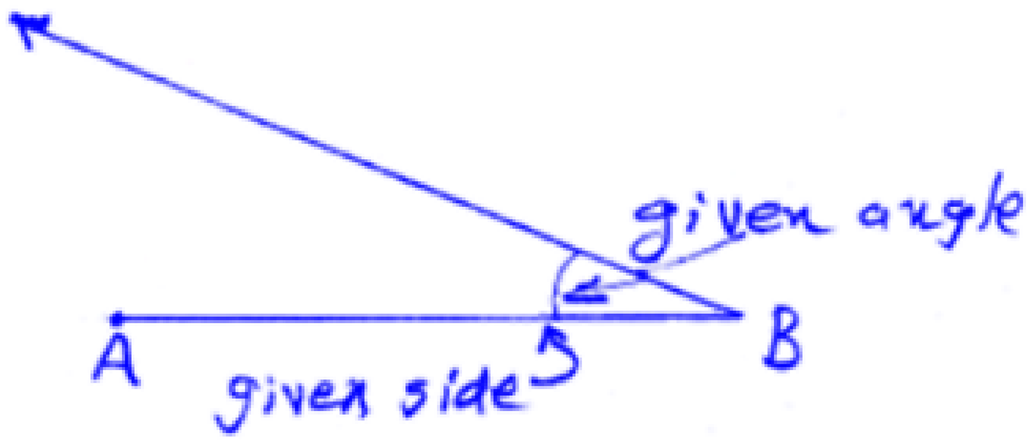
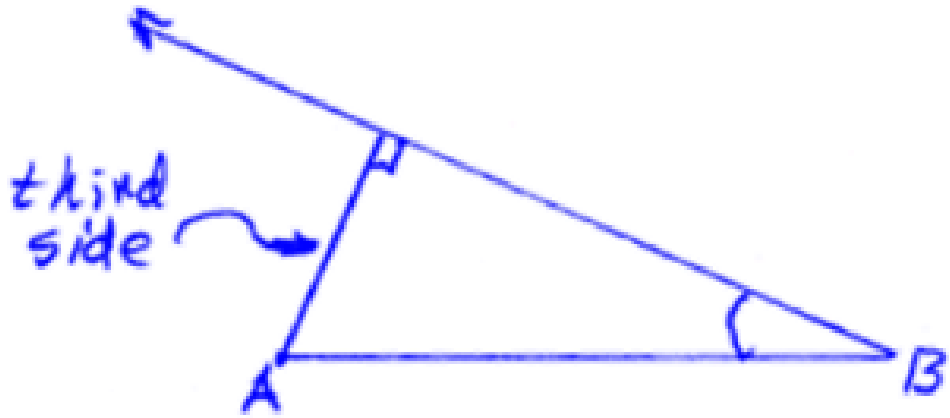
In the previous lesson the astute observer might have noticed that the three pieces of information initially given about a triangle never consisted of **two sides and a non-included angle**.

This is known as the **ambiguous case** since there are three possibilities for the solution.

First, consider being given a side (a line segment) and an angle at one end of that segment as shown here:

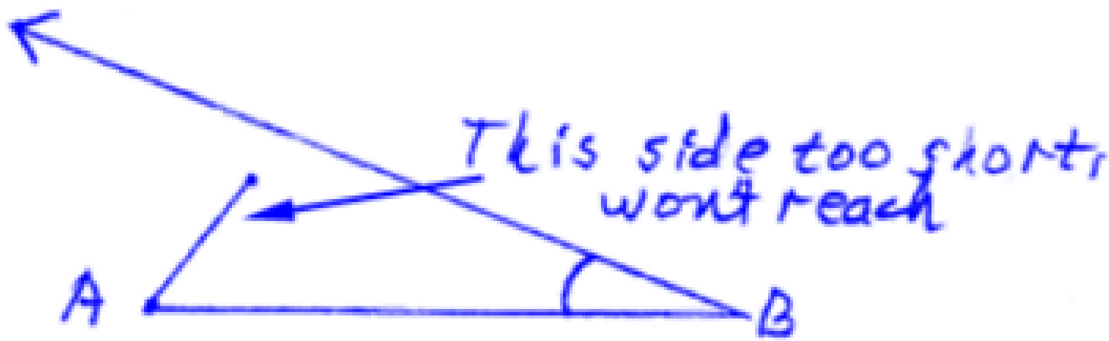


Case 1: Consider a third side that is opposite the angle (**now we have two sides and a non-included angle**) that is just barely long enough to reach the other side of the angle in a **perpendicular** fashion.



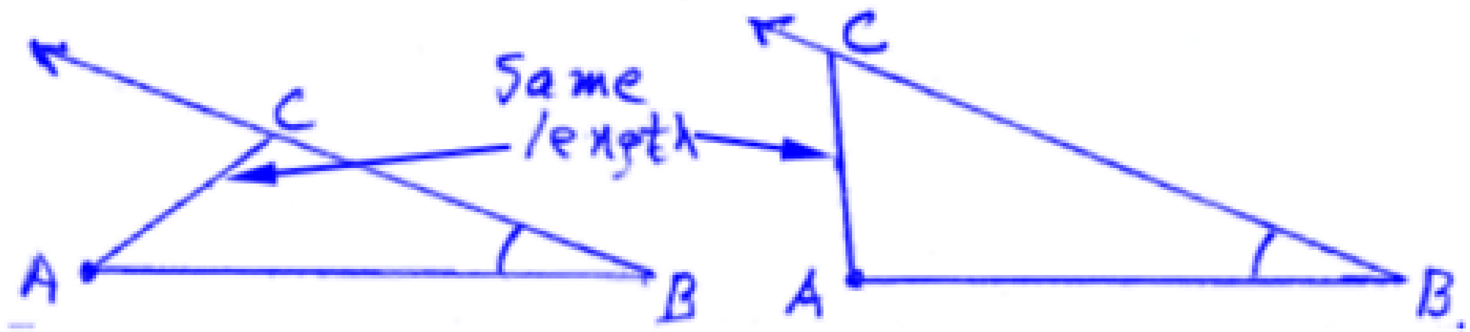
Mathematically, you will be alerted to this when the sine calculation of an angle yields a value of 1 (remember, sin 90° = 1).

Case 2: Consider a third side that is opposite the angle and is **too short** to reach the other side of the angle.



Mathematically, you will be alerted to this unpleasant possibility when the sine or cosine calculation of an angle yields an impossible value (outside the acceptable range, $-1 \le \text{value} \le 1$).

Case 3: Consider a third side (that is opposite the angle) that can actually touch the other side of the angle in **two places**.



Two different triangles are possible, and it must be decided from the physical situation represented by the problem if both solutions are acceptable or if one must be rejected.

Yesterday we discussed how to determine if a triangle had zero, one, or two solutions. We did so with a helpful chart found on page 320.

IF Angle $< 90°$ and $a<b$ then

$a<b \sin A$ NO SOLUTION	$a=b \sin A$ ONE SOLUTION	$a>b \sin A$ TWO SOLUTIONS

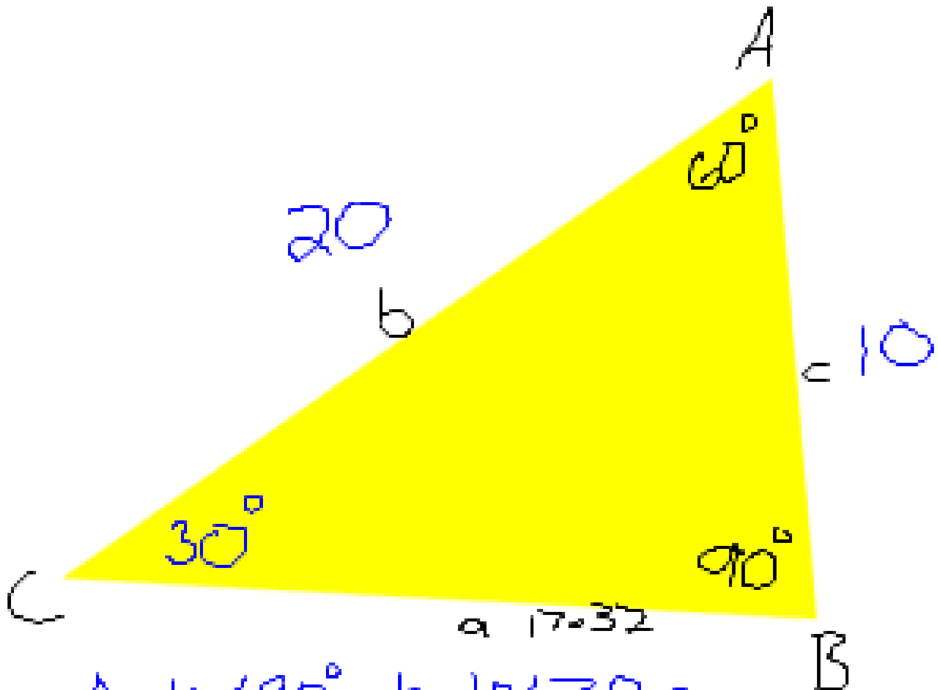
IF Angle $< 90°$ and $a \geq b$ then

ONE SOLUTION

IF Angle $\geq 90°$ then

$a \leq b$ NO SOLUTION	$a>b$ ONE SOLUTION

Example 1: Solve triangle ABC where b = 20, c = 10, C = 30°.



Angle $< 90°$ & $10 < 20$ so

$10 \boxed{=} 20 \sin 30°$

Hence 1 △.

$$\frac{10}{\sin 30°} = \frac{20}{\sin B}$$

$$\sin B = 20/(10/\sin 30°)$$

$$B = \sin^{-1}(20/(10/\sin 30°))$$

$$B = 90°$$

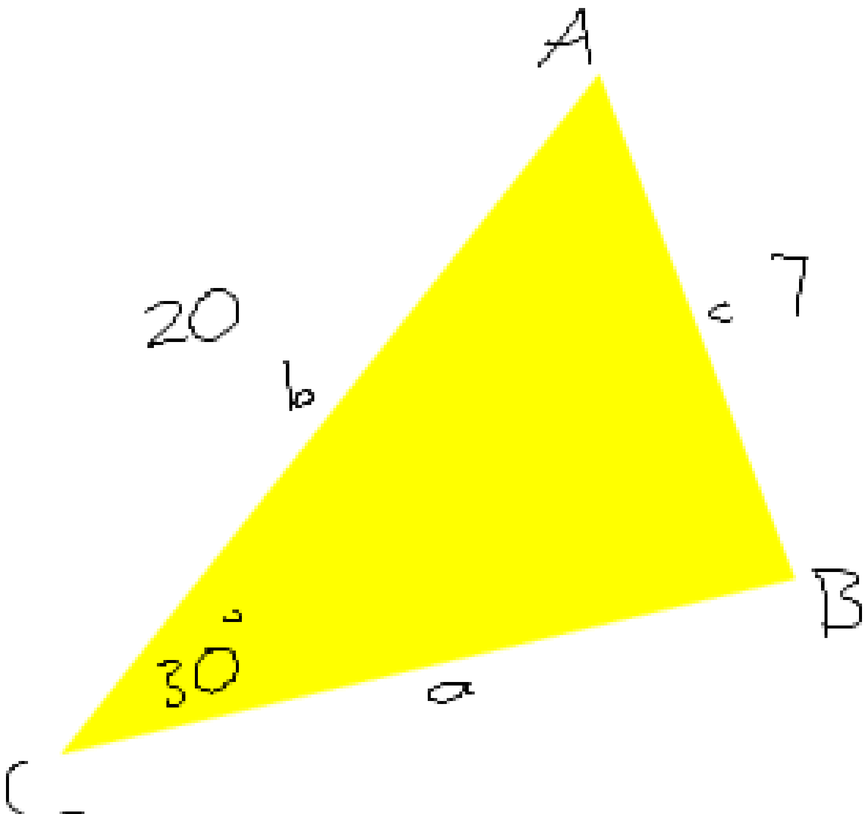
Thus $A = 60°$

$$\frac{10}{\sin 30°} = \frac{a}{\sin 60°}$$

so

$$a = 17.32$$

Example 2: Solve triangle ABC where b = 20, c = 7, C = 30°.

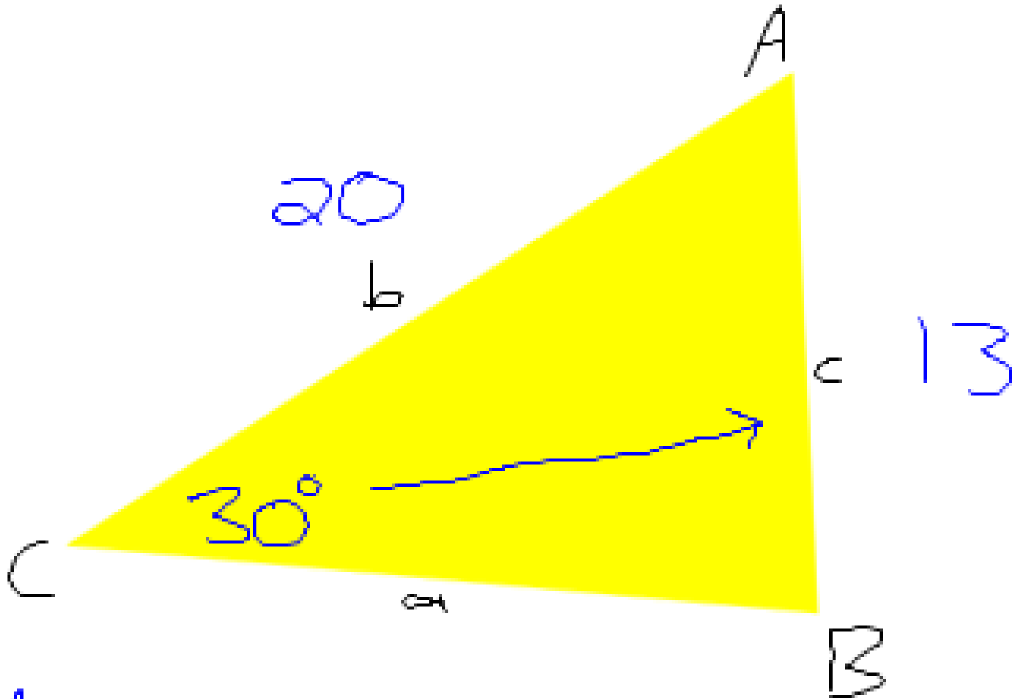


Angle $< 90°$ + $7 < 20$

$7 \boxed{<} 20 \sin 30$

Thus No Δ exists

Example 3: Solve triangle ABC where $b = 20$, $c = 13$, $C = 30^\circ$.

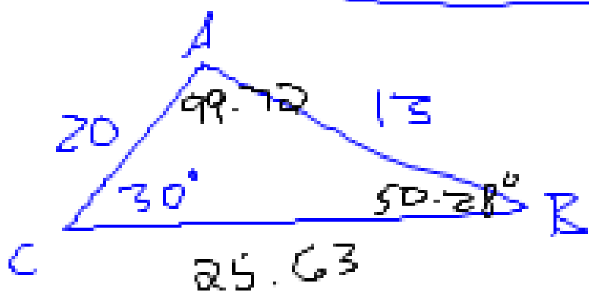


Angle $< 90^\circ$ & $13 < 20$

$13 \boxed{>} 20 \sin 30^\circ$

2 Δ's

Triangle 1



$$\frac{13}{\sin 30^\circ} = \frac{20}{\sin B}$$

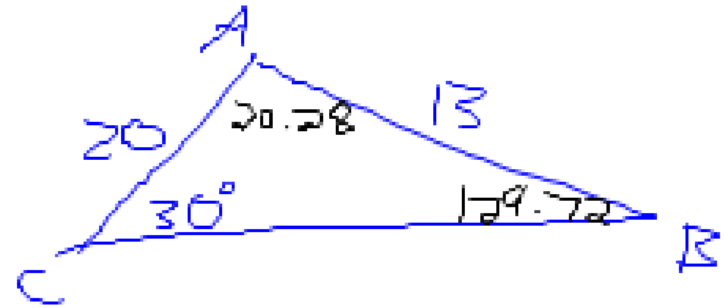
$$\sin B = 20 \Big/ \left(13 / \sin 30^\circ \right)$$

$$B = 50.28^\circ$$

$$\frac{13}{\sin 30^\circ} = \frac{a}{\sin 99.72^\circ}$$

$$a = 25.63$$

Triangle 2

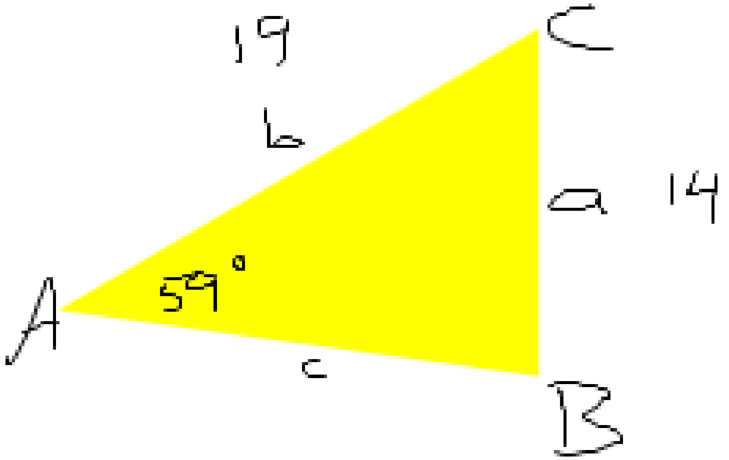


$$B = 180 - 50.28$$
$$= 129.72$$

$$\frac{13}{\sin 30^\circ} = \frac{a}{20.28^\circ}$$

$$a = 9.01$$

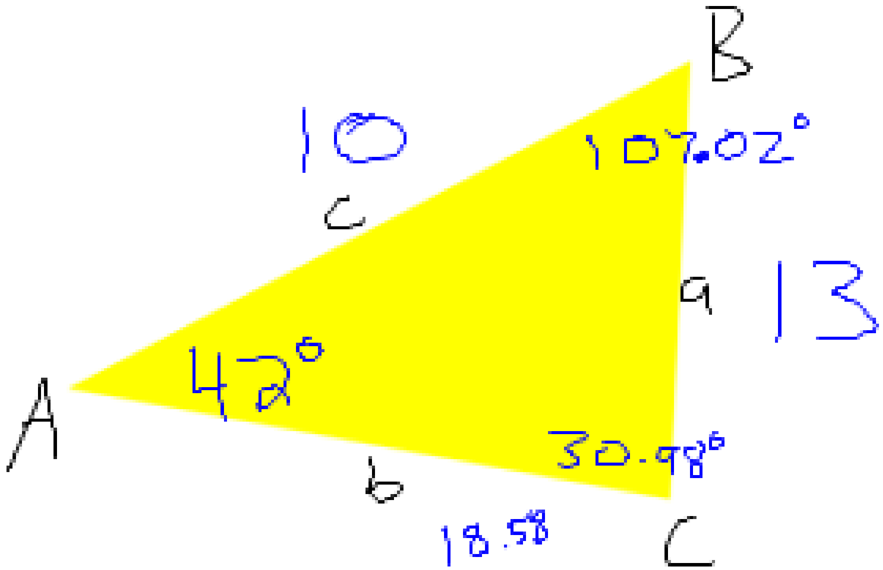
Example 4: A = 59°, a = 14, and b = 19



$14 \boxed{<} 19 \sin 59°$

No sol.

Example 5: A= 42°, a=13, c=10



$$\frac{13}{\sin 42^\circ} = \frac{10}{\sin C}$$

$$\sin C = 10/(13/\sin 42^\circ)$$

$$C = 30.98^\circ$$

1 $\triangle$

$$\frac{13}{\sin 42^\circ} = \frac{b}{\sin 107.02}$$

$$b = 18.58$$

Example 6: $B=39°$, $a=30$, $b=25$

Left to reader