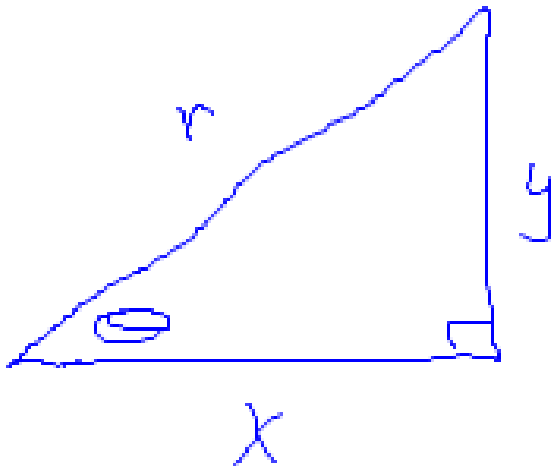
Real World Applications Involving Vectors

Vectors can be used to describe any type of force, distance, motion, ect.

Once again, we will solve all problems using the component method.

To determine the x and y components, use right triangle trigonometry.



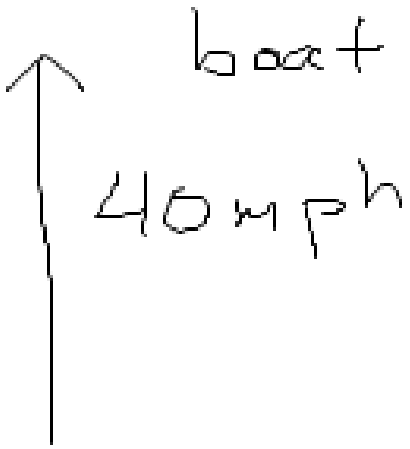
$r\cos\theta = \frac{x}{r}$ or

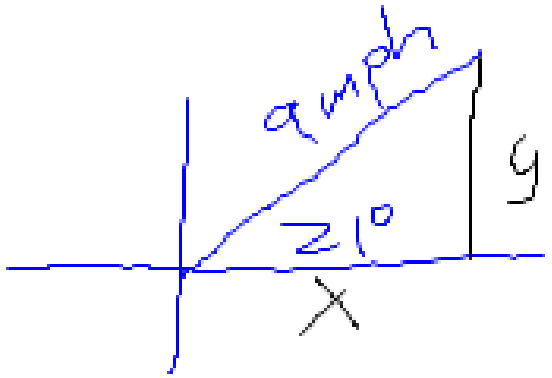
$x = r\cos\theta$

$r\sin\theta = y/r$ or

$y = r\sin\theta$

Example 1: A boat is headed north at 40 mph. The current is 9 mph at an angle of 21 degrees. Determine the speed and direction of the boat.





$x = 9 \cos 21^\circ$

$y = 9 \sin 21^\circ$

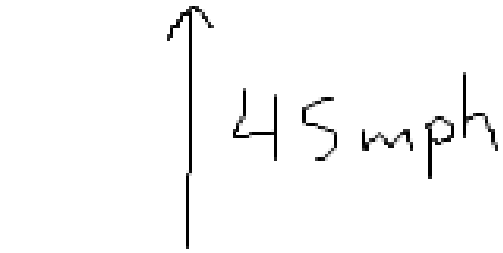
$\langle 0, 40 \rangle$

$+ \langle 8.4, 3.2 \rangle$

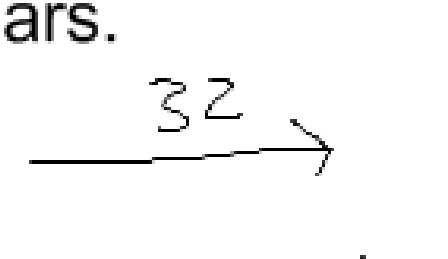
Boat + water $\langle 8.4, 43.2 \rangle$

44.0 mph at 79°

Example 2: Two cars head into an intersection. One car, headed north at 45 mph, strikes another car that was headed east at 32 mph. What are the magnitude and direction of the conjoined cars.



$\langle 0, 45 \rangle$



$\langle 32, 0 \rangle$

$+ \langle 32, 0 \rangle$

Both cars $\langle 32, 45 \rangle$

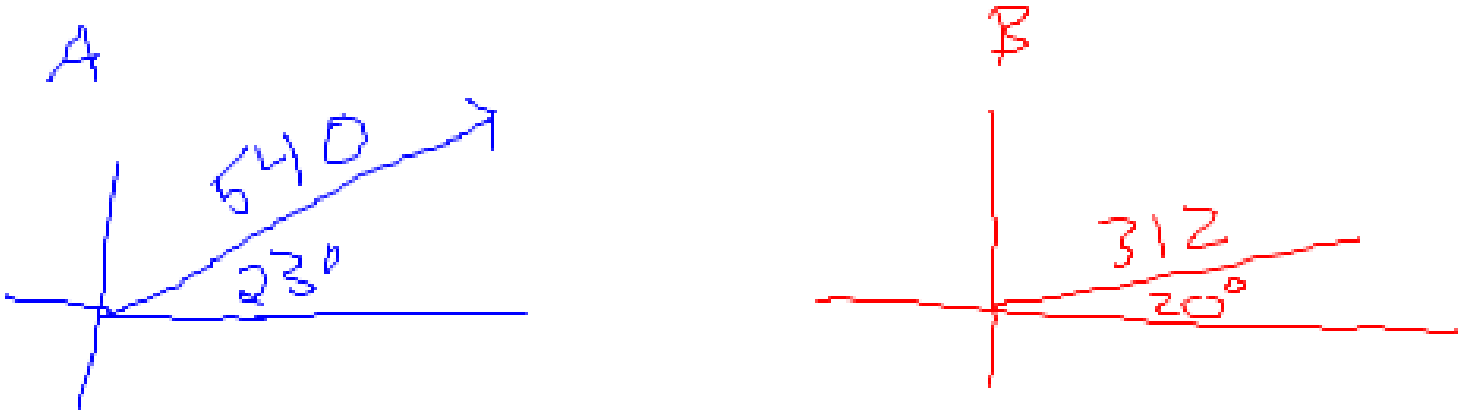
$\sqrt{32^2 + 45^2}$

55.22

55.22 mph at 54.6°

$\tan^{-1}(45/32) = 54.6$

3. Add vectors A and B using the component method. Vector A has a magnitude of 540 and an angle of elevation of 23°. Vector B has a magnitude of 312 and an angle of elevation of 20°.



$x = 540 \cos 23^\circ = 497$

$y = 540 \sin 23^\circ = 211$

$x = 312 \cos 20^\circ = 293$

$y = 312 \sin 20^\circ = 107$

$\vec{A} + \vec{B} = \langle 790, 318 \rangle$

$= \underline{852} \text{ at } \underline{22^\circ}$

$\sqrt{790^2 + 318^2}$

852

$\tan^{-1}\left(\frac{318}{790}\right) \approx$

22°

4. An object is moving with a speed of 180 m/sec in an x-y coordinate system. Its direction is that described by a 137° angle in standard position. What are the *X* and *Y* components of the velocity of the object?

$X = 180 \cos 137^\circ = -131.6$ mph

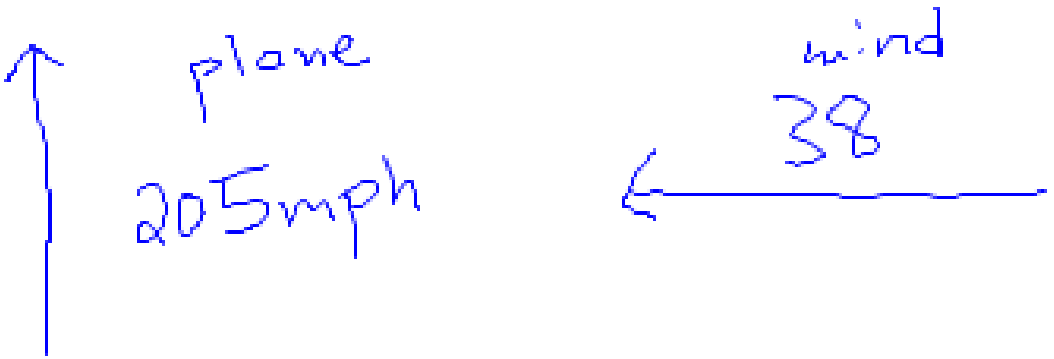
$y = 180 \sin 137^\circ = 122.8$ mph

8. A football is thrown due south at 5 m/sec from a car that is traveling east at 30 m/sec. Find the speed of the ball relative to the ground and its direction of travel.

Left to reader

9. An airplane is headed due north with a speed of 205 mph. The wind is blowing from the east at a speed of 38 mph. What is the navigational heading and speed of the plane's shadow on the ground?

N 8.8° W



$\langle 0, 205 \rangle + \langle -38, 0 \rangle$

plane + wind $= \langle -38, 205 \rangle =$ 208.4mph at 98.88°

$\sqrt{(-38)^2 + (205)^2}$

208.4

$\tan^{-1}\left(\frac{205}{-38}\right) = -81.12$

$+180$

98.88