An Application of the FTC

The way to determine the derivative of an integral.

As a consequence of the **Fundamental Theorem of Calculus:**

If *f* and *g* are continuous on [a, b], then

$\int_a^{g(x)} f(t)dt$ has a derivative everywhere in [a, b], and

$$\frac{d}{dx}\int_a^{g(x)} f(t)dt = f\big(g(x)\big)\, g'(x)$$

Example 1: If $h(x) = \int_4^{5x} \sin(t)\, dt$, find $h'(x)$.

Example 2: Find $\frac{d}{dx}\int_0^{\sec^2 x} \frac{1}{\sqrt{t^2-7}}\,dt$

If *f*, *g*, and *h* are continuous on [a, b], then

$\int_{h(x)}^{g(x)} f(t)dt$ has a derivative everywhere in [a, b], and

$$\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)dt = f\big(g(x)\big)\,g'(x) - f(h(x))\,h'(x)$$

Example 3: If $Q(x) = \int_{2x}^{x^2} \tan(e^t)\, dt$, then find $Q'(x)$.