

## Dear Family,

The next Unit in your child's mathematics class this year is **What Do You Expect?** Students will gain an understanding of experimental and theoretical probabilities and the relationship between them. The Unit also makes important connections between probability and rational numbers, geometry, statistics, science, and business.

### ▶ Unit Goals

Students will learn to find probabilities by conducting trials and collecting experimental data, and also by analyzing situations to determine theoretical probabilities. Students will be using fractions, decimals, and percents to describe how likely certain events are.

To explore probability, students experiment with coins, number cubes, spinners, and paper cups. They examine simple games of chance to determine whether they are fair. Students analyze basketball free-throw success rates to determine average points per attempt. They also use a tree diagram or organized list to determine which team has a better chance of winning a 7-game series.

### ▶ Homework and Having Conversations About The Mathematics

You can help with homework and encourage sound mathematical habits during this Unit by asking questions such as:

- *What are the possible outcomes that can occur for the events in this situation?*
- *How could I determine the experimental probability of each of the outcomes?*
- *Is it possible to determine the theoretical probability of each of the outcomes?*
- *If so, what are these probabilities?*
- *How can I use probabilities to answer questions or make decisions about this situation?*

You can help your child with his or her work for this Unit in several ways:

- Discuss examples of statements or situations in everyday experiences that relate to the likelihood of certain events. Examples might include weather forecasting, the chances of a baby being a girl, the chances of your favorite college team winning a championship, or the likelihood of winning the lottery.
- Look at sports statistics with your child, and ask questions such as how a batting average or a free-throw average can be used to predict the likelihood of a hit the next time at bat or making two free-throw attempts.
- Look over your child's homework and make sure all questions are answered and that explanations are clear.

In your child's notebook, you can find worked-out examples, notes on the mathematics of the Unit, and descriptions of the vocabulary words.

### ▶ Common Core State Standards

While all of the Standards for Mathematical Practice are developed and used by students throughout the curriculum, particular attention is paid to *constructing viable arguments* and *critiquing the reasoning of others* as students make conjectures about the probability of events and games. *What Do You Expect?* focuses largely on the Statistics and Probability domain, and also includes work from the Ratio and Proportional Relationships domain.

A few important mathematical ideas that your child will learn in *What Do You Expect?* are on the next page. As always, if you have any questions or concerns about this Unit or your child's progress in class, please feel free to call.

Important Concepts	Examples																																								
<p><b>Probability</b> A number from 0 to 1 that describes the likelihood that an event will occur.</p>	<p>If a bag contains a red marble, a white marble, and a blue marble, then the probability of drawing a red marble is 1 out of 3 or <math>\frac{1}{3}</math>. We would write: <math>P(\text{red}) = \frac{1}{3}</math>.</p>																																								
<p><b>Theoretical Probability</b> A probability obtained by analyzing a situation. If all the <b>outcomes</b> (possible results) are equally likely, theoretical probability is the ratio of the number of outcomes you are interested in to the total number of outcomes.</p>	<p>If a number cube has six sides with the possible outcomes of rolling 1, 2, 3, 4, 5, or 6, then the probability of rolling a 3 is 1 out of 6.</p> <p><math>P(\text{Rolling a 3}) =</math>  <math display="block">\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1 \text{ (there is 1 number 3 on the cube)}}{6 \text{ (there are 6 possible outcomes)}}</math></p>																																								
<p><b>Experimental Probability</b> A probability found as a result of an experiment. This probability is the relative frequency of the <b>event</b> (a set of outcomes)—that is, the ratio of the number of times the event occurred compared to the total number of <b>trials</b> (one round of an experiment). Experimental probabilities are used to predict behavior over the long run.</p>	<p>You could find the experimental probability of getting a head (H) when you toss a coin by tossing the coin several times and keeping track of the outcomes. If you tossed a coin 50 times and heads occurred 23 times, the relative frequency of heads would be <math>\frac{23}{50}</math>.</p> <p><math>P(H) = \frac{\text{number of times the event occurred}}{\text{number of trials}} = \frac{\text{number of heads}}{\text{total number of tosses}} = \frac{23}{50}</math></p>																																								
<p><b>Random Events</b> In mathematics, <i>random</i> means that any particular outcome is unpredictable, but the long-term behavior exhibits a pattern.</p>	<p>When you roll a number cube, the number that will result is uncertain on any one particular roll, but over a great many rolls, each number will occur about the same number of times.</p>																																								
<p><b>Strategies for Finding Outcomes</b> When situations involve more than one action, we need to generate the outcomes in a systematic way. An organized list or tree diagram is particularly useful.</p>	<table border="0" style="width: 100%;"> <tr> <td style="text-align: center; vertical-align: top;"> <p><b>Organized List</b></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Coin 1</th> <th>Coin 2</th> <th>Outcome</th> </tr> </thead> <tbody> <tr><td>H</td><td>H</td><td>H-H</td></tr> <tr><td>H</td><td>T</td><td>H-T</td></tr> <tr><td>T</td><td>H</td><td>T-H</td></tr> <tr><td>T</td><td>T</td><td>T-T</td></tr> </tbody> </table> </td> <td style="text-align: center; vertical-align: top;"> <p><b>Tree Diagram</b></p> </td> </tr> </table>	<p><b>Organized List</b></p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Coin 1</th> <th>Coin 2</th> <th>Outcome</th> </tr> </thead> <tbody> <tr><td>H</td><td>H</td><td>H-H</td></tr> <tr><td>H</td><td>T</td><td>H-T</td></tr> <tr><td>T</td><td>H</td><td>T-H</td></tr> <tr><td>T</td><td>T</td><td>T-T</td></tr> </tbody> </table>	Coin 1	Coin 2	Outcome	H	H	H-H	H	T	H-T	T	H	T-H	T	T	T-T	<p><b>Tree Diagram</b></p>																							
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<p><b>Area Model</b> A diagram in which fractions of the area correspond to probabilities in a situation. Area models are particularly helpful when the outcomes being analyzed are not equally likely, and larger areas can represent the more likely outcomes. Area models are also most helpful for outcomes involving more than one stage, such as <i>roll a die</i> and then <i>flip a coin</i>.</p>	<p>If there are three blue blocks and two red blocks in a container and one block is drawn out at a time, without replacing the block drawn each time, the area model at the right shows that the probability of getting two red blocks is <math>\frac{2}{20}</math> or <math>\frac{1}{10}</math>.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td colspan="2"></td> <td colspan="5" style="text-align: center;"><b>First Draw</b></td> </tr> <tr> <td colspan="2"></td> <td></td> <td style="text-align: center;">B</td> <td style="text-align: center;">B</td> <td style="text-align: center;">B</td> <td style="text-align: center;">R</td> <td style="text-align: center;">R</td> </tr> <tr> <td rowspan="4" style="vertical-align: middle;"><b>Second Draw</b></td> <td style="text-align: center;">B</td> <td style="border: 1px solid black;">BB</td> <td style="border: 1px solid black;">BB</td> <td style="border: 1px solid black;">BB</td> <td style="border: 1px solid black;">BR</td> <td style="border: 1px solid black;">BR</td> </tr> <tr> <td style="text-align: center;">B</td> <td style="border: 1px solid black;">BB</td> <td style="border: 1px solid black;">BB</td> <td style="border: 1px solid black;">BB</td> <td style="border: 1px solid black;">BR</td> <td style="border: 1px solid black;">BR</td> </tr> <tr> <td style="text-align: center;"><math>\frac{R}{B}</math></td> <td style="border: 1px solid black;">BR</td> <td style="border: 1px solid black;">BR</td> <td style="border: 1px solid black;">BR</td> <td style="border: 1px solid black;">RB</td> <td style="border: 1px solid black;">RB</td> </tr> <tr> <td style="text-align: center;">R</td> <td style="border: 1px solid black;">BR</td> <td style="border: 1px solid black;">BR</td> <td style="border: 1px solid black;">BR</td> <td style="border: 1px solid black;">RR</td> <td style="border: 1px solid black;">RR</td> </tr> </table>			<b>First Draw</b>								B	B	B	R	R	<b>Second Draw</b>	B	BB	BB	BB	BR	BR	B	BB	BB	BB	BR	BR	$\frac{R}{B}$	BR	BR	BR	RB	RB	R	BR	BR	BR	RR	RR
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<p><b>Expected Value or Long-Term Average</b> The average payoff over many trials.</p>	<p>A game is played with two number cubes. You score 2 points when a sum of 6 is rolled, 1 point for a sum of 3, and 0 points for anything else. If you roll the cubes 36 times, you could expect to get a sum of 6 about five times and a sum of 3 about twice. This means that you could expect to score <math>(5 \times 2) + (2 \times 1) = 12</math> points for 36 rolls, an average of <math>\frac{12}{36} = \frac{1}{3}</math> point per roll. This is the expected value (or long-term average) of one roll.</p>																																								
<p><b>Law of Large Numbers</b> Experimental data gathered over many trials should produce probabilities that are close to the theoretical probabilities.</p>	<p>For 1 million flips, exactly 50% heads is improbable. But for 1 million flips, it would be extremely unlikely for the percent of heads to be less than 49% or more than 51%.</p>																																								