13, write the coordinate rule with the given center and scale factor.

12.

center : (−2, −1) , $k = \frac{1}{6}$

____

13.

center : (−7, −3) , k = 2

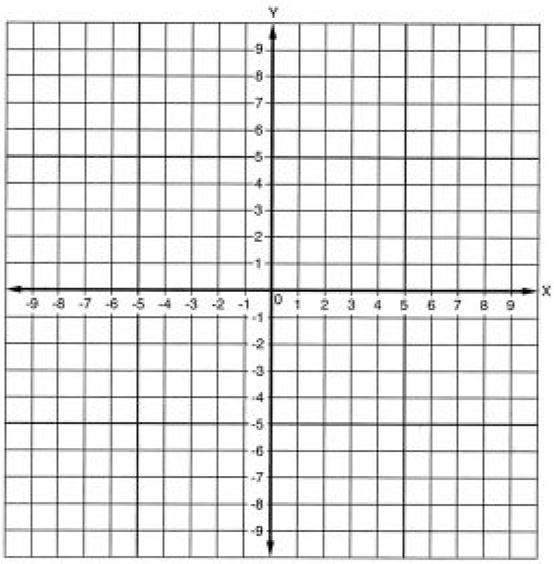
____

For 14 and 15, find the center of the dilation. Graph the pre-image and image.

14.

A(−3, 5), B(1, 4), C(0, −1), D(−2, −2) are dilated to A′(−31, 12), B′(1, 4), C′(−7, −36), D′(−23, −44), k = 8

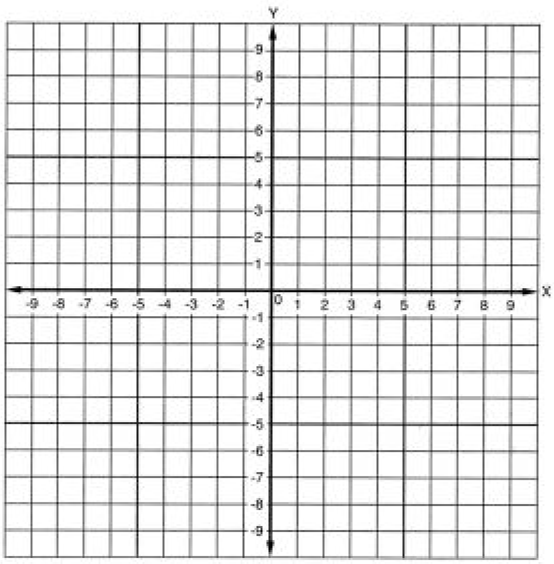
Center = ____________________



15.

E(3, –6), F(9, –6), G(6, –3) are dilated to E′(1, –8), F′(11, –8), G′(6, –3), $k = \frac{5}{3}$

Center = ______________________



For 16 and 17, work backwards to find the original coordinates.

16.

U′(2, –10), V′(–4, –8), W′(5, –3), X′(0, 0)

center : (0, –10) , k = 0.2

U : ______________ , V : ______________

W : ______________ , X : ______________

17.

X′(−11, 1), Y′(−3, −7), Z′(−7, −3)

center : (−7, −7) , k = 4

X : ______________ , Y : ______________

Z : ______________

Geometry Garden

The culminating task is based on the properties of triangles and quadrilaterals. You will create a map or illustration that encompasses the new properties that we have learned. Your map or illustration must include the following items:

- At least one example of a triangle inequality
- At least one set of congruent triangles
- At least one specific type of quadrilateral

For your map or illustration you will submit your visual AND an explanation for the map or illustration that includes an explanation/justification for why the geometric shapes or properties are useful and how it is related to the context of your map or illustration. Let the Culminating Task Rubric guide your work. If you need additional guidance, refer to the Culminating Task example.

This task may be completed using computer software or by hand on paper. As always, please ask any questions that may come up along the way, and do your best.

Real Mathematics: Application

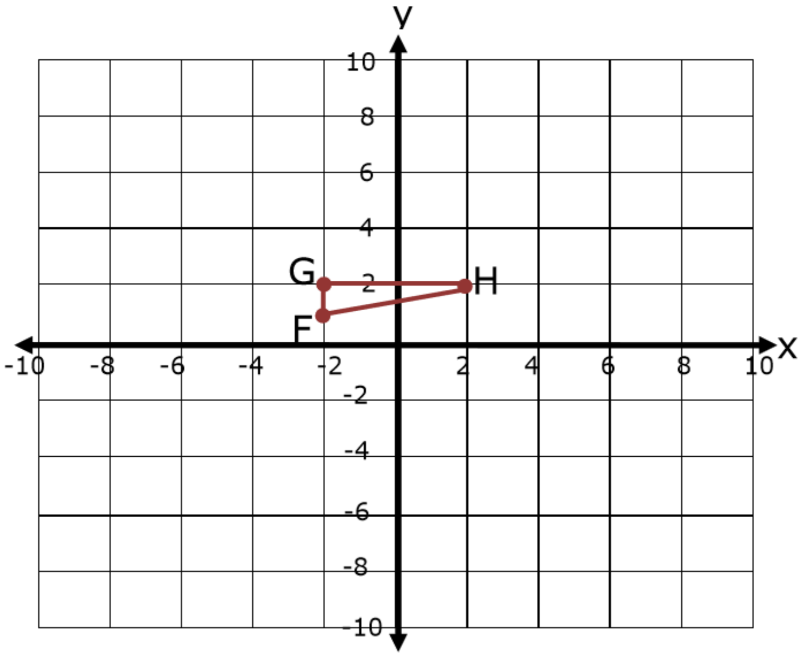
Map Culminating Task Rubric					
	5	**4**	**3**	**2**	**1**
Congruent Triangles	Congruent triangles are present in the map with an explanation for why congruent triangles are useful in the context of the map or illustration.	Congruent triangles are present with an explanation for why the triangles are useful only loosely tied to the context of the map or illustration.	Triangles are present with no proof that they are congruent although an explanation for their use that is relevant to the context is present.	Triangles are present with no proof that they are congruent with an explanation for their use only loosely tied to the context.	Triangles are present with no proof that they are congruent with no explanation for their use.
Triangle Inequalities	At least one example of a triangle inequality is present in the map or illustration with an explanation for the inequality's usefulness in the context of the map or illustration.	A triangle inequality is present with an explanation for why the inequality is useful only loosely tied to the context of the map or illustration.	A triangle inequality is present but with no proof of it with an explanation for the inequality's usefulness in the context of the map or illustration.	A triangle inequality is present but with no proof of it with an explanation that is only loosely tied to the context of the map or illustration.	A triangle inequality is present but with no proof of it with no explanation of its use.
Quadrilateral	At least one example of a specific quadrilateral is present in the map or illustration with an explanation for its usefulness in the context of the map or illustration.	A specific quadrilateral is present with an explanation for why the quadrilateral is useful only loosely tied to the context of the map or illustration.	A specific quadrilateral is present but with no specifics sited to determine the type of quadrilateral. The explanation for its usefulness is in the context of the map or illustration.	A specific quadrilateral is present but with no specifics sited to determine the type of quadrilateral. The explanation for its usefulness is only loosely tied to the context.	A specific quadrilateral is present but with no specifics sited to determine the type of quadrilateral. There is no explanation for the quadrilateral.
Presentation	The map or illustration has a cohesive theme, is neatly done and makes use of the various geometric shapes and properties we have learned.	The map or illustration has a cohesive theme, is neatly done and makes use of most of the various geometric shapes and properties we have learned.	The map or illustration has a cohesive theme, is neatly done and makes use of the some of the geometric shapes and properties we have learned.	The map or illustration has a cohesive theme, is not neatly done and makes use of only some of the geometric shapes and properties we have learned.	The map or illustration has no cohesive theme, is not neatly done and makes use of only some of the geometric shapes and properties we have learned.

Working with Dilations

Desmos Activity

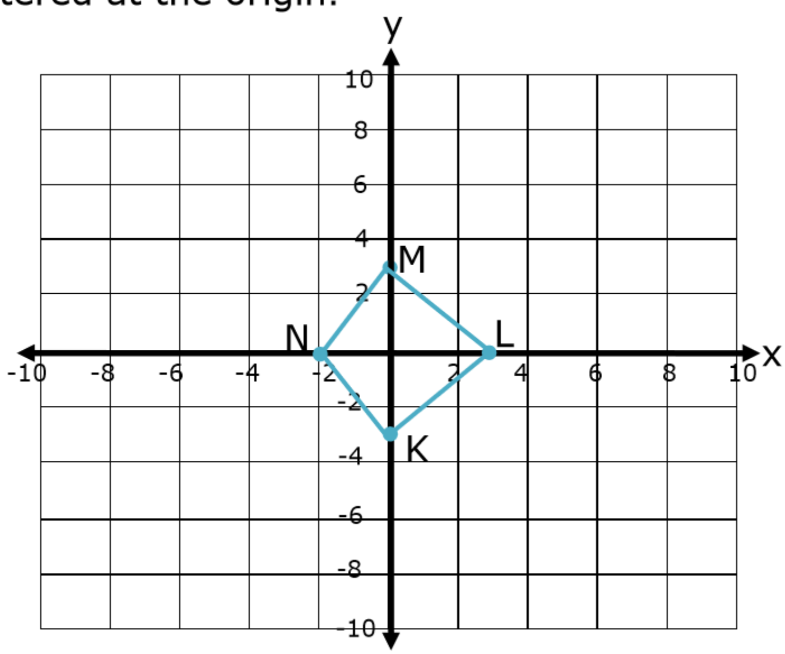
1.

Graph the image of triangle FGH after a dilation with a scale factor of 5, centered at the origin.

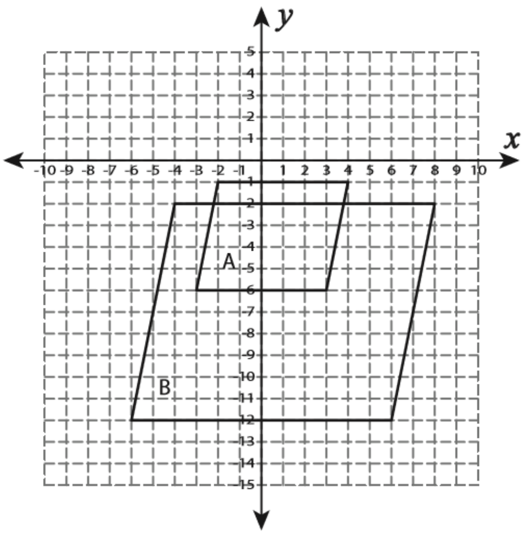


2.

Graph the image of quadrilateral KLMN after a dilation with a scale factor of 3, centered at the origin.



3.



Scale factor = ____________

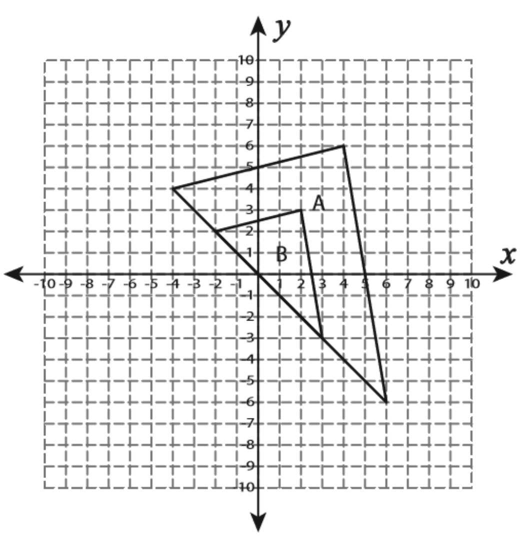
For 4 and 5 state whether the scale factor is a reduction or an enlargement.

4.

Scale factor = 0.2

____

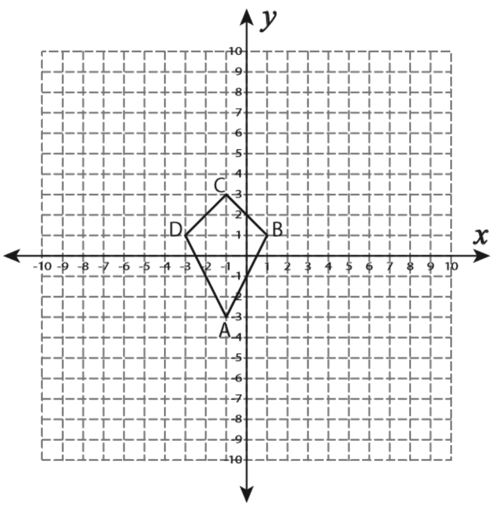
5.



____

6. Find the dilated coordinates with the given scale factor. Draw the dilated image.

Scale factor = 3

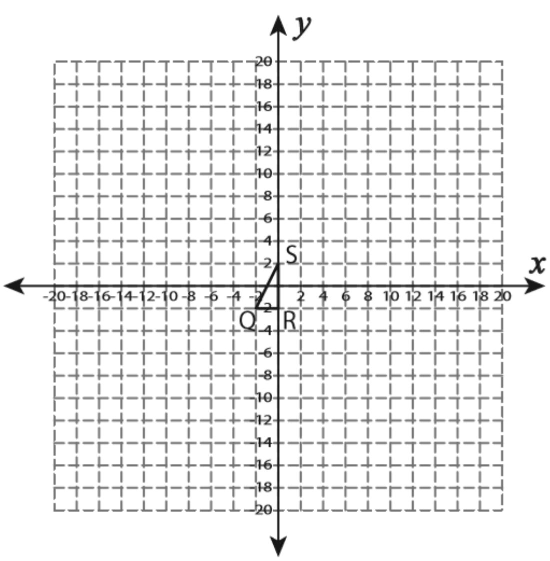


A′ : ______________ , B′ : ______________

C′ : ______________ , D′ : ______________

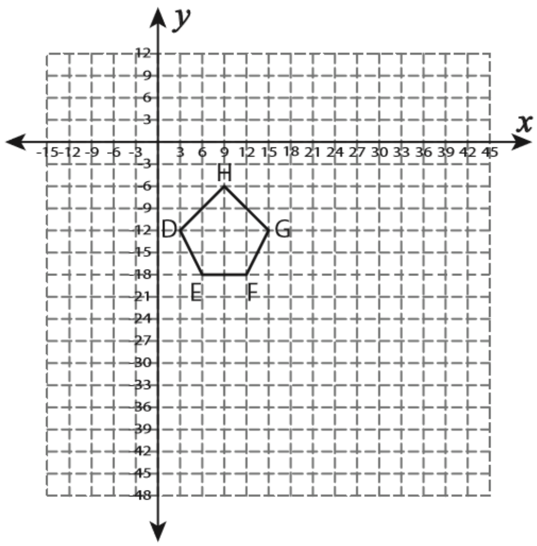
7. Draw the dilated image with the given scale factor.

Scale factor = 9



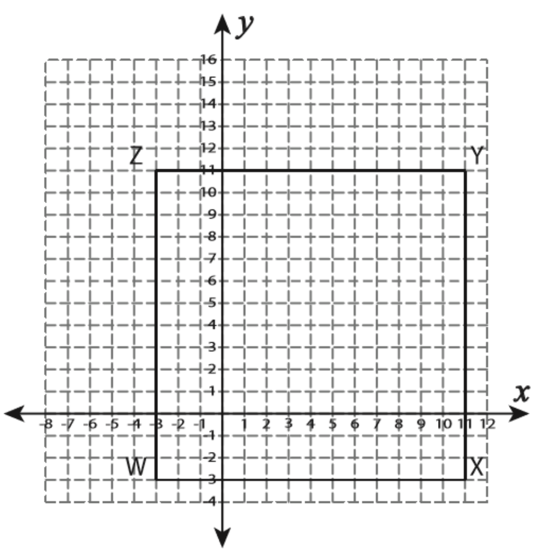
8. Draw the dilated image with the following center and scale factor.

center : (9, −12) , k = 4



9. Draw the dilated image with the following center and scale factor.

center : (−3, −3) , k = $\frac{3}{7}$



For 10 and 11, find the new coordinates given the center and scale factor.

10.

T(4, 1), U(4, –8), V(–5, –4), W(–4, –1)

center : (–6, –4) , k = 0.9

T′ : ______________ , U′ : ______________

V′ : ______________ , W′ : ______________

11.

A(–1, –9), B(–4, –6), C(2, 9), D(5, –3)

center : (8, –6) , $k = \frac{1}{3}$

A′ : ______________ , B′ : ______________

C′ : ______________ , D′ : ______________

For 12 and 13, write the coordinate rule with the given center and scale factor.

12.

center : (1, 0) , k = 9

____

13.

center : (–5, 10) , $k = \frac{3}{5}$

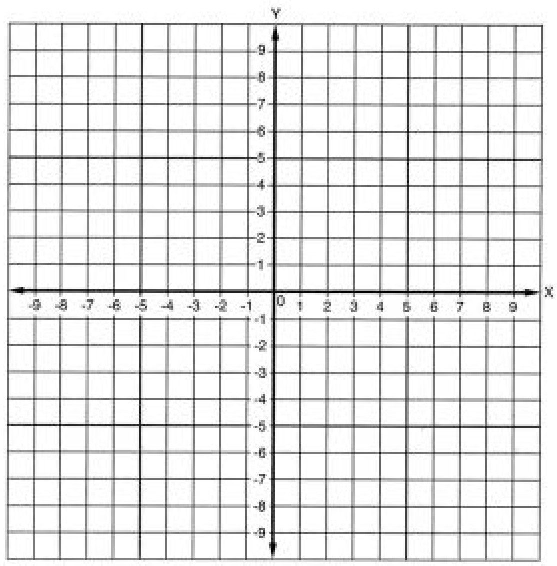
____

For 14 and 15, find the center of the dilation. Graph the pre-image and image.

14.

S(1, 1), T(−5, 3), U(−7, 1), V(−5, −3) are dilated to S′(−2, −2), T′(−5, −1), U′(−6, −2), V′(−5, −4), k = 0.5

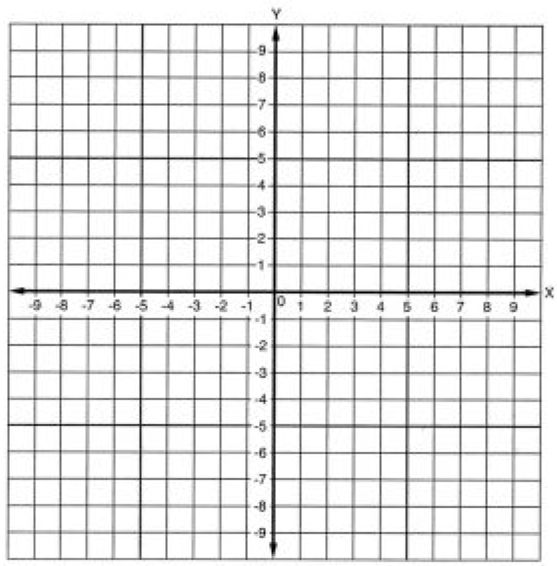
Center = ____________________



15.

K(9, 6), L(12, 5), M(7, –2), N(5, 3) are dilated to K′(13, 24), L′(22, 21), M′(7, 0), N′(1, 15), k = 3

Center = ____________________



For 16 and 17, work backwards to find the original coordinates.

16.

R′(3, 8), S′(9, 11), T′(12, 14), U′(15, 17)

center : (–12, 8) , $k = \frac{3}{4}$

R : ______________ , S : ______________

T : ______________ , U : ______________

17.

K'(5, 3), L'(11, 9), M'(8, –6), N'(14, –9)

center : (2, 6) , k = 1.5

K : ______________ , L : ______________

M : ______________ , N : ______________

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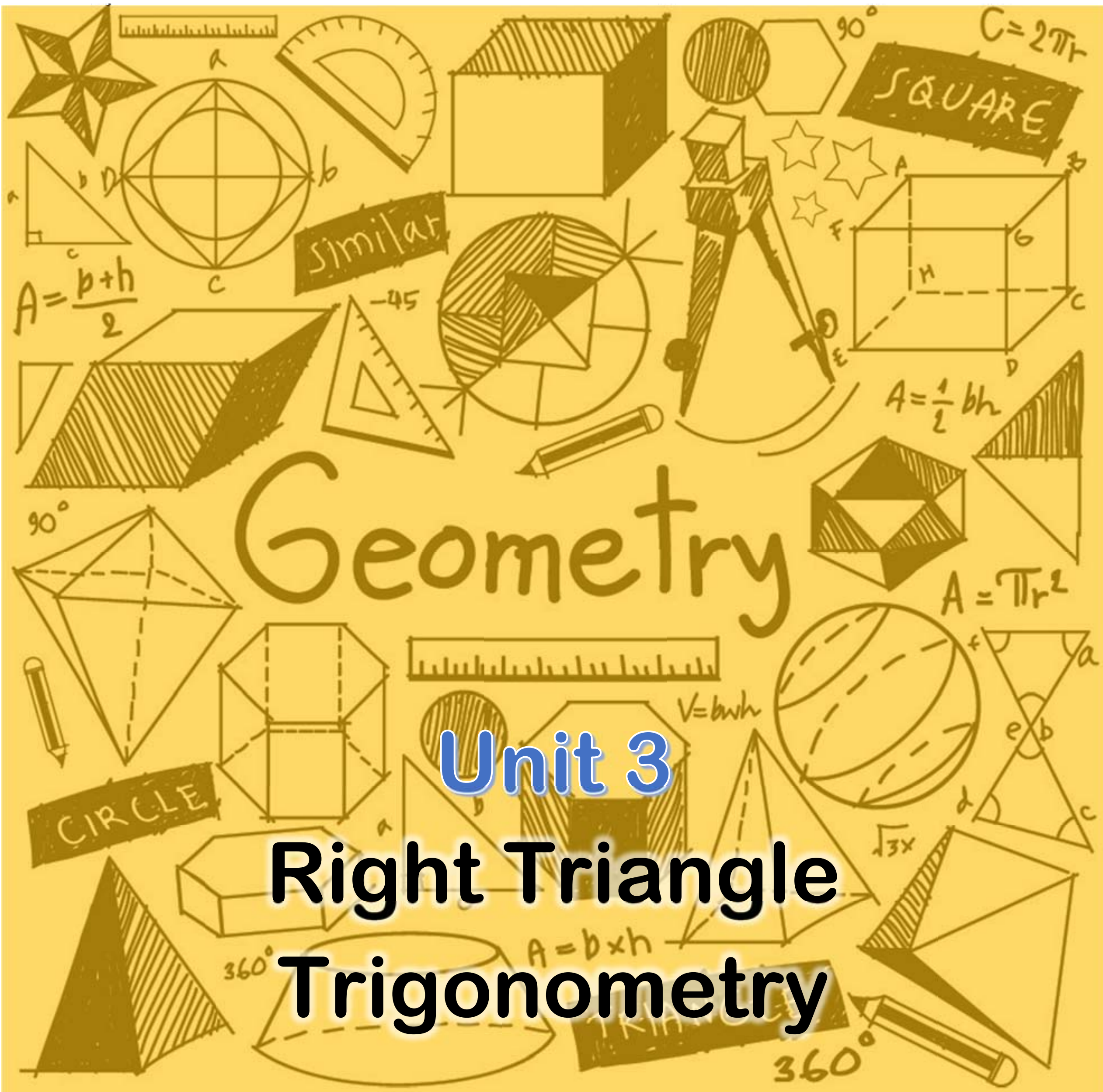
MATHEMATICS DEPARTMENT



Unit 3

Right Triangle Trigonometry

HOUSTON COUNTY
BOARD OF EDUCATION
HIGH-ACHIEVING TEACHERS

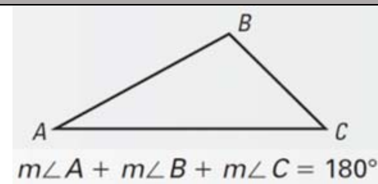
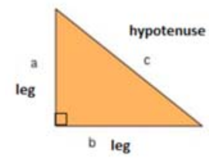
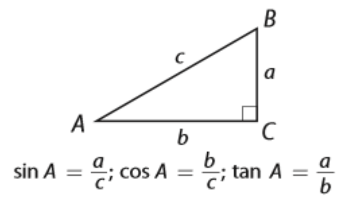
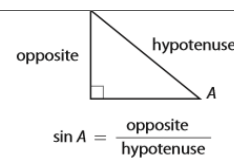
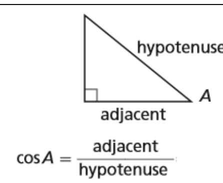
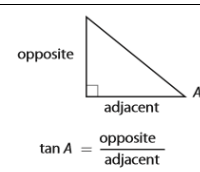
MATHEMATICS
DEPARTMENT

Introduction to Right Triangle Trigonometry – Vocabulary			
Term	**Definition**	**Notation**	**Diagram/Visual**
Triangle Sum Theorem			
Pythagorean Theorem			
Trigonometric Functions			
Ratio			
Sine			
Cosine			
Tangent			

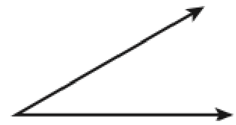
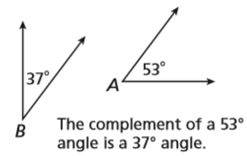
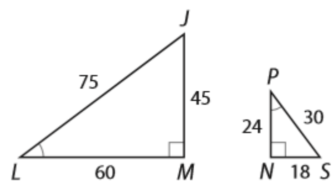
Relationships in Right Triangles – Vocabulary			
Term	**Definition**	**Notation**	**Diagram/Visual**
Acute Angle	______________ ______________ ______________		
Complementary Angles	______________ ______________ ______________		
Similar Figures	______________ ______________ ______________		

sing Right Triangle Trigonometry to Solve for Unknown Sides and Angles – Vocabulary			
Term	**Definition**	**Notation**	**Diagram/Visual**
Inverse Trigonometric Functions	______________ ______________ ______________		
30-60-90 Right Triangle	______________ ______________ ______________		
45-45-90 Right Triangle	______________ ______________ ______________		

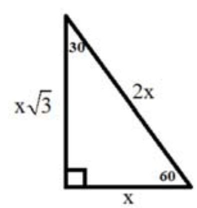
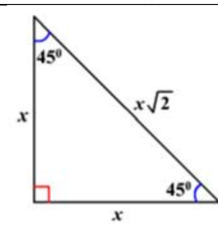
Angles of Elevation and Depression – Vocabulary			
Term	**Definition**	**Notation**	**Diagram/Visual**
Angle of Elevation	____________ ____________ ____________		
Angle of Depression	____________ ____________ ____________		
Alternate Interior Angles Theorem	____________ ____________ ____________		

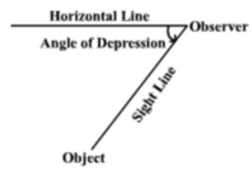
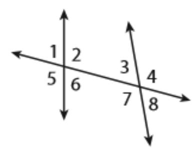
Introduction to Right Triangle Trigonometry – Vocabulary			
Term	**Definition**	**Notation**	**Diagram/Visual**
Triangle Sum Theorem	The Triangle Sum Theorem states that if you add all three interior angles, those are the angles inside the triangle, they would add up to 180 degrees.		
Pythagorean Theorem	A right triangle consists of two legs and a hypotenuse. The two legs meet at a 90° angle and the hypotenuse is the longest side of the right triangle and is the side opposite the right angle. The sum of the legs squared is equal to the hypotenuse squared.	$a^2 + b^2 = c^2$	
Trigonometric Functions	A ratio of two sides of a right triangle. ____ ____		
Sine	In a right triangle, the ratio of the length of the leg opposite ∠A to the length of the hypotenuse.		
Cosine	In a right triangle, the cosine of angle A is the ratio of the length of the leg adjacent to angle A to the length of the hypotenuse.		
Tangent	In a right triangle, the ratio of the length of the leg opposite ∠A to the length of the leg adjacent to ∠A.		

Relationships in Right Triangles – Vocabulary

Term	Definition	Notation	Diagram/Visual
Acute Angle	An angle that measures greater than 0° and less than 90°.		
Complementary Angles	Two angles whose measures have a sum of 90°.		
Similar Figures	Two figures whose corresponding angles are congruent and whose corresponding side lengths are proportional.		

Using Right Triangle Trigonometry to Solve for Unknown Sides and Angles – Vocabulary

Term	Definition	Notation	Diagram/Visual
Inverse Trigonometric Functions	The measure of an angle whose trigonometric ratio is known.		If $\cos A = x$, then $\cos^{-1} x = m\angle A$.
30-60-90 Right Triangle	A 30-60-90 triangle is special because of the relationship of its sides. The hypotenuse is equal to twice the length of the shorter leg, which is the side across from the 30 degree angle. The longer leg, which is across from the 60 degree angle, is equal to multiplying the shorter leg by the square root of 3.		
45-45-90 Right Triangle	In every 45-45-90 triangle, the lengths of the two legs are always equal, and the ratio of the length of the hypotenuse to the length of a leg is always square root 2 to 1.		

Angles of Elevation and Depression – Vocabulary			
Term	**Definition**	**Notation**	**Diagram/Visual**
Angle of Elevation	The term angle of elevation denotes the angle from the horizontal upward to an object. An observer's line of sight would be above the horizontal		
Angle of Depression	The term angle of depression denotes the angle from the horizontal downward to an object. An observer's line of sight would be below the horizontal.		
Alternate Interior Angles Theorem	For two lines intersected by a transversal, a pair of nonadjacent angles that lie on opposite sides of the transversal and between the other two lines.	 ∠3 and ∠6 are alternate interior angles.	

Introduction to Right Triangle Trigonometry – Vocabulary			
Term	**Definition**	**Notation**	**Diagram/Visual**
Triangle Sum Theorem			
Pythagorean Theorem			
Trigonometric Functions			
Ratio			
Sine			
Cosine			
Tangent			

Introduction to Right Triangle Trigonometry – Guided Notes

GUIDED NOTES

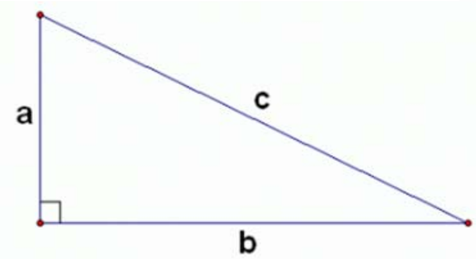
Things we already know about right triangles…

Triangle Sum Theorem

The sum of the three interior angles of a triangle is 180°.

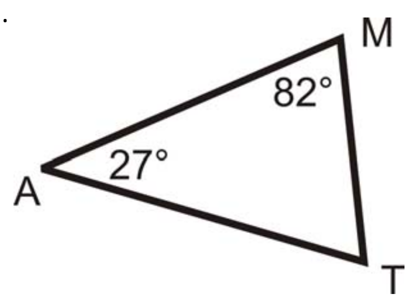
$m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Pythagorean Theorem



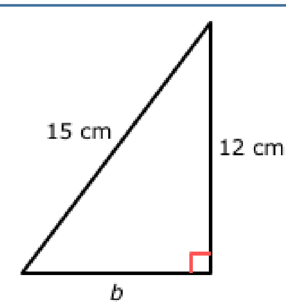
$a^2 + b^2 = c^2$

1.



$m\angle T =$ ______

2.



What is the length of the missing leg?

$b =$ ☐ centimeters

So…

- We know how to find a missing angle measure when we are given two ______________________________ by using ____.
- We know how to find a missing side length when we are given two_________________________________ by using ____.

What do we do when we are given 1 angle measure and 1 side length?

<u>Trigonometric Functions</u> are our way to connect side lengths and angle measures. The basic trigonometric functions are sine, cosine, and tangent.

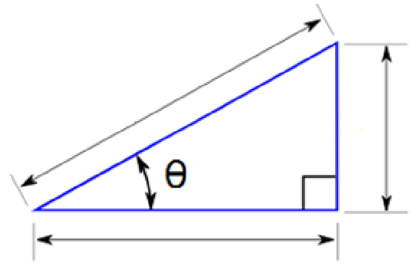
$$\sin\theta = \frac{opp}{hyp} \quad \cos\theta = \frac{adj}{hyp} \quad \tan\theta = \frac{opp}{adj}$$

Right Triangle Trigonometry Ratios

$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

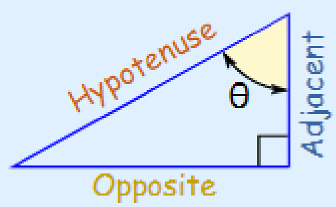
$\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

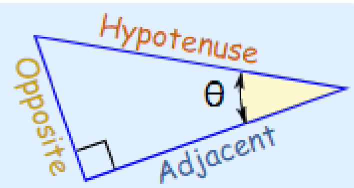


Label each side of the right triangle.

Adjacent is always next to the angle

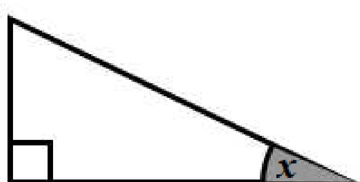
And **Opposite** is opposite the angle

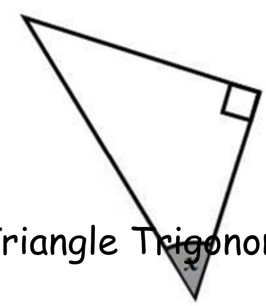




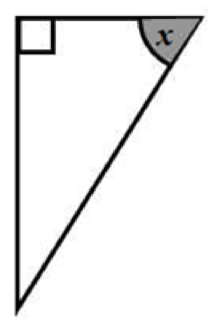
Example! Before we begin finding specific trig ratios, we need to identify which sides are the "opposite", "adjacent", and "hypotenuse" of an angle.

Label the sides of each right triangle as "opposite", "adjacent", and "hypotenuse" of angle *x*. You can abbreviate "opp", "adj", and "hyp".

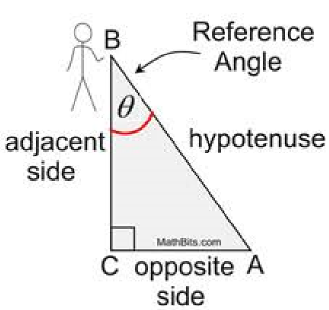


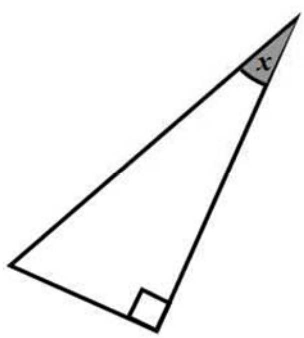


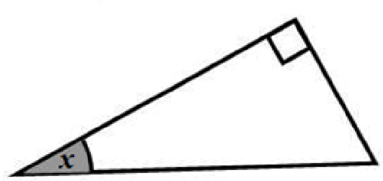
Right Triangle Trigonometry Ratios

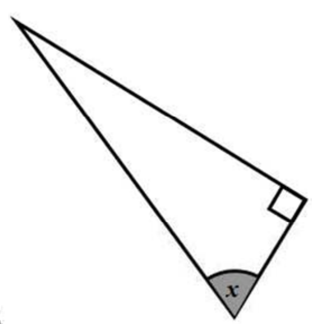


SELF CHECK **Label the sides of each right triangle as "opposite", "adjacent", and "hypotenuse" of angle *x*. You can abbreviate "opp", "adj", and "hyp".**





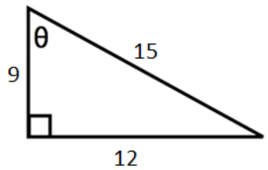




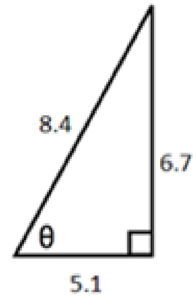
Example! Now that we can successfully identify the "opposite", "adjacent", and "hypotenuse" of an angle, let's find the sine, cosine, and tangent of specific angles.

Find the indicated trig ratio for each right triangle below.

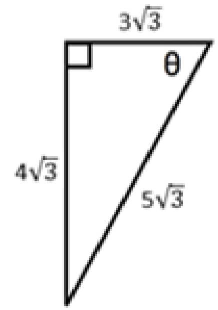
1. **sin θ =**



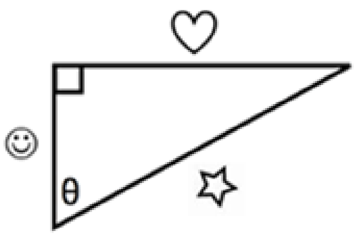
2. cos θ =



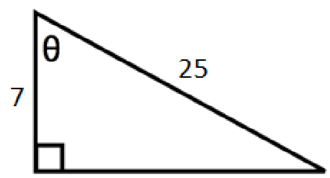
3. tan θ =



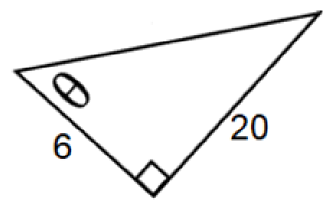
4. sin θ =



5. _______θ =



6. _______θ =



Let's see how well you can take what you have learned so far about right triangle trigonometry to answer different types of questions.

Given $\triangle ABC$ with m$\angle C$ = 90° and cos A = 8/17 answer the following questions.

1. Draw and label $\triangle ABC$ described above.

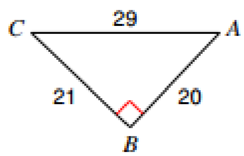
2. Find the following: sin B=____________ ; tan A= ____________ ; tan B= ____________

3. What do you notice about cos A and sin B?

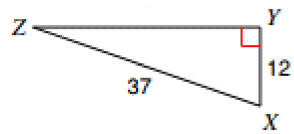
4. What do you notice about tan A and tan B?

SELF CHECK **Answer the following.**

1. tan A =



2. __________ Z =



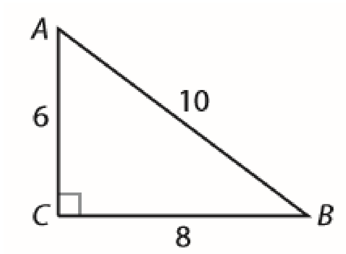
3. Let ΔABC be a right triangle where $m\angle C = 90°$. If tan A = 4/3, sin B = _____ (Hint: Draw and label a triangle.)

In ΔABC with $m\angle C = 90°$, what is true about angles A and B?

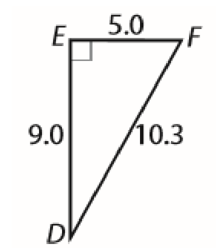
Introduction to Right Triangle Trigonometry - Practice

In each right triangle, find the specified trigonometric ratio. Simplify if possible.

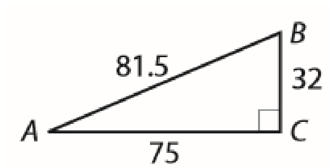
1. sin A =



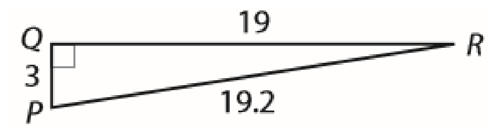
2. cos F =



3. tan B =



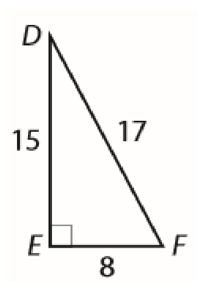
4. sin P =



For problems 5 & 6, give both trigonometric ratios.

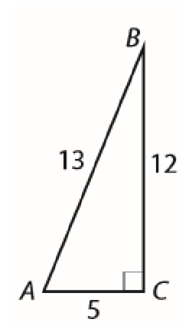
5. sin F =

cos D =



6. sin A =

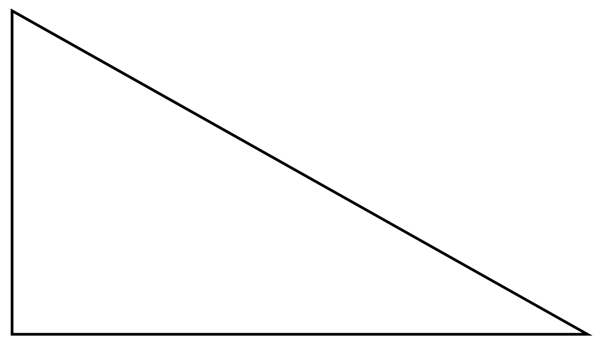
cos B =



What do you notice about the sin F and cos D in problem #5? sin A and cos B in problem #6?

Introduction to Right Triangle Trigonometry – Application

Given ΔABC with $m\angle B = 90°$ and $\sin 23° = 5/13$, find each unknown angle measure and each unknown side length. Label the triangle below showing your answers. Show all work.

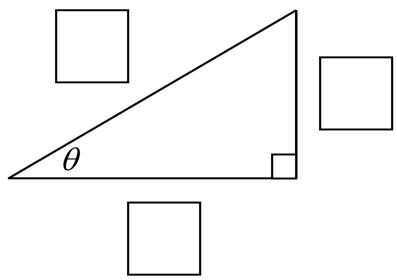


Introduction to Right Triangle Trigonometry – Homework

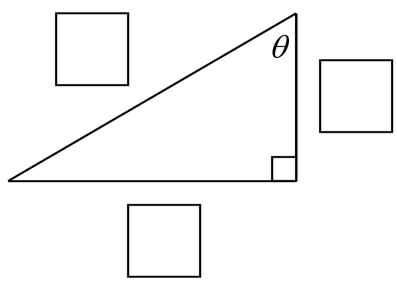
1. Label the following names of the sides as opposite, hypotenuse, or adjacent in the correct positions for each triangle:

O = Opposite H = Hypotenuse A = Adjacent

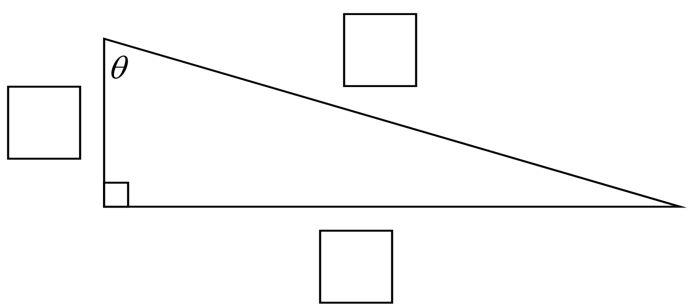
a.



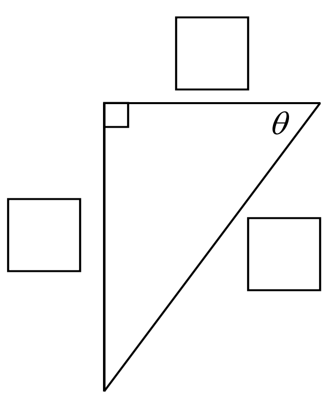
b.



c.

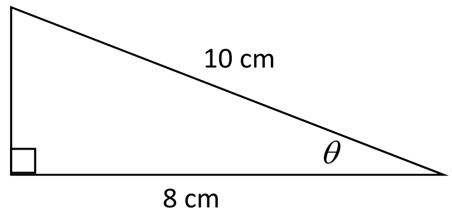


d.



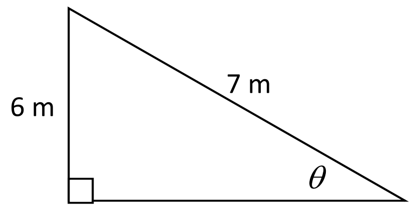
2. Choose the correct trigonometric function and the appropriate trig ratio for each of the following right triangles:

a.



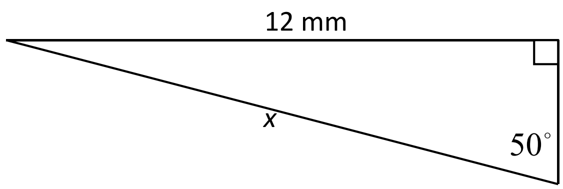
$\square\ \theta = \dfrac{\square}{\square}$

b.



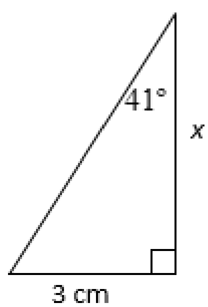
$\square\ \theta = \dfrac{\square}{\square}$

c.



$\square\ 50^\circ = \dfrac{\square}{\square}$

d.



$\square\ 41^\circ = \dfrac{\square}{\square}$

3. When a ladder is rested against a tree (height = x), the foot of the ladder is 2 m from the base of the tree and forms an angle of 47° with the ground.

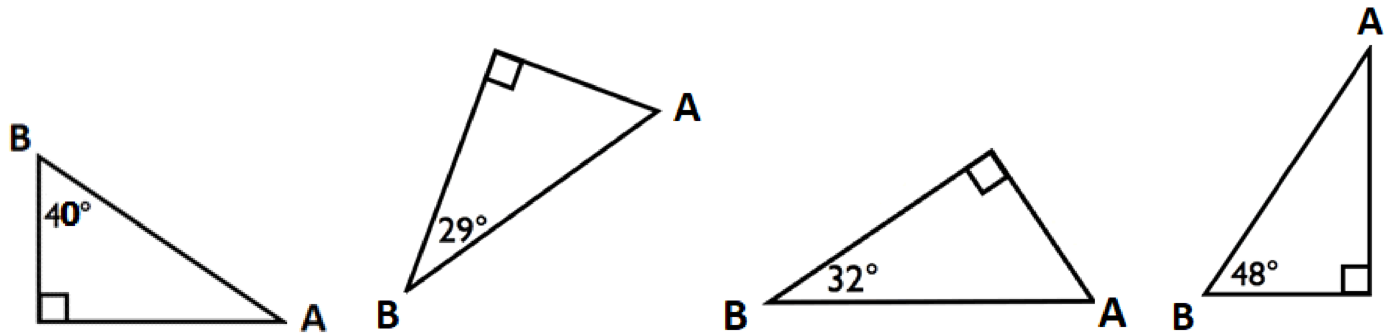
a. Draw a right triangle representing this scenario.

b. Set up the appropriate trig ratio that would determine the height of the tree.

Relationships in Right Triangles – Vocabulary			
Term	**Definition**	**Notation**	**Diagram/Visual**
Acute Angle	______________ ______________ ______________		
Complementary Angles	______________ ______________ ______________		
Similar Figures	______________ ______________ ______________		

Relationships in Right Triangles – Guided Notes

Find the unknown angle measure in each triangle below:

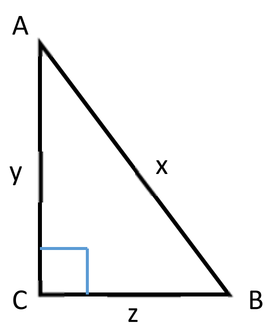


What is the relationship between ∠A and ∠B in each triangle? ____

Is this relationship true for the acute angles in any right triangle? ____

> **Know it! Note** The acute angles in a right triangle are ALWAYS ______________________.

~~~~~~



Identify the following for ΔABC to the left.

The side adjacent to ∠A is
The side adjacent to ∠B is
The side opposite ∠A is
The side opposite ∠B is

Write each of the following ratios for ΔABC above:

sin A =	cos A =	tan A =
sin B =	cos B =	tan B =

What do you notice about the sin A and cos B? ____

What do you notice about the sin B and cos A? ____

What do you notice about the tan A and tan B? ____
~~~~~~

Putting It All Together…

Know it! Note

Three important relationships:

1) The acute angles in a right triangle are ______________________________.

2) The sine of an angle is ________________ to the cosine of its complement.

3) The tangents of two complementary angles are _____________________.

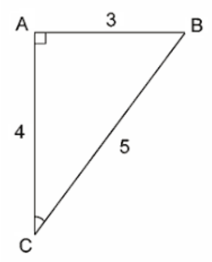
Example! **Use the relationships you have learned to find the sine, cosine, or tangent of complementary angles.**

1. sin 29° = cos _______
2. cos 47° = sin _______
3. If tan 37° = ¾ , then tan 53°= _______.
4. Given that sin 38° ≈ 0.616, write the cosine of a complementary angle.
5. Given that cos 73° ≈ 0.292, write the sine of a complementary angle.
6. Given that tan 71° = $\frac{35}{12}$, write the tangent of a complementary angle.

SELF CHECK **Use the relationships you have learned to find the sine, cosine, or tangent of complementary angles.**

1. Given that cos 60° = 0.5, write the sine of a complementary angle.
2. Given that sin 45° ≈ 0.707, write the cosine of a complementary angle.
3. If tan 30° = $\frac{\sqrt{3}}{3}$, what is the value of tan 60°?

Similarity and Right Triangles



1. ΔABC above is dilated by a scale factor of 2 to create ΔLMN. Sketch and label the dilated ΔLMN.

2. What is the relationship between ΔABC and Δ LMN? (*Think about what type of figures dilations create*.)

3. Using what you know about similarity, what is true about the measures of the triangles' corresponding angles and corresponding sides?

4. Fill in the blanks with the correct angles. $\angle A \cong$ ________ ; $\angle B \cong$ ________ ; $\angle C \cong$ ________

5. Write each of the following ratios for ΔABC.

sin B =	cos B =	tan B =
sin C =	cos C =	tan C =

6. Write each of the following ratios for ΔLMN.

sin M =	cos M =	tan M =
sin N =	cos N =	tan N =

What do you notice about sin B and sin M?____

What do you notice about cos B and cos M?____

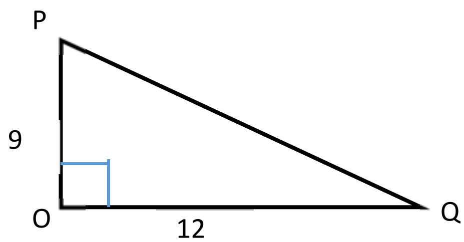
What do you notice about cos C and cos N?____

What do you notice about tan C and tan N?____

Putting It All Together…

Know it! Note: The sine, cosine, and tangent for the corresponding angles of similar right triangles are ______________________.

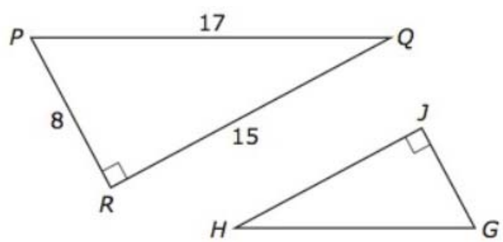
Example! **Use what you have learned about the similarity and right triangles to answer the following questions.**



1) Find the length of PQ.

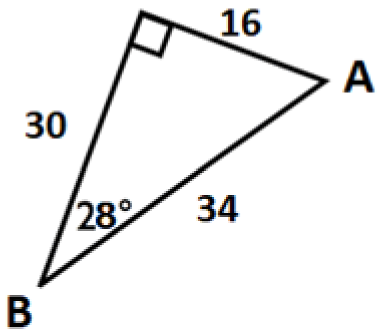
2) ΔOPQ is dilated by a scale factor of 3 to form ΔRST. What is the value of sin T?

SELF CHECK **Use what you have learned about the similarity and right triangles to answer the following questions.**



1. $\Delta PQR \sim \Delta GHJ$. Based on this information, what is tan *H*?

Relationships in Right Triangles – Practice



1. Write each of the following ratios for the triangle.

sin 28° =	sin 62° =
cos 28° =	cos 62° =
tan 28° =	tan 62° =

2. What do you notice about ∠ A and ∠ B? ____

3. What do you notice about the sin 28° and cos 62°? ____

4. What do you notice about the cos 28° and sin 62°? ____

5. What do you notice about the tan 28° and tan 62°? _______________________________________

6. If sin 50° = cos θ, what is the value of θ?

7. If $\tan 30° = \frac{1}{\sqrt{3}}$, what is the value of tan 60°?

~~~~~~

8. The sine of an angle is equal to the cosine of ____.

   a) the same angle b) its supplementary angle c) its complementary angle d) no other angle

9. The tangents of complementary angles are ____.

   a) equal b) reciprocals c) opposites d) not related

10. The sine of 52° would have the same value as the cosine of what angle measure? _________________

11. Given the following: sin(30°) = ½ , $\cos(30°) = \frac{\sqrt{3}}{2}$, and $\tan(30°) = \frac{1}{\sqrt{3}}$

    a) What angle is complementary to 30°? ________________________

    b) Determine the sine, cosine, and tangent of the complementary angle of 30°.
~~~~~~

12. Given that $tan(\alpha) = \frac{2}{3}$ and $tan(\beta) = \frac{3}{2}$, what can be said about angles α and β?

13. Given that α and β are complementary angles, if $\sin(\alpha) = \frac{x}{y}$, $\cos(\alpha) = \frac{z}{y}$, and $\tan(\alpha) = \frac{x}{z}$, find each of the following:

a) sin β = b) cos β = c) tan β =

14. Given that $sin(34°) = 0.559$ and $cos(\theta) = 0.559$, what is the value of θ? ____________

15. If $tan(20°) = \frac{9}{25}$, then tan (70°) would have what value? ________________

16. Given that α and β are complementary angles, if $\sin(\alpha) = \frac{1}{\sqrt{10}}$, $\cos(\alpha) = \frac{3}{\sqrt{10}}$, and $\tan(\alpha) = \frac{1}{3}$, find each of the following:

a) sin β = b) cos β = c) tan β =

Relationships in Right Triangles – Application

1. There are certain special angles where it is possible to give the exact value of sine and cosine. These are the angles that measure $0°$, $30°$, $45°$, $60°$, and $90°$; these angle measures are frequently seen.

   a. Learn the following sine and cosine values of the key angle measurements.

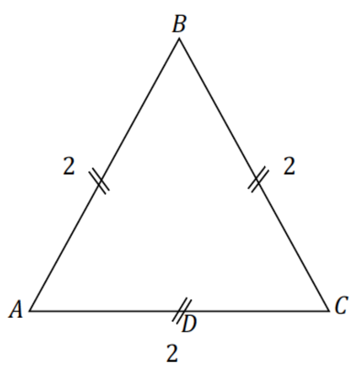
$\boldsymbol{\theta}$	$\mathbf{0°}$	$\mathbf{30°}$	$\mathbf{45°}$	$\mathbf{60°}$	$\mathbf{90°}$
Sine	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

We focus on an easy way to remember the entries in the table. What do you notice about the table values?

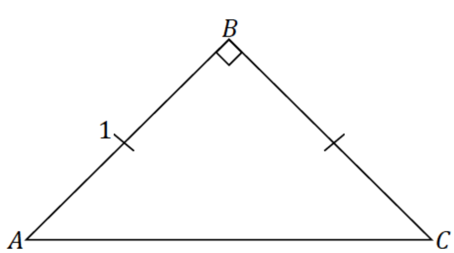
This is easily explained because the pairs $(0,90)$, $(30,60)$, and $(45,45)$ are the measures of complementary angles.
So, for instance, $\sin 30 = \cos 60$.

The sequence $0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$ may be easier to remember as the sequence $\frac{\sqrt{0}}{2}, \frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{4}}{2}$.

   b. $\triangle ABC$ is equilateral, with side length 2; D is the midpoint of side $\overline{AC}$. Label all side lengths and angle measurements for $\triangle ABD$. Use your figure to determine the sine and cosine of 30 and 60.



   c. Draw an isosceles right triangle with legs of length 1. What are the measures of the acute angles of the triangle? What is the length of the hypotenuse? Use your triangle to determine sine and cosine of the acute angles.

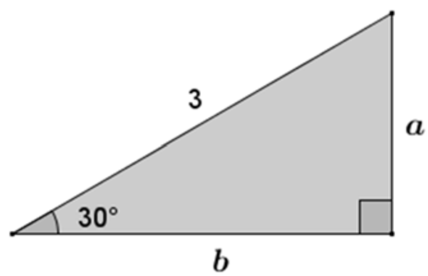


Parts (b) and (c) demonstrate how the sine and cosine values of the mentioned special angles can be found.

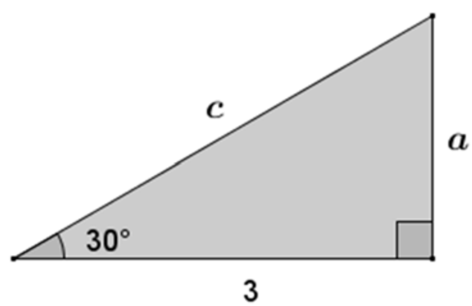
These triangles are common to trigonometry; we refer to the triangle in part (b) as a 30–60–90 triangle and the triangle in part (c) as a 45–45–90 triangle.

30–60–90 Triangle, side length ratio $1:2:\sqrt{3}$	**45–45–90 Triangle, side length ratio** $1:1:\sqrt{2}$
$2:4:2\sqrt{3}$	$2:2:2\sqrt{2}$
$3:6:3\sqrt{3}$	$3:3:3\sqrt{2}$
$4:8:4\sqrt{3}$	$4:4:4\sqrt{2}$
$x:2x:x\sqrt{3}$	$x:x:x\sqrt{2}$

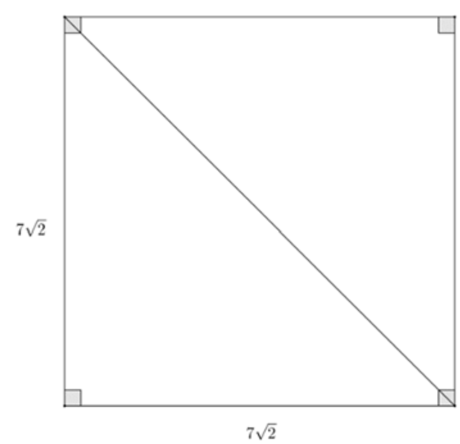
2. Find the missing side lengths in the triangle.



3. Find the missing side lengths in the triangle.

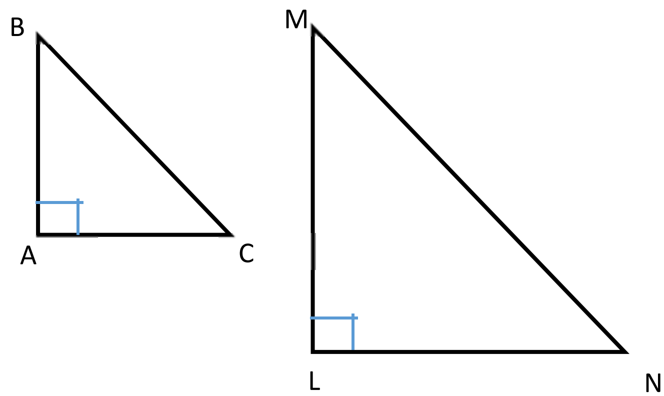


4. A square has side lengths of $7\sqrt{2}$. Use sine or cosine to find the length of the diagonal of the square. Confirm your answer using the Pythagorean theorem.



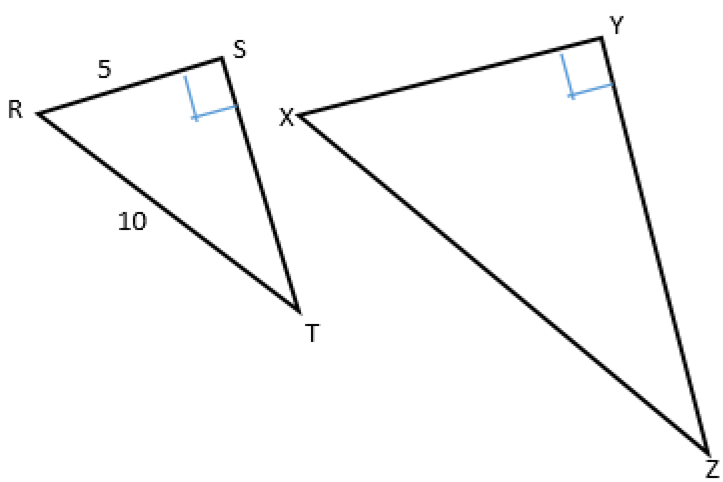
Relationships in Right Triangles – Homework

1) $\triangle ABC$ and $\triangle LMN$ are similar and $\sin(B) = \frac{7}{15}$.



a) Use the Pythagorean Theorem to find the length of AB.	
b) What is the value of cos N?	c) Determine the value of sin M.
d) What is the value of the tangent of N?	e) The tangent of N is equal to the tangent of what other angle?

2) Let $\triangle RST$ and $\triangle XYZ$ be similar triangles.



a) Determine sin Z.
b) Find cos X.
c) What is the value of tan Z?

For #3 – #6, angle A and B are complementary angles in a right triangle.

3) If the cos B = 8/17, what is the sin A?

4) If the sin A = 4/5, what is the cos B?

5) If the tan B = 12/5, what is the tan A?

6) If the tan A = 5/7, what is the sin A?

~~~~~~

7) sin 84° = cos _______

8) cos 67° = sin _______

9) Given that sin 60° ≈ 0.866, write the cosine of a complementary angle.

10) Given that cos 26° ≈ 0.899, write the sine of a complementary angle.
~~~~~~

Using Right Triangle Trigonometry to Solve for Unknown Sides and Angles – Vocabulary			
Term	**Definition**	**Notation**	**Diagram/Visual**
Inverse Trigonometric Functions	____________ ____________ ____________		
30-60-90 Right Triangle	____________ ____________ ____________		
45-45-90 Right Triangle	____________ ____________ ____________		

Using Right Triangle Trigonometry to Solve for Unknown Sides and Angles – Guided Notes

<u>FIND AN UNKNOWN SIDE</u>

When given one angle measure and one side length…

① Determine the correct trigonometric ratio to use based on the given information and the SIDE you want to find.

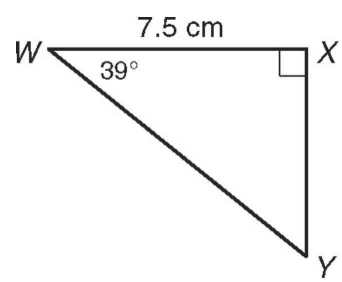
② Write the trig ratio equation and substitute the given values and a variable for the unknown side.

③ Solve for *x*.

④ Use a calculator to evaluate the expression.

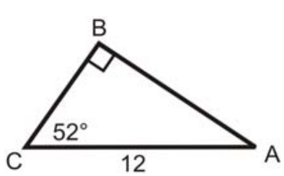
~~~~~~

Find *WY*.

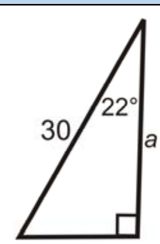


$\cos W = \frac{\text{adjacent leg}}{\text{hypotenuse}}$	Determine which trigonometric ratio to use.
$\cos 39° = \frac{7.5}{x}$	Substitute the given values
$x \cdot \cos 39° = 7.5$	Multiply both sides by x
$x = \frac{7.5}{\cos 39°}$	Solve for x.
$WY \approx 9.65$ cm	Use a calculator to evaluate the expression.

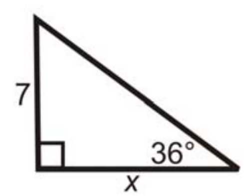
Find the length of the <u>MISSING SIDE</u> by setting up a trig ratio equation.



1. Find *AB*.

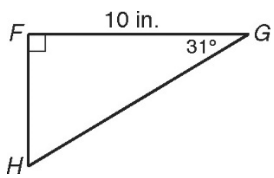


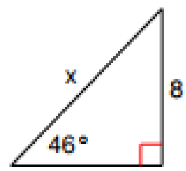
2. Find the value of *a*.

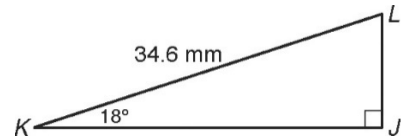


3. Solve for *x*.
~~~~~~

SELF CHECK **Find the length of the MISSING SIDE by setting up a trig ratio equation.**



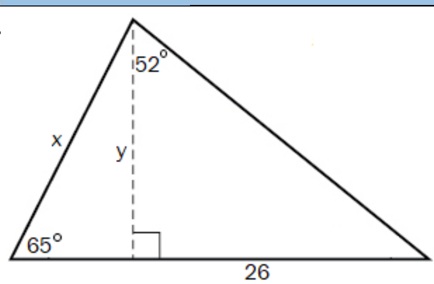




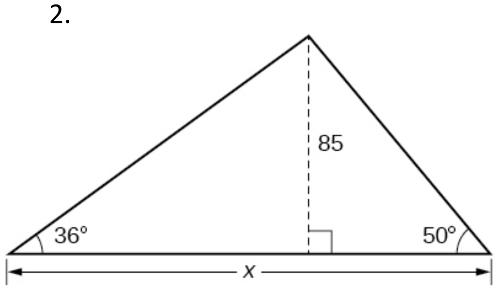
1. Find *FH*.
2. Find the value of *x*.
3. Find *JK*.

Example! **Find the length of *x* and *y*.**

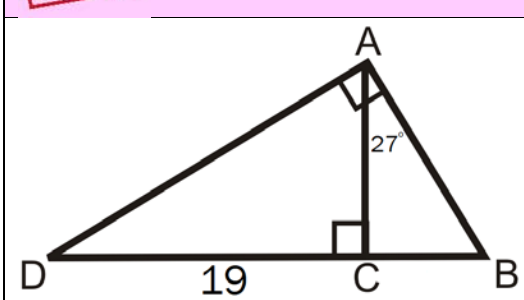
1.



2.



SELF CHECK **Find *AD* and *AB*.**



FIND AN UNKNOWN ANGLE

When given two side lengths…

① Determine the INVERSE trig ratio to use based on the given information and the ANGLE you want to find.

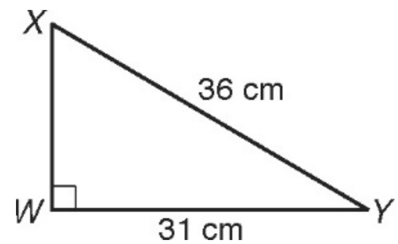
② Write the inverse trig ratio equation and substitute the given values.

③ Use a calculator to evaluate the expression.

Inverse Trigonometric Functions

$\sin^{-1}\left(\frac{opp}{hyp}\right) = x°$ $\cos^{-1}\left(\frac{adj}{hyp}\right) = x°$ $\tan^{-1}\left(\frac{opp}{adj}\right) = x°$

Find m∠*Y*.

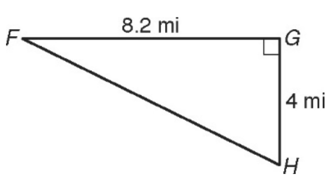


$\cos^{-1}\left(\frac{adj}{hyp}\right) = x°$ Determine which trigonometric ratio to use.

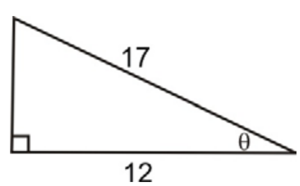
$\cos^{-1}\left(\frac{31}{36}\right) = m\angle Y$ Substitute the given values.

$m\angle Y \approx 30.56°$ Use a calculator to evaluate the expression.

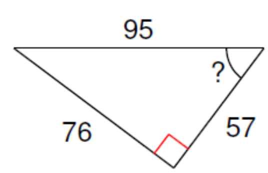
Example! **Find the measure of the MISSING ANGLE by using an inverse trig ratio equation.**



1. Find m∠*GHF*.



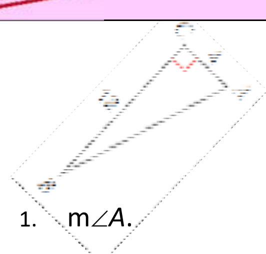
2. What is the value of θ?



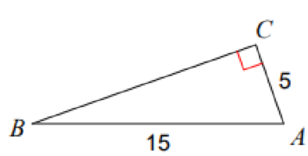
Sometimes you can choose either of the three.

3. Find the indicated angle measure.

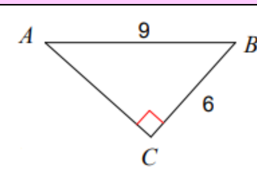
SELF CHECK **Find the measure of the MISSING ANGLE by using an inverse trig ratio equation.**



1. m∠*A*.



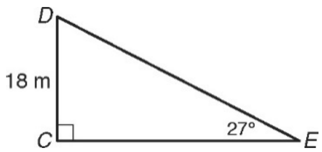
2. Find m∠*CBA*.



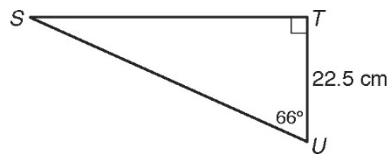
3. Find m∠*ABC*.

Using Right Triangle Trigonometry to Solve for Unknown Sides and Angles – Practice

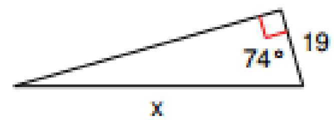
Find the indicated measure. Round to the nearest hundredth.



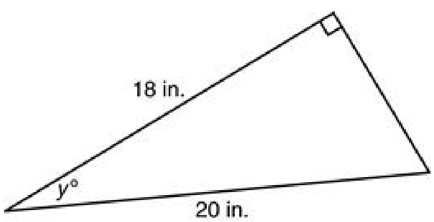
1. Find *DE*.



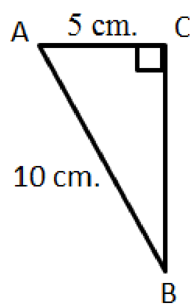
2. Find *ST*.



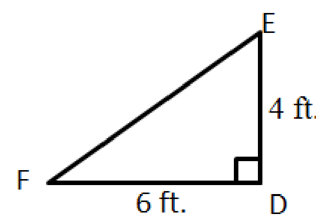
3. Find the value of *x*.



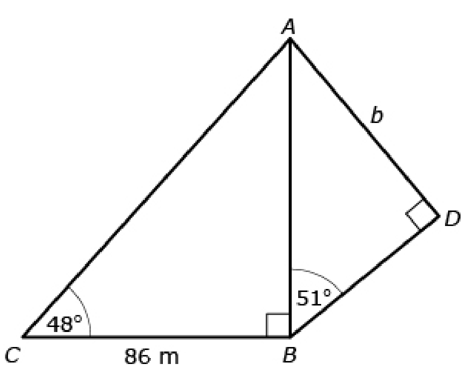
4. Find the value of *y*.



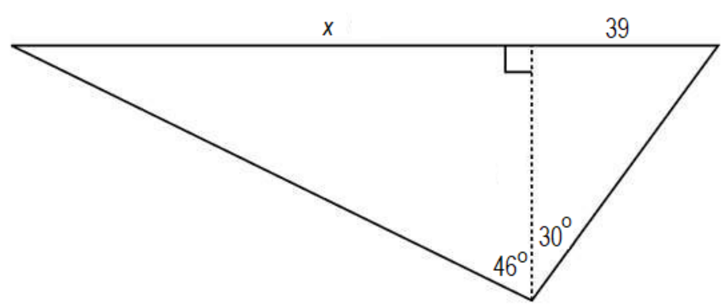
5. Find ∠*ABC*.



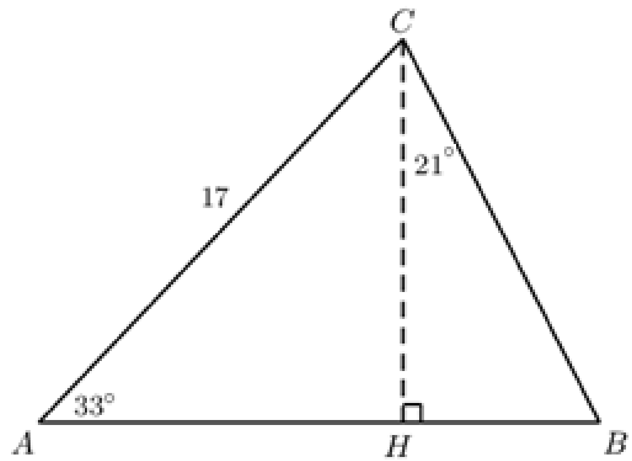
6. Find m∠*E*.



7. Find the value of b.



8. Find the value of *x*.



8. Find each measure. Round to the nearest hundredth.

AH = _________ m∠*ACH* = _________

HB = _________ m∠*ABC* = _________

CH = _________ m∠*ACB* = _________

BC = _________

Using Right Triangle Trigonometry to Solve for Unknown Sides and Angles – Application

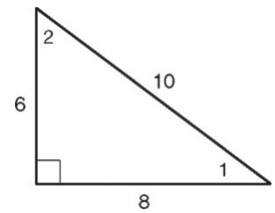
Extend and Apply

Use the trigonometric ratio sin *A* = 0.8 to determine which angle of the triangle is ∠*A*.

$\sin \angle 1 = \dfrac{\text{leg opposite } \angle 1}{\text{hypotenuse}}$ $\sin \angle 2 = \dfrac{\text{leg opposite } \angle 2}{\text{hypotenuse}}$

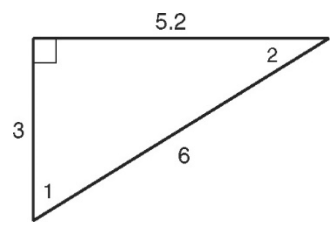
$= \dfrac{6}{10}$ $= \dfrac{8}{10}$

$= 0.6$ $= 0.8$



Since $\sin A = \sin \angle 2$, ∠2 is ∠*A*.

Use the given trigonometric ratio to determine which angle of the triangle is ∠*A*.



1. $\sin A = \dfrac{1}{2}$

____

2. $\cos A = \dfrac{13}{15}$

____

3. $\cos A = 0.5$

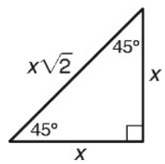
____

4. $\tan A = \dfrac{15}{26}$

____

Special Right Triangles

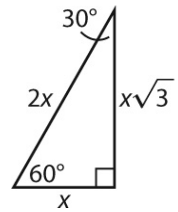
The legs of an isosceles right triangle have the same length. Each acute angle has a measure of 45°.



The lengths of the sides of a **45°-45°-90° triangle** have a set ratio.

Leg	Leg	Hypotenuse
x	x	$x\sqrt{2}$
1	1	$\sqrt{2}$

The hypotenuse of a 30°-60°-90° triangle is always twice the length of the short leg.



The lengths of the sides of a **30°-60°-90° triangle** have a set ratio.

Short Leg	Long Leg	Hypotenuse
x	$x\sqrt{3}$	$2x$
1	$\sqrt{3}$	2

Use a special right triangle to write each trigonometric ratio as a fraction.

1. sin 45°

____

2. cos 30°

____

3. cos 45°

____

4. tan 60°

____

5. tan 45°

____

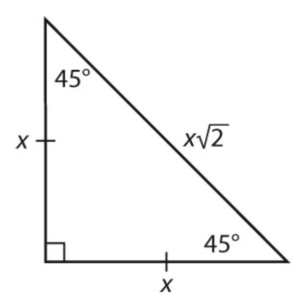
6. sin 30°

____

An isosceles-right triangle is a right triangle with two congruent legs. The base angles of an isosceles-right triangle both measure 45°, so another name for this triangle is a 45-45-90 triangle. Both legs are the same length. The hypotenuse length is the leg length times $\sqrt{2}$.

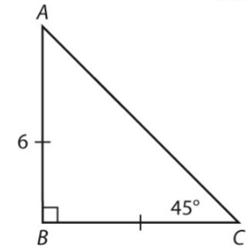
The sine, cosine, and tangent of 45° can be calculated from the triangle, using the ratios.

$$\sin 45° = \cos 45° = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \tan 45° = \frac{x}{x} = 1$$



Find the given side lengths and angle measurements for triangle *ABC*.

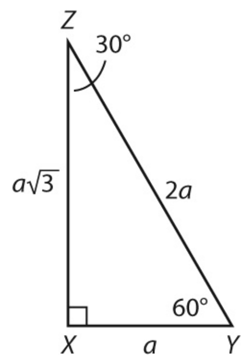
7. $BC =$ ______ 8. $AC =$ ______ 9. $m\angle A =$ ______



Another special right triangle is the 30-60-90 triangle like triangle *XYZ* in the figure.

The length of the hypotenuse is double the length of the shorter leg, and the other leg's length is $\sqrt{3}$ times the length of the shorter leg.

The sine, cosine, and tangent of 30° and 60° can be calculated using these ratios.



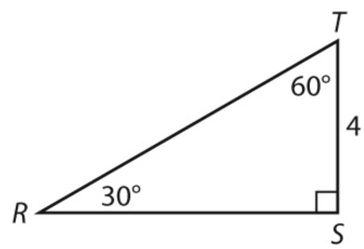
Find the indicated values from the figure.

10. $RT =$ ______ 11. $RS =$ ______

12. $\sin 30° =$ ______ 13. $\cos 30° =$ ______

14. $\sin 60° =$ ______ 15. $\cos 60° =$ ______

16. $\tan 30° =$ ______ 17. $\tan 60° =$ ______



Right Triangles in the Coordinate Plane

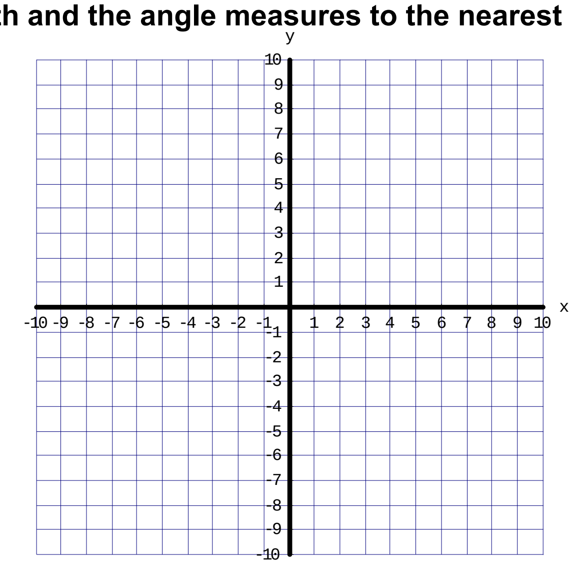
Plot the points. Find the side lengths to the nearest hundredth and the angle measures to the nearest whole degree.

M(–5, 1), *N*(1, 1), *P*(–5, 7)

MN = ______ $m\angle M =$ ______

MP = ______ $m\angle N =$ ______

NP = ______ $m\angle P =$ ______



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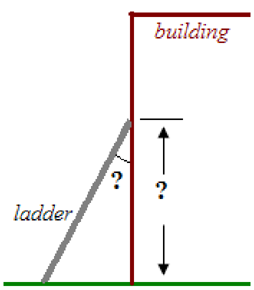
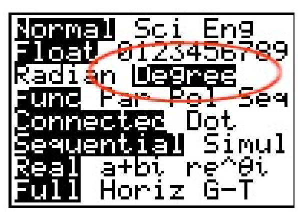
Using Right Triangle Trigonometry to Solve for Unknown Sides and Angles – Application

1. A ladder is leaning against the outside wall of a building. The figure at the right shows the view from the end of the building, looking directly at the side of the ladder. The ladder is exactly10 feet long and makes an angle of 60° with the ground. If the ground is level, what angle does the ladder make with the side of the building? How far up the building does the ladder reach (give an exact value and then approximate to the nearest inch)? Hint: Use a known trigonometric ratio in solving this problem.



The first problem in this task involves trigonometric ratios in special right triangles, where the values of all the ratios are known exactly. However, there are many applications involving other size angles. Graphing calculators include keys to give values for the sine and cosine functions with very accurate approximations for all trigonometric ratios of degree measures greater than 0° and less than 90°. You should use calculator values for trigonometric functions, as needed, for the remainder of this task.



In higher mathematics, it is standard to measure angles in radians. The issue concerns you now because you need to make sure that your calculator is in **degree mode** (and not radian mode) before you use it for finding values of trigonometric ratios. If you are using any of the IT-83/84 calculators, press the MODE button, then use the arrow keys to highlight "Degree" and press enter. The graphic at the right shows how the screen will look when you have selected degree mode. To check that you have the calculator set correctly, check by pressing the TAN key, 45, and then ENTER. The answer should be 1. If you are using any other type of calculator, find out how to set it in degree mode, do so, and check as suggested above. Once you are sure that your calculator is in degree mode, you are ready to proceed to the remaining items of the question.

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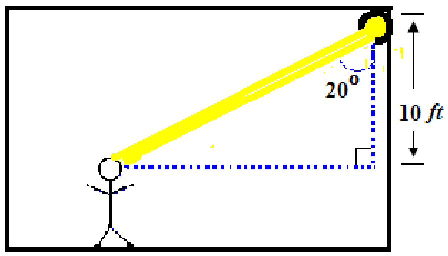
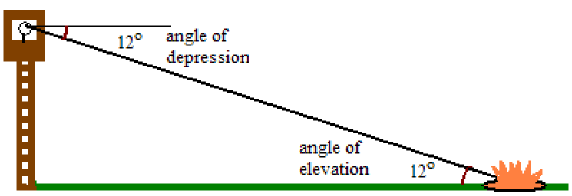
2. The main character in a play is delivering a monologue, and the lighting technician needs to shine a spotlight onto the actor's face. The light being directed is attached to a ceiling that is 10 feet above the actor's face. When the spotlight is positioned so that it shines on the actor's face, the light beam makes an angle of 20° with a vertical line down from the spotlight. How far is it from the spotlight to the actor's face? How much further away would the actor be if the spotlight beam made an angle of 32° with the vertical?



3. A forest ranger is on a fire lookout tower in a national forest. His observation position is 214.7 feet above the ground when he spots an illegal campfire. The angle of depression of the line of site to the campfire is 12°. (See the figure below.)



Note that an angle of depression is measured down from the horizontal; in order to look down at something; you need to lower, or depress, the line of sight from the horizontal. We observe that the line of sight makes a transversal across two horizontal lines, one at the level of the viewer (such as the level of the forest ranger), and one at the level of the object being viewed (such as the level of the campfire). Thus, the angle of depression looking down from the fire lookout tower to the campfire, and the angle of elevation is the angle looking up from the campfire to the tower. The type of angle that is used in describing a situation depends on the location of the observer.

The angle of depression is equal to the corresponding angle of elevation. Why?

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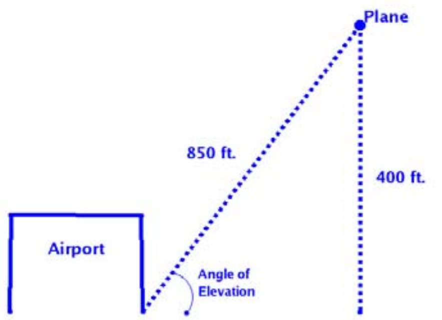
4. An airport is tracking the path of one of its incoming flights. If the distance to the plane is 850 ft. (from the ground) and the altitude of the plane is 400 ft, then

a. What is the sine of the angle of elevation from the ground at the airport to the plane (see figure at the right)?

b. What is the cosine of the angle of elevation?



c. Now, use your calculator to find the measure of the angle itself. Pressing "2nd" followed by one of the trigonometric function keys finds the degree measure corresponding to a given ratio. Press 2nd, SIN, followed by the sine of the angle from *part a*. What value do you get?

d. Press 2nd, COS, followed by the cosine of the angle from *part b*. What value do you get?

Did you notice that, for each of the calculations in *parts c-d,* the name of the trigonometric ratio is written with an exponent of -1? These expressions are used to indicate that we are starting with a trigonometric ratio (sine and cosine,) and going backwards to find the angle that gives that ratio. You'll learn more about this notation later. For now, just remember that it signals that you are going backwards from a ratio to the angle that gives the ratio.

e. Why did you get the same answer each time?

f. To the nearest hundredth of a degree, what is the measure of the angle of elevation?

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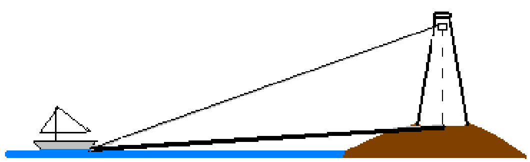
5. The top of a billboard is 40 feet above the ground. What is the angle of elevation of the sun when the billboard casts a 30-foot shadow on level ground?

6. An observer in a lighthouse sees a sailboat out at sea. The angle of depression from the observer to the sailboat is 6°. The base of the lighthouse is 50 feet above sea level and the observer's viewing level is 84 feet above the base. (See the figure at the right, which is not to scale.)

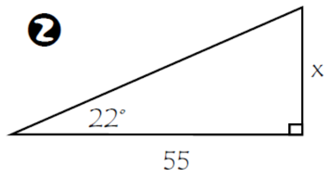
What is the distance from the sailboat to the observer?

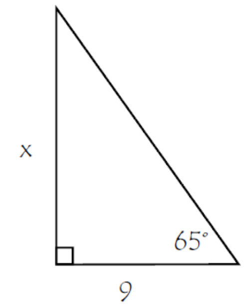


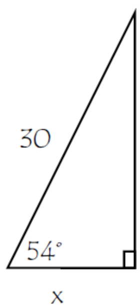
Using Right Triangle Trigonometry to Solve For Unknown Sides and Angles – Homework

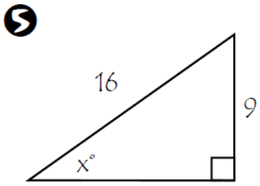
Solve for *x*.

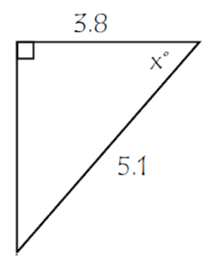
1. $\sin 27° = \frac{x}{8}$

2. 

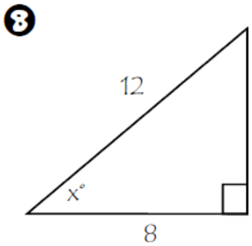
3. 

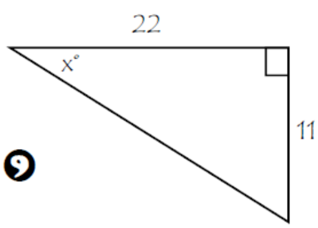
4. 

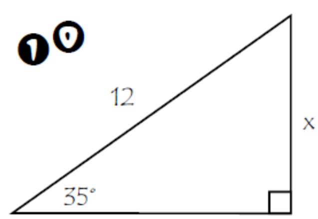
5. 

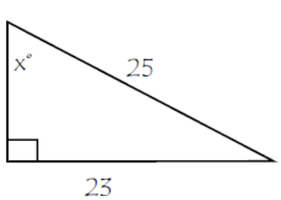
6. 

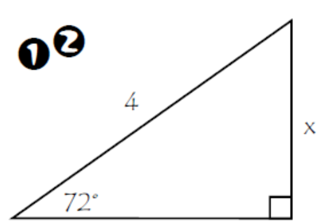
7. $\tan 18° = \frac{x}{75}$

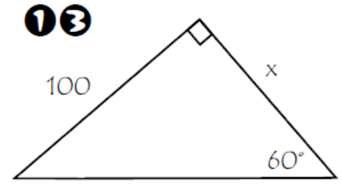
8. 

9. 

10. 

11. 

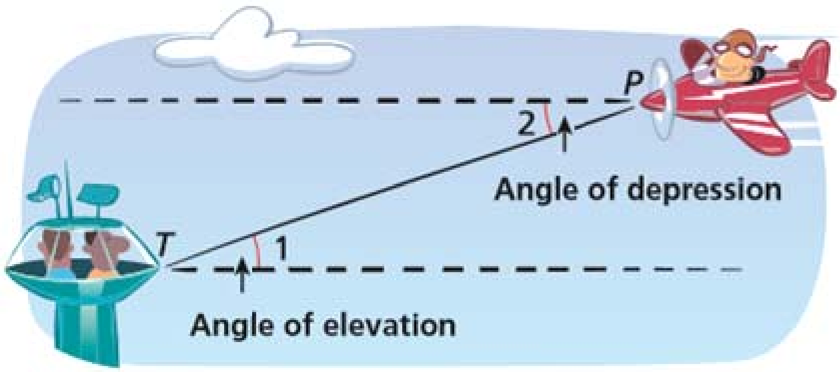
12. 

13. 

Angles of Elevation and Depression – Vocabulary			
Term	**Definition**	**Notation**	**Diagram/Visual**
Angle of Elevation	______________ ______________ ______________		
Angle of Depression	______________ ______________ ______________		
Alternate Interior Angles Theorem	______________ ______________ ______________		

Angles of Elevation and Depression – Guided Notes

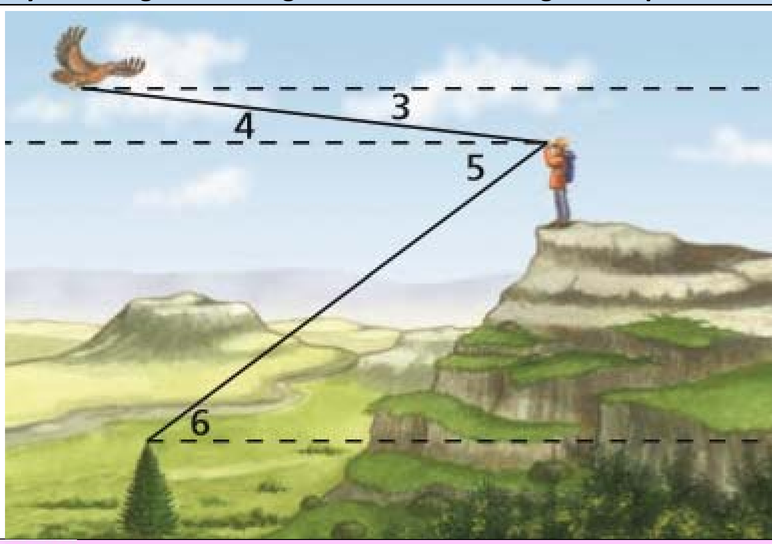
GUIDED NOTES



- An angle of elevation is the angle formed by a horizontal line and a line of sight to a point_________________the line. In the picture above, ∠_____ is an angle of elevation.
- An angle of depression is the angle formed by a horizontal line and a line of sight to a point_______________the line. In the picture above, ∠_____ is an angle of depression.
- Since horizontal lines are parallel, ∠1 ≅ ∠2 by the ____. Therefore, the angle of elevation from one point is congruent to the angle of depression from the other point.

Example! Before we begin solving problems involving angles of elevation and depression, let's practice identifying them.

Identify each angle as an angle of elevation or angle of depression.



∠3 is formed by a horizontal line and a line of sight *below* the line so ∠3 is an ___________________________________.

∠4 is formed by a horizontal line and a line of sight *above* the line so ∠4 is an ___________________________________.

SELF CHECK **Use the diagram above to identify each angle as an angle of elevation or angle of depression.**

1. ∠5 ____
2. ∠6 ____

Example! To find an unknown distance when given an angle of elevation, we will use what we learned about finding an unknown side with trigonometric functions.

An air traffic controller at an airport sights a plane at an angle of elevation of 41°. The pilot reports that the plane's altitude is 4000 ft. What is the horizontal distance between the plane and the airport? Round to the nearest hundredth of a foot.

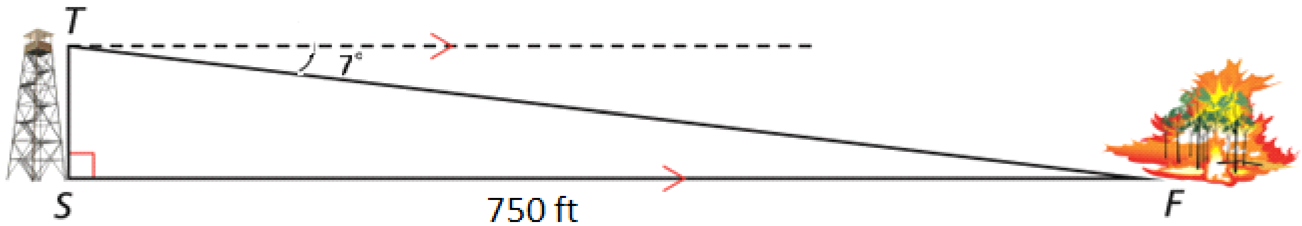
①Draw and label a triangle to represent the given information.	②Write a trig equation based on the given info and the side you want to find.	③Solve the trig equation.

SELF CHECK **Suppose the plane in the problem above is at an altitude of 3500 ft and the angle of elevation from the airport to the plane is 29°. What is the horizontal distance between the plane and the airport? Round to the nearest hundredth of a foot.**

Questions To Ponder **Explain why in the real world the distance between the plane and the airport will be slightly different than your calculations?**

Example! To find an unknown distance when given an angle of depression, we will use what we learned about finding an unknown side with trigonometric functions.

A forest ranger in an observation tower sees a fire 750 ft. away. The angle of depression to the fire is 7°.

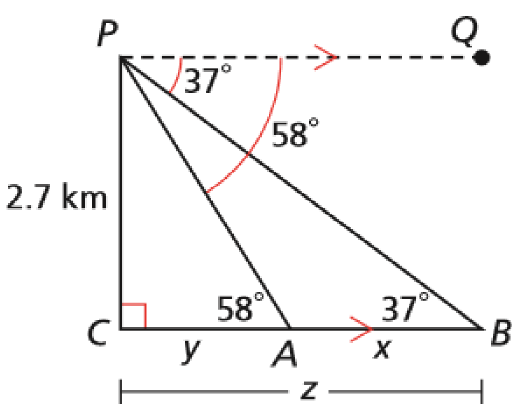


By the Alternate Interior Angles Theorem, what angle in the triangle is 7°? _______________

What is the height of the tower? Round to the nearest hundredth of a foot.

SELF CHECK **Suppose the ranger sees another fire and the angle of depression to the fire is 3°. What is the horizontal distance to this fire? Round to the nearest hundredth of a foot.**

Example! **A pilot flying at an altitude of 2.7 km sights two control towers directly in front of her. The angle of depression to the base of one tower is 37°. The angle of depression to the base of the other tower is 58°. What is the distance between the two towers? Round to the nearest tenth of a kilometer.**

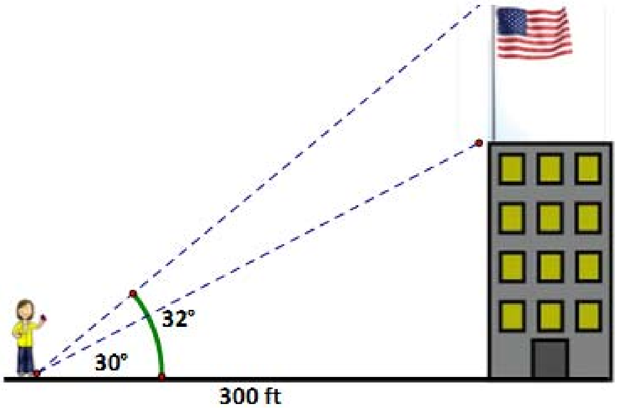


Step 1: Find *y*.

Step 2: Find *z*.

Step 3: Find *x* by subtracting *z* and *y*.

Example! **A flagpole is at the top of a building. You are standing 300 ft. from the base of the building where the angle of elevation of the top of the pole is 32°, and the angle of elevation of the bottom of the pole is 30°. Determine the length of the flagpole (to the hundredths place).**

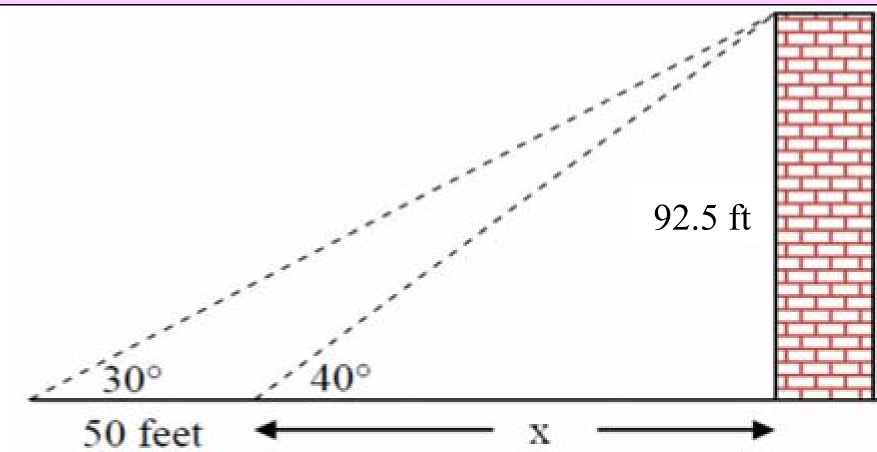


Step 1: Find the height of the building.

Step 2: Find the height of the building and flagpole together.

Step 3: Find Step1 and Step 2.

SELF CHECK **An observer on the ground looks up to the top of a building at an angle of elevation of 30°. After moving 50 feet closer, the angle of elevation is now 40°. How far was the observer from the building before he moved closer?**

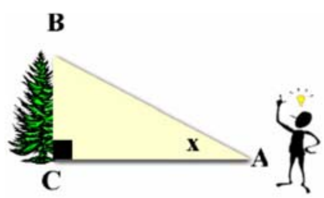


Angles of Elevation and Depression – Practice

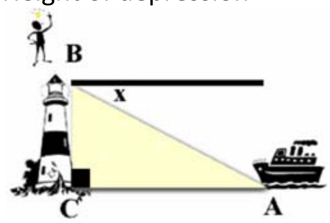
For the questions below, draw the appropriate triangle and label the sides. Then solve the problem.

Remember:

Height of elevation



Height of depression



1. A photographer shines a camera light at a particular painting forming an angle of 40° with the camera platform. If the light is 58 feet from the wall where the painting hangs, how high above the platform is the painting?

2. Liola drives 16 km up a hill that is at a grade of 10°. What horizontal distance, to the nearest tenth of a kilometer, has she covered?

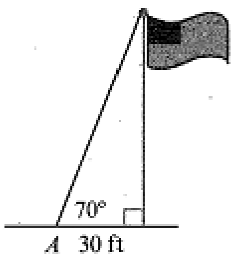
3. To find the height of a tower, a surveyor positions a transit that is 2 meters tall at a spot 95 meters from the base of the tower. She measures the angle of elevation to the top of the tower to be 32°. What is the height of the tower to the nearest meter?

4. An 8 foot ladder is leaning against a wall. The bottom of the ladder is 33 inches from the bottom of the wall. What is the measure of the angle that the ladder forms with the ground?

5. A campsite is 9.41 miles from a point directly below the mountain top. If the angle of elevations is 12° from the camp to the top of the mountain, how high is the mountain?

6. At a point 42.3 feet from the base of a building, the angle of elevation of the top is 75°. How tall is the building?

7. In the diagram, a period is standing at point A. To the nearest tenth of a food how tall is the flag point?



8. You are standing on a bridge looking down at a boat on the water. Your angle of depression is 30 degree. The bridge is 200 feet tall. What is the distance you would have to shoot to hit a can in the boat?

Angles of Elevation and Depression – Task

Calculating the Angle of Elevation of the Sun

Trigonometry is all around us! Right triangles can be found in many daily situations. In this task you will apply your knowledge of trigonometry to shadows in order to calculate the angle of elevation to the sun at different times of day.

To calculate the angle of elevation of the sun, use the following procedure:

- Measure your height and the length of the shadow you cast at two different times of day (at least 3 hours apart) on two different days.
- Record the times and measurements (with units).
- Draw a sketch of the right triangle in this scenario.
- Label the sides of your sketch with your measurements and angle of elevation.
- Solve for the angle of elevation while clearly showing all of your steps.

Do not forget that you need to do the procedure above for two different times of day on two different days!

~~~~~~

Once you have completed the calculations above, answer the following questions:

1. In several sentences, explain the right triangle drawings you used to model the shadow scenario. Make sure it answers the following:
   a) What does each side length signify? In particular, explain what does the hypotenuse in your drawing signify in the real world?

   b) Explain why the angle you chose is the angle of elevation of the sun.

2. From your measurements, when was your shadow the longest? What was the angle of elevation of the sun at this time?

3. From your measurements, when was your shadow the shortest? What was the angle of elevation of the sun at this time?
~~~~~~

4. During what time of day do you think the shadow you cast will be longest? Use your knowledge of trigonometric ratios in your explanation. (Hint: Make sure you mention the angle of elevation, your height and a trig ratio that will help explain why the shadow is longest at this time of day).

5. During what time of day do you think the shadow you cast will be the shortest? Use your knowledge of trigonometric ratios in your explanation. (Use the hint from above, except this time you are explaining why the shadow is shortest at this time of day.)

Below is a checklist of all the components you will need to submit for grading:

- ☐ Tables of your height, shadow length (with units) and time of day. (9 pts)
- ☐ Sketches of the four right triangles. (14 pts)
- ☐ Label the sketches with your measurements and angle of elevation. (9 pts)
- ☐ Use the correct trigonometric ratio. (5 pts)
- ☐ Show all steps and calculations when solving for the angle of elevation. (14 pts)
- ☐ Answers to questions #1-5. (25 pts)

Hypsometer Actvity

How to Make a Hypsometer

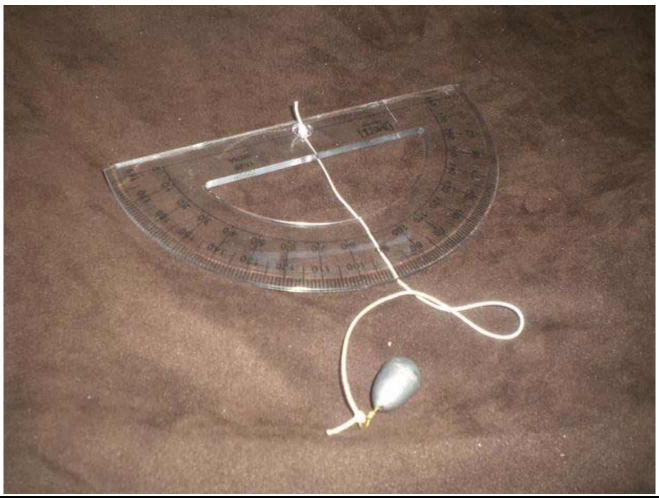
List of Materials:

protractor
drinking straw
string, fishing line, yarn, etc.
glue or tape
weight

Step 1: Attach a straw to the protractor by laying it on the straightedge of the protractor. You can attach the straw to the protractor with glue or tape.

Step 2: Tie the weight to the string. This is used to determine perpendicularity.

Step 3: Attach the other end of the string to our protractor by inserting it through the hole found where the x and y axis meet. Tie a knot to secure it and use scissors to snip off the excess string. Allow the string to dangle freely.



How do I use the hypsometer? (For this example we are using the height of the school)

Step 1: Pick a job

Job Title	**Name**
1. Measurer – measures the distances from objects to looker	
2. Looker – looks through the hypsometer at the tops of objects (needs eye height measured before starting)	
3. Reader – reads angle of elevation from hypsometer	
4. Recorder – records distances and angles of elevation	

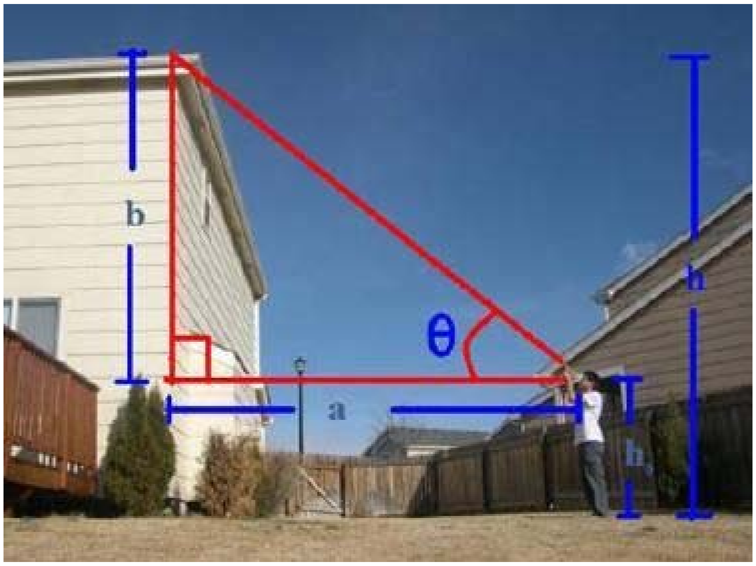
Eye height of looker (inches): __________

For all objects measured, the Measurer should measure some horizontal distance from the object for the Looker to stand. The Looker should use the hypsometer to look up to the top of the object. The Reader should read what angle the hypsometer says. The Recorder should record both pieces of data.

Step 2: From the wall of the school, measure how far away you are standing. Write this down as your base length.

Step 3: Looking through the straw of the hypsometer, find the roof of the school building and look at which degree the line falls on. If you read the degree measure on the protractor at the point where the string dangles, how would you determine the angle of elevation? _______

Step 4: Use your right triangle trig knowledge to find the height of the building. Don't forget to take into consideration the height of the "looker!"



Now let's measure some objects!

Object	Distance (inches)	Angle of Elevation (degrees)
1. flag pole		
2. light post		
3. garage		
4. football goal post		

Results: Heights in feet (divide your answer by 12)

1. light post	3. garage
2.flag pole	4. football goal post

Angles of Elevation and Depression – Homework

Tell whether each statement is true or false. If false, explain why.

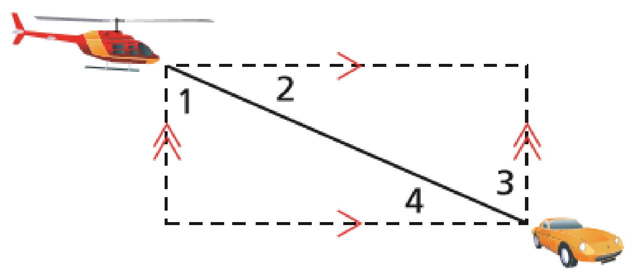
1. The angle of elevation from your eye to the top of a tree increases as you walk toward the tree.

2. If you stand at street level, the angle of elevation to a building's tenth-story window is greater than the angle of elevation to one of its ninth-story windows.

3. As you watch a plane fly above you, the angle of elevation to the plane gets closer to 0° as the plane approaches the point directly overhead.

4. An angle of depression can never be more than 90°.

Use the diagram for questions #5 and #6.

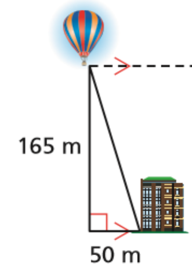


5. Which angles are not angles of elevation or angles of depression?

6. The angle of depression from the helicopter to the car is 30°. Find the following angle measures.
   m∠1 = __________ m∠2 = __________ m∠3 = __________ m∠4 = __________

7. Marion is observing the launch of a space shuttle from the command center. When she first sees the shuttle, the angle of elevation to it is 16°. Later, the angle of elevation is 74°. If the command center is 1 mi from the launch pad, how far did the shuttle travel while Marion was watching? Round to the nearest tenth of a mile.

8. An observer in a hot-air balloon sights a building that is 50 m from the balloon's launch point. The balloon has risen 165 m. What is the angle of depression from the balloon to the building? Round to the nearest hundredth of a degree.



9. To measure the height of a rock formation, a surveyor places her transit 100 m from its base and focuses the transit on the top of the formation. The angle of elevation is 76°. The transit is 1.5 m above the ground. What is the height of the rock formation?

10. A surveyor finds that the angle of elevation to the top of a 1000 ft tower is 67°.
    a. To the nearest foot, how far is the surveyor from the base of the tower?

    b. How far back would the surveyor have to move so that the angle of elevation to the top of the tower is 55°? Round to the nearest hundredth.

11. Two students are using shadows to calculate the height of a pole. One says that it will be easier if they wait until the angle of elevation to the sun is exactly 45°. Explain why the student made this suggestion.

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Science Technology Engineering Mathematics

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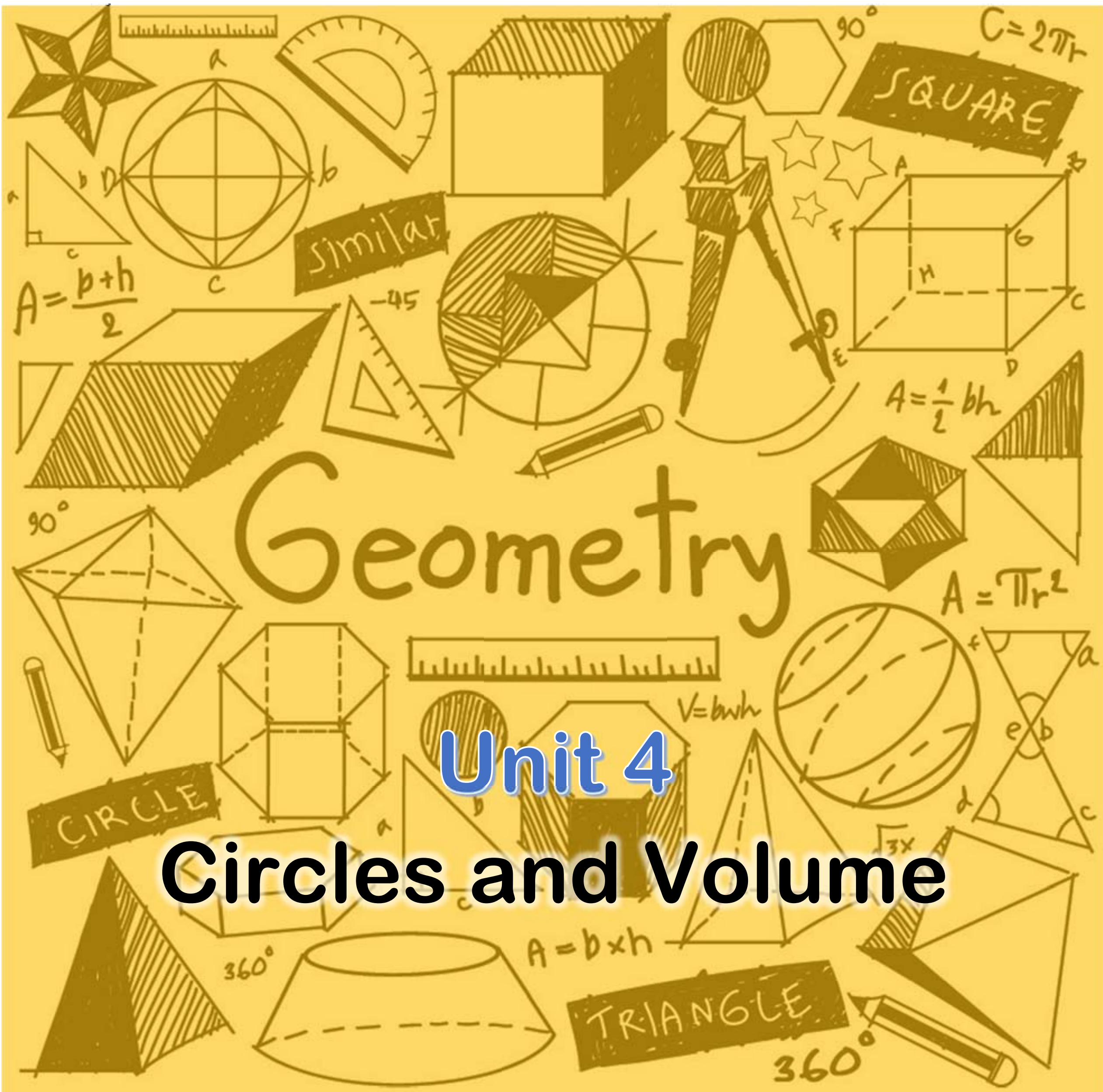

Unit 4
Circles and Volume

HOUSTON COUNTY
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MATHEMATICS
DEPARTMENT

Science Technology Engineering Mathematics

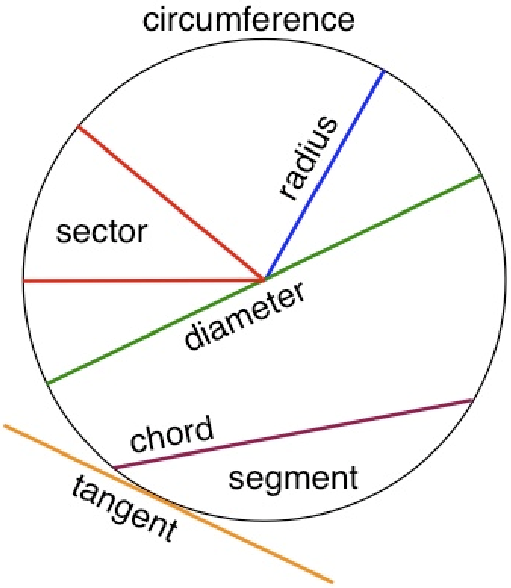
Lesson Name: G.U4.C1.A.01.**Vocab.CircleBasics**

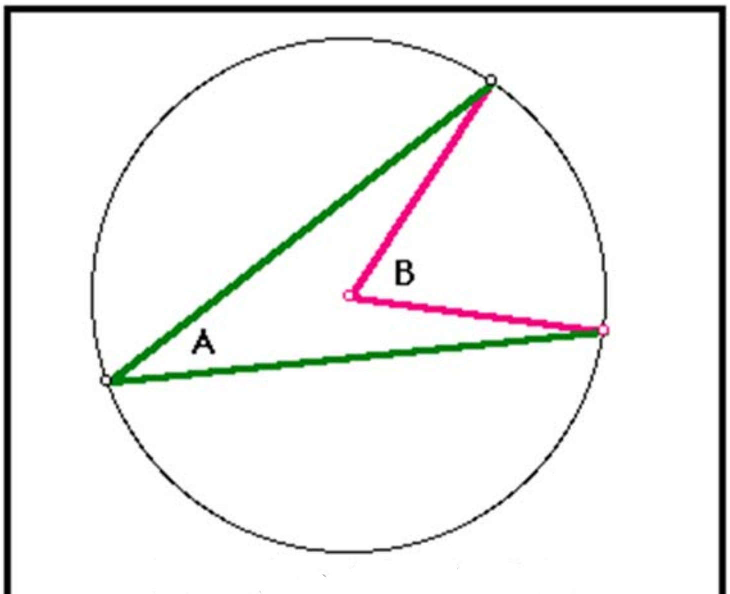
Term	Definition	Notation	Diagram/Visual
Circle			
Radius			
Diameter			
Circumference			
Area (of a circle)			
Chord			
Secant			

Lesson Name: G.U4.C1.A.01.**Vocab.CircleBasics**

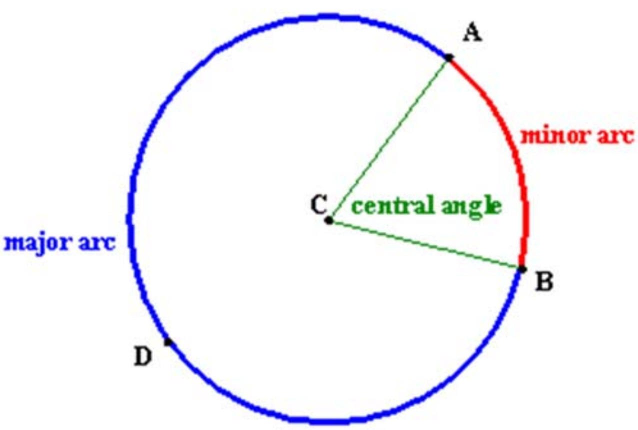
Term	Definition	Notation	Diagram/Visual
Tangent			
Point of Tangency			
Central Angle			
Inscribed Angle			
Minor Arc			
Major Arc			
Semi Circle			

Parts of a Circle

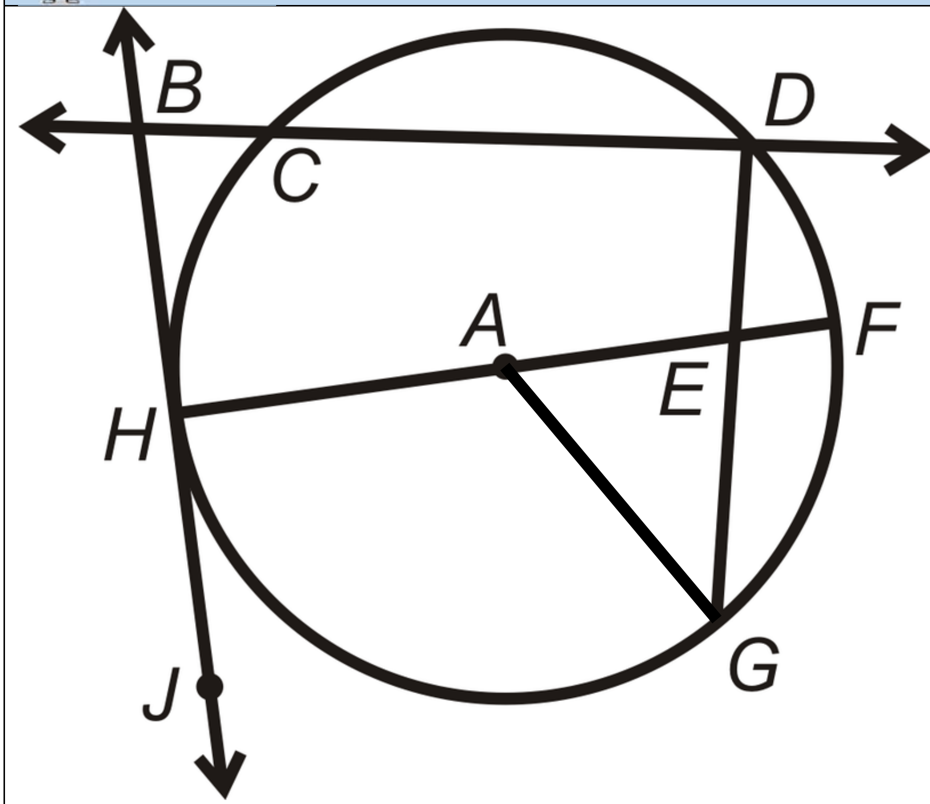




A is an inscribed angle, B is a central angle



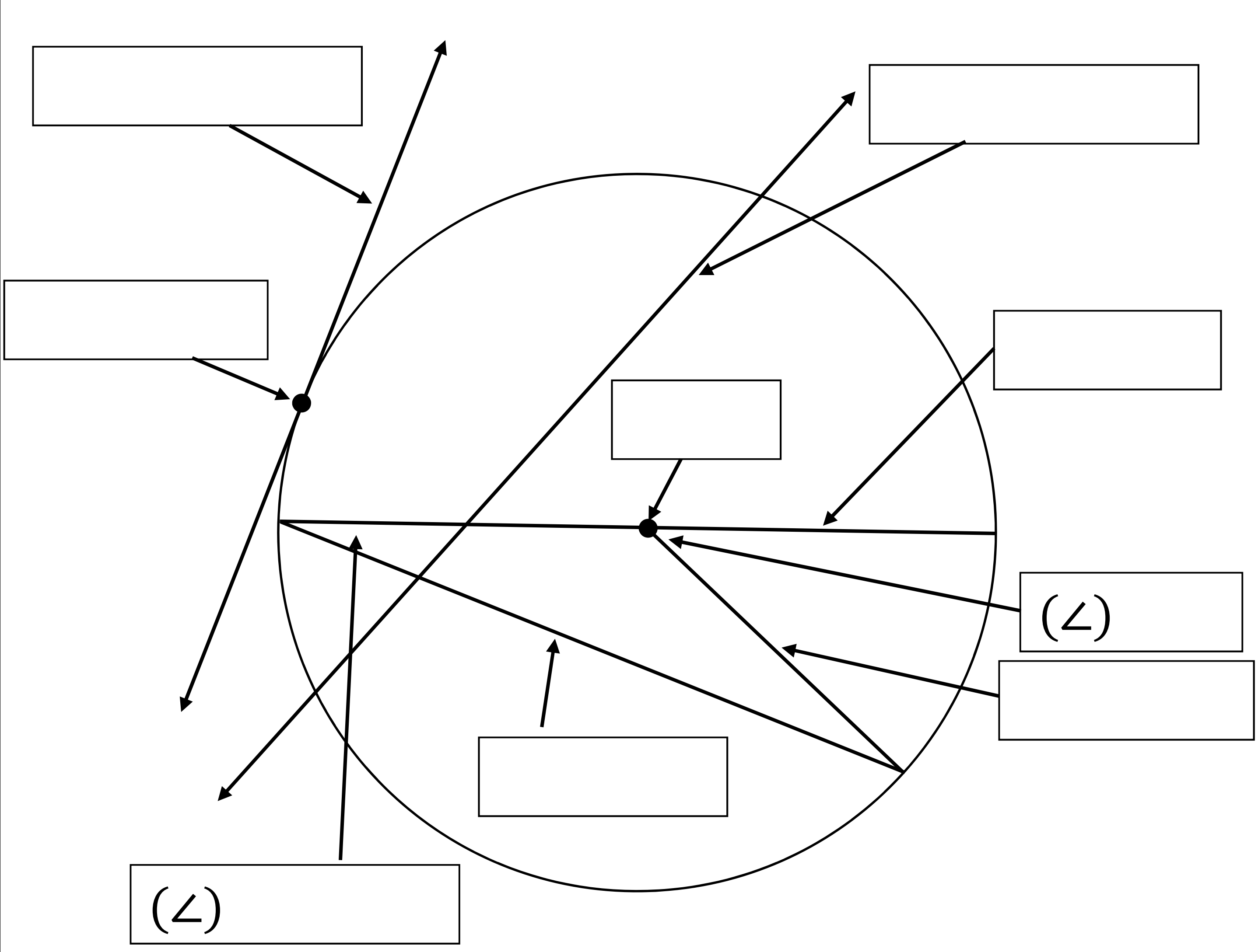
Example!



1. Use the vocabulary to name the different parts of the circle:

Circle	
Radius	
Diameter	
Chord	
Secant	
Tangent	
Point of Tangency	
Major Arc	
Minor Arc	
Semi Circle	
Inscribed Angle	
Central Angle	

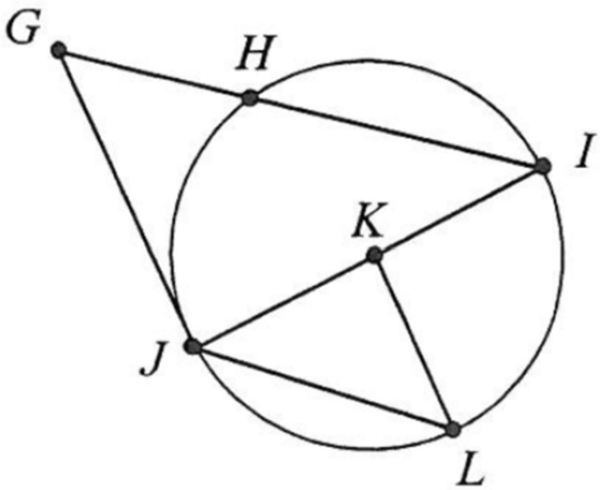
SELF CHECK



What is the relationship between the radius and diameter?

What is the relationship between a chord, secant, and diameter?

What is the relationship between a minor arc, major arc, and semi circle?



Center:__________

Radius:__________

Chord:__________

Diameter:__________

Secant:__________

Tangent:__________

Point of Tangency:__________

Minor Arc:__________

Major Arc:__________

Semicircle:__________

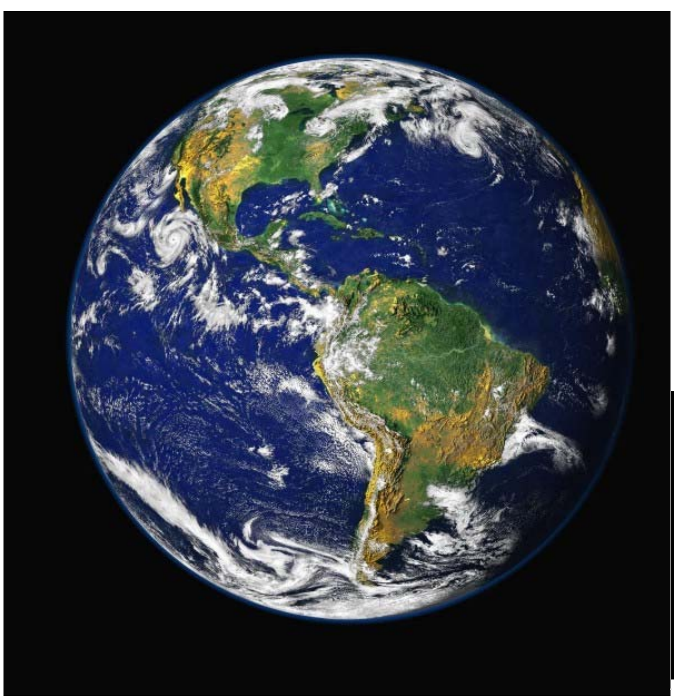
Central Angle:__________

Inscribed Angle:__________

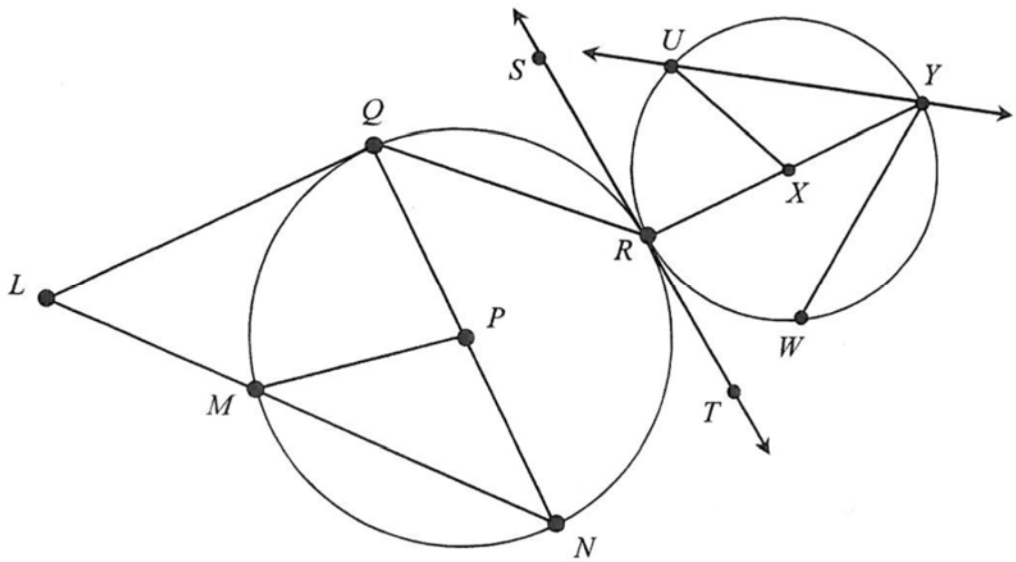
Lifting the Rope

Imagine wrapping the circumference of a tennis ball and basketball with pieces of string so that the pieces of string wrap perfectly around the balls. Now, imagine lifting the entire pieces of string 1 foot off of each ball. Which ball requires more string to “close” string circle that is 1 foot off of the ball?

Imagine wrapping the circumference of the Moon and Earth with pieces of string so that the pieces of string wrap perfectly around the surfaces (we’ll imaging the surfaces are smooth). Now, imagine lifting the entire pieces of string 1 foot off of each surface. Which surface requires more string to “close” the string circle? How do these compare to the amount of string needed to “close” the string circles off of the tennis and basketball?



Directions: Use your vocabulary and guided notes to classify each circle part. Parts may be used more than once.



1. $\overline{LQ}$ ______________________

2. $\overline{WY}$ ______________________

3. $\angle NQR$ ______________________

4. X ______________________

5. $\widehat{RWU}$ ______________________

6. $\overleftrightarrow{ST}$ ______________________

7. $\overline{PN}$ ______________________

8. $\angle UXY$ ______________________

9. $\widehat{MQ}$ ______________________

10. $\overline{QN}$ ______________________

11. R ______________________

12. $\overleftrightarrow{UY}$ ______________________

13. $\widehat{QRN}$ ______________________

14. $\angle MPQ$ ______________________

15. $\overline{QR}$ ______________________

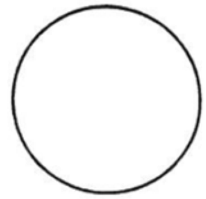
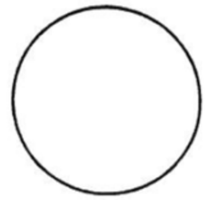
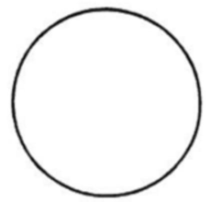
16. $\angle UYR$ ______________________

17. $\widehat{WY}$ ______________________

18. $\overline{LN}$ ______________________

19. $\overline{UX}$ ______________________

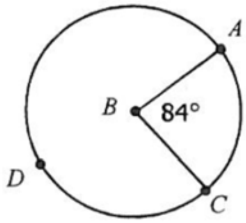
20. $\widehat{RUY}$ ______________________

Term	Definition	Notation	Diagram/Visual
Central Angle			
Inscribed Angle			
Inscribed Quadrilateral			

Central Angles, Inscribed Angles, and Inscribed Quadrilaterals

GUIDED NOTES

Central Angles



- A **central angle** is an angle with its vertex at the ___________ of the circle and its two sides are _____________. The sum of all central angles in a circle is __________.

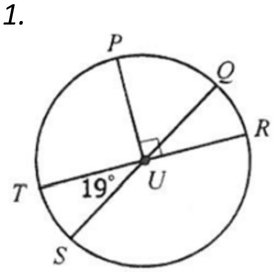
  Example: _____________

- The **degree of the arc** formed by the endpoints of a central angle is ____________ to the degree of the central angle.

  $m\overset{\frown}{AC}$ = ________; $m\overset{\frown}{ADC}$ = _________

Example!

1.



$m\overset{\frown}{PQ}$ = ________

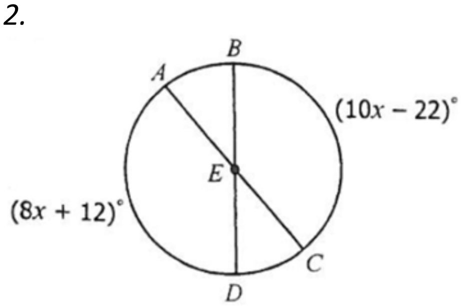
$m\overset{\frown}{SR}$ = ________

$m\overset{\frown}{QRT}$ = ________

$m\overset{\frown}{PSR}$ = ________

$m\overset{\frown}{PS}$ = ________

2.



x = ________

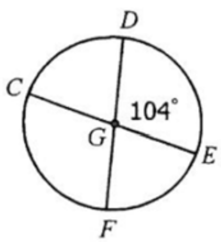
$m\overset{\frown}{AD}$ = ________

$m\overset{\frown}{CD}$ = ________

$m\overset{\frown}{BDC}$ = ________

SELF CHECK

1.



$m\widehat{DE}$ = ________

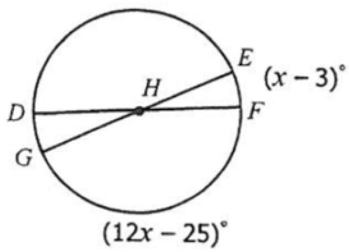
$m\widehat{FE}$ = ________

$m\widehat{DEF}$ = ________

$m\widehat{CFD}$ = ________

$m\widehat{DFE}$ = ________

2.



x = ________

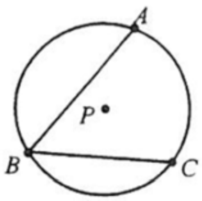
$m\widehat{DE}$ = ________

$m\widehat{EF}$ = ________

$m\widehat{DFG}$ = ________

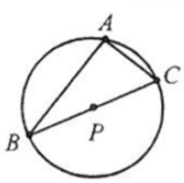
GUIDED NOTES

Inscribed Angles



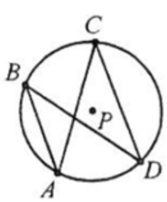
***m∠ABC* =** ____________

- An **inscribed angle** is an angle with its vertex ______ the circle with two sides that are _____________.
- An **intercepted arc** is the arc that lies between the ______________ of an inscribed angle.
- The **degree of the inscribed angle** is equal to _________ the measure of its **intercepted arc**.



If an inscribed angle **intercepts a diameter**, then it is a _____________ angle.

***m∠BAC* =** ____________

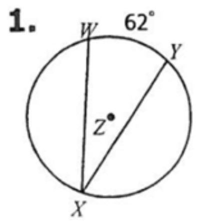


If two inscribed angles intercept the **same arc**, then the angles are ___________________.

***m∠ABD* =** ____________

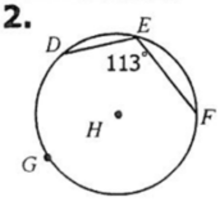
Example!

1.



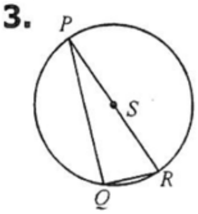
$m\angle WXY =$ ______

2.



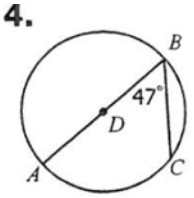
$m\widehat{DGF} =$ ______

3.



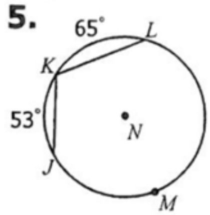
$m\angle PQR =$ ______

4.



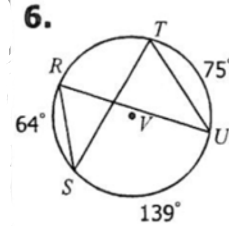
$m\widehat{BC} =$ ______

5.



$m\angle JKL =$ ______

6.

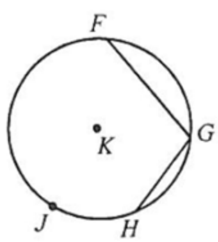


$m\angle RST =$ ______

$m\angle RUT =$ ______

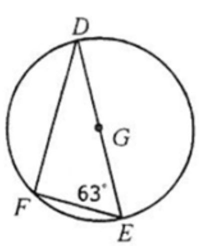
7.

If $m\angle FGH = (6x + 21)^\circ$ and $m\widehat{FJH} = (17x - 28)^\circ$, find $m\widehat{FJH}$.



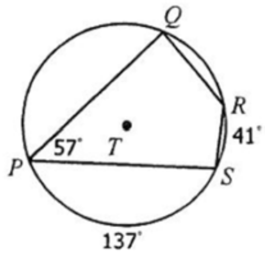
SELF CHECK

1.



$m\widehat{FE} =$ ______

2.



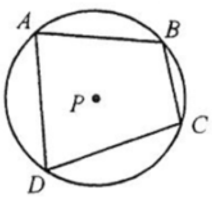
$m\angle Q$ = ______

$m\angle R$ = ______

$m\angle S$ = ______

GUIDED NOTES

Inscribed Quadrilaterals

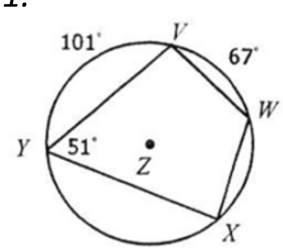


If a quadrilateral is inscribed in a circle, then its opposite angles are ____________________.

____________________ and ____________________

Example!

1.

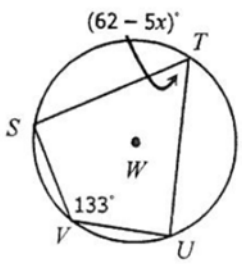


$m\angle V$ = ______

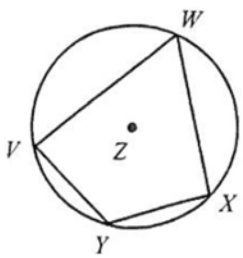
$m\angle W$ = ______

$m\angle X$ = ______

2. Solve for x.

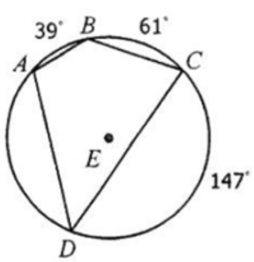


3.
If $m\angle W = (5x + 1)^\circ$ and $m\angle Y = (13x - 37)^\circ$, find $m\angle Y$.



SELF CHECK

1.



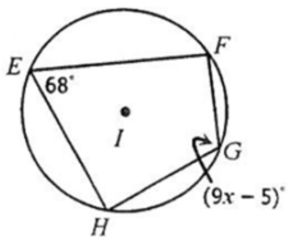
$m\angle A =$ ______

$m\angle B =$ ______

$m\angle C =$ ______

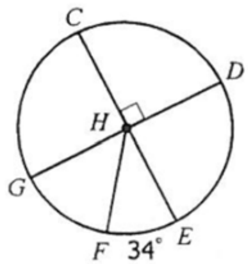
$m\angle D =$ ______

2. Solve for x.



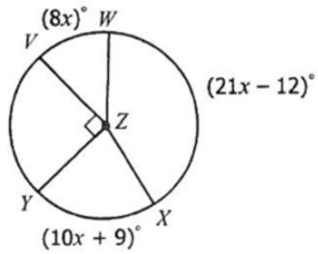
If a central angle and an inscribed angle intercept the same arc, what will be the relationship between the angle measures?

1.



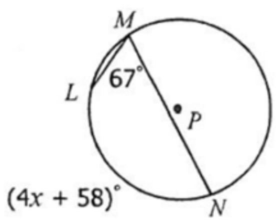
$m\overset{\frown}{CD}$ = ________
$m\overset{\frown}{FD}$ = ________
$m\overset{\frown}{DCF}$ = ________
$m\overset{\frown}{GDF}$ = ________

2.



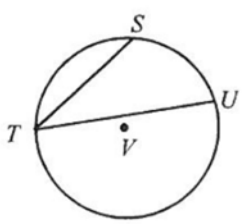
x = ________
$m\overset{\frown}{WX}$ = ________
$m\overset{\frown}{YW}$ = ________
$m\overset{\frown}{YX}$ = ________
$m\overset{\frown}{VXW}$ = ________

3. Solve for x.

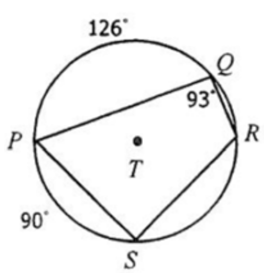


4.

If $m\angle STU = (5x - 16)^\circ$ and $m\overset{\frown}{SU} = (12x - 50)^\circ$, find $m\angle STU$.



5.



$m\angle P$ = ________
$m\angle R$ = ________
$m\angle S$ = ________

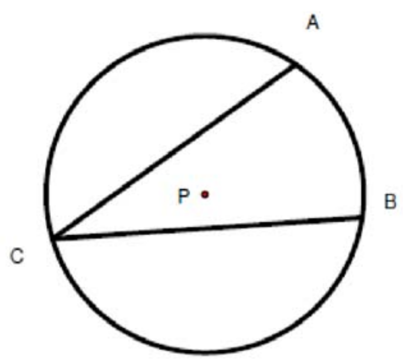
Investigating Angle Relationships in Circles

In this task, you will be investigating, discovering, and proving two theorems that involve circles and their inscribed angles. Afterwards, you will be expected to memorize and apply these theorems, and several others that you will be shown, to solve problems.

Part 1: Inscribed Angles

Definition: an inscribed angle is an angle whose vertex lies on the circle and whose sides are chords of the circle.

In ○P, ∠ACB is an inscribed angle.



1. Sketch another inscribed angle in ○P.

2. Now, you need to investigate the measure of an inscribed angle and its intercepted arc by following your teacher's instructions.

3. Write your conjecture here:

Remember that a conjecture is not a theorem until it has been proved.

Part 2: Quadrilaterals Inscribed in a Circle

4. Define quadrilateral.

A polygon is inscribed in a circle when every vertex of the polygon is on the circle.

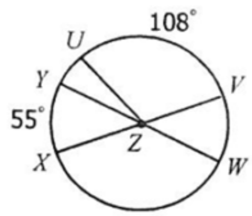
5. Sketch a picture of a circle P with an inscribed quadrilateral ABCD.

6. Now, you will investigate the relationships among the angles of the quadrilateral inscribed in a circle.

7. Write your conjecture here:

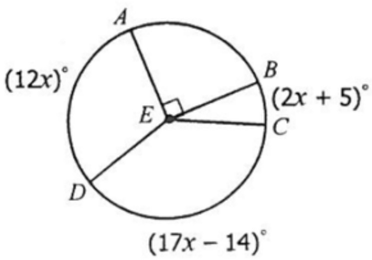
8. Write a proof of the theorem using your sketch from above.

1.



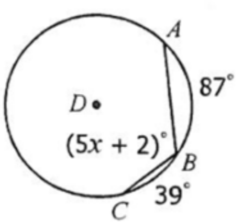
$m\widehat{YU}$ = ________
$m\widehat{XW}$ = ________
$m\widehat{XVW}$ = ________
$m\widehat{VW}$ = ________
$m\widehat{YWU}$ = ________

2.



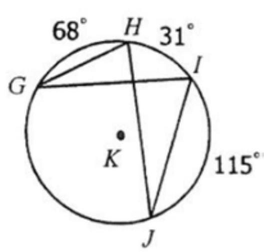
x = ________
$m\widehat{AD}$ = ________
$m\widehat{BC}$ = ________
$m\widehat{DC}$ = ________
$m\widehat{DBC}$ = ________

3. Find the value of x.



x = ________

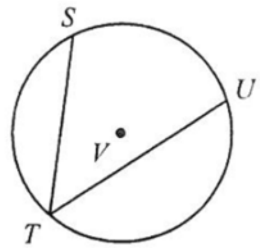
4.



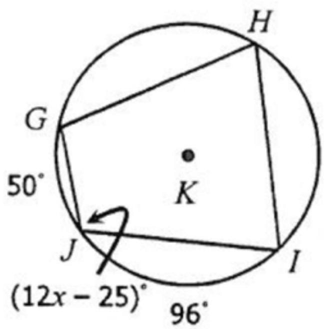
$m\angle GHJ$ = ______
$m\angle GIJ$ = ______

5.

If $m\angle STU = (13x - 6)^\circ$, $m\overset{\frown}{SU} = (21x + 8)^\circ$, and $m\overset{\frown}{UT} = 143^\circ$, find $m\overset{\frown}{ST}$.

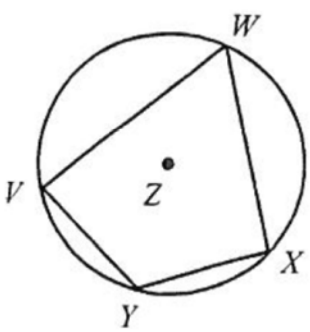


6. Solve for x.



7.

If $m\angle W = (5x + 1)^\circ$ and $m\angle Y = (13x - 37)^\circ$, find $m\angle Y$.



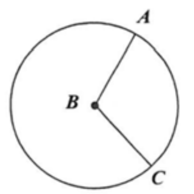
Term	Definition	Notation	Diagram/Visual
Arc Measure			
Overlapping Arcs			
Intersecting Chords or Secants on the interior			
Intersecting Chords/Tangents/ Secants on the circle			
Intersecting Secants on the exterior			
Intersecting Secants and Tangents on the exterior			
Intersecting Tangents on the exterior			

Arc and Angle Measures

GUIDED NOTES

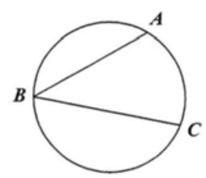
ARC & ANGLE MEASURES IN CIRCLES

Central Angles



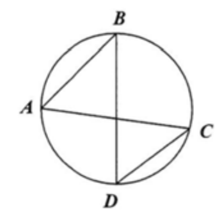
$m\angle ABC = m\overset{\frown}{AC}$

Inscribed Angles



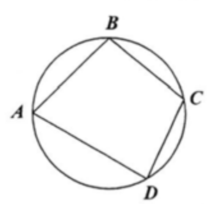
$m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$

Overlapping Arcs



$m\angle ABD = m\angle ACD$

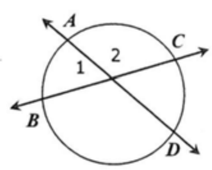
Inscribed Quadrilaterals



$m\angle A + m\angle C = 180°$

$m\angle B + m\angle D = 180°$

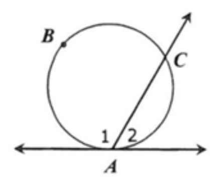
Intersecting Chords or Secants (on the Interior)



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AC} + m\overset{\frown}{BD})$

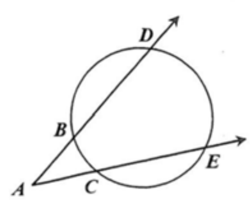
Intersecting Tangents & Chords/Secants (on the circle)



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{ABC})$

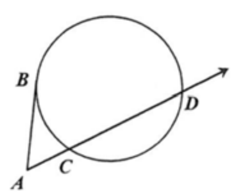
$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AC})$

Intersecting Secants (on the Exterior)



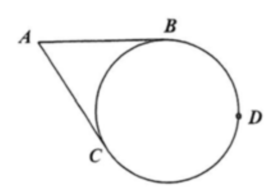
$m\angle A = \frac{1}{2}(m\overset{\frown}{DE} - m\overset{\frown}{BC})$

Intersecting Secants & Tangents (on the Exterior)



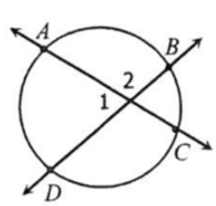
$m\angle A = \frac{1}{2}(m\overset{\frown}{BD} - m\overset{\frown}{BC})$

Intersecting Tangents (on the Exterior)



$m\angle A = \frac{1}{2}(m\overset{\frown}{BDC} - m\overset{\frown}{BC})$

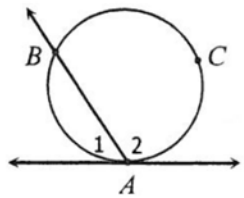
If two secants or chords intersect inside a circle, then the measure of the angle formed is equal to half the sum of the measures of the intercepted arcs.



$m\angle 1 =$ ______________________

$m\angle 2 =$ ______________________

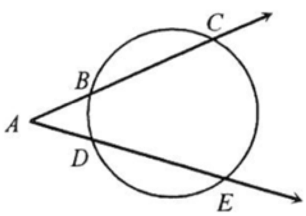
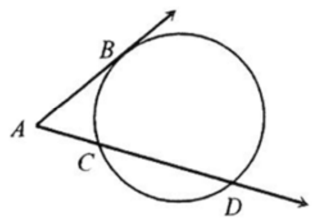
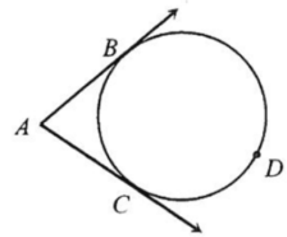
If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is equal to half the measure of its intercepted arc.



$m\angle 1$ = ______________________

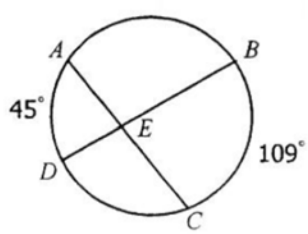
$m\angle 2$ = ______________________

If secants and/or tangents intersect on the exterior of a circle, then the measure of the angle formed is equal to half the difference of the intercepted arcs.

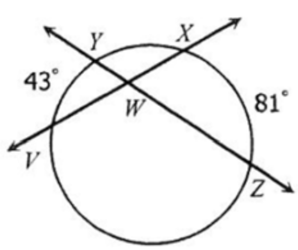
TWO SECANTS	SECANT & TANGENT	TWO TANGENTS
		
$m\angle A$ = ______________	$m\angle A$ = ______________	$m\angle A$ = ______________

Example!

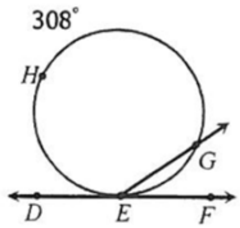
1.
$m\angle AED$



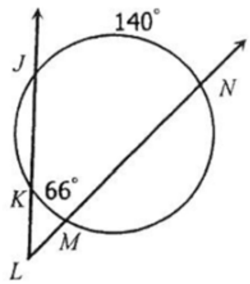
2.
$m\angle YWX$



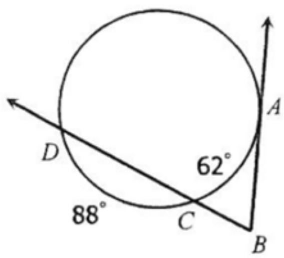
3.
$m\angle DEG$



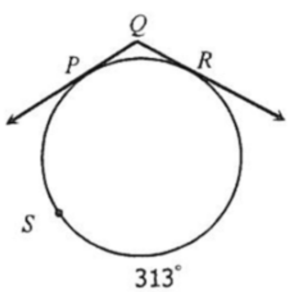
4.
$m\angle KLM$



5.
$m\angle ABC$

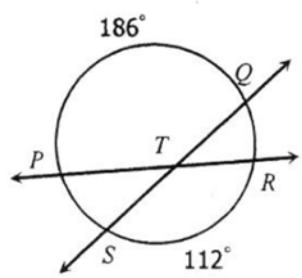


6.
$m\angle PQR$

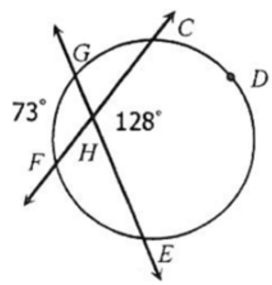


SELF CHECK

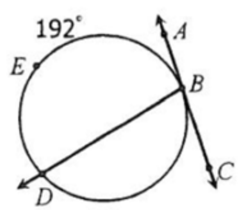
1.
$m\angle STR$



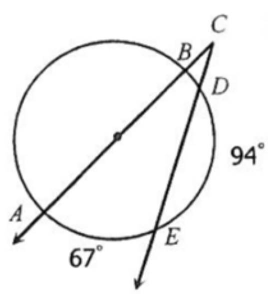
2.
$m\overset{\frown}{CDE}$



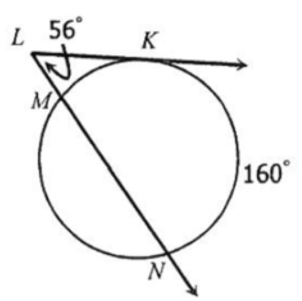
3.
$m\angle DBC$



4.
$m\angle BCD$



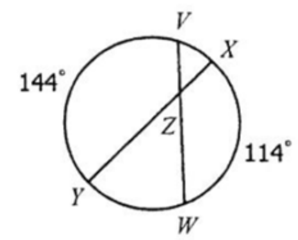
5.
$m\overset{\frown}{MK}$



What patterns/relationships do you see with the different formulas? Does the location where the lines intersect make a difference?

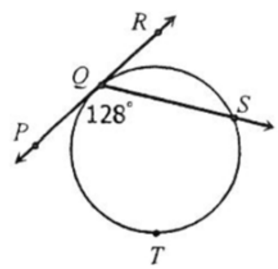
1.

$m\angle YZV$



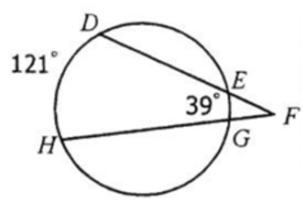
2.

$m\widehat{QS}$



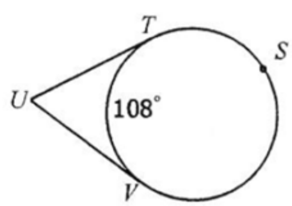
3.

$m\angle EFG$



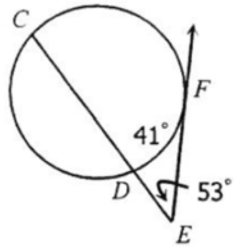
4.

$m\angle TUV$

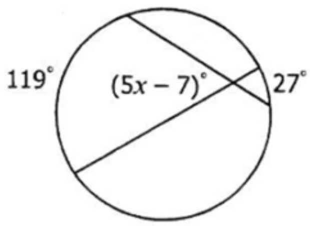


5.

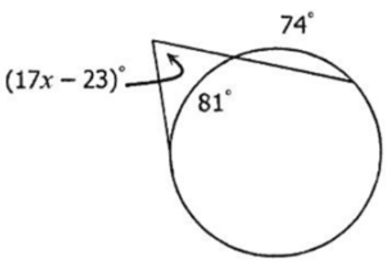
$m\widehat{CF}$



6. Solve for x.

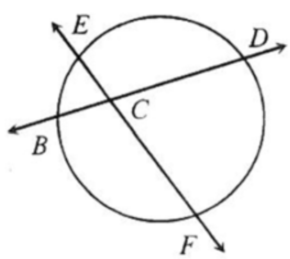


7. Solve for x.



8.

If $m\overset{\frown}{ED} = (9x - 3)^\circ$, $m\overset{\frown}{BF} = (15x - 39)^\circ$, and $m\angle BCF = (11x - 9)^\circ$, find $m\overset{\frown}{ED}$.



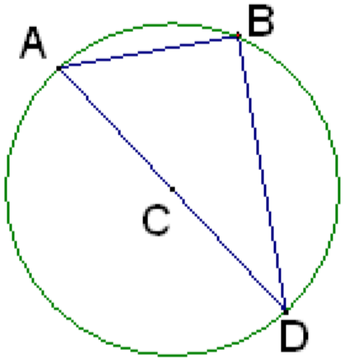
Investigating Angle Relationships in Circles (continued from previous task)

Part 3: Graphic Organizer for Angle Theorems

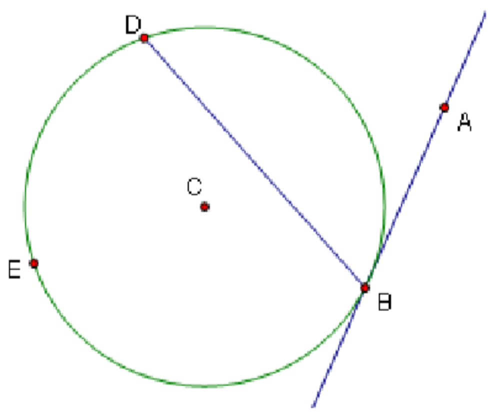
Location of the Vertex	Picture	Theorem
Inside the circle **At the Center**		
Not at the center		
Outside of the circle		
On the circle		

Part 4: Apply these theorems to solve these special cases of inscribed angles.

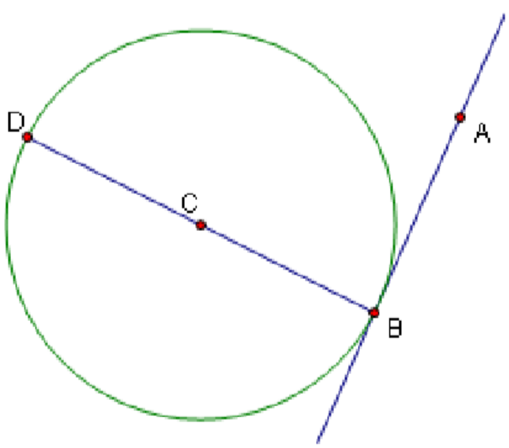
1. Find the m∠ABD, the inscribed angle of ○C.



2. Find the m∠ABD, the inscribed angle of ○C, if $m\widehat{BED} = 300°$.

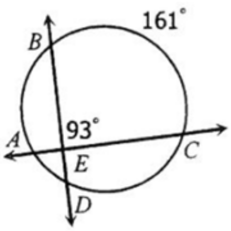


3. Find the m∠ABD, the inscribed angle of ○C.



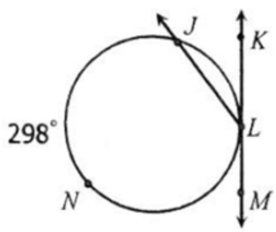
1.

$m\widehat{AD}$



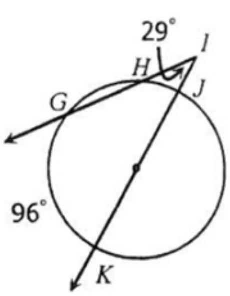
2.

$m\angle JLK$



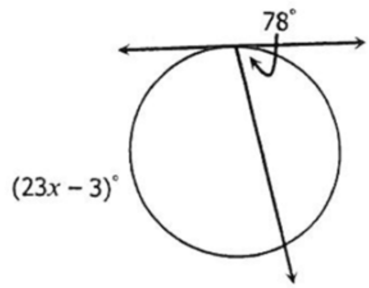
3.

$m\widehat{HJ}$



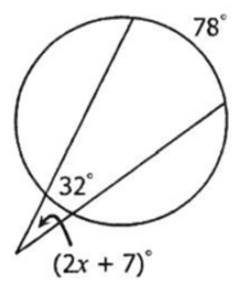
4.

Solve for x.



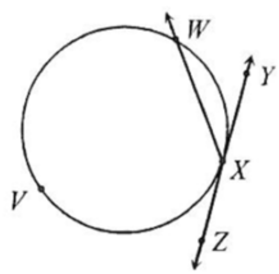
5.

Solve for x.



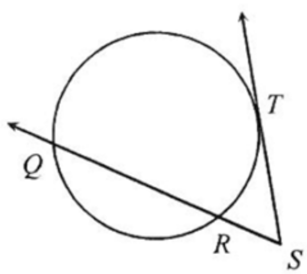
6.

If $m\widehat{WVX} = (13x + 9)^\circ$ and $m\angle WXZ = (5x + 36)^\circ$, find $m\angle WXY$.



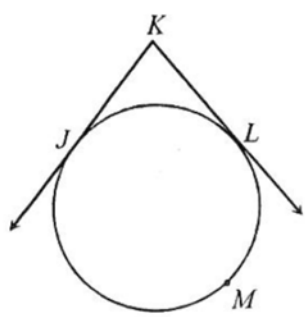
7.

If $m\widehat{QT} = (27x + 3)^\circ$, $m\widehat{RT} = (9x - 5)^\circ$ and $m\angle RST = (10x - 2)^\circ$, find $m\widehat{RT}$.



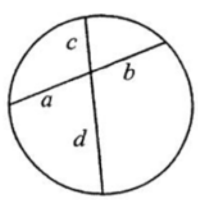
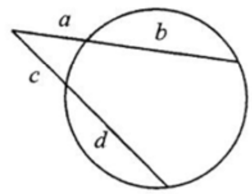
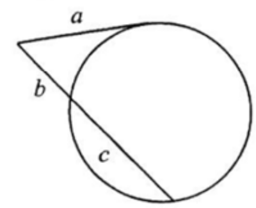
8.

If $m\angle JKL = (8x - 6)^\circ$ and $m\widehat{JML} = (25x - 13)^\circ$, find $m\widehat{JML}$.



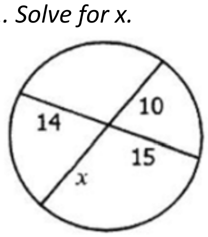
Segment Lengths

GUIDED NOTES

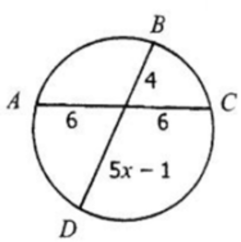
TYPE 1	TYPE 2	TYPE 3
Intersecting chords (or secants) inside the circle. 	Intersecting secants outside the circle. 	Intersecting secant and tangent outside the circle. 

Example!

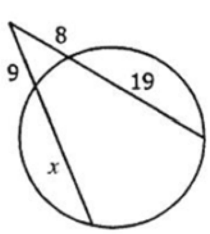
1. Solve for *x*.



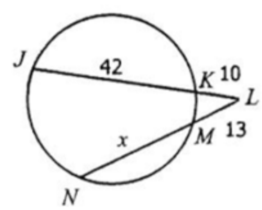
2. Find *BD*.



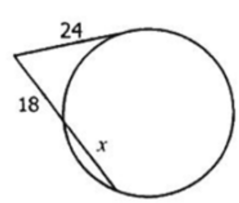
3. Solve for *x*.



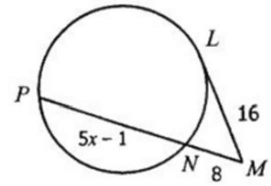
4. Find *NL*.



5. Solve for *x*.

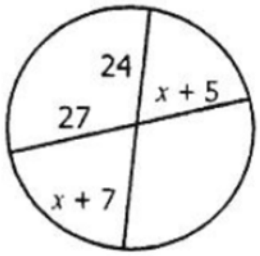


6. Find *PN*.

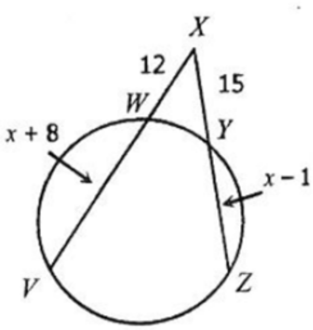


SELF CHECK

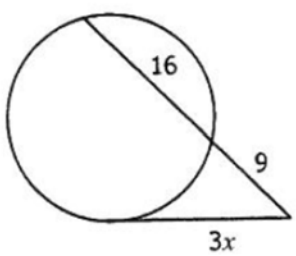
1. Solve for x.



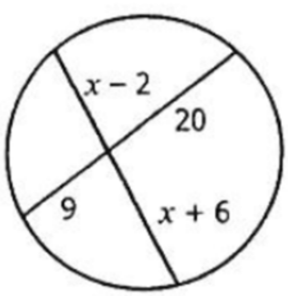
2. Find YZ.



3. Find x.



4. Solve for x.

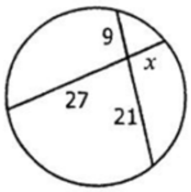


1. In what type of intersection might you need factor the equation like you have done in the past?

2. What is the difference between a tangent and a secant?

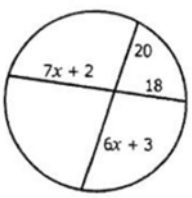
1.

Solve for x.



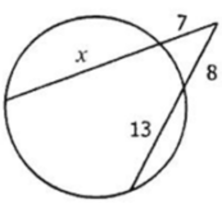
2.

Solve for x.



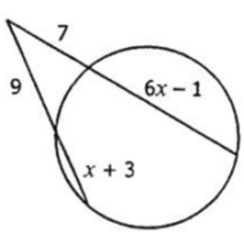
3.

Solve for x.



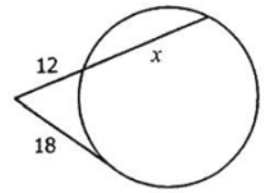
4.

Solve for x.



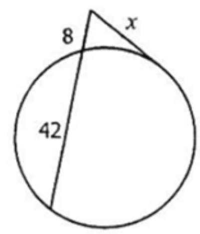
5.

Solve for x.



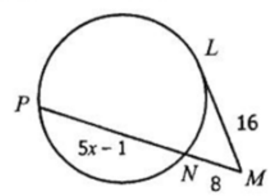
6.

Solve for x.



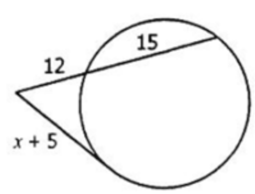
7.

Find PN.



8.

Solve for x.

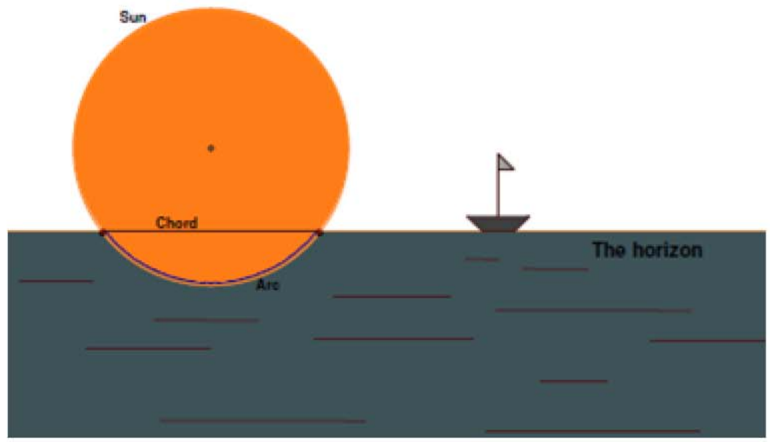


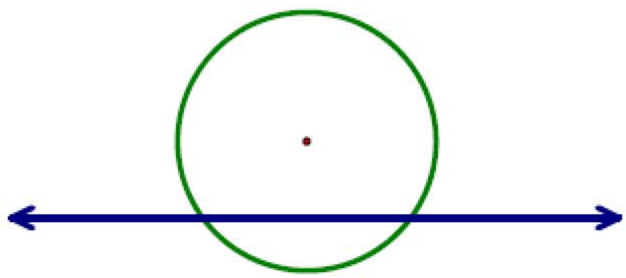
Chords, Secants, and Tangents

Part 1: Sunrise on the First Day of the New Year

It is customary for people in Asia to visit the seashores on the eastern sides of their countries on the first day of the year. While watching the sun rise over the ocean, visitors wish for good luck in the New Year.

As the sun rises, the horizon cuts the sun at different positions. Although a circle is not a perfect representation of the sun, we can simplify this scene by using a circle to represent the sun and a line to represent the horizon.





1. Using the simplified diagram above, sketch and describe the different types of intersections the sun and the horizon may have.

2. Definitions:
A **tangent line** is a line that intersects a circle in exactly one point.
A **secant line** intersects a circle in two points.

Do any of your sketches contain tangent or secant lines?
If so, label them.

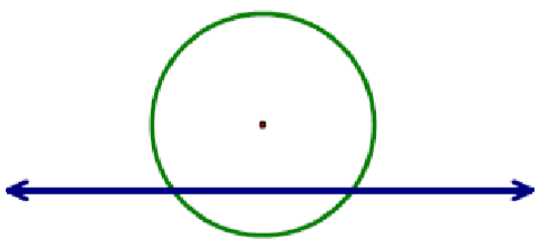
Is it possible for a line to intersect a circle in 3 points? 4 points?
Explain why or why not.

3. When a secant line intersects a circle in two points, it creates a chord. As you have already learned, a **chord** is a segment whose endpoints lie on the circle. How does a chord differ from a secant line?

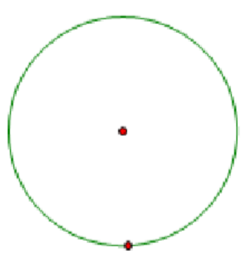
4. Look again at our representation of the sun and the horizon.

Let *d* represent the distance between the center of a circle and a line *l*. Let *r* represent the length of a radius of the circle.

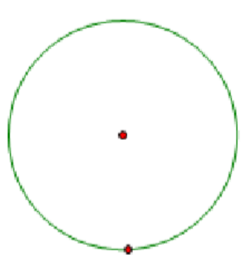
a. Draw and describe the relationship between *d* and *r* when *l* is a secant line



b. Draw and describe the relationship between *d* and *r* when *l* is a tangent line



c. Draw and describe the relationship between *d* and *r* when *l* does not intersect the circle



5. You just compared the length of a radius of a circle to the distance from the center of the circle to a tangent line. What does this comparison tell you about the relationship of a tangent line to a radius at the point of tangency?

Part 2: The Segment Theorems Graphic Organizer
In the remaining items of this task, we will work with the relationships between the lengths of the segments created when these lines intersect.

Picture	Type	Theorem
	2 tangents **vertex outside**	
	2 secants **vertex outside**	
	Secant and tangent **vertex outside**	
	2 secants **VERTEX INSIDE**	

Real Mathematics:
Application

Part 3: Apply the Theorems to Solve Problems

Sketch a picture for each problem, choose a theorem, set up an equation, and then solve.

1. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside a circle at point E. $AE = 2$, $CE = 4$, and $ED = 3$. Find EB.

2. A diameter of a circle is perpendicular to a chord whose length is 12 inches. If the length of the shorter segment of the diameter is 4 inches, what is the length of the longer segment of the diameter?

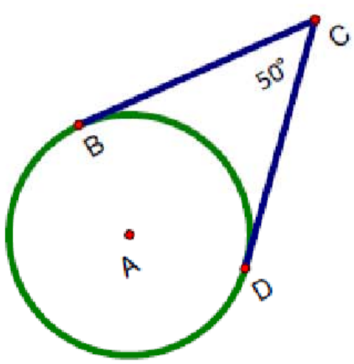
3. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside a circle at point E. $AE = 5$, CE $= 10$, $EB = x$, and $ED = x\text{-}4$. Find EB and ED.

4. Two secant segments are drawn to a circle from a point outside the circle. The external segment of the first secant segment is 8 centimeters and its internal segment is 6 centimeters. If the entire length of the second secant segment is 28 centimeters, what is the length of its external segment?

5. A tangent segment and a secant segment are drawn to a circle from a point outside the circle. The length of the tangent segment is 15 inches. The external segment of the secant segment measures 5 inches. What is the measure of the internal secant segment?

6. The diameter of a circle is 19 inches. If the diameter is extended 5 inches beyond the circle to point *C*, how long is the tangent segment from point *C* to the circle?

7. A satellite orbits the earth so that it remains at the same point above the Earth's surface as the Earth turns. If the satellite has a 50° view of the equator, what percent of the equator can be seen from the satellite?

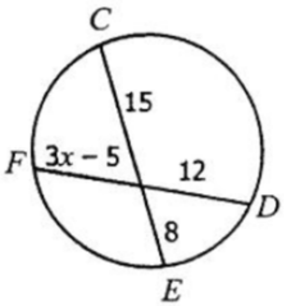


8. The average radius of the Earth is approximately 3959 miles.
a. How far above the Earth's surface is the satellite described in *Problem 7?*

b. What is the length of the longest line of sight from the satellite to the Earth's surface? Identify this line of sight using the diagram.

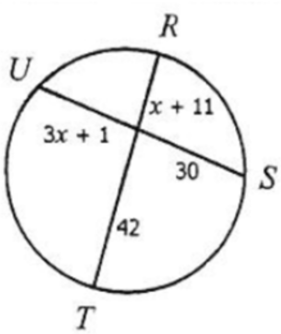
1.

Find FD.



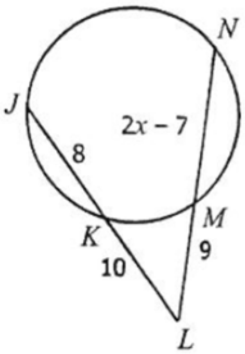
2.

Find US.



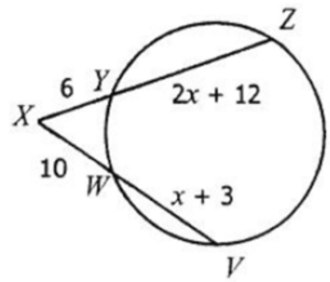
3.

Find MN.



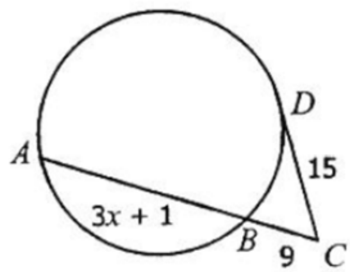
4.

Find YZ.



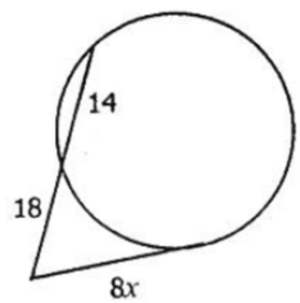
5.

Find AC.



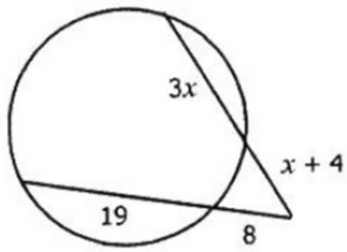
6.

Solve for x.



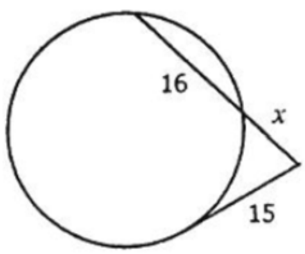
7.

Solve for x.



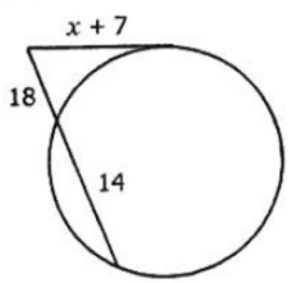
8.

Solve for x.



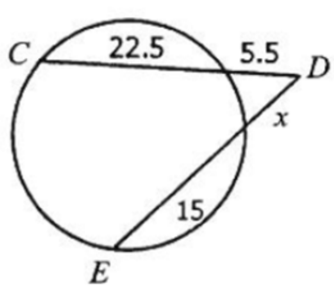
9.

Solve for x.



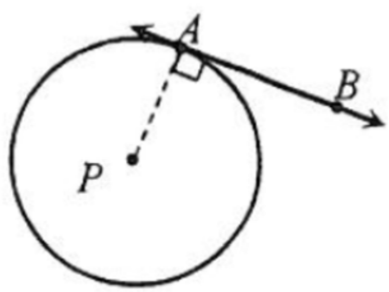
10.

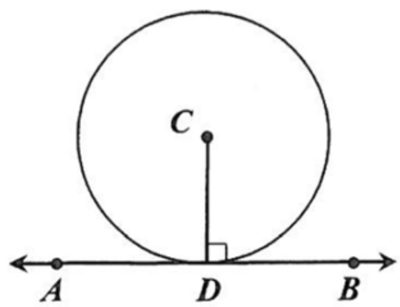
Solve for x.



Tangents

GUIDED NOTES





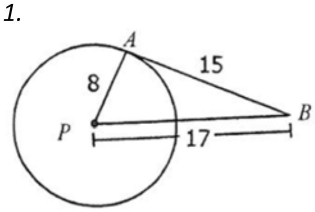
If $\overleftrightarrow{AB}$ is tangent to circle C, then $\overleftrightarrow{AB} \perp \overline{CD}$

- A **tangent line** intersects a circle at exactly ________ point, called the **point of tangency.**
- A line is tangent to a circle if and only if it is ______________________ to a _____________ drawn to the point of tangency.

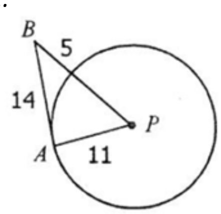
Example!

Determine if $\overline{AB}$ is tangent to circle P.

1.

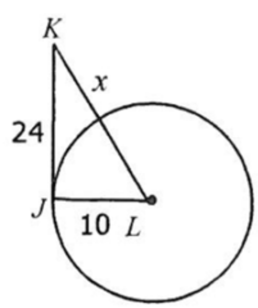


2.



3.

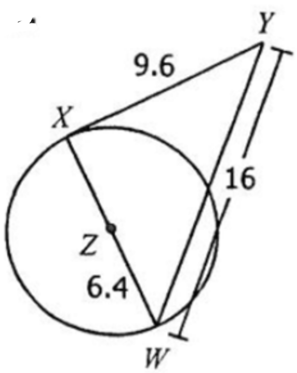
If $\overline{JK}$ is tangent to circle L, find x.



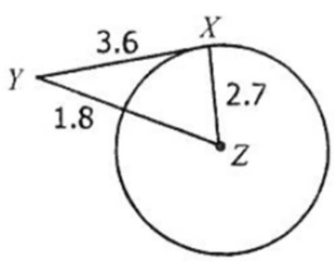
SELF CHECK

Determine if $\overline{XY}$ is tangent to circle Z.

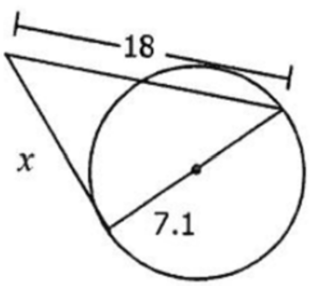
1.



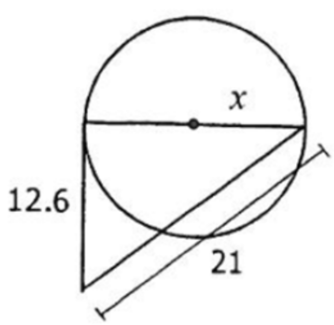
2.



3. Find the value of x. Assume the segments that appear to be tangent are tangent.



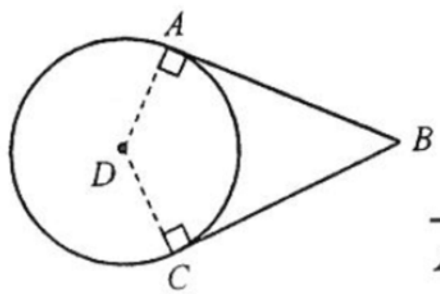
4. Find the value of x. Assume the segments that appear to be tangent are tangent.



Tangents

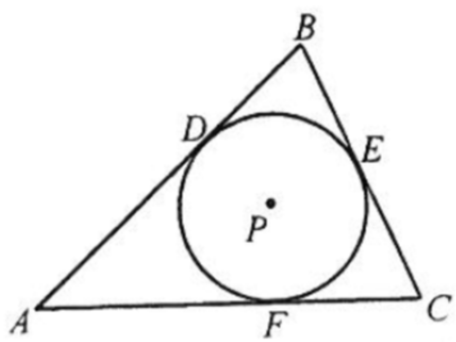
GUIDED NOTES

If **two segments from the same external point** are **tangent** to a circle, then they are **congruent**.



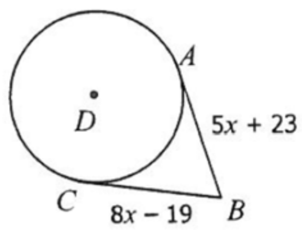
$\overline{AB} \cong \overline{CB}$

If a **polygon is circumscribed** around a circle, then **all sides are tangent**.

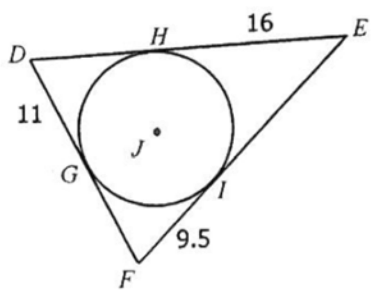


Example!

1. Find x.

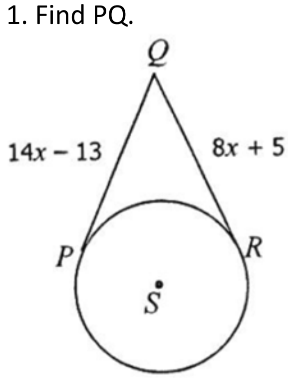


2. Find the perimeter of ΔDEF.

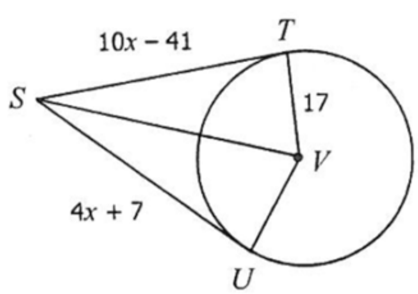


SELF CHECK

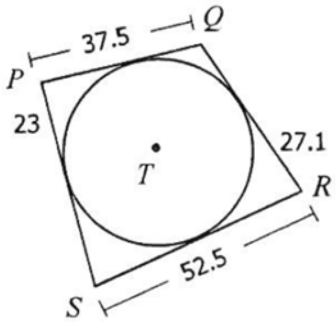
1. Find PQ.



2. Find SV.



3. Find the perimeter of PQRS.



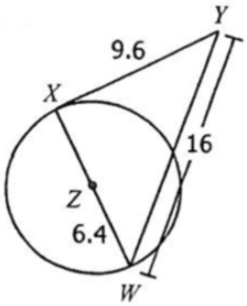
How do I know a line is tangent to the circle if the problem doesn't state it?

Geometry Unit 4 Concept 2

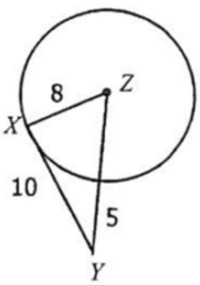
Lesson Name: G.U4.C2.E.03.**ClassWk.Tangents**

Determine if XY is tangent to Circle Z.

1.

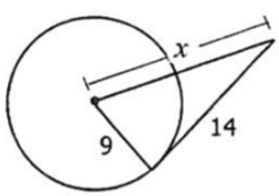


2.

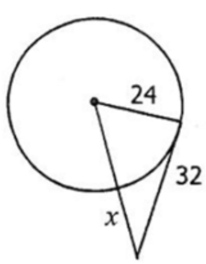


Find the value of x. Assume the segments that appear to be tangent are tangent.

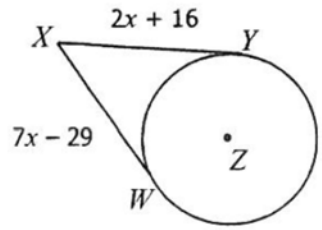
3.



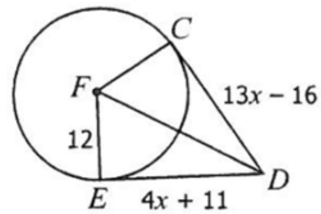
4.



5. Find WX.

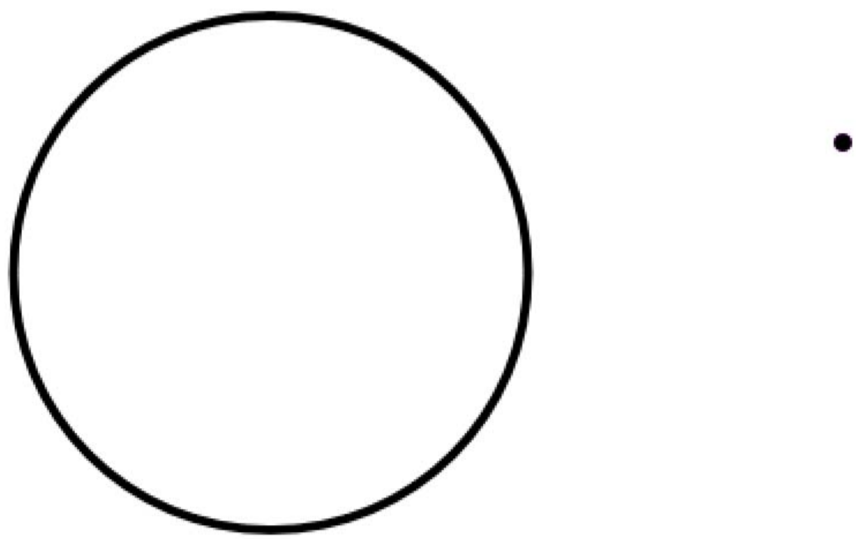


6. Find FD



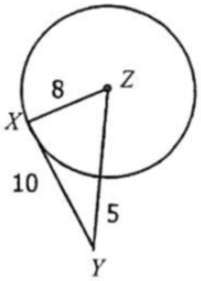
Euclidean Geometry Constructions

1. Using a Compass and Straight Edge, Construct a Tangent Line to a circle from a given exterior point.

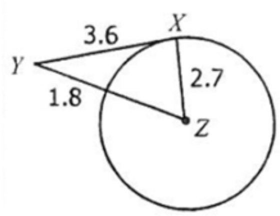


Determine if XY is tangent to Circle Z.

1.

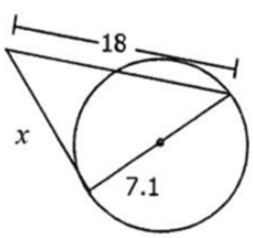


2.

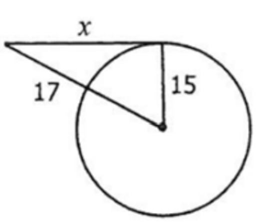


Find the value of x. Assume the segments that appear to be tangent are tangent.

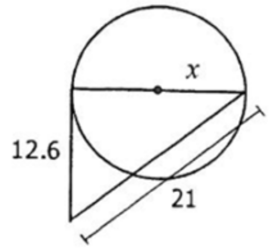
3.



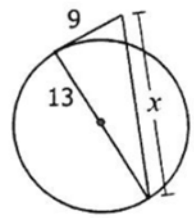
4.



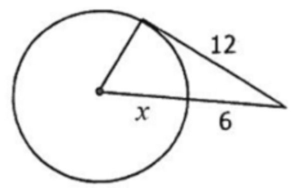
5.



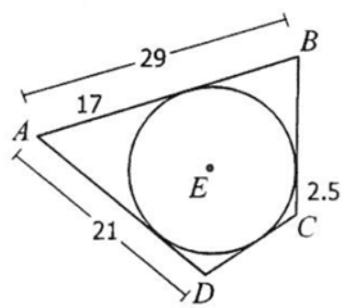
6.



7.

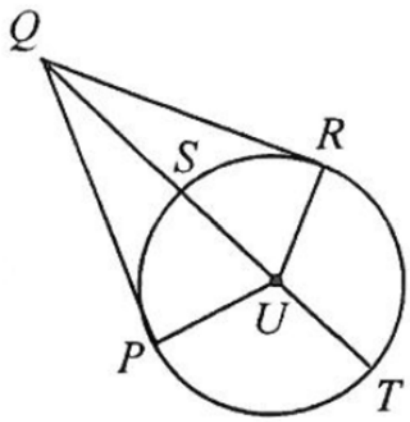


8. Find the perimeter of ABCD



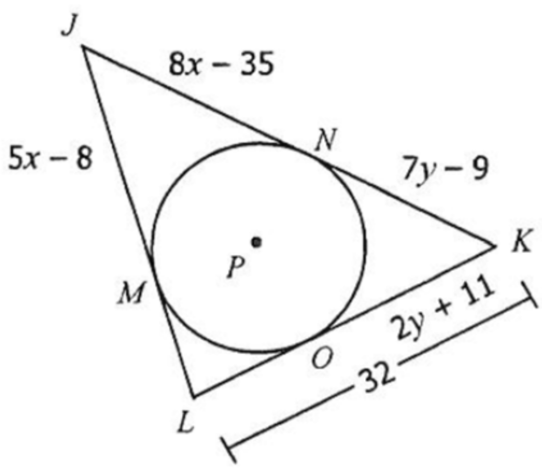
9.

If $PQ = 4x + 2$, $QR = 7x - 19$, and $QU = 34$, find ST.

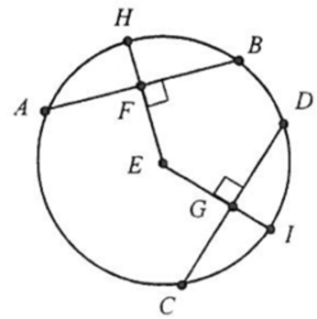


10.

Find the perimeter of ΔJKL.



Congruent Chords and Arcs



- Two chords are congruent if and only if:

a) ____

____________________ ↔ ____________________

b) ____

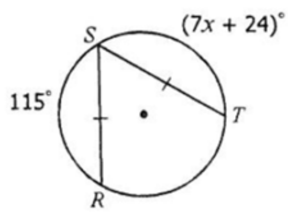
____________________ ↔ ____________________

- If a diameter or radius is ____________________ to a chord, then it ______________ the ______________ and its ___________.

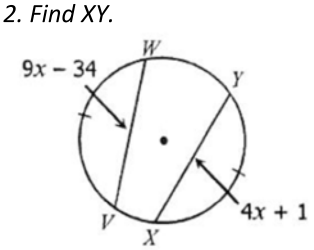
______________ → ______________ and ____________________

Example!

1. *Find x.*

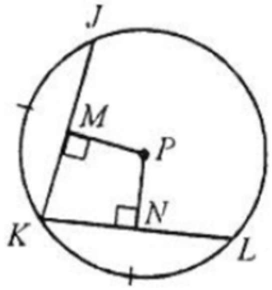


2. *Find XY.*



3.

If $MP = 5x - 34$ and $PN = 2x - 4$, find MP.



4.

In circle Z, if $RS = 18$, and $m\overset{\frown}{TY} = 42^\circ$, find each measure.

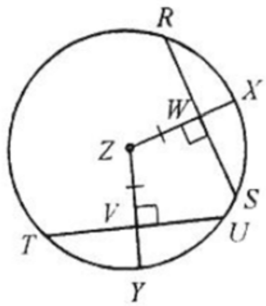
TV = ________

TU = ________

WS = ________

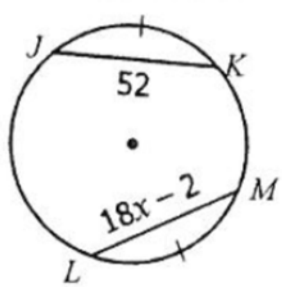
$m\overset{\frown}{YU}$ = ________

$m\overset{\frown}{RS}$ = ________



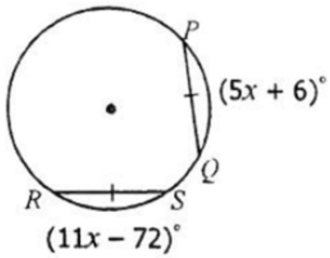
SELF CHECK

1. Find x.



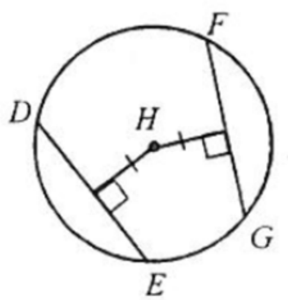
2.

Find $m\overparen{RS}$.



3.

If $DE = 11x + 15$ and $FG = 32x - 27$, find DE.



4.

In circle I, if $BG = 17$, and $m\overparen{CHA} = 256^\circ$, find each measure.

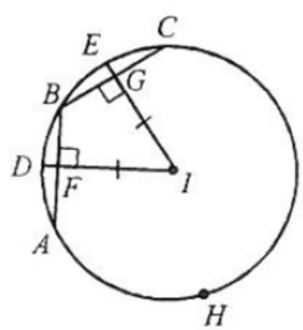
BC = ________

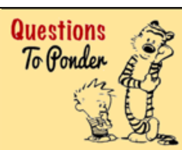
FB = ________

$m\overparen{AB}$ = ________

$m\overparen{BC}$ = ________

$m\overparen{EC}$ = ________

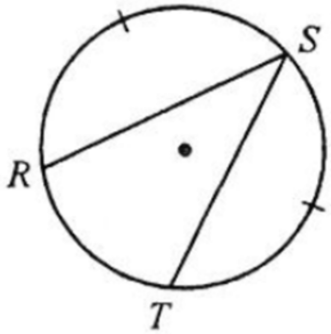




How do you prove that parallel chords cut off congruent arcs?

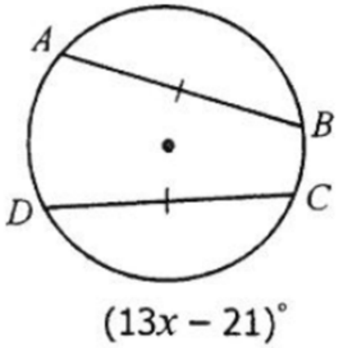
1.

If $RS = 59$ and $ST = 10x - 31$, find x.



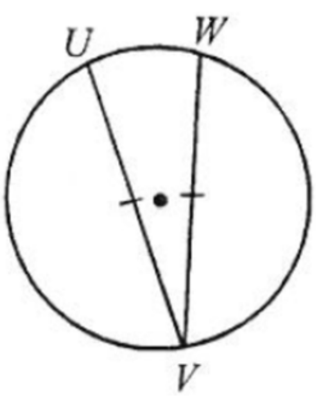
2.

If $m\overset{\frown}{AD} = 85^\circ$ and $m\overset{\frown}{BC} = 31^\circ$, find the value of x.



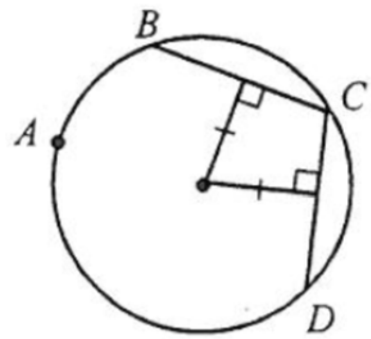
3.

If $m\overset{\frown}{UV} = (8x - 17)^\circ$ and $m\overset{\frown}{WV} = (5x + 52)^\circ$, find $m\overset{\frown}{WV}$.



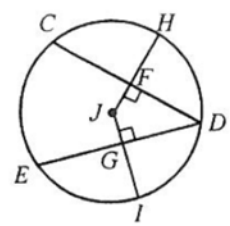
4.

If $m\overset{\frown}{BC} = (9x - 53)^\circ$ and $m\overset{\frown}{CD} = (2x + 45)^\circ$, find $m\overset{\frown}{BAD}$.



5.

If $JG = JF$, $GD = 13$, and $m\overset{\frown}{CD} = 136^\circ$, find each measure.



ED = ________

CF = ________

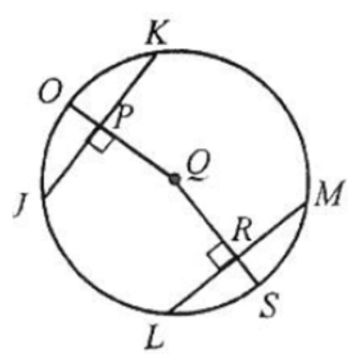
$m\overset{\frown}{ED}$ = ________

$m\overset{\frown}{HD}$ = ________

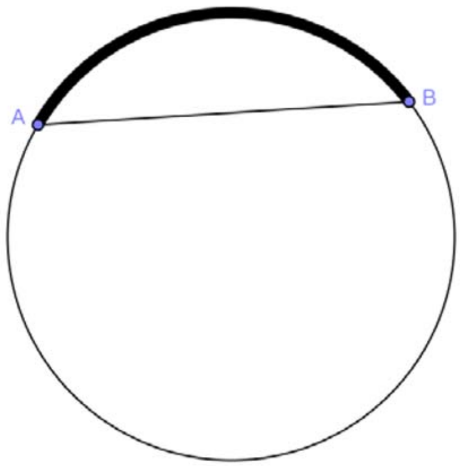
$m\overset{\frown}{CE}$ = ________

6.

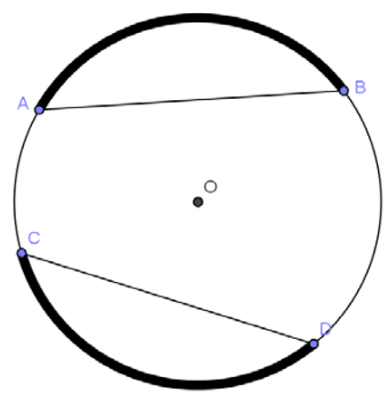
If $PQ = QR$, $JK = 3x + 23$ and $LM = 9x - 19$, find PK.



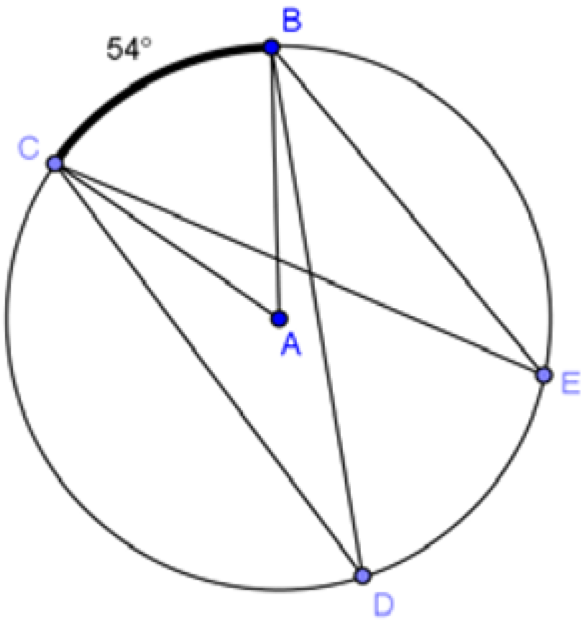
Arcs and Chords

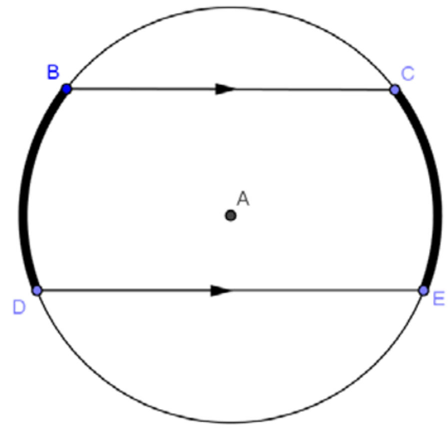


- Tell me what you see in this diagram.
- What do you notice about the chord and the minor arc?
- We say that arc $\widehat{AB}$ is subtended by chord $\overline{AB}$. Can you repeat that with me?
- What do you think we mean by the word "subtended"?
- Display circle at right. What can we say about arc $\widehat{CD}$?
- If $AB = CD$, what do you think would be true about $m\widehat{AB}$ and $m\widehat{CD}$?



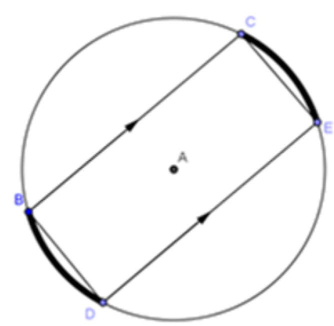
1. Given circle A with $m\widehat{BC} = 54°$ and $\angle CDB \cong \angle DBE$, find $m\widehat{DE}$. Explain your work.



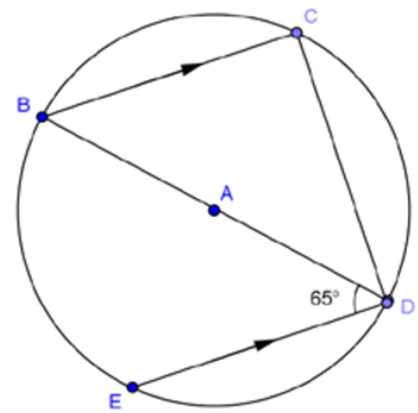


- What do you see in this diagram?
- What looks to be true about the arcs?
- This is true, and here is the theorem: *In a circle, arcs between parallel chords are congruent.*
- Let's prove this together. Construct a diameter perpendicular to the parallel chords.
- What does this diameter do to each chord?
- Reflect across the diameter (or fold on the diameter). What happens to the endpoints?
- What have we proven?
- Draw $\overline{CD}$. Can you think of another way to prove this theorem using properties of angles formed by parallel lines?

2. If two arcs in a circle have the same measure, what can you say about the quadrilateral formed by the four endpoints? Explain.



3. **Find the angle measure of $\widehat{CD}$ and $\widehat{ED}$.**

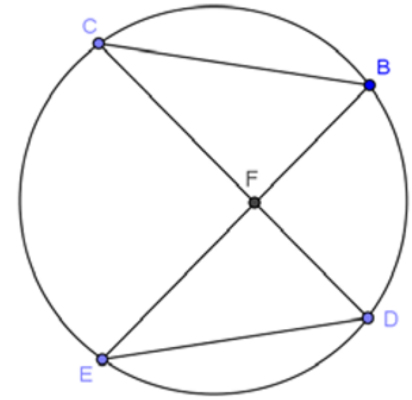


4. $m\widehat{CB} = m\widehat{ED}$ **and** $m\widehat{BC}:m\widehat{BD}:m\widehat{EC} = 1:2:4$**. Find**

a. $m\angle BCF$

b. $m\angle EDF$

c. $m\angle CFE$

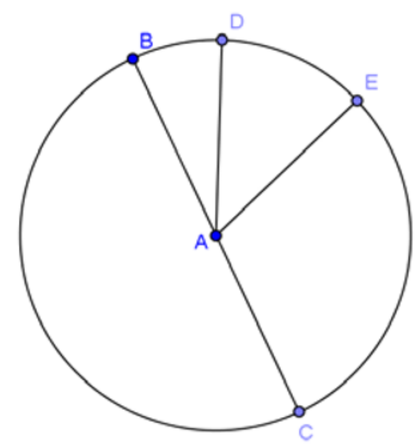


5. $\overline{BC}$ **is a diameter of circle** A**.** $m\widehat{BD}:m\widehat{DE}:m\widehat{EC} = 1:3:5$**. Find**

a. $m\widehat{BD}$

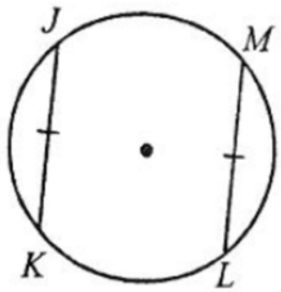
b. $m\widehat{DEC}$

c. $m\widehat{ECB}$



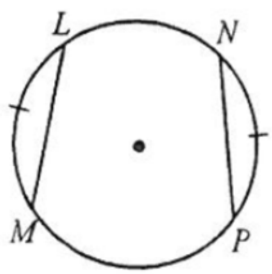
1.

If $m\widehat{JK} = (7x - 39)^\circ$ and $m\widehat{ML} = 87^\circ$, find x.



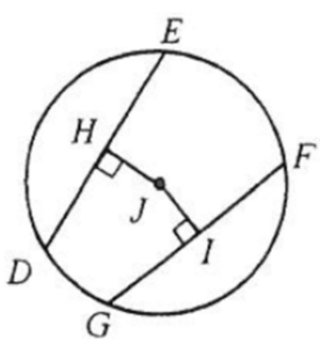
2.

If $LM = 41 - 2x$ and $NP = 7x + 5$, find LM.



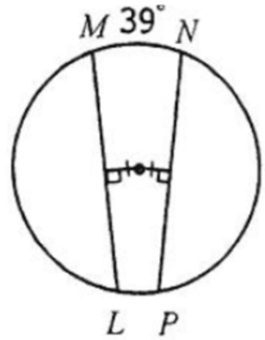
3.

If $DE = GF$, $HJ = 3x + 20$ and $JI = 15x - 64$, find JI.



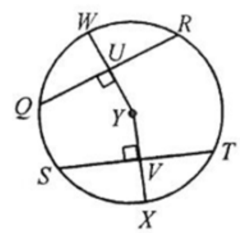
4.

If $m\widehat{LM} = (8x - 56)^\circ$ and $m\widehat{NP} = (5x + 22)^\circ$, find $m\widehat{LP}$.



5.

If $YU = YV$, $ST = 16$, $m\widehat{QS} = 34^\circ$, and $m\widehat{RT} = 98^\circ$, find each measure.



QU = ________

QR = ________

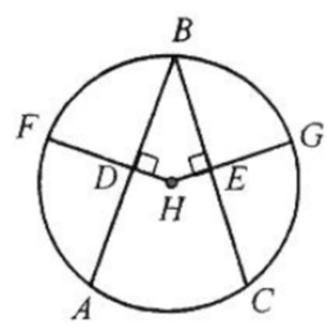
$m\widehat{ST}$ = ________

$m\widehat{QR}$ = ________

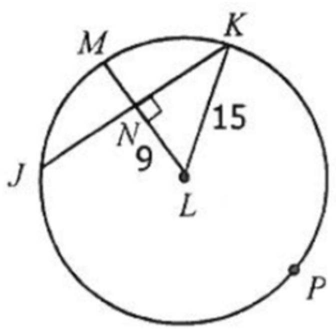
$m\widehat{XT}$ = ________

6.

If $DH = HE$, $m\widehat{BG} = (9x - 20)^\circ$ and $m\widehat{GC} = (5x + 28)^\circ$, find $m\widehat{AB}$.



For 7 – 10, use the following diagram.



7.

Find NK.

8.

Find $m\widehat{MK}$.

9.

Find JK.

10.

Find $m\widehat{JPK}$.

Arc Length and Sector Area			
Term	**Definition**	**Notation**	**Diagram/Visual**
Circumference			
Theta (θ)			
Arc			
Arc Measure			
Arc Length			
Area (of a circle)			
Sector			

Arc Length and Sector Area

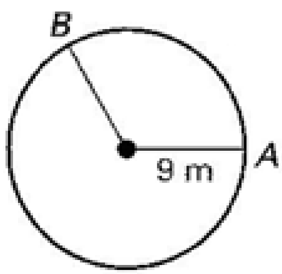
GUIDED NOTES

ARC LENGTH

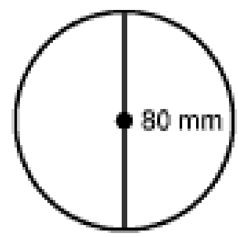
REVIEW: What information do you need to find the CIRCUMFERENCE of a circle? ___________________

What is the formula for finding the circumference? _______________________

Find the circumference of each circle below.

1. 

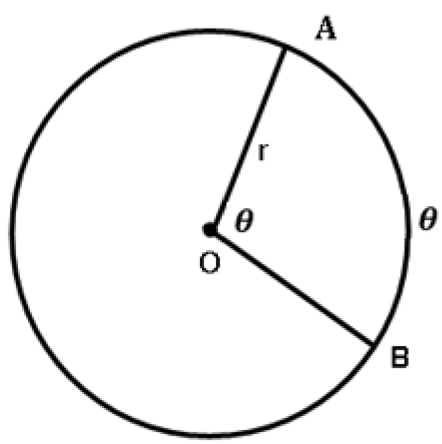
C =__________

2. 

C =__________

The formula for circumference is **C = ____.**
A circle has ______ degrees.

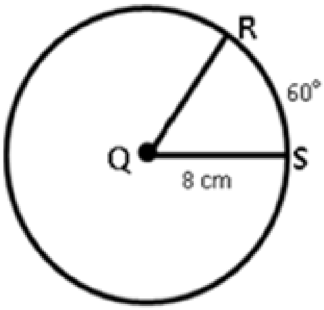
Remember: An arc is a **part** of the circumference of a circle. We will use this definition to breakdown the formula used to find the length of an arc.



$$\textbf{Arc Length} = \frac{\theta}{360}(2\pi r) = \frac{2\pi r\,\theta}{360}$$

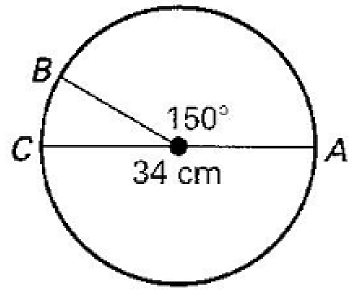
Example! **Find the arc length. Give the exact answer in terms of π and the approximate answer as a decimal to the hundredths place.**

Given $mRS = 60°$ and $QS = 8$ cm



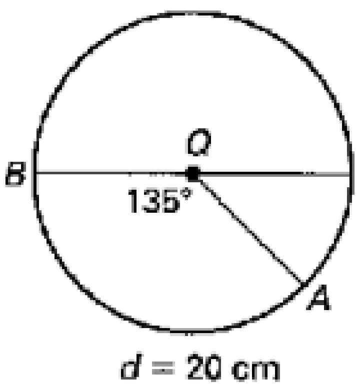
1. Length of $\overset{\frown}{RS}$ = ______≈______

Given $m\overset{\frown}{AB} = 150°$ and $AC = 34$ cm.

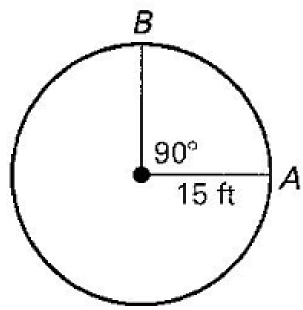


2. Length of $\overset{\frown}{AB}$ = ______≈______

SELF CHECK **Find the length of the arc AB in each circle. Give the exact answer in terms of π and the approximate answer as a decimal to the hundredths place.**



1. Length of $\overset{\frown}{AB}$ = ______≈______

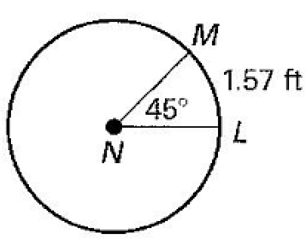


2. Length of $\overset{\frown}{AB}$ = ______≈______

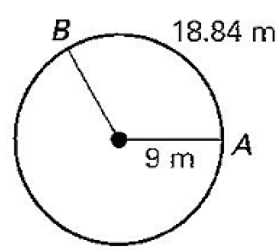
Example! **Sometimes we will be given the length of the arc and then asked to find the radius (r) or angle measure (θ).**

Find the indicated measure.

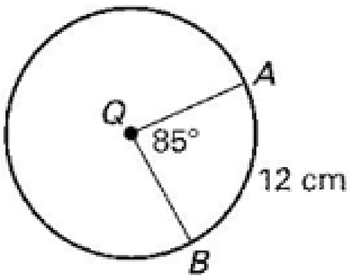
1. Radius of $\odot N$



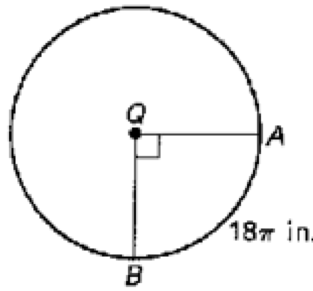
2. $m\overset{\frown}{AB}$



3. Diameter of circle *Q*.

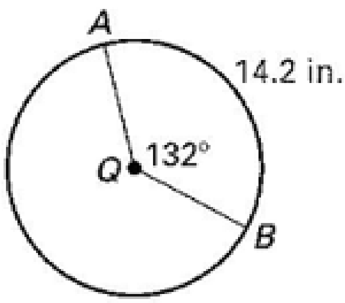


4. Length of *QB*.

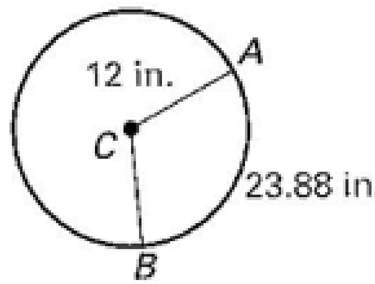


SELF CHECK **Find the indicated measure.**

1. Radius of ⊙Q



2. $m\overset{\frown}{AB}$



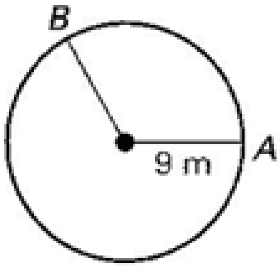
GUIDED NOTES

AREA OF A SECTOR

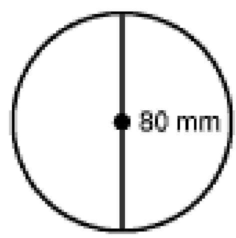
REVIEW: What information do you need to find the AREA of a circle? ________________________

What is the formula for finding the area of a circle? _______________________

Find the area of each circle below.



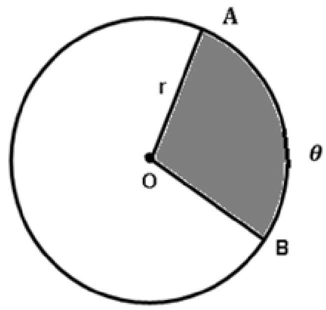
1. A =__________



2. A =__________

The formula for area is **A =** __________.
A circle has ________ degrees.

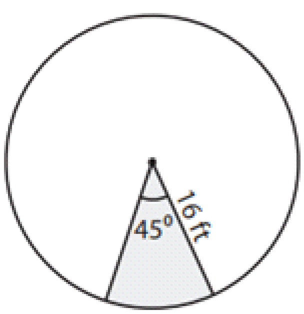
Remember: A sector arc is a **part** of the area of a circle. We will use this definition to breakdown the formula used to find the area of a sector.



$$\textbf{Sector Area} = \frac{\theta}{360}(\pi r^2) = \frac{\pi r^2 \theta}{360}$$

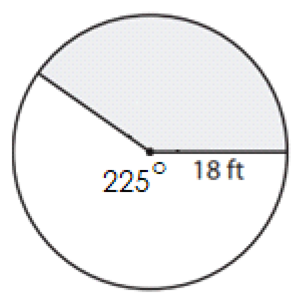
Example! **Find the area of the shaded sector. Give the exact answer in terms of π and the approximate answer as a decimal to the hundredths place.**

1.



A = _______≈_______

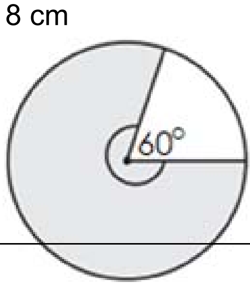
2.



A = _______≈_______

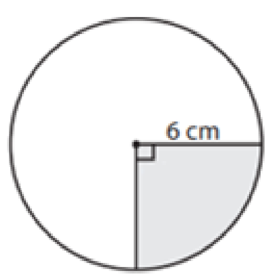
SELF CHECK **Find the area of the shaded sector. Give the exact answer in terms of π and the approximate answer as a decimal to the hundredths place.**

1. r = 8 cm



A = _______≈_______

2.



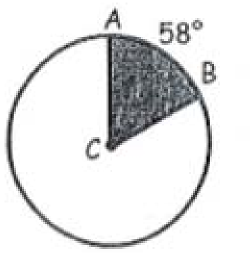
A = _______≈_______

Example!

Sometimes we will be given the area of a sector and then asked to find the radius (r) or angle measure (θ).

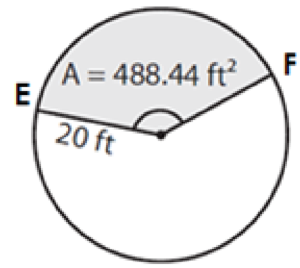
Find the indicated measure.

1. Area of sector: 36 in^2



CB = ______________

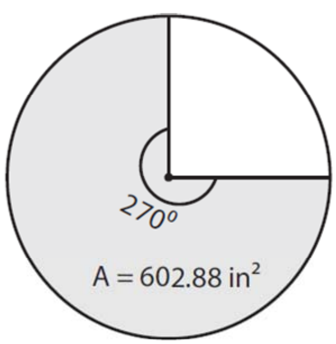
2.



$m\,\overset{\frown}{EF}$ = ______________

SELF CHECK **Find the indicated measures.**

1.

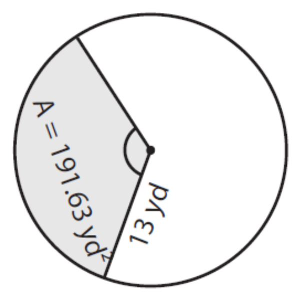


Radius = __________

Central angle = __________

Area of a sector = __________

2.



Radius = __________

Central angle = __________

Area of a sector = __________

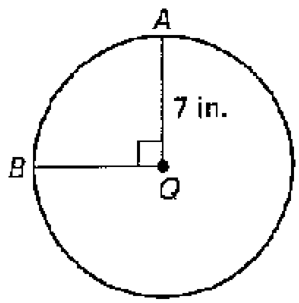
Arc Length and Sector Area

ARC LENGTH

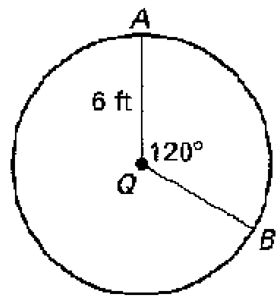
Find arc length in terms of π and to the hundredths place. Find all other measures to the nearest hundredths place.

Find the length of $\overset{\frown}{AB}$.

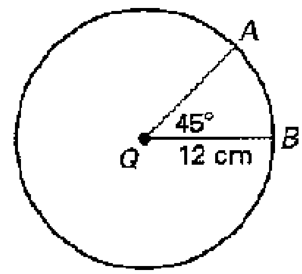
1.



2.



3.



In $\odot D$ shown below, $\angle ADC \cong \angle BDC$. Find the indicated measure.

4. $m\overset{\frown}{ACB}$

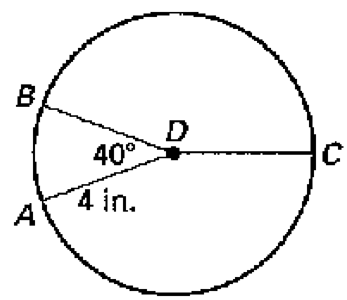
5. $m\overset{\frown}{CB}$

6. Length of $\overset{\frown}{ACB}$

7. Length of $\overset{\frown}{CB}$

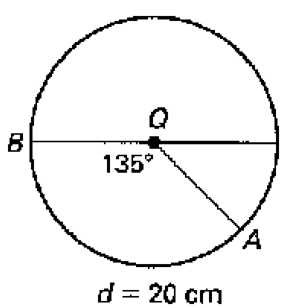
8. $m\overset{\frown}{ABC}$

9. Length of $\overset{\frown}{BAC}$

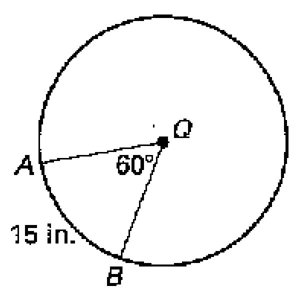


Find the indicated measure.

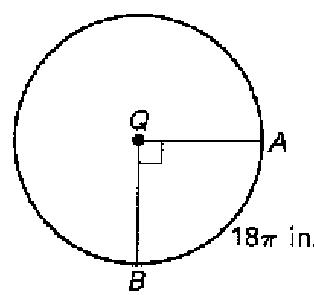
10. Length of $\overset{\frown}{AB}$



11. Circumference of $\odot Q$



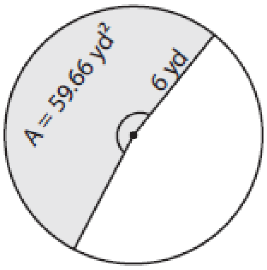
12. Radius of $\odot Q$



AREA OF A SECTOR

Find the sector area in terms of π and to the hundredths place. Find all other measures to the nearest hundredths place.

1)

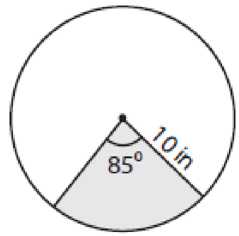


Radius = ___________

Central angle = ___________

Area of a sector = ___________

2)

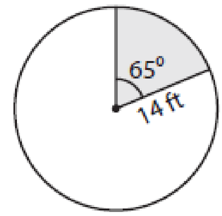


Radius = ___________

Central angle = ___________

Area of a sector = ___________

3)

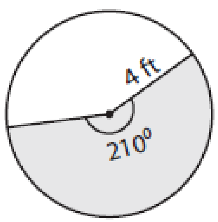


Radius = ___________

Central angle = ___________

Area of a sector = ___________

4)

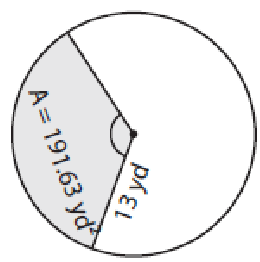


Radius = ___________

Central angle = ___________

Area of a sector = ___________

5)

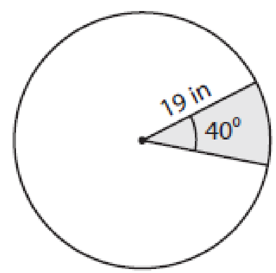


Radius = ___________

Central angle = ___________

Area of a sector = ___________

6)

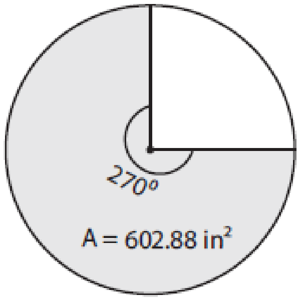


Radius = ___________

Central angle = ___________

Area of a sector = ___________

7)

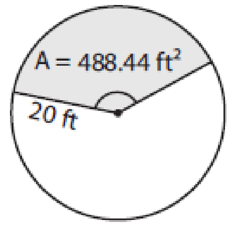


Radius = ___________

Central angle = ___________

Area of a sector = ___________

8)

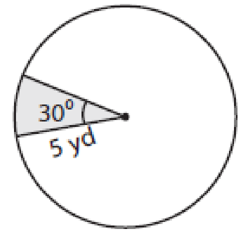


Radius = ___________

Central angle = ___________

Area of a sector = ___________

9)



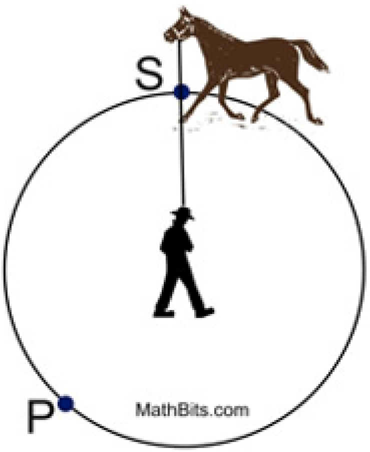
Radius = ___________

Central angle = ___________

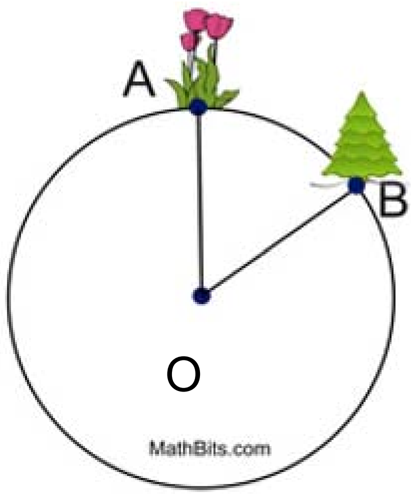
Area of a sector = ___________

Arc Length and Area of a Sector

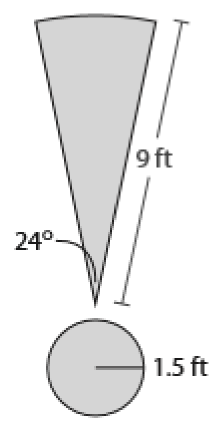
1) A horse on a lunge line travels in a circle around its trainer. The radius of the horse's circle is 24 ft. If the angle between points S and P on the circle is 140°, find the distance the horse travels in ft, rounded to the nearest foot.

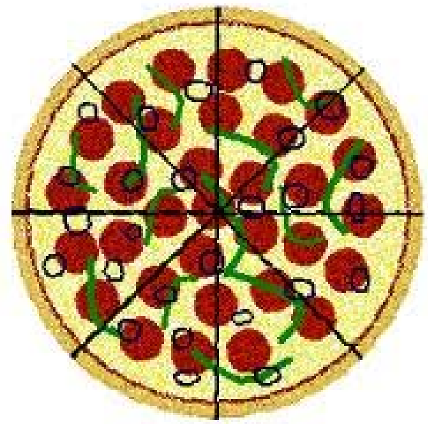


2) A sprinkler is equidistant from flower A and tree B. The sprinkler waters in a circular pattern. If the length of arc $\widehat{AB}$ is 12 feet and the radius of the circle is 10 feet, find the measure of the central $\angle AOB$, rounded to the nearest degree.



3) The exclamation point on a billboard consists of a circle sector and a circle. The radius of the sector is 9 ft., and the radius of the circle is 1.5 ft. The angle of the sector is 24°. What is the total area of the exclamation point on the billboard? Round to the nearest hundredth.





A pizzeria offers a lunch special of single pizza slices. All pizzas are cut into 8 equal slices.

PART A: Lunch special A is one slice from a 18-inch pizza for $3.99. How much pizza is in one slice of a 18-inch diameter pizza?

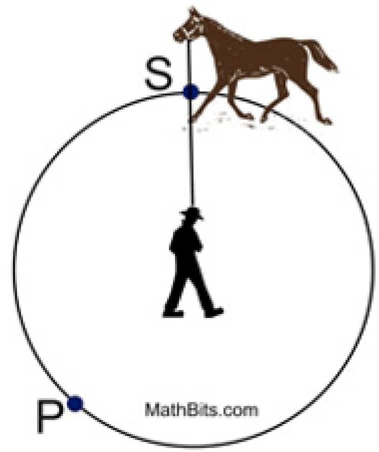
PART B: Lunch special B is two slices from a 12-inch pizza for $3.99. How much pizza is in two slices of a 12-inch diameter pizza?

PART C: Which lunch special is the better deal? Explain your reasoning.

PART D: Mrs. Smith orders three slices of the 18-inch diameter pizza for her children. The children eat the pizza and leave the crust for Mrs. Smith. If she were to measure the length, how long would the total amount of crust be that she eats?

Arc Length and Area of a Sector – Application

1) A horse on a lunge line travels in a circle around its trainer. The radius of the horse's circle is 24 ft. If the angle between points S and P on the circle is 140°, find the distance the horse travels in ft, rounded to the nearest foot.



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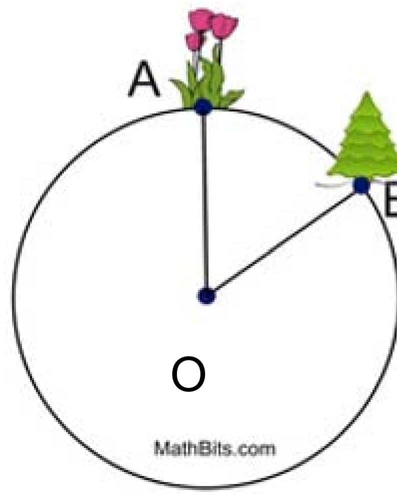
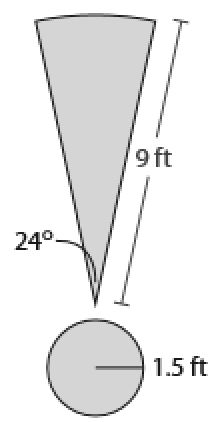
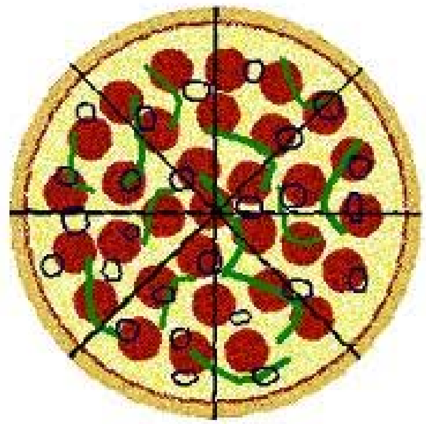


3) The exclamation point on a billboard consists of a circle sector and a circle. The radius of the sector is 9 ft., and the radius of the circle is 1.5 ft. The angle of the sector is 24°. What is the total area of the exclamation point on the billboard? Round to the nearest hundredth.





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Arc Length and Area of a Sector

Arc Length and Area of a Sector

Define Arc Length.

Define Sector of a Circle.

Part 1: Hands on Activity - COOKIE LAB

Materials: Large Soft Cookie, String, Protractor, Ruler, Knife, Paper Towel

1. Find the circumference of the cookie in cm using the string and the ruler.

   Circumference = _______cm

2. Find the measure of the diameter in cm.

   Diameter = ________cm

3. What is the ratio of the Circumference to the Diameter?

   $\frac{C}{d}$ = ______

4. The formula for Area of a circle is πr^2; where r = radius of circle

5. Find the Area of the cookie. ____________cm^2

Cut the cookie in half on the diameter. Then cut each half of the cookie into two ***unequal*** *sectors. You will have 4 different pieces of cookie.*

6. Using the protractor, find the Angle Measure of each sector's central angle.

   Angle 1 = _______° Angle 2 = _______°

   Angle 3 = _______° Angle 4 = _______°

You may now eat your cookie!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

Real Mathematics: Application

7. The formula for the **length of an arc** in a circle is $Arc\ Length = \frac{2\pi r\theta}{360}$

where r = radius of circle and θ= central angle

8. Using the Arc Length formula, find the measure of each sector's arc length.

Arc Length 1 = ________cm

Arc Length 2 = ________cm

Arc Length 3 = ________cm

Arc Length 4 = ________cm

9. What is the total length of the 4 arcs? _________cm

10. How does it compare to the circumference of the cookie?

11. The formula for the **Area of a Sector** is $Area\ of\ Sector = \frac{\pi r^2\theta}{360}$

where r = radius of circle and θ = central angle

12. Find the Area of each sector.

Area of sector 1 = _________cm^2

Area of sector 2 = _________ cm^2

Area of sector 3 = _________ cm^2

Area of sector 4 = _________ cm^2

13. What is the total area of the four sectors? _________ cm^2

14. How does it compare to the area of the original cookie?

15. Explain why the 4 arc lengths should add to the circumference of your circle. If they did not add to the circumference of your cookie, explain why they did not.

16. Explain why the 4 sector areas should add to the area of your cookie. If they did not sum to equal the area, explain why.

Part 2: Understanding the Formulas

Investigating the Area of a Circle

1. Cut out a Circle and Fold it then Cut it into at least 8 Congruent Sectors (or pizza slices).

2. Lay the slices next to each other to create a rectangle like shape.

3. Think about the dimensions of your "rectangle" in terms of the original circle's Circumference and radius. Sketch and label it here.

4. Since the Area of a Rectangle is found by multiplying the length of its base by its height, Find the Area of your "rectangle" by doing this calculation as well.

My Rectangle created from a circle has an approximate area of:

5. How does your formula compare with the formula you know to be the Area of a Circle?

Real Mathematics:
Application

Investigating Arc Length and the Area of a Sector

In this portion of the investigation you will look at the relationship between a central angle and its intercepted arc.

1. Decide how long you would like your radius to be. r = __________ (don't forget units!)

2. Using your compass, draw a circle with the above radius on a separate piece of paper. What is the circumference of your circle? Don't forget units!

3. Divide your circle, using a protractor, into four equal parts (hint: use two diameters). What is the measure of each central angle in the circle you constructed?

4. Write a fraction that compares the measure of the central angle to the total number of degrees in a circle. Then simplify this fraction.

5. Keeping in mind your answer above, what would be the length of the arc formed by one of the central angles?

6. Write a fraction that compares the arc length computed above to the total circumference of the circle. Then simplify this fraction.

7. Make a conjecture about the proportion of the measure of a central angle and the proportion of its intercepted arc? (Hint: What do you notice about the fraction you found in question 4 and the fraction you found in question 6?) **Come up with a general formula for this conjecture.**

8. Compare your findings and formula with your peers around you. Does your formula work with different sized circles?

Write your discovery as:

Arc Length Proportion **Arc Length Formula**

In the next portion of this investigation we will look at the relationship between the central angle and the sector area it creates.

Real Mathematics: Application

9. What is the area of your circle? Don't forget units!

10. Using your divided circle from the beginning of the investigation, what is the area of one of the sectors?

11. Write a fraction that compares the sector area computed above to the total area of the circle. Then simplify this answer.

12. What do you notice about the simplified fraction of the sector area and the fraction you found in problem 4 of the central angle?

13. Make a conjecture about the proportion of the measure of a central angle and the proportion of its sector area? **Come up with a general formula for this conjecture.**

14. Compare your findings and formula with your peers around you. Does your formula work with different sized circles?

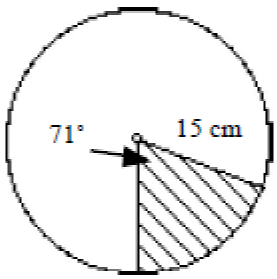
Write your discovery as:

Area of Sector Proportion **Area of Sector Formula**

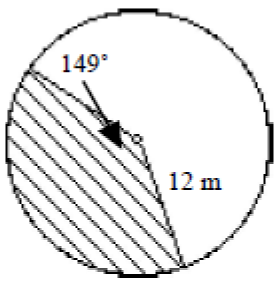
Arc Length and Sector Area

Find the length of the indicated arc, to the nearest hundredths.

1.

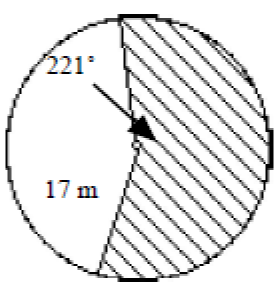


2.



Find the area of the indicated sector, to the nearest hundredths.

3.



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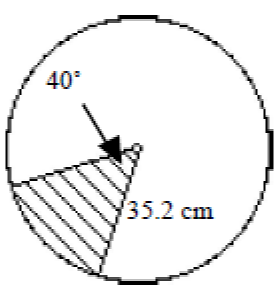
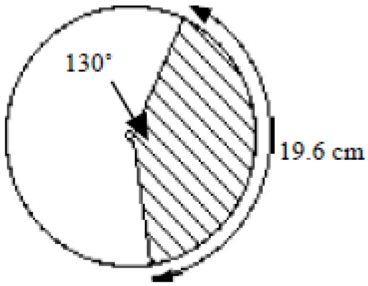


Find the radius. Round the nearest hundredths.

5.



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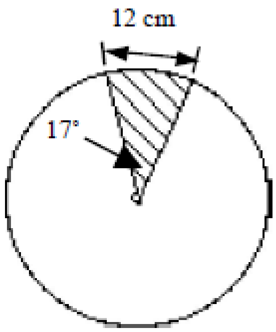

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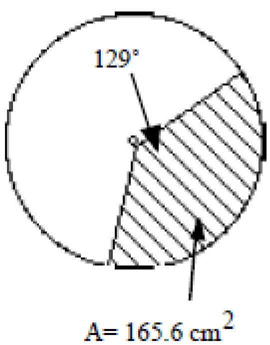

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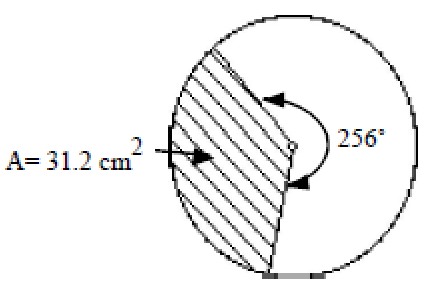
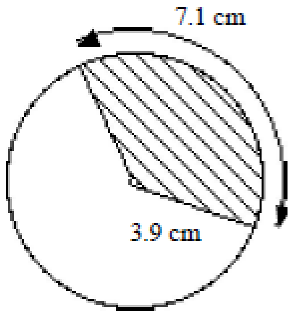


Find the measure of the shaded central angle. Round the nearest hundredths.

9.



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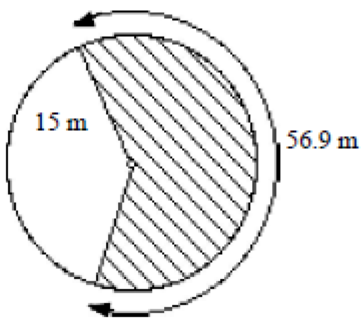

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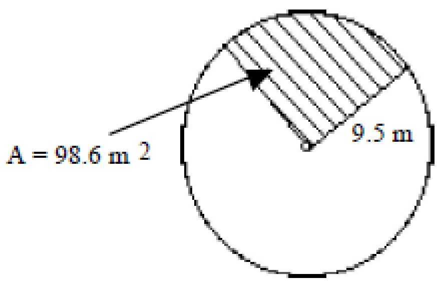

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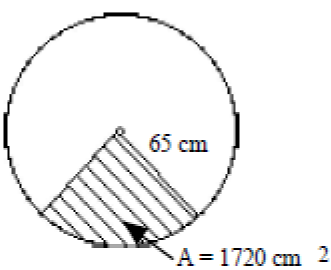
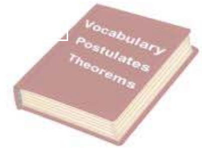


Term	Definition	Notation	Diagram/Visual
Area			
Volume			
Two-dimensional figure			
Three-dimensional figure			
Polygon			
Prism			
Cylinder			
Sphere			

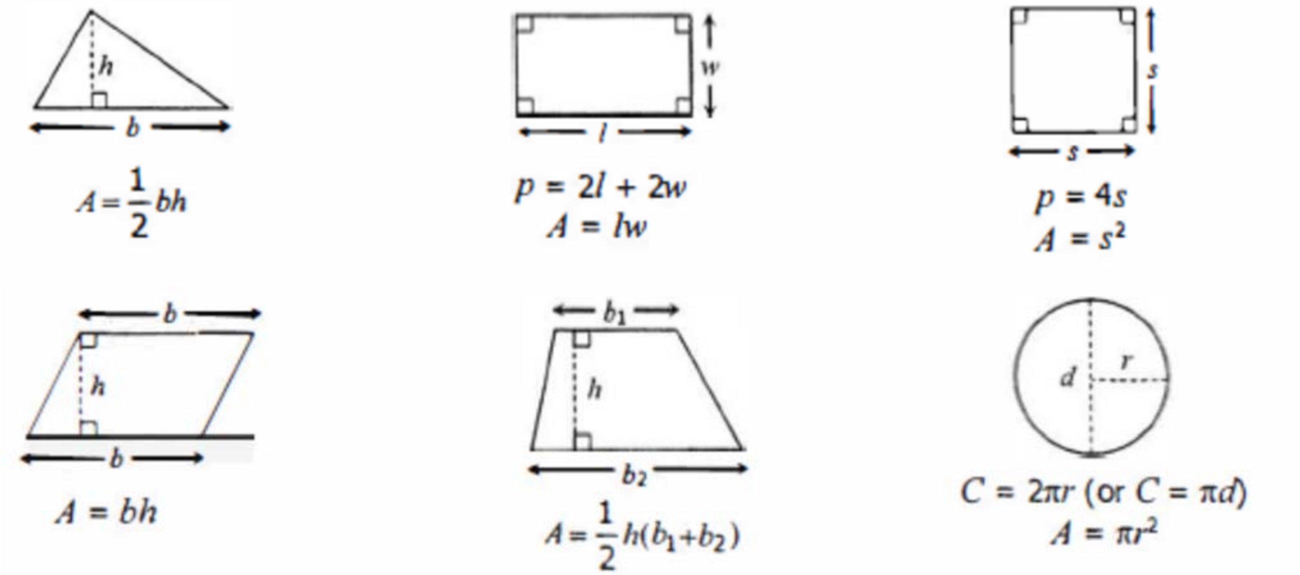


Pyramid			
Cone			
Square			
Rectangle			
Triangle			
Parallelogram			
Trapezoid			
Circle			

Composite Figure			

Revisiting Area

GUIDED NOTES



$A = \frac{1}{2}bh$

$p = 2l + 2w$
$A = lw$

$p = 4s$
$A = s^2$

$A = bh$

$A = \frac{1}{2}h(b_1 + b_2)$

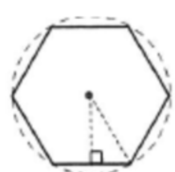
$C = 2\pi r$ (or $C = \pi d$)
$A = \pi r^2$

- A **composite figure** is a figure that can be ____________________ into ______________ that are basic plane figures. (Like triangles, parallelograms, trapezoids, rectangles, circles etc.)
- To find the **area of a composite figure**, ______________ up the regions and find each ____________, then find the ____________ of the areas.

To find the **area of a shaded region**, find the area of the _____________ figure, then __________________ the area of the non-shaded regions.

Parts of a Regular Polygon:

- The distance from the center of a polygon to any side is called the _________________.
- The _____________ of a regular polygon is the distance from the center to a vertex.



Formula for Area of a Regular Polygon: $A = \frac{1}{2}ap$

a = apothem
p = perimeter

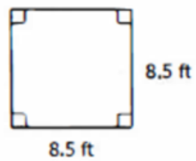
Create a **right triangle** by drawing the apothem and radius.

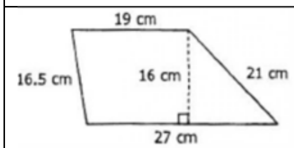
Find the **measure of the interior angle** where the radius intersects the vertex.
Use the interior angles sum formula ($S = (n-2) \cdot 180$), then divide by n, then by 2.

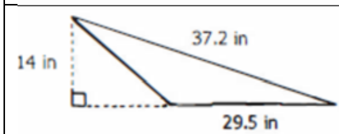
Use **special right triangles** (45°-45°-90°/30°-60°-90°), or **trigonometry** (SOH CAH TOA) to find the apothem and/or side lengths.

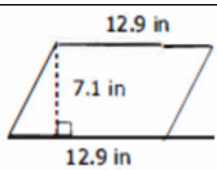
Example!

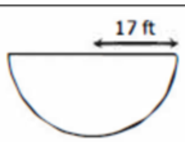
Find the area of each plane figure below.









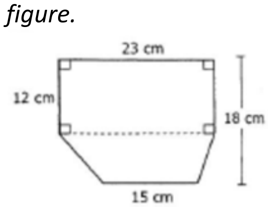


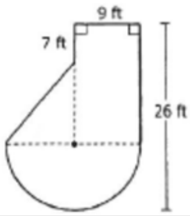
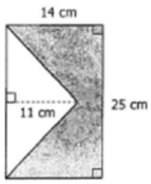
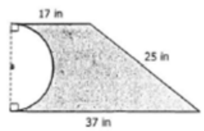
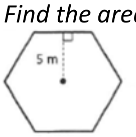
A triangle with an area of 45.5 cm^2 has a base that measures 14 cm. Find the height of the triangle.

A trapezoid with base measures of 23 km and 29 km has an area of 390 km^2. Find the height of the trapezoid.

Find the diameter of a circle with an area of 380.13 m^2.

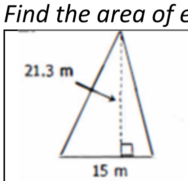
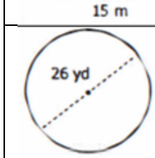
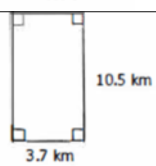
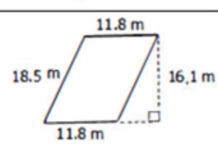
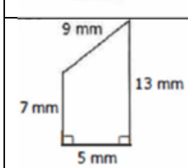
Find the total combined area of the composite figure.

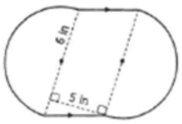
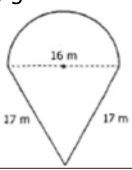
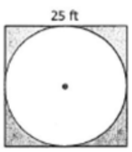
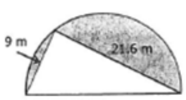


Find the total combined area of the composite figure. 	
Find the area of the shaded region. 	
Find the area of the shaded region. 	
Find the area. 	

SELF CHECK

Find the area of each plane figure below.

Find the total combined area of the composite figure. 	
Find the total combined area of the composite figure. 	
Find the area of the shaded  *region.*	
Find the area of the shaded region. 	
Find the area. 	

What connections do you see between the area formulas of squares, rectangles, and parallelograms?

Explain why the area formula for a triangle has a 1/2 in it.

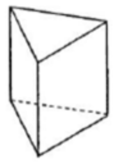
Explain the formula for the area of a trapezoid; what other formulas can you connect it to?

What is the difference between a radius and an apothem?

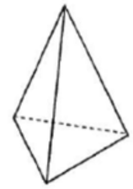
Volume of 3D Figures

GUIDED NOTES

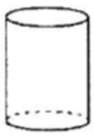
Prism
A solid with two bases that are congruent and parallel.
This figure is a triangular prism because the bases are triangles.



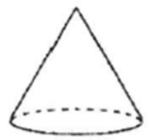
Pyramid
A solid with one base and faces that meet at a point.
This figure is a triangular pyramid because the base is a triangle.



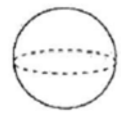
Cylinder
A prism with circular bases.

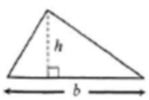


Cone
A pyramid with a circular base.

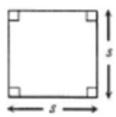


Sphere
A solid in which each point is equidistant from a center point.

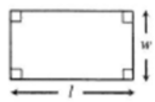




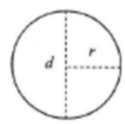
$A = \frac{1}{2}bh$



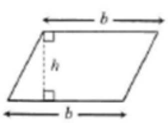
$p = 4s$
$A = s^2$



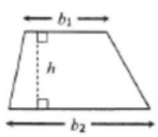
$p = 2l + 2w$
$A = lw$



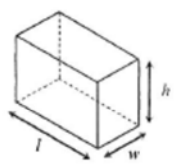
$C = 2\pi r$
(or $C = \pi d$)
$A = \pi r^2$



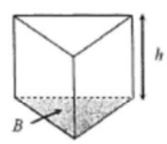
$A = bh$



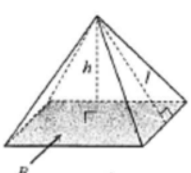
$A = \frac{1}{2}h(b_1 + b_2)$



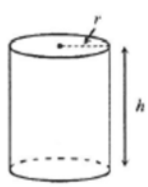
$V = lwh$
$SA = 2lw + 2lh + 2wh$



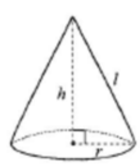
$V = Bh$
$LA = hp$
$SA = hp + 2B$



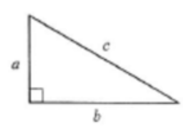
$V = \frac{1}{3}Bh$
$LA = \frac{1}{2}lp$
$SA = \frac{1}{2}lp + B$



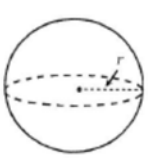
$V = \pi r^2 h$
$LA = 2\pi rh$
$SA = 2\pi r^2 + 2\pi rh$



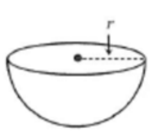
$V = \frac{1}{3}\pi r^2 h$
$LA = \pi rl$
$SA = \pi r^2 + \pi rl$



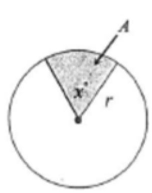
$a^2 + b^2 = c^2$



$V = \frac{4}{3}\pi r^3$
$SA = 4\pi r^2$

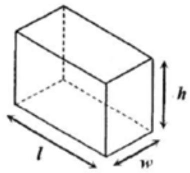


$V = \frac{2}{3}\pi r^3$
$SA = 3\pi r^2$



$A = \frac{x \cdot \pi r^2}{360}$

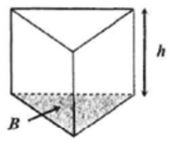
Rectangular Prisms (or Cubes)



$V = lwh$

l = length
w = width
h = height

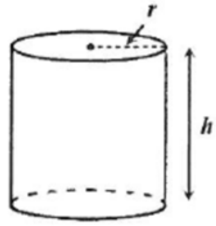
All Other Prisms



$V = Bh$

B = area of the base
h = height between bases

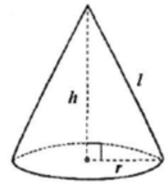
Cylinders



$V = \pi r^2 h$

r = radius
h = height

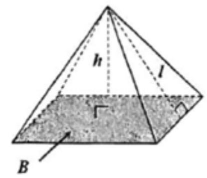
Cones



$V = \frac{1}{3}\pi r^2 h$

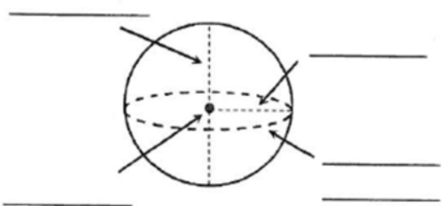
r = radius
h = height

Pyramids



$V = \frac{1}{3}Bh$

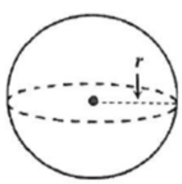
B = area of the base
h = height



A **sphere** is a solid in which each point is equidistant from a center point.

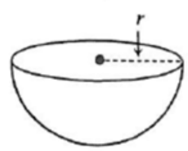
The **great circle** slices the sphere into two **hemispheres**.

Sphere



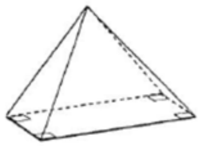
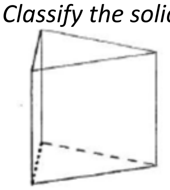
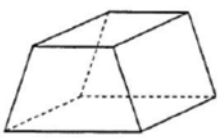
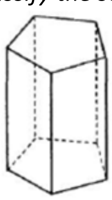
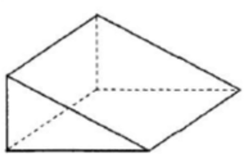
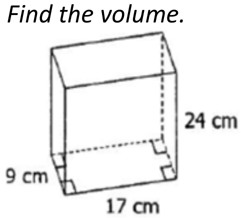
$V = \frac{4}{3}\pi r^3$

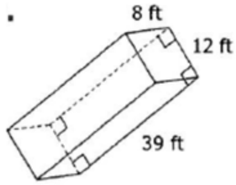
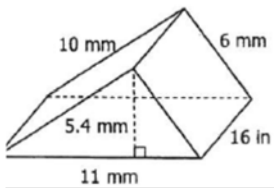
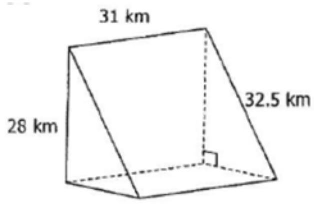
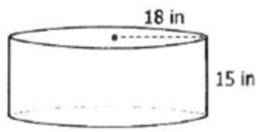
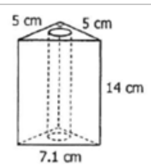
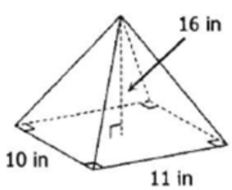
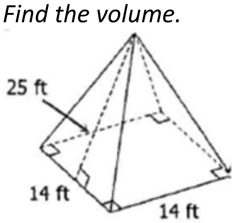
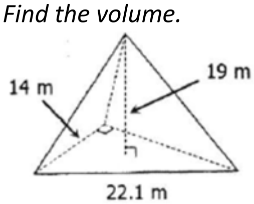
Hemisphere

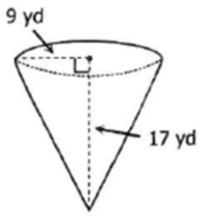
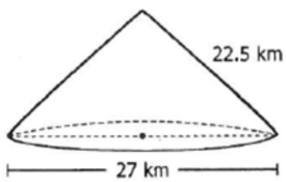
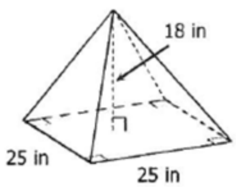
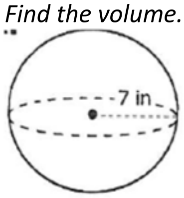
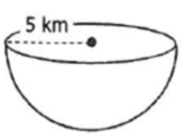
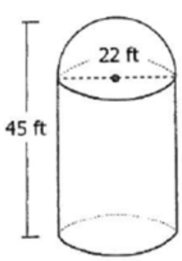


$V = \frac{2}{3}\pi r^3$

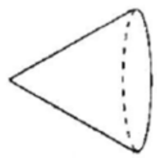
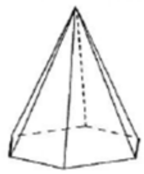
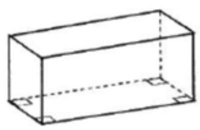
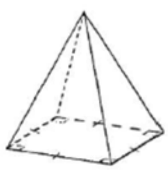
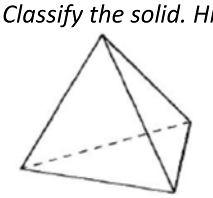
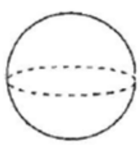
Example!

Classify the solid. Highlight the base. 	
Classify the solid. Highlight the base. 	
Classify the solid. Highlight the base. 	
Classify the solid. Highlight the base. 	
Classify the solid. Highlight the base. 	
Classify the solid. Highlight the base. 	
Find the volume. 	

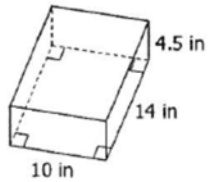
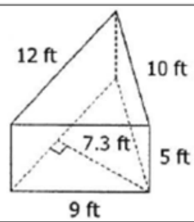
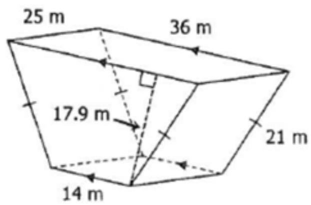
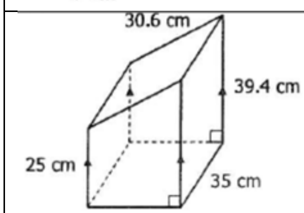
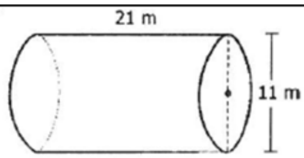
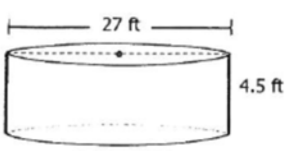
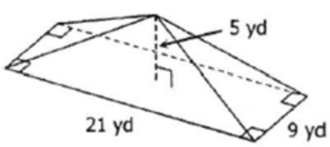
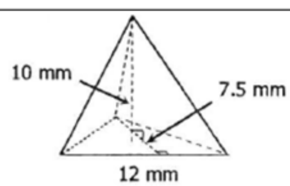
Find the volume.  	
Find the volume.  	
Find the volume.  	
Find the volume.  	
 	A cylinder with a 2 centimeter diameter is drilled through the triangular prism shown on the left. Find the volume of the prism.
Find the volume.  	
Find the volume.  	
Find the volume.  	

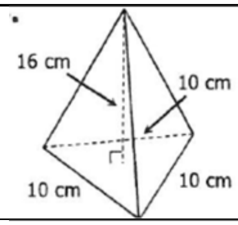
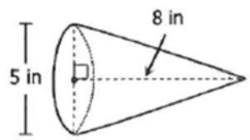
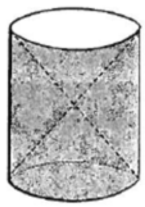
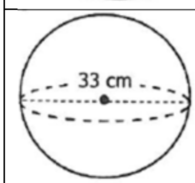
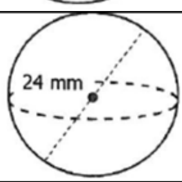
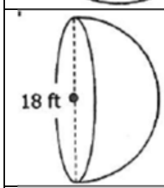
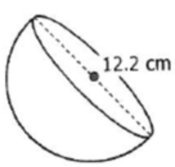
Find the volume. 	
Find the volume. 	
	Katylyn must build a sand castle in the form of a square pyramid for a project, as shown to the left. She bought 3 bags of sand, each containing 1200 in^3 of sand. Will she have enough sand to build the castle?
Find the volume. 	
Find the volume. 	
Find the volume of a sphere with a great circle area of 201.06 square inches.	
	The Henley's have a silo on their farm to store grain. Assuming the entire space is used, what is the maximum amount of grain that the silo can hold?

SELF CHECK

Classify the solid. Highlight the base. 	
Classify the solid. Highlight the base. 	
Classify the solid. Highlight the base. 	
Classify the solid. Highlight the base. 	
Classify the solid. Highlight the base. 	
Classify the solid. Highlight the base. 	

Find the volume of each figure.

	
	
	
	
	
	A diagram of Eric's pool is shown on the left. He plans to fill his pool to a depth of 4 feet with a garden hose that has an 80 ft^3 per hour flow rate. How many hours will it take to fill the pool?
	
	

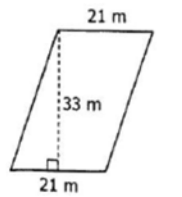
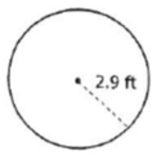
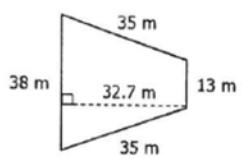
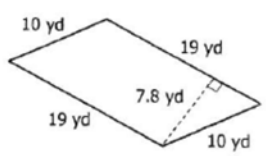
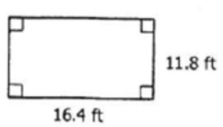
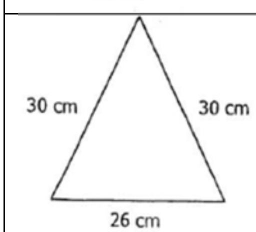
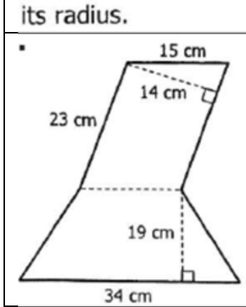
	
	
A cone has a volume of 1309 cm^3. If the height of the cone is 8 centimeters, find its radius.	
	The cylinder to the left contains two congruent hollow cones. If the cylinder's height is 20 inches and its diameter is 14 inches, find the volume of the solid.
	
	
	
	

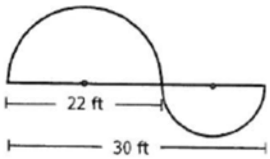
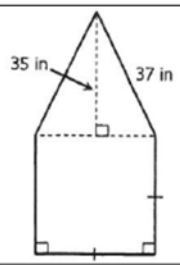
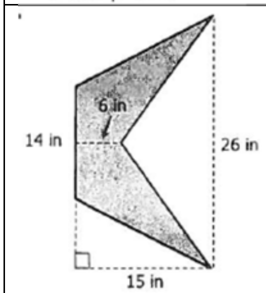
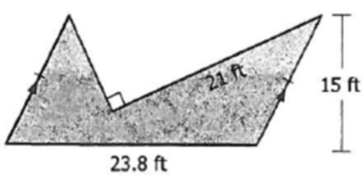
What is the connection between prisms and cylinders?

What is the connection between cones and pyramids?

Why do we use an uppercase B to denote the base of the 3D figure in the formulas?

Revisiting Area

	
	
	
	
	
	
A parallelogram with an area of 211.41 m^2 has a base that measures 24.3 m. Find its height.	
Find the length of the second base of a trapezoid with one base measuring 29 ft, a height of 9 ft, and an area of 193.5 ft^2.	
A semicircle has an area of 76.97 km^2. Find its radius.	
	

22 ft 30 ft	
35 in 37 in	
6 in 14 in 26 in 15 in	
21 ft 15 ft 23.8 ft	

Volume of 3D Figures

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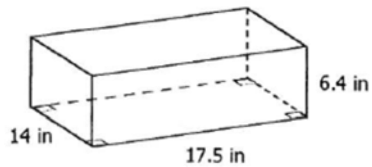
<tr><td>
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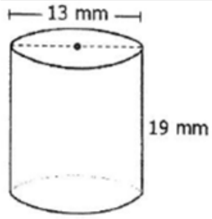
<tr><td>
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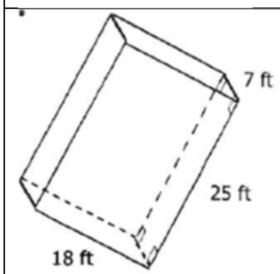
<tr><td>
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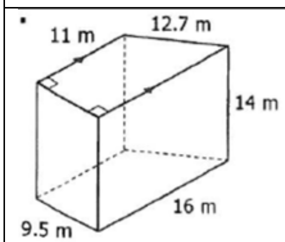
<tr><td>
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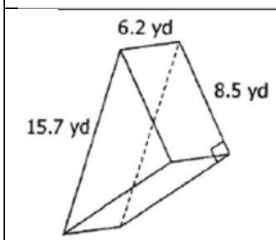
<tr><td>
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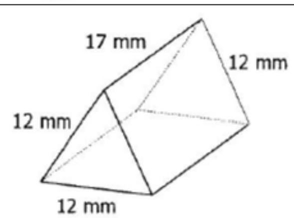
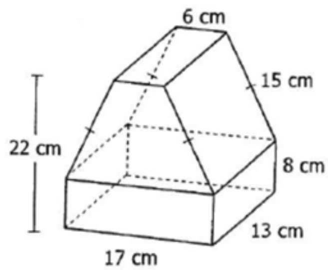
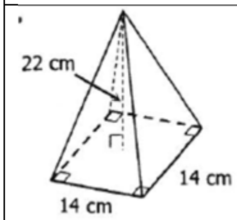
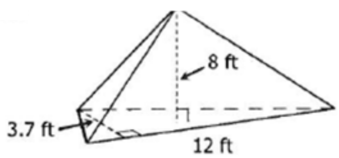
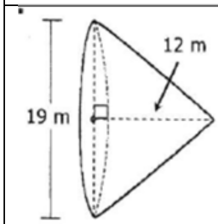
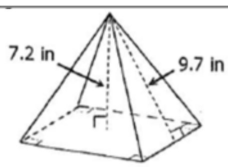
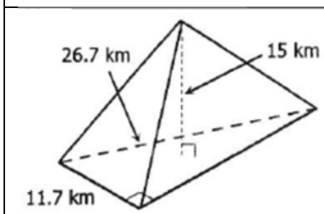
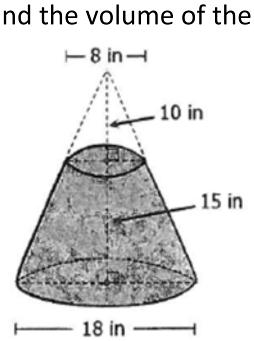
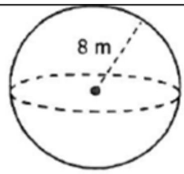
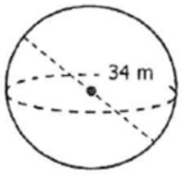
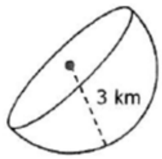
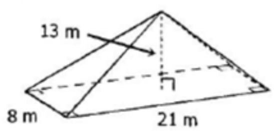
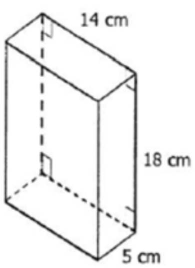
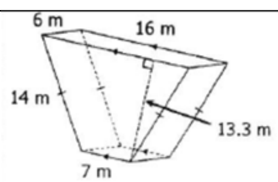
<tr><td>
</td><td></td></tr>
<tr><td>If a cylinder has a height of 7 inches and a volume of 2,908.33 in^3, find its diameter.</td><td></td></tr>
</table>

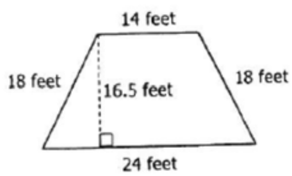
Find the total volume. 	
	
	
	
	
	
Find the volume of the shaded section. 	

	
	
	
Find the volume of a hemisphere with a great circle circumference of 37.7 centimeters.	
	
	
	

Volume

Revisiting Area Application

Mr. Payton would like to paint his deck. The dimensions of the deck are shown below. If one can pf paint covers 72 square feet, how many cans will he need to purchase for two coats of paint?



Volumes of Cylinders, Cones, Pyramids, and Spheres

Bonaventura Francesco Cavalieri (1598 – November 30, 1647) was an Italian mathematician. We will use his works as the basis for this task. Cavalieri's Principle states:

> The volumes of two solids are equal if the areas of corresponding sections drawn parallel to some given plane are equal.

This can best be understood by looking at a stack of pennies. The volume of two stacks of the same number of pennies is the same, even if one stack is not vertically aligned.

Let's apply this to derive the formula for the volume of a right cylinder. If you cut a super thin slice, or cross section, out of a cylinder what shape would result?

What is the Area Formula for that shape? $A =$

How many slices would you need to stack up to create your cylinder?

Complete the Volume of a Cylinder Formula:

$V = \textit{Area of the Base} \bullet \textit{height}$

$V = B \bullet h$

$V =$

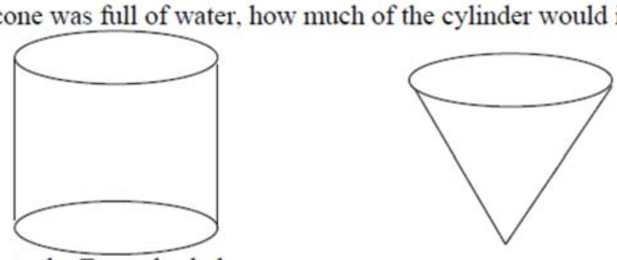
Cavalieri principle. *Encyclopedia of Mathematics*. URL: http://www.encyclopediaofmath.org/index.php?title=Cavalieri_principle&oldid=17072

Part 1: Understanding the Formulas

Let's investigate the relationship between a Cone and its corresponding Cylinder with the same height and radius. LABEL the height, h and radius, r on each diagram below.

1. If the cylinder was full of water and you poured it into the cone, how many times would it fill up the cone completely?

   If the cone was full of water, how much of the cylinder would it fill up?



2. Complete the Formulas below:

Volume of a Cylinder Formula vs. Volume of a Cone Formula

$V = Area\ of\ the\ Base \bullet height$

$V = B \bullet h$ $V =$

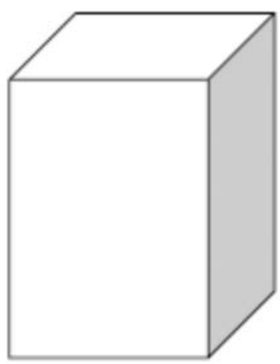
$V =$

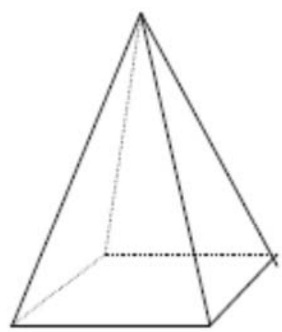
3. Describe the relationship between the formulas and the amount the cone filled the cylinder.

Let's investigate the relationship between a Pyramid and its corresponding Rectangular Prism with the same height, length, and width. LABEL the height, h, length, l, and width, w on each diagram below.

4. If the rectangular prism was full of water and you poured it into the pyramid, how many times would it fill up the pyramid completely?

   If the pyramid was full of water, how much of the rectangular prism would it fill up?





5. Complete the Formulas below:

Right Rectangular Prism Volume Formula vs. Right Pyramid Volume Formula

$V = Area\ of\ the\ Base \bullet height$

$V = B \bullet h$ $V =$

$V =$

6. Describe the relationship between the formulas and the amount the pyramid filled the prism.

Let's investigate the Volume of a Sphere Formula.

An image from
http://mathworld.wolfram.com/GeodesicDome.html

What is the formula for the Surface Area of a Sphere?

$SA =$

If that surface is divided up into triangles that are actually the bases of triangular pyramids that fill up the entire space inside of the sphere, then the Volume of the Sphere would equal the sum of the Volumes of all those pyramids.

Let's see if we can use this concept to derive the formula for the volume of a sphere:

$V_{pyramid} =$

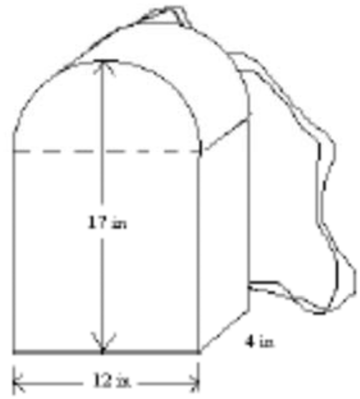
$V_{all\ the\ pyramids} =$

$=$

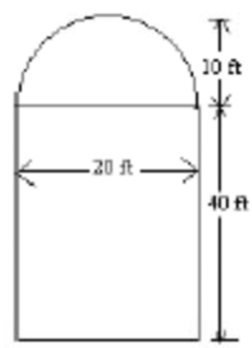
$V_{sphere} = \frac{4}{3}\pi r^3$

Part 2: Applications

1. Approximate the Volume of the Backpack that is 17in x 12in x 4in.



2. Find the Volume of the Grain Silo shown below that has a diameter of 20ft and a height of 50ft.

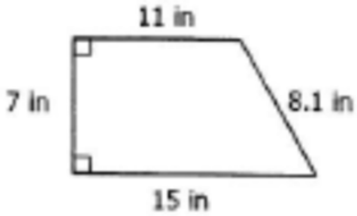


3. The diameter of a baseball is about 1.4 in.
   How much leather is needed to cover the baseball?
   How much rubber is needed to fill it?

4. The volume of a cylindrical watering can is $100cm^3$. If the radius is doubled, then how much water can the new can hold?

Revisiting Area

1.



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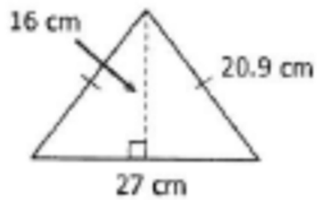

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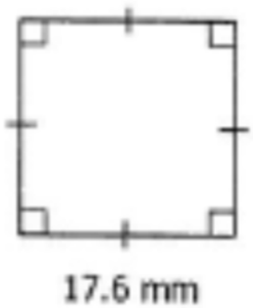

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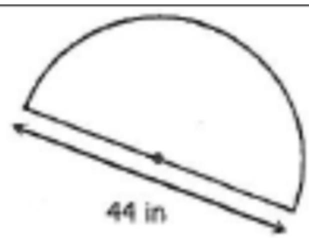

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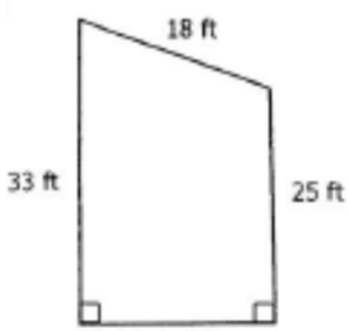
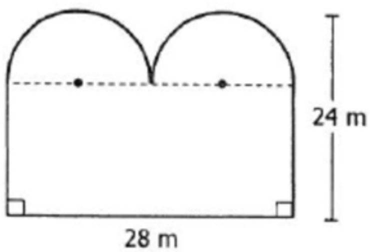
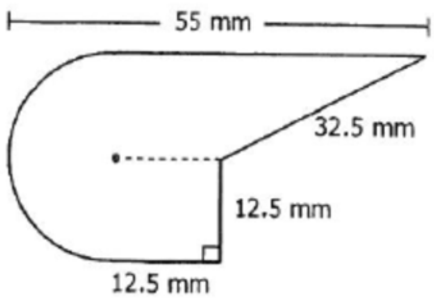
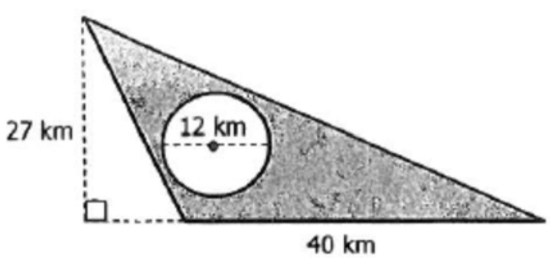
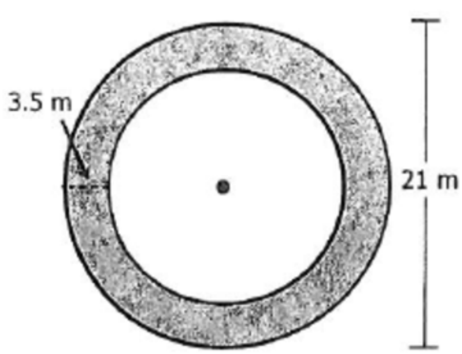
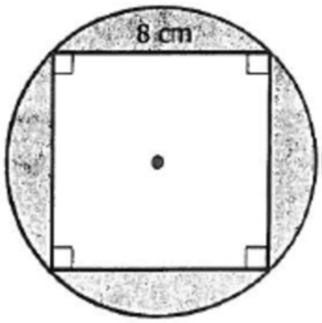
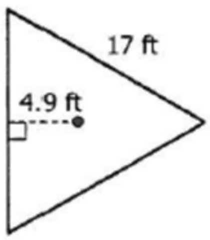
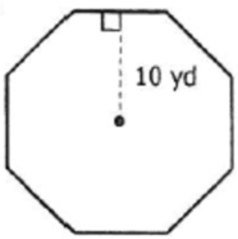


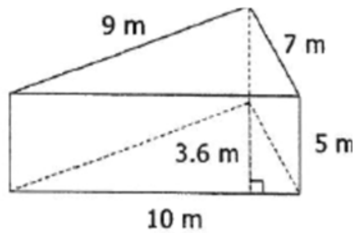
7. A triangle with an area of 202.5 mm^2 has a height that measures 15 mm. Find its base.	
8. A trapezoid with an area of 166.75 in^2 has bases that measure 21 in and 8 in. Find the height of the trapezoid.	
9. A rectangle has an area of 228 ft^2 and a length that measures 12 ft. Find the perimeter of the rectangle.	
10. Find the area of the composite figure. 	
11. Find the area of the composite figure. 	
12. Find the area of the shaded region. 	
13. Find the area of the shaded region. 	

14. Find the area of the shaded region. 	
15. 	
16. 	

Volume of 3D Figures

Find the volume of each figure

17.



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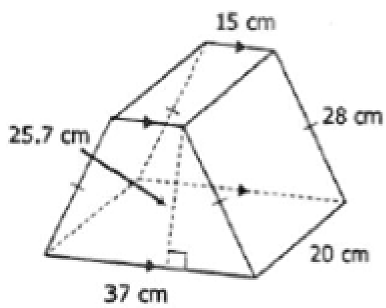

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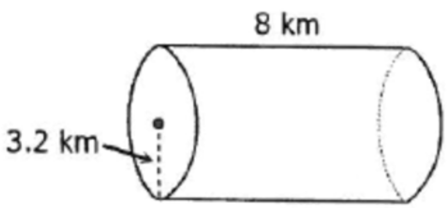

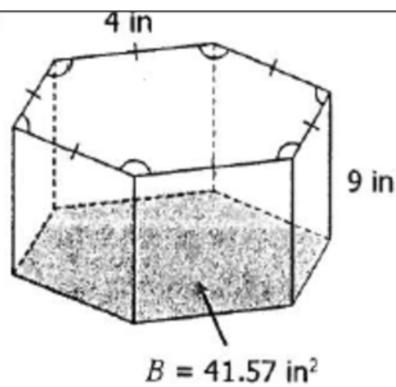

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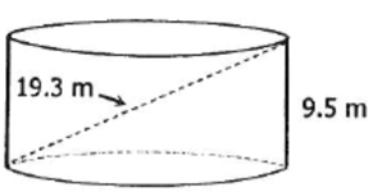
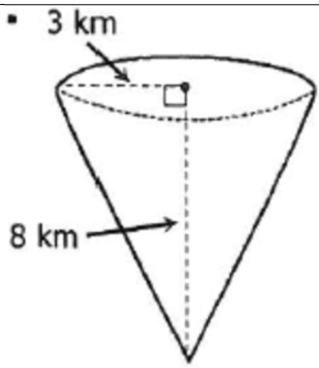


21.

<table>
<tr><td>22.

The volume of a rectangular prism is 655.2 ft^3. If the base of the prism is 9 feet by 5.2 feet, find the height of the prism.</td><td></td></tr>
<tr><td>23.



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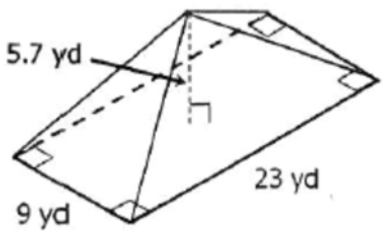

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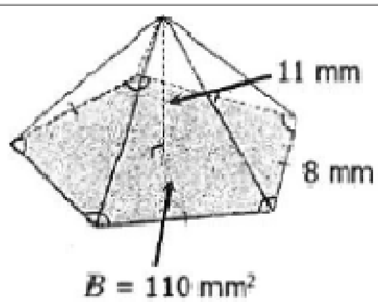

25.</td><td></td></tr>
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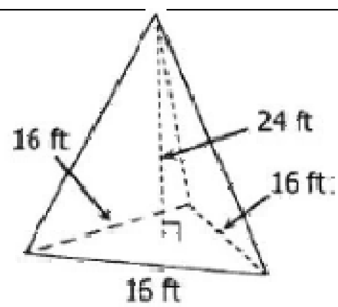

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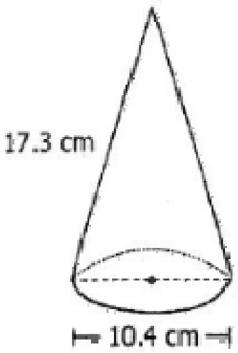
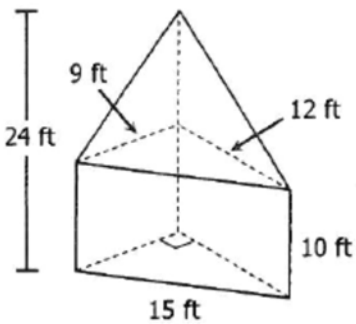
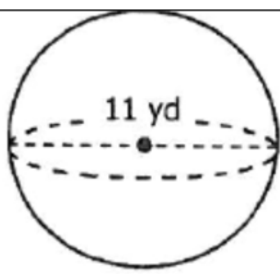
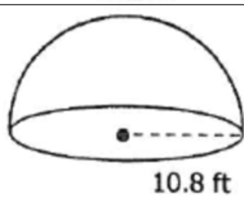
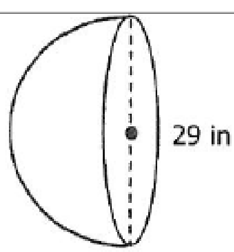
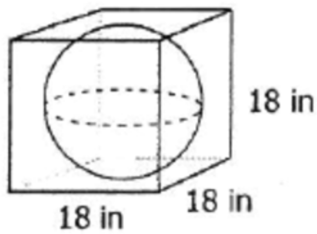
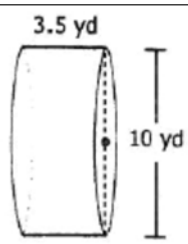
27. 	
 28. Find the total voume of this composite 3D figure. 29.	
30. 	
31. 	
32. 	

33. A hollow sphere sits snugly in a foam cube so that the sphere



touches each side of the cube. Find the volume of the foam.

34.



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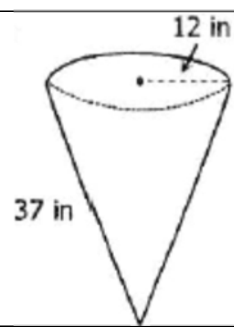

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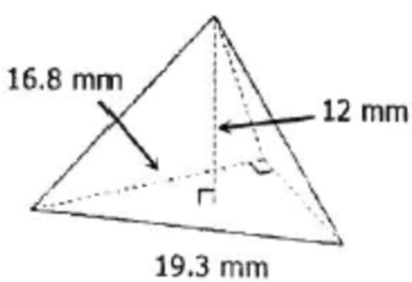


Density

Density is a ratio of two measurements of an object. In the past you might have seen the following formula for density:

$$d = \frac{m}{V}$$

This formula states that the density of an object is the ratio of its mass to its volume. If two objects have the same volume, but different masses, the object with the greater mass will be denser.

You can also think about the density of an area of land such as a city. In this case, you would look at the ratio of number of people to area of land.

$$population\ density = \frac{number\ of\ people}{area\ of\ land}$$

Example! Solve each word problem.
Reminder: You may have to calculate the Volume before using the density formula.

1. A cylinderical wood log has a diameter of 6cm, a hight of 17cm and a mass of 254kg. What is the density of the log?

2. Burlington, Vermont has an area of about 160 km^2 and a population of 109,000 people. What is the approximate population density of Burlington?

SELF CHECK

1.

A block of wood 3.0 cm on each side and has a mass of 27 g. What is the density of this block?

2. There are approximately 17,620 people per square mile in New York City. If New York City is 468 square miles, approximately how many people live in New York City?

What are the pieces of information I need to solve a density problem?

1. A gold nugget has a volume of 1.68 cm^3 and a mass of 32.4 g. Calculate the density of gold.

2. The density of silver is 10.5g/cm^3. What is the mass of a 23.6 cm^3 piece of silver?

3. Osmium has a density of 22.6 g/cm^3. What is the mass of a block of osmium that measures 1.01 cm x 0.233 cm x 0.648 cm?

4. Iron has a density of 7.9 g/cm^3. What is the mass of a cube of iron if the length of one side is 55.0mm?

5. What is the volume of a 63.4 g piece of metal with a density of 12.86 g/cm^3 ?

Use the following table to answer questions 6-

Cylindrical logs of wood.

Type of wood	Diameter (ft.)	Height (ft.)	Mass (lb)
Aspen	3.6	4.5	1,195
Juniper	3.0	6.0	1,487

6. What is the density of Aspen?

7. What is the density of Juniper?

8. Which type of wood is denser?

9. Julia runs a hotel. She wants to hold a party in the banquet hall of her hotel. The hall has a size of 70,000 square feet. She found the population density to 0.004 people per square foot. This included all people in the banquet hall. How many total people can attend the party in the banquet hall?

10. Mary purchased a new drawing box. The height, width and length of the box are respectively 12 cm, 6 cm, and 3 cm. She wants to fill the box with pencils. The density of a pencil is 0.03 pencils per cm^3. How many total pencils can she fit in the drawing box?

Density

1. Henry is a carpenter. He received an order to make an attractive and light weight metal frame door.

   He has two options: one is an iron frame door. The second is an aluminum frame door.

   The volume and density of an iron frame is respectively 1350 mL and 7.87 g/mL.

   The volume and density of an aluminum frame is respectively 2850 grams and 2.7 g/ml.

   The customer wants the lightest door frame possible. Which one is the right option for the door?

2. Norris threw a plastic toy in the tub for her dog. But there is a problem; the puppy doesn't like putting his face in water.

   If the weight of the toy is more than 350 grams it will sink.

   The density and volume of the toy is respectively 0.453 g/ml and 673.45 mL.

   Will the toy sink or float?

1. The density of silver is 10.5 g/cm^3. What is the volume of a 61.3 g piece of silver?

2. A wooden object has a mass of 10.782 g and occupies a volume of 13.72 mL. Calculate its density.

3. A liquid has a density of 2.67 g/mL. If you need to measure 1240 grams of the liquid, what volume should you measure?

4. The density of liquid bromine is 3.12 g/mL. What is the mass of 0.250 L of bromine?

5. A piece of metal weighing 100.0 g is placed into a cylinder containing 39.0 mL of water. The final volume is 45.8 mL. Calculate the density.

Use the following table to answer questions 6 – 8

Cylindrical wood logs

Types of wood	Diameter (cm)	Height (cm)	Mass (kg)
Douglas fir	6	17	254
American Redwood	8	12	271

6. What is the density of Douglas Fir?

7. What is the density of American Redwood?

8. Which one is denser?

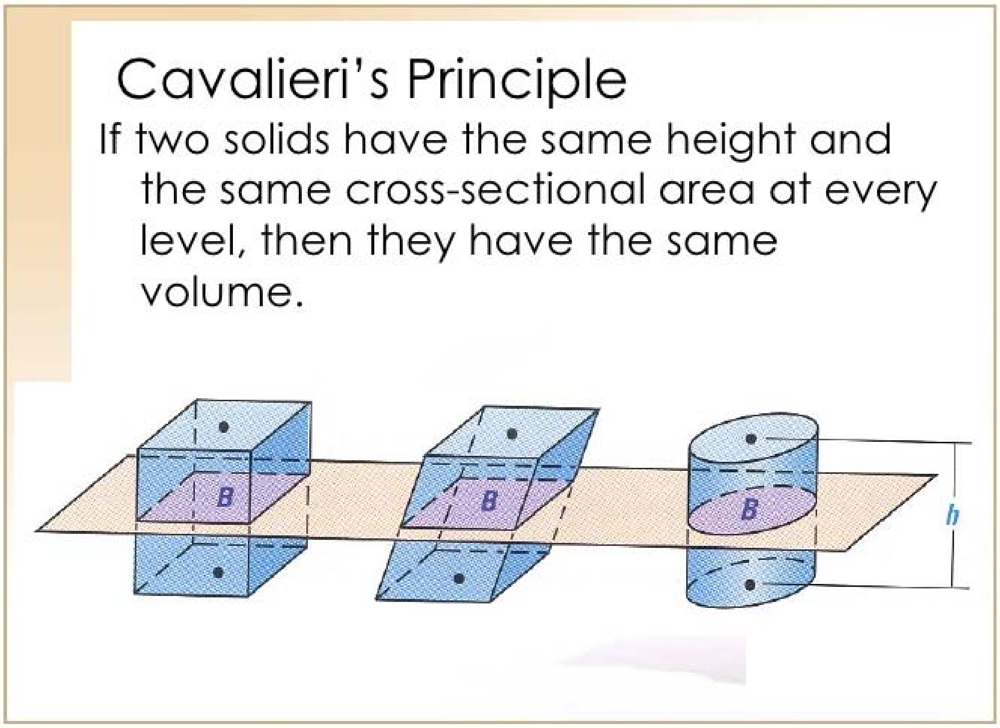
9. Eric has a trolley He wants to use that trolley to transport wooden blocks. The size trolley is 3,960,000 square centimeter. He found the wooden blocks to have a density of 0.0029 blocks per square centimeter. How many total wooden blocks can he carry in the trolley?

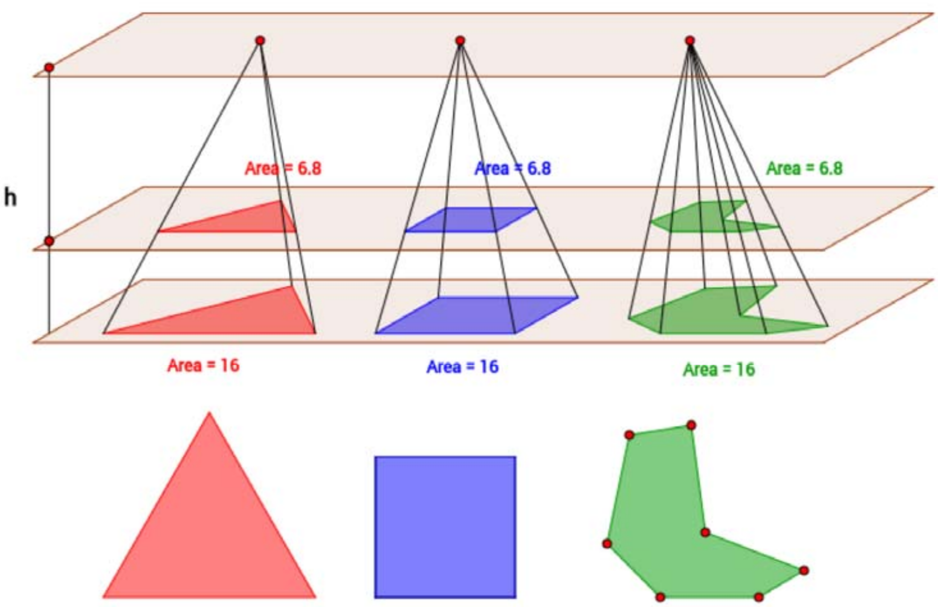
10. Frank purchased a new ice box having 2kg capacity. He wants it to be full with ice cubes. He measured the ice cubes to be 3.5 cm by 3.5 cm by 3.5 cm and have a density of 0.914 g/mL. How many cubes can he put in the box?

Term	Definition	Notation	Diagram/Visual
Cavalieri's Principle			
Cross Section			

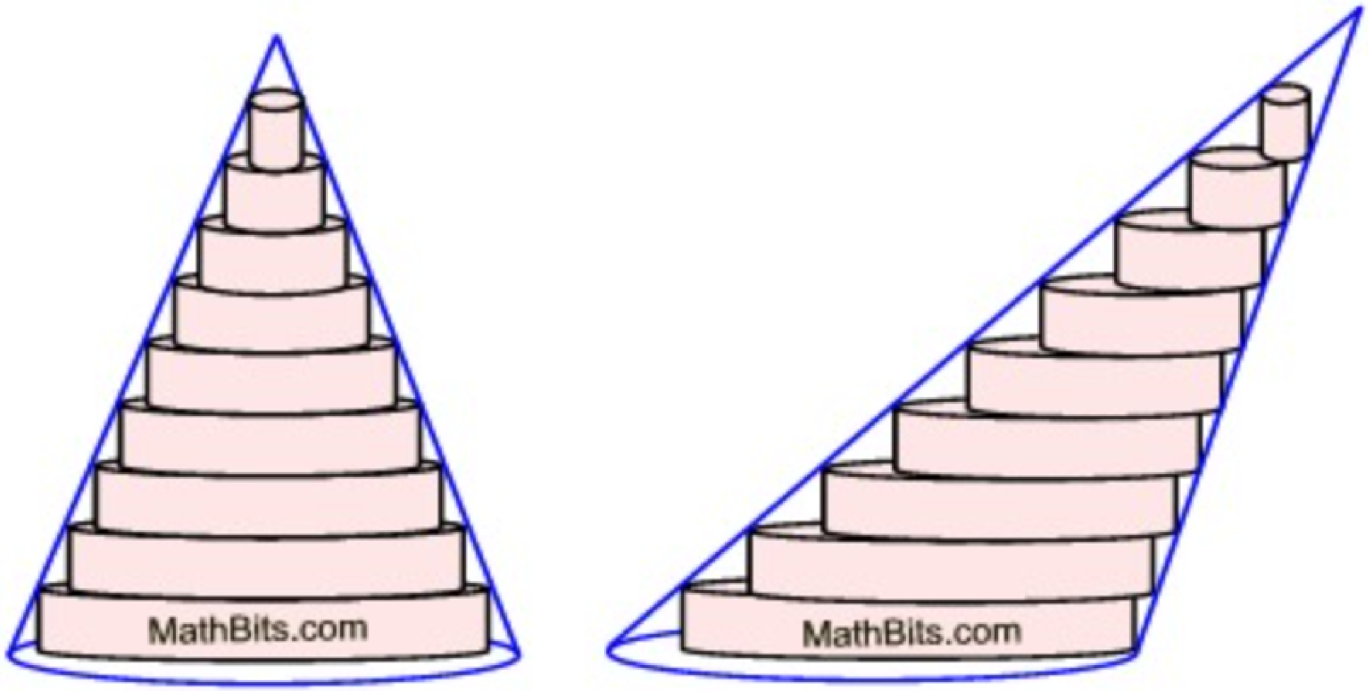
Cavalieri's Principle

GUIDED NOTES





The three pyramids have the same base area and the same height, and parallel cross sections at any level have the same area. Therefore they have equal volumes.

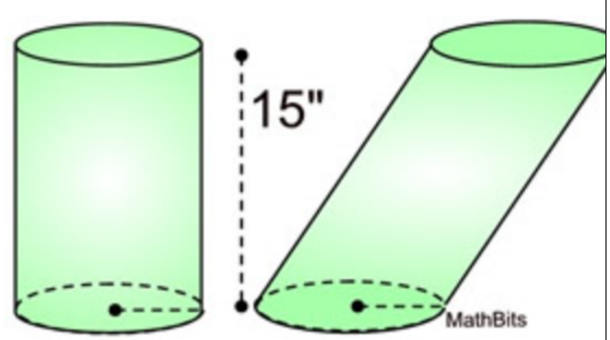


Example!

1.

A right circular cylinder and an oblique circular cylinder are given.

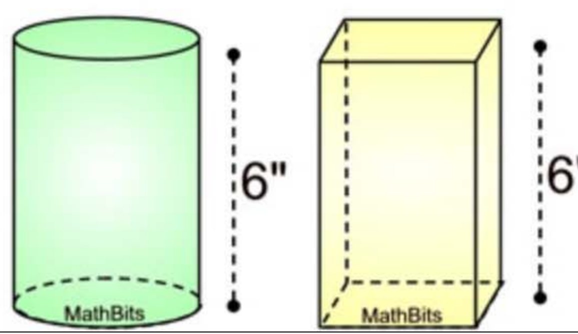
True or False? If the radii of both cylinders are equal, the volumes of the cylinders will be equal because they have the same height.



2.

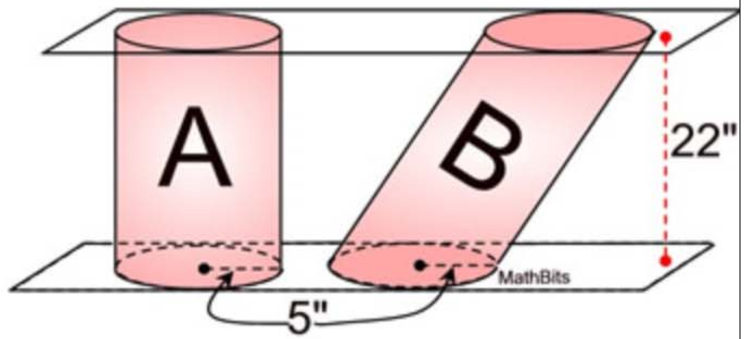
A right circular cylinder and a right rectangular prism are given.

True or False? Cavalieri's Principle does not apply to these solids because their bases are not the same shape.



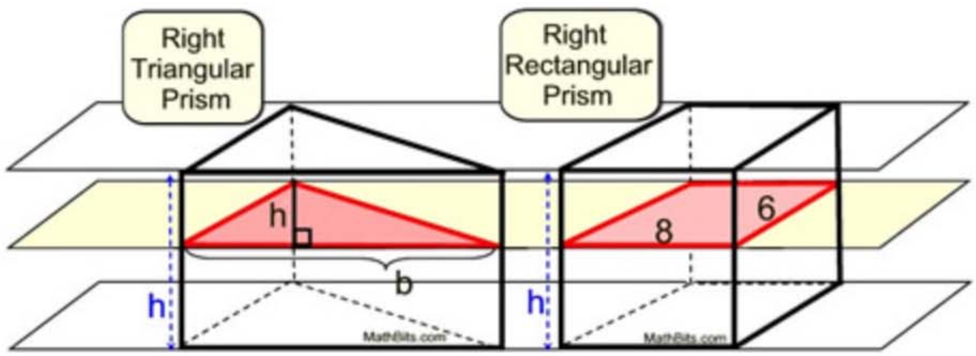
3.

A right circular cylinder, A, and an oblique circular cylinder, B, are shown at the right. Find the volume of cylinder B in cubic inches.



SELF CHECK

1.



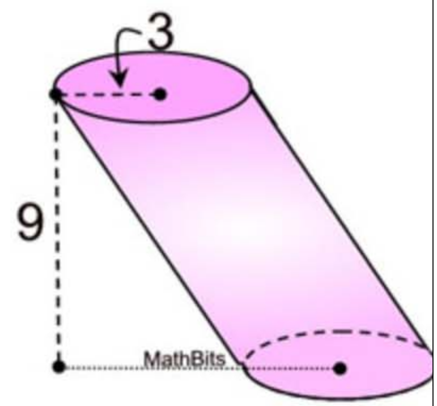
By Cavalieri's Principle, this right triangular prism and right rectangular prism have the same volume. If the center plane intersects the solids parallel to their bases, which of the following choices could be the base and height of the triangular cross section?

Choose:

- ○ $h = 4;\ b = 12$
- ○ $h = 8;\ b = 12$
- ○ $h = 4;\ b = 8$
- ○ $h = 8;\ b = 14$

2.

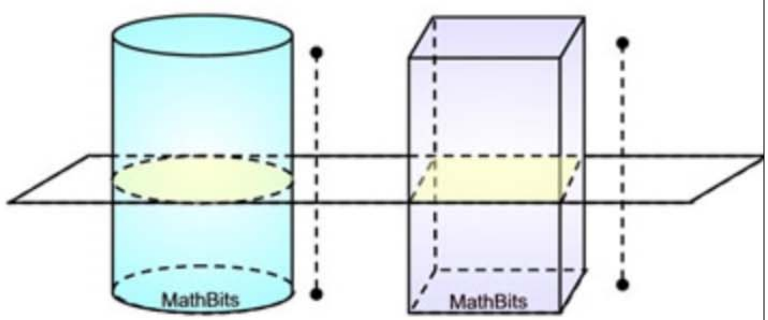
What is the volume of an oblique cylinder with a radius of 3 and a height of 9?



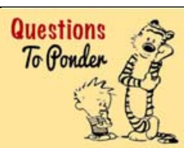
○ 27 units^3
○ 27π units^3
○ 81 units^3
○ 81π units^3

3.

The right circular cylinder and the right rectangular prism have the same heights and the same base areas. A plane, parallel to the bases slices the two solids.



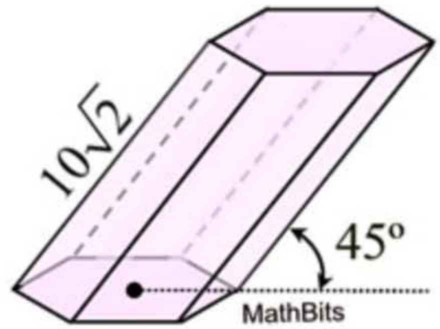
If the rectangular cross section has a base length of 16 inches and a width of 4π inches, what is the radius of the cylinder's cross section?



Why is the volume of two solids having the same cross section area and same height the same?

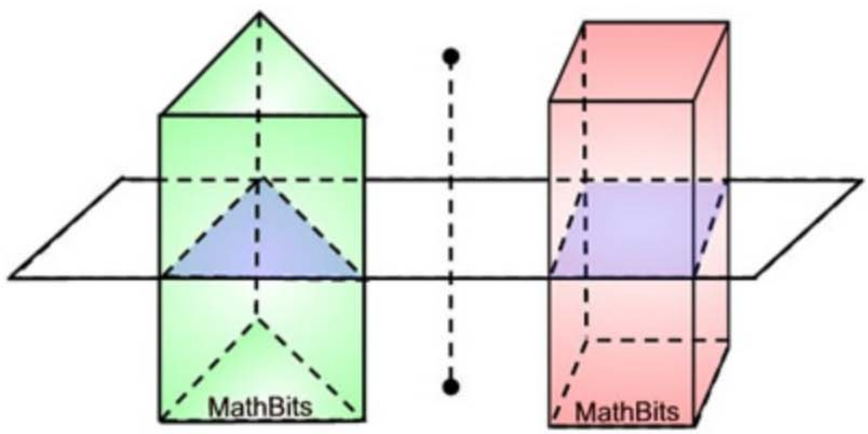
1.

An oblique hexagonal prism has a base area of 68 square meters and a lateral (slant) side length of $10\sqrt{2}$ meters. The lateral side makes an angle of 45º with the horizontal plane containing the base, as shown. What is the volume of the prism?



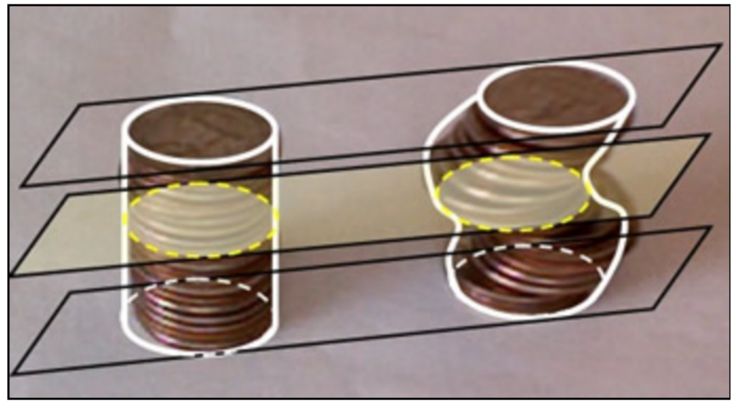
2.

A right triangular prism and a right square prism are given. The base of the triangular prism is an isosceles right triangle with a hypotenuse of $6\sqrt{2}$.



Both solids have a height of 10, and their bases have equal areas. A plane slices both solids parallel to their bases. Find the length of the side of the square prism's cross section.

3.



A series of coins are stacked to represent a right circular cylinder (on the left). The coins are then "slid" to represent a distorted cylinder (on the right). The same number of congruent coins was used in each stack.

Which of the following statements will be **TRUE** regarding these stacks of coins?

Check all that apply

□ The volume of both stacks will be the same.

□ The area of a cross section parallel to the bases will not be equal due to the distorted nature of the second stack.

□ The height of the distorted stack will be slightly larger than that of the straight stack.

□ Cavalieri's Principle can be used in this situation to verify that the volumes of the stacks are equal.

Rate of Change Lab

Materials: 2 paper snow cone cups per group, scissors, 1 cylindrical plastic cup, salt, 2 rulers with cm markings, lab sheet, dust buster for spills

Lab Sheet:

1. Measure the height of your cone. h = ________cm

2. Measure the diameter of your cone. d = ______cm r = _______cm

3. Snip off a small tip of your cone.

4. Place the second cone under the first, as a cap.

5. When the teacher instructs you, fill the cone with salt.

6. During 15 second intervals, you will be removing the overlay cone and letting the salt drip from the cone into your cup. You will then be recording the new height of your salt below. You will not be able to record the radius and volume at this time.

7. We will continue doing this until all of the groups have emptied their cones of salt.

8. Re-Record your original height and radius as Time Interval 1. This is the only radius, you know. You will have to calculate all the other radii and all of the volumes.

Time Interval	Height	Radius	Volume
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

9. Calculate the missing boxes on your chart.

10. Calculate the Rate of Change of the Volume from one interval to the next and record below.

Interval	per 15 sec	per second
1 to 2		
2 to 3		
3 to 4		
4 to 5		
5 to 6		
6 to 7		
7 to 8		
8 to 9		
9 to 10		
10 to 11		
11 to 12		
12 to 13		
13 to 14		
14 to 15		

11. Calculate the Average Rate of Change of the Volume.

12. Using your groups Average Rate of Change, how long would it take to fill your cylindrical cup, if the salt had to first go through your cone?

Radius of Cup: __________cm

Height of Cup: ___________cm

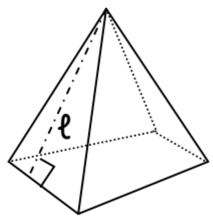
Volume of Cup: ____________cm^3

Length of time to fill cup: __________________seconds

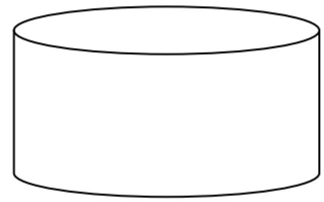
13. How many times would you need to fill your cone?

Practice. Determine what information you will need in order to find the volume for each object.

1.



2.



Solve. Find the volume of each figure.

3. Pyramid
   h = 10
   B = 16

4. Cone
   h = 8
   r = 3.5

5. Cylinder
   h = 8
   C = 4π

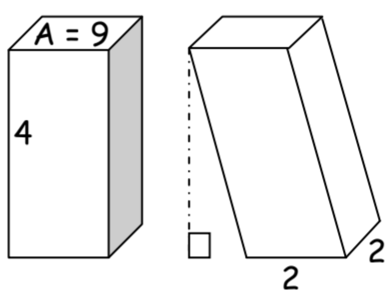
6. Sphere
   C = 8π

7. Rectangular Prism
   h = 4
   l = 8
   w = 2

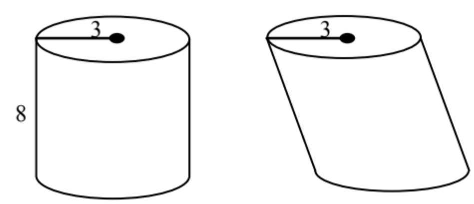
8. Cube
   w = 15

Use Cavalieri's principle to determine whether the objects pictured have the same volume. Explain your answer.

9.



10.



Term	Definition	Notation	Diagram/Visual
Axis			
Perspective			

2D Figure Rotations to Form 3D Forms with Volume

GUIDED NOTES

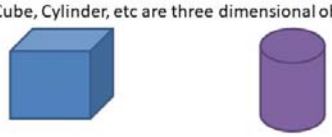
Review of Two-Dimensional and Three-Dimensional Figures

A Two-Dimensional (2D) shape is a shape that only has two dimensions: width and height.

Examples: Squares, Circles, Triangles, etc are two dimensional objects

A Three-Dimensional (3D) shape is a shape that has three dimensions: width, depth and height.

Examples: Cube, Cylinder, etc are three dimensional objects



2D → 3D

Three-Dimensional figures can be generated by rotating two-dimensional figures.

Don't Forget

"Rotation" means turning around a center.

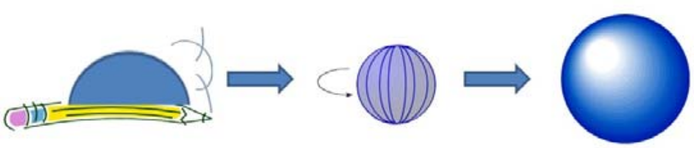
A three-dimensional object rotates always around an imaginary line called a *rotation axis*.

WHAT DID YOU GUESS?

If you guessed cone, you are correct! A cone is solid revolution of a right triangle around one of its legs.

WHAT 3D SHAPE IS PRODUCED IF WE ROTATE A SEMI-CIRCLE ?

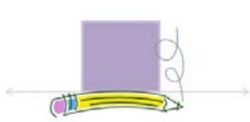
A sphere is solid revolution of a semi-circle around its diameter.



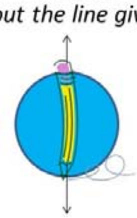
You try:

Given the shape below, determine the 3D solid formed by rotating the two-dimensional shape about the line given.

1.



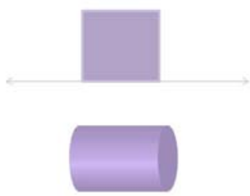
2.



You try:

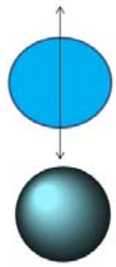
Given the shape below, determine the 3D solid formed by rotating the two-dimensional shape about the line given.

1.



A square rotated about the above line results in a right cylinder.

2.

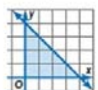


A circle rotated about the above line results in a sphere.

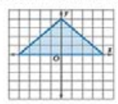
Another Way to Visualize the Rotation

If you do not want to cut out the shapes, and you still need help visualizing the rotation:

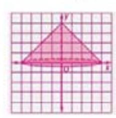
1. **Draw your shape and shade the region to be rotated.**



2. **Next, draw a reflection (mirror image) of the region about the axis or line of rotation.**



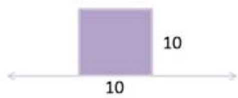
3. **Connect the vertices of the original image and its reflection using curved lines**

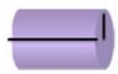


Word Problems

A square with area of 100 cm² is rotated to form a cylinder.
What is the volume of the cylinder?

Area = 100 cm^2
$s^2 = 100$
s = 10





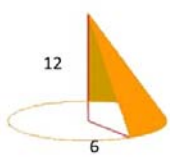
Volume = $3.14r^2$ h

Volume = $3.14(5)^2$ (10)

Volume ≈ 785

Word Problems

Given a cone with a radius of 6 ft and a height of 12 ft, find the area of the triangle formed by a perpendicular cross-section down through the cone's center.



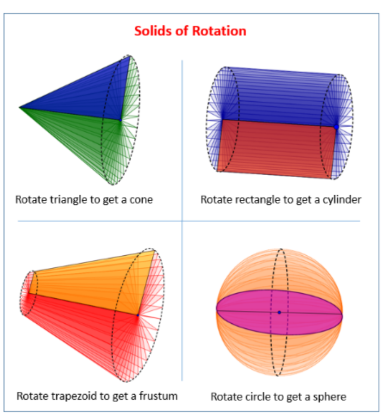
A= ½ bh

A = ½ (6)(12)

A = 36 square ft

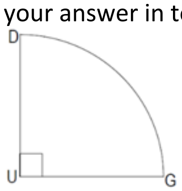
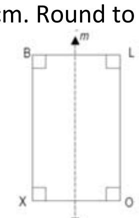
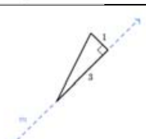
Visualize this concept with Geogebra.

https://www.geogebra.org/m/UMHXAvhb

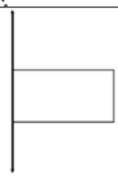
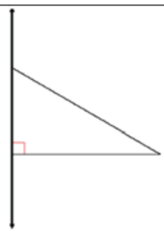
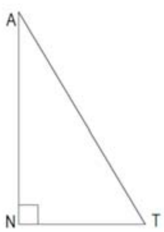
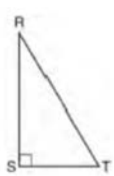


Example!

<table>
<tr><td>Rotating 2D shapes to form 3D shapes If AN = 8, NT = 6, and AT = 10 and triangle ANT is rotated about segment AN, what shape will be formed? Sketch the shape, label the measurements, and find the volume of the figure.

</td><td></td></tr>
<tr><td>Sketch the figure formed when sector DUG is rotated about segment UG. Describe the figure and find its volume if the radius of sector DUG is 8. Leave your answer in terms of pi.

</td><td></td></tr>
<tr><td>Sketch the figure formed when rectangle BLOX is rotated about line m, which is the perpendicular bisector of BL . Describe the figure and find its volume if BX is 87.3 cm and BL = 42.5 cm. Round to the nearest hundredth.

</td><td></td></tr>
<tr><td>
What solid 3D object is produced by rotating the triangle about line ***m?***</td><td></td></tr>
</table>

SELF CHECK

When each shape below is rotated about the bold line, a 3D shape is formed. Name the shape for each figure.	
	
	
	
	
Triangle ANT is rotates about segment AT, what shape will be formed? Sketch and describe the shape. 	
Which object is formed when right triangle RST shown below is rotated around leg $\overline{RS}$?  1 a pyramid with a square base 2 an isosceles triangle 3 a right triangle 4 a cone	

If the rectangle below is continuously rotated about side *w*, which solid figure is formed?

1 pyramid
2 rectangular prism
3 cone
4 cylinder

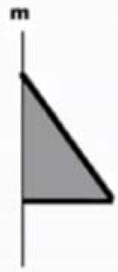
Cakes sold by Destiny's Deserts are made in the shape of cones or cylinders. The cones are 6" in diameter and 6" tall while the cylinders are 4" in diameter and 6" tall. Should Destiny charge the same amount for both cakes, charge more for the cone cakes, or more for the cylinder cakes? Justify your answer based on the amount of ingredients she will need for each cake. Start by sketching the two cake shapes and labeling them with their measurements.

Rotations of 2D Shapes

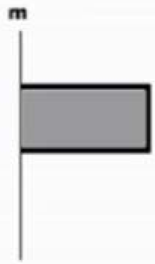
Example 1:

Describe the solid that is formed by rotating each of these figures about line m and sketch it.

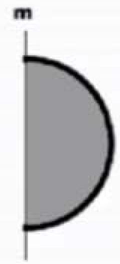
a)



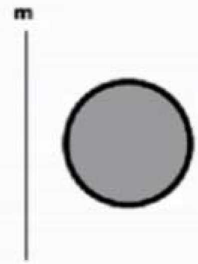
b)



c)



d)

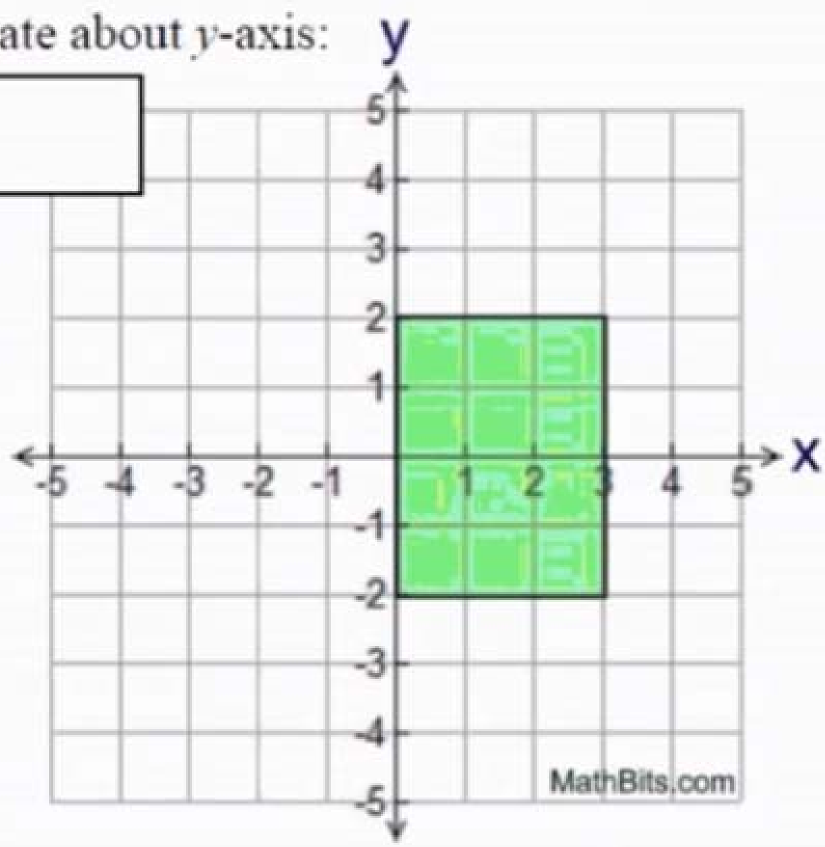


Name/Description   Name/Description   Name/Description   Name/Description

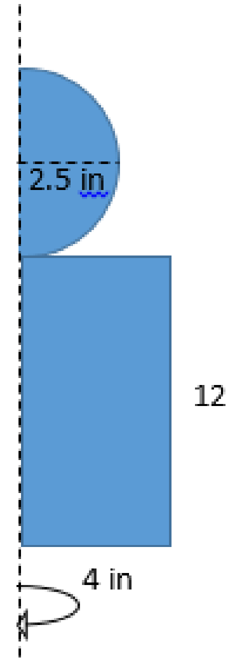
Example 2:

Rotate about y-axis:

V =

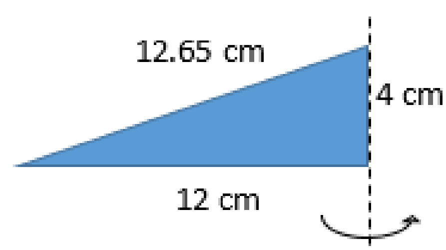


1) A trophy for an upcoming soccer tournament is created by rotating a rectangle and a circle around a vertical axis as shown below.
   a. Determine the total volume of the trophy.

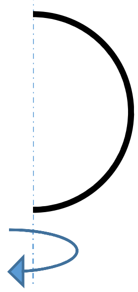


   b. The second place team will get a trophy in which **each dimension** is ¾ that of the winners' trophy. Determine the total volume of the 2nd place trophy.

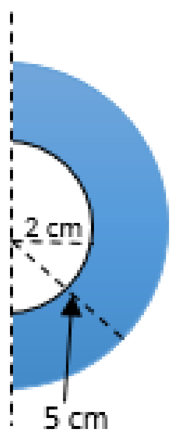
2) A solid is created by rotating the figure shown below about the given axis. Identify the solid of revolution that is formed and find its volume.



3) A volleyball is made by rotating a semicircle with a radius of 4.125 inches about an axis as shown below. Determine the total volume of air needed to fill a volleyball.

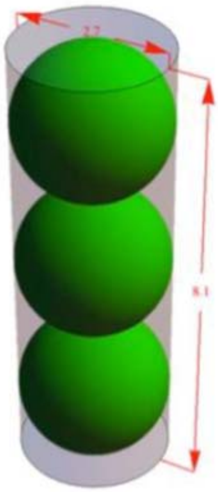


The disk below is rotated about the given axis to create a solid of revolution. Describe the solid that is created, then find its volume. Write your answers in terms of π a.



2D Figure Rotations to Form 3D Forms with Volume

The official diameter of a tennis ball, as defined by the International Tennis Federation, is at least 2.575 inches and at most 2.700 inches. Tennis balls are sold in cylindrical containers that contain three balls each. To model the container and the balls in it, we will assume that the balls are 2.7 inches in diameter and that the container is a cylinder the interior of which measures 2.7 inches in diameter and $3 \times 2.7 = 8.1$ inches high.



(a)Lying on its side, the container passes through an X-ray scanner in an airport. If the material of the container is opaque to X-rays, what outline will appear? What dimensions?

(b)If the material of the container is partially opaque to X-rays and the material of the balls is completely opaque to X-rays, what will the outline look like (still assuming the can is lying on its side)?

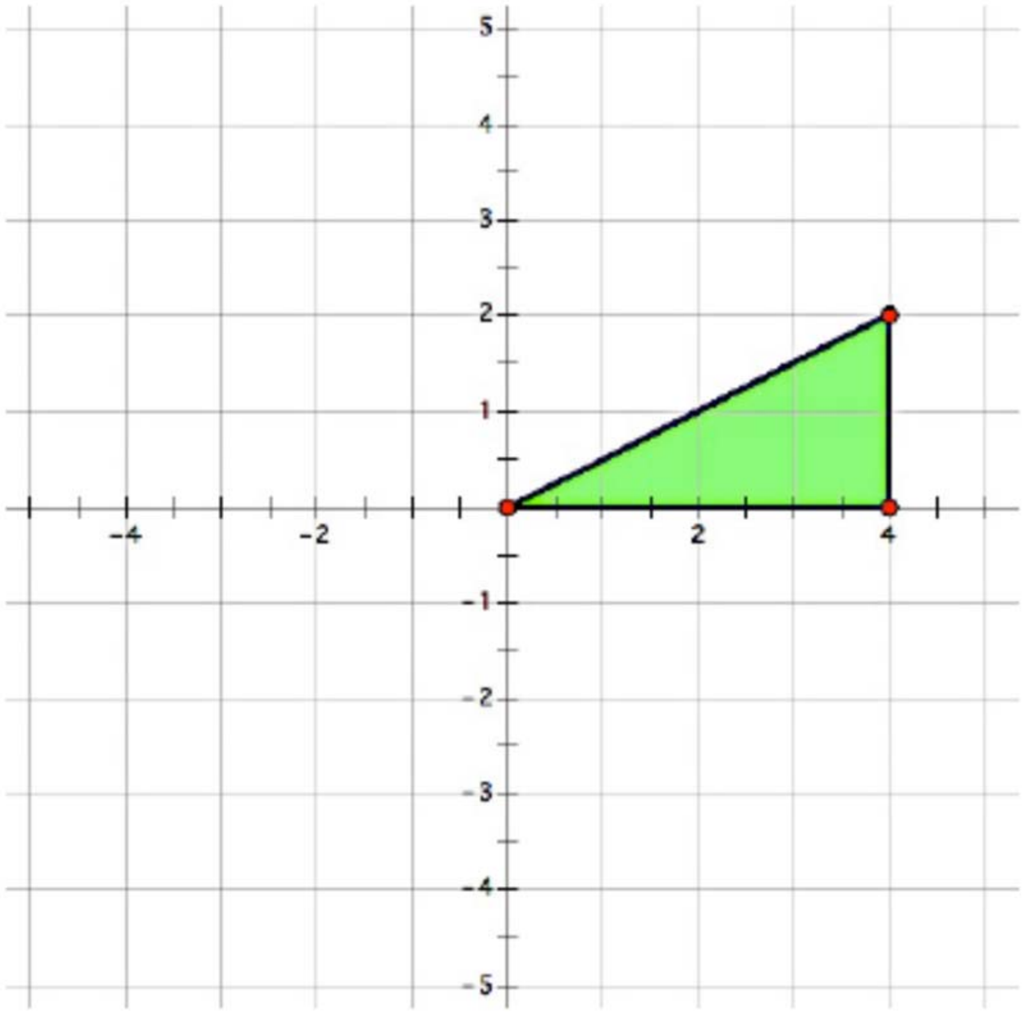
(c)The *central axis* of the container is a line that passes through the centers of the top and bottom. If one cuts the container and balls by a plane passing through the central axis, what does the intersection of the plane with the container and balls look like? (The intersection is also called a *cross section*. Imagine putting the cut surface on an ink pad and then stamping a piece of paper. The stamped image is a picture of the intersection.)
(d)If the can is cut by a plane parallel to the central axis, but at a distance of 1 inch from the axis, what will the intersection of this plane with the container and balls look like?

(e)If the can is cut by a plane parallel to one end of the can—a horizontal plane—what are the possible appearances of the intersections?

Formative Assessment Lesson: 2D Representations of 3D Objects

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project

http://map.mathshell.org/materials/download.php?fileid=1280



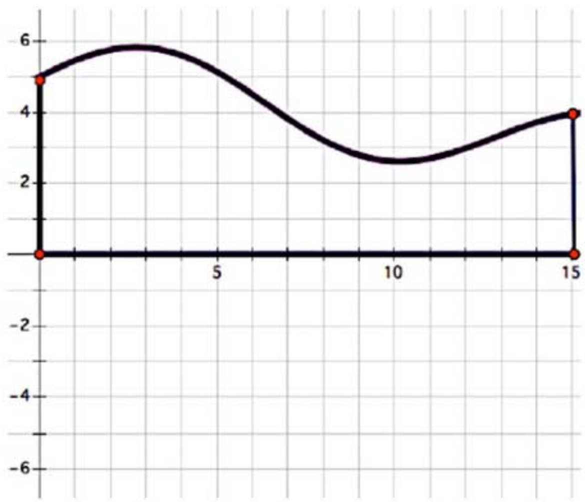
1. Draw and describe the solid of revolution formed by rotating this triangle about the x-axis.

2. Find the volume of the solid formed.

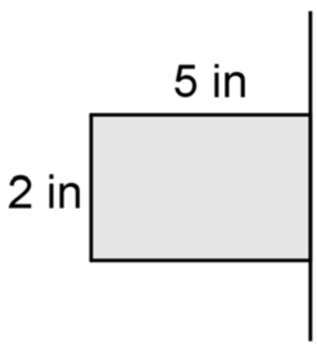
3. What would this figure look like if the triangle rotates rapidly about the y-axis? Draw and describe the solid of revolution formed by rotating this triangle about the y-axis.

4. Find the volume of the solid formed.

What about the following two-dimensional figure? Draw and describe the solid of revolution formed by rotating this figure about the x-axis.

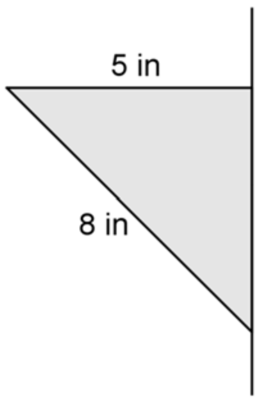


Use the picture below for #1-#3.



1. Describe the solid that is created when the figure above is rotated around the line.
2. Find the volume of the solid.
3. Identify at least 3 two dimensional shapes created by cross sections of the solid.

Use the picture below for #4-#6.

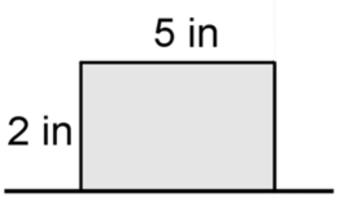


4. Describe the solid that is created when the figure above is rotated around the line.

5. Find the volume of the solid.

6. Identify at least 3 two dimensional shapes created by cross sections of the solid.

Use the picture below for #7-#9.

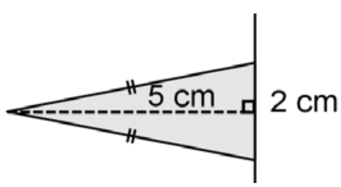


7. Describe the solid that is created when the figure above is rotated around the line.

8. Find the volume of the solid.

9. Identify at least 2 two dimensional shapes created by cross sections of the solid.

Use the picture below for #10-#12.

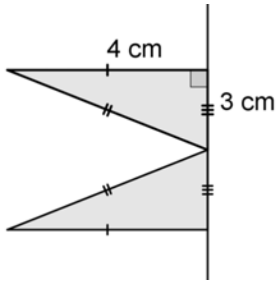


10. Describe the solid that is created when the figure above is rotated around the line.

11. Find the volume of the solid.

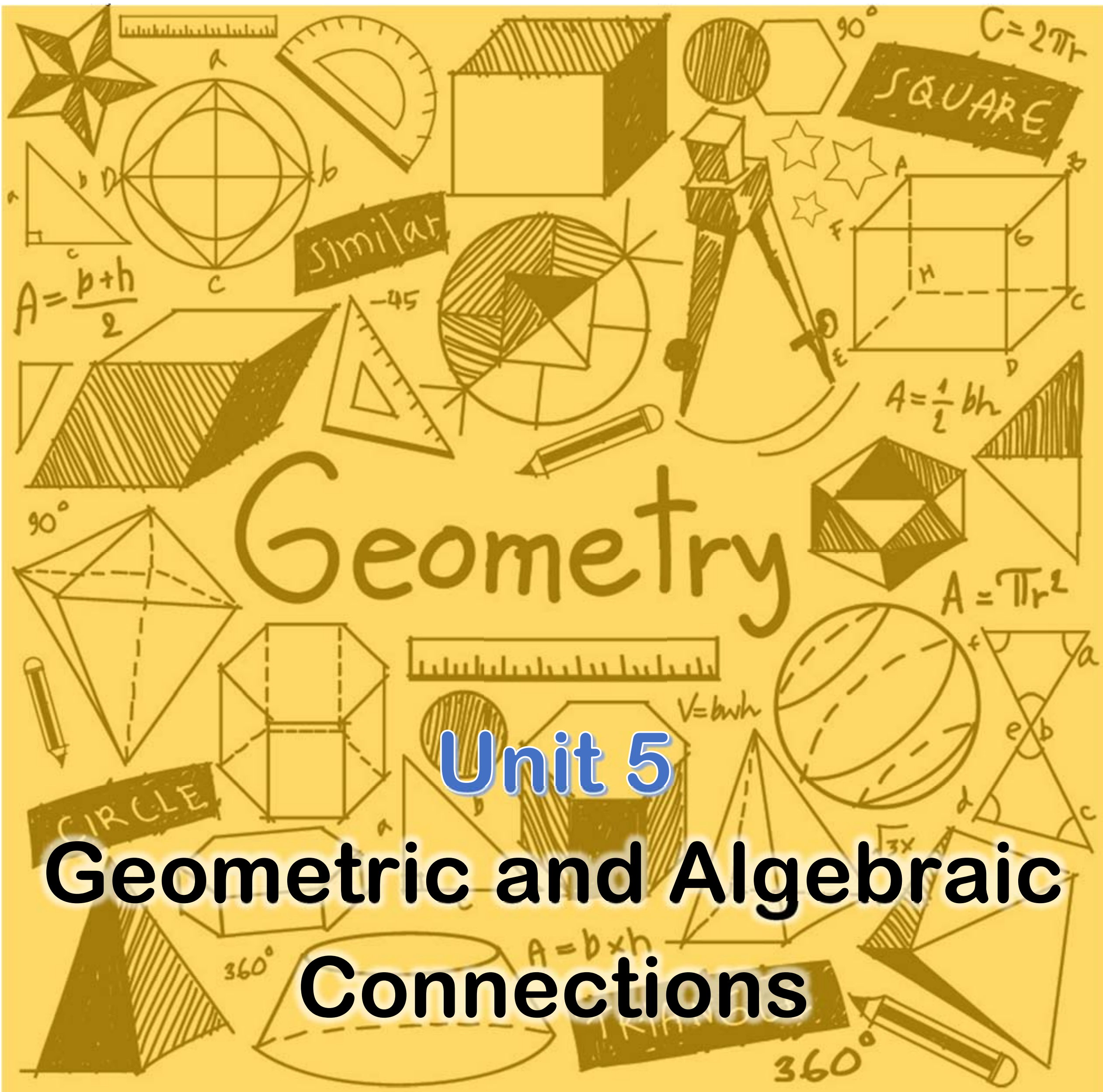
12. Identify at least 2 two dimensional shapes created by cross sections of the solid.

Use the picture below for #13-#15.



13. Describe the solid that is created when the figure above is rotated around the line.
14. Find the volume of the solid.
15. Identify at least 2 two dimensional shapes created by cross sections of the solid.

HOUSTON COUNTY BOARD OF EDUCATION
HIGH-ACHIEVING TEACHERS
MATHEMATICS DEPARTMENT



Unit 5

Geometric and Algebraic Connections

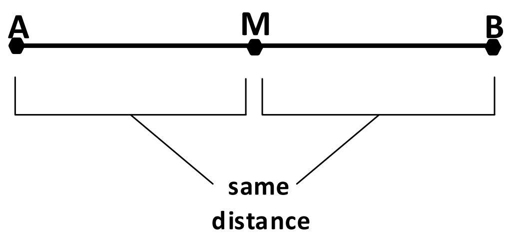
Lesson Name: G.U5.C1.A.01.**Vocab.MidPointDistance**

	Diagram/Visual		
Distance Formula			
Midpoint Formula			

Midpoint Formula

GUIDED NOTES

The **midpoint** of a segment is the one point on a segment that is the same distance from both of the endpoints of the segment.



For example, consider segment $\overline{AB}$ to the left.

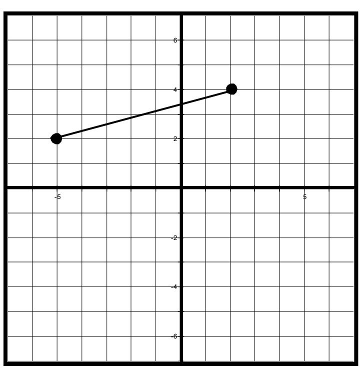
It has endpoints named *A* and *B*.

The midpoint of the segment is labeled *M*. It is the same distance from each of the endpoints.

Two Endpoints

Suppose one is given the coordinates of the endpoints of a line segment. Suppose they are (x_1, y_1) and (x_2, y_2). In order to find the coordinates of the midpoint, the **midpoint formula** can be used:

$$\text{Coordinates of midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Consider the segment graphed on the coordinate plane to the left. What are the coordinates of the two endpoints?

Endpoint 1: (-5, 2) Endpoint 2: (2, 4)

X_1, Y_1 X_2, Y_2

What are the coordinates of the midpoint of the segment?

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5+2}{2}, \frac{2+4}{2}\right) = \boxed{(-1.5, 3)}$$

One End Point and Midpoint

Suppose a segment has one endpoint with coordinates of (3, 9). Also, suppose the segment has a midpoint with coordinates of (10, 5). What are the coordinates of the other endpoint?

Note that the *x*-coordinate of one endpoint is given: $x_1 = 3$.

The *x*-coordinate of the other endpoint is unknown, so it will be known as x_2.

The *x*-coordinate of the midpoint is given - it is 10.

According to the midpoint formula: $\frac{x_1 + x_2}{2} =$ *x*-coordinate of the midpoint.

However, since x_1 and the *x*-coordinate of the midpoint are known, the expression can be rewritten as:

$$\frac{3 + x_2}{2} = 10.$$

Multiply both sides by 2 to eliminate the 2 in the denominator of the fraction on the left:

$$3 + x_2 = 20.$$

Now, subtract 3 from both sides to find the *x*-coordinate of the other endpoint:

$$\boxed{x_2 = 17}.$$

Similarly, to find the *y*-coordinate of the other endpoint, use the formula:

$$\frac{y_1 + y_2}{2} = \text{ } y\text{-coordinate of the midpoint}$$

Since $y_1 = 9$, and the *y*-coordinate of the midpoint is 5, the expression is rewritten as:

$$\frac{9 + y_2}{2} = 5.$$

After multiplying both sides by 2 and subtracting 9 from both sides, $\boxed{y_2 = 1}$.

Thus, the coordinates of the other endpoint are $\boxed{(17,1)}$.

Example!

1. Point A is located at (-2, 4), and point B is located at (1, 18).
   What are the coordinates of the midpoint of $\overline{AB}$?

   Midpoint = $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$ =

2. A segment has a midpoint located at (-2, -3) and an endpoint located at (-7, 2).
   What are the coordinates of the other endpoint?

   $\frac{x_1 + x_2}{2} =$ *x*-coordinate of the midpoint

   $\frac{y_1 + y_2}{2} =$ *y*-coordinate of the midpoint

SELF CHECK

Find the midpoint given the following points.

1. (2,5) and (8,7)
2. (4,-6) and (6,8)

Find the missing endpoint given the following endpoint and midpoint.

3. Endpoint 1 : (-2, 4) Midpoint : (3, -4)
4. Endpoint 1: (-4,7) Midpoint : (-1,5)

Distance Formula

GUIDED NOTES

Distance Formula: Given the two points (x_1, y_1) and (x_2, y_2), the distance *d* between these points is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Don't let the subscripts scare you. They only indicate that there is a "first" point and a "second" point; that is, that you have two points. Whichever one you call "first" or "second" is up to you. The distance will be the same, regardless.

Find the distance between the points (-2,-3) and (-4,4).

I just plug the coordinates into the Distance Formula:

$$d = \sqrt{(-4 - (-2))^2 + (4 - (-3))^2}$$

$$d = \sqrt{(-4 + 2)^2 + (4 + 3)^2}$$

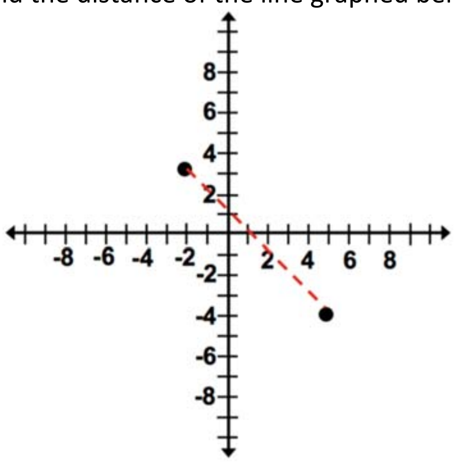
$$d = \sqrt{(-2)^2 + (7)^2}$$

$$d = \sqrt{4 + 49} = \sqrt{53} \approx 7.28$$

Example!

1. Find the distance between the points (6,2) and (3,-2)

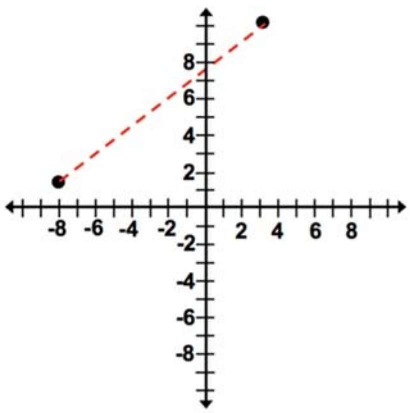
2. Find the distance of the line graphed below.



SELF CHECK

1. Find the distance between the points (-4,-5) and (-1,2)

2. Find the distance of the line graphed below.



Find the coordinates of the midpoint of the segment with the given endpoints.

1. Endpoint 1: (13, 12)

   Endpoint 2: (17, 5)

2. Point P is located at (-9, -5), and point Q is located at (-9, -8). What are the coordinates of the midpoint of $\overline{PQ}$?

3. Suppose a segment has one endpoint with coordinates of (0, 11). Also, suppose the segment has a midpoint with coordinates of (9, 10). What are the coordinates of the other endpoint?

Find the coordinates of the missing endpoint of the segment with the given endpoint and midpoint.

4. Endpoint 1: (13, 12)

   Midpoint: (7.5, 5)

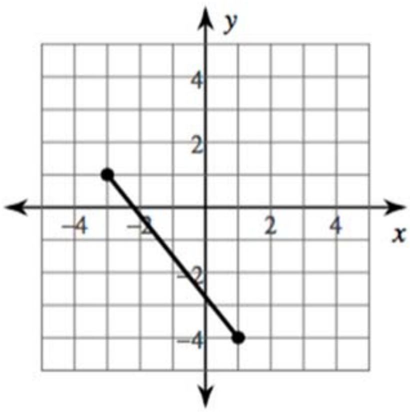
5. Endpoint 1 : (-1,9)

   Midpoint : (-9,-10)

6. Point K is located at (4.2, 0.3) and point L is located at (5.8, -0.6). What is the location of the midpoint of $\overline{KL}$?

7. The midpoint of a segment is 11 units from one endpoint. How long is the segment?

Use the graph below to answer questions 8-10

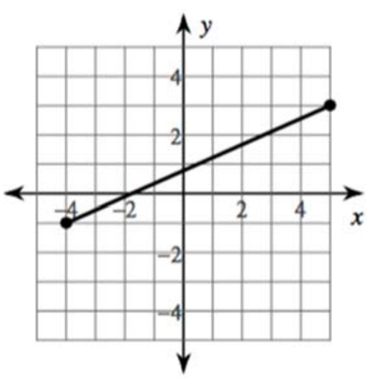


8. What are the end points of the line graphed?

9. What is the midpoint of the line graphed?

10. What is the distance of the line graphed?

Use the graph below to answer questions 11-13

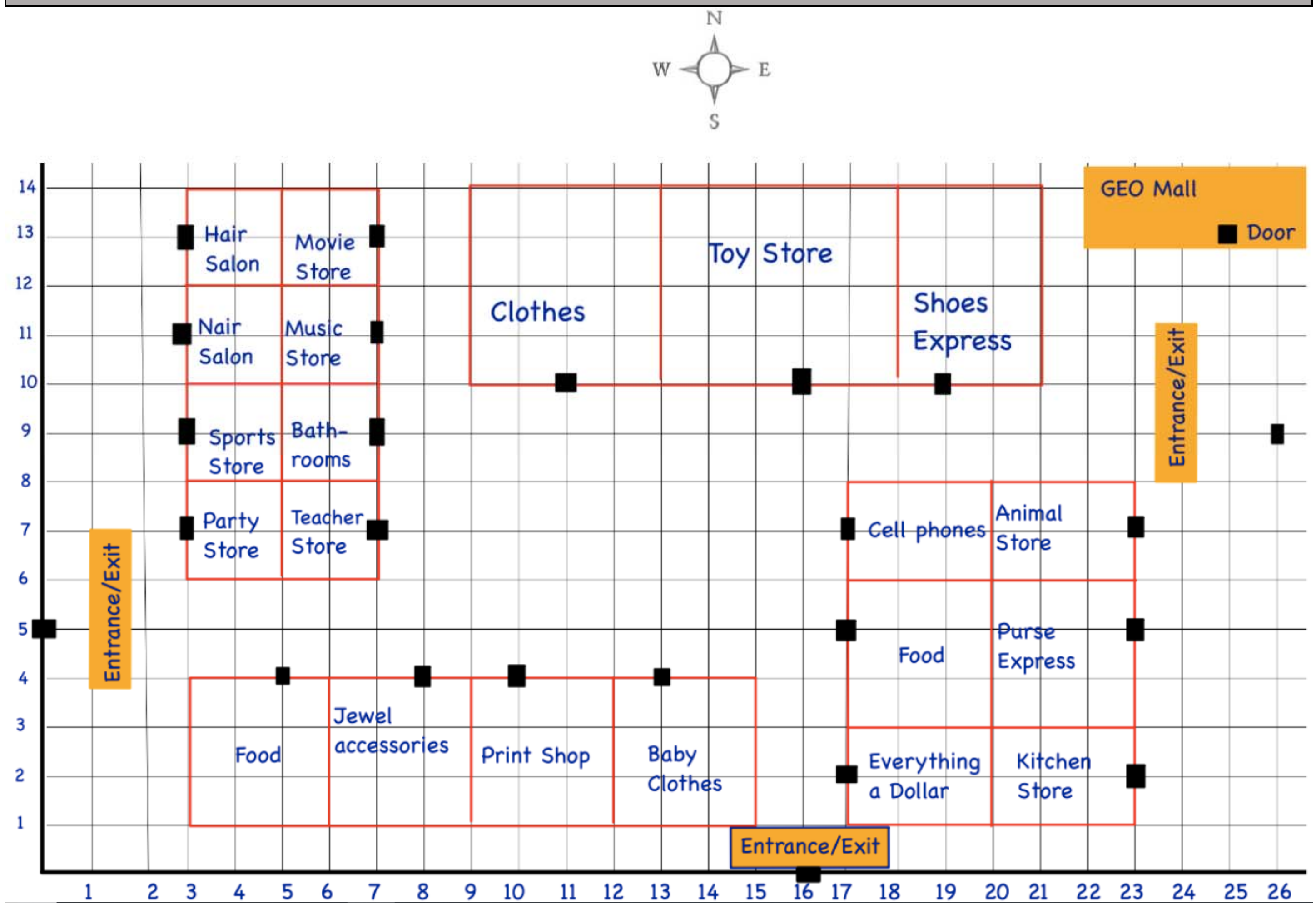


11. What are the end points of the line graphed?

12. What is the midpoint of the line graphed?

13. What is the distance of the line graphed?

Mall Distance



Activity Worksheet: Distance and Midpoint Exploration

Travis and Janelle want to take a trip to the mall to buy a toy for a friend who is moving. However, they have a very short time to shop for their friend. Janelle printed a layout of the mall to map out their route. Use the map to answer the following questions. The black dots represent the doors to the mall or stores. You must use straight lines to answer the questions. Show all computations that lead up to your solutions.

Janelle wants to not only go to the toy store but wants to stop at a couple other stores. She goes to the mall before Travis to meet up with him later. Janelle enters the mall at the East entrance to go to the Animal Store. After looking around for a while at the Animal store Travis calls her to make arrangements to meet.

1. Travis is at the West entrance and wants to meet Janelle at the midpoint of their two locations.
   a. Where would they meet? Use order pairs to represent their meeting location.

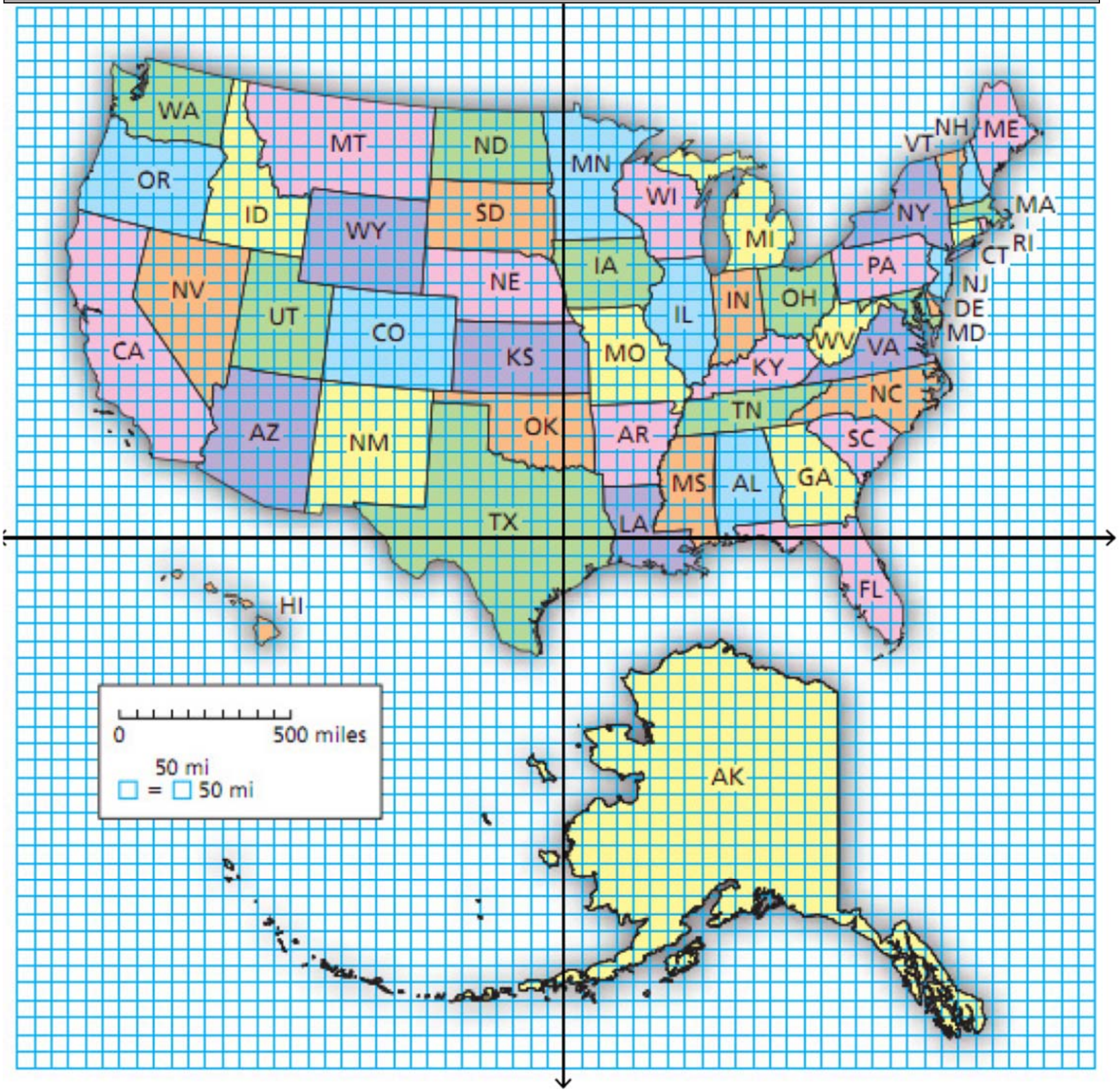
b. After meeting, are they closer to the Toy store or Music store? Include mathematical computations to justify your answer.

2. After meeting up, Janelle and Travis decide to get some food before shopping for a toy. What is the distance they will travel to get to the door of the closest food place?

3. After they finished eating, they went to the Toy Store, bought a great toy for their friend, then left the mall. After leaving the Toy store, Travis went back to the original meeting place then to the entrance he came in. What was Travis' total distance that he traveled from the time he entered the mall to the time he left? (Not to include any walking time while in the stores.) Illustrate Travis' route on the map provided.

Real Mathematics: Application

Travel the US



Real Mathematics: Application

You are planning a trip across the US. You have to travel through at least 10 states.

1. Find the total distance between the Start and Finish that you will travel
2. Split your trip up into 5 days. Find the distance you will travel each day (try to pick locations you would want to visit).
3. You MUST make a pit stop everyday for gas and food. You have to stop exactly half way everyday. Find your stopping points (aka your midpoint.)

Find the coordinates of the midpoint of the segment with the given endpoints.

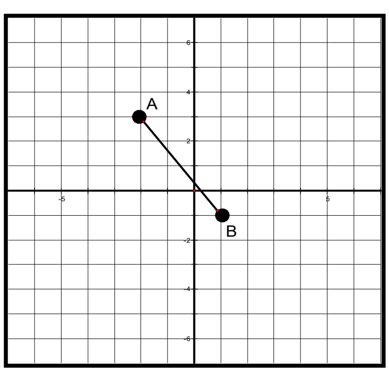
1. Endpoint 1: (-10, -2)
   Endpoint 2: (-1, -3)

Find the coordinates of the other endpoint of the segment with the given endpoint and midpoint.

2. Endpoint 1: (-10, -2)
   Midpoint: (-2.5, -9.5)

3. A segment on the coordinate plane has one endpoint located at (7, -7), and its midpoint is located at (17.5, -0.5). What are the coordinates of the other endpoint of the segment?

Use the graph below to answer questions 4-6



4. What are the coordinates of the end points?

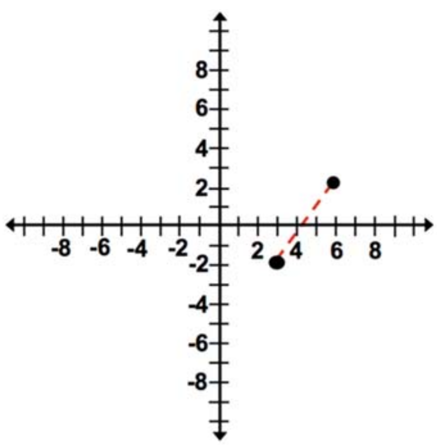
5. What are the coordinates of the midpoint of $\overline{AB}$?

6. What is the distance of $\overline{AB}$?

7. What is the distance between the points (-14, -1) and (10, -8) ?

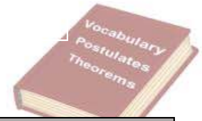
8. What is the distance between the points (-4,-5) and (-1,2) ?

Use the graph below to solve 9-10



9. What is the midpoint of the line?

10. How long is the line graphed above?



Term	Definition	Notation
Partitioning Formula (1)		
Partitioning Formula (2)		
Directed Line Segment		

Partitioning a Line Segment – Guided Notes

❖ To partition a line segment means to divide a segment into _________________ parts.

Draw points on the segments below to show each segment partitioned into the given number of sections.

3 Sections 

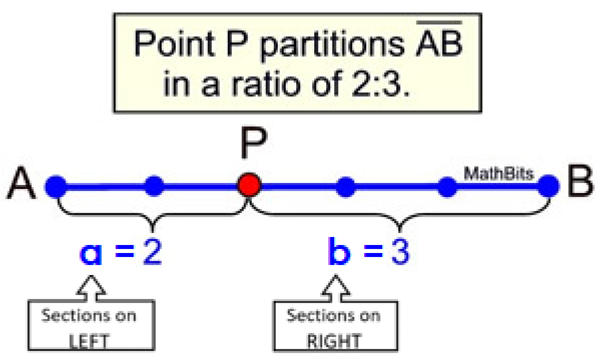
4 Sections 

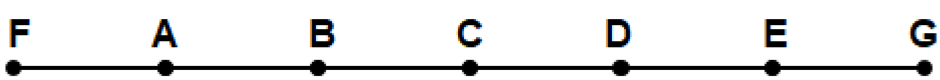
2 Sections 

UNDERSTANDING RATIOS

We may need to find the coordinates of the point that partitions a line segment into a given ***ratio***.

❖ A partitioning ratio represents the number of sections to the ______________ vs. the number of sections to the ________________ of a specific point, denoted a:b.





1. $\overline{FG}$ is partitioned into ______ sections.
2. What ratio of segments does point A divide $\overline{FG}$ into? ________________
3. What ratio of segments does point D divide $\overline{FG}$ into? ________________
4. What point divides $\overline{FG}$ into a ratio of 1:2? ________________
5. What point divides $\overline{FG}$ into a ratio of 5:1? ________________

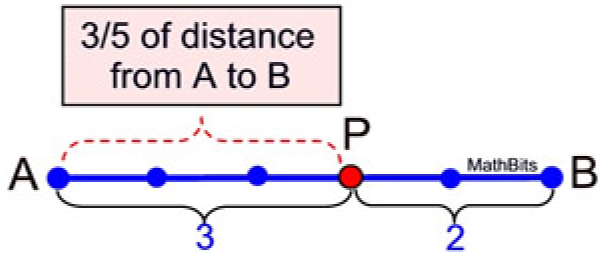
A ***directed line segment*** has direction associated with it. The order tells at which point to start and end. ***ORDER MATTERS!!***

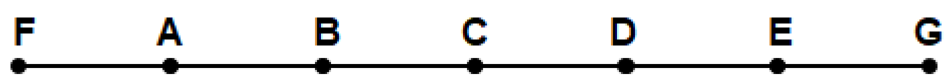
6. What ratio does point E divide the directed line segment $\overline{GF}$ into?* ________________

UNDERSTANDING FRACTIONAL DISTANCES

We may need to find the coordinates of the point that is a specific ***fractional distance*** from the beginning of a line segment.

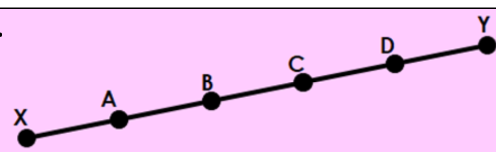
❖ The fractional distance represents the number of sections out of the ______________ number of sections a point is along a line segment.





7. What fractional distance is point A from F to G? ________________
8. What fractional distance is point D from F to G? ________________
9. What point is 1/3 of the way from F to G? ________________
10. What point is 5/6 of the distance from F to G? ________________
11. What fractional distance is point E from G to F?* ________________

SELF CHECK **Putting It All Together…**



1. Point B partitions $\overline{XY}$ into a ratio of ______________.
2. Point B is ________ of the way down $\overline{XY}$.
3. Point D partitions $\overline{YX}$ into a ratio of ______________.
4. Point D is ________ of the way down $\overline{YX}$.

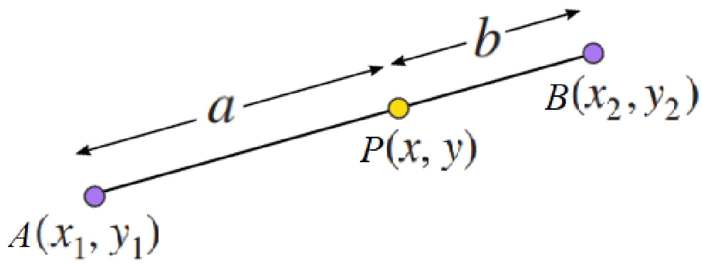
Questions To Ponder

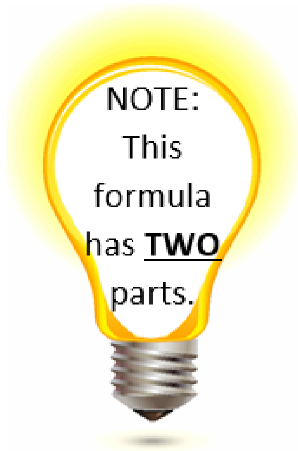
Using the ratio a:b, can you write the general form that represents the fractional distance?

To find the coordinates of a point that partitions a line segment into a given ratio, we use a formula.

The Formula

$$(x, y) = \left(x_1 + \frac{a}{a+b}(x_2 - x_1),\; y_1 + \frac{a}{a+b}(y_2 - y_1)\right)$$





The other Formula:

$$\left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a}\right)$$

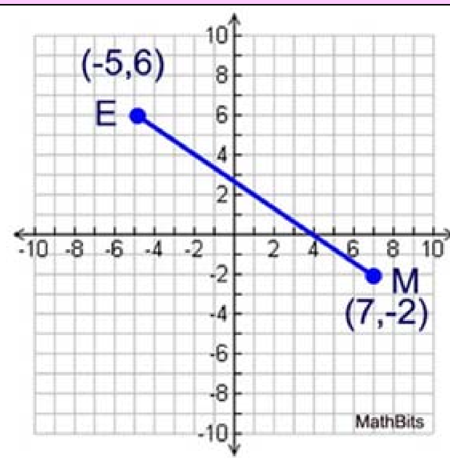
Example! **Find the point that partitions a line segment *ST* from S(3, 4) to T(5, 10) in the ratio 1:2. Draw a label an illustration of the line segment described showing the total number of sections and the location of the partitioning point.**

Example! **Find the coordinates of point *P* along the directed line segment *AB* that partitions *AB* so that $\frac{AP}{PB} = \frac{5}{3}$. Draw a label an illustration of the line segment described showing the total number of sections and the location of the partitioning point.**

Example! **Find the point P that is 2/3 of the way along AB if A is (2, 8) and B is (20, 100). Draw a label an illustration of the line segment described showing the total number of sections and the location of the partitioning point.**

SELF CHECK **Answer the following.**

1. Given $\overline{ME}$ as shown at the right. Find the coordinates of partition point *P* that divides directed segment $\overline{ME}$ into a 1:3 ratio.



Example! **Sometimes we will be given the coordinates of the partitioning point and one endpoint of a segment. In this situation you will have to use each part of the formula as an equation to solve for the unknown coordinates.**

The directed line segment *AB* is partitioned by point *P* so that the ratio of *AP* to *PB* is 5:1. If *A* has coordinates (-5, 4) and *P* has coordinates (5, -2), what are the coordinates of point *B*? Draw and label an illustration of the line segment described showing the total number of sections and the location of the partitioning point.

$$x = x_1 + \frac{a}{a+b}(x_2 - x_1) \qquad y = y_1 + \frac{a}{a+b}(y_2 - y_1)$$

Example! **Sometimes we will be given the coordinates of the partitioning point and one endpoint of a segment. In this situation you will have to use each part of the formula as an equation to solve for the unknown coordinates.**

Point *P* is ⅔ of the way along segment *XY*. Given that *X*(-2, 1) and *P*(2, 5), determine the location of *Y*? Draw a label an illustration of the line segment described showing the total number of sections and the location of the partitioning point.

$$x = x_1 + \frac{a}{a+b}(x_2 - x_1) \qquad\qquad y = y_1 + \frac{a}{a+b}(y_2 - y_1)$$

SELF CHECK **Line segment AB is divided by point C in the ratio 1:2. Point A is at (-6, 6) and point C is at (2, 4). What are the coordinates of point B? Draw a label an illustration of the line segment described showing the total number of sections and the location of the partitioning point.**

Partitioning A Line Segment – Practice

1. What ratio divides a segment AB in half? Illustrate your answer.

2. Given point A (4, 5) and point B (0, 0), find the coordinate of point P on the directed line segment AB that partitions $\overline{AB}$ in the ratio 3:2.

3. Given point A (-2, 4) and point B (7, -2), find the coordinate of point P on the directed line segment AB that partitions $\overline{AB}$ in the ratio 1:2.

4. Given point A (2, 4) and point B (8, 10), find the coordinate of point P on the directed line segment AB that partitions $\overline{BA}$ in the ratio 5:1.

5. Find the coordinate of point P that is ¾ of the way along the directed line segment from A(6, -5) to B(-3, 4).

6. Find the point C that is midway between A (3, 4) and B (-4, 1)

7. If point C partitions line segment AB in the ratio 1:4, in what ratio does point C partition $\overline{BA}$?

Challenge:

8. Find the ratio where point L partitioned segment JK.
Given the coordinates of J, K, and L as J (0, 0) K (6, 3) L (1.5, 0.75)

9. Line segment AB is divided by point C in the ratio 1:2. Point A is at (-6, 6) and point C is at (2, 4). What are the coordinates of point B?

This Graph is Lit

For 1-26, find each midpoint and connect them as you go to create an image.

1. (-8, 3) and (-4, -3)
2. (-10, 2) and (0, 4)
3. (-5, 1) and (-4, 7)
4. (0, -1) and (-9, 7)
5. (-12, 2) and (6, -1)
6. (-9, 20) and (5, -14)
7. (0.5, -1) and (1.5, 19)
8. (8, 30) and (-6, -16)
9. (-12, 7) and (20, -5)
10. (100, 19) and (-90, -15)
11. (8, -2.5) and (3, 10.5)
12. (-56, -99) and (72, 99)
13. (-3.5, -1) and (21.5, -5)
14. (10, -10) and (8, 0)
15. (59, -12) and (-43, -3)
16. (-4, -13) and (17, -5)
17. (-18, -1) and (28, -18)
18. (5.5, -7) and (0.5, -12)
19. (-2, -2) and (6, -18)
20. (-0.5, -268) and (2.5, 248)
21. (5, -11) and (-6, -8)
22. (12.5, -74) and (-16.5, 54)
23. (-22, -88) and (12, 70)
24. (-9.5, -9.5) and (-3.5, -4.5)
25. (0, 49) and (-14, -59)
26. (-38, -99) and (26, 99)

STOP

For 27 – 39, partition the segments in the given ratio. Connect the new points as you go to continue the image.

27. From (-4, 0) to (-10, 3), partition 2:1
28. From (-7, 0.5) to (-9, 4.5), partition 1:3
29. From (-11, -2) to (-2, 7), partition 4:5
30. From (-10, 0.5) to (-1, 9.5), partition 1:2
31. From (-5.5, -1) to (-10.5, 9), partition 2:3
32. From (-18, 6) to (2, -2), partition 1:1

STOP

33. From (0, -13) to (12, 7), partition 3:1
34. From (5, 4) to (17, 1), partition 1:2
35. From (4, 22) to (13, -14), partition 1:1
36. From (20.5, 15) to (-1.5, -7), partition 6:5
37. From (5.5, -6.5) to (10.5, 13.5), partition 2:3
38. From (0.5, 5.5) to (14.5, -1.5), partition 4:3
39. From (39, 20) to (-41, -28), partition 3:5

STOP

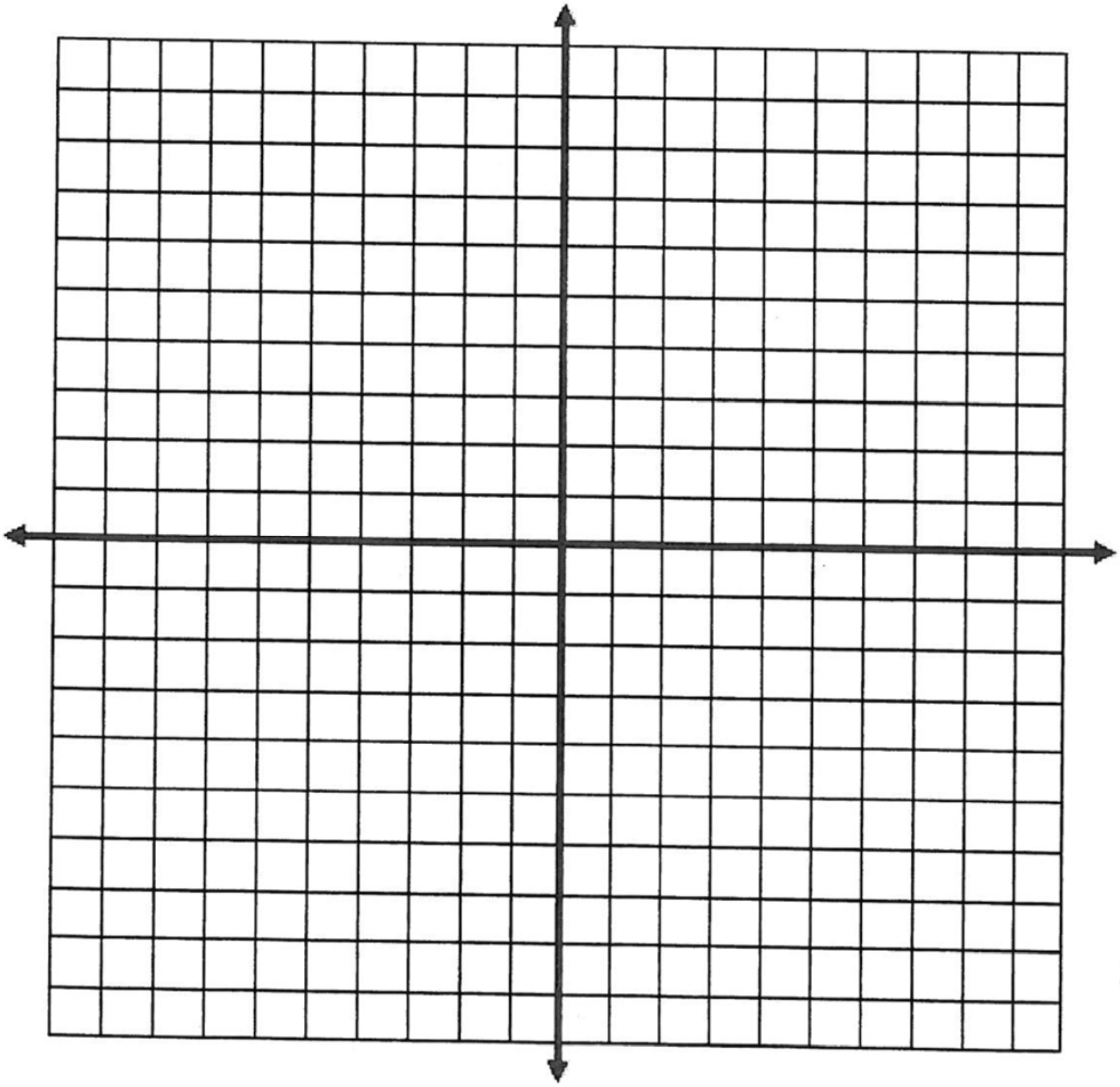
For 40 – 52, calculate the missing endpoint. Connect the new points as you go to complete the image.

40. Endpoint (-8, 11), midpoint (-6, 9)
41. Endpoint (0.5, 8.5), midpoint (-1.5, 7.5)
42. Endpoint (-20, 0), midpoint (-11.5, 3.5)
43. Endpoint (-1.5, -1), midpoint (-2, 3.5)
44. Endpoint (-25, 30), midpoint (-14, 20)
45. Endpoint (33, 100), midpoint (15, 54.5)
46. Endpoint (0, -12), midpoint (-2, -2.5)

STOP

47. Endpoint (-9, -1.5), midpoint (-2.5, 2.5)
48. Endpoint (-11.5, 17), midpoint (-3.5, 10.5)
49. Endpoint (-24, 0.5), midpoint (-10, 2)
50. Endpoint (19.5, 308), midpoint (11.5, 156)
51. Endpoint (-8.5, -24), midpoint (-2.5, -9.5)
52. Endpoint (288, -6.5), midpoint (146, 0)

STOP



New York City

The streets of New York City are laid out in a rectangular pattern, with all blocks approximately square and approximately the same size. **Avenues** run in a north–south direction, and the numbers increase as you move west. **Streets** run in an east–west direction, and the numbers increase as you move north.

Emily works at a building located on the corner of 9th Avenue and 61st Street in New York City. Her brother, Gregory, is in town on business. He is staying at a hotel at the corner of 9th Avenue and 43rd Street.

1. Gregory calls Emily at work, and they agree to meet for lunch. They agree to meet at a corner half way between Emily's work and Gregory's hotel. Then Gregory's business meeting ends early so he decides to walk to the building where Emily works.

a. How many blocks does he have to walk? Justify your answer using a diagram on grid paper.

b. After meeting Emily's coworkers, they walk back toward the corner restaurant halfway between Emily's work and Gregory's hotel. How many blocks must they walk? Justify your answer using your diagram.

2. After lunch, Emily has the afternoon off, so she walks back to the hotel with Gregory before turning to go to her apartment. Her apartment is three blocks north and four blocks west of the hotel.

a. At what intersection is her apartment building located?

b. How many blocks south of the restaurant will they walk before Emily turns to go to her apartment?

c. When Emily turns, what fraction of the distance from the restaurant to the hotel have the two of them walked? Express this fraction as a ratio of distance walked to distance remaining for Gregory.

3. Gregory and Emily are going to meet for dinner at a restaurant 5 blocks south of her apartment.

a. At which intersection is the restaurant located?

b. After dinner, they walk back towards her apartment, but stop at a coffee shop that is located three–fifths of the distance to the apartment. What is the location of the coffee shop?

By investigating the situations that follow, you will determine a procedure for finding a point that partitions a segment into a given ratio.

4. Here, you will find a point that partitions a directed line segment from $C(4, 3)$ to $D(10, 3)$ in a given ratio.

a. Plot the points on a grid. What is the distance between the points?

b. Use the fraction of the total length of CD to determine the location of Point A which partitions the segment from C to D in a ratio of 5:1. What are the coordinates of A?

c. Find point B that partitions a segment from C to D in a ratio of 1:2 by using the fraction of the total length of CD to determine the location of Point B. What are the coordinates of B?

5. Find the coordinates of Point X along the directed line segment YZ.

a. If $Y(4, 5)$ and $Z(4, 10)$, find X so the ratio is of YX to XZ is 4:1.

b. If $Y(4, 5)$ and $Z(4, 10)$, find X so the ratio is of YX to XZ is 3:2.

So far, the situations we have explored have been with directed line segments that were either horizontal or vertical. Use the situations below to determine how the procedure used for Questions 4 and 5 changes when the directed line segment has a defined, nonzero slope.

6. Find the coordinates of Point A along a directed line segment from $C(1, 1)$ to $D(9, 5)$ so that A partitions CD in a ratio of 3:1. *NOTE:* Since CD is neither horizontal nor vertical, the x and y coordinates have to be considered distinctly.

a. Find the x–coordinate of A using the fraction of the horizontal component of the directed line segment (i.e., the **horizontal** distance between C and D).

b. Find the y–coordinate of A using the fraction of the vertical component of the directed line segment (i.e., the **vertical** distance between C and D).

c. What are the coordinates of A?

7. Find the coordinates of Point A along a directed line segment from $C(3, 2)$ to $D(5, 8)$ so that A partitions CD in a ratio of 1:1. *NOTE:* Since CD is neither horizontal nor vertical, the x and y coordinates have to be considered distinctly.

a. Find the x–coordinate of A using the fraction of the horizontal component of the directed line segment (i.e., the **horizontal** distance between C and D).

b. Find the y–coordinate of A using the fraction of the vertical component of the directed line segment (i.e., the **vertical** distance between C and D).

c. What are the coordinates of A?

8. Now try a few more …

a. Find Point Z that partitions the directed line segment *XY* in a ratio of 5:3.
X(–2, 6) and *Y*(–10, –2)

b. Find Point *Z* that partitions the directed line segment *XY* in a ratio of 2:3.
X(2, –4) and *Y*(7, 2)

c. Find Point *Z* that partitions the directed line segment *YX* in a ratio of 1:3.
X(–2, –4) and *Y*(–7, 5) (Note the direction change in this segment.)

Back to Gregory and Emily….

9. When they finished their coffee, Gregory walked Emily back to her apartment, and then walked from there back to his hotel.

a. How many blocks did he walk?

b. If Gregory had been able to walk the direct path ("as the crow flies") to the hotel from Emily's apartment, how far would he have walked? Justify your answer using your diagram.

c. What is the distance Emily walks to work from her apartment?

d. What is the length of the direct path between Emily's apartment and the building where she works? Justify your answer using your diagram.

Determine a procedure for determining the distance between points on a coordinate grid by investigating the following situations.

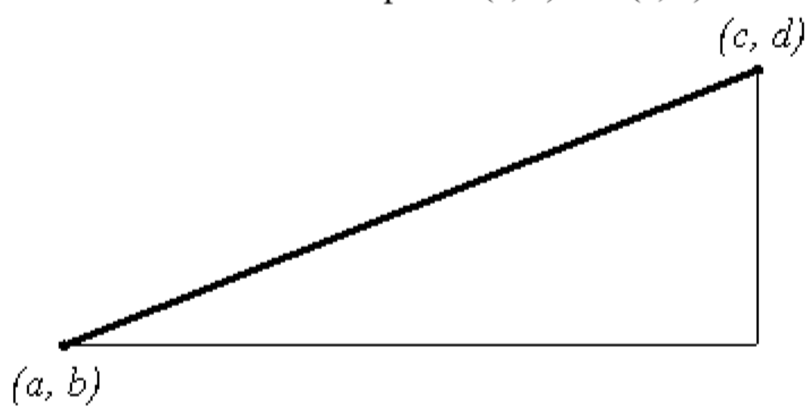
10. What is the distance between 5 and 7? 7 and 5? –1 and 6? 5 and –3?

11. Find a formula for the distance between two points, *a* and *b*, on a number line.

12. Using the same graph paper, find the distance between:

(1, 1) and (4, 4) (–1, 1) and (11, 6) (–1, 2) and (2, –6)

13. Find the distance between points (a, b) and (c, d) shown below.



14. Using your solutions from #13, find the distance between the point (x_1, y_1) and the point (x_2, y_2). Solutions written in this generic form are often called formulas.

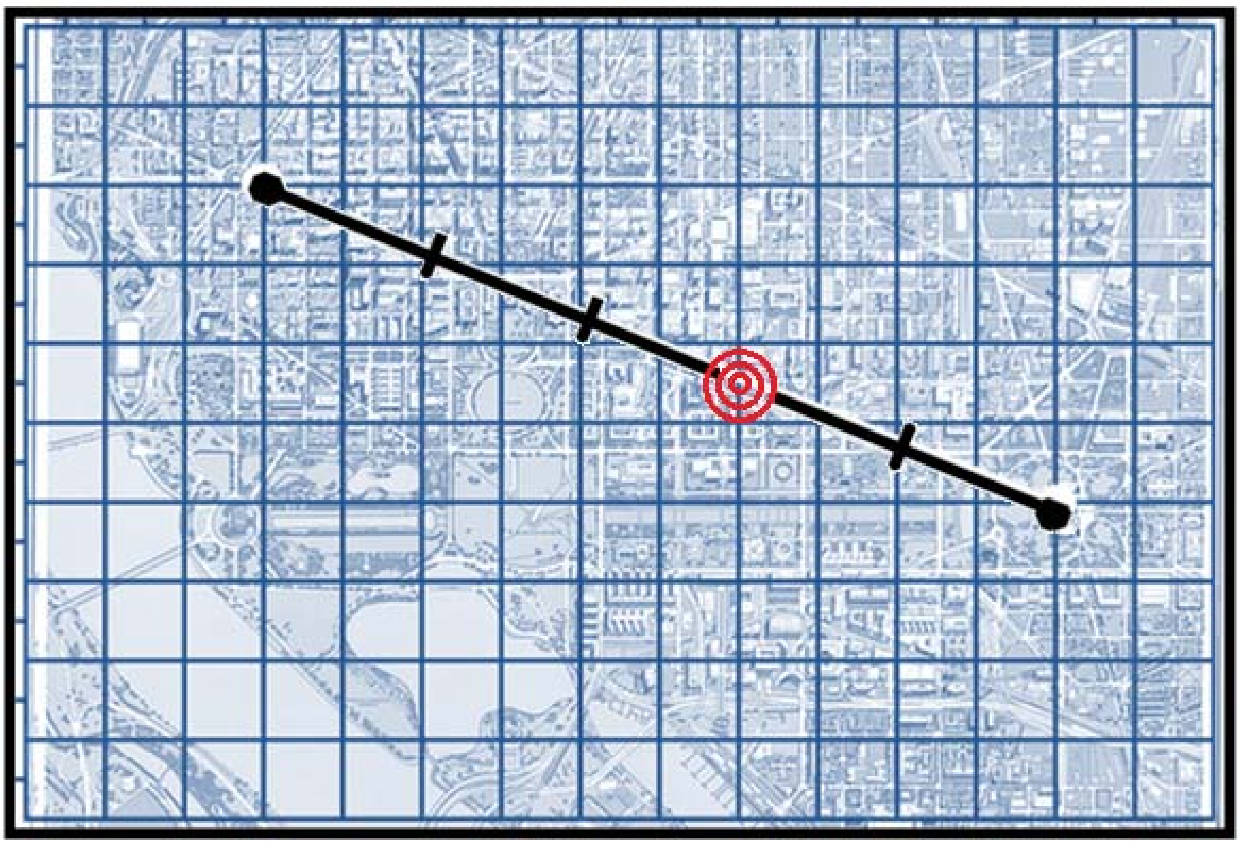
15. Do you think your formula would work for any pair of points? Why or why not?

Partitioning a Line Segment – Application

CALL OF DUTY

You are playing a war video game in which you must enter in the coordinates of your target. You are trapped in your Humvee in a parking garage, with only one roof-mounted Stinger Launcher on your vehicle. You have only one infrared homing missile left, and so this shot has to make contact with your target, otherwise you will face imminent "end of game."

Your terrain map tells you that your location is at (3, 8), and it shows that the location of the tower is (13, 4). Your range finder can triangulate the target location relative to the distance between your location and the tower. Your range finder indicates that your target is 3/5th of the distance between you and the tower (see picture, below). You must enter the coordinates of your target within a tenth of a km into your range finder in order for your Stinger's missile to annihilate your enemy.



Answer: ______________________________________

Raging River Task

You are a member of the Coast Guard Rescue Team that is stationed closest to the Mississippi River. There have been heavy rains in the area which has resulted in sudden flooding. You just received a call notifying you that some campers were washed into the raging river by the flood waters. Since the area is several miles long, you are sending 5 rescue teams immediately. It is your job to notify each team of their rescue location coordinates based on the map on the bottom of this page. Place the teams so they are equal distance apart to provide a swifter rescue!

Answers:

1st Point: ____________

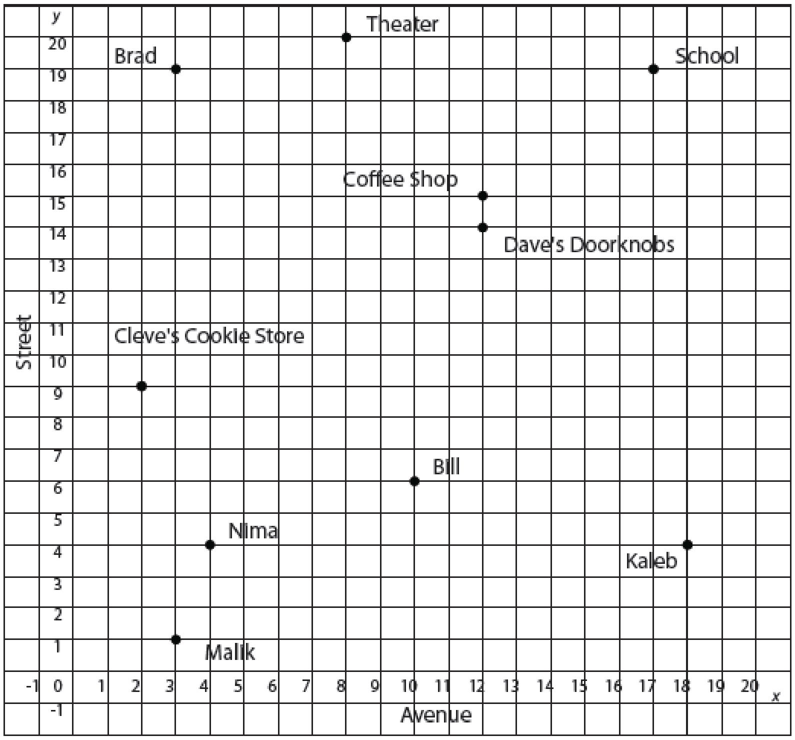
2nd Point: ____________

3rd Point: ____________

4th Point: ____________

5th Point: ____________

Use the map and the information given to solve each problem that follows.



1. Luis works at a theater on 8th Avenue and 20th Street. Kaleb lives at the corner of 18th Avenue and 4th Street. What is the location that is midway between them?

2. Nima lives at the corner of 4th Avenue and 4th Street. Bill lives at the corner of 10th Avenue and 6th Street. Their favorite bakery is located midway between them. Where is the bakery?

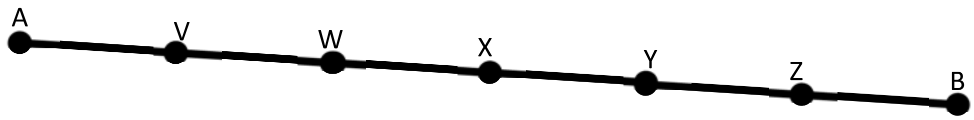
3. Cleve's Cookie Store is located at the corner of 2nd Avenue and 9th Street. Dave's Doorknobs is located at the corner of 12th Avenue and 14th Street. Located 1/5 of the distance from Cleve's Cookie Store is the post office. Where is the post office?

4. Malik and Brad both live on 3rd Avenue. Malik lives at the corner of 1st Street, and Brad lives at the corner of 19th Street. 2/3 the distance from Malik's apartment to Brad's apartment is a market. Where is the market?

5. The main entrance to the high school is located at the corner of 17th Avenue and 19th Street. On his way from school to the bank, Luis stops at the coffee shop located at 12th Avenue and 15th Street. The coffee shop is the midpoint of this trip. What is the location of the bank?

Partitioning a Line Segment – Homework

Use the diagram for questions #1 – #6.



1. Which point would partition the line segment $\overline{AB}$ in a ratio of 2:4? ____________

2. Which point would partition the line segment $\overline{AB}$ in a ratio of 5:1? ____________

3. Which point would partition the line segment $\overline{AB}$ in a ratio of 1:2? ____________

4. Which point would partition the line segment $\overline{AB}$ in a ratio of 2:1? ____________

5. Which point would partition the line segment $\overline{AB}$ in a ratio of 1:1? ____________

6. Write the formula for partitioning a line segment.

7. What does the formula for partitioning a line segment find? In other words, what do the numbers you get after using the formula tell you?

8. A point is ¾ of the way along a line segment. Write the ratio that expresses this partitioning.

9. A point is one fifth of the way along a line segment. Write the ratio that expresses this partitioning.

10. Point Q partitions a line segment PR in the ratio 2:4. Write the fraction that expresses how far point Q is along PR.

11. Point Y partitions a line segment XZ in the ratio 3:1. Write the fraction that expresses how far point Y is along XZ.

12. Given the points A(–1, 2) and B(7, 14), find the coordinates of the point P on directed line segment $\overline{AB}$ that partitions $\overline{AB}$ in the ratio 1:3.

13. Given the points A(-2, 4) and B(7, -2), find the coordinates of the point P on directed line segment $\overline{BA}$ that partitions $\overline{BA}$ in the ratio 1:2.

14. Given the points A(-3, -4) and B(5, 0), find the coordinates of the point P on directed line segment $\overline{AB}$ that partitions $\overline{AB}$ in the ratio 2:3.

15. Points Q(3, 4) and S form segment $\overline{QS}$. Point R$(4\frac{4}{5}, 7\frac{3}{5})$ partitions $\overline{QS}$ into a ratio of 3 : 2. What are the coordinates of point S?

16. Points A and C(8, 4) form segment $\overline{AC}$. Point B(6.6, 3.8) forms a ratio between points A and C that is 4:1. What are the coordinates of point A?

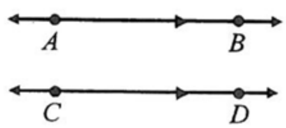
17. Point *M* is 1/5 of the way along segment *LN*. Given that *M* $\left(7, -\frac{2}{5}\right)$ and *N*(3, -2), determine the coordinates of *L*?

Term	Definition	Notation	Diagram/Visual
Parallel Lines			
Perpendicular Lines			
Tangent Line			

Parallel and Perpendicular Lines

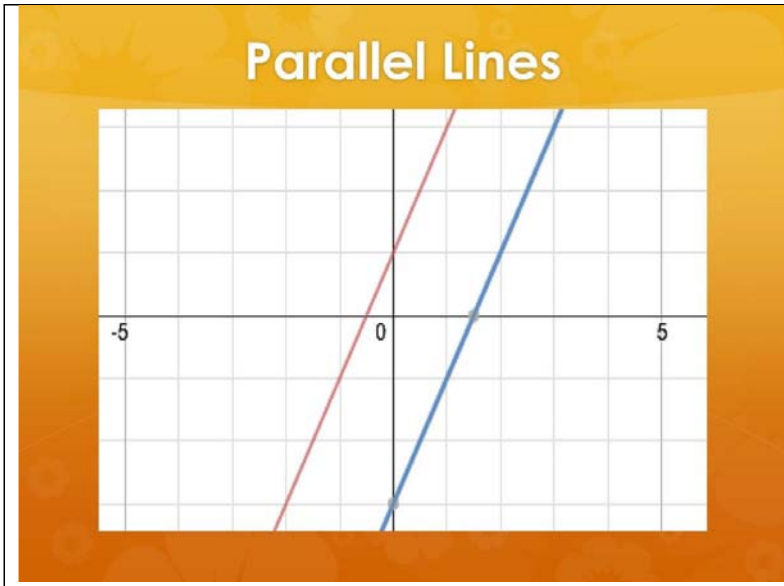
GUIDED NOTES

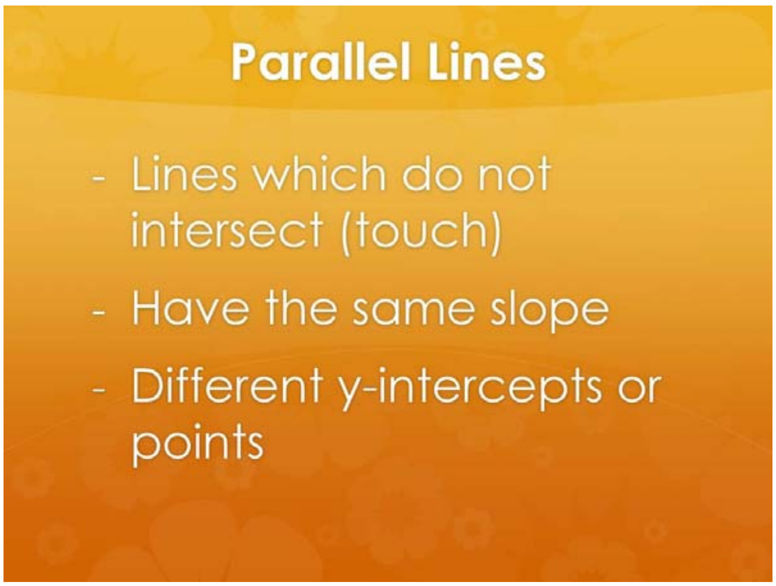
PARALLEL Lines



- ______________ lines that do not ______________.
- ___________ are used to indicate the lines are parallel.
- **Symbolic Notation:** ______________________

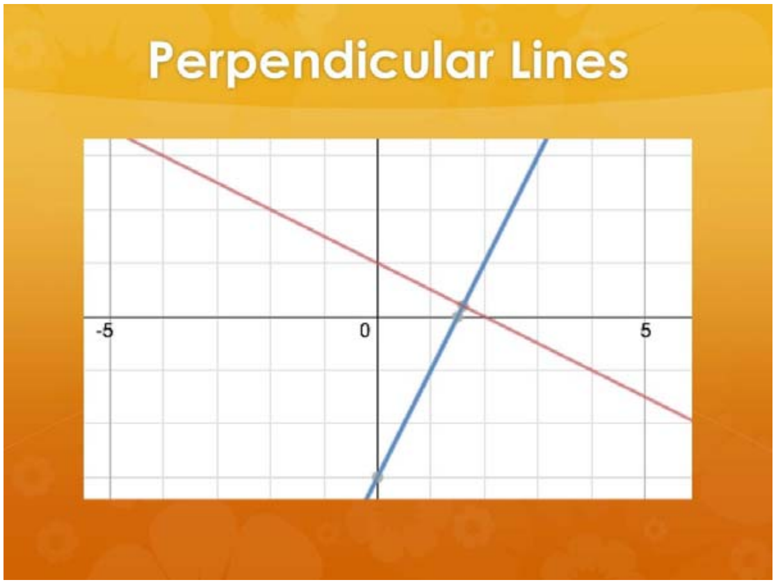
Parallel Lines	Parallel lines have the _____________ slope!
Perpendicular Lines	Perpendicular lines have _________________ ____________________ slopes!
Negative Reciprocals	
	Some examples: **1)** $\frac{3}{4}$ & **2)** 2 & **3)** $-\frac{7}{8}$ & **4)** 1 & **5)** 0 &

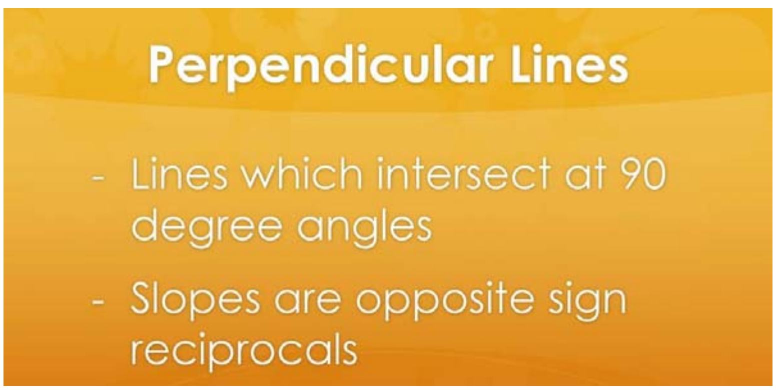




Parallel Lines

- In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.
- Any two vertical lines are parallel





Perpendicular

• In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

• Horizontal lines are perpendicular to vertical lines.

• If two lines are perpendicular to one another the slopes will be opposite reciprocals of one another.

The **equation of a straight line** is usually written this way:

$$y = mx + b$$

y = how far up

x = how far along

m = Slope or Gradient (how steep the line is)

b = value of **y** when **x=0**

How do you find "m" and "b"?

b is easy: just see where the line crosses the Y axis.

m (the Slope) needs some calculation:

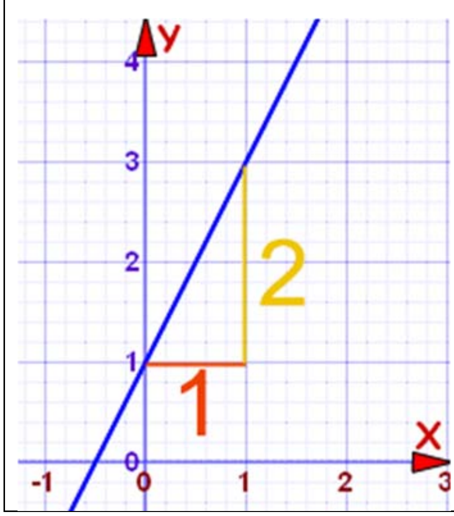
m = $\frac{change\ in\ y}{change\ in\ x}$

Sometimes the words "rise" and "run" are used.

Rise: how far up
Run: how far along
And so the slope "m" is: **m** = $\frac{rise}{run}$

Given a graph, use $m = \frac{rise}{run}$ Given ordered pairs, use $m = \frac{y_2 - y_1}{x_2 - x_1}$

Knowing this we can work out the equation of a straight line:



$m = \frac{2}{1} = 2$

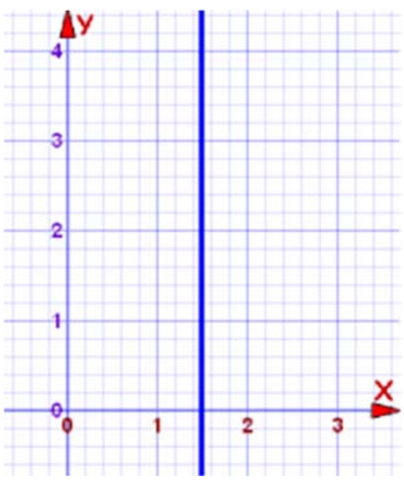
b = 1 *(value of **y** when x=0)*

So: y = 2x + 1

Positive or Negative Slope?

Remember, these are all equivalent fractions.

$$-\frac{3}{5} = \frac{-3}{5} = \frac{3}{-5}$$



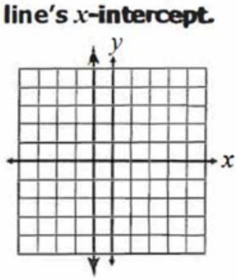
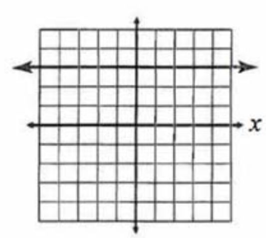
What is the equation for a vertical line?
The slope is **undefined** ... and where does it cross the Y-axis?

In fact, this is a **special case**, and you use a different equation, not "**y**=...", but instead you use "**x**=...".

Like this:

x = 1.5

Every point on the line has **x** coordinate **1.5**,
that is why its equation is **x = 1.5**

Vertical Lines	Horizontal Lines
A vertical line is written in the form $x = a$, where a represents the line's x-intercept.  **Equation:** ______________.	**A horizontal line is written in the form $y = a$, where a represents the line's y-intercept.**  **Equation:** ______________.

Example!

Fill in the chart with the missing slopes.

Slope of the Given Line	Slope of the Line Parallel to the Given Line	Slope of the Line Perpendicular to the Given Line
12		
$\frac{6}{5}$		
Undefined		
$-\frac{1}{3}$		

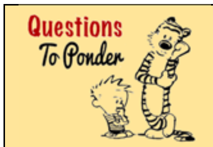
Are these lines parallel perpendicular or neither? How do we know?	
$y = x + 2$ $y = -x + 5$	
$2x - 3y = 3$ $-4x + 6y = -24$	

SELF CHECK

Fill in the chart with the missing slopes.

Slope of the Given Line	Slope of the Line Parallel to the Given Line	Slope of the Line Perpendicular to the Given Line
-5		
$\frac{4}{3}$		
2		
Zero		

Are these lines parallel perpendicular or neither? How do we know?	
$y = 3x + 9$ $y = \frac{1}{3}x - 4$	
$3x + 2y = 5$ $3y + 2x = -3$	



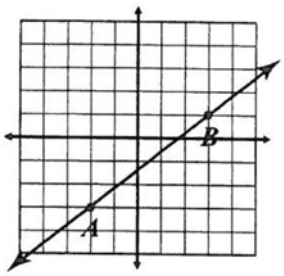
How might we use information about line relationships (parallel or perpendicular) to discover or analyze shapes like quadrilaterals in the coordinate plane?

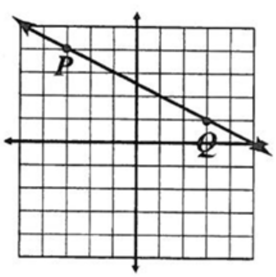
Classifying Slope

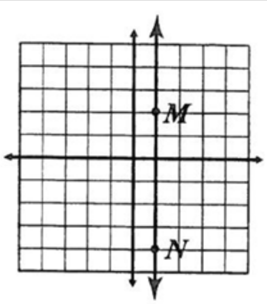
			

Find the slope:

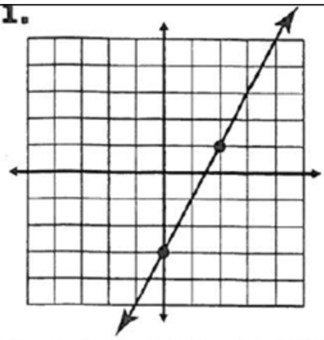
$m = \frac{rise}{run}$ $m = \frac{y_2 - y_1}{x_2 - x_1}$







1.



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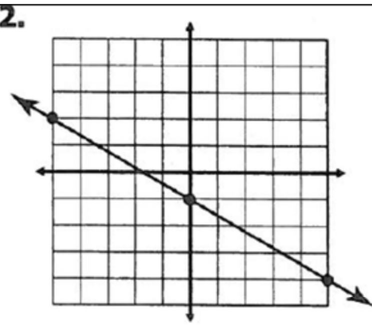

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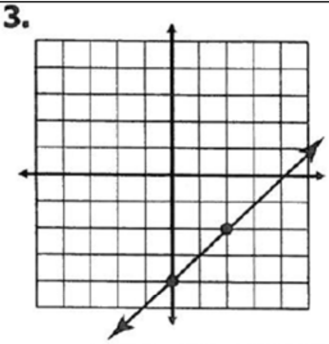

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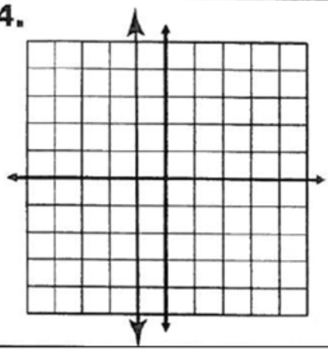

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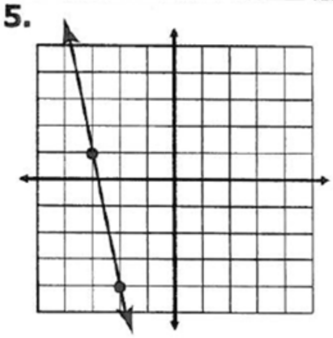

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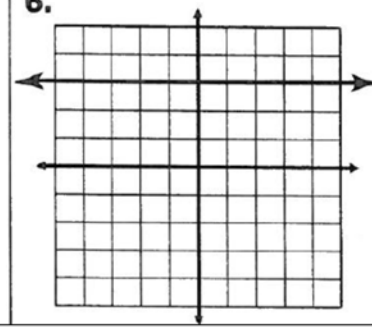


Find the slope of $\overleftrightarrow{AB}$: A(-2, 5) and B(6, -1)	
Find the slope of $\overleftrightarrow{GH}$: G(-11, -3) and H(-6, 7)	
Find the slope of $\overleftrightarrow{XY}$: X(9, -4) and Y(9, 2)	
Find the slope of $\overleftrightarrow{RS}$: R(-7, -3) and S(0, -3)	
(-1, -11) and (-6, -7)	
(-7, -13) and (1, -5)	
(8, 3) and (-5, 3)	
(15, 7) and (3, -2)	
(-5, -1) and (-5, -10)	
(-12, 16) and (-4, -2)	

Use the slope to determine if lines AB and CD are parallel, perpendicular, or neither.

	Slope of $\overleftrightarrow{AB}$	Slope of $\overleftrightarrow{CD}$	Types of Lines
A(-2, 3), B(2, 6), C(-1, 0), D(3, 3)			

	Slope of $\overleftrightarrow{AB}$	Slope of $\overleftrightarrow{CD}$	Types of Lines
A(0, 2), B(5, 4), C(1, 8), D(3, 3)			

A(-9, -12), B(-2, 2), C(-1, 6), D(-5, -2)	**Slope of $\overleftrightarrow{AB}$**	**Slope of $\overleftrightarrow{CD}$**	**Types of Lines**
A (-1, 8), B(2, 6), C(-1, 2), D(3, 3)	**Slope of $\overleftrightarrow{AB}$**	**Slope of $\overleftrightarrow{CD}$**	**Types of Lines**
A(0, -2), B(0, 7), C(3, -5), D(6, -5)	**Slope of $\overleftrightarrow{AB}$**	**Slope of $\overleftrightarrow{CD}$**	**Types of Lines**
A(3, 1), B(3, -4), C(-4, 1), D (-4, 5)	**Slope of $\overleftrightarrow{AB}$**	**Slope of $\overleftrightarrow{CD}$**	**Types of Lines**
Use the slope to determine if lines AB and CD are parallel, perpendicular, or neither.			
P(-9, -4), Q(-7, -1), R(-2, 5), S(-6, -1)	**Slope of $\overleftrightarrow{PQ}$**	**Slope of $\overleftrightarrow{RS}$**	**Types of Lines**
P(-4, 17), Q(l, -3), R(-9, 3), S(-5, 4)	**Slope of $\overleftrightarrow{PQ}$**	**Slope of $\overleftrightarrow{RS}$**	**Types of Lines**
15. P(12, -2), Q(S, -10), R(-4, 10), S(4, 3)	**Slope of $\overleftrightarrow{PQ}$**	**Slope of $\overleftrightarrow{RS}$**	**Types of Lines**
Convert to slope-intercept form.			
5x + 2y = -2			
3x - y = 5			
X - 4y = 0			
Determine whether the lines are parallel, perpendicular, or neither.			
y = 5x + 2 and y = 5x -1			
x + 2y = 2 and 6x - 3y = 21			
$y = -\frac{1}{4}x - 5$ and $2x + 8y = 56$			

$2x + 7y = 28$ $7x - 2y = 4$	
$y = -5x + 1$ $x - 5y = 30$	
$3x + 2y = 8$ $2x + 3y = -12$	
$4x + 9y = 18$ $y = 4x + 9$	
$5x - 10y = 20$ $y = -2x + 6$	
$-9x + 12y = 24$ $y = \frac{3}{4}x - 5$	
$x - 2y = 18$ $2x + y = 6$	
$x = 4$ $x = -6$	
$x = 1$ $y = -8$	

Slopes of Special Pairs

Parallel Lines

1. On an *xy*–plane, graph lines ℓ_1, ℓ_2, and ℓ_3, containing the given points. ℓ_1 contains points *A* (0, 7) and *B* (8, 9); ℓ_2 contains points *C* (0, 4) and *D* (8, 6); ℓ_3 contains points *E* (0, 0) and *F* (8, 2). Make sure to carefully extend the lines past the given points.

a. Find the distance between *A* and *C* and between *B* and *D*. What do you notice?

What word describes lines ℓ_1 and ℓ_2?

b. Find the distance between *C* and *E* and between *D* and *F*. What do you notice?

What word describes lines ℓ_2 and ℓ_3?

c. Find the distance between *A* and *E* and between *B* and *F*. What do you notice?

What word describes lines ℓ_1 and ℓ_3?

d. Now find the slopes of ℓ_1, ℓ_2, and ℓ_3.

What do you notice?

2. Now plot line ℓ_4 through points W (–1, 3) and X(–3, 6) and line ℓ_5 through points Y (–2, 1) and Z (–4, 4) carefully extending the lines across the y–axis.

a. Use a ruler to measure the distance from W vertically to ℓ_5. Then measure the distance from X vertically to ℓ_5. What do you notice?

b. What word describes these lines?

c. Find the slope of each line. What do you notice?

3. What appears to be true about the slopes of parallel lines?

4. Follow the steps below to prove this true for all pairs of parallel lines.

a. Let the straight lines ℓ and m be parallel. Sketch these on grid paper.

b. Plot any points U and V on line ℓ and the point W so that WV is the rise and UW is the run of the slope of line ℓ. (A straight line can have only one slope.)

That is, the slope of line ℓ is $\dfrac{WV}{UW}$.

c. Draw the straight line UW so that it intersects line m at point X and extends to include Point Z such that segment YZ is perpendicular to UW.

d. What is the slope of line m?

e. Line UZ is the ______________________ of the lines ℓ and m, so $\angle VUW$ and $\angle YXZ$

are ____ angles, so $\angle VUW$ ____ $\angle YXZ$.

f. Why is it true that $\angle UWV \cong \angle YXZ$?

g. Now, ΔUWV and ΔYXZ are similar, so the ratio of their sides is proportional. Write the proportion that relates the vertical leg to the horizontal leg of the triangles.

h. Note that this proportion shows the slope of line ℓ is the same as the slope of line m. **<u>Therefore, parallel lines have the same slope.</u>**

5. Write equations of two lines that are parallel to the line. $y = \frac{2}{3}x + 4$

6. Determine which of the following lines is / are parallel to $2x - 3y = 21$. Explain why.

a. $y = -\frac{2}{3}x + 2$ **b.** $-6x + 9y = 12$ **c.** $\frac{1}{3}x + y = 6$

d. $2x + 3y = 7$ **e.** $3y = 2x + 1$

7. Line m is parallel to the line $y = -\frac{1}{2}x + 2$ and contains the point (–6, 1). What is the equation of line m in slope–intercept form?

8. What is the equation of the line that passes through (5, 2) and is parallel to the line that passes through (0, 5) and (–4, 8)?

Perpendicular Lines

1. On a coordinate grid, graph the following pairs of lines. For each pair, answer: Do these lines intersect? If so, describe the angles formed at their intersection. Use a protractor if necessary. If not, describe the lines.

a. $y = -\frac{3}{4}x + 5$ and $y = \frac{4}{3}x + 1$

b. $y = 3x - 1$ and $y = -\frac{1}{3}x - 1$

c. $y = -7x + 2$ and $y = \frac{1}{7}x - 3$

d. $y = x$ and $y = -x - 8$

2. Create two equations that have the same type relationship as the lines in Question 1. Draw the lines on a grid to show this relationship. What characteristics do the equations of these lines possess?

3. Will all lines with these characteristics have the same graphical relationship? If so, prove it. If not, give a counterexample.

4. Use the relationship between slopes of perpendicular lines to answer the following questions.

a. Line m has the equation $y = \frac{5}{4}x + 1$. What is the slope of a line perpendicular to line m?

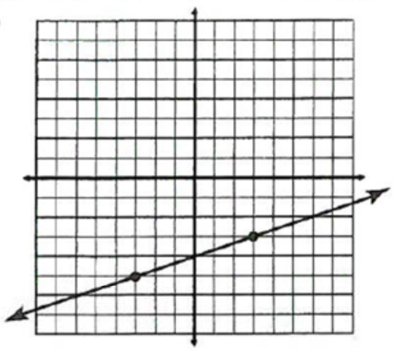
b. Write the equation of the line perpendicular to $y = -2x + 5$ whose y–intercept is 12.

c. Write the equation of the line perpendicular to $y = \frac{1}{5}x - 6$ which passes through the point (1, –3).

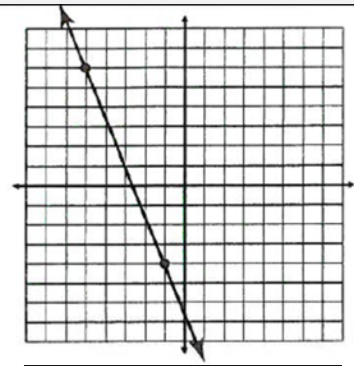
d. What is the equation of the line that passes through (5, 2) and is perpendicular to the line that passes through (0, 5) and (–4, 8)?

Find the slope:

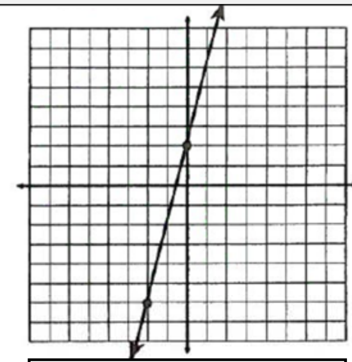
1.



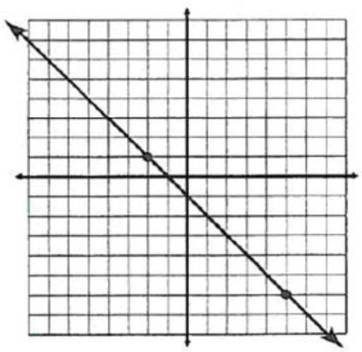
2.



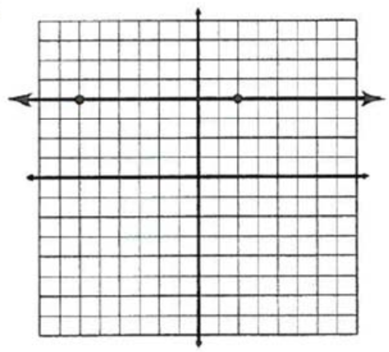
3.



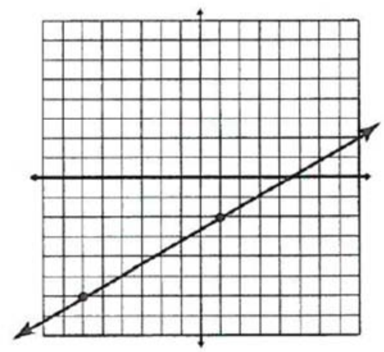
4.



5.



6.

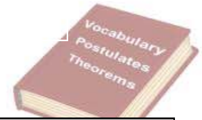


(-8, -11) and (17, 4)	
(10, -15) and (13, -17)	
(-6, -7) and (5, -7)	
(-4, -3) and (2, -9)	
(10, -2) and (10, 5)	
(-5, 3) and (19, -6)	
(-7, -12) and (1, -16)	
(-18, 0) and (-13, 1)	
(1, -11) and (-2, -4)	

HOMEWORK

Use the slope to determine if lines AB and CD are parallel, perpendicular, or neither.			
A(2, 3), B(-1, 4), C(-5, 3), D(-4, 6)	**Slope of** $\overleftrightarrow{AB}$	**Slope of** $\overleftrightarrow{CD}$	**Types of Lines**
A(-3, 13), B(4, -15), C(-2, 5), D(1, -7)	**Slope of** $\overleftrightarrow{AB}$	**Slope of** $\overleftrightarrow{CD}$	**Types of Lines**
A(9, 2), B(-1, 8), C(-5, 16), D(-8, 11)	**Slope of** $\overleftrightarrow{AB}$	**Slope of** $\overleftrightarrow{CD}$	**Types of Lines**
A(-3, 8), B(3, 2), C(7, 1), D(5, -1)	**Slope of** $\overleftrightarrow{AB}$	**Slope of** $\overleftrightarrow{CD}$	**Types of Lines**
A(5, -8), B(-2, -10), C(-6, -13), D(-2, 1)	**Slope of** $\overleftrightarrow{AB}$	**Slope of** $\overleftrightarrow{CD}$	**Types of Lines**
A(-4, 7), B(-2, 6), C(2, -2), D(-8, 3)	**Slope of** $\overleftrightarrow{AB}$	**Slope of** $\overleftrightarrow{CD}$	**Types of Lines**
Use the slope to determine if lines PQ and RS are parallel, perpendicular, or neither.			
P(-3, 14), Q(2, -1), R(4, 8), S(-2, -10)	**Slope of** $\overleftrightarrow{PQ}$	**Slope of** $\overleftrightarrow{RS}$	**Types of Lines**
P(-6, 16), Q(2, -4), R(-6, 14), S(-2, 4)	**Slope of** $\overleftrightarrow{PQ}$	**Slope of** $\overleftrightarrow{RS}$	**Types of Lines**
P(2, -1), Q(-3, -1), R(-11, 9), S(-7, 9)	**Slope of** $\overleftrightarrow{PQ}$	**Slope of** $\overleftrightarrow{RS}$	**Types of Lines**
Convert to slope-intercept form.			
x + y = 6			
2x - 4y = 28			
6x +8y= -16			

Determine whether the lines are parallel, perpendicular, or neither.	
x - y = 9 and y = x + 4	
4x - 6y = 12 and 3x + 2y = 10	
$y=-\frac{3}{2}x+4$ and $y=\frac{2}{3}x-7$	
$y = 3x - 7$ $y = 3x + 1$	
$y = -\frac{2}{5}x + 3$ $y = \frac{2}{5}x + 8$	
$y = -\frac{1}{4}x$ $y = 4x - 5$	
$y = -4x - 1$ $8x + 2y = 14$	
$x + y = 7$ $x - y = 9$	
$y = \frac{1}{3}x + 9$ $x - 3y = 3$	
$y = x - 3$ $x - y = 8$	
$10x + 8y = 16$ $5y = 4x - 15$	
$y = \frac{5}{3}x + 7$ $6x - 10y = 10$	



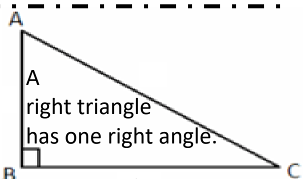
Term	Definition	Notation	Diagram/Visual
Pythagorean Theorem			
Isosceles Triangle			
Parallelogram			
Rectangle			
Rhombus			
Square			

Coordinate Geometry Proofs (Triangles and Parallelograms) – Guided Notes

❖ Coordinate geometry proofs employ the use of formulas such as Slope, Midpoint, and Distance.

Slope Formula	Midpoint Formula	Distance Formula
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	$(x, y) = \left(\dfrac{x_1 + x_2}{y_1 + y_2}\right)$	$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$
We use slope to show lines are parallel (‖)or perpendicular (⊥). ***Parallel lines have the same slope.** ***Perpendicular lines have opposite reciprocal slopes.**	We use midpoint to show lines bisect each other.	We use distance to show line segments have equal lengths.

PROOFS WITH TRIANGLES

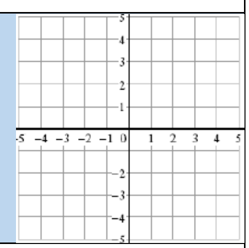


How can we prove a triangle is a RIGHT TRIANGLE?

By distances: Find the lengths of all three sides and then show the three lengths make the Pythagorean Theorem true.

By slopes: Find the slopes of all three sides and show that 2 sides have opposite reciprocal slopes, making them perpendicular to form a right angle.

Prove that *A* (0, 1), *B* (3, 4), *C* (5, 2) is a right triangle.



By distances…
Find the lengths of all three sides to show Pythagorean's Theorem is true.

$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$

____ $= \sqrt{(\quad\quad)^2 + (\quad\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

____ $= \sqrt{(\quad\quad)^2 + (\quad\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

____ $= \sqrt{(\quad\quad)^2 + (\quad\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

$a^2 + b^2 = c^2$

$(\quad\quad)^2 + (\quad\quad)^2 = (\quad\quad)^2$

Proof Statement:

Δ_______ is a right triangle because the ____ is true.

By slopes…
Find the slopes of all three sides to show opposite reciprocals.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} = \dfrac{(\quad) - (\quad)}{(\quad) - (\quad)} =$

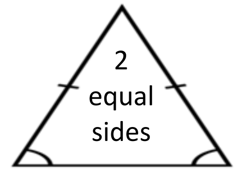
$m_{BC} = \dfrac{(\quad) - (\quad)}{(\quad) - (\quad)} =$

$m_{AC} = \dfrac{(\quad) - (\quad)}{(\quad) - (\quad)} =$

Proof Statement:

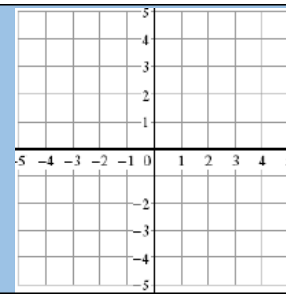
Δ_______ is a right triangle because __________ and __________ have opposite reciprocal slopes.

<u>How can we prove a triangle is an ISOSCELELS TRIANGLE?</u>



By distances: Find the lengths of all three sides and show that 2 sides have equal lengths.

Example! **Prove that *X*(1, 4) , *Y*(3, – 1) , and *Z*(– 1, – 1) is an isosceles triangle.**



By distances…

Find the lengths of all three sides to show 2 sides have equal lengths.

$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$

$\_\_\_\_ = \sqrt{(\quad\quad)^2 + (\quad\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

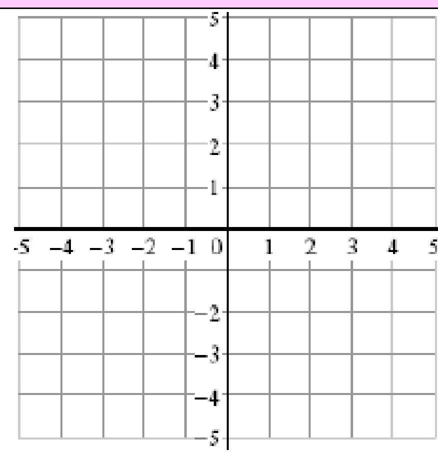
$\_\_\_\_ = \sqrt{(\quad\quad)^2 + (\quad\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

$\_\_\_\_ = \sqrt{(\quad\quad)^2 + (\quad\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

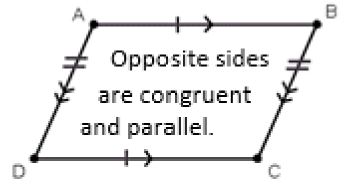
Proof Statement:

Δ_______ is an isosceles triangle because the ___________ = ___________.

SELF CHECK **Prove that *A* (-2, -2), *B* (5, -1), *C* (1, 2) is an isosceles right triangle.**
(***Make sure you show it is BOTH isosceles and right.***)



PROOFS WITH PARALLELOGRAMS

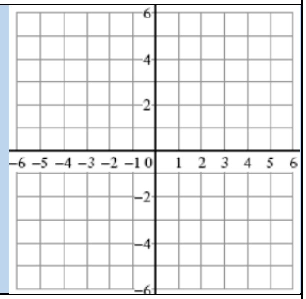


How can we prove a quadrilateral is a PARALLELOGRAM?

By distances: Find the lengths of all 4 sides and show that both pair of opposite sides are equal.

By slopes: Find the slopes of all 4 sides and show both pair of opposite sides have equal slopes, making them parallel.

Prove that the quadrilateral with the coordinates P(1, 1) , Q(2, 4) , R(5, 6) and S(4, 3) is a parallelogram.



By distances…
Find the lengths of all four sides to show opposite sides are equal.

$$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$$

$\_\_\_ = \sqrt{(\quad\quad)^2 + (\quad\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

$\_\_\_ = \sqrt{(\quad\quad)^2 + (\quad\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

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$\_\_\_ = \sqrt{(\quad\quad)^2 + (\quad\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

Proof Statement:

__________ is a parallelogram because both pair of opposite sides are equal, _______ = _______ and _______ = _______.

By slopes…
Find the slopes of all four sides to show opposite sides have equal slopes.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{PQ} = \frac{(\quad) - (\quad)}{(\quad) - (\quad)} =$$

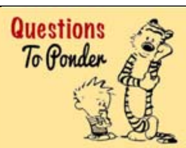
$$m_{RS} = \frac{(\quad) - (\quad)}{(\quad) - (\quad)} =$$

$$m_{PS} = \frac{(\quad) - (\quad)}{(\quad) - (\quad)} =$$

$$m_{QR} = \frac{(\quad) - (\quad)}{(\quad) - (\quad)} =$$

Proof Statement:

________ is a parallelogram because both pair of opposite sides are parallel, _______ ∥ _______ and _______ ∥ _______.



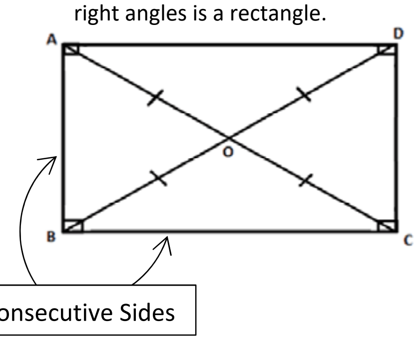
One property of parallelograms is that the diagonals bisect each other. How could you prove a quadrilateral is a parallelogram using this property?

How can we prove a quadrilateral is a RECTANGLE?

- Diagonals of a rectangle are equal.
- Any parallelogram with all four right angles is a rectangle.

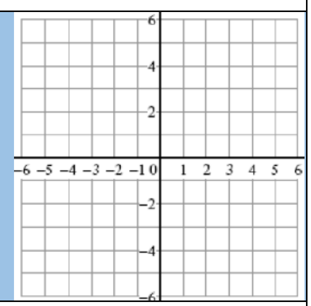
By distances: Find the lengths of all 4 sides and show both pair of opposite sides are equal. Then find the lengths of the diagonals and show the diagonals are equal.

By slopes: Find the slopes of all 4 sides and show both pair of opposite sides have equal slopes, making them parallel. Then show the slopes of two consecutive sides are opposite reciprocals, making them perpendicular to form a right angle.



NOTE: Proving a quadrilateral is a rectangle starts with first proving it is a parallelogram.

Prove a quadrilateral with vertices G(1, 1) , H(5, 3) , I(4, 5) and J(0, 3) is a rectangle.



By distances…
Show opposite sides are equal and diagonals are equal.

$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$

Sides

____ $= \sqrt{(\quad)^2 + (\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

____ $= \sqrt{(\quad)^2 + (\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

____ $= \sqrt{(\quad)^2 + (\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

____ $= \sqrt{(\quad)^2 + (\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

Diagonals

____ $= \sqrt{(\quad)^2 + (\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

____ $= \sqrt{(\quad)^2 + (\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

Proof Statement:
__________ is a parallelogram because both pair of opposite sides are equal, _______ = _______ and _______ = _______.

By slopes…
Show opposite sides have equal slopes and consecutive sides have opposite reciprocal slopes.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{GH} = \frac{(\quad) - (\quad)}{(\quad) - (\quad)} =$$

$$m_{JI} = \frac{(\quad) - (\quad)}{(\quad) - (\quad)} =$$

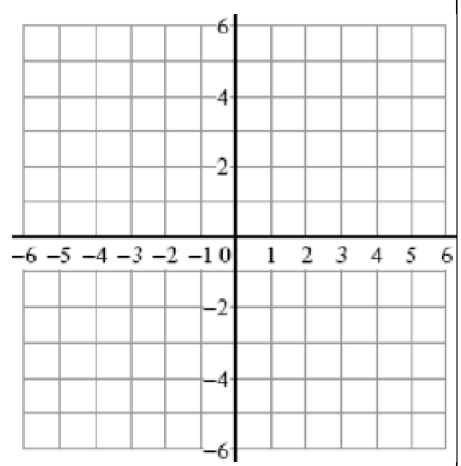
$$m_{GJ} = \frac{(\quad) - (\quad)}{(\quad) - (\quad)} =$$

$$m_{HI} = \frac{(\quad) - (\quad)}{(\quad) - (\quad)} =$$

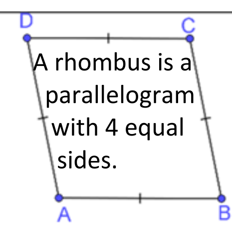
Proof Statement:
________ is a parallelogram because both pair of opposite sides are parallel, _______ ǁ _______ and _______ ǁ _______.

__________ is a rectangle because it is a ___________________________ whose diagonals are equal, _______ = _______.	__________ is a rectangle because it is a ___________________________ with right angles, _______ ⊥ _______.

SELF CHECK **Prove that quadrilateral ABCD with the vertices A(2,1), B(1,3), C(-5,0), and D(-4,-2) is a rectangle.**

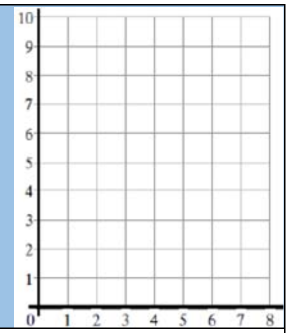


How can we prove a quadrilateral is a RHOMBUS?



By distances: Find the lengths of all 4 sides and show all sides are equal.

Example! **Prove that a quadrilateral with the vertices A(-1, 3) , B(3, 6) , C(8, 6) and D(4, 3) is a rhombus.**



By distances…

Find the lengths of all 4 sides to show they have equal lengths.

$$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$$

____ $= \sqrt{(\quad)^2 + (\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

____ $= \sqrt{(\quad)^2 + (\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

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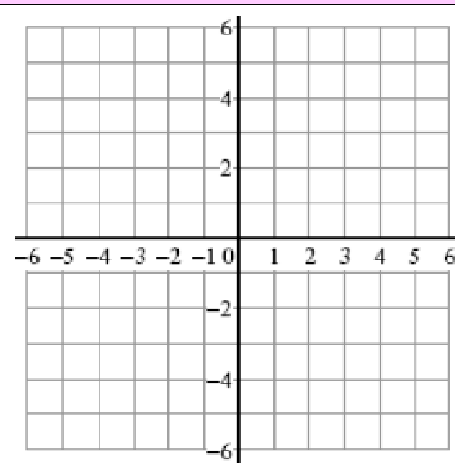
____ $= \sqrt{(\quad)^2 + (\quad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

Proof Statement:

_______ is a rhombus because all four sides are equal, _______ = _______ = _______ = _______.

SELF CHECK **Prove that quadrilateral DAVE with the vertices D(2, 1) , A(6, – 2) , V(10, 1) , and E(6, 4) is a rhombus.**



Questions To Ponder

One property of a rhombus is that the diagonals are perpendicular. Explain how could you prove a parallelogram is a rhombus using this property?

<u>**How can we prove a quadrilateral is a SQUARE?**</u>

By distances: Find the lengths of all 4 sides and show all sides are equal. Then find the lengths of the diagonals and show the diagonals are equal.

A square is a parallelogram that is both a rectangle and a rhombus.

Example! **Prove that the quadrilateral with vertices A(– 1, 0) , B(3, 3), C(6, – 1) and D(2, – 4) is a square.**

By distances…

Show all sides are equal and diagonals are equal.

$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$

Sides

____ $= \sqrt{(\qquad)^2 + (\qquad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

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Diagonals

____ $= \sqrt{(\qquad)^2 + (\qquad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

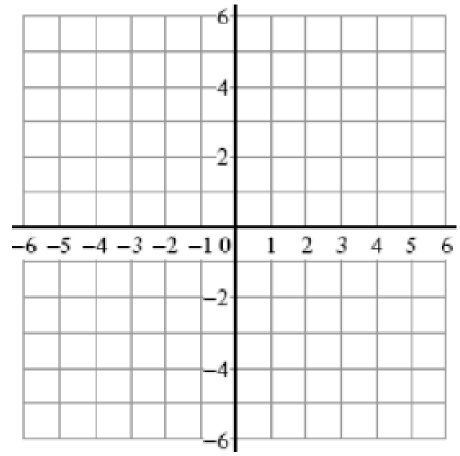
____ $= \sqrt{(\qquad)^2 + (\qquad)^2} = \sqrt{(\quad) + (\quad)} = \sqrt{\quad}$

Proof Statement:

__________ is a rhombus because all sides are equal, _______ = _______ = _______ = _______. It is a rectangle because the diagonals are equal, _______ = _______. So, __________ is a square because it is both a __________________________ and a _______________________.

SELF CHECK

Prove that a quadrilateral with vertices J(2, -1) , K(-1, -4) , L(-4, -1) and M(-1, 2) is a square.

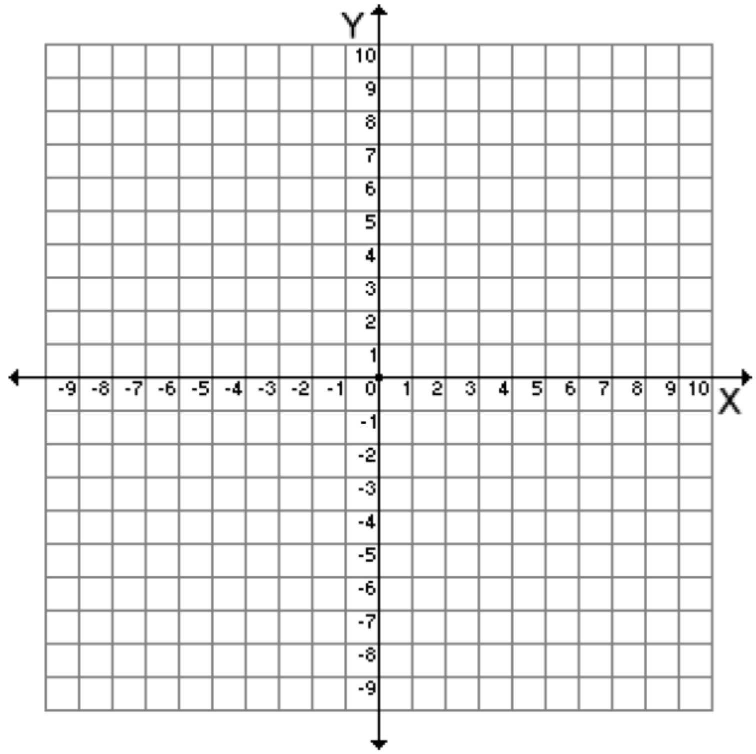


Questions To Ponder

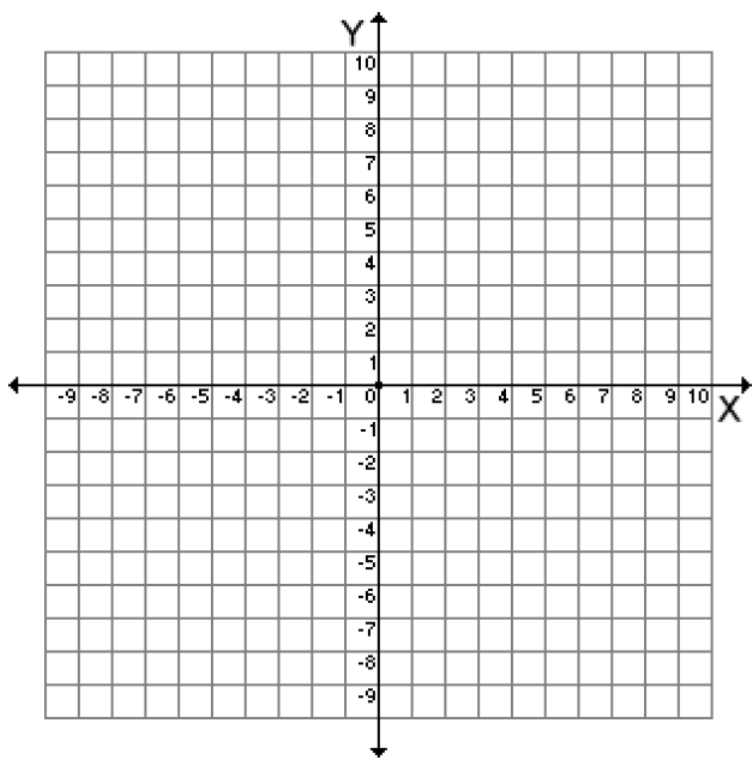
How can you prove a quadrilateral is a square using only the properties related to diagonals?

Coordinate Geometry Proofs (Triangles and Parallelograms) - Practice

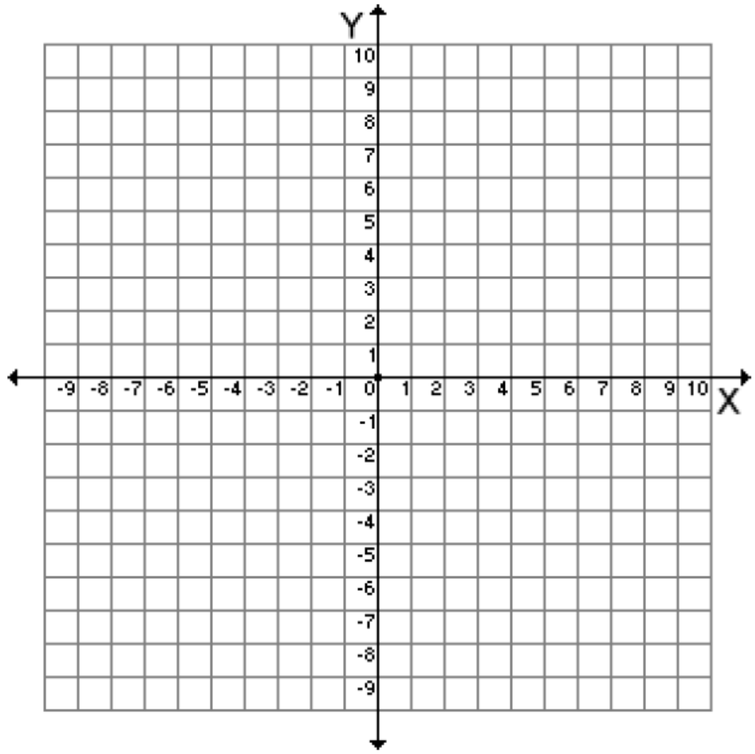
1. Prove that the polygon with coordinates A(1, 1), B(4, 5), and C(4, 1) is a right triangle.



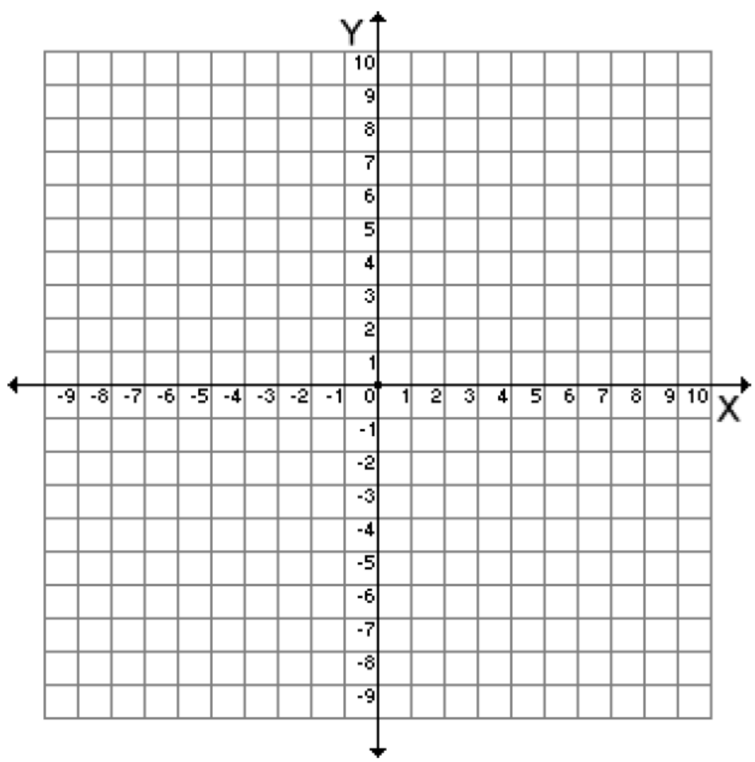
2. Prove that the polygon with coordinates A(4, -1), B(5, 6), and C(1, 3) is an isosceles right triangle.



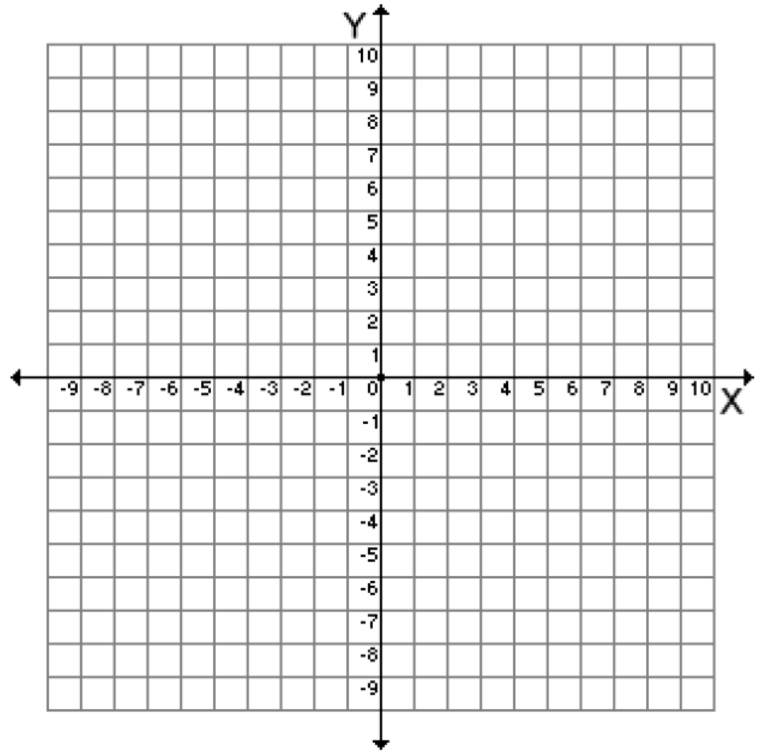
3. Prove that the quadrilateral with the coordinates P(1,1), Q(2,4), R(5,6) and S(4,3) is a parallelogram.



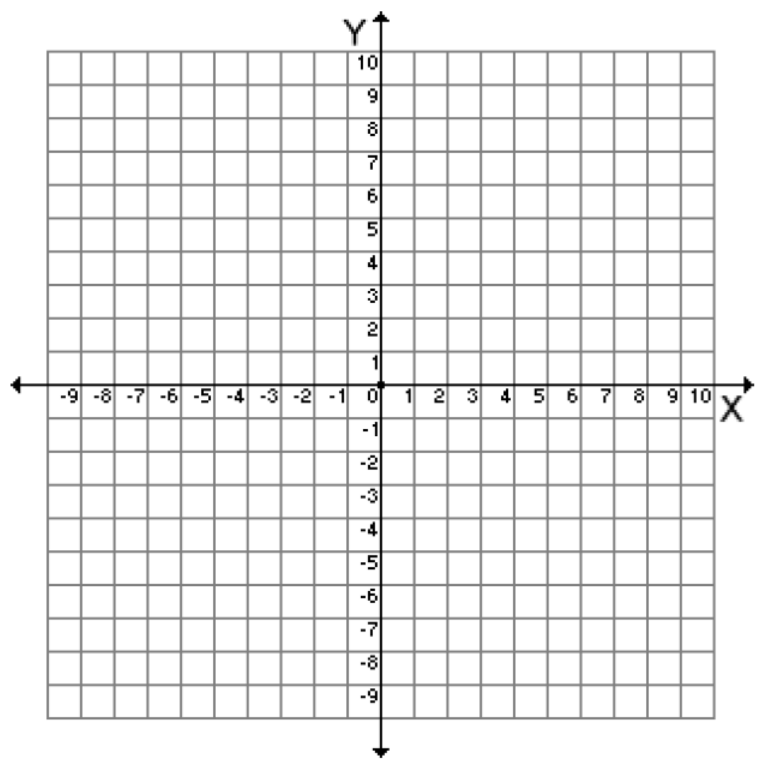
4. Prove that quadrilateral PLUS with the vertices P(2,1), L(6,3), U(5,5), and S(1,3) is a rectangle.



5. Prove that quadrilateral GHIJ with the vertices G(-2,2), H(3,4), I(8,2), and J(3,0) is a rhombus.

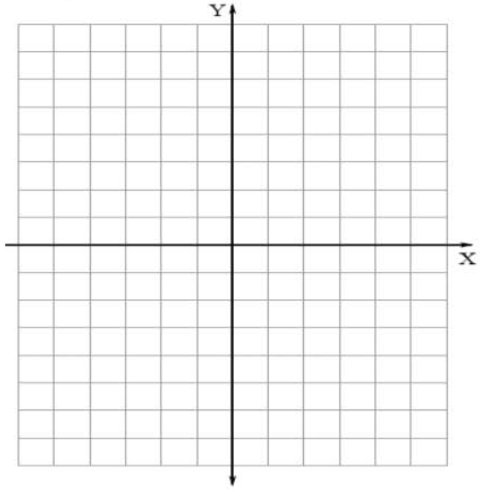


6. Prove that a quadrilateral with vertices J(2,-1), K(-1,-4), L(-4,-1) and M(-1, 2) is a square.



Proving Quadrilaterals in a Coordinate Plane

Plot points A = (-3, -1), B = (-1, 2), C = (4, 2), and D = (2, -1).



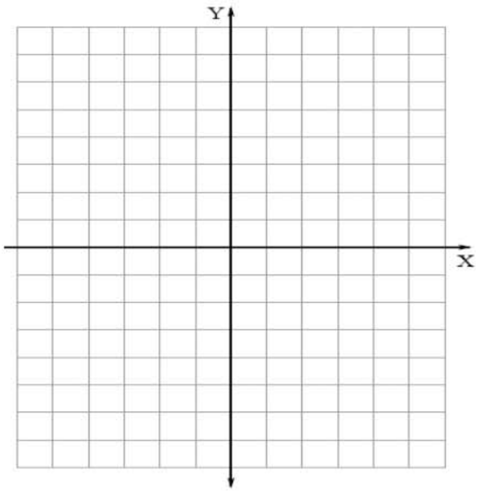
1. What specialized geometric figure is quadrilateral ABCD? Support your answer mathematically.

2. Draw the diagonals of ABCD. Find the coordinates of the midpoint of each diagonal. What do you notice?

3. Find the slopes of the diagonals of ABCD. What do you notice?

4. The diagonals of ABCD create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points E = (1, 2), F = (2, 5), G = (4, 3) and H = (5, 6).



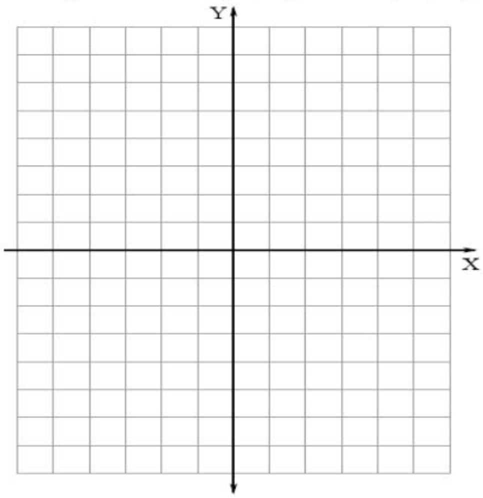
5. What specialized geometric figure is quadrilateral EFHG? Support your answer mathematically using two different methods.

6. Draw the diagonals of EFHG. Find the coordinates of the midpoint of each diagonal. What do you notice?

7. Find the slopes of the diagonals of EFHG. What do you notice?

8. The diagonals of EFHG create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points P = (4, 1), W = (-2, 3), M = (2,-5), and K = (-6, -4).



9. What specialized geometric figure is quadrilateral PWKM? Support your answer mathematically.

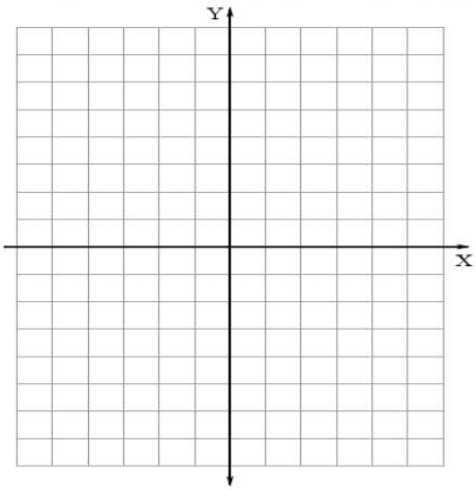
10. Draw the diagonals of PWKM. Find the coordinates of the midpoint of each diagonal. What do you notice?

11. Find the lengths of the diagonals of PWKM. What do you notice?

12. Find the slopes of the diagonals of PWKM. What do you notice?

13. The diagonals of ABCD create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

Plot points A = (1, 0), B = (-1, 2), and C = (2, 5).



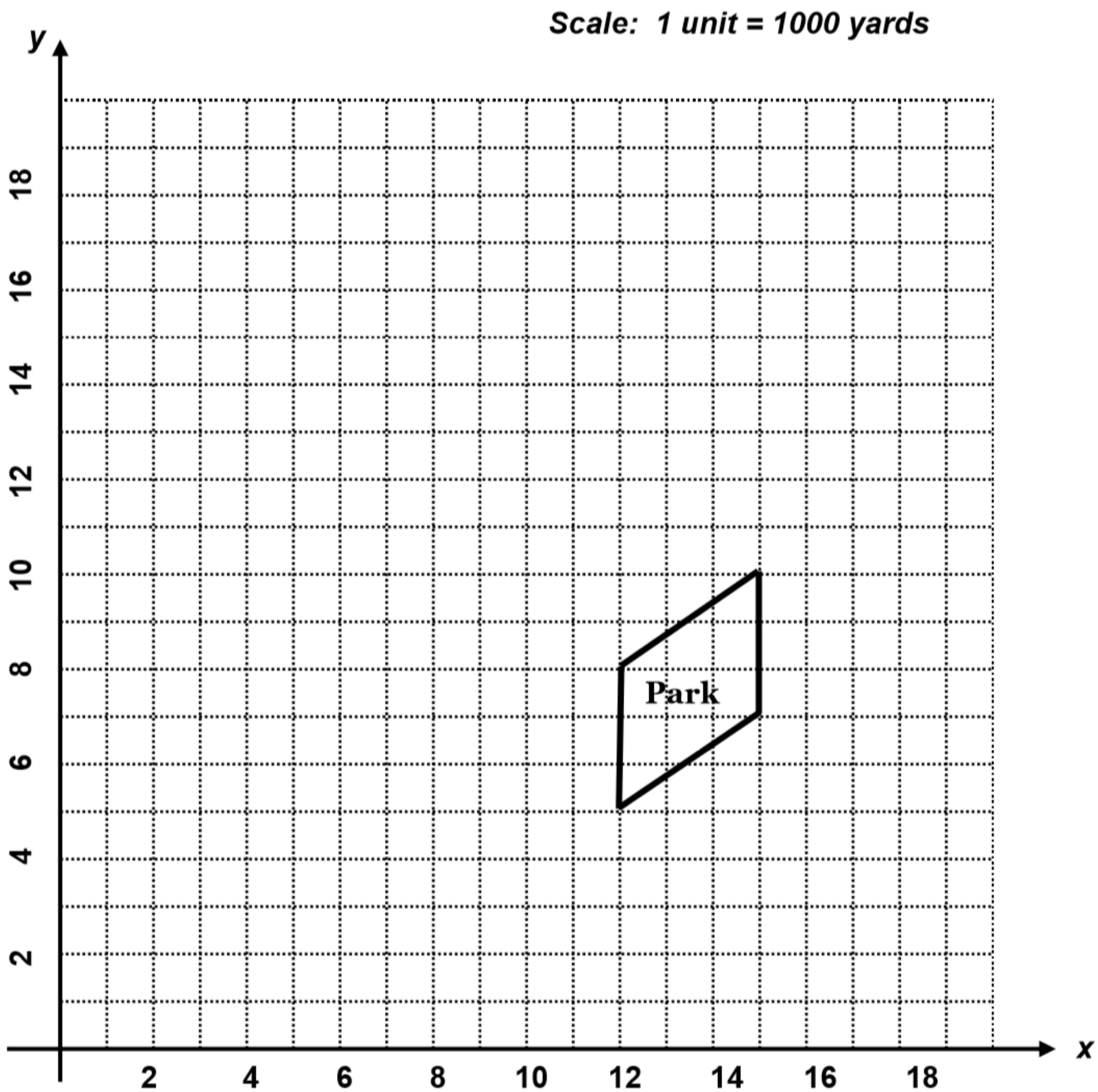
14. Find the coordinates of a fourth point D that would make ABCD a rectangle. Justify that ABCD is a rectangle.

15. Find the coordinates of a fourth point D that would make ABCD a parallelogram that is not also a rectangle. Justify that ABCD is a parallelogram but is not a rectangle.

Euler's Village

You would like to build a house close to the village of Euler. There is a beautiful park just outside the village, and the road you would like to build your house on begins right at the town square and goes by this park.

The road follows an approximately north east direction as you leave town and continues for 3,000 feet. It passes right by a large shade tree located approximately 200 yards east and 300 yards north of the town square. There is a stretch of the road, between 300 and 1200 yards to the east of town, which currently has no houses. This stretch of road is where you would like to locate your house. All water supplies are linked to town wells and the closest well to this part of the road is 500 yards east and 1200 yards north of the town square.



1. How far from the well would it be if the house was located on the road 300 yards east of town? 500 yards east of town? 1,000 yards east of town? 1,200 yards east of town? (For the sake of calculations, assume the house is exactly *on* the road.)

2. The cost of the piping leading from the well to the house is a major concern. Where should you locate your house in order to have the shortest distance to the well? (*Remember: the shortest distance between a line and a point is the length of the segment perpendicular to the line that passes through the point*). Justify your answer mathematically.

3. If the cost of laying pipes is $22.50 per linear yard, how much will it cost to connect your house to the well?

4. You also want to install a swimming pool on the line with the pipes. You want the front edge of the pool to be $\frac{3}{5}$ the distance from the road to the well. What are the coordinates of the front corner of the swimming pool?

5. The builder of your house is impressed by your calculations and wants to use the same method for placing other houses. Describe the method you used. Would you want him to place the other houses in the same manner?

6. Write a formula that the builder could use to find the cost of laying pipes to any house along this road. How would you have to change your formula for another road?

7. One day you were wondering what geometric shape the park formed, so you looked at the map and took some measurements. Using those measurements, prove what specific geometric shape is formed by the park boundaries.

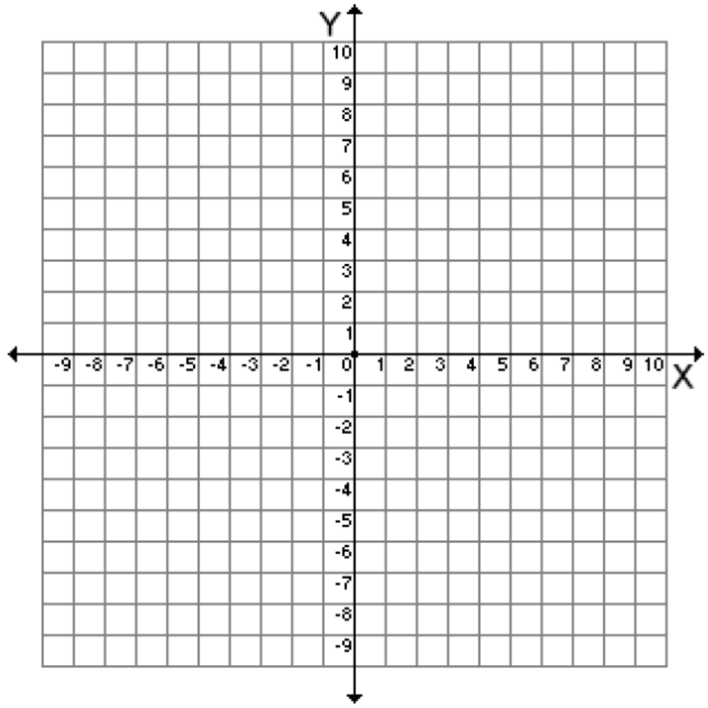
8. Your service club noticed that the fence around the park needed to be replaced. How much fencing would be needed?

9. While the fence was being replaced, some damage occurred to a portion the grass in the park. The entire park is covered with grass, but only ¼ of the total area needs to be replaced. Grass seed is sold in bags that cover 5000 square yards each. How many bags of grass seed would your club need to purchase to reseed the entire park?

Coordinate Geometry Proofs (Triangles and Parallelograms) – Homework

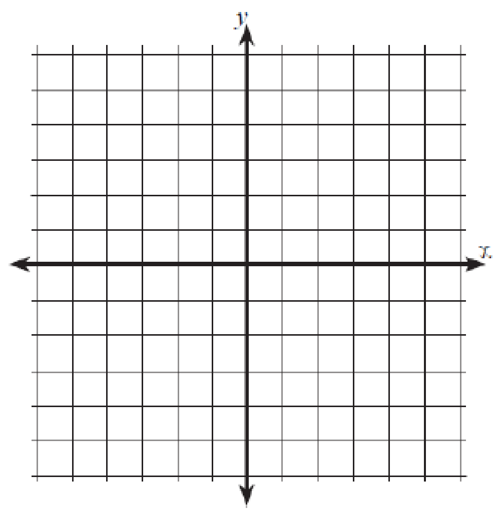
1. The coordinates of the vertices of triangle *SUE* are *S*(–2 , –4) , *U*(2, –1) and *E*(8, –9). Using coordinate geometry prove that…

   a) Δ*SUE* is a right triangle.

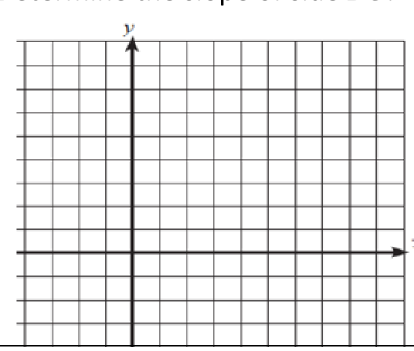


   b) Δ*SUE* is not an isosceles right triangle.

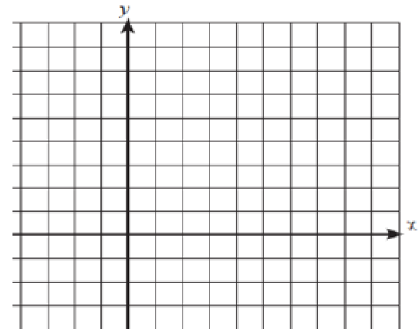
2. Prove or disprove that a figure defined by four points: $A(-1, -1)$, $B(5, -1)$, $C(5, -4)$ and $D(-1, -4)$ is a rectangle.



3. The coordinates of two vertices of square $ABCD$ are $A(2, 1)$ and $B(4, 4)$. Determine the slope of side $\overline{BC}$.



4. The coordinates of the vertices of parallelogram $ABCD$ are A$(-3, 2)$, $B(-2, -1)$, $C(4, 1)$ and $D(3, 4)$. The slopes of which line segments could be calculated to show that *ABCD* is a rectangle?



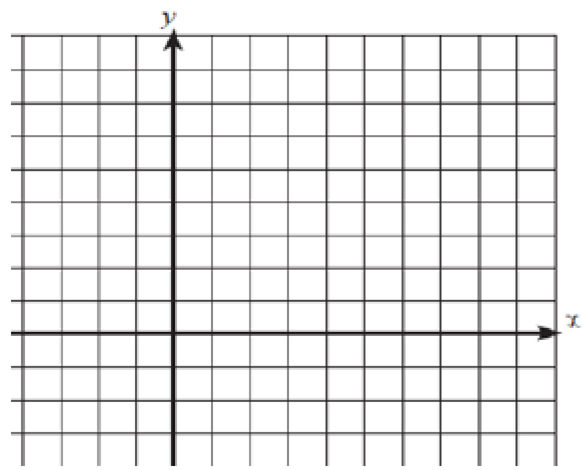
   a) $\overline{AB}$ and $\overline{DC}$

   b) $\overline{AB}$ and $\overline{BC}$

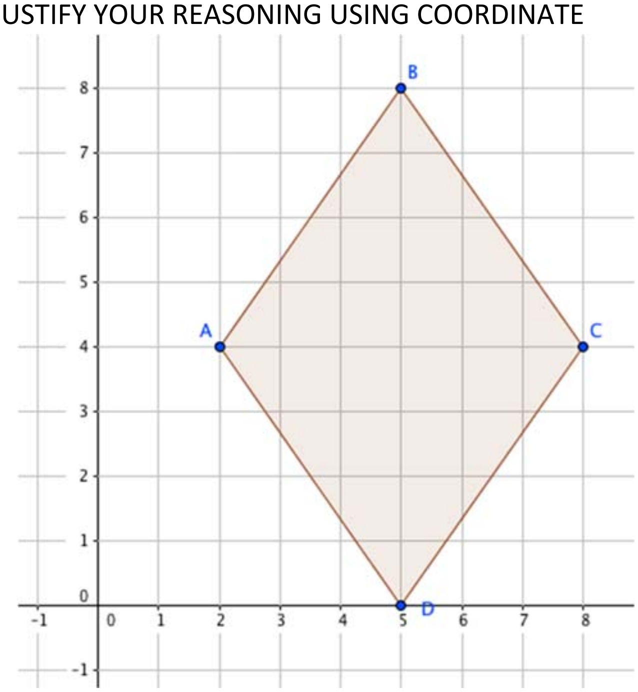
   c) $\overline{AD}$ and $\overline{BC}$

   d) $\overline{AC}$ and $\overline{BD}$

5. Given: A(−2, 2) , B(6, 5) , C(4, 0), and D(−4, −3). Prove $ABCD$ is a parallelogram but ***not*** a rectangle.



6. Prove that the figure is a rhombus that is not a square. JUSTIFY YOUR REASONING USING COORDINATE GEOMETRY.



Term	Definition	Notation	Diagram/Visual
Center of a Circle			
Diameter			
Radius			
Standard form of a Circle			

Equations of a Circle

GUIDED NOTES

Equation of a Circle

Standard Form: ______________________

(__________ is the ____________ and _____ is the ____________)

Example!

Write the equation of the circle with the given information.

1.

Center: (5, 3), Radius: 2 ______________________

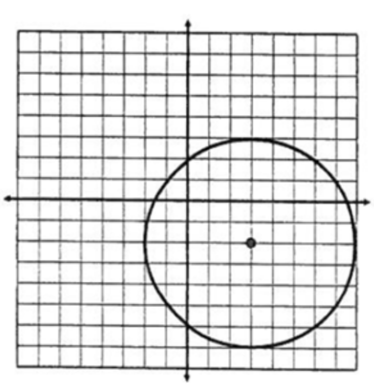
2.

Center: (0, 9), Diameter: 32 ______________________

3.

Center: (15, -6), Radius: $\sqrt{158}$ ______________________

4.



5.

Center: (9, 10), Point on Circle: (7, 4)

6.

Center: (-1, 4)
Circumference: 6π

7.

Center: (-8, -11)
Area: 16π

8.

$(x-16)^2+(y-6)^2=1$
Translated 4 left, 2 up

9.

Center: $(-15, 9)$
Tangent to $x=-17$

Given the equation of the circle, identify the center and radius/diameter.

10.

$(x-5)^2 + (y-1)^2 = 64$

Center: ______________

Radius: __________

11.

$(x+7)^2 + (y-2)^2 = 324$

Center: ______________

Diameter: __________

SELF CHECK

Write the equation of the circle with the given information.

1.

Center: (-1, 7), Radius: 14 ________________________________

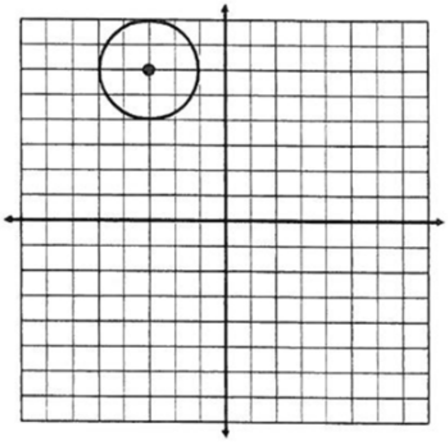
2.

Center: (-2, -11), Diameter: 18 ________________________________

3.

Center: (-4, 0), Radius: $\sqrt{47}$ ________________________________

4.



5.

Center: (-2, 0); Point on Circle: (-9, -4)

6.

Center: (9, 12)
Circumference: 17π

7.

Center: (2, -2)
Area: 361π

8.

$(x+5)^2 + (y+7)^2 = 36$
Translated 5 left, 4 down

9.

Center: $(-2, 12)$
Tangent to $x = -5$

Given the equation of the circle, identify the center and radius/diameter.

10.

$x^2 + (y + 10)^2 = 42.25$

Center: ________________

Diameter: ____________

11.

$(x - 6)^2 + (y + 11)^2 = 58$

Center: ________________

Radius: ____________

Challenge! Write the equation of a circle with diameter endpoints of (-14, 1) and (-10, 9).

Completing the Square

GUIDED NOTES

Some quadratic equations in the form of $ax^2 + bx + c = 0$ can be solved easily by factoring. For example, the equation $x^2 + 6x - 16 = 0$ can be factored easily to $(x+8)(x-2) = 0$ to give solutions of $x = -8$ *and* $x = 2$

When a quadratic equation cannot be factored using integers, you have two options. You can use the quadratic formula of you can use a method called **completing the square**. When a = 1, completing the square is the way to go (when a >1, use the quadratic formula).

Example 1: Solve $x^2 + 8x - 10 = 0$ by completing the square.

Since it cannot be factored using integers, Write the equation in the form $ax^2 + bx = -c$	$x^2 + 8x - 10 = 0$ $x^2 + 8x = 10$
Find $\frac{1}{2}$ of b and add the square of that number $(\frac{b}{2})^2$ to both sides of the equation	Think $b = 8$ $\frac{1}{2}b = 4$ and $4^2 = 16$ $x^2 + 8x = 10$ $x^2 + 8x + 16 = 10 + 16$
The left side is now a perfect square trinomial (PST), so factor it.	$(x+4)(x+4) = 26$ $(x+4)^2 = 26$

Example!

Write the equation in standard form.

1. $8x + x^2 - 2y = 64 - y^2$

2. $137 + 6y = -y^2 - x^2 - 24x$

SELF CHECK

Write the equation in standard form.

1. $8x + 32y + y^2 = -263 - x^2$

Proving a point is on, inside, or outside a circle

GUIDED NOTES

To tell if a point is **INSIDE** of a circle….

Step 1 – Find the center and radius of the circle.

Step 2 – Find the distance from the center to the point.

**If the distance is ________________________the radius, then the point is _______________ of the circle. **

To tell if a point is **OUTSIDE** of a circle….

Step 1 – Find the center and radius of the circle.

Step 2 – Find the distance from the center to the point.

**If the distance is ________________________the radius, then the point is _______________ of the circle. **

To tell if a point is **ON** a circle….

Step 1 – Find the center and radius of the circle.

Step 2 – Find the distance from the center to the point.

**If the distance is ________________________the radius, then the point is _______________ the circle. **

OR

Step 1 – Plug the point in to the equation for (x, y).

Step 2 – Simplify both sides of the equation.

If the left and right side are _______________, then the point is ____________ the circle.

Example!

1. Is the point (–1, –6) inside, outside, or on the circle with equation $(x + 1)^2 + (y - 4)^2 = 121$

2. Is the point (–4, 2) inside, outside, or on the circle with equation $(x - 6)^2 + (y - 1)^2 = 81$?

3. Is the point (2, 3) inside, outside, or on the circle with equation $(x - 7)^2 + y^2 = 34$?

4. Is the point (7, – 3) on the circle with equation $x^2 + y^2 + 10x – 4y = 140$?

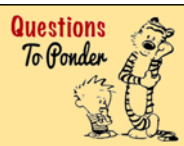
SELF CHECK

1. Prove whether the following points lie ON, INSIDE or OUTSIDE the circle defined by $(x + 2)^2 + (y – 3)^2 = 36$

a) P (-1,2)

b) Q (-5,8)

c) R (3,-2)



What is all the information you need to write the equation of a circle?

How do you know if a point is outside, inside, or on a circle?

Write the equation of the circle with the given information.

1.

Center: (2, 8), Radius: 3 ______________________________

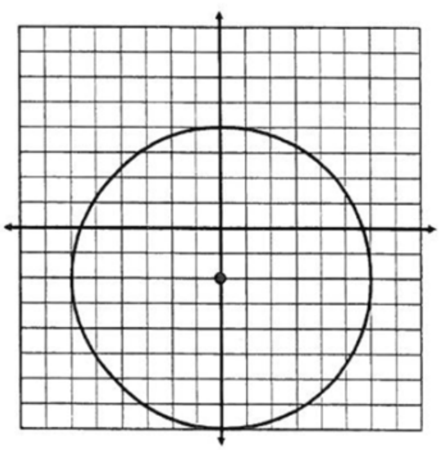
2.

Center: (-9, -7), Diameter: 34 ______________________________

3.

Center: (0, -16), Radius: $\sqrt{15}$ ______________________________

4.



5.

Center: (-10, -4), Point on Circle: (4, -2)

6.

Center: (10, 7), Circumference: 14π

7.

Center: (-4, -5), Area: 400π

8.

$(x+5)^2+(y+7)^2=36$
Translated 5 left, 4 down

9.

Center: $(-2, 12)$
Tangent to $x=-5$

10.

Ends of a diameter: $(-17, -9)$ and $(-19, -9)$

Given the equation, identify the center and radius or diameter.

11.

$(x+6)^2+y^2=90.25$ **Center:** ______________; **Diameter:** ____________

12.

$(x-2)^2+(y+13)^2=150$ **Center:** ______________; **Radius:** ____________

Write the equation in standard form.

13.

$$28y+x^2+y^2=-349+26x$$

14.

$$18x + y^2 + x^2 = -14y - 114$$

Is the point T on, inside, or outside the given circle?

15.

$$(x-3)^2 + (y-2)^2 = 7$$

$$T\,(-4, 3)$$

16.

Center: $(0, 1)$
Point on Circle: $(0, 5)$

$$T\,(4, 1)$$

17.

$$x^2 + y^2 + 2x - 6y + 1 = 0$$

$$T\,(0, 4)$$

Radio Station Listening Areas

1. Radio signals emitted from a transmitter form a pattern of concentric circles. Write equations for three concentric circles.

2. Randy listens to radio station WYAY from Atlanta. Randy's home is located 24 miles east and 32 miles south of the radio station's transmitter. His house is located on the edge of WYAY's maximum broadcast range.

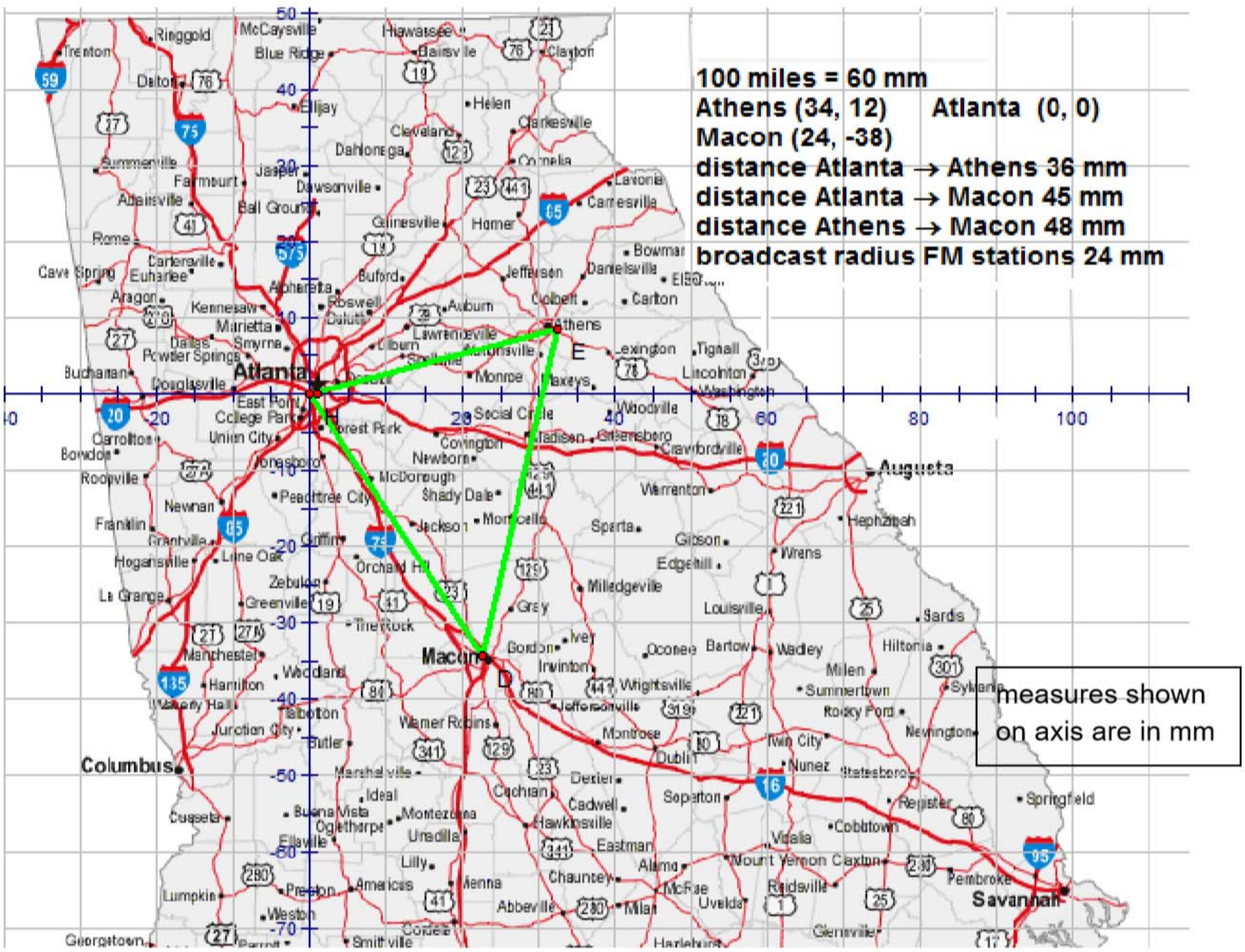
a. When a radio signal reaches Randy's house, how far has it traveled? Sketch WYAY's listening area of the partial map of Georgia given. On the map let Atlanta's WYAY have coordinates (0, 0) and use the scale as 100 miles = 60 mm.

b. Find an equation which represents the station's maximum listening range.

c. Determine four additional locations on the edge of WYAY's listening area, give coordinates correct to tenths.

3. Randy likes to listen to country music. Several of his friends have suggested that in addition to WYAY, he try station WXAG in Athens and WDEN in Macon. WYAY, WXAG, and WDEN are FM stations which normally have an average broadcast range of 40 miles. Use the map included with the indicated measures to answer the following questions.

   a. Given the location of Randy's home, can he expect to pick up radio signals from WXAG and WDEN? Explain how you know.



Write the equation of the circle with the given information.

1.

Center: (5, -13), Radius: 11 ______________________

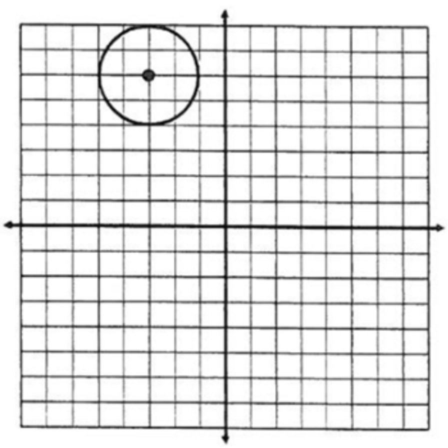
2.

Center: (-8, 0), Diameter: 5 ______________________

3.

Center: (12, -1), Radius: $\sqrt{209}$ ______________________

4.



5.

Center: (5, -7), Point on Circle: (-3, -1)

6.

Center: (0, -1), Circumference: 25π

7.

Center: (-6, 2), Area: π

8.

$(x + 5)^2 + (y - 7)^2 = 1$ Translated 5 left, 3 up

9.

Find the equation of the circle centered at (2, –3) and tangent to $x = 8$.

10.

Ends of a diameter: $(-3, 11)$ and $(3, -13)$

Given the equation, identify the center and radius or diameter.

11.

$(x - 9)^2 + (y - 4)^2 = 36$ **Center:** ______________; **Radius:** ______________

12.

$(x + 1)^2 + (y - 1)^2 = 196$ **Center:** ______________; **Diameter:** ______________

Write the equation in standard form.

13.

$$y^2 + 144 - 2x = 24y - x^2$$

14.

$$0 = -109 - x^2 + 10y - 20x - y^2$$

Is the point T on, inside, or outside the given circle?

15.

$$(x+2)^2 + (y-2)^2 = 4$$

$T\ (-1, 2)$

16.

Center: $(3, 2)$
Point on Circle: $(7, 2)$

$T\ (5, 0)$

17.

$$x^2 + y^2 + 2x - 6y + 1 = 0$$

$T\ (0, 4)$

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DEPARTMENT

Science Technology Engineering Mathematics

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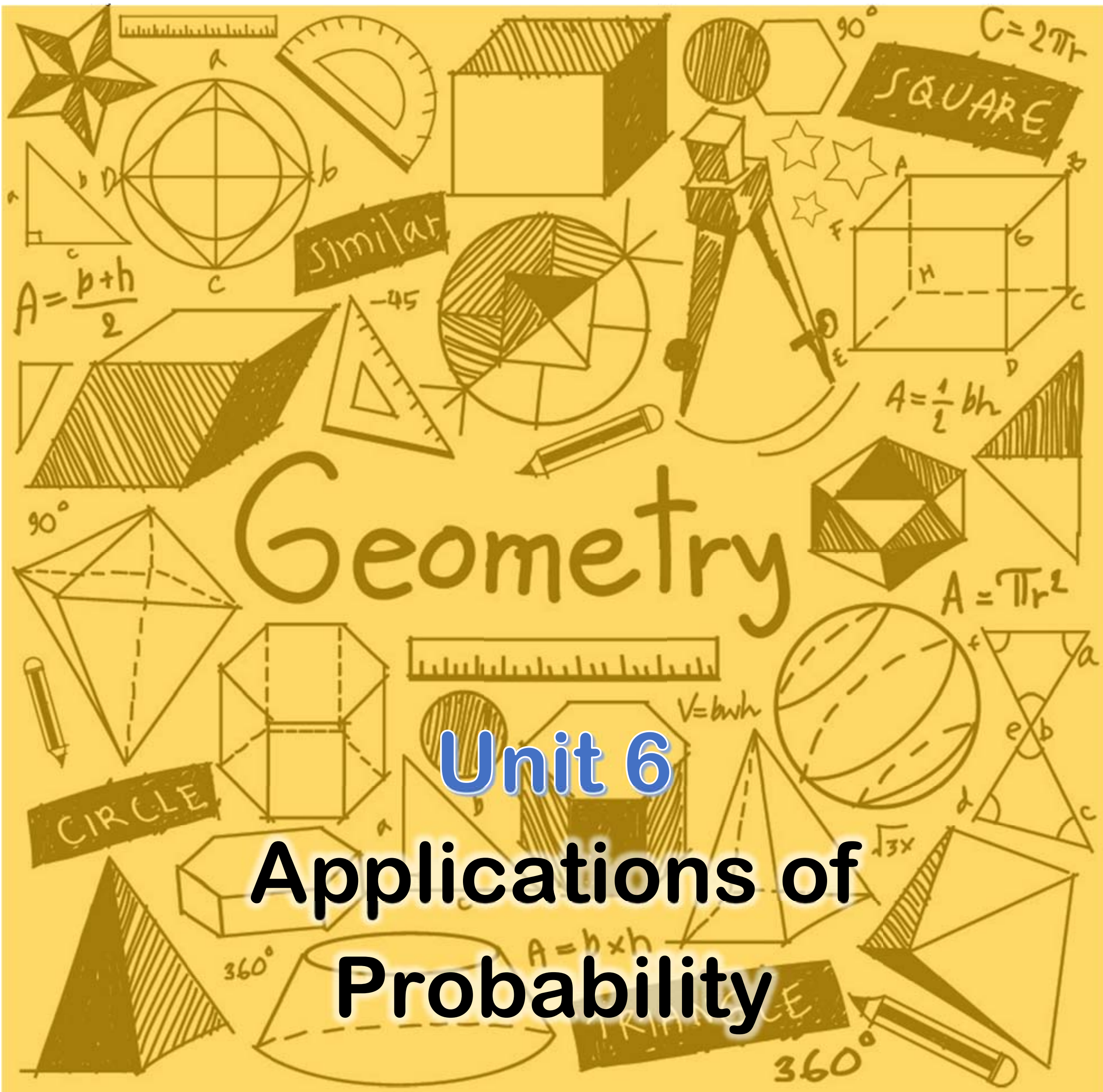

Unit 6
Applications of Probability

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MATHEMATICS
DEPARTMENT

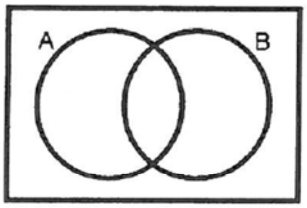
Diagram/Visual		
Addition Rule		
Complement		
Conditional Property		
Element		
Intersection of Sets		
Mutually Exclusive Event		
Outcome		
Overlapping Events		
Sample Space		
Set		

Subset			
Union of Sets			
Venn Diagram			

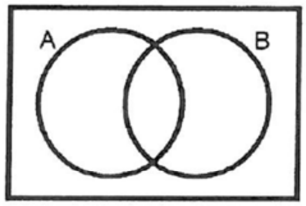
Venn Diagrams

GUIDED NOTES

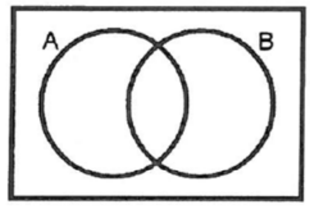
shade the regions corresponding to the sets



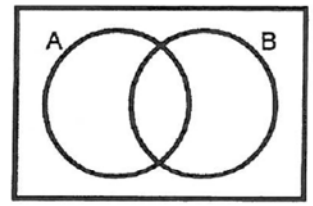
A



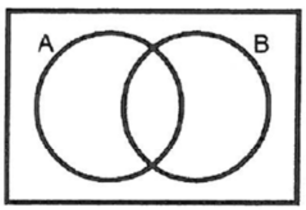
$A \cup B$



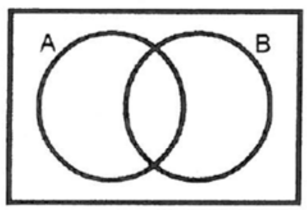
$A \cap B$



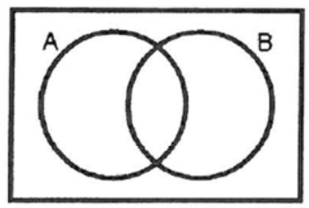
B'



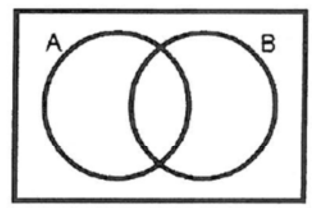
$A \cup B'$



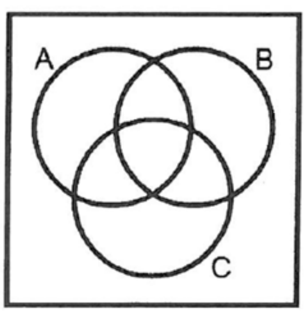
$A' \cap B$



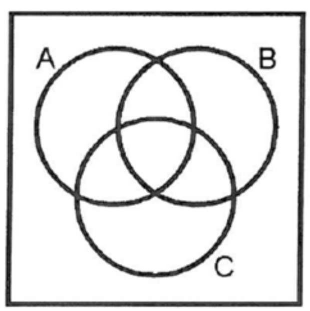
$A' \cap B'$



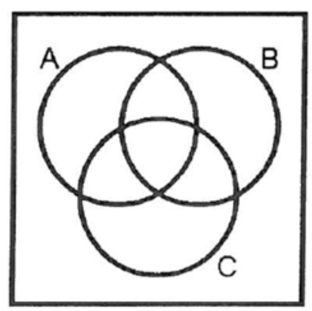
$(A \cup B)'$



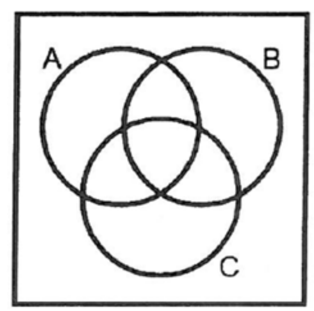
$A \cap C$



C'



$B \cup C$



$A \cap C'$

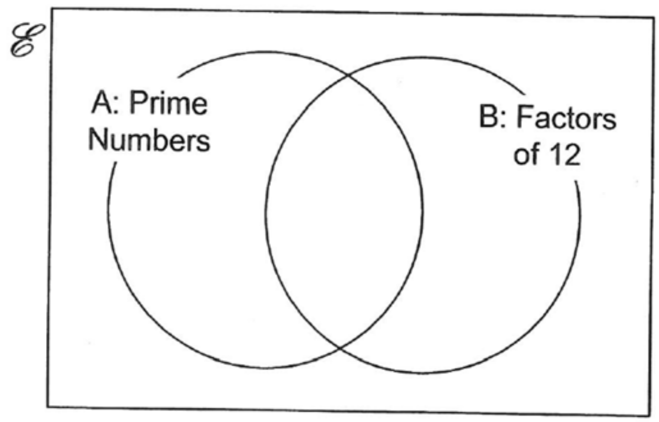
Example!

1.

① **Place each element in the correct section of the Venn diagram.**

1	2	3	4	5	6
7	8	9	10	11	12

Elements ℰ



② **Shade the TRUE statements.**

A has 5 elements	B has the same number of elements as B'
B has 4 elements	B has twice as many elements as A ∩ B
A ∩ B has 9 elements	There are 4 elements in A'
A ∪ B has 9 elements	¼ of the elements are outside both circles

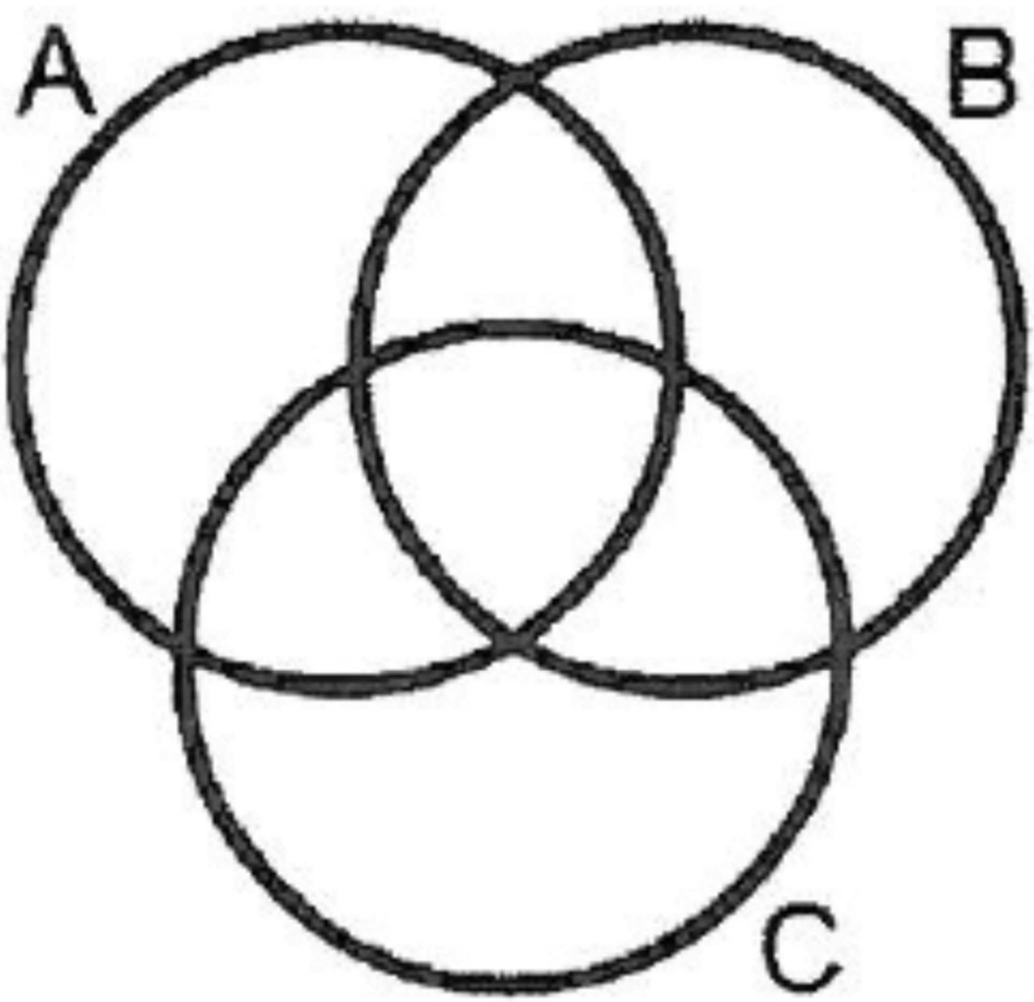
③ **One of the numbers is chosen at random. Choose the correct probabilities.**

$P(A)$	
$P(B)$	
$P(A')$	
$P(B')$	
$P(A \cap B)$	
$P(A \cup B)$	

$\frac{1}{6}$ 75% $\frac{7}{12}$ $\frac{5}{12}$ 0.5 $1 - P(B)$

2.
Let A be the numbers from 1 - 20
Let B be perfect squares from 1 - 20
Let C be multiples of 3 from 1 - 20

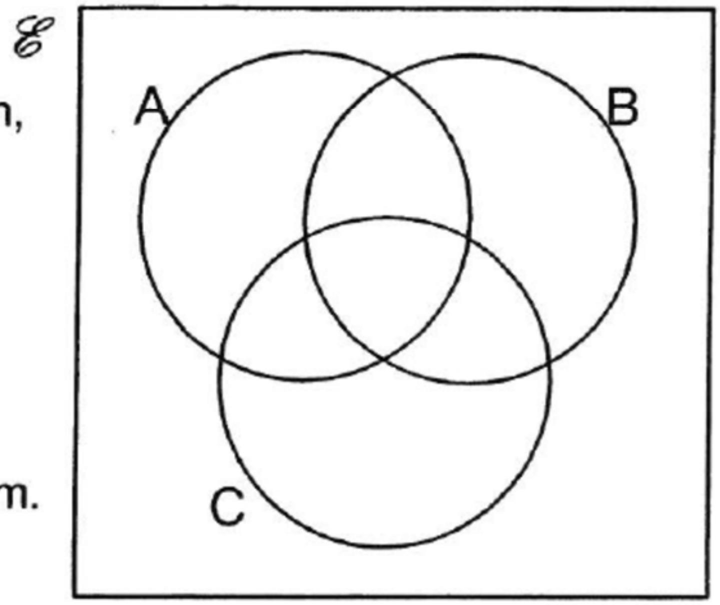
Fill in the Venn Diagram



3.

Students at a college can take part in different extra-curricular activities, including Aerobics classes, Badminton and Chess club.

In a group of 140 students,
7 do all three activities,
34 do aerobics and badminton,
22 do aerobics and chess,
19 do badminton and chess,
76 do aerobics,
62 do badminton,
46 do chess.



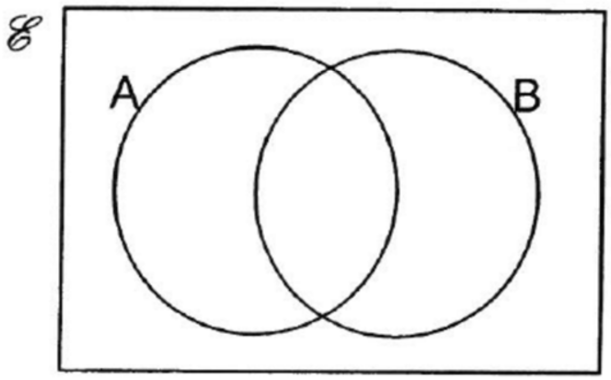
(a) Complete the Venn diagram.

(b) How many students do none of the three activities?

SELF CHECK

1.

$\mathscr{E}$ = {8,9,10,11,12,13,14,15,16}
A = {even numbers}
B = {square numbers}
(a) Complete the Venn diagram.



(b) An element is picked at random. Work out:

P(A') =

P(B') =

P(A ∩ B) =

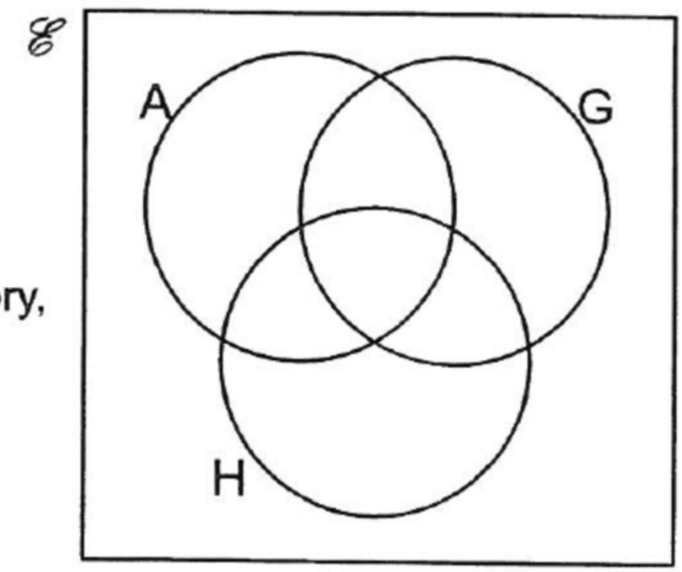
P(A ∪ B) =

2.

Students at a school can choose to study GCSEs in Art, Geography and History.

In a group of students,
12% study all of the subjects,
19% study Art and Geography,
23% study Art and History,
27% study Geography and History,
40% study Art,
54% study Geography,
2% study none of the subjects.

(a) Complete the Venn diagram.

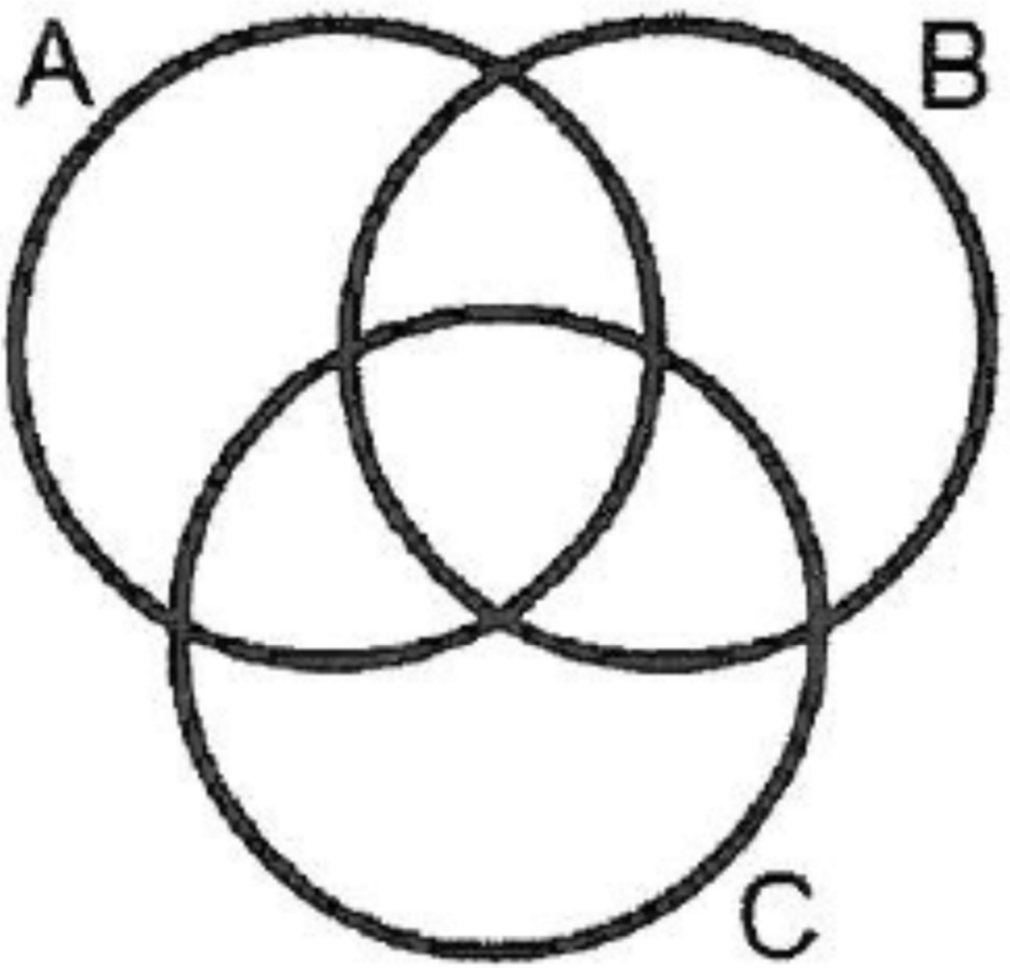


(b) What proportion of the Art students also study History?

3.

Let A be the numbers from 16-35
Let B be multiples of 5 from 16-35
Let C be odd numbers from 16-35

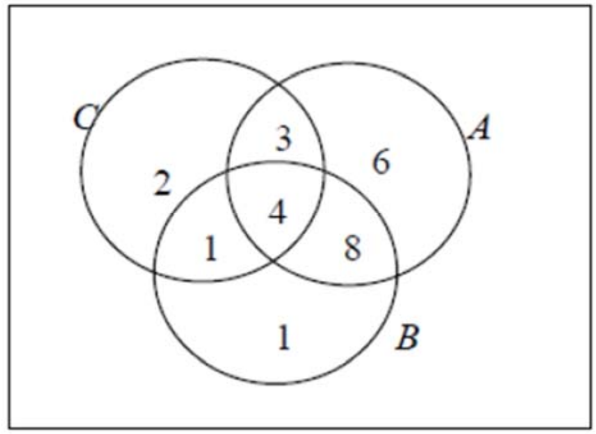
Fill in the Venn Diagram



Where could I apply Venn Diagrams into my other subjects?

1. The Venn diagram shows the number of players at a sports club who take part in various sporting activities, where:

A = {members who do archery};
B = {members who play badminton};
C = {members who take cross country};



Find the number of members who:
(a) take part in cross country;

(b) take part in more than one activity;

(c) play badminton but do not take part in cross country;

(d) do not do archery.

2. A poll was taken of the leisure time activities of 90 students.

60 students watch TV (T), 60 students read (R), 70 students go to the cinema (C).

26 students watch TV, read **and** go to the cinema.
20 students watch TV and go to the cinema only.
18 students read and go to the cinema only.
10 students read and watch TV only.

(a) Draw a Venn diagram to illustrate the above information.

(b) Calculate how many students:

(i) only watch TV;

(ii) only go to the cinema.

Real Mathematics: Application

How Odd

Part 1 – For this task you will need a pair of six-sided dice. In Part 1, you will be concerned with the probability that one (or both) of the dice show odd values.

1. Roll your pair of dice 30 times, each time recording a success if one (or both) of the dice show an odd number and a failure if the dice do not show an odd number.

Number of Successes	Number of Failures

2. Based on your trials, what would you estimate the probability of two dice showing at least one odd number? Explain your reasoning.

3. You have just calculated an *experimental probability*. 30 trials is generally sufficient to estimate the *theoretical probability*, the probability that you expect to happen based upon fair chance. For instance, if you flip a coin ten times you expect the coin to land heads and tails five times apiece; in reality, we know this does not happen every time you flip a coin ten times.

a. A lattice diagram is useful in finding the theoretical probabilities for two dice thrown together. An incomplete lattice diagram is shown to the right. Each possible way the two dice can land, also known as an **outcome**, is represented as an ordered pair. (1, 1) represents each die landing on a 1, while (4, 5) would represent the first die landing on 4, the second on 5. Why does it have 36 spaces to be filled?

Dice Lattice

(1,1)	(1,2)	(1,3)	(,)	(,)	(,)
(2,1)	(,)	(,)	(,)	(,)	(,)
(,)	(,)	(,)	(,)	(,)	(,)
(,)	(,)	(,)	(,)	(,)	(,)
(,)	(,)	(,)	(,)	(,)	(,)
(,)	(,)	(,)	(,)	(,)	(,)

b. Complete the lattice diagram for rolling two dice.

The 36 entries in your dice lattice represent the *sample space* for two dice thrown. The sample space for any probability model is all the possible outcomes.

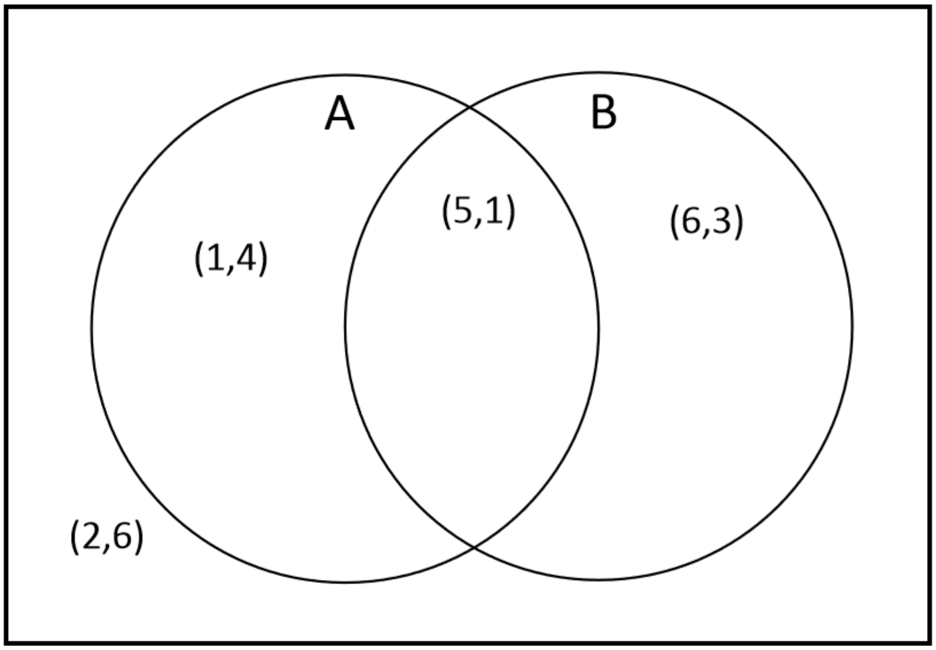
c. It is often necessary to list the sample space and/or the outcomes of a set using *set notation*. For the dice lattice above, the set of all outcomes where the first roll was a 1 can be listed as: $\{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)\}$. This set of outcomes is a **subset** of the set because all of the *elements* of the subset are also contained in the original set. Give the subset that contains all elements that sum to 9.

d. What is the probability that the sum of two die rolled will be 9?

e. Using your lattice, determine the probability of having at least one of the two dice show an odd number.

4. The different outcomes that determine the probability of rolling odd can be visualized using a Venn Diagram, the beginning of which is seen below. Each circle represents the possible ways that each die can land on an odd number. Circle A is for the first die landing on an odd number and circle B for the second die landing on odd. The circles overlap because some rolls of the two dice are successes for both dice. In each circle, the overlap, and the area outside the circles, one of the ordered pairs from the lattice has been placed. $(1,4)$ appears in circle A because the first die is odd, $(6,3)$ appears in circle B because the second die is odd, $(5,1)$ appears in both circles at the same time (the overlap) because each die is odd, and $(2,6)$ appears outside of the circles because neither dice is odd.

a. Finish the Venn Diagram by placing the remaining 32 ordered pairs from the dice lattice in the appropriate place.



b. How many outcomes appear in circle A? (Remember, if ordered pairs appear in the overlap, they are still within circle A).

c. How many outcomes appear in circle B?

d. The portion of the circles that overlap is called the **intersection**. The notation used for intersections is $\cap$. For this Venn Diagram the intersection of A and B is written $A \cap B$ and is read as "A intersect B" or "A and B." How many outcomes are in $A \cap B$?

e. When you look at different parts of a Venn Diagram together, you are considering the **union** of the two outcomes. The notation for unions is $\cup$, and for this diagram the union of A and B is written $A \cup B$ and is read "A union B" or "A or B." In the Venn Diagram you created, $A \cup B$ represents all the possible outcomes where an odd number shows. How many outcomes are in the union?

f. Record your answers to b, c, d, and e in the table below.

b. Circle A	c. Circle B	d. $A \cap B$	e. $A \cup B$

g. How is your answer to e related to your answers to b, c, and d?

h. Based on what you have seen, make a conjecture about the relationship of A, B, $A \cup B$ and $A \cap B$ using notation you just learned.

i. What outcomes fall outside of $\boldsymbol{A \cup B}$ (outcomes we have not yet used)? Why haven't we used these outcomes yet?

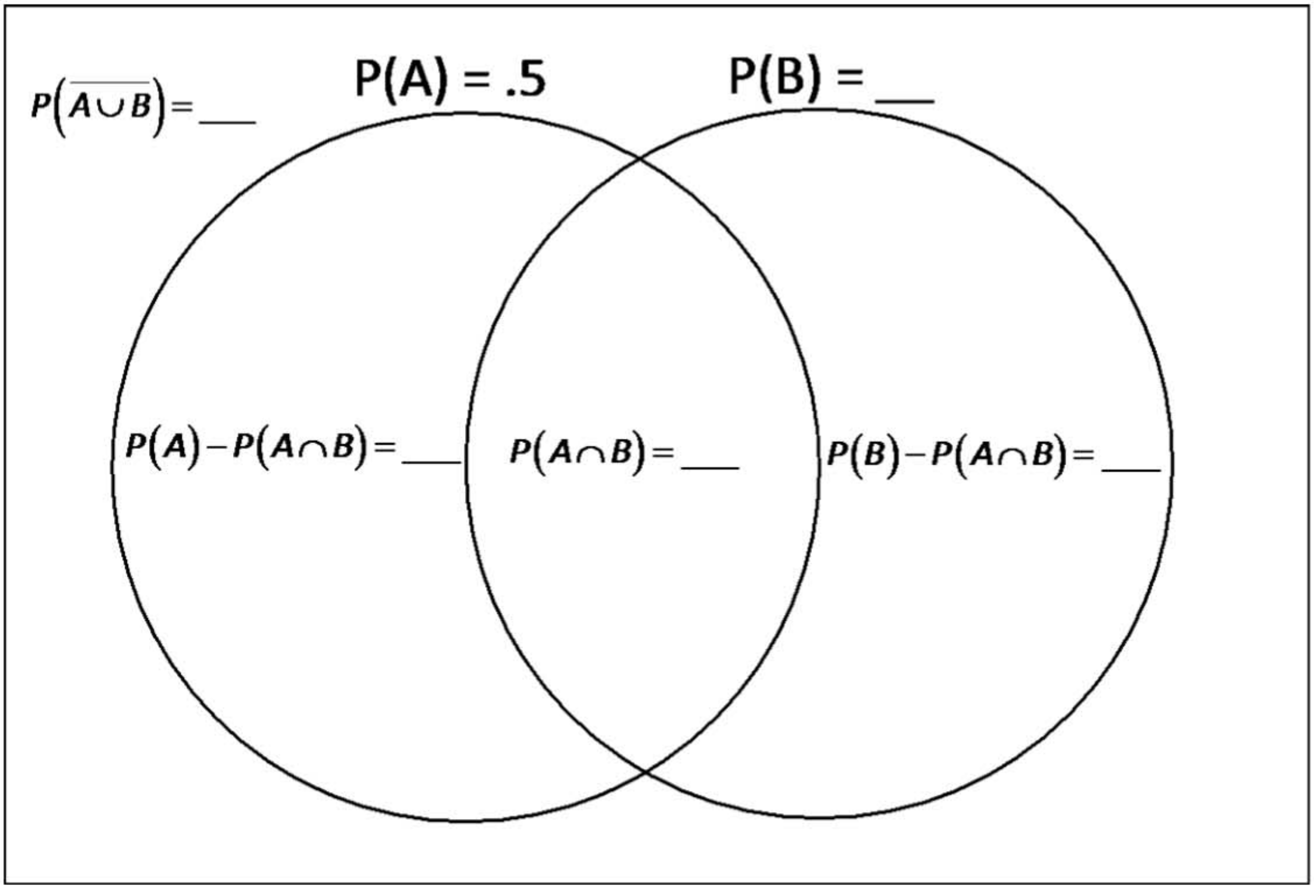
In a Venn Diagram the set of outcomes that are *not* included in some set is called the complement of that set. The notation used for the complement of set A is $\overline{A}$, read "A bar", or ~A, read "not A". For example, in the Venn Diagram you completed above, the outcomes that are outside of $A \cup B$ are denoted $\overline{\mathbf{A \cup B}}$.

j. Which outcomes appear in $\overline{\mathbf{A}} - \mathbf{B}$?

k. Which outcomes appear in $\overline{B} - \left(\overline{A \cup B}\right)$?

5. The investigation of the Venn Diagram in question 4 should reveal a new way to see that the probability of rolling at least one odd number on two dice is $\frac{27}{36} = \frac{3}{4}$. How does the Venn diagram show this probability?

6. Venn Diagrams can also be drawn using probabilities rather than outcomes. The Venn diagram below represents the probabilities associated with throwing two dice together. In other words, we will now look at the same situation as we did before, but with a focus on probabilities instead of outcomes.



a. Fill in the remaining probabilities in the Venn diagram.

b. Find $P(A \cup B)$ and explain how you can now use the probabilities in the Venn diagram rather than counting outcomes.

c. Use the probabilities in the Venn diagram to find $P(\overline{B})$.

d. What relationship do you notice between $P(B)$ and $P(\overline{B})$? Will this be true for any set and its complement?

Part 2 – Venn diagrams can also be used to organize different types of data, not just common data sets like that generated from rolling two dice. In this part of the task, you'll have an opportunity to collect data on your classmates and use a Venn diagram to organize it.

1. Music is a popular topic amongst high school students, but one in which not all can agree upon. Let's say we want to investigate the popularity of different genres of music in your math class, particularly, Hip Hop and Country music. What genre of music do you enjoy listening to: Hip Hop, Country, or Neither?

2. Each student should identify themselves by their 3 initials (first, middle, last). Any student who listens to both Country and Hip Hop may be listed in both categories. Record results of the class poll in the table.

Hip Hop (HH)	Country (C)	Neither (N)

3. Draw a Venn diagram to organize your outcomes. (*Hint: Students listed in both the Hip Hop and Country categories should be identified first prior to filling in the diagram.)*

4. Find $P(HH)$.

5. Find $\mathbf{P}\left(\overline{\mathbf{C}}\right)$.

6. Find $P(HH \cap C)$.

7. Find $P(HH \cup C)$.

8. In part 1, you found the relationship between A, B, $A \cup B$, and $A \cap B$ to be $A \cup B = A + B - A \cap B$. In a similar way, write a formula for $P(A \cup B)$.

9. Now find $P(HH \cup C)$ using the formula instead of the Venn diagram. Did you get the same answer as you did in f above?

10. In what situation might you be forced to use the formula instead of a Venn diagram to calculate the union of two sets?

Part 3 – Now that you have had experience creating Venn Diagrams on your own and finding probabilities of events using your diagram, you are now ready for more complex Venn Diagrams.

1. In this part of the task, you will be examining data on the preference of social networking sites based on gender. Again, you will collect data on students in your class, record the data in a *two-way frequency table*, and then create a Venn Diagram to organize the results of the poll. Which social networking site do you prefer?

2. Record results from the class poll in the table.

	Twitter (T)	Facebook (FB)
Female (F)		
Male (M)		

3. Draw a Venn Diagram to organize your outcomes. *(Hint: Notice that male and female will not overlap and neither will Twitter and Facebook).*

4. Find $\mathbf{P(T \cup M)}$.

5. What is another way to write the probability of $\mathbf{P(T \cup M)}$ using a complement?

6. Find $\mathbf{P(\overline{FB} \cap F)}$.

7. Find $\mathbf{P(T \cap M) + P(\overline{T \cup M})}$.

1. 60 people were asked if they read either of two local newspapers, the Chronicle and the Echo.

12 people read both newspapers

36 people read the Chronicle

25 people read the Echo

Create a Venn Diagram with the above information and answer the following questions.

How many people read neither?

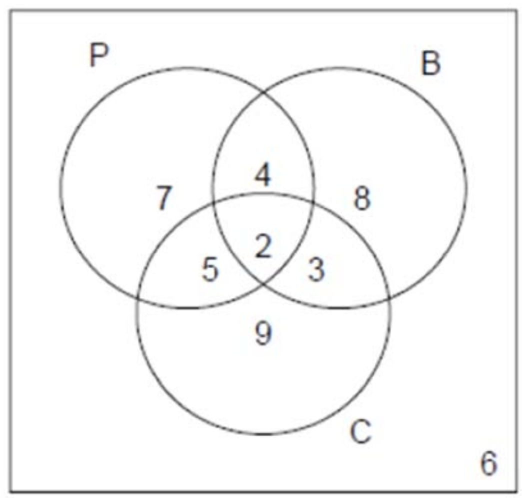
2.

The Venn diagram shows the numbers of pupils in a school according to whether they study the sciences Physics (P), Chemistry (C) or Biology (B).

(a) Write down the number of pupils that study Chemistry only.

(b) Write down the number of pupils that study **exactly** two sciences.

(c) Write down the number of pupils that do not study Physics.



Term	Definition	Notation	Diagram/Visual
Dependent Events			
Independent Events			
Multiplication Rule for Independent Events			

Probability of Independent and Dependent Events, and Conditional	
"With replacement"	means you put the item back.
"Without replacement"	means don't put it back once you remove it.
Independent Event (2 or more events)	Event A and Event B are independent if they do ______ affect each other. Ex. Landing on heads after tossing a coin AND rolling a 5 on a 6-sided die. Ex. Choosing a marble from a jar AND landing on tails after tossing a coin. Ex. Getting a 3 from a deck of cards, replacing it, AND THEN choosing an ace. Ex. Rolling a 4 on a single 6-sided die, AND then rolling a 1 on a second roll of the die. **<u>Probability of Two Independent Events</u>:** **P(A and B) = P(A) • P(B)** **"AND" (for multiple events) → ____________** Ex: Events *A* and *B* are independent. Find the indicated probability. a. P(A) = 0.3, P(B) = 0.5, P(A and B) = ? b. P(A) = ?, P(B) = 0.9, P(A and B) = 0.45 Ex: Adam goes to the grocery store to buy cereal. The shelves contain 9 boxes of Brand A and 6 boxes of Brand B. He selects one brand at random & then puts it back. Another person does the same thing. What is the probability they both selected Brand A? Ex. A bag contains 4 green, 3 blue, and 5 yellow marbles. What is the probability of selecting a green marble, replacing it, then selecting a yellow marble?
Dependent Event	Event A and Event B are dependent if Event A affects the probability of Event B. Ex. Choosing items from a container without replacement Ex. Choosing people to from a committee Ex. Choose a 3 from a deck of cards, not replacing it and then choosing an ace as the second card. Ex. Speeding, and then getting a speeding ticket. **<u>Probability of Two Dependent Events</u>:** **P(A & B) = P(A) • P(B after A)**

Figure out the first probability. Figure out the second probability and how it is affected by the first event. Then, multiply them together.

Ex: Tasha's bowl contains 4 red, 6 green and 3 brown candies. She randomly chooses and keeps 3 candies from the bowl. What is the probability she will choose all brown?

Ex: From a standard deck of 52 cards, *2 cards* are selected. What is the probability:

<u>with replacement?</u> <u>without replacement?</u>

a) 2 black cards are selected

b) 1 red card and then 1 spade
in that order

<u>Conditional Probability</u>

the probability that B will occur given that A has **<u>already</u>** occurred is written P(B|A)

Narrow down your total outcomes from the event that has already occurred. Then take the new probability based on what's left.

Ex: Let *n* be a randomly selected integer from 1 to 20. Find the indicated probability.

a) *n* is 2 given that it is even.

b) *n* is prime given that it has two digits.

c) *n* is odd given that it is even.

1. You randomly choose a marble, put it back, then randomly choose another marble. Are the events "choose a red marble first" and "choose a blue marble second" *independent* or *dependent*?

2. A drawer contains 15 socks, 7 blue and 8 white. You close your eyes and pull out a blue sock first, then a white sock, without replacing the blue sock. Are these events independent or dependent?

3. Tara is playing a game at a carnival where she picks a rubber duck from a pond. There are 12 ducks in the pond for which there is no prize and 4 ducks that will award a prize. What is the probability that Tara picks a prize-winning duck, replaces the duck in the pond, then picks another prize-winning duck?

4. A drawer contains 12 socks, 7 blue and 5 white. You close your eyes and pull out a blue sock first, then a white sock, after replacing the blue sock. Are these events independent or dependent?

SELF CHECK

Jeffrey's mother has 10 orange juice boxes, 7 grape juice boxes, and 3 lemonade juice boxes in the cooler for Jeffrey and his friends.

1. Jeffrey randomly takes a juice box from the cooler, then randomly chooses another juice box without replacing the first. Find the probability that both juice boxes are grape.

2. Find the probability that both juice boxes are lemonade when the first juice box chosen is not replaced.

What are key terms to let you know if an event is independent or dependent?

1. Determine which of the following are examples of independent or dependent events.

a. Rolling a 5 on one die and rolling a 5 on a second die.
b. Choosing a cookie from the cookie jar and choosing a jack from a deck of cards.
c. Selecting a book from the library and selecting a book that is a mystery novel.
d. Choosing an 8 from a deck of cards, replacing it, and choosing a face card.
e. Choosing a jack from a deck of cards and choosing another jack, without replacement.

2. A coin and a die are tossed. Calculate the probability of getting tails and a 5.

3. In Tania's homeroom class, 9% of the students were born in March and 40% of the students have a blood type of O+. What is the probability of a student chosen at random from Tania's homeroom class being born in March and having a blood type of O+?

4. What is the probability of tossing 2 coins one after the other and getting 1 head and 1 tail?

5. 2 cards are chosen from a deck of cards. The first card is replaced before choosing the second card. What is the probability that they both will be clubs?

f. 2 cards are chosen from a deck of cards. The first card is replaced before choosing the second card. What is the probability that they both will be face cards?

g. If the probability of receiving at least 1 piece of mail on any particular day is 22%, what is the probability of not receiving any mail for 3 days in a row?

h. Jonathan is rolling 2 dice and needs to roll an 11 to win the game he is playing. What is the probability that Jonathan wins the game?

i. Thomas bought a bag of jelly beans that contained 10 red jelly beans, 15 blue jelly beans, and 12 green jelly beans. What is the probability of Thomas reaching into the bag and pulling out a blue or green jelly bean and then reaching in again and pulling out a red jelly bean? Assume that the first jelly bean is not replaced.

j. For question 10, what if the order was reversed? In other words, what is the probability of Thomas reaching into the bag and pulling out a red jelly bean and then reaching in again and pulling out a blue or green jelly bean without replacement?

k. What is the probability of drawing 2 face cards one after the other from a standard deck of cards without replacement?

l. There are 3 quarters, 7 dimes, 13 nickels, and 27 pennies in Jonah's piggy bank. If Jonah chooses 2 of the coins at random one after the other, what is the probability that the first coin chosen is a nickel and the second coin chosen is a quarter? Assume that the first coin is not replaced.

m. For question 12, what is the probability that neither of the 2 coins that Jonah chooses are dimes? Assume that the first coin is not replaced.

n. Jenny bought a half-dozen doughnuts, and she plans to randomly select 1 doughnut each morning and eat it for breakfast until all the doughnuts are gone. If there are 3 glazed, 1 jelly, and 2 plain doughnuts, what is the probability that the last doughnut Jenny eats is a jelly doughnut?

15. Steve will draw 2 cards one after the other from a standard deck of cards without replacement. What is the probability that his 2 cards will consist of a heart and a diamond?

16. A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

17. A penny and a nickel are tossed. Find the probability that the penny Shows heads, given that the nickel shows heads.

18. A die is tossed. Find $P(\textit{less than } 5 \mid \textit{even})$.

19. A number is selected, at random, from the set $\{1,2,3,4,5,6,7,8\}$.

Find : $P(odd)$

20. Using the information from problem 19 find $P(prime|odd)$.

The Land of Independence

Part 1 – Confirming Independence

By developing a full picture of conditional probability in the previous task, you were able to conclude that events that occur without regard to conditions, independent events, are defined by the equation $P(A \cap B) = P(A) \cdot P(B)$. This equation is known as *necessary and sufficient*. It works exactly like a biconditional statement: two events A and B are independent if and only if the equation $P(A \cap B) = P(A) \cdot P(B)$ is true.

1. Based upon the definition of independence, determine if each set of events below are independent.

   a. $P(A) = 0.45, P(B) = 0.30, P(A \cap B) = 0.75$

   b. $P(A) = 0.12, P(B) = 0.56, P(A \cap B) = 0.0672$

   c. $P(A) = \frac{4}{5}, P(B) = \frac{3}{8}, P(A \cap B) = \frac{7}{40}$

   d. $P(A) = \frac{7}{9}, P(B) = \frac{3}{4}, P(A \cap B) = \frac{7}{12}$

2. Determine the missing values so that the events A and B will be independent.

   a. $P(A) = 0.55, P(B) =$ ______ $, P(A \cap B) = 0.1375$

   b. $P(A) =$ ______ $, P(B) = \frac{3}{10}, P(A \cap B) = \frac{1}{7}$

Part 2 – Independence and Inference

With knowledge of probability and statistics, statisticians are able to make *statistical inferences* about large sets of data. Based upon what you have learned in this unit, you have the knowledge necessary to make basic inferences.

Much of the data collected every 10 years for the Census is available to the public. This data includes a variety of information about the American population at large such as age, income, family background, education history and place of birth. Below you will find three different samples of the Census that looks at comparing different aspects of American life. Your job will be to use your knowledge of conditional probability and independence to make conclusions about the American populace.

Gender vs. Income – Has the gender gap closed in the world today? Are men and women able to earn the same amount of money? The table below organizes income levels (per year) and gender.

	Under $10,00	Between $10,000 and $40,000	Between $40,000 and $100,000	Over $100,000	
Male	15	64	37	61	
Female	31	73	14	58	

By finding different probabilities from the table above, make a determination about whether or not income level is affected by gender. Investigate whether your conclusion is true for all income levels. Show all the calculations you use and write a conclusion using those calculations.

Real Mathematics: Application

Bills vs. Education – When you grow up, do you think the amount of schooling you have had will be at all related to the amount of money you have to pay out in bills each month? Below is a table that compares two variables: the highest level of education completed (below a high school diploma, a high school diploma, or a college degree) and the amount paid for a mortgage or rent each month.

	Pays under $500	Pays between $500 and $1000	Pays over $1000	
Below high school	57	70	30	
High school diploma	35	47	11	
College degree	24	62	40	

By determining the probabilities of each education level and the probabilities of housing costs, you should be able to decide whether or not these two variables are independent. Show all the calculations you use, and write a conclusion about the *interdependence* of these two variables.

Gender vs. Commute – What else might gender affect? Is your commute to work related to whether or not you are male or female? The data below allows you to investigate these questions by presenting gender data against the minutes needed to commute to work each day.

	Under 30 minutes	Between 30 minutes and an hour	Over an hour	
Male	65	24	15	
Female	64	22	7	

By finding various probabilities from the table above, decide whether or not a person's gender is related to their commute time to work. Write your conclusion below and include any relevant calculations.

1. In a survey at a football game, 50 of 75 male fans and 40 of 50 female fans said that they favor the new team mascot. If 1 male and 1 female are randomly selected, what is the probability that both favor the new mascot?

2. Find the probability of drawing the given cards from a standard deck of 52 cards with replacement and without replacement.

		Numbers	Face Cards
BLACK	Spade	A 2 3 4 5 6 7 8 9 10	J Q K
BLACK	Club	A 2 3 4 5 6 7 8 9 10	J Q K
RED	Heart	A 2 3 4 5 6 7 8 9 10	J Q K
RED	Diamond	A 2 3 4 5 6 7 8 9 10	J Q K

	(INDEPENDENT)	(DEPENDENT)
	With replacement	Without replacement
a) A club, then a spade		
b) A queen, then an ace		
c) A face card, then a 6		
d) A 10, then 2		
e) A king, then a queen, then a jack		
f) A spade, then a club, then another spade		
f) Three hearts in a row		

3. One bag contains 2 green marbles and 4 white marbles, and a second bag contains 3 green marbles and 1 white marble. If Trent randomly draws one marble from each bag, what is the probability that they are both green?

4. On a certain day the chance of rain is 80% in San Francisco and 30% in Sydney. Assume that the chance of rain in the two cities is independent. What is the probability that it will NOT rain in either city? (It will not rain in BOTH cities.)

5. A math teacher is randomly distributing 15 rulers with centimeter labels and 10 rulers without centimeter labels. What is the probability that the first ruler she hands out will have centimeter labels and the second ruler will NOT have labels?

6. A basket contains five apples and seven peaches. You randomly select one piece of fruit and eat it. Then you randomly select another piece of fruit. What is the probability that the first piece of fruit is an apple and the second piece is a peach?

7. You roll a fair six-sided die twice. What is the probability the first roll shows a five and the second roll shows a six?

CONDITIONAL PROBABILITY:

8. Let *n* be a randomly selected integer from 1 to 20. Find the indicated probability.

List the integers:

a) *n* is 2 given that it is even.

b) n is 5 given that it is less than 8.

c) *n* is prime given that it has two digits.

d) n is odd given that it is prime.

9. A box contains three blue marbles, five red marbles, and four white marbles. If one marble is drawn at random, find:

   a) $P(blue \mid not\ white)$

   b) $P(not\ red \mid not\ white)$

10. A number is selected randomly from a container containing all the integers from 10-50. Find:

   a) $P(even \mid greater\ than\ 40)$

   b) $P(greater\ than\ 40 \mid even)$

Two Way Tables

GUIDED NOTES

What is a Table?

Have you ever seen something like this when you are watching sports? What do all the numbers, words and spaces mean?

	Team A	Team B
Points	1	2
Penalties	2	0
Bonus	0	0

This table is telling us the number of points, penalties and bonuses that both Team A and Team B have. The way it's displayed side by side is comparing the numbers for both teams. In this lesson, you'll learn what a table is and how to read one.

Tables are used to record facts in an organized way. A table is composed of lines that form boxes. We say the boxes across the page are **rows** and the boxes going down the page are **columns**. The first row and first column in a table will have **headers** that help identify the information in the table. Numbers that match the labels at the top of each column and the left of each row will be placed i the table. In the Cats and Dogs Table, the column headers are 'Cat' and 'Dog' and the row headers are 'Black' and 'White.' The number under the 'Cat' column are the number of cats. The numbers in the 'Black' row are the number of animals that have black fur. On this table, there are also totals at the bottom and far right.

	Cats	Dogs	Total
Black	2	2	4
White	3	1	4
Total	5	3	8

How to Read a Table

A table can be read from left to right or from top to bottom. If you read a table across the row, you read the information from left to right. In the Cats and Dogs Table, the number of black animals is **2 + 2 = 4**. You'll see that those are the numbers in the row directly to the right of the word 'Black.' If you read a table down a column, you read the information from top to bottom. In this case, the number of cats is **2 + 3 = 5**.

Reading a table down one column or across one row is very useful. Another great way to use a table is to pick one row header and one column header to ask a question: How many black cats are there? In this table, follow the row labeled 'Black' to the right until you find the column that says 'Cats.' The box that lines up with both headers has a number that will answer this question: **2**.

Example!

	Wimbledon	US Open	Total
5-18	12	38	50
19-40	20	30	50
41+	35	15	50
Total	67	83	150

How many of these tennis fans prefer Wimbledon?	
How many tennis fans age 5-18 prefer the US Open?	
How many tennis fans 40 and younger prefer Wimbledon?	
How many tennis fans over 40 prefer the US Open?	
How many tennis fans prefer the US Open?	
How many tennis fans over 18 prefer Wimbledon?	
What is the probability of randomly selecting a tennis fan that prefers the US Open?	
What is the probability of randomly selecting a tennis fan in the 5-18 age group?	
What is the probability of randomly selecting a tennis fan in the 5-18 age group that prefers the US Open?	
What is the probability of randomly selecting a tennis fan who prefers the US Open given that the fan is 18 or under?	
What is the probability of randomly selecting a tennis fan in the 19-40 age group given the fan prefers Wimbledon?	
Accoding to this table, a 55 year old tennis fan would be more likely to prefer: *A. Wimbledon* *B. The US Open* *C. Equally likely to prefer both.*	

SELF CHECK

	Steak	Chicken
5-13	20	56
14-25	32	40
26+	55	47

A local restaurant surveyed their customers based on age and preference of chicken or steak. The results were compiled in a 2-way frequency table. Let S={Steak Lovers}, C={Chicken Lovers}, A_5={Customers ages 5-13}, A_{14}= {Customers ages 14-25}, and A_{26}= {Customers ages 26+}. Answer the probability questions as reduced fractions.

1. How many members are in A_{14}?	
2. How many members are in the complement of A_{14}?	
3. How many members are in $A_{26} \cap S$?	
4. How many members are in $A_{26} \cup S$?	
5. How many members are in $(A_{26} \cup S)'$?	
6. What is P(C)?	
7. What is $P(\overline{C})$?	
8. What is $P(C \cup A_5)$?	
9. What is $P(A_{26} \mid C)$?	
10. What is $P(S \mid A_{14})$?	
11. What is $P(A_5 \mid S)$?	
12. According to this table a 12-year customer would be more likely to prefer: A. Chicken B. Steak C. Is equally likely to prefer both	

Why is reading the way a question is written important? (Before analyzing data in a two-way table)?

Give some examples of data misinterpretation from not clearly reading the question.

Use the 2-way frequency table to answer each. Express probabilities as a reduced fraction.

	Physics	Anatomy
Seniors	24	40
Juniors	48	16

1. How many students are represented in the 2-way table?	
2. How many students are in Physics?	
3. How many students are Juniors?	
4. How many students are seniors or in anatomy?	
5. How many students are seniors and in Physics?	
6. What is the probability of randomly selecting a student who is Physics?	
7. What is the probability of randomly selecting a student who is a Junior?	
8. What is the probability of randomly selecting a student who is a junior or in Physics?	
9. What is the probability of randomly selecting a student who is a junior and in Physics?	
10. What is the probability of randomly selecting a student who is a junior, given that the student is in Anatomy?	
11. What is the probability of randomly selecting a student who is in Physics given that the student is a senior?	
12. What is the probability of selecting 2 different students that are both Juniors?	

	Cat Owner	Dog Owner	Total
Male	48	56	104
Female	64	32	96
Total	112	88	200

13. How many members are in M?	
14. How many members are in the complement of M?	
15. How many members are in $D \cup F$?	
16. How many members are in $(D \cup F)'$?	
17. What is $P(D \cap F)$?	
18. What is $P(D \cap F)'$?	
19. What is $P(F)$?	
20. What $P(\overline{F})$?	
21. What is $P(F\|C)$?	
22. What is $P(D\|M)$?	
23. What is $P(M\|C)$?	
24. A local pet store is giving away 2 gift cards to the people in this table. The cards are worth $100 each. What is the probability a dog owner wins both if the same person can win both prizes?	

For 25 – 29, use the following table.

	Cat Owner	Dog Owner	Total
Male	12	24	36
Female	4	8	12
Total	16	32	48

25. What is the probability of randomly selecting a pet owner who is female and owns a cat?

26. What is the probability of randomly selecting a pet owner who is female or owns a cat?

27. What is the probability of randomly selecting a pet owner who owns a cat given the owner is male?

28. If two randomly selected owners are picked for a free pet food trial, what is the probability both winners are females who own a cat? Assume the same owner can't win twice.

29. According to the data, which gender is more likely to own a dog?

For 30 – 33, use the following table.

	Wimbledon	US Open
5 – 18	6	9
19 – 40	12	15
41 +	8	10

30. What is the probability of randomly selecting a tennis fan who prefers Wimbledon?

31. What is the probability of randomly selecting a tennis fan between the ages of 5 and 18?

32. What is the probability of randomly selecting a tennis fan 41 years or older who prefers the US Open?

33. What is the probability of randomly selecting a tennis fan who likes Wimbledon given the fan is older than 40?

For 34 – 39, use the following table.

	X	*Y*	*Z*
A	*8*	*4*	*18*
B	*16*	*32*	*18*

34. What is P(A∪X)?

35. What is P(A∩X)?

36. What is P$(B|X)$?

37. What is P$(Z|A)$?

38. According to this data, which event is more likely to occur?

   a. $P(Z|A)$

   b. P$(Z|B)$

   c. These events are equally likely to occur.

39. According to this data, which event is more likely to occur?

   a. $P(A|Z)$

   b. P$(B|Z)$

   c. These events are equally likely to occur.

Are we positive?

The following task was adapted from an Op-Ed piece for the NYTimes.com entitled “Chances Are” by Steven Strogatz. http://opinionator.blogs.nytimes.com/2010/04/25/chances-are/

We know that modern medicine is rich with biology, chemistry and countless other branches of science. While mathematics does not make a lot of headlines for changing the way we think about our health and well-being, there are many ways in which mathematics is informing and improving the way scientists and doctors approach the world of medicine.

Consider the following data for a group of 1000 women. Of these women, 8 are known to have breast cancer. All 1000 undergo the same test to determine whether or not they have breast cancer, and 77 test positive. 7 of the 8 that have breast cancer test positive and 70 of those that do not have breast cancer test positive.

1. Based on the data presented above, do you think the test for breast cancer is effective at identifying women who have breast cancer? Why or why not?

2. Organize the data using the table below.

Women with Breast Cancer	Test Positive
	Test Negative

Women without Breast Cancer	Test Positive
	Test Negative

3. Find the following probabilities:

a. The probability that a woman tests positive given that she has breast cancer.

b. The probability that a woman tests positive given that she does not have breast cancer.

4. Determine whether or not the events of having breast cancer and testing positive for breast cancer are independent. Show all relevant calculations.

The analysis you have done so far should seem straightforward. Testing for breast cancer is successful at identifying those women who have it. It is reassuring to know that testing positive for breast cancer is not independent of having breast cancer, as this seems to indicate that screening for breast cancer is an effective way to identify when a woman actually has breast cancer.

There is one aspect of this analysis that needs further inspection. The probabilities that you have found have the condition that a woman does or does not have breast cancer. In reality, a woman knowing this before getting tested is highly unlikely. The point of getting tested is to find out! While the results you have found seem sound, it will be good to find the probabilities from a more realistic standpoint.

5. Now you will look through the lens of a woman who tests positive for breast cancer.

a. Find the probability that a woman has breast cancer given that her test result is positive.

b. What seems strange about this result to you?

c. Compare this probability to what you calculated in question 3a. What is causing these probabilities to be so different?

6. Let's also look through the lens of a woman who tests negative.

a. Find the probability that a woman does not have breast cancer given that her test result is negative.

b. What does this result indicate?

7. This task focused specifically on medical tests for breast cancer. It is not a stretch to say that the efficacy of most medical tests is similar to that of what you have investigated here. Write a paragraph that discusses the use and effectiveness of medical tests in regards to the probability theory that underlies them.

1. 80 students each study one Science. The table shows some information about these students

a. Complete the table

	Biology	Chemistry	Physics	Total
Female	18			47
Male			19	
Total		21	33	80

b. What is the probability that the student studies Physics?

c. What is the probability that the student is male and does not study biology?

d. What is the probability that the student is female and studies Chemistry?

e. What is the probability that the student is not female?

f. What is the probability that the student does not study Biology?

For 2 – 4, Use the following table.

	Drama	*PE*	*Band*	*Total*
Juniors	*10*	*50*	*45*	*105*
Seniors	*20*	*30*	*45*	*95*
Total	*30*	*80*	*90*	*200*

2. What is the probability of randomly selecting a student enrolled in PE given the student is a senior?

3. What is the probability of randomly selecting a student who is a senior given the student is enrolled in band?

4. According to the data, which class (juniors or seniors) is more likely to participate in band?

For 5 – 10, fill in the missing values in the two-way tables and answer the questions.

	Horses	*Cows*	*Total*
Male	*24*		*32*
Female			
Total	*40*		*80*

5. How many male cows are represented in the table?

6. How many female horses are represented in the table?

7. How many female animals are represented in the table?

8. What is the probability of randomly selecting a cow?

9. What is the probability of randomly selecting a cow given the animal is male?

10. What is the probability of randomly selecting a female given the animal is a horse?

For 11 – 16, fill in the missing values in the two-way tables and answer the questions.

	Drama	*PE*	*Band*	*Total*
Juniors	*20*	*30*		*150*
Seniors				
Total		*100*	*120*	*300*

11. How many drama students are represented in the table?

12. How many seniors are in band?

13. How many seniors are represented in the table?

14. What is the probability of randomly selecting a student in PE?

15. What is the probability of randomly selecting a student in PE given they are a junior?

16. According to the data, which class (juniors or seniors) is more likely to take drama?

For 17 – 23, fill in the missing values in the two-way tables and answer the questions.

	Wimbledon	*US Open*
5 – 18	*18*	
19 – 40		*8*
41 +		

17. 50 tennis fans from each age group were surveyed. If a total of 84 liked Wimbledon, how many liked the US Open?

18. 50 tennis fans from each age group were surveyed. If a total of 84 liked Wimbledon, how many in the 41+ age group liked the US Open?

19. 50 tennis fans from each age group were surveyed. If a total of 84 liked Wimbledon, how many in the 19 – 40 age group liked Wimbledon?

20. 50 tennis fans from each age group were surveyed. If a total of 84 liked Wimbledon, what is the probability of selecting a US Open fan?

21. 50 tennis fans from each age group were surveyed. If a total of 84 liked Wimbledon, which age group is likely to prefer the US Open?

a. 5 – 18

b. 19 – 40

c. 41+

d. Cannot be determined

22. 50 tennis fans from each age group were surveyed. A total of 84 liked Wimbledon. If 2000 tennis fans aged 5 – 18 attended a tennis camp, how many campers would this table predict would like the US Open?

23. 50 tennis fans from each age group were surveyed. A total of 84 liked Wimbledon. At the age 50 and over senior nationals, there are 4000 participants. How many of these participants would this table predict to like Wimbledon?

For 24 – 28, fill in the missing values in the two-way tables and answer the questions.

	X	*Y*	*Z*	*Total*
A			*20*	
B	*24*		*25*	*65*
Total	*74*		*45*	*150*

24. What is P(A∩Y)?

25. What is $P(B|X)$?

26. What is $P(Y|A)$?

27. If this data were used to predict an outcome for a larger set of data, how many would belong to set A if 5000 were the total amount?

28. If this data were used to predict an outcome for a larger set of data, how many would belong to set Y if 4000 were the total amount?