REPAIR OR REPLACE?
MATH 6TH GRADE

Describe the context of your task here. Separate the parts of the task into A, B, C, etc.

Bob is a machining technician at a local company that manufactures gears and pinions for trucks. His machine cleans the gears that they manufacture for assembling rear axles for big trucks. Bob and his coworkers were becoming frustrated because the cleaning machine kept malfunctioning. When they examined their maintenance records for the past year, they discovered that the most common problems with the machine were attributed to sensors not working properly. They calculated that they had, in the past 7 months, spend $76,000 for replacement sensors.

A. PARTS (SENSORS)

1. Bob wanted to calculate the cost of replacing the sensors in dollars per month for the past 7 months. Use ratios and unit rate to show the cost per month to replace the sensors. Be sure to show your work and explain your answer using ratio language.

2. Assuming the amounts spent per month were equal, construct a table to show how much in dollars was spent after 1 month, 2 months, 3 months, 4 months, 5 months, 6 months, and 7 months. Be sure to write an equation based on your table where x is the month and y is the accumulated amount in dollars spend after each month. You may round to the nearest dollar.

3. If this trend continued, make a prediction of approximately how much money would be spent for the replacement sensors over a one year period. Use ratios and explain your answer in ratio language.

4. The company spent $76,000 for replacement sensors over this 7 month time-frame. Each sensor costs $95. How many sensors did they buy in 7 months? Show and explain your work.

5. Bob calculated that they bought 124 sensors per month. Do you agree or disagree with Bob? Explain.

6. The company's historical data indicated that during the time the machine was torn up, they could have potentially cleaned 23,000 gears in those 7 months. They plan to recover 75% of these gears when the machine is fixed. How many gears do they predict they can recover? Show and explain your work.

B. LABOR (WORKERS PERFORMING JOBS)

1. If the machine had worked properly during these 7 months, they could have potentially made 23,000 gears. Records indicate that Bob and his coworkers can clean 1000 gears per 8 hour working shift. How many 8 hour shifts of workers did not get to complete their jobs?

2. How many total hours of work were lost because of this defective machine? (Hint: 8 hours of work per shift.) Show your work and explain.

3. The total amount paid for labor to these workers to perform other jobs not related to cleaning the gears during the 7 months the machine was down was $7728. Using the above information, calculate the amount paid by the company to the workers per hour. Use ratios and unit rate and explain in ratio language.

C. PARTS AND LABOR COSTS COMBINED

1. Bob researches the cost of buying a new machine. He finds that a new machine would cost $75,000. He concludes that the company would save money over time, if they just bought a new machine and stopped buying replacement sensors. He recommends that the company purchase a new machine. Considering what you found to be the cost of these sensors (the parts) and the cost of the labor (workers completing their jobs), do you agree or disagree with Bob? Show your work and explain why.

2. Consider the total cost of replacement sensors ($76,000) and the cost of the labor paid ($7728) to run this machine for 7 months. If they did buy a new machine, how much money could potentially be saved in 1 year? How much money could be saved over a 3 year time frame? How much money could be saved over a 5 year time frame? Explain how you arrived at your answers.

Common Core State Standards

List the Common Core State Standards (and math practices if applicable) associated with your task.

Common Core State Standards:

Understand ratio concepts and use ratio reasoning to solve problems*.*

1. *Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.*
2. *Understand the concept of a unit rate a/b associated with a ratio a:b and use rate language in the context of a ratio relationship.*
3. *Use ratio and rate reasoning to solve real-world and mathematical problems.*
   a. *Make tables of equivalent ratios relating quantities with whole number measurements and find missing values in the tables. Use tables to compare ratios.*
   b. *Solve unit rate problems.*
   c. *Find a percent of a quantity as a rate per 100.*
   d. *Use ratio reasoning to convert measurement units.*

Compute with multi-digit numbers.

2. *Fluently divide multi-digit numbers using the standard algorithm.*
3. *Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm.*

Apply and extend previous understandings of arithmetic to algebraic expressions.

2. *Write, read and evaluate expressions in which letters stand for numbers.*
   a. *Write expressions that record operations with numbers and with letters standing for numbers.*

6. *Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.*

9. *Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express on quantity, thought of as a dependent variable, in terms of the other quantity, thought of as the independent variable Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.*

Math Practices:

1. *Make sense of problems and persevere in solving them.*
2. *Construct viable arguments and critique the reasoning of others.*
3. *Model with mathematics.*
4. *Use appropriate tools strategically.*
5. *Attend to precision.*
6. *Look for and express regularity in repeated reasoning.*

Essential Understandings

What key insights should students take from participating in this task?

Students should now have a practical application for calculating ratio, unit rate and percentage problems. They should be able to think logically through each process and anticipate a reasonable answer for their problem.

Comparing Quantities: (6.RP.A.1) Two quantities can be compared using addition/subtraction or multiplication/division. Forming a ratio is a way of comparing two quantities multiplicatively. Reasoning with ratios involves attending to and coordinating two quantities.

Unit Rate: (6.RP.A.2) When the ratio of ab is scaled up or down to a/b/1, a/b to 1 is referred to as a unit rate.

Using Models: (6RP.A.3) Real-world relationships involving ratios can be modeled with a number of representations, e.g., diagram, table, graph, or ratio; however, in such representation, both quantities in the relationship must be scaled multiplicatively. A ratio can be scaled up using multiplication because the two quantities vary in such a way that one of them is a constant multiple of the other; a ratio can be scaled down using division, since division by some number, q, is the equivalent of multiplication by the multiplicative inverse of q, 1/q.

Solve Unit Rate Problems: (6.RP.A 3a) Two unit rates are associated with a multiplicative relationship a and b: a/b to 1 and b/a to 1. Each unit rate reveals different information about real-world problems associated with the relationship.

Possible Solutions/Solution Paths

What solutions or solution paths are acceptable in achieving a correct response for this task? Be sure to address all parts of the task.

A. SENSORS (PARTS))

1. *Students could set up two ratios and find the unit rate.*
   Cost/month $76,000/7 = $10,857/1
   Students may also decide to just divide $76,000/7= $10,857

2. *Months(x) Dollars(y)* *EQUATION: 10,857x = y*

1	*$10,857*
2	*$21,714*
3	*$32,571*
4	*$43,428*
5	*$54,285*
6	*$65,142*
7	*$75,999*

3. *Students could set up equivalent ratios, remembering to convert the unit of measurement from one year to 12 months.*
   Cost/months $76,000/7 = c/12 c= 130,285.714 (Round)
   Students may also find the unit rate and multiply by 12.

4. *Multiply: $76,000 / $95 = 800 sensors*

5. *Students should disagree with Bob. To arrive at this conclusion, students should have possibly set up ratios comparing sensors/months. 800/7 = s/1 where s = 114.29. Students may have simply divided 800/7. They should arrive at the answer that the company bought approximately 114 sensors in one month,* ***not*** *the 124 sensors per month that Bob calculated.*

6. *Students could have multiplied 23,000(75%) to arrive at the answer of 17,250 gears. They could have also set up ratios comparing part to whole: x / 23,000 = 75 / 100. Solve the ratio by cross multiplying. Students may have also recognized that 75% is equivalent to ¾ and they could have divided 23,000 by 4 and then multiplied that number by 3 to find 3 out of 4 parts of the total 23,000.*

B. LABOR (WORKERS PERFORMING JOBS)

1. *Students could divide the total 23,000 gears that could have been made In those 7 months by 1000 gears to find the number of shifts that were interrupted. 23 shifts.*

2. *Students should multiply 23 shifts by 8 hours each. 23 x 8 = 184 hours.*

3. *Students could divide the total $7728 by 184 hours to conclude that the company paid the workers for this machine a total of $42 per hour. They could also set up ratios to find the unit rate. Dollars / Hours $7728 / 184 = d / 1. Solve using cross multiplication and dividing.*

C. PARTS AND LABOR COSTS COMBINED

1. *Students should agree with Bob. The cost of buying replacement sensors (parts) for this machine and paying labor for this machine for the last 7 months exceeded the cost of purchasing a new machine.*

   Parts : $76,000
   Labor: + $ 7,728
   _____

   Total: $ 83,728 (for 7 months of operating this machine)

2. *The cost of purchasing the new machine is $75,000. For the first 7 months, the cost is $83,728. Divide $83,728 by 7 to get the unit rate for operating cost per month. Answer: $11,961.14 per month.*
   To find the potential operating cost of this machine for one year, multiply $11,961.14 by 12 = $143,533.68 per year. Students may simply multiply this number ($143,533.68) by 3 = ($430,601.04) then multiply this number ($143,533.68) by 5 = ($717,668.40) to find cost projections for 3 years and 5 years. They might also use ratios comparing Cost /Year. They might also construct a table to show operating costs for years 1 through 5, concluding that an equation to represent the table would be $143,533.68x = y, where x is the year and y is the total cumulative operating costs for each successive year.

Additional Teacher Information
Add any additional notes that will help the teacher execute the task including necessary manipulatives, equipment, etc., and possible students misconceptions that may need to be addressed. *Students need to be aware that the sensors are the parts needed to repair the machine and labor represents the money paid to workers to complete their jobs.*