

Polar and Rectangular Coordinates

As seen with our examples concerning vectors, it is possible to convert between polar and rectangular coordinates using the following formulas:

$$x = r \cos \theta \quad \text{and} \quad y = r \sin \theta$$

$$r^2 = x^2 + y^2 \quad \text{or} \quad r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Example 1: Find the rectangular coordinates of each point:

$$\text{a) } (6, 30^\circ) \longrightarrow (5.2, 3)$$

$$x = 6 \cos 30^\circ$$

$$y = 6 \sin 30^\circ$$

$$\text{b) } (-2, 140^\circ) \longrightarrow (1.53, -1.3)$$

$$x = -2 \cos 140^\circ$$

$$y = -2 \sin 140^\circ$$

$$\text{c) } \left(4, \frac{\pi}{3}\right) = (4, 60^\circ) \longrightarrow (2, 3.46)$$

$$\frac{\pi}{3} \cdot \frac{180}{\pi} = 60^\circ$$

Example 2: Find the rectangular coordinates of each point:

a) $(2, -20^\circ) \rightarrow (1.87, -0.68)$

b) $(1, \pi/4) \rightarrow (.707, .707)$

c) $(9, 42^\circ) \rightarrow (6.69, 6.02)$

Example 3: Find the polar coordinates of each point:

a) $(3, 4) \rightarrow (5, 53.1^\circ)$

$$r = \sqrt{3^2 + 4^2} = 5$$
$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1$$

b) $(6, 9) \rightarrow (10.8, 56.3^\circ)$

$$r = \sqrt{6^2 + 9^2} = 10.8$$

$$\theta = \tan^{-1}\left(\frac{9}{6}\right) = 56.3$$

c) $(7, -3) \rightarrow (7.6, 336.8^\circ)$

$$\sqrt{7^2 + (-3)^2}$$

$$\tan^{-1}\left(\frac{-3}{7}\right) = -23.2 + 360$$

Example 4: Find the polar coordinates of each point:

a) $(2, 2) \rightarrow (2.82, 45^\circ)$

b) $(5, -3) \rightarrow (5.8, 329.1^\circ)$

c) $(-6, 2) \rightarrow (6.3, 161.2)$

We can also use the formulas from the first slide to manipulate the equations from one form to another:

Guided Example: $r = 6 \cos \theta$

$$r \cdot r = \underline{6r \cos \theta}$$

$$x^2 + y^2 = 6x$$

Example 5: Convert the following equation from polar to rectangular form.

$$r = 4r \cos \theta + 3r \sin \theta$$

$$\underline{r^2} = 4 \underline{r \cos \theta} + 3 \underline{r \sin \theta}$$

$$x^2 + y^2 = 4x + 3y$$

Example 6: Convert the following equation from polar to rectangular form.

$$r^2 = 5^2$$

$$x^2 + y^2 = 25$$

Example 7: Convert the following equation from rectangular to polar form.

$$\underline{x^2 + y^2} = \underline{5x} + \underline{9y}$$

$$r^2 = 5r \cos \theta + 9r \sin \theta$$

$$r = 5 \cos \theta + 9 \sin \theta$$

Example 8: Convert the following equation from rectangular to polar form.

$$\underline{y} = 8$$

$$r \sin \theta = 8$$

$$r = 8 / \sin \theta$$

Homework: Worksheet