Task: Modeling Production Output
PreCalculus, Advanced Algebra & Trigonometry (10^{th}-12^{th})

***Task Description**:*
For this task students will be asked to investigate the production output of a machine at Otics. The daily production quota must be met and will not change; however, the hourly output of the machine will fluctuate. Students will have to model the hourly output and determine if the daily quota can be met.

Prerequisite Knowledge:
- Students should be able to do mathematical modeling (i.e. recognizing which family of functions a plotted graph corresponds to).
- Students must be able to read, interpret, and create graphs.

Suggested Units:
- Graphing Trigonometric Functions / Modeling Trigonometric Functions
- Sigma notation / Series
- Reading / Interpreting graphs
- Connecting the graph of the derivative with the graph of the function
- Differentiation and / or Integration
- Modeling with regression
    - Linear Functions: $y=mx+b$
    - Quadratic Functions: $y=Ax^2+Bx+C$ or $y=A(x-h)^2+k$
    - Cubic Functions: $y=Ax^3+Bx^2+Cx+D$ or $y=A(x-h)^3+k$
    - Even Polynomials: $y= A(x-h)^{even}+k$
    - Odd Polynomials: $y= A(x-h)^{odd}+k$
    - Trigonometric Functions: $y=A \text{ trig } B(x-C)+D$
    - Exponential Functions: $y=A * B^{x-C}+D$

Lesson Paths:
This lesson can be adapted to fit various mathematical topics. The functions that are chosen will vary depending on the chose topic.

Note: IF the class has covered all of these topics, this could be a group-based activity with each presenting their results.

Note: The student work required depends on the objective for that lesson.
Trig: accurately model the machine's output with an equation
Sigma Notation / Series: predict the machine's output for the day
Derivatives: create a graph of the parts per hour and the overall parts
Integrals: create a total output graph from the given rates of production
Graphing: change the sample problem so that the students must get all the required information from a graph or table

Sample Problem:

Toyota has placed an order for roller rockers (small engine part). Though the machine is programmed to run at a constant rate of 1 part per 1.5 seconds, this number varies throughout the day. At various times throughout the day the Team Leader measures the machine's output and records the data in the output log:

Start of shift: 3600 parts per hour
3^{rd} hour: 4800 parts per hour
6^{th} hour: 3600 parts per hour
9^{th} hour: 2400 parts per hour
12^{th} hour: 3600 parts per hour
15^{th} hour: 4800 parts per hour
18^{th} hour: 3600 parts per hour
21^{st} hour: 2400 parts per hour
24^{th} hour: 3600 parts per hour

Note: These values may be changed to fit the desired context / objective.

A. Plot the given information to show the production rate of each hour.

B. Based from the given rates, how many parts will be produced after:

- 3 hours
- 6 hours
- 10 hours
- 12 hours
- 24 hours

C. Plot the information from part B on a separate graph.

D. Create an equation to model the machine's production rate. Use this equation to predict the production rate for the 30^{th}, 40^{th}, and 48^{th} hour. Graph this equation on the same coordinate plane as part A.

E. Create an equation to model the machine's total output for each hour. Graph this equation on the same coordinate plane as part C.

F. Use your equation from part E to predict the total output for each hour listed in part B.

G. Use Sigma Notation to represent this sum

H. Explain why the machine's rate varies throughout the day.

Common Core State Standards

Advanced Algebra & Trigonometry

Algebra – Sequences & Series – 1, 2
Algebra – Conic Section – 2
Functions – Building Functions – 1
Functions - Interpreting Functions – 4, 7
Functions - Graphing Trig Functions – 1, 4
Statistics & Data - Model with data - 1, 2, 3

PreCalculus:

There is little discernable difference between the standards for precalculus than advanced algebra and trigonometry.

Essential Understandings

Students should be able to model real world situations with the mathematics they have encountered. They should see the connection between the actual situation, symbolic representation (algebra), and graphical representation. Then they can use that information to make predictions.

The students should understand the role of A, B, C, & D in the standard trig equation y= A trig B(x-C)+D. This may change if the given data is not trigonometric in nature.

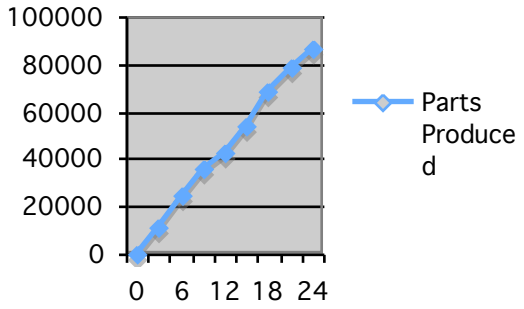
If the lesson is done in conjunction with a unit on sequences, sums, and series, students should understand the difference between the individual terms of the sequence and the partial sums of the sequence.

Possible Solutions/Solution Paths

The instructions are rather clear. Acceptable answers will have the correct equations and hit all of the relevant data points. The answers for B part may vary based on how the students interpret the data; however, after the equations have been generated that answers should be the same.

A. A correct plot of the numbers given should look similar to this graph. The students should notice that the pattern is that of a since wave; however, the amplitude, period, and vertical shift have been altered. At this point the graph does not need to be connected, but it should be labeled. *Be sure to check that this is a graph of **parts per hour.***

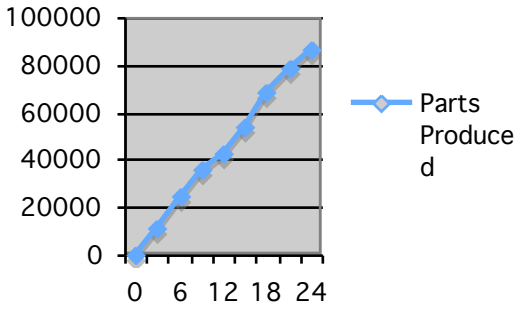
B. Answers for this section will vary depending on the students' understanding of trigonometric graphs. Though the approach will not be emphasized, students who may have some calculus connections may approach this by using left-hand, right-hand, midpoint, or trapezoidal sums. The suggested approach will be to use an Excel Spread sheet.



Hours Ran	Production Rate	Total
3	3600	10800
3	4800	14400
3	3600	10800
3	2400	7200
		43200

The students may notice that the values cycle twice. Hence, the total output by this method is 86,400 pieces. Other methods may produce different results, but the values should be close.

C. The key for this section is that the students are focusing on the **total number of parts produced.** This is not parts per hour. The answers for this section will depend on the method chosen for part B. Depending on the scale chosen for their graphs, the students may believe that the output function is linear.



D. To create the intended equation students must understand the transformation of trig functions, in particular amplitude, period, and vertical shift. Then the students must use that function to predict future production values. Some of the values chosen can be found by recognizing the pattern while others must be found by a formula.

R(t) = 1200 sin (pi/6) x + 3600

R(30)= 3600

R(40)= 4640 NOTE: This requires correct rounding

R(48) = 3600

E. The solution to this problem will depend on part B & C; however, if this lesson is present close to a unit on sequences, sum, or series, the students may quickly create the formula:

$$\sum_{x=0}^{23}\left(1200\sin\left(\frac{\pi x}{6}\right)+3600\right)=86\,400$$

The students should be cautious with the index of summation. Most problems students encounter in the textbook begin with x=1. Also, student will be tempted to end the series at 24, but a quick count would show that this is actually 25 hours. The answer achieved by either of these two mistakes would disagree with the answers obtained in previous steps.

An additional approach would be for the students to model the situation with a linear equation.

P(t) = 3600 t

H. Answers will vary heavily.

Additional Teacher Information

There will be some confusion over the difference between a production total and the rate of production.

Depending on the class, a worksheet may be necessary to guide the students through the process.

Though I don't recommend using 86,400 blocks to represent the problems, a scaled physical representation of the problem could help students grasp the problem. If 1 block = 1200 parts, students should be able to understand the problem. One should note, that though this may help the students understand, the will still be expected to represent the problem with graphs and equations.