Section 2 Part A (calculator):

1. The slope of a function at any point (x, y) is $\frac{e^x}{e^x+1}$. The point $(0, 2\ln 2)$ is on the graph of f.

(a) Write an equation of the tangent line to the graph of f at $x = 0$.

$$f'(x) = \frac{e^x}{e^x+1} \Rightarrow f'(0) = \frac{e^0}{e^0+1} = \frac{1}{1+1} = \frac{1}{2} \Rightarrow y = m(x - x_1) + y_1$$

$$\Rightarrow y = \frac{1}{2}(x-0) + 2\ln 2 \Rightarrow y = \frac{1}{2}x + 2\ln 2$$

(b) Use the tangent line in part (a) to approximate $f(0.1)$ to the nearest thousandth.

$$f(0.1) \approx y = \frac{1}{2}(0.1) + 2\ln 2 \approx 1.436$$

(c) Solve the differential equation $\frac{dy}{dx} = \frac{e^x}{e^x+1}$ with the initial condition $f(0) = 2\ln 2$.

$$f(x) = \int \frac{e^x}{e^x+1}\,dx \qquad \text{let} \quad u = e^x + 1, \quad du = e^x\,dx$$

$$\Rightarrow \int \frac{1}{u}\,du = \ln u + C \Rightarrow \ln(e^x+1) + C$$

$$2\ln 2 = \ln(e^0+1) + C$$

$$2\ln 2 = \ln(2) + C$$

$$2\ln 2 - \ln 2 = C$$

$$\ln 2 = C$$

$$\Rightarrow f(x) = \ln(e^x+1) + \ln 2$$

(d) Use the solution in part (c) and find $f(0.1)$ to the nearest thousandth.

$$\Rightarrow f(0.1) = \ln(e^{0.1}+1) + \ln 2 \approx 1.438$$

2. The temperature in a greenhouse from 7:00 p.m. to 7:00 a.m. is given by $f(t) = 96 - 20\sin\left(\frac{t}{4}\right)$, where $f(t)$ is measured in Fahrenheit and t is measured in hours.

(a) What is the temperature of the greenhouse at 1:00 a.m. to the nearest degree Fahrenheit?

$$f(6) = 96 - 20\sin\left(\frac{6}{4}\right) \approx 76°F$$

(b) Find the average temperature between 7:00 p.m. and 7:00 a.m. to the nearest tenth of a degree Fahrenheit.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{12}\int_0^{12}\left(96 - 20\sin\frac{t}{4}\right)dt$$

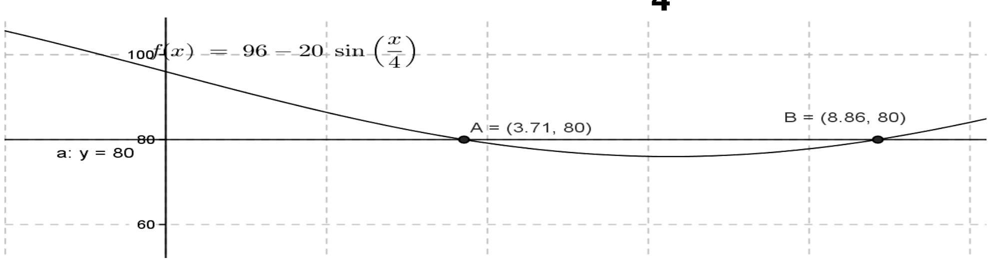
$$= \frac{1}{12}\left(96t - 4(20)\left(-\cos\left(\frac{t}{4}\right)\right)\right)\Bigg|_0^{12} = \frac{1}{12}\left(96t + 80\left(\cos\left(\frac{t}{4}\right)\right)\right)\Bigg|_0^{12}$$

$$= \frac{1}{12}\left(96(12) + 80\left(\cos\left(\frac{12}{4}\right)\right) - 0 + 80(\cos(0))\right)$$

$$= \frac{1}{12}(1152 + (-79.199\ldots) - 0 + 80) \approx 82.7°F$$

(c) When the temperature of the greenhouse drops below $80°F$, a heating system will automatically be turned on to maintain the temperature at a minimum of $80°F$. At what value of t to the nearest tenth is the heating system turned on?

With a calculator find where $y_1 = 96 - 20\sin\frac{t}{4}$ *and* $y_2 = 80$ intersect *at* $t \approx 3.7$ *and* $t \approx 8.9$



So between $3.7 \le t \le 8.9$ or between 10:42PM and 3:54AM the heat will be on.

(d) The cost of heating the greenhouse is \$0.25 per hour for each degree. What is the total cost to the nearest dollar to heat the greenhouse from 7:00 p.m. and 7:00 a.m.?

$$0.25\int_{3.7}^{8.9}\left(80 - \left(96 - 20\sin\frac{t}{4}\right)\right)dt = 0.25\int_{3.7}^{8.9}\left(-16 + \left(20\sin\frac{t}{4}\right)\right)dt$$

$$= 0.25\left(-16t - 80\cos\left(\frac{t}{4}\right)\right)\Bigg|_{3.7}^{8.9} = 0.25\left(-16(8.9) - 80\cos\left(\frac{8.9}{4}\right) + 16(3.7) + 80\cos\left(\frac{3.7}{4}\right)\right)$$

= ***\$3.41*** so it will cost \$3.41 to heat the greenhouse from 7PM to 7AM

3. A particle is moving along a straight line. The velocity of the particle for $0 \le t \le 30$ is shown in the table below for selected values of t.

t (sec)	0	3	6	9	12	15	18	21	24	27	30
$v(t)$ (m/sec)	0	7.5	10.1	12	13	13.5	14.1	14	13.9	13	12.2

(a) Using the midpoints of five subintervals of equal length, find the approximate value of $\int_0^{30} v(t)dt$.

$$\Delta t = \frac{30-0}{5} = 6 \quad \int_0^{30} f(t)dt = 6[v(3)+v(9)+v(15)+v(21)+v(27)]$$
$$= 6[7.5+12+13.5+14+13] = 6[60] = 360$$

(b) Using the result in part (a), find the average velocity over the interval $0 \le t \le 30$.

$$f_{ave} = \frac{1}{30-0}\int_0^{30} v(t)dt = \frac{1}{30}[360] = 12\ {}^{m}/_{sec}$$

(c) Find the average acceleration over the interval $0 \le t \le 30$.

$$\frac{v(b)-v(a)}{b-a} = \frac{v(30)-v(0)}{30-0} = \frac{12.2-0}{30-0} = \frac{12.2}{30} \approx 0.407\ {}^{m}/_{sec^2}$$

(d) Find the approximate acceleration at $t = 6$.

$$\frac{v(b)-v(a)}{b-a} = \frac{v(9)-v(3)}{9-3} = \frac{12-7.5}{9-3} = \frac{4.5}{6} = 0.75\ {}^{m}/_{sec^2}$$

(e) During what intervals of time is the acceleration negative?

Decreases from 18 seconds to 30 seconds, so negative acceleration from $18 \le t \le 30$ *sec onds*

Section 2 Part B (no calculator):

4. See figure 1T-13.

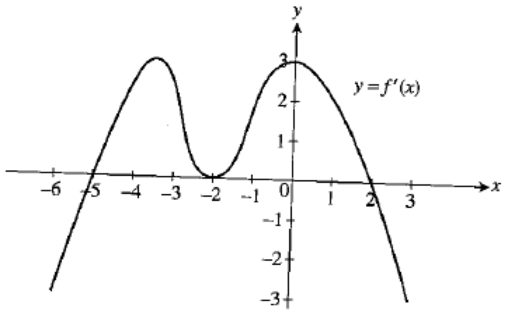
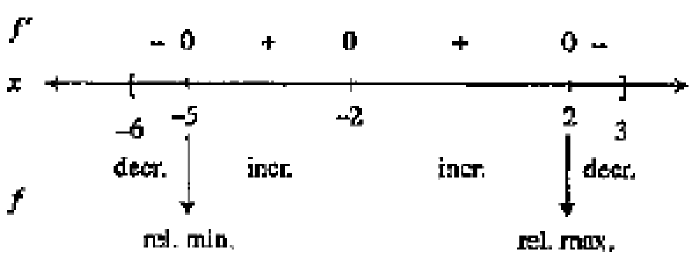


Figure 1T-13

The graph of f', the derivative of a function f, for $-6 \le x \le 3$ is shown in Figure 1T-13.

(a) At what value(s) of x does f have a relative maximum value? Justify your answer.

At x=2 since it increases from (-5,2) and decreases from (2,3)

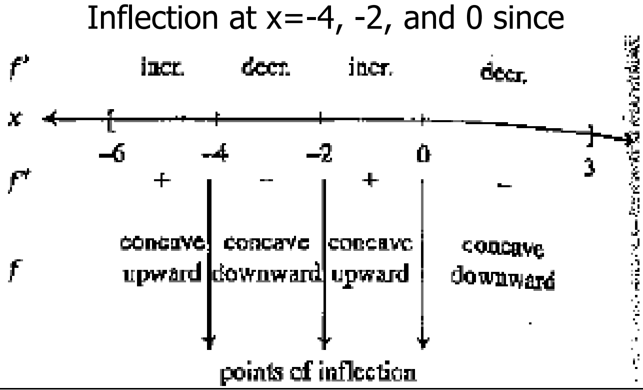


(b) At what value(s) of x does f have a relative minimum value? Justify your answer.

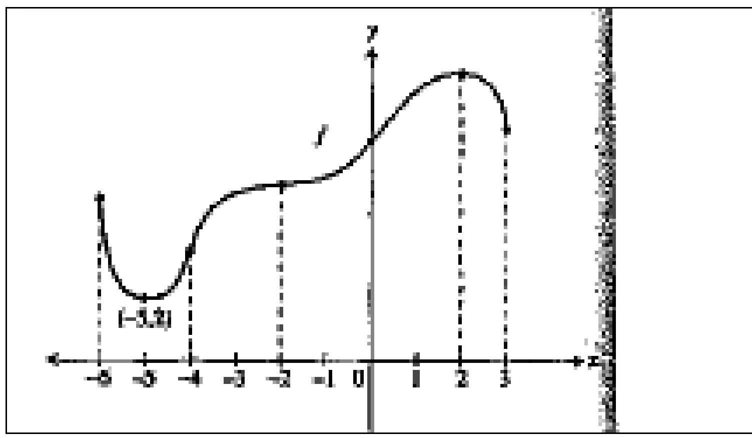
At x=-5 since it decreases from (-6,-5) and increases on (-5,2) at x=-5 there is a relative minimum.

(c). At what value(s) of x does the function have a point of inflection? Justify your answer.

Inflection at x=-4, -2, and 0 since



(d) If $f(-5) = 2$, draw a possible sketch of f on $-6 \le x \le 3$.



5. Given the equation $y^2 - x + 2y - 3 = 0$:

(a) Find $\frac{dy}{dx}$.

$$2yy' - 1 + 2y' - 0 = 0 \Rightarrow 2yy' + 2y' = 1 \Rightarrow y'(2y + 2) = 1 \Rightarrow y' = \frac{1}{2y + 2}$$

(b) Write an equation of the line tangent to the graph of the equation at the point $(0, -3)$.

$$\Rightarrow m = \frac{1}{2(-3) + 2} = -\frac{1}{4} \Rightarrow y = -\frac{1}{4}(x - x_1) + y_1 \Rightarrow y = -\frac{1}{4}(x - 0) - 3 \Rightarrow y = -\frac{1}{4}x - 3$$

(c) Write an equation of the line normal to the graph of the equation at the point $(0, -3)$.

Normal means perpendicular to tangent so: $\Rightarrow y = 4x - 3$

(d) The line $y = \frac{1}{4}x + 3$ is tangent to the graph at point P. Find the coordinates of point P.

$$y' = \frac{1}{2y + 2} = \frac{1}{4} \Rightarrow 4 = 2y + 2 \Rightarrow y = 1 \Rightarrow (1)^2 - x + 2(1) - 3 = 0 \Rightarrow x = 0$$

so the point P is at (0,1).

6. Let R R be the region enclosed by the graph of $y = x^2$ and the line $y = 4$.

(a) Find the area of region R.

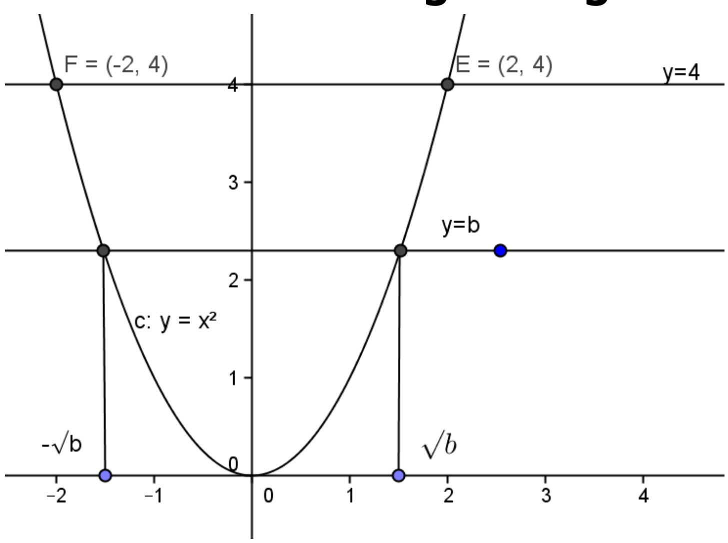
$$\int_{-2}^{2} 4 - x^2 dx = \left[4x - \frac{1}{3}x^3\right]_{-2}^{2} = \left[4(2) - \frac{1}{3}(2)^3 - 4(-2) + \frac{1}{3}(-2)^3\right] = \left[8 - \frac{8}{3} + 8 - \frac{8}{3}\right] = \frac{32}{3}$$

(b) If the line $x = a$ divides region R into two regions of equal area, find a.

Since $y = x^2$ is an even function, x=0 divides R into two regions of equal area. Thus a=0.

(c) If the line $y = b$ divides the region R into two regions of equal area, find b.

$$Area\ R_1 = Area\ R_2 = \frac{32}{3} \div 2 = \frac{16}{3}$$



$$\int_{-\sqrt{b}}^{\sqrt{b}} b - x^2 dx = 2\int_0^{\sqrt{b}} b - x^2 dx \Rightarrow 2\left[bx - \frac{1}{3}x^3\right]_0^{\sqrt{b}} = 2\left[b\sqrt{b} - \frac{1}{3}\left(\sqrt{b}\right)^3 - 0\right]$$

$$\Rightarrow 2\left[\frac{2}{3}\left(b\sqrt{b}\right)\right] = \frac{4}{3}\left(b\sqrt{b}\right) \Rightarrow \frac{4}{3}\left(b\sqrt{b}\right) = \frac{16}{3}$$

$$\Rightarrow \left(b\sqrt{b}\right) = 4 \Rightarrow b^{3/2} = 4 \Rightarrow b = 4^{2/3}$$

(d) If region R is revolved about the x-axis, find the volume of the resulting solid.

$$\pi\int_{-2}^{2} \left(4^2 - \left(x^2\right)^2\right) dx = \pi\int_{-2}^{2} \left(16 - x^4\right) dx$$

$$= \pi\left(16x - \frac{1}{5}x^5\right)_{-2}^{2} = \pi\left[16(2) - \frac{1}{5}(2)^5 - 16(-2) + \frac{1}{5}(-2)^5\right] = \pi\left[32 - \frac{32}{5} + 32 - \frac{32}{5}\right]$$

$$= \frac{256\pi}{5}$$