# Solving Problems involving Logs/Exps

Log theorems:

- $\log_b(PQ) = \log_b P + \log_b Q$
- $\log_b(P/Q) = \log_b P - \log_b Q$
- $\log_b P^n = n \log_b P$

These theorems must be memorized.

Recall that logarithms and exponentials are inverses of each other.

We use the inverses when solving for x.

Just remember, there can only be at most one log/exp expression on each side of the = sign

Example 1: $\ln(x-3) + \ln(x-1) = \ln(3x+6)$

$$\ln\left((x-3)(x-1)\right) = \ln(3x+6)$$

$$(x-3)(x-1) = 3x+6$$

$$x^2 - 4x + 3 = 3x + 6$$

$$x^2 - 7x - 3 = 0$$

Solve by CTS or QF

Left to Reader

# Example 2: $\log_2(x-2) + 5 = \log_2(x+1)$

$$5 = \log_2(x+1) - \log_2(x-2)$$

$$5 = \log_2\left(\frac{x+1}{x-2}\right)$$

$$2^5 = \frac{x+1}{x-2}$$

$$\frac{32}{1} = \frac{x+1}{x-2}$$

$$32(x-2) = x+1$$

$$32x - 64 = x+1$$

$$31x = 65$$

$$x = 65/31$$

# Example 3: $\log_2(x^2 - 12) - 3 = \log_2(-x + 1)$

$$-3 = \log_2(-x+1) - \log_2(x^2-12)$$

$$-3 = \log_2\left(\frac{-x+1}{x^2-12}\right)$$

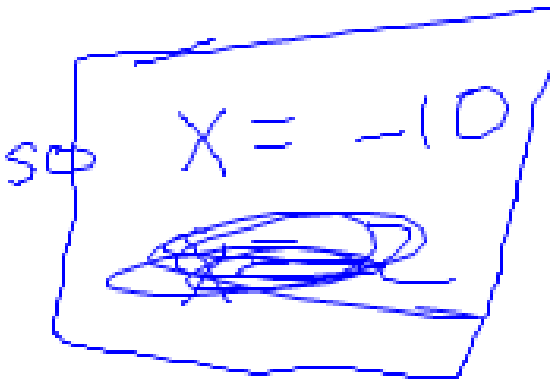
$$2^{-3} = \frac{-x+1}{x^2-12} \quad \text{or} \quad \frac{-x+1}{x^2-12} \times \frac{1}{8}$$

$$x^2 - 12 = -8x + 8$$

$$x^2 + 8x - 20 = 0$$

$$(x+10)(x-2) = 0$$

so $x = -10$

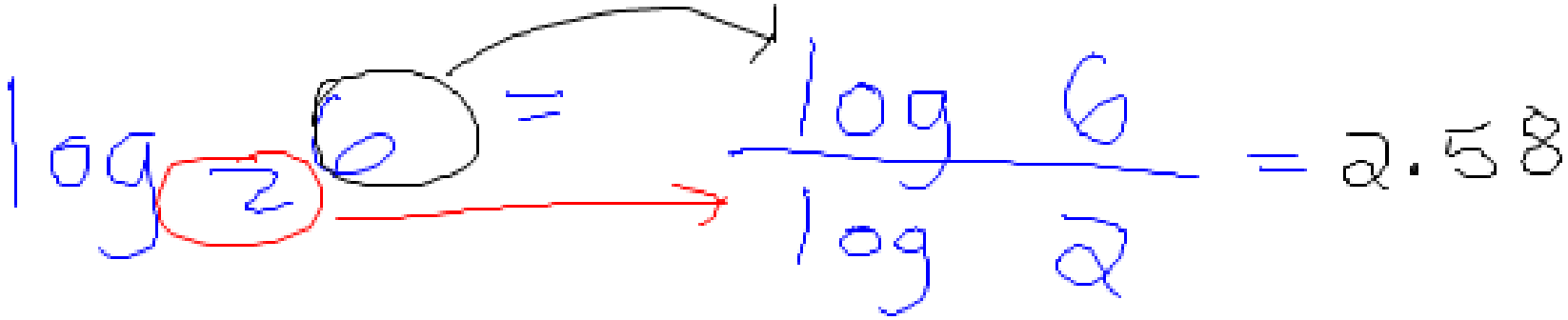


Change of Base Theorem:
$\log_a b$ is equal to $\log b / \log a$

$$\log_3 7 = \frac{\log 7}{\log 3} = 1.77$$

changing to a common log
"base 10"

Example 4: Use change of base to evaluate $\log_2 6$ and $\log_5 20$.




$$\log_5 20 = \frac{\log 20}{\log 5} = 1.86$$

We can also use logs and natural logs (ln) to solve exponential functions.

# Example 5: $12 = 2e^x - 10$

$$\frac{22}{2} = \frac{2e^x}{2}$$

Get $e^x$ by itself

$$11 = e^x$$

$$\ln(11) = \ln e^x$$

Take ln of each side

$$\ln(11) = x$$

$$x \approx 2.397$$

# Example 6: $5^x = 22^{x+1}$

$\ln 5^x = \ln 22^{x+1}$

Take ln of each side

"Bring power down"

$x \cdot \ln 5 = (x+1) \ln 22$

$x \cdot \ln 5 = x \ln 22 + \ln 22$

$x \cdot \ln 5 - x \ln 22 = \ln 22$

$x(\ln 5 - \ln 22) = \ln 22$

$\frac{x(\ln 5 - \ln 22)}{\ln 5 - \ln 22} = \frac{\ln 22}{(\ln 5 - \ln 22)}$

$x \approx -2.086$

# Example 7: $5^x = 3^2$

$$5^x = 9$$

$$\ln 5^x = \ln 9$$

$$x \cdot \ln 5 = \ln 9$$

$$x = \frac{\ln 9}{\ln 5} \approx 1.36$$