Related Rates

It is possible to use related rates to solve real-world type of problems, such as the following:

Ex 1: Water drains from a rectangular 10X8X6 foot tank at a rate of 2 cubic feet per minute. At what rate is the height of the tank changing?

Ex 2: A pepple is dropped in a pond causing ripples in the form of concentric circles. The radius of the outer ripple is increasing at a rate of 1 foot per second. At what rate is the area of the disturbed water changing?

From the two examples above we can develop the following guidelines in solving a "related rate" problem:

- Determine what is "given" in terms of a rate (almost invariably a derivative w.r.t. time).
- Determine "what is being asked for"... typically a rate that is also a derivative w.r.t. time.
- Usually the resulting equation will have two independent variables. Get rid of one of the variables by:
    - finding an equation that relates the two variables, and
    - then eliminating one of the variables with a substitution from this equation.

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Ex 3: Sand is bring poured onto a conical pile at a rate of 15 ft^3/min. The resulting friction of the sliding grains of sand forces the ratio of the height to raduis to be 4/5. What is the rate at which the height of the pile is changing when the height is 20 ft.?

Example 4:

4. A 12 ft ladder is leaned against a wall. The top of the ladder remains in contact with the wall while the bottom of the ladder is pulled horizontally along the floor with a speed of 1.5 ft/sec. With what speed is the top of the ladder descending when the top is 4 feet above the ground?

Example 5:

5. Two roads cross each other at right angles. A car on the north-south road is 1 mi north of the intersection and is approaching the intersection at 60 mph. Another car traveling 40 mph on the east-west road is 2 mi west of the intersection and headed west. How fast is the distance between the two cars changing?

Example 6: A television camera at ground level is filming the lift-off of a space shuttle that is rising vertically according to the equation $s(t) = 50t^2$, where s is measured in feet and t is measured in seconds. The camera is 2000 feet from the launch pad. Find the rate of change of the elevation of the camera at 10 seconds after lift off.