

# The Polar Coordinate System

**Polar Coordinate System:** Recording the position of an object using the distance from a fixed point and an angle with a fixed ray from that point

The polar axis is horizontal from the pole (the positive x axis) and the fixed point (pole) is located at the origin.

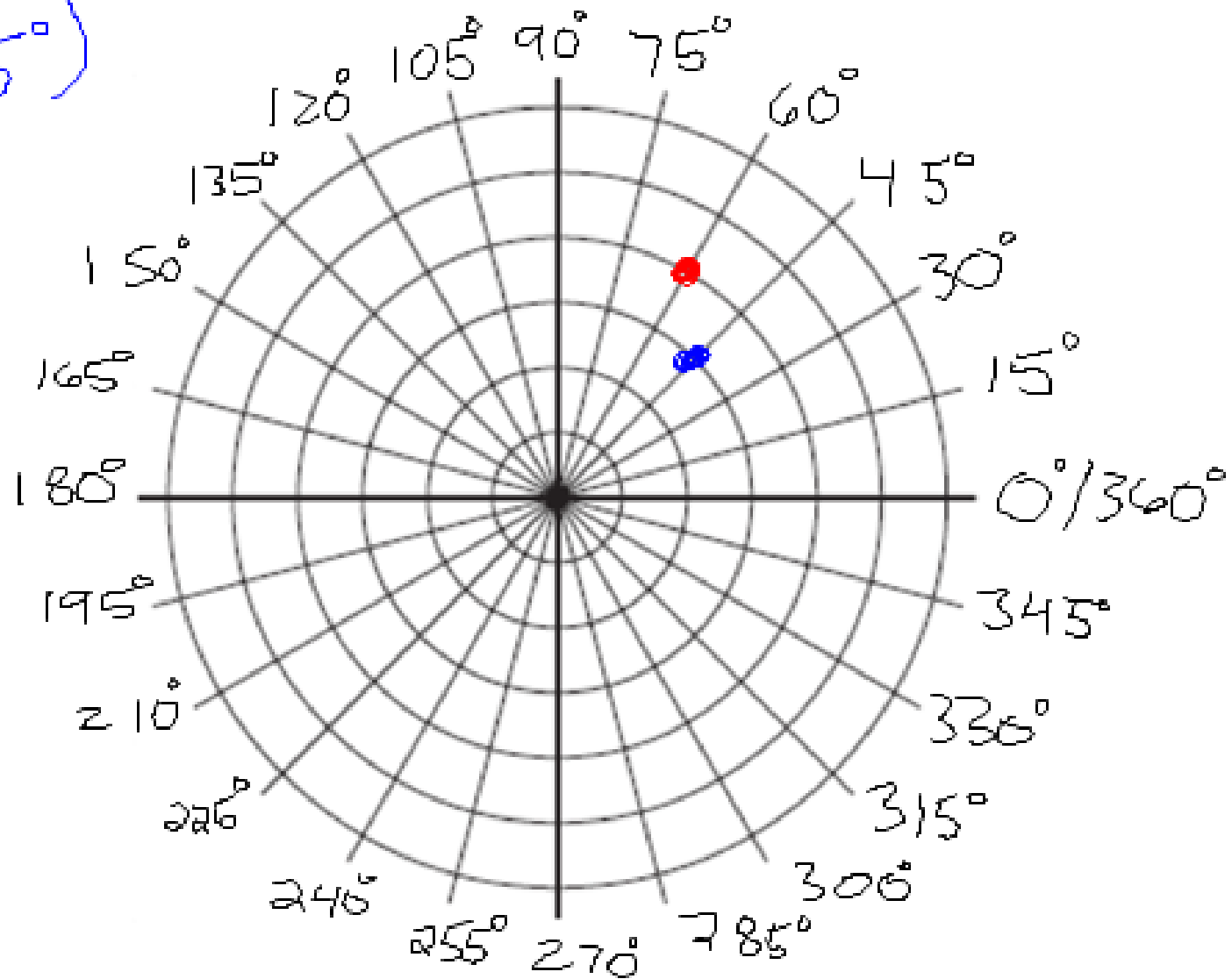
A polar point is represented as

$(r, \theta)$

The polar grid is represented in 15 degree intervals.  
The circles represent a unit distance.

Plot  $(3, 45^\circ)$

$(4, 60^\circ)$



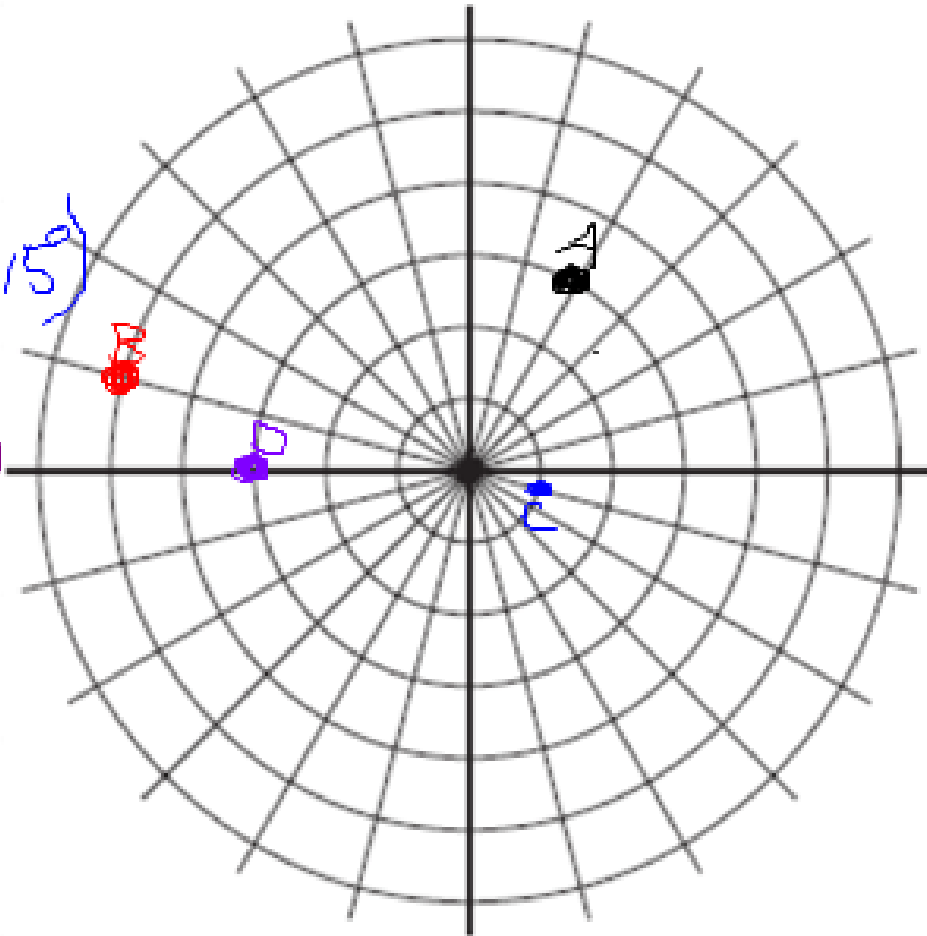
Example 1: Plot the following polar points:

a:  $(3, 60^\circ)$

b:  $(5, 165^\circ)$

c:  $(1, \frac{7\pi}{4}) = (1, 315^\circ)$

d:  $(3, \pi) = (3, 180^\circ)$



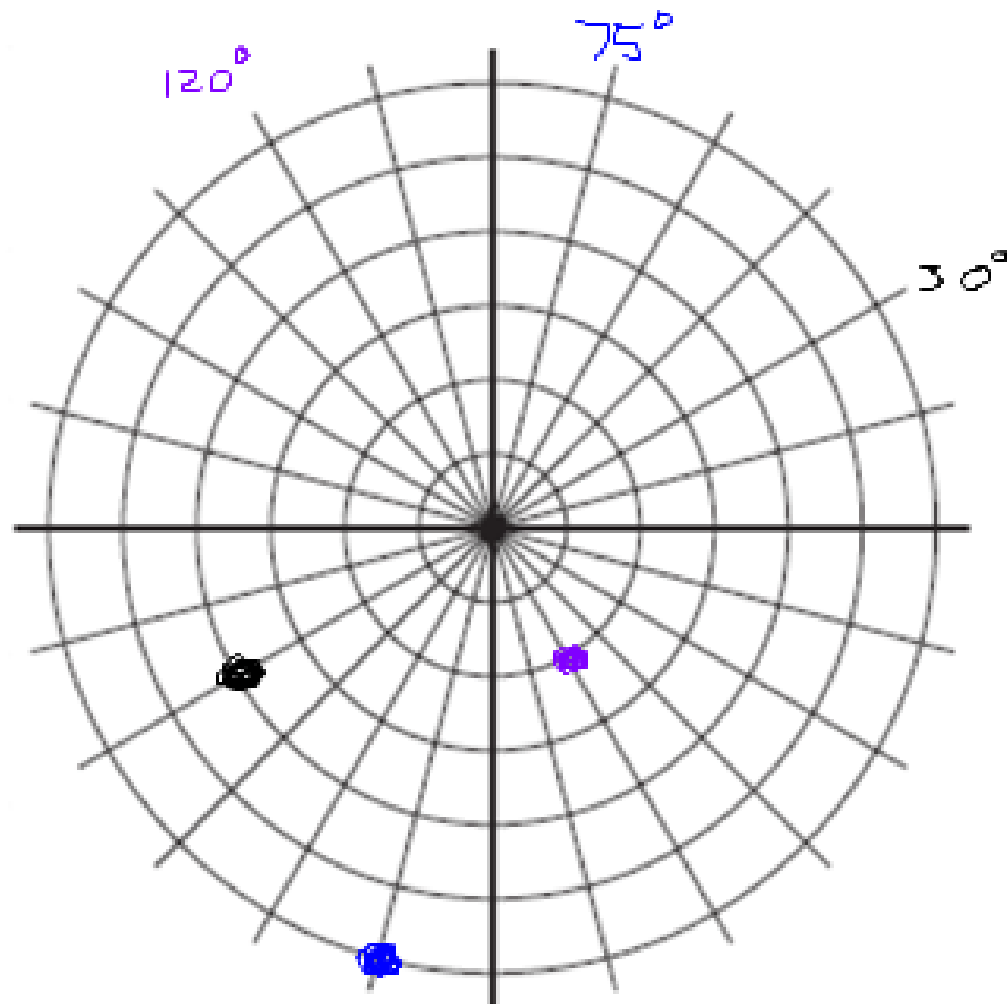
Points can also be plotted using a negative value for  $r$  as seen below.

Example 2: Plot the following points:

a:  $(-2, 120^\circ)$

b:  $(-4, 30^\circ)$

c:  $(-6, 75^\circ)$



Since we can define points with negative values of  $r$ , and we can rotate around the circle more than once, we can represent similar points that share the same location (kind of like co-terminal angles).

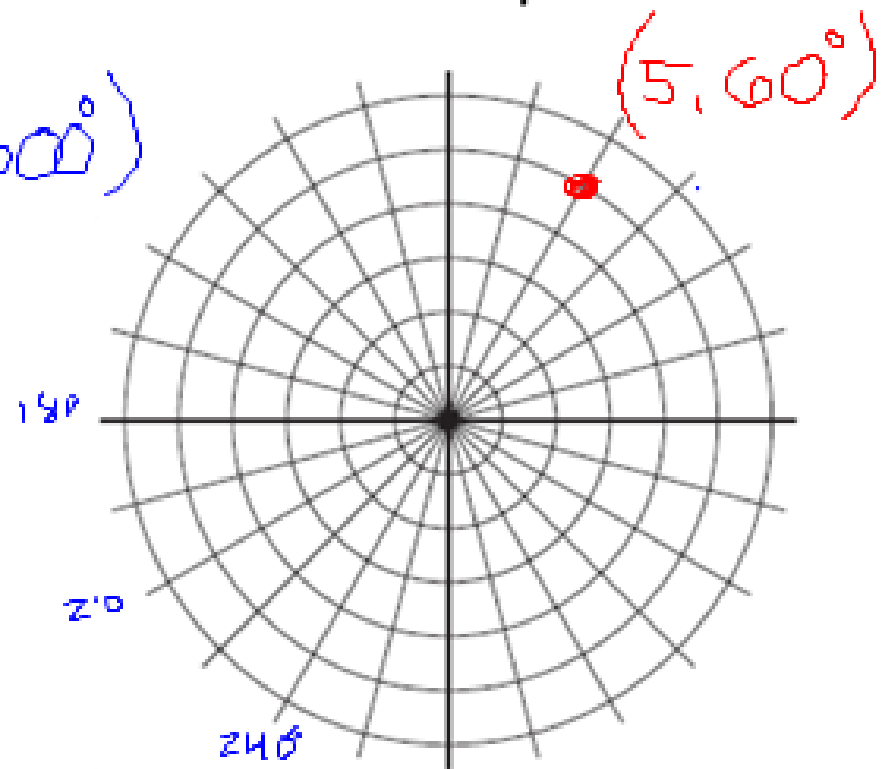
Below are 5 ways to represent the same point:

$$(-5, 240^\circ) \quad (-5, 60^\circ)$$

$$(5, 420^\circ)$$

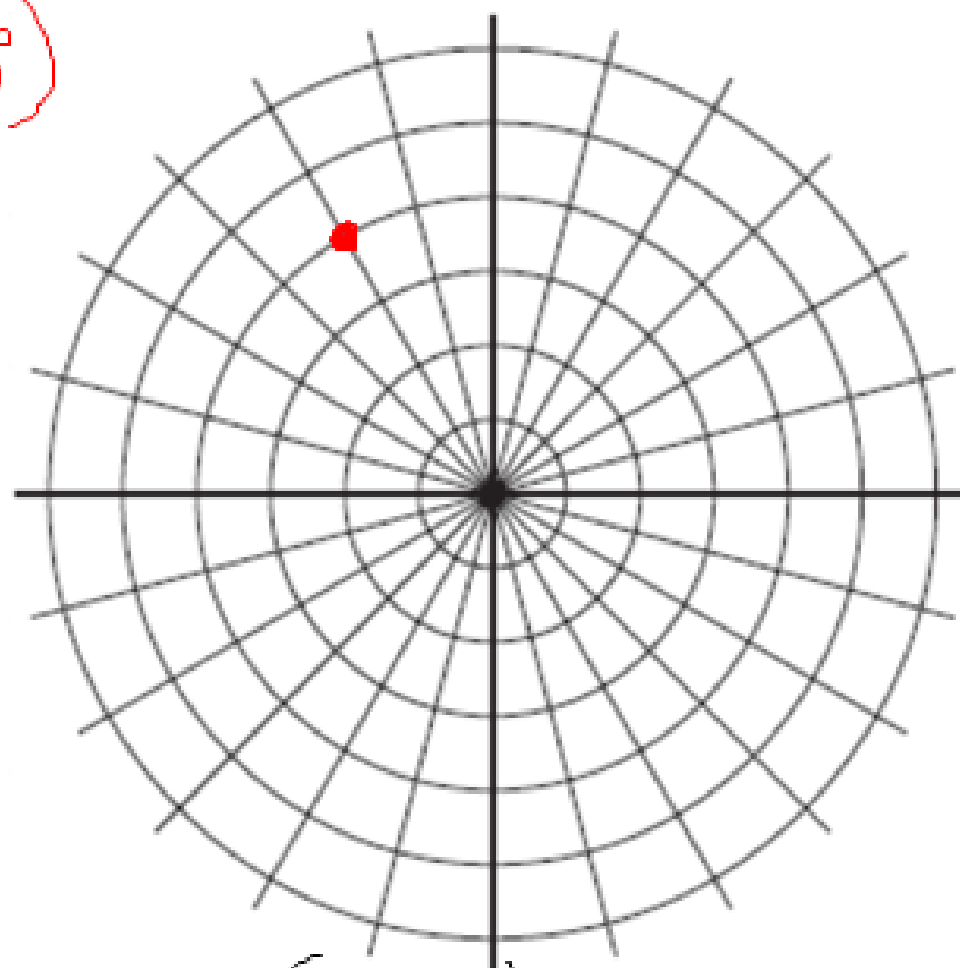
$$(5, 780^\circ)$$

$$(5, -300^\circ)$$



Example 3: Name four similar points to the point graphed below:

Point  $(4, 120^\circ)$



1)  $(4, 120^\circ)$

2)  $(4, 156^\circ)$

3)  $(4, -240^\circ)$

4)  $(-4, 300)$

also  $(4, \frac{2\pi}{3})$

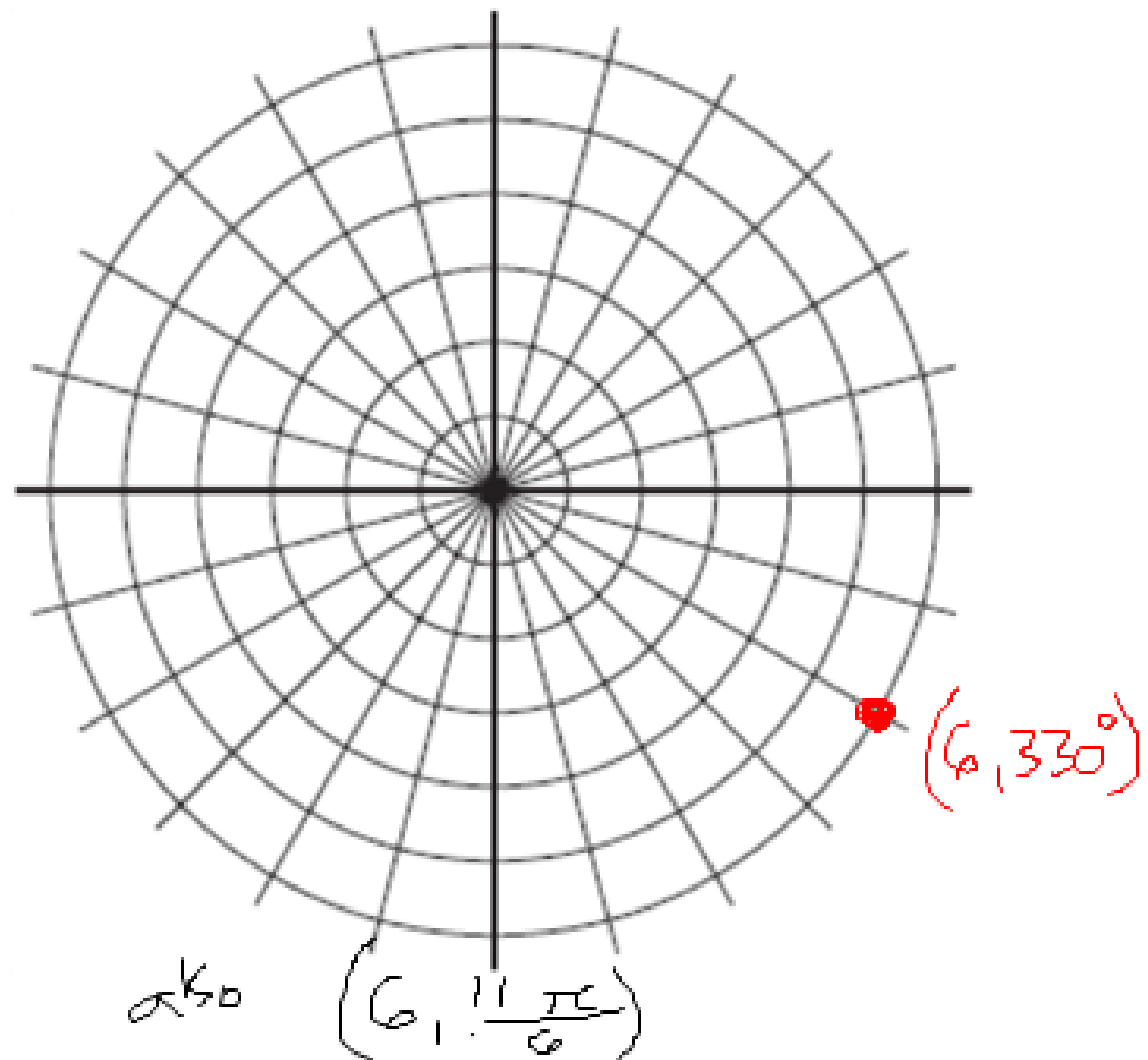
Example 4: Name four similar points to the point graphed below:

1)  $(6, 1050^\circ)$

2)  $(6, -30^\circ)$

3)  $(-6, 150^\circ)$

4)  $(6, 1770^\circ)$



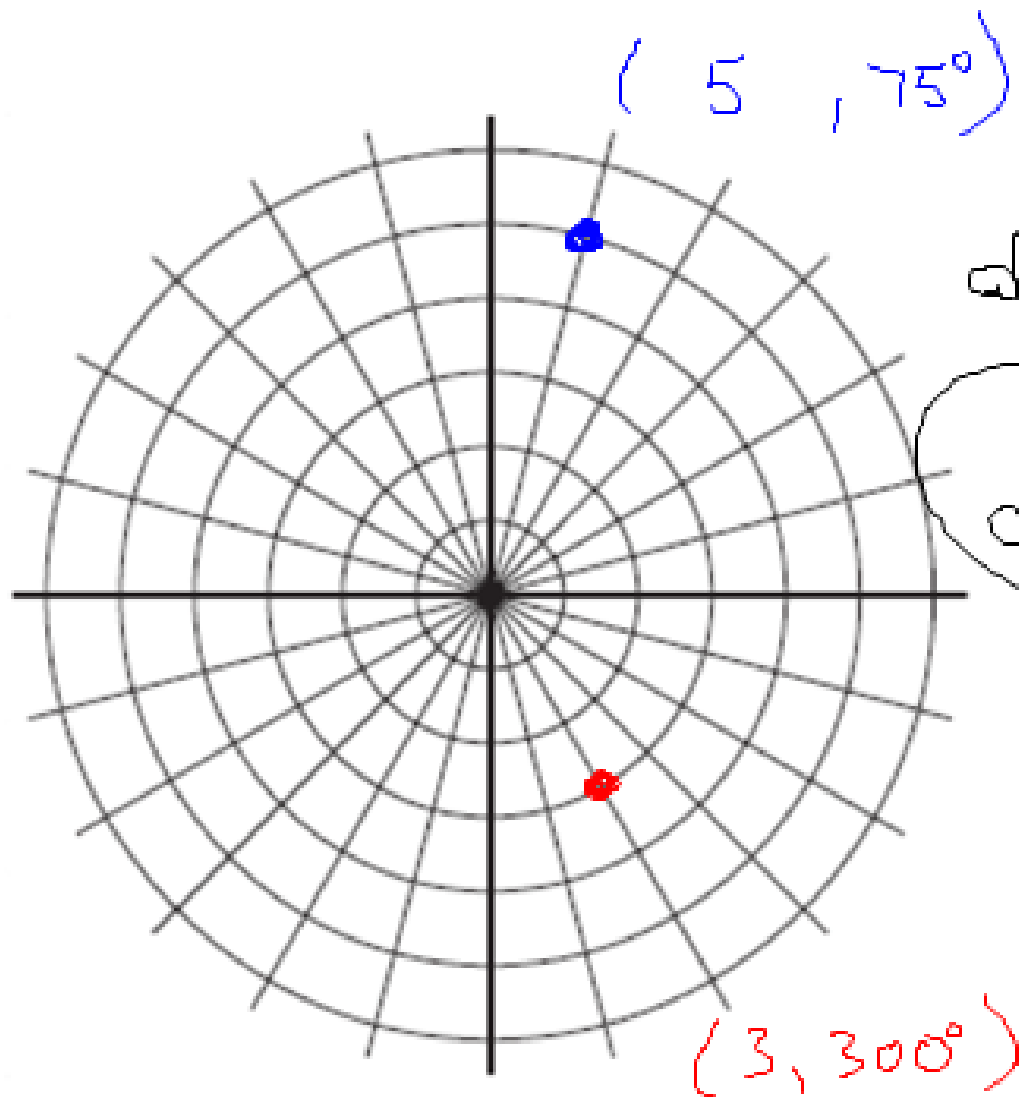
We can determine the distance between two polar points using the Polar Distance Formula. This formula should look very familiar.

$$d = \sqrt{(r_1)^2 + (r_2)^2 - 2(r_1)(r_2)\cos(\theta_2 - \theta_1)}$$

"Law of Cosines"



Example 5: Determine the distance between the following points:

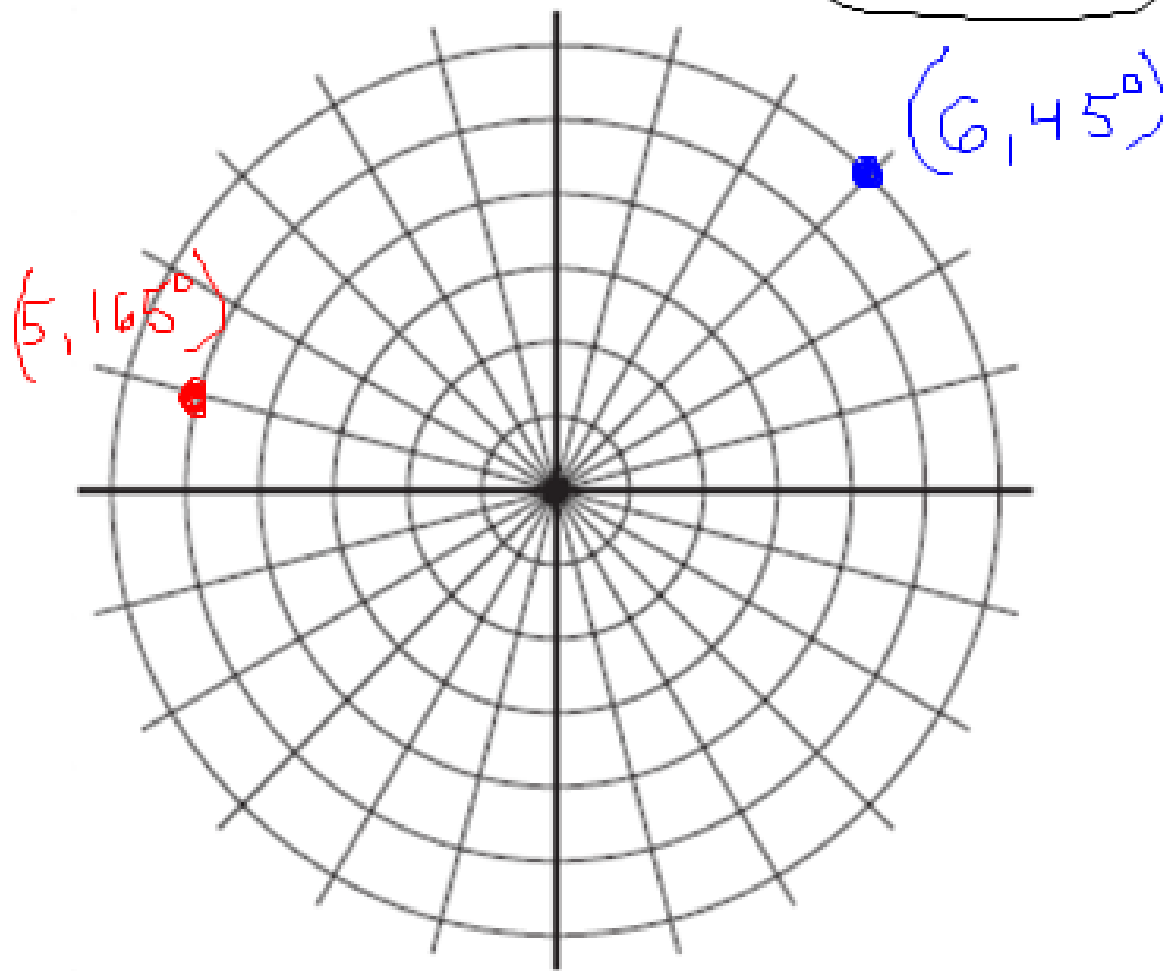


$$d = \sqrt{5^2 + 3^2 - 2(3)(5)\cos(225^\circ)}$$

$$d = 7.43$$

Example 6: Determine the distance between the following points:

$$d = \sqrt{6^2 + 5^2 - 2(6)(5) \cos(120^\circ)}$$
$$= 9.53$$

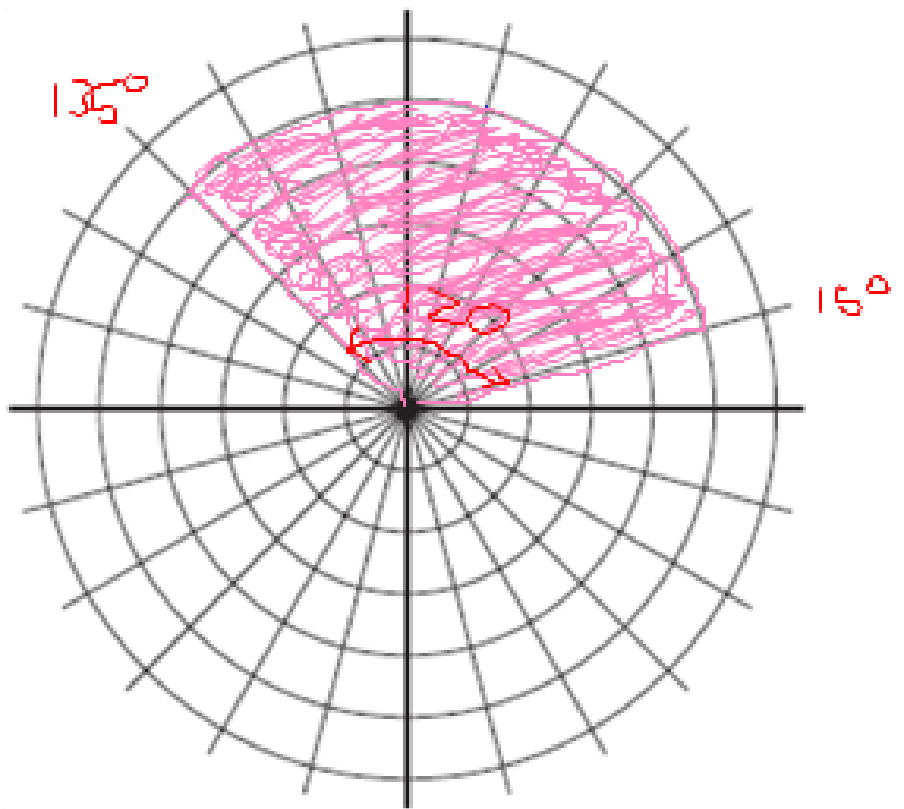


Finally, we can calculate the area of a sector (slice of pizza) using the following formula:

Example 7: Determine the area of the sector shaded below.

$$A = \pi r^2 \left( \frac{\theta}{360} \right)$$

$$A = \pi (5)^2 \cdot \left( \frac{120}{360} \right)$$
$$= 26.2$$



Homework: p 558 #16-31, 42-45, 49, 50, 52