DeSoto
COUNTY SCHOOLS

Yearlong Algebra I

Week 3

NAME _____ DATE _____ PERIOD ____

8-4 Study Guide and Intervention *(continued)*

Factoring Trinomials: $ax^2 + bx + c$

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve some equations of the form $ax^2 + bx + c = 0$.

Example **Solve $12x^2 + 3x = 2 - 2x$. Check your solutions.**

$12x^2 + 3x = 2 - 2x$	Original equation
$12x^2 + 5x - 2 = 0$	Rewrite equation so that one side equals 0.
$(3x + 2)(4x - 1) = 0$	Factor the left side.
$3x + 2 = 0$ or $4x - 1 = 0$	Zero Product Property
$x = -\frac{2}{3}$ $\quad x = \frac{1}{4}$	Solve each equation.

The solution set is $\left\{-\frac{2}{3}, \frac{1}{4}\right\}$.

Since $12\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) = 2 - 2\left(-\frac{2}{3}\right)$ and $12\left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right) = 2 - 2\left(\frac{1}{4}\right)$, the solutions check.

Exercises

Solve each equation. Check your solutions.

1. $8x^2 + 2x - 3 = 0$

2. $3n^2 - 2n - 5 = 0$

3. $2d^2 - 13d - 7 = 0$

4. $4x^2 = x + 3$

5. $3x^2 - 13x = 10$

6. $6x^2 - 11x - 10 = 0$

7. $2k^2 - 40 = -11k$

8. $2p^2 = -21p - 40$

9. $-7 - 18x + 9x^2 = 0$

10. $12x^2 - 15 = -8x$

11. $7a^2 = -65a - 18$

12. $16y^2 - 2y - 3 = 0$

13. $8x^2 + 5x = 3 + 7x$

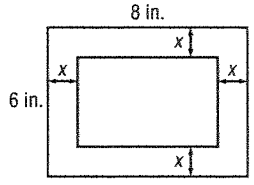
14. $4a^2 - 18a + 5 = 15$

15. $3b^2 - 18b = 10b - 49$

16. The difference of the squares of two consecutive odd integers is 24. Find the integers.

17. GEOMETRY The length of a Charlotte, North Carolina, conservatory garden is 20 yards greater than its width. The area is 300 square yards. What are the dimensions?

18. GEOMETRY A rectangle with an area of 24 square inches is formed by cutting strips of equal width from a rectangular piece of paper. Find the dimensions of the new rectangle if the original rectangle measures 8 inches by 6 inches.



NAME _____ DATE _____ PERIOD _____

8-5 Study Guide and Intervention

Factoring Differences of Squares

Factor $a^2 - b^2$ The binomial expression $a^2 - b^2$ is called the **difference of two squares**. The following pattern shows how to factor the difference of squares.

Difference of Squares	$a^2 - b^2 = (a - b)(a + b) = (a + b)(a - b)$.

Example 1 **Factor each binomial.**

a. $n^2 - 64$

$n^2 - 64$
$= n^2 - 8^2$ Write in the form $a^2 - b^2$.
$= (n + 8)(n - 8)$ Factor.

b. $4m^2 - 81n^2$

$4m^2 - 81n^2$
$= (2m)^2 - (9n)^2$ Write in the form $a^2 - b^2$.
$= (2m - 9n)(2m + 9n)$ Factor.

Example 2 **Factor each polynomial.**

a. $50a^2 - 72$

$50a^2 - 72$
$= 2(25a^2 - 36)$ Find the GCF.
$= 2[(5a)^2 - 6^2)]$ $25a^2 = 5a \cdot 5a$ and $36 = 6 \cdot 6$
$= 2(5a + 6)(5a - 6)$ Factor the difference of squares.

b. $4x^4 + 8x^3 - 4x^2 - 8x$

$4x^4 + 8x^3 - 4x^2 - 8x$ Original polynomial
$= 4x(x^3 + 2x^2 - x - 2)$ Find the GCF.
$= 4x[(x^3 + 2x^2) - (x + 2)]$ Group terms.
$= 4x[x^2(x + 2) - 1(x + 2)]$ Find the GCF.
$= 4x[(x^2 - 1)(x + 2)]$ Factor by grouping.
$= 4x[(x - 1)(x + 1)(x + 2)]$ Factor the difference of squares.

Exercises

Factor each polynomial if possible. If the polynomial cannot be factored, write *prime*.

1. $x^2 - 81$

2. $m^2 - 100$

3. $16n^2 - 25$

4. $36x^2 - 100y^2$

5. $49x^2 - 32$

6. $16a^2 - 9b^2$

7. $225c^2 - a^2$

8. $72p^2 - 50$

9. $-2 + 2x^2$

10. $-81 + a^4$

11. $6 - 54a^2$

12. $8y^2 - 200$

13. $4x^3 - 100x$

14. $2y^4 - 32y^2$

15. $8m^3 - 128m$

16. $6x^2 - 25$

17. $2a^3 - 98ab^2$

18. $18y^2 - 72y^4$

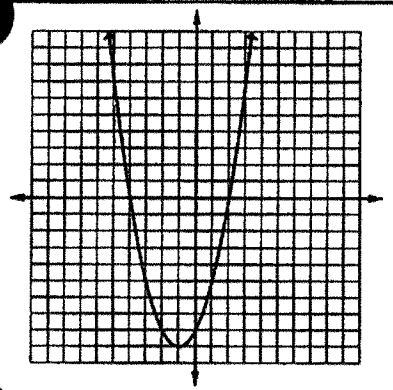
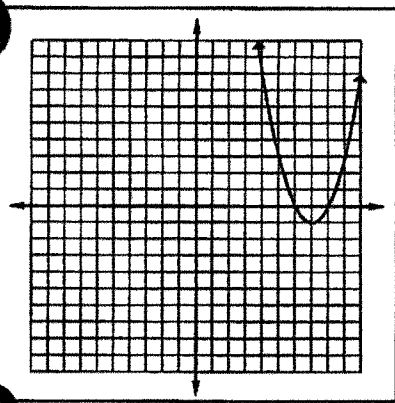
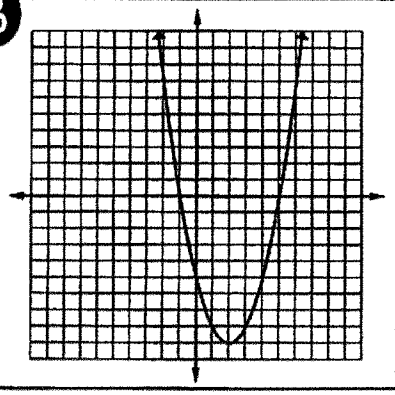
19. $169x^3 - x$

20. $3a^4 - 3a^2$

21. $3x^4 + 6x^3 - 3x^2 - 6x$

Representing QUADRATIC EQUATIONS

Quadratic equations can be represented in vertex form, standard form, or factored form.
Complete each row of the chart below with the given information.

	GRAPH	VERTEX FORM	STANDARD FORM	FACTORED FORM
1		$y=(x+1)^2-9$	$y=(x+1)(x+1)-9$ $y=x^2+x+x+1-9$ $y=x^2+2x-8$	$y=(x+4)(x-2)$
2		$y=(x-7)^2-1$	$y=(x-7)(x-7)-1$ $y=x^2-7x-7x+49-1$ $y=x^2-14x+48$	$y=(x-8)(x-6)$
3		$y=(x-2)^2-9$	$y=(x-2)(x-2)-9$ $y=x^2-2x-2x+4-9$ $y=x^2-4x-5$	$y=(x-5)(x+1)$

Problems

Factor each difference of squares.

1. $x^2 - 16$
2. $x^2 - 25$
3. $64m^2 - 25$
4. $4p^2 - 9q^2$
5. $9x^2y^2 - 49$
6. $x^4 - 25$
7. $64 - y^2$
8. $144 - 25p^2$
9. $9x^4 - 4y^2$

Factor each perfect square trinomial.

10. $x^2 + 4x + 4$
11. $y^2 + 8y + 16$
12. $m^2 - 10m + 25$
13. $x^2 - 8x + 16$
14. $a^2 + 8ab + 16b^2$
15. $36x^2 + 12x + 1$
16. $25x^2 - 30xy + 9y^2$
17. $9x^2y^2 - 6xy + 1$
18. $49x^2 + 1 + 14x$

Factor completely.

19. $9x^2 - 16$
20. $9x^2 + 24x + 16$
21. $9x^2 - 36$
22. $2x^2 + 8xy + 8y^2$
23. $x^2y + 10xy + 25y$
24. $8x^2 - 72$
25. $4x^3 - 9x$
26. $4x^2 - 8x + 4$
27. $2x^2 + 8$

Answers

1. $(x + 4)(x - 4)$
2. $(x + 5)(x - 5)$
3. $(8m + 5)(8m - 5)$
4. $(2p + 3q)(2p - 3q)$
5. $(3xy + 7)(3xy - 7)$
6. $(x^2 + 5)(x^2 - 5)$
7. $(8 + y)(8 - y)$
8. $(12 + 5p)(12 - 5p)$
9. $(3x^2 + 2y)(3x^2 - 2y)$
10. $(x + 2)^2$
11. $(y + 4)^2$
12. $(m - 5)^2$
13. $(x - 4)^2$
14. $(a + 4b)^2$
15. $(6x + 1)^2$
16. $(5x - 3y)^2$
17. $(3xy - 1)^2$
18. $(7x + 1)^2$
19. $(3x + 4)(3x - 4)$
20. $(3x + 4)^2$
21. $9(x + 2)(x - 2)$
22. $2(x + 2y)^2$
23. $y(x + 5)^2$
24. $8(x + 3)(x - 3)$
25. $x(2x + 3)(2x - 3)$
26. $4(x - 1)^2$
27. $2(x^2 + 4)$

SOLVING QUADRATIC EQUATIONS: Methods Comparison

	FACTORING	COMPLETING THE SQUARE	THE QUADRATIC FORMULA
1	$x^2 - 14x + 48 = 0$ $(x-8)(x-6)=0$ $x-8=0$ | $x-6=0$ $x=8$ | $x=6$ $x=\{6,8\}$	$x^2-14x = -48$ $(-7)^2 = 49$ $x^2-14x+49 = -48+49$ $\sqrt{(x-7)^2} = \sqrt{1}$ $x-7 = \pm 1$ $x = 7 \pm 1$ $x=\{6,8\}$	$x = \frac{14 \pm \sqrt{(-14)^2 - 4(1)(48)}}{2(1)}$ $x = \frac{14 \pm \sqrt{196-192}}{2}$ $x = \frac{14 \pm \sqrt{4}}{2}$ $x = \frac{14 \pm 2}{2}$ $x=\{6,8\}$
2	$x^2 + 19 = 4 - 8x$ $x^2+8x+15=0$ $(x+3)(x+5)=0$ $x+3=0$ | $x+5=0$ $x=-3$ | $x=-5$ $x=\{-5,-3\}$	$x^2+8x = -15$ $4^2 = 16$ $x^2+8x+16 = -15+16$ $\sqrt{(x+4)^2} = \sqrt{1}$ $x+4 = \pm 1$ $x = -4 \pm 1$ $x=\{-5,-3\}$	$x = \frac{-8 \pm \sqrt{8^2-4(1)(15)}}{2(1)}$ $x = \frac{-8 \pm \sqrt{64-60}}{2}$ $x = \frac{-8 \pm \sqrt{4}}{2}$ $x = \frac{-8 \pm 2}{2}$ $x=\{-5,-3\}$
3	$2x^2 + 4x = 48$ $2x^2+4x-48=0$ $2(x^2+2x-24)=0$ $2(x+6)(x-4)=0$ $2\neq 0$ | $x+6=0$ | $x-4=0$ $x=-6$ | $x=4$ $x=\{-6,4\}$	$\frac{2x^2+4x}{2} = \frac{48}{2}$ $x^2+2x = 24$ $1^2 = 1$ $x^2+2x+1 = 24+1$ $\sqrt{(x+1)^2} = \sqrt{25}$ $x+1 = \pm 5$ $x = -1 \pm 5$ $x=\{-6,4\}$	$x = \frac{-4 \pm \sqrt{4^2-4(2)(-48)}}{2(2)}$ $x = \frac{-4 \pm \sqrt{16+384}}{4}$ $x = \frac{-4 \pm \sqrt{400}}{4}$ $x = \frac{-4 \pm 20}{4}$ $x=\{-6,4\}$

SOLVING QUADRATICS REVIEW

METHOD 1: FACTORING

1. $x^2 - 6x - 40 = 0$

2. $x^2 - 14x + 49 = 0$

3. $10x^2 - 25 = x^2$

4. $12x^2 + 8x = 0$

5. $2x^2 + 80 = 26x$

6. $4x^2 + 5x + 1 = 0$

METHOD 2: SQUARE ROOTS

7. $x^2 = 225$

8. $4x^2 - 1 = 48$

9. $x^2 - 3 = 95$

10. $\frac{3}{4}x^2 + 1 = 31$

FACTORING SHORTCUTS 8.1.5

Although most factoring problems can be done with generic rectangles, there are two special factoring patterns that, if recognized, can be done by sight. The two patterns are known as the **Difference of Squares** and **Perfect Square Trinomials**. The general patterns are as follows:

Difference of Squares: $a^2x^2 - b^2y^2 = (ax + by)(ax - by)$
Perfect Square Trinomial: $a^2x^2 + 2abxy + b^2y^2 = (ax + by)^2$

Example 1

Difference of Squares	**Perfect Square Trinomials**
$x^2 - 49 = (x + 7)(x - 7)$	$x^2 - 10x + 25 = (x - 5)^2$
$4x^2 - 25 = (2x - 5)(2x + 5)$	$9x^2 + 12x + 4 = (3x + 2)^2$
$x^2 - 36 = (x + 6)(x - 6)$	$x^2 - 6x + 9 = (x - 3)^2$
$9x^2 - 1 = (3x - 1)(3x + 1)$	$4x^2 + 20x + 25 = (2x + 5)^2$

Example 2

Sometimes removing a common factor reveals one of the special patterns:

$8x^2 - 50y^2 \Rightarrow 2(4x^2 - 25y^2) \Rightarrow 2(2x + 5y)(2x - 5y)$

$12x^2 + 12x + 3 \Rightarrow 3(4x^2 + 4x + 1) \Rightarrow 3(2x + 1)^2$

Name: _____ **Unit 8:** Quadratic Equations

Date: _____ **Bell:** _____ **Homework 14:** Projectile Motion

1. When a cannonball is fired, the equation of its pathway can be modeled by $h = -16t^2 + 128t$.

a) Find the maximum height of the cannonball.

b) Find the time it will take for the cannonball to reach the ground.

2. When Joey dives off a diving board, the equation of his pathway can be modeled by $h = -16t^2 + 15t + 12$.

a) Find Joey's maximum height.

b) Find the time it will take for Joey to reach the water.

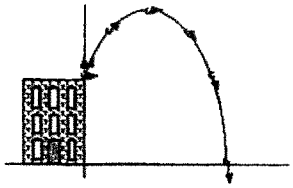
3. A toy rocket is launched from a platform that is 48 feet high. The rocket's height above the ground is modeled by $h = -16t^2 + 32t + 48$.

a) Find the maximum height of the rocket.

b) Find the time it will take for the rocket to reach the ground.

4. At the end of the school year, Rachel and Amber threw their Algebra textbooks off the top of a 12-story building. The equation of the pathway of each book is given below. By how many seconds does Rachel's textbook beat Amber's textbook to the ground?

Rachel: $h = -16t^2 + 36t + 160$ **Amber:** $h = -16t^2 + 50t + 160$



PROJECTILE MOTION *Problems*

1. A soccer ball is kicked from the ground with an initial upward velocity of 90 feet per second. The equation $h = -16t^2 + 90t$ gives the height h of the ball after t seconds.

   a. Find the maximum (vertex) height of the ball.

   $t = \frac{-90}{2(-16)} = \frac{-90}{-32} \approx 2.81$

   $h = -16(2.81)^2 + 90(2.81) \approx \boxed{126.56}$

   b. How many seconds will it take for the ball to reach the ground? ($h = 0$)

   $-16t^2 + 90t = 0$

   $-2t(8t - 45) = 0$

   $t \neq 0$ | $\boxed{t = 5.625}$

1a. 126.56 ft

b. 5.625 sec

2. An apple is launched directly upward at 64 feet per second from an 80-foot tall platform. The equation for this apple's height h at time t seconds after launch is $h = -16t^2 + 64t + 80$.

   a. Find the maximum height of the apple.

   $t = \frac{-64}{2(-16)} = \frac{-64}{-32} = 2$

   $h = -16(2)^2 + 64(2) + 80 = \boxed{144}$

   b. How many seconds will it take for the apple to reach the ground?

   $-16t^2 + 64t + 80 = 0$

   $-16(t^2 - 4t - 5) = 0$

   $-16(t - 5)(t + 1) = 0$

   $-16 \neq 0$ | $\boxed{t = 5}$ | $t \neq -1$

2a. 144 ft

b. 5 sec

3. In science class, the students were asked to create a container to hold an egg. They would then drop this container from a window that is 25 feet above the ground. If the equation of the container's pathway can be modeled the equation $h = -16t^2 + 25$, how long will it take the container to reach the ground?

   a. Find the maximum height of the container.

   $t = \frac{0}{2(-16)} = 0$

   $h = -16(0)^2 + 25 = \boxed{25}$

   b. How many seconds will it take for the container to reach the ground?

   $0 = -16t^2 + 25$

   $16t^2 = 25$

   $\sqrt{t^2} = \sqrt{\frac{25}{16}}$ $\quad t = \pm\frac{5}{4}$ $\quad t = \boxed{1.25}$

3a. 25 ft

b. 1.25 sec

METHOD 3: COMPLETING THE SQUARE

11. $x^2 - 18x + 65 = 0$

12. $x^2 + 16x + 55 = 3$

13. $2x^2 + 9x = x - 14$

14. $9x^2 = 18x + 40$

METHOD 4: QUADRATIC FORMULA

15. $x^2 + 12x - 24 = 0$

16. $-x^2 - 3x + 9 = 0$

17. $-x^2 + 4x = 9$

18. $3x^2 - 9x = x - 5$