NC Math 4 Collaborative Instructional Framework

The following Collaborative Instructional Framework is meant to serve as a guide for teachers and districts as they organize the curriculum for the school year. Unlike traditional pacing guides, the instructional framework consists of clusters of standards that are meant to be adapted to various schools and contexts. The instructional framework used research on students' learning progression in mathematics to create and order clusters of standards that are taught together. While there is a strongly suggested order for teaching the clusters, we recognize that schools differ in their contexts and may wish to switch the order around. In those cases, we have given guidance regarding alternative clusterings; however, we note when certain clusters need to be taught in a certain order.

The Collaborative Instructional Framework was created over a five-month period, beginning in July. Twenty-eight individuals from classroom teachers, district leaders, and university faculty worked together to a) read research about pacing guides, student learning progressions, and standards, b) determine the best clusterings based upon research, when possible, and c) wrote this draft of the framework. The team surveyed educators across the state to view the draft and provide feedback on the instructional framework. As a team, the members read each piece of feedback gathered, discussed whether the feedback aligned with what research says is best for math learning, and determined how to address the feedback to improve the instructional framework. The members of the Fourth Course Framework Team include: Lauren Baucom, Margaret Borden, Chad Broome, Stefanie Buckner, Alicia Conklin, Arren Duggan, Dr. Cyndi Edgington, Charles Hall, Emily Hare, Maria Hernandez, Michael Hoyes, Patrick Kosal, Hema Lalwani, Dr. Katie Mawhinney, Dr. Allison McCulloch, Emily Myers, Christina Pennington, Todd Rackowitz, Martha Ray, Audrea Saunders, Dr. Catherine Schwartz, Gayle Scott, Chase Tuttle, Jennifer Williams, Carmen Wilson, Dr. Travis Weiland, Dr. P. Holt Wilson, and Bill Worley. These mathematics professionals represent the four main regions of NC as well as urban, rural, and charter schools. Special thanks to Joseph Reaper and Lisa Ashe from NC DPI for providing guidance and checking for consistency among the framework and DPI resource documents.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Standards for Mathematical Practice are critical ways of acting and communicating in classrooms that should be instilled in students throughout the school year. Whether students are learning to reason proportionally or statistically, they should be obliged to make sense of the problems posed (SMP1) and create a mathematical solution that can contribute to their peers' and their own learning (SMP3). When solving a problem, such as which company is the cheapest when comparing the prices of t- shirts, students should be able to create a viable argument for their choice, with mathematical evidence to defend their solution (SMP3). Students should be able to move among various representations, reasoning quantitatively with symbols (SMP2), and create models of both every day and mathematical situations they encounter (SMP4). Teachers should provide opportunities for students to reason with a variety of tools (SMP5), including technologies that are specific to mathematics (e.g., calculators, Desmos, GeoGebra, etc.). Attending to precision (SMP6) is a practice in which students attempt to present clear arguments, definitions, and meanings for symbols as they explain their reasoning to others. Finding patterns and structure is crucial throughout the standards as students attempt to mathematize complex problem situations (SMP7). Finally, students should attempt to find regularity in repeated reasoning (SMP8), as with this repetition they are able to generalize their findings from one instance to multiple instances.

NC Math 4 Cluster Sequencing

The clusters are recommended using the progression in the framework, but this is not the only possible progression teachers may use. Please look in the "What is the Mathematics?" and the "Important Considerations" for notes about the purposeful sequencing of the clusters, if another cluster is desired. Also, continue to focus on how the Standards of Mathematical Practice can be incorporated with these content clusters. If your school or district is considering changing the order of the clusters, we suggest keeping most of the sequence with the following two alternative orderings:

Recommended Order	Alternative Order A	Alternative Order B
Building Mathematical Community with Parent Functions & Key Features Functions Unit Logarithmic Functions Unit Trigonometry Unit Exploratory Data Analysis Unit Probability Distributions Unit Statistical Inference Unit ACT Prep Unit	Building Mathematical Community with Parent Functions & Key Features *Logarithmic Functions Unit* *Trigonometry Unit* *Functions Unit* Exploratory Data Analysis Unit Probability Distributions Unit Statistical Inference Unit ACT Prep Unit	Building Mathematical Community with ACT Prep Math Unit Functions Unit *(including Parent Functions & Key Features)* Logarithmic Functions Unit Trigonometry Unit Exploratory Data Analysis Unit Probability Distributions Unit Statistical Inference Unit

The recommended order of the clusters is based on research about the mathematical progressions of learning. In the Functions Unit, the topic of regression is used as a structure (SMP7) for students to utilize to determine goodness-of-fit when given bivariate data. This concept is further developed as students are introduced to Logarithmic Functions and Trigonometric Functions in the subsequent units.

In Alternative Order A, the Functions Cluster serves as a capstone unit for the study of functions across the courses Math 1 - 4. The concept of regression would need to be removed from the Logarithmic and Trigonometry Clusters, and then included within the Functions Unit, if this is the ordering that your school or district chooses.

In Alternative Order B, the ACT Prep cluster (complex numbers, matrices, and vectors) serves as the content for the Building Mathematical Community Unit. This order allows students to be exposed to the mathematics of the ACT prep Cluster at the beginning of the course. It is important to note that the ACT Prep cluster is placed at the end with the following note: *This unit does not have to be taught at the end of the course. These standards can be scattered anywhere it* *works for you to teach them. They do not fit necessarily in any of the other units cohesively, but that does not mean you couldn't throw them in where you have time or want to.* Alternative Order B serves to remind educators of their autonomy in placing the ACT Prep standards where they best fit for their context.

Course Pacing Overview

Unit Name	Duration
Building Mathematical Community with Parent Functions & Key Features	1 Week
Functions Unit (piecewise, composition of functions & regression)	2-3 Weeks
Logarithmic Functions Unit	1-2 weeks
Trigonometry Unit	2-3 Weeks
Exploratory Data Analysis Unit	2-3 weeks
Probability Distributions Unit	2-3 weeks
Statistical Inference Unit	2-3 weeks
ACT Prep Unit (Complex numbers, matrices & vectors)	2-3 days

Unit Name: Building Mathematical Community with Parent Functions & Key Features

Duration: 1 Week

Standards (Content):
It is recommended that the first week of the school year be spent engaging students with open-ended mathematics problems designed to support the students' growth mindset. This first week is also an opportune time for setting up the classroom expectations and norms for collaborating with classmates and participating in whole class discussions. Using this week strategically, this cluster is designed to also review prior knowledge involving Parent Functions & Key Features of functions. This grounds the work of building mathematical community in a content area that will serve both the needs of the students (to review) and the teacher (in terms of pacing).

What is the Mathematics?

- Recognize constant, linear, exponential, quadratic, square root, cubic, and absolute value functions from multiple representations including tables, graphs, function rules, and verbal descriptions.
- Discuss the key features (including domain, range, intervals where the function is increasing, decreasing, positive, or negative, end behavior, minimum and maximum) from Math 1-3 courses.
- Develop mathematicians with positive attitudes about their ability to do mathematics by:
    - Creating opportunities to develop an appreciation for mistakes
    - Seeing mistakes as opportunities to learn
    - Teaching students to take responsibility for their learning
- Develop mathematicians who respect others by:
    - Demonstrating acceptance, appreciation, and curiosity for different ideas and approaches
    - Establishing procedures and norms for productive mathematical discourse
    - Consider other solution paths
- Develop mathematicians with a mindset for problem solving by:
    - Encouraging student authority and autonomy when problem solving
    - Emphasizing questioning, understanding, and reasoning about math, not just doing math for the correct answer
    - Asking follow-up questions when students are both right and wrong
    - Allowing students to engage in productive struggle and moving them along by questioning, not telling

Important Considerations:
For success, significant time should be spent setting up the classroom culture. This includes:

- Developing classroom norms for communication (ex: non-verbal signals, listening and speaking expectations, talk moves for math discussions)
- Developing math routines

- Setting various expectations for the structure of the math block (ex: expectations for whole class instruction, cooperative learning, independent learning, effective integration of technology, etc.)
- Math discourse needs explicit modeling and practice. This includes students:
  - Sharing their thinking
  - Actively listening to the ideas of others
  - Connecting to others' ideas
  - Asking questions to clarify understanding
- Mathematical norms: http://www.youcubed.org/wp-content/uploads/Positive-Classroom-Norms2.pdf

Formative Assessments/Tasks:

- Parent Functions Card Sort http://www.mrseteachesmath.com/2014/12/parent-functions-matching-activity.html
- Parent Function Polygraph: https://teacher.desmos.com/polygraph/custom/560ad6907701c30306330608
- https://www.insidemathematics.org/sites/default/files/materials/sorting%20functions.pdf
- Exponential Marble Slides: https://teacher.desmos.com/activitybuilder/custom/566b317b4e38e1e21a10aafb
- Linear Marble Slides: https://teacher.desmos.com/activitybuilder/custom/566b31734e38e1e21a10aac8
- Quadratic Marble Slides: https://teacher.desmos.com/activitybuilder/custom/566b31784e38e1e21a10aade
- Which One Doesn't Belong: http://wodb.ca/graphs.html Graphs 16, 22, 27
- Key Features of a Function exploration: https://teacher.desmos.com/activitybuilder/custom/564b8b6fa8e7fefa0bad36b7
- See https://www.youcubed.org/ for suggested activities on building classroom community

Unit Name: Functions Unit
Duration: 2-3 Weeks
Standards (Content): **AF.4:** Understand the properties and key features of piecewise functions. **AF.4.1:** Translate between algebraic and graphical representations of piecewise functions (linear, exponential, quadratic, polynomial, square root, rational, radical, logarithmic). **AF.4.2:** Construct a piecewise function to model a contextual situation. **AF.1:** Apply properties of function composition to build new functions from existing functions. **AF.1.1**: Execute algebraic procedures to compose two functions. **AF.1.2:** Execute a procedure to determine the value of a composite function at a given value when the functions are in algebraic, graphical, or tabular representations. **AF.5:** Understand how to model functions with regression. **AF.5.1:** Construct regression models of linear, quadratic, exponential, logarithmic, & sinusoidal functions of bivariate data using technology to model data and solve problems. **AF.5.2:** Compare residuals and residual plots of non-linear models to assess the goodness-of-fit of the model.
What is the Mathematics? ● Students will use functions that they have previously learned in Math 1-3 to model and build piecewise functions. Since they have already seen graphs and equations of piecewise functions in Math 3, students are now translating between algebraic and graphical representations. This means they can be given a graph of a piecewise function and create the function rule. This is an opportunity to build on the discussion of parent functions done in the previous unit. Students will continue discussing key features and can evaluate the piecewise function here. As students looked at domain and range in the first unit, the development of piecewise will emphasize the importance of domain. ● Students will also use these piecewise functions to model real-world situations. Students could look at financial situations (budgets, tax brackets, pricing, salaries, commission, parking rates, etc), temperatures, velocities, growth charts, roller coasters, etc. ● Students will use the knowledge of evaluating functions at a specific value to extend into composition of functions. They can build on the discussion of function families to compose and get new functions. ● The standard as written does not intend for students to use composition to assess inverse functions. This could become an extension for honors students if there is time available since they have discussed inverse functions in Math 3. ● Students will use regression to model bivariate data. As a statistical thinker, students need to

understand part of the process when exploring data by first trying to get a look at what the data is doing. They will create a scatterplot of the data and try to describe the overall trend they are seeing. Then decide what types of functions might be appropriate to model the data. THEN, they will try to fit a model. This might be done on a graphing calculator or through other statistical technology. After students fit a model, they must decide how appropriate that model is for the data. It is extremely important for students to realize in the statistical process that this is part of explaining why and convincing others that their model is a good fit. In addition to the original scatterplot and the mathematical context, the residuals and residual plots are further evidence students will be using to justify that they have found the best fit model.

- Students will use residuals and residual plots because **r and r^2 do not apply** as measures of goodness-of-fit to all functions.
  - Video - R^2 explained: https://www.youtube.com/watch?v=IMjrEeeDB-Y

Important Considerations:

- This unit provides students an opportunity to review functions in terms of piecewise-defined functions. An approach to rewriting these functions might include the use of parent functions and function transformations that they talked about extensively in Math 1-3.
- This unit provides opportunities to bring in financial literacy components (tax brackets, compound interest on investments, debt & savings, income calculation, hourly wages, payroll deductions, budgeting).
- For this unit, the recommended functions to use for piecewise, composition, and regression are linear, exponential, quadratic, polynomial, and square root functions.
  - Note: The use of step functions in piecewise could also be an honors extension if desired.
- Unit Progression: There is always flexibility in a course structure. Below addresses two ways that this unit could be placed in this course:
  - In the current progression with this unit serving as Unit 2, students will expand on the ideas from Unit 1 with parent functions and continue discussing key features & characteristics while also using them to build new functions such as piecewise and composition. When introducing regression, this will be a review of what was previously taught in Math 1. Students will use the functions being addressed in this unit to explore appropriate regression models. This topic will continue to be spiraled throughout the course as students explore logarithmic and trigonometric functions later.
  - An alternate placement of this unit could be after students have explored logarithmic and trigonometric functions. Teachers can use this unit as a capstone for all functions learned in this course and in previous courses. If choosing this pathway, teachers should realize that it is not an expectation that students use logarithmic or trigonometric functions in standards **AF.4** and **AF.1**.

- Regardless of where this unit is placed, the length of time allotted for this unit is because it is the capstone of students' study of functions discussed in NCMath 1-3. The length allows time for a deeper understanding of function types and how they can be applied to piecewise functions & composition.

Formative Assessments/Tasks:
Piecewise Functions Exploration:
https://teacher.desmos.com/activitybuilder/custom/5bf2ee0cb5573d0c04696261

Choosing a Regression Model: **http://bit.ly/regression_models**

Unit Name: Logarithmic Functions Unit

Duration: 1-2 Weeks

Standards (Content):

AF.3: Apply the properties and key features of logarithmic functions.

- **AF.3.1:** Execute properties of logarithms to simplify and solve equations algebraically.
- **AF.3.2:** Implement properties of logarithms to solve equations in contextual situations.
- **AF.3.3:** Interpret key features of a logarithmic function using multiple representations.

AF.5: Understand how to model functions with regression.

- **AF.5.1:** Construct regression models of linear, quadratic, exponential, **logarithmic**, & sinusoidal functions of bivariate data using technology to model data and solve problems.

What is the Mathematics?

- In Math 3, students solved exponential equations algebraically by converting them to logarithmic form, but they did not use properties of logarithms. They have also not seen natural logarithms or exponential equations of base *e*. In this course, students will develop the properties of logarithms, use those properties (of any base, including *e*) to solve logarithmic equations algebraically, and analyze the graph and key features of logarithmic functions.
  - "Students are **not** expected to know or use the properties of logarithms, *e*, or natural logs to solve problems. These can be extension topics, but are beyond the scope of the NC Math 3 standards." NC.M3-F.LE.4
- Students will use rules of exponents to derive the properties of logs.
- After deriving the properties of logs, students will use those properties to solve equations. Students will also be expected to solve equations in the context of real-world problems.
- Students will need to understand and interpret the key features of log functions (intercepts, end behavior, domain, range, and intervals where the function is increasing and decreasing).
  - Continuing the ideas in Units 1 & 2.
- Students will use regression to model bivariate data. Just as in the Functions Unit, students will need to determine which type of function best fits the data by using scatter plots, residuals, and residual plots. This can be done using a graphing calculator or other statistical technology. Students should be able to justify why they have found the best fit model.
  - See example below in the Tasks section.

Important Considerations:

- AF.3.1 Make the conceptual connection between exponential rules and properties of logs
- AF.3.3 Use various representations (tables and graphs) to develop the idea of inverses and key features. Trying to develop the concept that for y = log (x), it means y is the power of 10 that you need to find x. *See the Brief for Math 3 Unit 2 Exponential & Logarithmic Functions.*

- Since the first known instance of the slide rule in 1622, changing bases between logarithms to solve problems has been considered important procedural knowledge for math learners to master. In recent years, graphing technology has added the ability to complete change of base between logarithms without knowing this procedure. The use of technology now allows students to use a logarithm with any base as the TI-84 has a logBASE option and Desmos allows you to type any base subscript. We do not negate that it is important for students to understand the conceptual knowledge behind how graphing technology is programmed to complete this procedure, but do suggest that the change of base property is not as important as it once was and the other properties of logarithms are a more relevant focus. For instance, students will need to understand how to condense logarithms into one log in order to solve equations, but would still be able to solve exponential equations without needing change of base.
    - For example, students need condensing properties to solve $log_2(x) + log_2(x-2) = 3$ Students do not need change of base to solve $3^{x-5} = 7$because they can convert to logarithmic form as in Math 3 or can take log_3 of both sides.
- Please include **natural log** in problems that discuss logarithms and context of logs.
- This Unit could be a time to bring in those financial literacy components (tax brackets, compound interest on investments, debt and savings, income calculation, hourly wages, payroll deductions, budgeting)

Formative Assessments/Tasks:
MVP Secondary Math 3 Module 2 Logarithmic Functions (Honors) covers much of the content of this unit

Logarithmic Regression Example: http://bit.ly/Log-Regression

Unit Name: Trigonometry Unit

Duration: 2-3 Weeks

Standards (Content):

AF.2: Apply properties of trigonometry to solve problems involving all types of triangles.

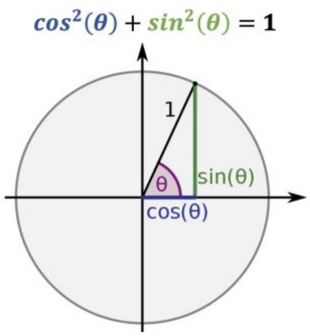
- **AF.2.1:** Translate trigonometric expressions using the reciprocal and Pythagorean identities.
- **AF.2.2:** Implement Law of Sines and Law of Cosines to solve problems.
- **AF.2.3:** Interpret key features (amplitude, period, phase shift, vertical shifts, midline, domain, range) of models using sine and cosine functions in terms of a context.

AF.5: Understand how to model functions with regression.

- **AF.5.1:** Construct regression models of linear, quadratic, exponential, logarithmic, & **sinusoidal** functions of bivariate data using technology to model data and solve problems.

What is the Mathematics?

- In Math 2, students worked with right triangle trig ratios. It is important that teachers take the time to develop the reciprocal and Pythagorean identities in this course. For AF.2.1 students will make connections to right triangle trig to develop the reciprocal identities (csc, sec, cot) as well as to develop Pythagorean Identities from the basis in Math 3 where they explored (cosx, sinx)



Note: Connect to prior knowledge of NC.M3.F-TF.2a-b. Remember at this level we ***do not expect students to memorize*** *the special coordinates of Unit Circle but rather we are using the understanding that the x-coordinate is the cosine value and y-coordinate is the sine value.*

- Students will extend their thinking of right triangle trig to establish relationships between the angles and side lengths for any triangle. Take the time here to facilitate an investigation of the law of sines and cosines to build conceptual understanding.
- In NC.M3.F-TF.5, students worked with key features (amplitude, period, vertical shifts, midline, domain, range) of sine functions in terms of a context. AF2.3 extends this learning to examine phase shifts and the cosine function. In this standard, students are interpreting the key features from graphs, tables, equations, and context. The context of this work is very

important and this is a good space to allow students to have fun with the mathematics in real-world applications.

- Students will use regression to model bivariate data. Just as in the Functions Unit, students will need to explore data and determine which function best fits the model of a scatter plot. This might be done on a graphing calculator or through other statistical technology. The residuals and residual plots ARE the evidence students will be using to prove they have found the best fit model. While this is the first point you have done regression for sinusoidal functions, this is the perfect place to spiral functions in from previous units.

Important Considerations:

- For trigonometric identities, going beyond the Pythagorean and reciprocal trig identities is not in the scope of this course. These trig identities are not meant to be complex multi-step precalculus or calculus level simplifying expressions problems. Instead the focus is on giving students opportunities to be exposed to the pythagorean and reciprocal identities and also set them up for possibly taking Precalculus in the future. When this is being taught teachers should also connect back to students previously learned concepts and right triangle trigonometry and the Pythagorean theorem.

Formative Assessments/Tasks:

Match My Trig Graph:
https://teacher.desmos.com/activitybuilder/custom/56a26aada55a6568199d1c5c

MVP Secondary Math Trig Functions, Equations, and Identities

Unit Name: Exploratory Data Analysis Unit

Duration: 2-3 Weeks

Standards (Content):

SP.1: Create statistical investigations to make sense of real-world phenomenon.

SP.1.1: Construct statistical questions to guide explorations of data in context.

[For example, a question that: anticipates variability, is answerable with data, states the population under consideration, states the attribute under investigation, and is clear enough to guide the analysis of the data]

SP.1.2: Design sample surveys and comparative experiments using sampling methods and to collect and analyze data to answer a statistical question.

SP.1.3: Organize large datasets of real world contexts (i.e. datasets that include 3 or more measures and have sample sizes >200) using technology (i.e. spreadsheets, dynamic data analysis tools) to determine: types of variables that are in the data set, possible outcomes for each variable, statistical questions that could be asked of the data, and types of numerical and graphical summaries could be used to make sense of the data.

SP.1.4: Interpret non-standard data visualizations from the media or scientific papers to make sense of real-world phenomenon.

[This standard is not referring to traditional graphs such as a histogram, dot plot, boxplot, bar graph, pie chart, or stem and leaf plot]

What is the Mathematics?

- When teaching statistics it is important to realize that statistics is generally considered a distinct, but heavily overlapping discipline, with mathematics. With these differences come some important considerations in teaching statistics compared to teaching mathematics. To begin, context is central to the practice of statistics. As a result it is important that all work in the statistics units is situated in a context and ideally ones that are real and relevant to your students. Another big difference is the omnipresence of variability. Statistics involves studying data that vary meaning that it is important to consider variability in the practice of statistics, which comes from many sources including measurement, natural, induced, and sampling. This means in teaching statistics it is important to always describe the variability in a distribution using both numerical summaries such as standard deviation, Interquartile range, and the five-number summary, as well as graphical summaries that help you to see the variability in the data and look for patterns in that variability. Another difference that falls out of the omnipresence of variability is the consideration of uncertainty. In terms of teaching, this means that the language we use in statistics is much less deterministic, normative or certain. In other words, we do not talk about proof so much as creating well-reasoned arguments with data-based evidence. The measurement of uncertainty is also where the field of probability comes into play which is mathematical in nature.
- Core to teaching statistics is that because statistics is a methodological discipline it should be taught as an investigative process that includes formulating statistical questions, collecting data

or finding data that can be used to answer your questions, analyzing data to make sense of the data related to a question posed, and finally interpreting the data and data analysis in context to answer a statistical question. One of the main goals of this unit is to emphasize the statistical investigation cycle and build upon and make connections between the statistics concepts that students have learned in their previous mathematics coursework. Exploring real data and asking and answering statistical questions should be the drive of each lesson in this unit.

- In relation to statistical questions in the investigative cycle, In middle school (6.SP.1), students learn how to identify statistical questions as well as determine whether or not a question is a statistical question. Statistical questions are questions that allow for data collection and will have variability in the data. Issues that may arise in a question include vagueness, biased tones, wordiness, confusing answer choices, unreasonable or unrealistic responses, or deterministic responses. Additionally, in Math 3, students are asked *what statistical question could you answer using the data*. In this class, students will be expected to construct their own statistical questions, in which they identify their population of interest and the aspect of that population that they wish to study. This is one of the foundations of statistics. When creating statistical questions researchers suggest using the following criterion:
    - The variable(s) of interest is/are clear and available.
    - The population of interest is clear.
    - The intent is clear.
    - The question can be answered with the data.
    - The question is one that is worth investigating – is interesting, has a purpose.
    - The question allows for analysis to be made of the whole group. (Arnold, 2013, p.111)

  When having students create their own statistical questions it is important to give them opportunities to share with other students and have them evaluate the questions based on the criterion, which connects to SMP3.
- In relation to collecting data in the statistical investigation cycle a foundational pillar of statistics is a good experimental or sampling design for data collection. An important consideration in data collection is what is the population under consideration as well as how can a representative sample be drawn from that population such that statistical inference is possible. An important note here is that collecting data does not necessarily mean collecting raw data every class period. Collecting data also involves looking at data that has already been collected and evaluating how it could be used to address a statistical question posed. In teaching data collection it is ideal to provide examples in contexts students are familiar with such as considering their school as a population and ways of sampling from that population such as stratified random samples such that grade levels are strata, or perhaps cluster sampling where classes during a particular period are clusters that are randomly selected. In collecting a representative sample it is ideal to have a randomized sample to reduce bias and make statistical inferences. To highlight this to students it is useful to have them collect data in non-random ways to then see the bias in data collected and then show examples of how to collect data randomly. There are multiple ways to collect a randomized sample: simple random sampling, stratified random sampling, cluster random sampling, and systematic random sampling. There are also non-randomized sampling methods that should be avoided when possible, due to bias, but are necessary for some studies: convenience sampling, volunteer

response bias, and non-response bias.

- Simple Random Sampling: A simple random sample is a sample that is taken by randomly selecting individuals from a list that represents the population (university listserv, for example). This can be done by pulling names out of a hat, flipping a coin, using a random number generator, or using a random digit table. It is important to note that individuals cannot be repeated in a sample.
- Stratified Random Sampling: A stratified random sample is a sample that is taken by randomly selecting individuals from lists that represent strata. A strata is a group of individuals that has something in common with each other and different from every other strata. For instance, you could stratify a sample on grade level (freshman, sophomore, junior, or senior), where each individual in the freshman group is the same age and different ages than the other groups, and then you would randomly select individuals from each group. The idea here is that the sample needs to include members from each group, so you collect this type of sample to ensure that all members are represented.
- Cluster Random Sampling: A cluster random sample is a sample that is taken by randomly selecting clusters of individuals that represent the whole and (usually) surveying all of the individuals in that cluster. In this case, the groups (clusters) should be similar in make-up from cluster to cluster, and the individuals in the cluster should represent the population. An example of this would be to randomly select homerooms in a school and survey all of the individuals in the room.
- Systematic Random Sampling: A systematic random sample is a sample taken by randomly selecting a starting point on a list of all of the individuals in the population, and then taking individuals that are every nth number of individuals away from each other. For instance, a systematic random sample of published phone numbers may come from randomly selecting a starting point in the phone book and then using every tenth phone number. This type of sampling is generally being abandoned by statisticians but is still an efficient way of collecting a random sample.
- Non-randomized sampling methods that should be **avoided,** due to bias:
  - Convenience Sampling: A convenience sample is a sample taken by surveying individuals that are conveniently located. An example of this would be standing outside of a store in a shopping center and asking everyone that walks by. This type of sampling would most likely result in undercoverage bias because this is not guaranteed to capture a representative sample of the population of interest. For instance, you might miss all of the people who do all of their shopping online or don’t go shopping.
  - Voluntary Response Bias: Volunteer response bias results from placing a survey in a location where individuals must volunteer to respond. This includes asking people to call in on a radio, asking them to go online and take the survey, asking them to send an email or leave a review. This type of sample relies on individuals actively working to respond to a survey, so it risks only having extreme responses.
  - Nonresponse Bias: This occurs when an attempt at a census (surveying every

individual in the population) is done, and individuals who don't respond are part of a common group that doesn't get represented in the sample. An example of when this might occur is sending out an email blast that people need to respond to. Potentially, people who are very busy will not be represented because they will never have the time to respond, among other groups who might not respond.

  - Randomized Experimental Design requires both the randomized sampling from above AND random assignment of treatments and placebos. This might look like: "For this experiment, a simple random sample was used to select 100 individuals, then the treatment was randomly assigned to fifty individuals and the placebo assigned to the rest."
  - Other considerations in experimental design would be to design single-blind and double-blind situations, where either the individual is unaware of which treatment they are receiving, the researcher is unaware of which treatment each individual received, or both (double-blind).
- In terms of analyzing data most analysis should be done with the support of technology, which relates to SMP5. Most studies in this day and age collect large amounts of data, sometimes multiple years' worth of data values (see important considerations section for links to large data set archives), and then use programs to run statistical analyses. In this unit, students should learn how to input a large data set into one of these programs, such as CODAP, Tuva, Excel, or Desmos, and then have those technologies produce the numerical summaries and graphical displays (which have been covered heavily in grades 6-12) that students have deemed appropriate for this data set.
  - When discussing the data set, students should be able to determine if a variable is quantitative (numerical, where an average would make sense) or categorical (qualitative, often in the form of words, but could be something like jersey numbers or zip codes, where an average would not make sense).
  - Then, they should further be able to determine if a quantitative variable is discrete or continuous and if a categorical variable is ordinal (ordered, such as rate it 1-5, with 1 at the lowest and 5 is the highest) or non-ordinal, and which numerical summaries and graphical displays would be reasonable to use to describe each type of variable data.
  - Numerical summaries for quantitative variables include but are not limited to mean, median, minimum, maximum, range, quartile 1, quartile 3, interquartile range (quartile 3-quartile 1), and standard deviation. Numerical summaries for categorical variables include but are not limited to frequencies (counts of individuals in each category), and relative frequencies (proportions of individuals in each category). .
  - Graphical displays include but are not are not limited to box plots, histograms, bar charts, pie charts (not a statistical favorite), segmented bar charts, scatterplots, frequency plots, dot plots, etc.
  - This is also an opportunity for students to practice constructing appropriate statistical questions, but this time they would do so using the large data set, rather than designing their own data collection methods.
- In terms of interpreting data it is important that all interpretations are based on specific

evidence from the data. Furthermore it is important to not overgeneralize!! At best, surveys and observational studies allow you to observe some relationships and possible differences that are occurring in the data. Experiments attempt to pinpoint specific relationships by controlling other factors that might affect the relationship, and potentially could allow for conversation about causation, but a good statistician will never state cause and effect because even the best experiments have the potential to miss an important feature affecting the relationship. In this unit all of the generalizations and conclusions should be descriptive and not go beyond the sample collected.

- In the media and workplaces, many different visual displays are used to describe data, and are often different from the standard representations that have been discussed previously. Students need to be able to analyze these different displays, based on the scales and descriptions that are provided on the image. In the important considerations section, there is a link to non-standard visual displays that have been used in the media recently, but feel free to explore your local newspapers and social media to find non-standard graphs and images to analyze.
- For further reading on the teaching of statistics we recommend you consult the Guidelines for Assessment and Instruction in Statistics Education from the American Statistical Association (https://www.amstat.org/asa/files/pdfs/GAISE/GAISEPreK-12_Full.pdf). Also there are good resources on the NC2ML webpage including research briefs that would be good resources to consider.

Important Considerations:

- Consider that some of these standards are not intended to be a standalone day of instruction. For example, S.1.4 would be better done by incorporating it throughout the unit and having classroom discussions on these nontraditional data visualizations.
- Teachers do have access to statistical technology. Some suggestions are CODAP (free and online), Tuva (partially free and online), Excel, and Desmos (free and online). A neat option for technology-based schools is R: because this is a true technology platform used in the field of statistics and it's free and would fit well in conjunction with teaching computer sciences or computational thinking.
- **SP.1** is intended to ensure that, throughout every piece of this unit, teachers are helping students explicitly make connections to the other pieces of the statistical analysis cycle that they are doing. To learn more about this, go to the k-12 GAISE report (https://www.amstat.org/asa/files/pdfs/GAISE/GAISEPreK-12_Full.pdf) and read the first chapter (pages 11-21). You can also read NC2ML's research briefs for Statistical Reasoning and Literacy (https://www.nc2ml.org/wp-content/uploads/2019/01/BRIEF-67-stats-MS.pdf) and Math 1-3
    - Math 1: https://www.nc2ml.org/wp-content/uploads/2018/10/BRIEF-6-V2-1.pdf
    - Math 2: https://www.nc2ml.org/wp-content/uploads/2018/10/BRIEF-12-V2.pdf
    - Math 3: https://www.nc2ml.org/wp-content/uploads/2018/03/BRIEF-20.pdf)

Resources (Open Access):

- Amazing list of teaching statistics resources updated frequently https://www.amstat.org/asa/files/pdfs/EDU-CommonCoreResources.pdf
- Webinars on teaching statistics https://www.amstat.org/asa/education/K-12-Statistics-Education-Webinars.aspx
- Real-life graphical and numerical displays for S.1.4
  - https://www.nytimes.com/column/whats-going-on-in-this-graph
  - https://www.amstat.org/ASA/Whats-Going-on-in-this-Graph.aspx
  - https://www.pollingreport.com/
  - https://www.gapminder.org/
  - https://fivethirtyeight.com/
- Large data sets:
  - https://www.ncdc.noaa.gov/data-access
  - https://www.dataquest.io/blog/free-datasets-for-projects/
  - https://github.com/awesomedata/awesome-public-datasets
  - https://ww2.amstat.org/censusatschool/
  - https://www.gapminder.org/
  - https://www.census.gov/data.html
  - https://collegescorecard.ed.gov/data/
  - https://www.cia.gov/library/publications/the-world-factbook/
  - https://www.ipums.org/
  - https://fivethirtyeight.com/
  - https://www.pewresearch.org/
- Lesson Plans
  - https://www.amstat.org/asa/education/stew/home.aspx
  - https://www.engageny.org/common-core-curriculum
  - https://tasks.illustrativemathematics.org/content-standards
  - https://www.mathematicsvisionproject.org/
  - https://www.introdatascience.org/
  - https://www.openintro.org/stat/index.php?stat_book=isrs

Resources (Closed Access):

- Friday Institute has 2 Massive Open Online Courses that can be taken for free online that help teach statistics conceptually.
  - https://www.fi.ncsu.edu/projects/teaching-statistics-through-inferential-reasoning-mooc-ed/
  - https://www.fi.ncsu.edu/projects/teaching-statistics-through-data-investigations-mooc-ed/

Unit Name: Probability Distributions Unit

Duration: 2 Weeks

Standards (Content):

SP.3: Apply probability distributions in making decisions in uncertainty.

> **SP.3.1:** Implement discrete probability distributions to model a random phenomenon and make decisions (for example, expected value of playing a game, etc).
> **SP.3.2:** Implement the binomial distribution to model situations and make decisions.
> **SP.3.4:** Implement the normal distribution as a probability distribution to determine the likelihood of events occurring.

What is the Mathematics?

- In teaching probability distributions it is helpful to begin with discrete probability distributions, which can be considered theoretically (based on assumptions about the physical world) and experimentally (based on collecting data from repeated trials). Oftentimes, students are pushed to the theoretical understanding in statistics too fast. It is helpful for students to start with a hands-on activity to create an initial understanding. Then, they can move to a simulation model. Because this is built on the idea of doing the activity many, many times, the simulation helps bridge to the theoretical because we cannot complete the hands-on activity a large (or infinite) amount of times. Lastly, students will move to the theoretical understanding.
- A discrete probability distribution is the distribution of probabilities for the outcomes of a discrete random variable, which is a variable that can take one of a countable list of distinct values. Students will need to understand that in a discrete probability distribution all probabilities are less than or equal to one and the sum of probabilities is equal to one. Also the expected value of a discrete probability distribution is the mean, or what would happen, on average, over a long series of trials calculated by $\Sigma x_i p_i$,where x_i represents the value of outcome i and p_i represents the probability of outcome i. Students can also think of the expected value as a weighted average.
    - A special type of discrete random variable is a uniform random variable, which is one where the probability of every value is the same. For example, the value of rolling one fair six-sided die.
    - A special type of discrete random variable is a binomial random variable, which applies to situations where the variable describes a binomial experiment. A binomial experiment is an experiment consisting of a fixed number of independent trials (n) each with two possible outcomes, success and failure, and the same probability of success (p). For example, flipping a coin four times and recording the number of times

it lands on heads would be a binomial experiment because there is a fixed number of trials (i.e. four), they are independent (i.e. the result of one flip does not impact the results of the next), there are only two outcomes (i.e. heads or tails), and the probability of success is the same for each trial (i.e. 0.5). Students will need to be able to recognize when a situation is binomial and use the binomial distribution to find the probability of events. There are some nice probabilities to the binomial distribution including that the expected value is np and the standard deviation is sqrt(np(1-p)). In teaching the binomial distribution it is important to eventual show using simulation that as you continue to increase n, the shape of the distribution begins to create a bell shaped curve which is a powerful basis for then moving into discussing the normal distribution, which is a continuous distribution but can model a binomial distribution under certain condition. An advanced connection for honors students that can be made is to Riemann sums and integrals since the area under a probability function is the probability of an event or events occurring.

  - When teaching discrete probability distributions it is important to focus on what they tell us about a situation. Some good examples to use with students are what the expected value tells you in a financial situation such as paying to play a game of chance and also in terms of weighted averages such as their GPA. Also when considering discrete random variables by hand it is important that the number of possible values the variable takes does not become too large (ideally less than 12 or so) as the calculations become tedious.

- A continuous probability distribution is one that tells the probability of intervals of values for a continuous random variable. A continuous random variable is one that can take any value in and interval of collection of intervals. A continuous probability distribution is generally modeled in the form of a smooth continuous curve where the area underneath the curve on an interval represents the probability of that interval of values occurring. There are many different continuous random variables but the focus of this unit is on the normal distribution. Students will need to understand that the normal distribution is a continuous probability distribution and that the total area under the curve is 1. Teachers should use this opportunity to teach students about standardized scores (z-scores) and explain how standardized scores allow for comparisons when original scales are different (for example: ACT math scores and SAT math scores).
  - Introduce the Empirical Rule (68-95-99.7 Rule) as a visual of probability under the Normal curve with standard deviations. This leads to the Standardized Normal Distribution with a mean of 0 and standard deviation of 1. Students can find probabilities with 1, 2, or 3 standard deviations away from the mean. Then, this will lead into the question, "What happens when I'm at something other than 1, 2, or 3

standard deviations? What about 1.5?" Therefore, there is a need for a z-score (standardized score) and technology to calculate these probabilities. (WE ARE NOT using a z-table for this at all. Technology has taken the place of this outdated way of calculating the probabilities.) *[See video resources below] **Desmos will allow students to do NormalDist. This does provide a nice visual to build conceptual understanding.*

- It is helpful for students to sketch the normal curve with appropriately shaded regions when you are calculating the probability. This will reinforce the connection and understanding of the probability to the area under the shaded region of the normal curve. Students will use technology to calculate normal curve probabilities.

- ***Teacher Instructional Support:***
  - How to use the Empirical Rule: https://youtu.be/cgxPcdPbujI
  - Z-scores and Normal Distributions: https://youtu.be/NY2zWGBXBhU
  - Using TI-84 for NormalCDF and invNorm (SUPER Important since we don't want the teachers to use the Z-table) https://youtu.be/S5n0pJ1gPDQ
  - Binomial Distribution: Explains what a Binomial Distribution is and how the binomial formula is used (gives a pretty detailed explanation but is somewhat dry. I wouldn't necessarily show it to students, but it can teach the teacher.) https://youtu.be/qIzC1-9PwQo
  - How to do Binomial PDF and CDF on a TI-84. (Does not explain what to do when you need to find X>n) https://youtu.be/IngLJs6T1Qw

Important Considerations:

- S.3.1 Decision making is important here and can possibly connect to financial math (gambling - in casino games the expected value is always negative, ensuring that the casino will make money over time).
- A good resource would be to use Desmos to calculate the binomial probability. While the TI-84 will let them find a binomialpdf and a binomialcdf, there are sometimes struggles for students in understanding that the binomcdf only produces area equivalent to less than or equal to a number. So Desmos would make this easier, since students can set the bounds.
- S.3.4 Ensure that students understand that the normal distribution is the z-distribution, that students know how to calculate a z-score, and then how to interpret the probability that is produced by the calculator (normalcdf).

Formative Assessments/Tasks:
https://tasks.illustrativemathematics.org/content-standards/HSS/MD/A/2/tasks/1023

Unit Name: Statistical Inference Unit

Duration: 2-3 Weeks

Standards (Content):

SP.2: Apply informal and formal statistical inference to make sense of, and make decisions in, meaningful real-world contexts.

- **SP.2.1:** Design a simulation to create a sampling distribution that can be used in making informal statistical inferences.
- **SP.2.2:** Construct confidence intervals of population proportions in the context of the data.
- **SP.2.3:** Implement a one-proportion z-test to determine if an observed proportion is significantly different from a hypothesized proportion.

SP.3: Apply probability distributions in making decisions in uncertainty.

- **SP.3.3:** Recognize from simulations of sampling distributions of sample proportions that a normal distribution can be used as an approximate model in certain situations.

What is the Mathematics?

- In considering sampling distributions we need to remember that they are a distribution of statistics of samples of the same size, they are not distributions of individuals like what we have been looking at up until this point. In simulating repeated sampling to create a sampling distribution we will initially use the value of a sample statistic to answer the question, “What are the possible values of the population *parameter*?” To answer this question the focus initially will be to build on students' understanding of simulation from Math III to focus now on constructing a simulation to create a sampling distribution. Sampling distributions can be conceptually difficult for students to understand and this unit will continue to build on students' previous experiences with them in Math III. Because we are focusing on proportions, one challenge conceptually is that the sampling distribution of a categorical variable is quantitative. To help students understand simulating repeated sampling to create a sampling distribution, it would be helpful to begin with a physical simulation. For example, ask your students to respond to a question that will provide categorical data such as, “How would you rate the importance of washing your hands frequently: not important, somewhat important, or very important?” Have them respond on a sticky note and then have them post their sticky notes on the board and as a class organize the sticky notes in such a way you could see how many students responded for each of the possible outcomes (i.e. some kind of bar graph). Next you want to shift students thinking from just describing the class to making inferences to a larger population such as the school. To do so we must consider a proportion rather than a count. Furthermore, we would expect there to be variation from sample to sample so to estimate what is happening in the population we would need to understand how much variability we might expect by chance. To be able to answer the statistical question of “What proportion of students at ____ High School believe it is very important to wash their hands frequently?” We can begin by treating our sample as a population and then collecting a random sample with replacement of the same size as what our sample was that we started with. Then record the proportion of successes in the case of this example the proportion of responses in the resample that are “very important.” Next draw a sample by random again, and again record the sample proportion. Continue this process several times for students to then see this is tedious and begin to think about how technology could be used here instead. You could then have students think about how to come up with another way to physically simulate drawing a sample (i.e. table of random numbers, drawing responses from a hat, etc.). Then build toward students thinking about how they might use technology such as Excel, CODAP, or Desmos to simulate drawing random samples.

  - Applet for simulations http://www.rossmanchance.com/applets/OneSample.html
  - CODAP works well for simulating repeated sampling for creating a sampling distribution (https://codap.concord.org/).
  - For many good resources specific to simulation see pages 32 and 33 of https://www.amstat.org/asa/files/pdfs/EDU-CommonCoreResources.pdf

- For considering when sampling distributions can be modeled by the normal distribution simulations are useful to visualize what a distribution of sample statistics would look like and to then see the conditions necessary for using the normal distribution to model a sampling distribution.
  - We are focused only on recognizing that a sampling distribution can be modeled by the Normal distribution when the sample size is sufficiently large ($np \geq 10$ and $n(1-p) \geq 10$) and observations are independent, but do not expect students to start doing probability with a sampling distribution. This is in AP Statistics. In this course, once students recognize that the sampling distribution would be approximately Normal, they could explain the difference between the original population, the sample measure collected, and the sampling distribution. Furthermore, the focus in this course is not to know what to do when a situation could not be modeled by a normal distribution. Such cases are the focus of college level statistics courses and beyond the scope of what is necessary here.
- In Math 3, students constructed a margin of error using a given formula: 2(standard deviation of a sampling distribution) or by approximating how far from the center they would have to go to capture approximately 95% of the samples in a sampling distribution. You can start with informally building confidence intervals from the simulated sampling distributions created earlier in the unit to build conceptually what a confidence interval is. To construct a confidence interval, students will build on this knowledge by taking a sample statistic and adding and subtracting that margin of error. Confidence intervals tell us how confident we can be that a population *parameter* (the true proportion) is included in the constructed interval. For instance, if our sample proportion, p-hat, is 0.4, the standard deviation of the statistic, p-hat, could be calculated using $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. If the sample size were n = 100, the standard deviation would be about 0.049. In the previous unit, students learned 1) approximately 95% of the area

under a normal curve is within 2 standard deviations of the mean, and 2) a sampling distribution for p-hat is approximately normal with mean p-hat and standard deviation $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. Thus a 95% confidence interval for p-hat would be 0.4 $\pm$0.049 or (0.351, 0.449). We would say that "we are 95% confident that the true population proportion is between 0.351 and 0.449". It is important to emphasize that the confidence is in the method used, not in this particular interval. Since the interval created from the sample proportion and there is natural variability from sample to sample, the interval will fluctuate from sample to sample. If we took many samples, approximately 95% of the intervals constructed would capture the true population proportion. It would be INCORRECT to say that the probability that the population parameter falls between 0.351 and .0449 is .95.

  - PBS Crash Course to understanding Confidence Intervals https://youtu.be/yDEvXB6ApWc

- For more support on this:
  - https://istats.shinyapps.io/ExploreCoverage/ (interactive site to explore CI)
- While we use a confidence interval to estimate a population parameter, we use significance tests to evaluate the evidence provided by data about some claim concerning the population parameter. Introducing students to hypothesis testing helps them to understand what we mean by "statistically significant". A simple introductory activity would be to tell students that you are going to walk around the room and flip a coin for each of them: tails they are not selected, heads they get some reward. As you walk around, flip the coin and announce the result (always "Tails") without allowing students to actually see the coin. After a certain amount of time, students are going to start to question the fairness of the coin. This will prompt a conversation about "at what point would you be convinced the coin is not fair, that this phenomenon is not occurring just due to chance?" The p-value of a hypothesis test is the probability of getting sample results as extreme or more extreme than the sample results, assuming the null hypothesis is true. One general rule of thumb is that if the p-value is less than 5%, we would be convinced that it is highly unlikely for this outcome to have happened just due to chance and we, therefore, have evidence against the null hypothesis in favor or the alternative hypothesis.
- It is important to emphasize to students that we never prove a null hypothesis is true and we never accept an alternative hypothesis. We only have evidence against the null hypothesis (in which case we reject the null hypothesis) or we do not have evidence against the null hypothesis (in which case we fail to reject the null hypothesis).
- It is also important to emphasize that the level of significance is not always .05. Depending on the context in a real-life application, it might be desirable that the p-value be .01, for example, in order to reject the null hypothesis.

Important Considerations:

- It is important that you understand how to create a simulation that is able to demonstrate repeated sampling in order to build a sampling distribution.
- Informally check the conditions and/or make sure to use the conditions language, such as "the conditions have been met." The conditions are: 1) the sample is a random sample from the population of interest (this is to avoid bias: systematically overestimating or underestimating the population proportion), 2) the population is at least 10 times the sample size (this allows us to use the formula for standard deviation of the sample proportion; since we are sampling without replacement, having such a large population to sample from means that the probability of success does not really change significantly from one draw to the next), 3) np (or n∗ p-hat) and n(1-p) (or n∗ (1-phat)) are both at least 10 (in the last unit we learned that the sampling distribution of p-hat is approximately normal if np and n(1-p) are both at least 10).
- When calculating a standardized test statistic for a one-proportion z-test, we assume the null hypothesis is true: $z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}}$ where p_o is the proportion from the null hypothesis.
- Friday Institute has 2 Massive Open Online Courses that can be taken for free online that help teach statistics conceptually.
  - https://www.fi.ncsu.edu/projects/teaching-statistics-through-inferential-reasoning-mooc-ed/
  - https://www.fi.ncsu.edu/projects/teaching-statistics-through-data-investigations-mooc-ed/
- Against All Odds Videos - https://www.learner.org/series/against-all-odds-inside-statistics/
- S.3.3 Use simulations of sampling distributions!!! And begin to develop a conceptual understanding of a sampling distribution.

Formative Assessments/Tasks:
https://www.statsmedic.com/intro-day2 **(**The "1 in 6 wins" activity is an introduction to the thinking behind a one proportion z test)

Unit Name: ACT Prep
Duration: 2-3 Days used throughout course
Standards (Content): **N.1:** Apply properties and operations with complex numbers. **N.1.1:** Execute procedures to add and subtract complex numbers. **N.1.2:** Execute procedures to multiply complex numbers. **N.2:** Apply properties and operations with matrices and vectors. **N.2.1:** Execute procedures of addition, subtraction, multiplication, and scalar multiplication on matrices. **N.2.2:** Execute procedures of addition, subtraction, and scalar multiplication on vectors.
What is the Mathematics? ● In Math 2, students learned about complex numbers in the context of simplifying expressions with a negative value under a square root and solving quadratic equations. In this course, students will simplify complex numbers using addition, subtraction, and multiplication. ● Students will use the operations of addition, subtraction, multiplication and scalar multiplication to combine matrices. ● The connection can be made between operations of matrices and vectors because vectors can be represented as a matrix.
Important Considerations: ● This unit is purely for the *procedural knowledge* of complex numbers, matrices, and vectors. ● These standards are included across fourth level courses in order to prepare students for the ACT due to the state legislature. ● ***NOTE:*** This unit does not have to be taught at the end of the course. These standards can be scattered anywhere it works for you to teach them. They do not fit necessarily in any of the other units cohesively, but that does not mean you couldn't throw them in where you have time or want to.
Formative Assessments/Tasks: Neat Matrix Multiplication Task: https://www.openmiddle.com/matrix-multiplication/ Complex Numbers: https://www.openmiddle.com/complex-number-products/ Possible Homework Assignment: Add, Subtract, Multiply Complex Numbers Matrix Multiplication Task