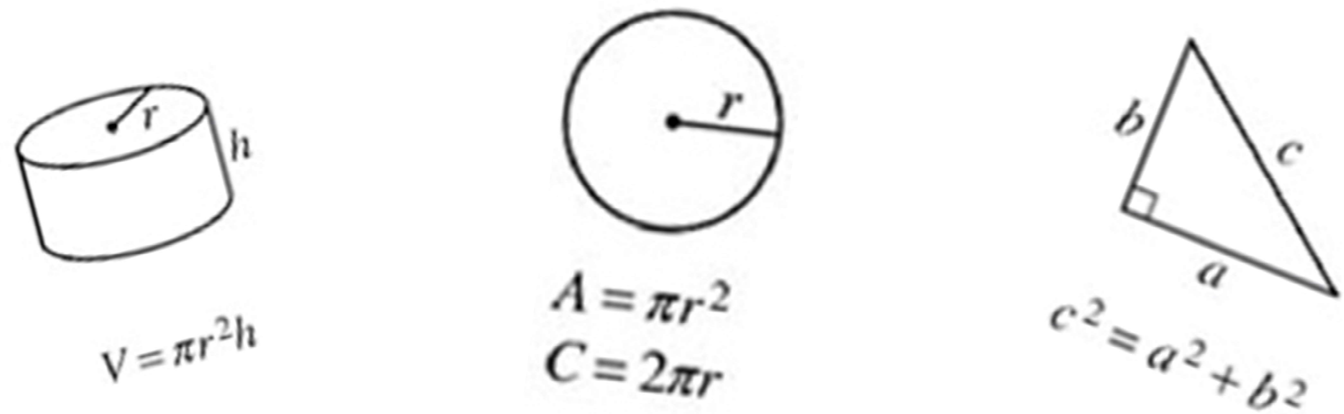


Mr. Mahlmann

GEOMETRY

Distance Learning

Take-Home Packet #2: 4/20 - 5/1/20

<u>Options for Submitting Work:</u>

1) Take pictures of your work, and send to Mr Mahlmann via either:
   a) As an attachment in *"Remind"*
   b) As an attachment to his email: carl.mahlmann@dcsms.org
2) Drop off at the Drop-Off bin in front of the school building

Students Note: When you finish reading this lesson, you'll be able to complete the section from the Fun Binder titled: "Polygons 8.1.1 – 8.1.5". DO NOT do problem #s 28, 29, 30 or Ex. 4

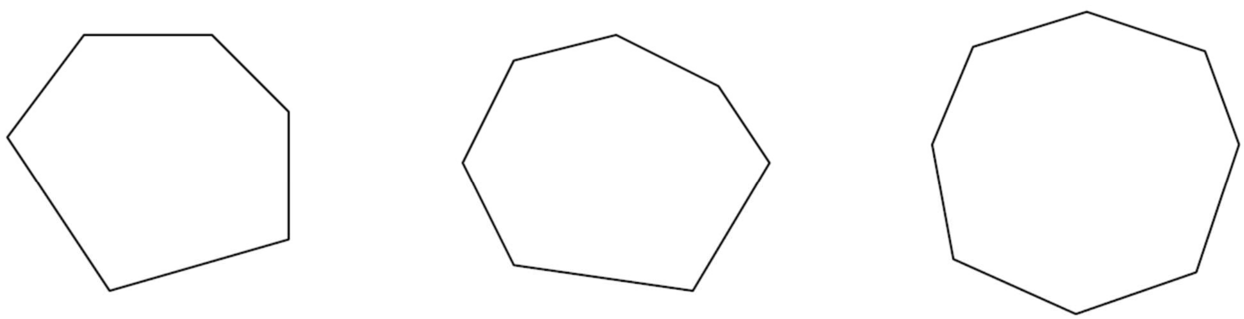
1) <u>Interior Angles of Polygons (Sum of)</u>

OK, so, we should all remember what the interior angles of a triangle add up to – right, Eli?

That's right; they add up to 180^{o}. But, we all know your always full of questions, and the question your asking right now is: What about other polygons? What is the sum of the interior angles of a hexagon, heptagon, or octagon, for instance?

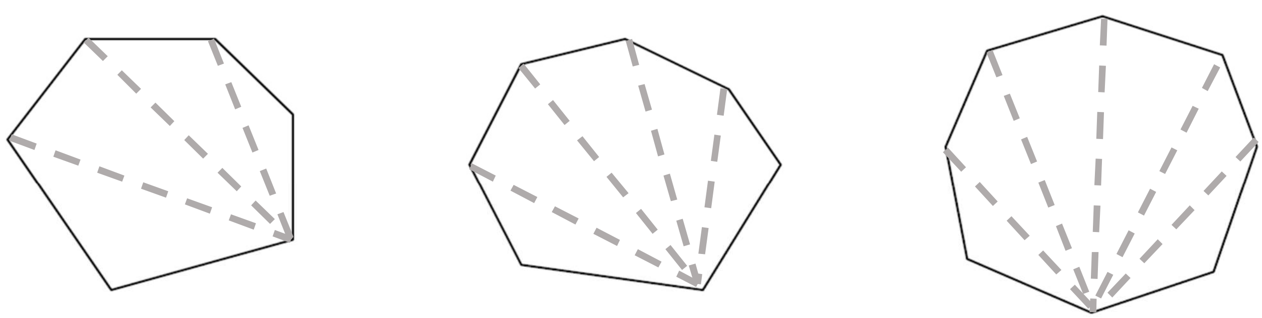
Well, Eli, I'm glad you asked. Because that's what we're going to learn right here & now.

Look at the Polygons below. How many sides does each Polygon have?



$n = 6$ $n = 7$ $n = 8$

Now breakdown the Polygons into Triangles. (Because that's our favorite, shape, right?)



$n = 6$ $n = 7$ $n = 8$

Now count the number of triangles in each Polygon.

What do you notice about the number of Triangles in each Polygon and the number of sides each has?

# Sides	# Triangles
6	4
7	5
8	6

So, did you notice that the number of triangles is always 2 less than the number of sides?

Well, that's true of any Polygon; the number of triangles into which it can be broken down will always be 2 less than the number of sides. For instance, how many triangles can a decagon be broken down into?

That's right – 8! Because a decagon has 10 sides.

Now, we know that the sum of the interior angles of each Triangle is 180^{o}. Therefore, all we have to do to find the sum of all the angles of each polygon is multiply the number of Triangles in each by 180^{o}.

For example, to find the sum of the angles in the Hexagon above, all we do is multiply 180 x 4 (the # of Triangles in it), and we get: 720^{o}.

Now, you try it: what's the sum of the interior angles of the Heptagon (7-sided Polygon)?

That's right! It's 900^{o}. (180 x 5).

So, from what we learned here, we can derive a formula to find the sum of the angles of any Polygon:

"n" = # of sides.

So, if I were to ask you (and I am…) the sum of the angles of a 100-gon, what would be the answer?

Well just plug 100 into the formula: 180(n-2); 180(100-2); 180(98); = 17,640^{o}. (Man, that's hot…!)

Now *you* try: What's the sum of the interior angles of a 125-gon?

That's right! It's 22,140^{o}. 180(n-2); 180(125-2); 180(123); = 22,140^{o}

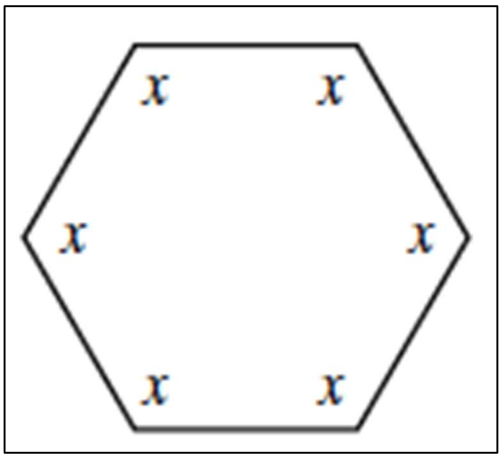
Now, let's reverse what we did above. Suppose we're given the sum of the Interior Angles of a Polygon, and asked to find the # of sides?

For example, what if we're told the sum of the Interior Angles is 900. What is the # of sides?

Well, just take our formula: Sum = 180(n – 2), and plug in 900 for the Sum:

1) 900 = 180(n – 2)
2) 900 = 180n – 360
3) 1260 = 180n
4) 7 = n (The shape is a Heptagon!)

2) <u>Interior Angles of Regular Polygons (Each)</u>



OK. Now we know how to find the sum of the Interior Angles of any Polygon [180(n – 2)]. But what if we want to find the measure of each Interior Angle?

First, we'll work with Regular Polygons. (Remember – a Regular Polygon is one in which all sides are equal to each other and all angle are equal to each other.) So, how would we find the measure of each Interior Angle in the Regular Hexagon at left?

1) Well, first find the sum: 180(n – 2) = 180(6 – 2) = 180(4) = 720.

2) Since we know each angle in a Regular Hexagon is equal to each other, we just divide the sum by the # of angles: 720 ÷ 6 = 120.

3) So each Interior Angle in this case measures 120^{o}.

4) We can come up with a formula for this. It would be:

What is the Formula to find measure of each Interior ∠ of a Regular Polygon?	$\frac{180(n-2)}{n}$

3) Interior Angles of Irregular Polygons (Each)

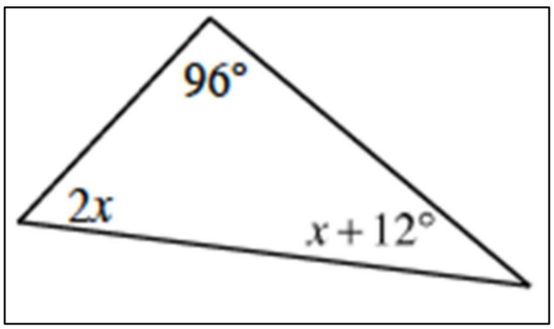
OK. So, now we know how to find:

1) The sum of the Interior Angles of any Polygon [**180(n – 2)]**

2) The measure of each Interior Angle of any Regular Polygon **[180(n – 2)] ÷ n**

But, I know what Miracle is thinking right now. She's thinking: "Mr Mahlmann, not all Polygons are Regular. What if we're trying to find the measures of the Interior Angles of Irregular Polygons?"

As usual, a great question. Well, I'll answer that question with another question: How would you find the measures of the angles in the shape below?

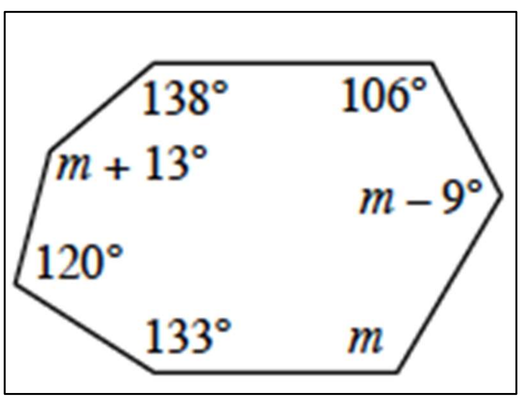


Well, we've done this many times. We all know that the Interior Angles add up to 180º. Therefore, we just set up an equation where we add each expression together, setting them equal to 180º, right?

This way we solve for "x" and plug that value back into each expression to find the measure of each angle.

So, we would use the same exact process to find the measures of the Interior Angles of any Irregular Polygon.

For example: Let's say we want to find the measure of each Interior Angle of the Irregular Polygon below. Well, the first thing we need to do is find the sum of each Interior Angle. And now, we know how to do that, don't we? That's right! Use the formula

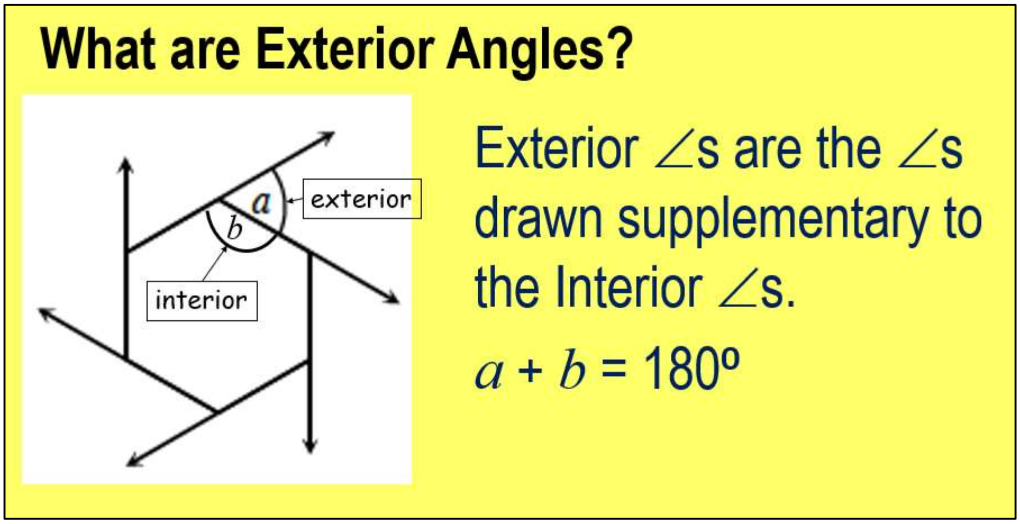


1) Sum = 180(n – 2)
2) Sum = 180(7 – 2)
3) Sum = 180(5)
4) Sum = 900
5) So now, just as we did with the Triangle above, we add all the expressions representing the measure of each Interior Angle together, and set them equal to the Sum; which, in this case, is 900.

6) 138 + 106 + m – 9 + m + 133 + 120 + m + 13 = 900
7) 501 + 3m = 900
8) 3m = 399
9) m = 133
10) Now that we've solved for "m" (=133), we just plug that value back into each expression where "m" appears to find the measure of each angle.

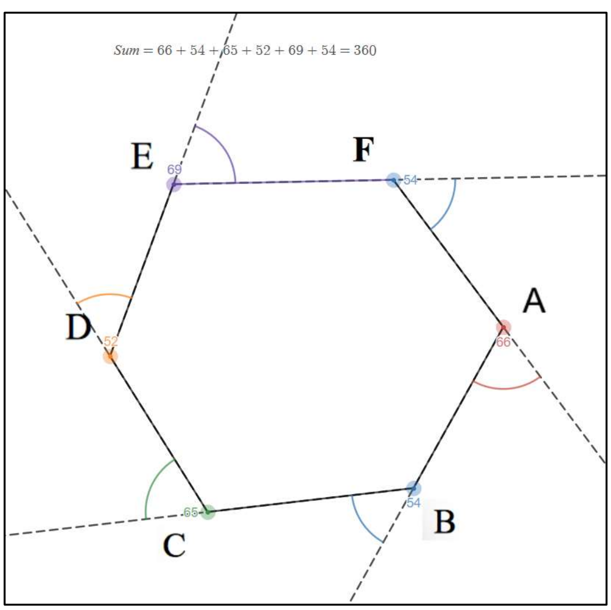
4) <u>Exterior Angles of Polygons</u>

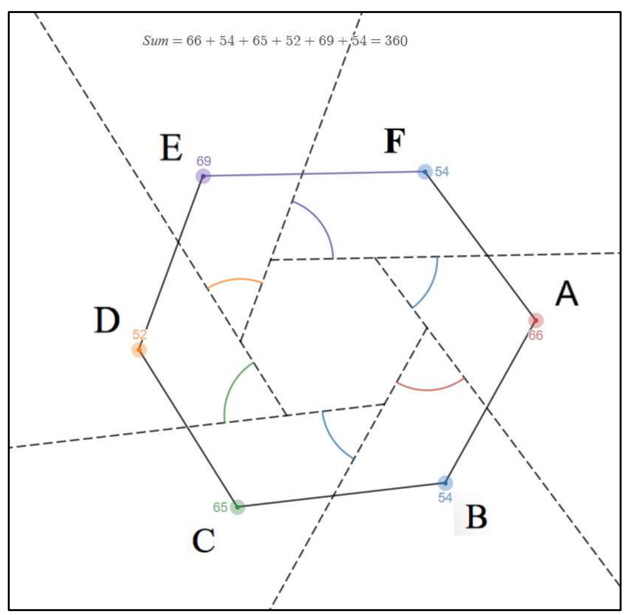
First of all, we need to know what Exterior angles are. They are:

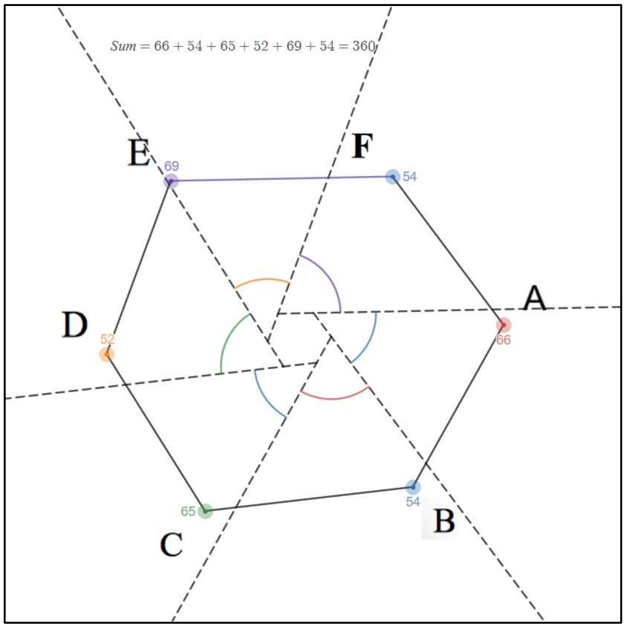


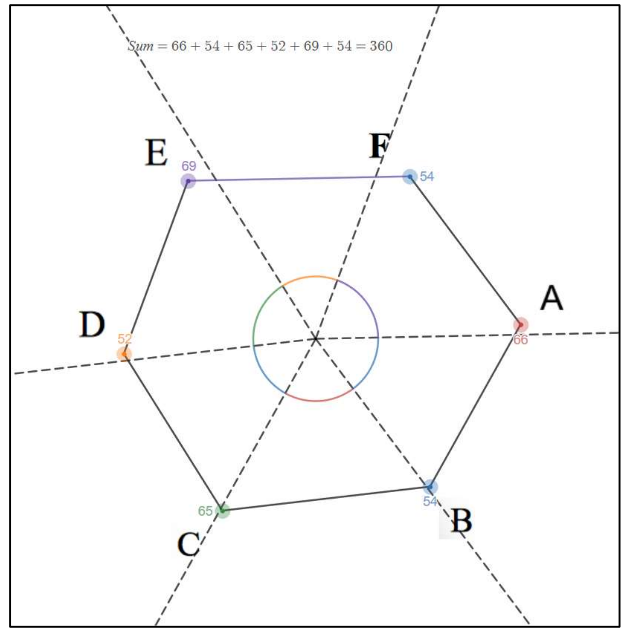
We already know how to find the sum of the Interior Angles of Polygons [180 (n – 2)]. So, the question now is: how do we find the sum of the angles of Exterior Angles? Well, the answer is a lot simpler than you might think.

Notice what happens we move the Exterior Angles to the center of the Polygon.* What kind of shape do the angles form? That's right! A Circle! (Kinda looks like the shutter on a camera lens… doesn't it?) And how many degrees are there in a Circle? Yup… 360!









**If any of you want to play around with this yourselves, using more than just hexagons, go to: https://www.desmos.com/calculator/vv8c615wkr*

It's lotsa fun!

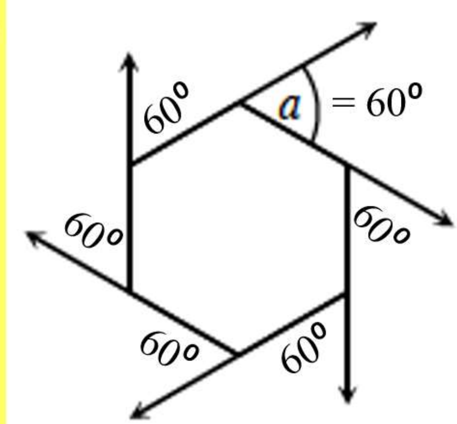
Now, we used a hexagon in the diagram above, but this is true for any Polygon:

The sum of Exterior ∠s of Polygons will always be 360°!

Therefore, to find each Exterior ∠ of a Regular** Polygon, divide 360 by the number of ∠s.

$$360 \div n$$

In this case, there are 6 exterior ∠s. Therefore, each exterior ∠ measures 360 ÷ 6 = 60°.



**Remember: a Regular Polygon is one in which all sides are equal to each other and all angles are equal to each other.

Now *you* try: What is the sum of the Exterior Angles of a Pentagon? An Octagon? A Decagon? A 100-gon?

That's right! They're all 360^o! It doesn't matter how many sides the Polygon has, the Exterior Angles will always add up to 360^o! Pretty, cool, Huh?

Now, what is the measure of each exterior angle of a Regular Pentagon? Regular Octagon? Regular Decagon? Regular 100-gon?

Well, since those Polygons are Regular, we know that each Exterior Angle in each Polygon is equal to the other. So, just divide 360 by the number of Exterior Angles in each. (Note: the # of Exterior Angles is equal to the # of sides…) So, each Exterior Angle in each of those Regular Polygons measure as follows:

Regular Polygon	**Sum of Exterior Angles**	**÷**	**# Exterior Angles**	**Measure of Each Exterior Angle**
Pentagon	360	÷	5	72
Octagon	360	÷	8	45
Decagon	360	÷	10	36
100-gon	360	÷	100	3.6

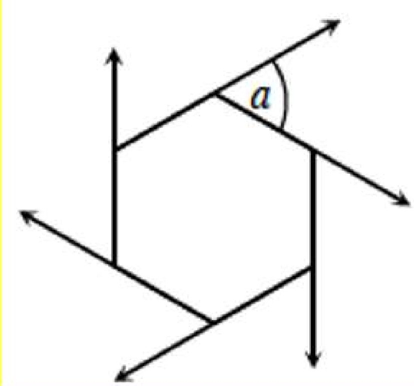
So now I know exactly what Eli is thinking. He's thinking: "Mr Mahlmann, if I know: a) how to find the measure of the Exterior Angles; and b) that the Exterior Angles and Interior Angles are supplementary, then why do I have to use the 180(n-2) formula? All I have to do is find the measure of the Exterior Angle, and then subtract that from 180 to find the measure of the Interior Angle!"

Well, Eli, as usual, you're right. I, too, find that to be a much quicker way to do it. So:

Another way to find the measure of interior ∠s of Regular Polygons:

1) Find the measure of the Exterior ∠s by dividing 360 by the number of sides.

2) Subract the measure of the Exterior ∠ from 180.



For example, to find the measure of each Interior Angle in the Regular Hexagon above, we would:

1) Find the measure of each Exterior Angle (which we already know is 60° from our work on the previous page);

2) Subtract 60 from 180 = 120º. So, each Interior Angle = 120º.

3) The formula for this would be: **180 – (360 ÷ *n*)**

4) We could have gotten the same answer using 180(n-2)/n: 180(6 – 2)/6 = 180(4)/6 = 720/6 = 120. But that takes a bit longer.

Thanks for the great question, Eli!

5) Interior & Exterior Angles of Polygons (Summary)

Polygon Formulas you must know:			
1	Sum of Interior ∠s	**180(*n* - 2)**	
2	Each Interior ∠ of Regular Polygon	$\frac{180(n-2)}{n}$ Or	**180 – (360 ÷ *n*)**
3	Sum of Exterior ∠s	**360°**	
4	Each Exterior ∠ of Regular Polygon	**360 ÷ *n***	

You should now be ready to work on:

a) Fun Binder Section 8.1.1 — 8.1.5

NOTE: DO NOT work on problem #s: 28, 29, 30 or Example 4

b) Polygon Angles – ACT Problems