Week One

Yearlong Algebra

Desoto County Schools

Factoring Methods for $ax^2 + bx + c$

ANY METHOD: Before factoring, factor out the GCF
ANY METHOD: After factoring, check by multiplying to verify original polynomial

Guess and Check

1. Factor out GCF.
2. Draw parentheses.
3. Find factors of a; find factors of c.
4. Try different pairings of factors until a pair works.

Factor $10x^2 + 21x + 8$

Factors of 10: 1·10 and 2·5; Factors of 8: 1·8, 2·4

$(1x + 1)(10x + 8)$ No
$(1x + 8)(10x + 1)$ No
$(1x + 2)(10x + 4)$ No
$(1x + 4)(10x + 2)$ No
$(5x + 8)(2x + 1)$ Yes

Grouping

1. Factor out GCF.
2. Multiply ac.
3. Find two factors of ac that add or subtract to b.
4. Split bx term into sum of those two numbers.
5. Group first two terms and last two terms (reverse distribute).
6. Factor the common polynomial.

Factor $10x^2 + 21x + 8$

10·8 = 80. Factors of 80 that add to 21: 16 and 5
$10x^2 + 16x + 5x + 8$
$2x(5x + 8) + (5x + 8)$
$(5x + 8)(2x + 1)$

Box Method

1. Factor out GCF.
2. Draw a 2x2 box.
3. Put first term (ax^2) in top left, last term (c) in bottom right
4. Multiply ac; find factors of ac that add to middle term b. Put these terms in top right and bottom left boxes.
5. Factor the GCF from each row and column.
6. These values make up the factors!

Factor $10x^2 + 21x + 8$

10·8 = 80. Factors of 80 that add to 21: 16 & 5

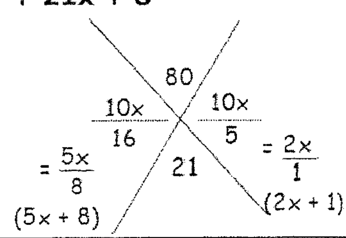
	5x	8
2x	$10x^2$	16x
1	5x	8

Factors: $(5x + 8)(2x + 1)$

Diamond Method

1. Factor out GCF.
2. Draw a big X. Multiply ac and put in top of x; put b in bottom of x.
3. Find two factors of ac that add to b.
4. Put those factors on the left and right of the "X," but make as denominators of fractions.
5. Make leading coefficient multiplied by variable as the numerator of the fraction.
6. Reduce fractions if possible. These are your factors!

Factor $10x^2 + 21x + 8$



Slide and Divide

1. Factor out GCF.
2. Multiply ac and rewrite the trinomial with a leading coefficient of 1 and the third term as the product of ac ("slide").
3. Factor using strategies when leading coefficient is 1 (type III factoring).
4. Divide each numerical term by the original leading coefficient, and reduce to simplest form ("divide").
5. Multiply the terms in each set of parentheses by the LCD of the two terms.

Factor $10x^2 + 21x + 8$

ac = 80; rewrite: $x^2 + 21x + 80$
Factor: $(x + 16)(x + 5)$
Divide by 10: $(x + \frac{16}{10})(x + \frac{5}{10})$
Reduce: $(x + \frac{8}{5})(x + \frac{1}{2})$
Multiply by LCD: $(5x + 8)(2x + 1)$

FACTORING QUADRATICS 8.1.1 through 8.1.4

Chapter 8 introduces students to rewriting quadratic expressions and solving quadratic equations. Quadratic functions are functions which can be rewritten in the form $y = ax^2 + bx + c$ (where $a \neq 0$) and when graphed, create a U-shaped curve called a parabola.

There are multiple methods that can be used to solve quadratic equations. One of them requires factoring the quadratic expression first. In Lessons 8.1.1 through 8.1.4, students factor quadratic expressions.

In previous chapters, students used algebra tiles to build "generic rectangles" of quadratic expressions. In the figure below, the length and width of the rectangle are $(x + 2)$ and $(x + 4)$. Since the area of a rectangle is given by (base)(height) = area, the area of the rectangle in the figure below can be expressed as a *product*, $(x + 2)(x + 4)$. But the small pieces of the rectangle also make up its area, so the area can be expressed as a *sum*, $4x + 8 + x^2 + 2x$, or $x^2 + 6x + 8$. Thus students wrote $(x + 2)(x + 4) = x^2 + 6x + 8$.

In the figure at right, the length and width of the rectangle, which are $(x + 2)$ and $(x + 4)$, are *factors* of the quadratic expression $x^2 + 6x + 8$, since $(x + 2)$ and $(x + 4)$ multiply together to produce the quadratic expression $x^2 + 6x + 8$. Notice that the $4x$ and the $2x$ are located diagonally from each other. They are like terms and can be combined and written as $6x$.

$+4$	$4x$	8
x	x^2	$2x$
	x	$+ 2$

The factors of $x^2 + 6x + 8$ are $(x + 2)$ and $(x + 4)$.

The ax^2 term and the c term are always diagonal to one another in a generic rectangle. In this example, the ax^2 term is $(1x^2)$ and the c term is the constant 8; the product of this diagonal is $1x^2 \cdot 8 = 8x^2$. The two x-terms make up the other diagonal and can be combined into a sum since they are like terms. The b of a quadratic expression is the *sum* of the coefficients of these factors: $2x + 4x = 6x$, so $b = 6$. The product of this other diagonal is $(2x)(4x) = 8x^2$. *Note that the products of the two diagonals are always equivalent.* In the textbook, students may nickname this rule "Casey's Rule," after the fictional character Casey in problem 8-4.

To factor a quadratic expression, students need to identify the coefficients of the two x-terms so that the products of the two diagonals are equivalent, and also the sum of the two x-terms is b. Students can use a "diamond problem" to help organize their sums and products. For more information on using a diamond problem and generic rectangle to factor quadratic expressions, see the Math Notes box in Lesson 8.1.4.

For additional information, see the Math Notes boxes in Lessons 8.1.1 through 8.1.4. For additional examples and more practice, see the Checkpoint 10B materials at the back of the student textbook.

Example 1

Factor $x^2 + 7x + 12$.

Sketch a generic rectangle.

Place the x^2 and the 12 along one diagonal.

	12
x^2	

Find two terms whose product is $12x^2$ and whose sum is $7x$. In this case, $3x$ and $4x$. (Students are familiar with this situation as a "diamond problem" from Chapter 1.)

$3x$	12
x^2	**$4x$**

Write these terms along the other diagonal. Either term can go in either diagonal space.

Determine the base and height of the large outer rectangle by using the areas of the small pieces and finding the greatest common factor of each row and column.

$+3$	$3x$	12
x	x^2	$4x$
	x +	4

Write the sum as a product (factored form).

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

Example 2

Factor $x^2 + 7x - 30$.

Sketch a generic rectangle.
Place the x^2 and the -30 along one diagonal.

	-30
x^2	

Find two terms whose product is $-30x^2$ and whose sum is $7x$. In this case, $-3x$ and $10x$.

$-3x$	-30
x^2	**$10x$**

Write these terms along the other diagonal. Either term can go in either diagonal space.

Determine the base and height of the large outer rectangle by using the areas of the small pieces and finding the greatest common factor of each row and column.

-3	$-3x$	-30
x	x^2	$10x$
	x +	10

Write the sum as a product (factored form).

$$x^2 + 7x - 30 = (x - 3)(x + 10)$$

Example 3

Factor $x^2 - 15x + 56$.

	56
x^2	

Sketch a generic rectangle.
Place the x^2 and the 56 along one diagonal.

$\mathbf{-8x}$	56
x^2	$\mathbf{-7x}$

Find two terms whose product is $56x^2$ and whose sum is $-15x$. Write these terms as the other diagonal.

-8	$-8x$	56
x	x^2	$-7x$
	x	-7

Determine the base and height of the large outer rectangle by using the areas of the small pieces and finding the greatest common factor of each row and column.

Write the sum as a product (factored form).

$$x^2 - 15x + 56 = (x - 7)(x - 8)$$

Example 4

Factor $12x^2 - 19x + 5$.

Sketch a generic rectangle.

Place the $12x^2$ and the 5 along one diagonal.

$\mathbf{-15x}$	5
$12x^2$	$\mathbf{-4x}$

→

-5	$-15x$	5
$4x$	$12x^2$	$-4x$

Find two terms whose product is $60x^2$ and whose sum is $-19x$. Write these terms as the other diagonal.

Find the base and height of the rectangle. Check the signs of the factors.

Write the sum as a product (factored form). $(3x - 1)(4x - 5) = 12x^2 - 19x + 5$

Example 5

Factor $3x^2 + 21x + 36$.

Note: If a common factor appears in all the terms, it should be factored out first.
For example, $3x^2 + 21x + 36 = 3(x^2 + 7x + 12)$.

Then $x^2 + 7x + 12$ can be factored in the usual way, as in Example.
$x^2 + 7x + 12 = (x + 3)(x + 4)$.

Then, since the expression $3x^2 + 21x + 36$ has a factor of 3,
$3x^2 + 21x + 36 = 3(x^2 + 7x + 12) = 3(x + 3)(x + 4)$.

Problems

1. $x^2 + 5x + 6$	2. $2x^2 + 5x + 3$	3. $3x^2 + 4x + 1$	4. $3x^2 + 30x + 75$
5. $x^2 + 15x + 44$	6. $x^2 + 7x + 6$	7. $2x^2 + 22x + 48$	8. $x^2 + 4x - 32$
9. $4x^2 + 12x + 9$	10. $24x^2 + 22x - 10$	11. $x^2 + x - 72$	12. $3x^2 - 20x - 7$
13. $x^3 - 11x^2 + 28x$	14. $2x^2 + 11x - 6$	15. $2x^2 + 5x - 3$	16. $x^2 - 3x - 10$
17. $4x^2 - 12x + 9$	18. $3x^2 + 2x - 5$	19. $6x^2 - x - 2$	20. $9x^2 - 18x + 8$

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8-3 Study Guide and Intervention

Factoring Trinomials: $x^2 + bx + c$

Factor $x^2 + bx + c$ To factor a trinomial of the form $x^2 + bx + c$, find two integers, m and n, whose sum is equal to b and whose product is equal to c.

Factoring $x^2 + bx + c$	$x^2 + bx + c = (x + m)(x + n)$, where $m + n = b$ and $mn = c$.

Example 1 **Factor each trinomial.**

a. $x^2 + 7x + 10$

In this trinomial, $b = 7$ and $c = 10$.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

Since $2 + 5 = 7$ and $2 \cdot 5 = 10$, let $m = 2$ and $n = 5$.

$x^2 + 7x + 10 = (x + 5)(x + 2)$

b. $x^2 - 8x + 7$

In this trinomial, $b = -8$ and $c = 7$. Notice that $m + n$ is negative and mn is positive, so m and n are both negative. Since $-7 + (-1) = -8$ and $(-7)(-1) = 7$, $m = -7$ and $n = -1$.

$x^2 - 8x + 7 = (x - 7)(x - 1)$

Example 2 **Factor $x^2 + 6x - 16$.**

In this trinomial, $b = 6$ and $c = -16$. This means $m + n$ is positive and mn is negative. Make a list of the factors of -16, where one factor of each pair is positive.

Factors of −16	Sum of Factors
1, −16	−15
−1, 16	15
2, −8	−6
−2, 8	6

Therefore, $m = -2$ and $n = 8$.

$x^2 + 6x - 16 = (x - 2)(x + 8)$

Exercises

Factor each trinomial.

1. $x^2 + 4x + 3$
2. $m^2 + 12m + 32$
3. $r^2 - 3r + 2$
4. $x^2 - x - 6$
5. $x^2 - 4x - 21$
6. $x^2 - 22x + 121$
7. $c^2 - 4c - 12$
8. $p^2 - 16p + 64$
9. $9 - 10x + x^2$
10. $x^2 + 6x + 5$
11. $a^2 + 8a - 9$
12. $y^2 - 7y - 8$
13. $x^2 - 2x - 3$
14. $y^2 + 14y + 13$
15. $m^2 + 9m + 20$
16. $x^2 + 12x + 20$
17. $a^2 - 14a + 24$
18. $18 + 11y + y^2$
19. $x^2 + 2xy + y^2$
20. $a^2 - 4ab + 4b^2$
21. $x^2 + 6xy - 7y^2$

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8-3 Skills Practice

Factoring Trinomials: $x^2 + bx + c$

Factor each trinomial.

1. $t^2 + 8t + 12$

2. $n^2 + 7n + 12$

3. $p^2 + 9p + 20$

4. $h^2 + 9h + 18$

5. $n^2 + 3n - 18$

6. $x^2 + 2x - 8$

7. $y^2 - 5y - 6$

8. $g^2 + 3g - 10$

9. $s^2 + 4s - 12$

10. $x^2 - x - 12$

11. $w^2 - w - 6$

12. $y^2 - 6y + 8$

13. $x^2 - 8x + 15$

14. $b^2 - 9b + 8$

15. $c^2 - 15c + 56$

16. $-4 - 3m + m^2$

Solve each equation. Check your solutions.

17. $x^2 - 6x + 8 = 0$

18. $b^2 - 7b + 12 = 0$

19. $m^2 + 5m + 6 = 0$

20. $d^2 + 7d + 10 = 0$

21. $y^2 - 2y - 24 = 0$

22. $p^2 - 3p = 18$

23. $h^2 + 2h = 35$

24. $a^2 + 14a = -45$

25. $n^2 - 36 = 5n$

26. $w^2 + 30 = 11w$