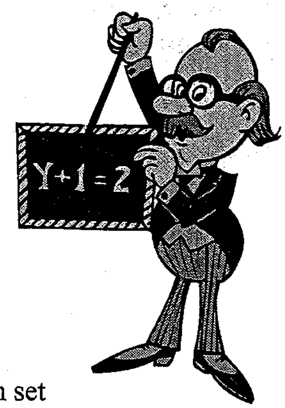
Algebraic Equations

Section 5.1
Substitution Principle



Algebraic equations have a combination of constants, coefficients, and variables on both sides of an equals sign. An equation says that the quantity on the left is equal to the quantity on the right.

Variable
Coefficient
$3a + 2 = 11$ Equals sign
Constants

The variables have one or more replacement values called a solution set that will make the equation true. Finding that solution set is called evaluating an equation. It can also be called simplifying an equation. By applying the rules or principles of mathematics, you can determine the solution(s) to an equation.

Look at the algebraic equations below. The x, y, t, and w are variables that stand for unknown values. Can you determine what those values have to be to make the equations true?

$$x + 5 = 15 \qquad y - 2 = 6 \qquad t \times 4 = 8 \qquad w \div 2 = 7$$

If you determined that x must be 10, y must be 8, t is 2, and w is 14, you are right. If you aren't sure if these are the right values, you can use the substitution principle to check.

Substitution Principle

You've already practiced the substitution principle by replacing a variable with a value. Now let's use it in another application — checking answers. Because solutions to an equation are values for the variables that make the statement true, you can also think of the solution set as replacements for the variables. If you know the value of a variable, you can substitute, or replace, the variable with the number.

Substitution Principle
If two quantities are equal, then one quantity can be substituted for the other quantity.

With algebraic equations, you seldom know the value of a variable ahead of time. One of the most useful parts of the substitution principle is in checking the solutions you get when you simplify equations. When you have a replacement value for a variable, you can substitute it into the equation to see if both sides are equal for that solution.

In fact, when you find replacement values for variables in an equation, you should always use the substitution rule to check your solutions.

Let's look at some examples.

Section 5.1, continued
Substitution Principle

Example 1: **Evaluate the equation $a + 3 = 9$ for $a = 6$.**

Since a and 6 are equal, you can replace a with 6 in the equation.

You can combine 6 and 3 for a total of 9. As you can see the statement $9 = 9$ is true. Six is a solution for a in the equation.

$$a + 3 = 9 \text{ for } a = 6$$
$$6 + 3 \stackrel{?}{=} 9$$
$$9 = 9$$

Example 2: **For the equation $4t = 8$, is $t = 4$?**

Substitute 4 for t.

Do the math. Both sides are NOT equal, so four is not a solution to this equation.

$$4t = 8 \text{ for } t = 4$$
$$(4)(4) \stackrel{?}{=} 8$$
$$16 \neq 8$$

Practice

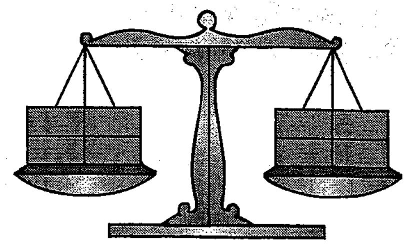
Use the substitution principle to determine whether or not the solution of each variable is correct. If it is correct, place a C in the blank. If it is incorrect, place an I in the blank.

Examples: __I__ $x + 2 = 7, x = 4$ — $4 + 2 \neq 7$ __C__ $2x + 1 = 3, x = 1$ — $2(1) + 1 = 3$

_____ 1. $x + 3 = 6, x = 1$	_____ 2. $3r = 9, r = 3$	_____ 3. $m - 2 = 5, m = 6$
_____ 4. $2a - 5 = 7, a = 5$	_____ 5. $4c + 3 = 11, c = 2$	_____ 6. $3d - 2 = 12, d = 5$

Algebraic Equations

Section 5.2
The Addition Principle



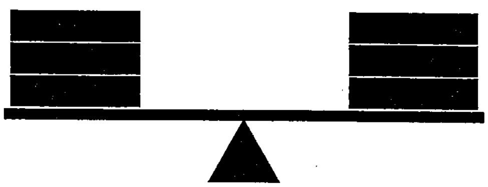
Look again at the algebraic equations below.

$x + 5 = 15$ $y - 2 = 6$ $t \times 4 = 8$ $w \div 2 = 7$

You can usually do these types of problems in your head. But when equations get more complicated, it gets harder and takes more time to determine the value of a variable by guessing. To save time, you can use a systematic way to solve equations that involves two principles: the addition principle and the multiplication principle. Let's look at the addition principle first.

Addition Principle

Both sides of an equation are equal. If you want to make a change to one side of an equation, you also have to make the same change to the other side of the equation. Think of an equation as a balance with an equal number of bricks on both sides.



If you want to add a brick to either side, you would have to add one to both sides to keep it balanced, or equal.

To take away a brick from one side, you would have to take a brick from the other side as well.

In algebra, adding or subtracting a quantity from both sides of an equation is called the Addition Principle.

> **Addition Principle**
>
> **The same number may be added (or subtracted) from both sides of any equation, and the solution remains the same.**

This can be a very useful rule if you want to determine the value of a variable in an equation. Let's take a look.

Example 1: **Given the equation $a + 2 = 12$, determine the value of a.**

Solution: One way to get this equation in the form of "$a =$ " would be to eliminate the +2 by adding the opposite, –2, to both sides of the equation.

$$\begin{array}{r} a + 2 = 12 \\ -2 = -2 \\ \hline a = 10 \end{array}$$

According to the Addition Principle, you can add –2 to the left side as long as you add the same to the right side.

Adding the opposite to both sides leaves only the variable on the left, which means the variable is equal to whatever quantity is on the right. In this case, it would be 10. Check using the substitution principle.

$$\begin{array}{r} 10 + 2 = 12 \\ 12 = 12 \end{array}$$

Example 2: **Given the equation $m - 2 = 12$, determine the value of m.**

The trick is to always add the opposite. In this case, the opposite of -2 would be $+2$. When you add $+2$ to both sides, the -2 is eliminated. Now you have only the variable on the left side of the equation and only a constant on the right side of the equation.

$$\begin{array}{r} m - 2 = 12 \\ +2 = +2 \\ \hline m = 14 \end{array}$$

Make sure you check your answer using the substitution principle.

$$\begin{array}{r} 14 - 2 = 12 \\ 12 = 12 \end{array}$$

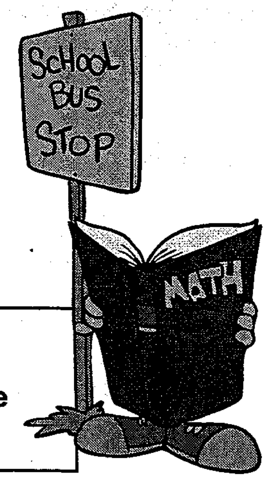
Practice

Use the addition principle to solve the following equations. Show your work and write your final answers in the blanks provided. Be sure you check each answer by using the substitution principle.

1. $b + 4 = 12$ ______	2. $a - 7 = 12$ ______	3. $m - 6 = -2$ ______
4. $c + 7 = 2$ ______	5. $r - 2 = -3$ ______	6. $x + 5 = 19$ ______
7. $x - 10 = 5$ ______	8. $y - 9 = -4$ ______	9. $t + 7 = -7$ ______

Algebraic Equations

Section 5.3
The Multiplication Principle



Not only can you add and subtract from both sides of an equation, you can multiply and divide as well.

Multiplication Principle

Both sides of an equation can be multiplied (or divided) by the same number, and the solution remains the same.

The multiplication principle is used to eliminate coefficients in front of a variable. But first, here's a reminder about the Identity Property of Multiplication.

Identity Property of Multiplication

The **Identity Property** says that you can multiply a number by its reciprocal to equal one. In case you don't remember reciprocals, the easiest way to think of them is to write the numbers in rational form. You remember rational numbers — they're fractions.

Take the number three. To write three as a rational number, divide it by 1. You can divide any number by one without changing the value.

Now that three is a fraction, you make the reciprocal by flipping the numerator and the denominator. The reciprocal of three over one is one over three or one third.

When you multiply a number by it's reciprocal, you get one!

$$3 = \frac{3}{1} \longrightarrow \left(\frac{3}{1}\right) \longrightarrow \frac{1}{3}$$

rational form — reciprocal

$$\frac{3}{1} \times \frac{1}{3} = \frac{3}{3} \text{ or } 1$$

Examples:

$\frac{2}{3} \rightarrow \frac{3}{2}$	$2 \rightarrow \frac{1}{2}$	$\frac{1}{4} \rightarrow 4$	$-\frac{2}{3} \rightarrow -\frac{3}{2}$	$-2 \rightarrow -\frac{1}{2}$	$-\frac{1}{4} \rightarrow -4$

Note: The reciprocal of a negative number is also negative.

Practice 1

Write the reciprocal of each number in the blanks provided.

1. 5	2. $\frac{2}{3}$	3. 10	4. -3	5. $-\frac{3}{5}$	6. $-\frac{1}{4}$
______	______	______	______	______	______

Example 1: **Given the equation $3m = 12$, determine the value of m.** $3m = 12$

This equation is saying that three times a number represented by the letter m equals 12. To solve it means to find the value of m. So how do you eliminate the coefficient of the variable? The simplest way would be to make the coefficient into one. As you have just seen, you can do that by using the Identity Property of Multiplication.

If you want to make the coefficient into one, multiply the coefficient by the reciprocal, which is one third. But according to the multiplication principle, you have to multiply both sides by one third. If it helps you to see it, change all the terms into fractions.

Step 1: To help you see the math, first rewrite the original equation in rational form. $\frac{3}{1}m = \frac{12}{1}$

Step 2: Multiply both sides by the reciprocal of the coefficient. $\frac{1}{3} \times \frac{3}{1}m = \frac{12}{1} \times \frac{1}{3}$

Step 3: Do the math. By using the reciprocal, the coefficient becomes $1m$ or just m. You are left with 4 on the other side, so $m = 4$. $\frac{3}{3}m = \frac{12}{3}$

$m = 4$

Step 4: Use the substitution principle to check the solution. $3(4) = 12$

$12 = 12$

> **Shortcut:** If all these fractions are confusing to you, think in terms of division instead of multiplication. In the example $3m = 12$, divide both sides by 3. You'll get the right answer. But it works only when you have a whole number coefficient.

Example 2: **Given the equation $\frac{1}{2}b = 10$, determine the value of b.**

The steps are the same for this problem. As long as you understand rational numbers, you don't have to write everything out as fractions. $\frac{1}{2}b = 10$

Step 1: Multiply both sides by the reciprocal of the coefficient. In this case, the reciprocal is 2. The whole number 2 and the 2 in the denominator of the fraction cancel out. $\cancel{2} \times \frac{1}{\cancel{2}}b = 10 \times 2$

$b = 20$

Step 2: Do the math. $\frac{1}{\cancel{2}}(\overset{10}{\cancel{20}}) = 10$

Step 3: Check your solution. $10 = 10$

Practice 2

Use the multiplication principle to solve the following equations. Show your work and write your final answers in the blanks provided. Be sure you check each answer by using the substitution principle.

1. $3m = 9$ ______	2. $2a = -2$ ______	3. $\frac{1}{3}x = 2$ ______
4. $14c = 28$ ______	5. $-2x = 6$ ______	6. $-\frac{1}{2}m = 2$ ______
7. $8x = -16$ ______	8. $\frac{1}{4}t = 8$ ______	9. $\frac{1}{5}y = -2$ ______
10. $-\frac{1}{4}k = -12$ ______	11. $-5w = -25$ ______	12. $-4x = 32$ ______

Answers as Fractions

In the real world, answers don't always work out to be whole numbers or integers. Many times, solving an algebra problem results in a fraction. Don't let fractions bother you. Simply reduce them to lowest terms or convert them to a mixed number when necessary.

Example 5: **Solve the equation $\frac{3}{4}x = 4$ for x.**

Step 1: The coefficient in front of x is $\frac{3}{4}$, so multiply both sides of the equation by its reciprocal $\frac{4}{3}$.

$$\frac{3}{4}x = 4$$

$$\left(\frac{4}{3}\right)\left(\frac{3}{4}\right)x = \left(\frac{4}{3}\right)\left(\frac{4}{1}\right)$$

Step 2: Do the math. When you get an improper fraction (the numerator is larger than the denominator), you may need to convert the fraction to a mixed number.

$$x = \frac{16}{3} \text{ or } 5\frac{1}{3}$$

Step 3: Check your solution. To make the math easier, use the solution in the form of an improper fraction. When multiplying fractions, remember that you can simplify by canceling like terms that appear in the numerator and denominator.

$$\left(\frac{3}{4}\right)\left(\frac{16}{3}\right) = 4 \quad \left(\frac{\cancel{3} \cdot \cancel{16}^{\,4}}{\cancel{4} \cdot \cancel{3}}\right) = 4$$

Practice 3

Use the multiplication principle to solve the following equations. Show your work and write your final answer in the blank provided. Reduce fractions and convert improper fractions to mixed numbers.

1. $3x = 7$ __________	2. $5y = -18$ __________	3. $\frac{3}{4}x = 9$ __________
4. $\frac{2}{5}a = 3$ __________	5. $-\frac{2}{3}x = -7$ __________	6. $\frac{4}{5}y = -2$ __________

Mixed Practice

Use either the addition principle or the multiplication principle to solve the following equations. If the answer is a fraction, reduce it to lowest terms. Convert any improper fractions to mixed numbers.

1. $-5x = 10$ ______	2. $3 + y = 14$ ______	3. $2t = -18$ ______
4. $-4x = -15$ ______	5. $-9x = 3$ ______	6. $y - 7 = 14$ ______
7. $\frac{1}{3}x = -9$ ______	8. $-\frac{3}{4}a = -5$ ______	9. $\frac{2}{3}t = 6$ ______

Algebraic Equations

Section 5 Review

Answer each question below. Darken the circle that represents the correct answer.

1. For which of the following equations is $a = -2$ a solution?

I. $3a = -6$
II. $4 - a = 2$
III. $a + 5 = 7$

A I only
B II only
C I and III only
D I, II, and III

Ⓐ Ⓑ Ⓒ Ⓓ

2. For which of the following equations is $x = 3$ a solution?

I. $2x - 2 = 4$
II. $2 - x = 3x + 14$
III. $5x + 9 = 6x + 6$

A I only
B I and II only
C I and III only
D I, II, and III

Ⓐ Ⓑ Ⓒ Ⓓ

3. What is the solution of the following equation?

$$\frac{2}{3}x = -1$$

A $x = -\frac{1}{3}$
B $x = -1\frac{1}{2}$
C $x = 1\frac{1}{3}$
D $x = -3$

Ⓐ Ⓑ Ⓒ Ⓓ

4. What is the solution of the following equation?

$$x + 5 = -5$$

A $x = 0$
B $x = -1$
C $x = -10$
D $x = -25$

Ⓐ Ⓑ Ⓒ Ⓓ

5. What is the solution of the following equation?

$$\frac{2}{5}x = 8$$

A $x = 3\frac{1}{5}$
B $x = 5$
C $x = 20$
D $x = 40$

Ⓐ Ⓑ Ⓒ Ⓓ

6. What is the solution of the following equation?

$$a - 4 = -7$$

A $a = -3$
B $a = -11$
C $a = 1\frac{3}{4}$
D $a = 28$

Ⓐ Ⓑ Ⓒ Ⓓ