DeSoto
COUNTY SCHOOLS

Algebra 2

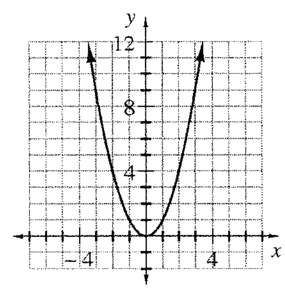
Week 4

TRANSFORMATIONS OF $f(x) = x^2$ 2.1.1 – 2.1.5

In order for the students to be proficient in modeling data or contextual relationships, they must easily recognize and manipulate graphs of various functions. Students investigate the general equation for a family of quadratic functions, discovering ways to shift and change the graphs. Additionally, they learn how to quickly graph a quadratic function when it is written in graphing form. For further information see the Math Notes box in Lesson 2.1.4.

Example 1

The graph of $f(x) = x^2$ is shown at right. For each new function listed below, explain how the new graph would differ from this original graph.

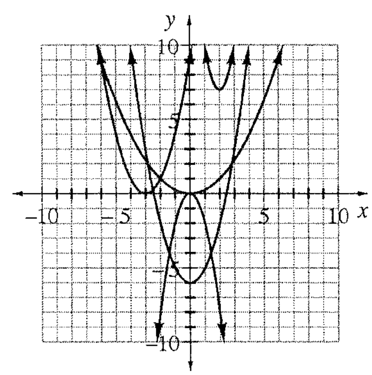


$g(x) = -2x^2$ $\quad$ $h(x) = (x+3)^2$ $\quad$ $j(x) = x^2 - 6$

$k(x) = \frac{1}{4}x^2$ $\quad$ $l(x) = 3(x-2)^2 + 7$

Every function listed above has something in common: they all have 2 as the highest power of x. This means that all of these functions are quadratic functions, and all will form a parabola when graphed. The only differences will be in the direction of opening (up or down), the size (compressed or stretched), and/or the location of the vertex.

The "–2" in $g(x) = -2x^2$ does two things to the parabola. The negative sign changes the parabola's direction so that it will open downward. The "2" stretches the graph making it appear skinnier. The graph of $h(x) = (x + 3)^2$ will have the same shape as $f(x) = x^2$, open upward, and have a new location: it will move to the **left** 3 units. The graph of $j(x) = x^2 - 6$ will also have the same shape as $f(x) = x^2$, open upward and be shifted **down** 6 units.



The function $k(x) = \frac{1}{4}x^2$ does not move, still opens upward, but the $\frac{1}{4}$ will compress the parabola, making it appear "fatter."
The last function, $l(x) = 3(x-2)^2 + 7$, combines all of these changes into one graph. The "3" causes the graph to be skinnier and open upward, the "–2" causes it to shift to the **right** 2 units, and the "+ 7" causes the graph to shift **up** 7 units. All these graphs are shown above. Match the equation with the correct parabola.

Example 2

For each of the quadratic equations below, where is the vertex?

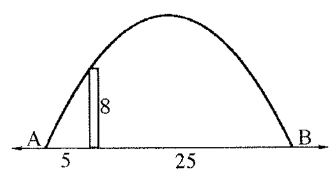
$f(x) = -2(x+4)^2 + 7$ $\qquad$ $g(x) = 5(x-8)^2$ $\qquad$ $h(x) = \frac{3}{5}x^2 - \frac{2}{5}$

For a quadratic equation, the vertex is the **locator point**. It gives you a starting point for graphing the parabola quickly. The vertex for the quadratic equation $f(x) = a(x-h)^2 + k$ is the point (h, k). For $f(x) = -2(x+4)^2 + 7$ the vertex is $(-4, 7)$. Since $g(x) = 5(x-8)^2$ can also be written $g(x) = 5(x-8)^2 + 0$, the vertex is $(8, 0)$. We can rewrite $h(x) = \frac{3}{5}x^2 - \frac{2}{5}$ as $h(x) = \frac{3}{5}(x-0)^2 - \frac{2}{5}$ to see that its vertex is $\left(0, \frac{2}{5}\right)$.

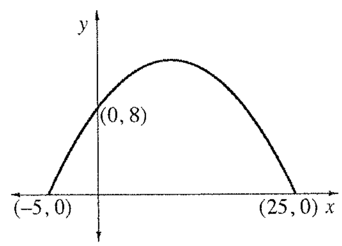
Example 3

In a neighborhood water balloon battle, Dudley has developed a winning strategy. He has his home base situated five feet behind an eight-foot fence. 25 feet away on the other side of the fence is his nemesis' camp. Dudley uses a water balloon launcher, and shoots his balloons so that they just miss the fence and land in his opponent's camp. Write an equation that, when graphed, will model the trajectory (path) of the water balloon.

As with many problems, it is most helpful to first draw a sketch of the situation. The parabola shows the path the balloon will take, starting five feet away from the fence (point A) and landing 25 feet past the fence (point B).



There are different ways to set up axes for this problem, and depending where you put them, your answer might be different. Here, the y-axis will be at the fence. With the axes in place, we label any coordinates we know. This now shows all of the information we have from the problem description. If we can find the coordinates of the vertex (highest point) of this parabola, we will be able to write the equation of it in graphing form. Parabolas are symmetric, therefore the vertex will be half-way between the two x-intercepts. The total distance between points A and B is 30 units, so half is 15. Fifteen units from point A is the point $(10, 0)$. We know the equation will be in the form $y = a(x-10)^2 + k$, with $a < 0$. Also, k must be greater than eight since the vertex is higher than the y-intercept of $(0, 8)$. The parabola passes through the points $(0, 8)$, $(-5, 0)$ and $(25, 0)$. We will use these points in the equation we have so far and see what else we can find.



Using the point $(0, 8)$ we can substitute the x- and y-values in to the equation and write:

$$8 = a(0-10)^2 + k$$
or
$$8 = 100a + k$$

This equation has two variables, which means we need another (different) equation with a and k to be able to solve for them. Using the point $(-5, 0)$:

$$0 = a(-5-10)^2 + k$$
or
$$0 = 225a + k$$

Begin solving by subtracting the second equation from the first:

$$\begin{array}{r} 8 = 100a + k \\ -(0 = 225a + k) \\ \hline 8 = -125a \\ a = -\frac{8}{125} \end{array}$$

Substitute this value for the variable a back into one of the two equations above to find k.

$$8 = 100\left(-\tfrac{8}{125}\right) + k$$
$$8 = -\tfrac{32}{5} + k$$
$$k = \tfrac{72}{5}$$

The equation for the path of a water balloon is $y = -\frac{8}{125}(x-10)^2 + \frac{72}{5}$. You should graph this on your graphing calculator to check.

Problems

Find the x- and y-intercepts of each of the following quadratic equations.

1. $y = x^2 + 4x + 3$
2. $y = x^2 + 5x - 6$
3. $y = 2x^2 - 7x - 4$
4. $y = -3x^2 - 10x + 8$
5. $y = 16x^2 - 25$
6. $y = 6x - 12$

Find the error in each of the following solutions. Then find the correct solution to the problem.

7. Solve for x if $3x^2 + 6x + 1 = 0$.

   $a = 3, b = 6, c = 1$

   $$x = \frac{6 \pm \sqrt{6^2 - 4(3)(1)}}{2(3)} = \frac{6 \pm \sqrt{36-12}}{6} = \frac{6 \pm \sqrt{24}}{6} = \frac{6 \pm 2\sqrt{6}}{6} = \frac{3 \pm \sqrt{6}}{3}$$

8. Solve for x if $-2x^2 + 7x + 5 = 0$

   $a = -2, b = 7, c = 5$

   $$x = \frac{-7 \pm \sqrt{7^2 - 4(-2)(5)}}{2(-2)} = \frac{-7 \pm \sqrt{49-40}}{-4} = \frac{-7 \pm 3}{-4}$$

   $x = \frac{-4}{-4} = 1$ or $x = \frac{-10}{-4} = 2.5$

Find the vertex of each of the following parabolas by "averaging the x-intercepts" or "completing the square." Then write each equation in graphing form, and sketch the graph.

9. $y=-2x^2+4x+1$

10. $y=x^2+10x+19$

11. $y=(x+7)(x-3)$

12. $y=2(x+6)^2-1$

For each situation, write an appropriate equation that will model the situation effectively.

13. When Twinkle Toes Tony kicked a football, it landed 100 feet from where he kicked it. It also reached a height of 125 feet. Write an equation that, when graphed, will model the path of the ball from the moment it was kicked until it first touched the ground.

14. When some software companies develop software, they do it with "planned obsolescence" in mind. This means that they plan on the sale of the software to rise, hit a point of maximum sales, then drop and eventually stop when they release a newer version of the software. Suppose the curve showing the number of sales over time is parabolic and that the company plans on the "life span" of its product to be six months, with maximum sales reaching 1.5 million units. Write an equation that best fits this data.

15. A new skateboarder's ramp just arrived at Bungey's Family Fun Center. A cross-sectional view shows that the shape is parabolic. The sides are 12 feet high and 15 feet apart. Write an equation that, when graphed, will show the cross section of this ramp.

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Date ____________________ Period ______

List the correct information about the graphs of the functions specified by each equation.

	equation	vertex	maximum or minimum	line of symmetry	x-intercept	comparison to $x = y^2$
1.	$y = x^2 + 1$	(0, 1)	minimum	$x = 0$	none	same
2.	$y = 2x^2 + 3$					
3.	$y = \frac{1}{2}x^2 - 2$					
4.	$y = -6x^2 - 3$					
5.	$y = \frac{2}{3}x^2 + 5$					
6.	$y = (x + 3)^2$					
7.	$y = (x - 2)^2$					
8.	$y = -2(x + 1)^2$					
9.	$y = -\frac{2}{3}(x - 4)^2$					
10.	$y = 3(x - 5)^2$					

NAME____ DATE________________________________ CLASS_____

ACT MATH SKILLS PREP – RATIONAL EXPRESSIONS

1. For all $x > 0$, $\frac{2}{x} - \frac{3}{5} = ?$

A. $\frac{10-3x}{5x}$

B. $\frac{5-3x}{5x}$

C. $\frac{2-3x}{x}$

D. $\frac{7}{x}$

E. None of these

2. In the equation $y = \frac{3}{(3+x)}$, x represents a positive integer. As x gets larger without bound, the value of y:

A. gets closer and closer to 0
B. gets closer and closer to 1
C. gets closer and closer to 3
D. remains constant
E. gets larger and larger

3. For all $x > 3$, $\frac{3x - x^2}{x^2 - x - 6} = ?$

A. $\frac{x}{x+2}$

B. $\frac{x}{x-2}$

C. $\frac{3-x}{x+2}$

D. $-\frac{x}{x+2}$

E. $-\frac{x}{x-2}$

4. If $m \neq -n$, what are the real values of x that make the following inequality true?

$$\frac{xm + xn}{2m + 2n} < 0$$

A. $x < -2$

B. $x < -1$

C. $x < -\frac{1}{2}$

D. $x < 0$

E. $x < 2$

5. If x is any number other than 2 and 3, then $\frac{(x-2)(x-3)}{(x-3)(2-x)} = ?$

A. -2

B. -1

C. 2

D. $-\frac{1}{2}$

E. $\frac{1}{2-x}$

6. For all $x \neq 6$, $\frac{x^2 - 36}{(x-6)^2} = ?$

A. $\frac{1}{x-6}$

B. $\frac{x-6}{x+6}$

C. $\frac{1}{x+6}$

D. $x - 6$

E. $\frac{x+6}{x-6}$