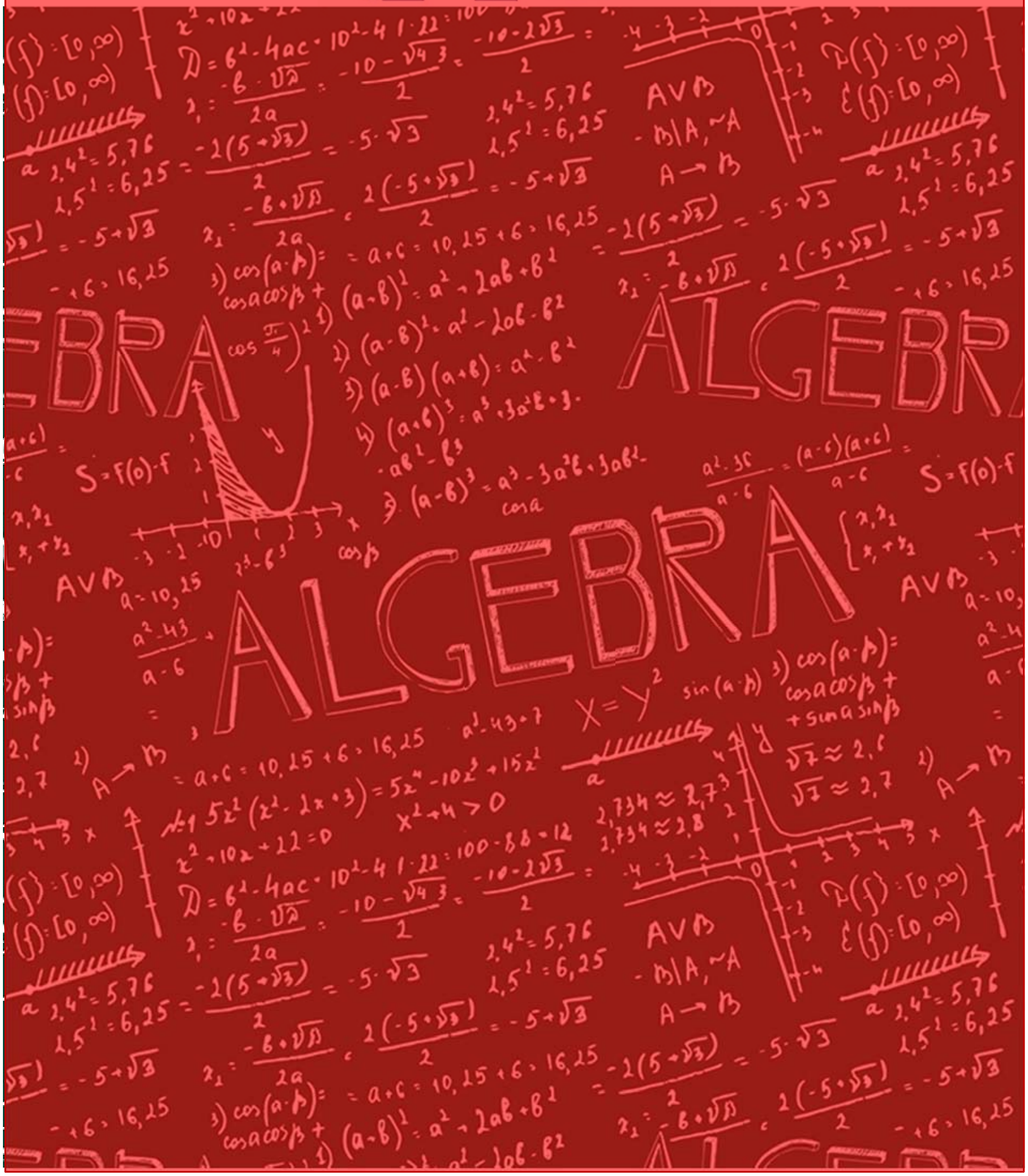
HOUSTON COUNTY
BOARD OF EDUCATION
HIGH-ACHIEVING TEACHERS

MATHEMATICS
DEPARTMENT



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MATHEMATICS
DEPARTMENT

Science Technology Engineering Mathematics

Algebra 1

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Unit 1

Relationships Between Quantities

Algebra 1

Unit 1: Relationships Between Quantities

Term	Definition	Notation	Diagram/Visual
Ratio			
Proportion			
Scale			
Dimensional Analysis			
Expression			
Term			
Coefficient			
Factor			

monomial	______________________ ______________________ ______________________
polynomial	______________________ ______________________ ______________________
Distributive property	______________________ ______________________ ______________________
Closure Property	______________________ ______________________ ______________________
square	______________________ ______________________ ______________________
Square root	______________________ ______________________ ______________________
Rational numbers	______________________ ______________________ ______________________
Irrational numbers	______________________ ______________________ ______________________
Product Property of Radicals	______________________ ______________________ ______________________

Quotient Property of Radicals	

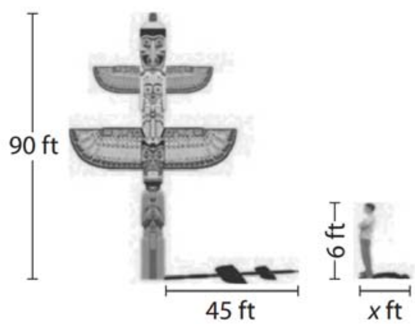
Units of Measure Graphically and Situationally

Using Ratios and Proportions to Solve Problems

Ratios and proportions are very useful when solving real-world problems. A ratio is a comparison of two numbers by division. An equation that states that two ratios are equal is called a proportion.

GUIDED NOTES

A totem pole that is 90 feet tall casts a shadow that is 45 feet long. At the same time, a 6-foot-tall man casts a shadow that is x feet long. The man and the totem pole are both perpendicular to the ground, so they form right angles with the ground. The sun shines at the same angle on both, so similar triangles are formed. Find the length of the man's shadow in feet.



Questions To Ponder

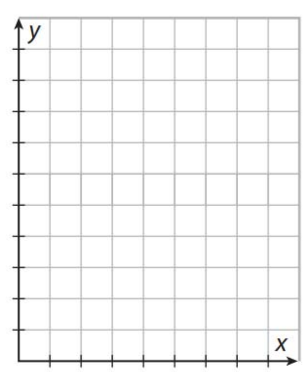
What is another ratio that could be written for this problem? Use it to write and solve a different proportion to find the length of the man's shadow in feet. Explain why your new proportion and solution are valid.

GUIDED NOTES

Graphing a Proportional Relationship

To graph a proportional relationship, first find the unit rate, then create scales on the x- and y-axes and graph points.

Simon sold candles to raise money for the school dance. He raised a total of $25.00 for selling 10 candles. Find the unit rate (amount earned per candle). Then graph the relationship.



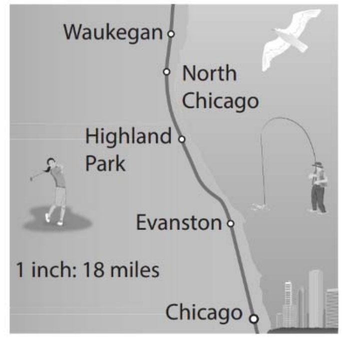
GUIDED NOTES

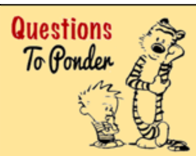
Using Scale Drawings and Models to Solve Problems

A <u>*scale*</u> *is the ratio of any length in a scale drawing or scale model to the corresponding actual length. A drawing that uses a scale to represent an object as smaller or larger than the original object is a scale drawing. A three-dimensional model that uses a scale to represent an object as smaller or larger than the actual object is called a scale model.*

Use the map to answer the following question.
The actual distance from Chicago to Evanston is 11.25 mi.

What is the distance on the map?





When you solve a proportion, why is the equation still true even after multiplication is used on it?

GUIDED NOTES

Using Dimensional Analysis to Convert Measurements

<u>*Dimensional analysis*</u> *is a method of manipulating unit measures algebraically to determine the proper units for a quantity computed algebraically. The comparison of two quantities with different units is called a rate. The ratio of two equal quantities, each measured in different units, is called a conversion factor.*

Dimensional analysis works because of the Multiplicative Identity Property. This property states that any number can be multiplied by 1 and maintain its value. In dimensional analysis, you are multiplying by a fraction whose numerator and denominator are equivalent amounts, such as 16 ounces and 1 pound. Whenever the numerator and denominator are equivalent, the fraction is equal to 1.

A large adult male human has about 12 pints of blood. Use dimensional analysis to convert this quantity to gallons.

The length of a building is 720 in. Use dimensional analysis to convert this quantity to yards

Questions To Ponder

When setting up a conversion factor, do you put the given units in the numerator or in the denominator? Why?

How do you ensure that the result of your calculation is in the desired unit?

GUIDED NOTES

Using Dimensional Analysis to Convert and Compare Rates

Use dimensional analysis to determine which rate is greater.

During a cycling event for charity, Amanda traveled 105 kilometers in 4.2 hours and Brenda traveled at a rate of 0.2 mile per minute. Which girl traveled at a greater rate?

A box of books has a mass of 4.10 kilograms for every meter of its height. A box of magazines has a mass of 3 pounds for every foot of its height. Which box has a greater mass per unit of height? Use 1 lb = 0.45 kg and 1 m = 3.28 ft. Round your answer to the nearest tenth.

Questions To Ponder

Why is it important to convert rates to the same units before comparing them?

SELF CHECK

1. Alan's go-kart travels 1750 feet per minute, and Barry's go-kart travels 21 miles per hour. Whose go-kart travels faster? Round your answer to the nearest tenth.
2. A scale model of a human heart is 196 inches long. The scale is 32 to 1. How many inches long is the actual heart? Round your answer to the nearest whole number.

Use dimensional analysis to convert the measurements. Round answers to the nearest tenth.

3. 7500 seconds ≈ hours
4. 3 feet ≈ meters
5. 4 inches ≈ yards
6. Give three examples of proportions. How do you know they are proportions? Then give three nonexamples of proportions. How do you know they are not proportions?
7. If a scale is represented by a ratio less than 1, what do we know about the actual object? If a scale is represented by a ratio greater than 1, what do we know about the actual object?

	Customary Units	
Length	12 in	1 ft
	3 ft	1 yd
	5280 ft	1 mi
Weight	16 ounces (oz)	1 pound (lb)
	2000 lb	1 ton (T)
Capacity	8 fl oz	1 c
	2 c	1 pt
	2 pt	1 qt
	4 qt	1 gal

	Metric Units Kilo Hecta Deka (base) deci centi milli
Length	Km Hm Dm meters dm cm mm
Weight	Kg Hg Dg grams dg cg mg
Capacity	KL HL DL Liters dL cL mL

	Customary Units	Metric Units
Length	1 in	2.54 cm
	1 ft	0.3048 m
	1 mi	1.609 km
Weight	1 oz	28.350 g
	1 lb	0.454 kg
Capacity	1 fl oz	29.574 mL
	1 gal	3.785 L

Time	
60 seconds	1 minute
60 minutes	1 hour
24 hours	1 day
7 days	1 week
12 months	1 year
52 weeks	1 year
365 days	1 year

Independent Practice

Example!

Multiplying by 1 does not change the value of a number: $33 \bullet 1 = 33$

So, multiplying by a fraction equal to 1 does not change the value either: $33 \bullet \frac{5}{5} = 33$

Multiplying by 1 is like converting rates.

Example
Convert 150 feet per minute to miles per hour.

Step 1 Convert feet to miles.

$\frac{150\text{ ft}}{1\text{ min}} \bullet 1$

1 mile = 5280 ft

$$\frac{150\,\cancel{\text{ft}}}{1\text{ min}} \bullet \frac{1\text{ mi}}{5280\,\cancel{\text{ft}}} = \frac{150\text{ mi}}{5280\text{ min}}$$

Step 2 Convert minutes to hours.

$\frac{150\text{ mi}}{5280\text{ min}} \bullet 1$

$$\frac{150\text{ mi}}{5280\,\cancel{\text{min}}} \bullet \frac{60\,\cancel{\text{min}}}{1\text{ h}} = \frac{9000\text{ mi}}{5280\text{ h}} = 1.7\frac{\text{mi}}{\text{h}}$$

1. Convert 8 milliliters to fluid ounces. Use 1 mL ≈ 0.034 fl oz.	2. Convert 12 kilograms to pounds. Use 1 kg ≈ 2.2 lb.
3. The dwarf sea horse Hippocampus zosterae swims at a rate of 52.68 feet per hour. Convert this speed to inches per minute	**4.** Tortoise A walks 52.0 feet per hour and tortoise B walks 12 inches per minute. Which tortoise travels faster? Explain
5. The pitcher for the Robins throws a baseball at 90.0 miles per hour. The pitcher on the Bluebirds throws a baseball 121 feet per second. Which pitcher throws a baseball faster? Explain.	6. A soft-serve ice cream machine makes 1200 gallons per hour. Convert this rate to cups per minute.
7. You were riding your bike at 30 mph. Convert your rate to feet per minute.	8. A trip to downtown Seattle from Bellevue is 13.5 miles. How many inches is this?

The Leaky Faucet

Kim has a leaky faucet in her Kitchen.

Kim estimates that the faucet in her kitchen drips at a rate of 1 drop every 2 seconds. She wants to know how many times the faucet drips in a week.

1. Help her find how many seconds in a week.
$$\left[\frac{7\ days}{1\ week}\cdot\frac{24\ hours}{1\ day}\cdot\frac{60\ minutes}{1\ hour}\cdot\frac{60\ seconds}{1\ minute}=\frac{?_{seconds}}{1\ week}\right]$$

2. Set up an expression multiplying two fractions to find the number of time her faucet drips per week.
$$\underline{\hspace{3cm}}\cdot\underline{\hspace{3cm}}=\frac{?\ drops}{1\ week}$$

3. She wants to know how much water her leaky faucet wastes in a year. How many seconds are in a year?
$$\left[\frac{365\ days}{1\ year}\cdot\frac{24\ hours}{1\ day}\cdot\frac{60\ minutes}{1\ hour}\cdot\frac{60\ seconds}{1\ minute}=\frac{?_{seconds}}{1\ year}\right]$$

4. Set up an expression multiplying two fractions to find the number of time her faucet drips per year.
$$\underline{\hspace{3cm}}\cdot\underline{\hspace{3cm}}=\frac{?\ drops}{1\ year}$$

5. Kim estimates that approximately 575 drops fill a 100 milliliter bottle. Estimate how much water her leaky faucet wastes in a year. Set up an expression multiplying three fractions to find the volume in liters her faucet drips per year.
$$\underline{\hspace{3cm}}\cdot\underline{\hspace{3cm}}\cdot\underline{\hspace{3cm}}\cdot\frac{?\ liters}{1\ year}$$

The Leaky Faucet – 3 Act Task Option

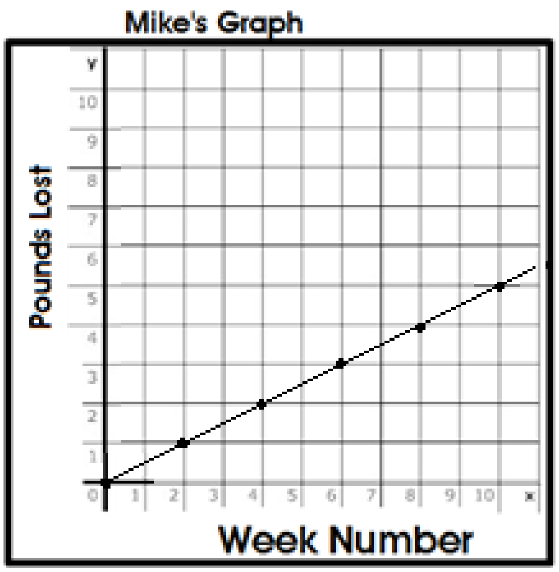
Real Mathematics:

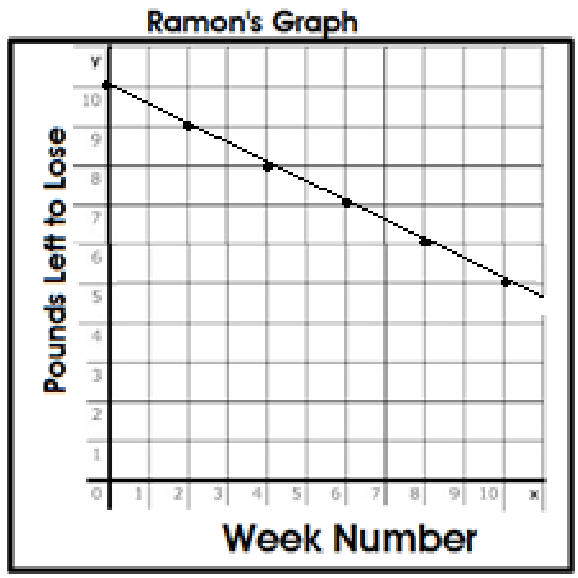
The Largest Loser

This is the pre-assessment for the FAL for concept one.

Mike and Ramon are both contestants in a weight loss contest. Part of the contest requires that they graph their progress. See their charts, below.

1. You are one of the contest judges. Look carefully at the two graphs and make a case for which contestant you think should be named the winner that you think would be convincing to the other judges.





2. One of your fellow judges wants to know how fast each contestant lost weight. How could you explain how he can calculate each contestant's rate of weight loss?

Homework #____1_____

1. The scale on a map of Virginia shows that 1 inch represents 30 miles. The actual distance from Richmond, VA, to Washington, D.C., is 110 miles. On the map, how many inches are between the two cities?	2. Sam is building a model of an antique car. The scale of his model to the actual car is 1:10. His model is 18 1/2 inches long. How long is the actual car?
3. A certain cooking oil has a density of 0.91 grams per milliliter. Which of the following series of calculations correctly determines the mass, in kilograms, of 15 liters of this oil? Ⓐ $15\,\text{L} \cdot \frac{1\,\text{L}}{1000\,\text{mL}} \cdot \frac{0.91\,\text{g}}{1\,\text{mL}} \cdot \frac{1\,\text{kg}}{1000\,\text{g}}$ Ⓑ $15\,\text{L} \cdot \frac{1000\,\text{mL}}{1\,\text{L}} \cdot \frac{0.91\,\text{g}}{1\,\text{mL}} \cdot \frac{1000\,\text{g}}{1\,\text{kg}}$ Ⓒ $15\,\text{L} \cdot \frac{1000\,\text{mL}}{1\,\text{L}} \cdot \frac{1\,\text{mL}}{0.91\,\text{g}} \cdot \frac{1000\,\text{g}}{1\,\text{kg}}$ Ⓓ $15\,\text{L} \cdot \frac{1000\,\text{mL}}{1\,\text{L}} \cdot \frac{0.91\,\text{g}}{1\,\text{mL}} \cdot \frac{1\,\text{kg}}{1000\,\text{g}}$	4. A police officer saw a car traveling 1,800 feet in 30 seconds. The speed limit on the road is 55 miles per hour. Was the car speeding? Explain
5. Write an expression that converts 50 liters per minute into millimeters per second.	6. The perimeter of a square picture frame is 48 inches. One side length of the frame is 30.48 centimeters. How many centimeters are in an inch?

Parts of Expressions and Equations

An expression is a mathematical phrase that contains operations, numbers, and/or variables. The terms of an expression are the parts that are being added. A coefficient is the numerical factor of a variable term. There are both numerical expressions and algebraic expressions. A numerical expression contains only numbers while an algebraic expression contains at least one variable.

GUIDED NOTES

a. Identify the terms and the coefficients of the expression $8p + 2q + 7r$.

b. Identify the terms and coefficients of the expression $18 - 2x - 4y$.

c. Identify the terms and coefficients in the expression $2x + 3y - 4z + 10$

Tickets to an amusement park are \$60 for adults and \$30 for children. If a is the number of adults and c is the number of children, then the cost for a adults and c children is $60a + 30c$.

d. What are the terms of the expression?

e. What are the factors of $60a$?

f. What are the factors of $30c$?

g. What are the coefficients of the expression?

h. Interpret the meaning of the two terms of the expression.

The price of a case of juice is \$15.00. Fred has a coupon for 20 cents off each bottle in the case. The expression to find the final cost of the case of juice is $15 - 0.2b$, wherein b is the number of bottles.

i. What are the terms of the expression?

j. What are the factors of each term?

k. Do both terms have coefficients? Explain. What are the coefficients?

l. What does the expression $15 - 0.2b$ mean in the given situation?

Questions To Ponder

1. Sally identified the terms of the expression 9a + 4b - 18 as 9a, 4b, and 18. Explain her error.

2. What is the coefficient of b in the expression b + 10? Explain.

Example!

Interpreting Algebraic Expressions in Context

In many cases, real-world situations and algebraic expressions can be related. The coefficients, variables, and operations represent the given real-world context.

Curtis is buying supplies for his school. He buys p packages of crayons at \$1.49 per package and q packages of markers at \$3.49 per package. What does the expression 1.49p + 3.49q represent?

Interpret the meaning of the term 1.49p. What does the coefficient 1.49 represent?
The term 1.49p represents the cost of p packages of crayons. The coefficient represents the cost

of one package of crayons, $1.49.
Interpret the meaning of the term 3.49q. What does the coefficient 3.49 represent?
The term 3.49q represents the cost of q packages of markers. The coefficient represents the cost of one package of markers, $3.49.

Interpret the meaning of the entire expression.

The expression 1.49p + 3.49q represents the total cost of p packages of crayons and q packages of markers.

SELF CHECK Interpret the algebraic expression corresponding to the given context.

1. George is buying watermelons and pineapples to make fruit salad. He buys w watermelons at $4.49 each and p pineapples at $5 each. What does the expression 4.49w + 5p represent?

2. Sandi buys 5 fewer packages of pencils than p packages of pens. Pencils costs $2.25 per package and pen costs $3 per package. What does the expression 3p + 2.25(p - 5) represent?

3. A bicyclist travels 1 mile in 5 minutes. If m represents minutes, what does the expression $\frac{m}{5}$ represent?

Example!

Modeling Expressions in Context

The table shows some words and phrases associated with the four basic arithmetic operations. These words and phrases can help you translate a real-world situation into an algebraic expression.

Operation	Words	Examples
Addition	the sum of, added to, plus, more than, increased by, total, altogether, and	1. A number increased by 2 2. The sum of *n* and 2 3. $n + 2$
Subtraction	less than, minus, subtracted from, the difference of, take away, taken from, reduced by	1. The difference of a number and 2 2. 2 less than a number 3. $n - 2$
Multiplication	times, multiplied by, the product of, percent of	1. The product of 0.6 and a number 2. 60% of a number 3. $0.6n$
Division	divided by, division of, quotient of, divided into, ratio of,	1. The quotient of a number and 5 2. A number divided by 5 3. $n \div 5$ or $\frac{n}{5}$

Ex. the price of an item plus 6% sales tax

Price of an item + 6% sales tax
$p + 0.06p$

Algebra 1 U1.C2.Lesson A. **Parts of Expressions and Equations**

The algebraic expression is $p + 0.06p$, or $1.06p$

Use the Distributive Property to show why $p + 0.06p = 1.06p$.

SELF CHECK

Write an algebraic expression to model the given context. Give your answer in simplest form.

4. the number of gallons of water in a tank, that already has 300 gallons in it, after being filled at 35 gallons per minute for *m* minutes

5. the original price *p* of an item less a discount of 15%

6. A man's age today is 2 years more than three times the age his son will be 5 years from now. Let a represent the son's age today. Write an expression to represent the man's age today. Then find his age if his son is now 8 years old.

7. Let n represent an even integer. Write an expression for the sum of that number and the next three even integers after it. Simplify your expression fully.

Re-Teaching and Independent Practice

Example!

A **numerical expression** contains operations and numbers.	$4 + 3 - 1$ is a numerical expression.
An **algebraic expression** contains operations, numbers, and at least one variable.	$5x + 3 - 7y$ is an algebraic expression.
Terms are the parts of the expression that are separated by addition signs. Rewrite subtraction to show addition.	Rewrite $5x + 3 - 7y$ to show addition: $5x + 3 + (-7y)$. The terms are $5x$, 3, and $-7y$.
A **coefficient** is a number that multiplies a variable.	5 is the coefficient of x. -7 is the coefficient of y. 3 does not multiply a variable, so it is not a coefficient.

Example

Write an expression that reflects the situation.

Jacques has 8 more DVDs than Erik. Write an expression for the number of DVDs Jacques has.

Erik has d DVDs. Jacques has 8 more DVDs. The word "more" shows addition.

Jacques has $d + 8$ DVDs.

Identify the terms and coefficients of each expression.

1. $4a + 3c + 8$

   terms: ____________

   coefficients: ________

2. $9b + 6 + 2g$

   terms: ____________

   coefficients: ________

3. $f + 15 + 2.7g$

   terms: ____________

   coefficients: ________

4. $7p - 3r + 6 - 5s$

   terms: ____________

   coefficients: ________

5. $3m - 2 - 5n + p$

   terms: ____________

   coefficients: ________

6. $6w - 3 + 6.4x - 1.9y$

   terms: ____________

   coefficients: ________

Explain the Error A student wrote that there are two terms in the expression $3p - (7 - 4q)$. Explain the student's error.

Interpret the meaning of the expression.

7. Frank buys p pounds of oranges for \$2.29 per pound and the same number of pounds of apples for \$1.69 per pound. What does the expression $2.29p + 1.69p$ represent?

 ____

8. Kathy buys p pounds of grapes for \$2.19 per pound and one pound of kiwi for \$3.09 per pound. What does the expression $2.19p - 3.09$ represent?

 ____

Write an expression to represent each situation.

9. Eliza earns \$400 per week plus \$15 for each new customer she signs up. Let c represent the number of new customers Eliza signs up. Write an expression that shows how much she earns in a week.

 ____

10. Max's car holds 18 gallons of gasoline. Driving on the highway, the car uses approximately 2 gallons per hour. Let h represent the number of hours Max has been driving on the highway. Write an expression that shows how many gallons of gasoline Max has left after driving h hours.

 ____

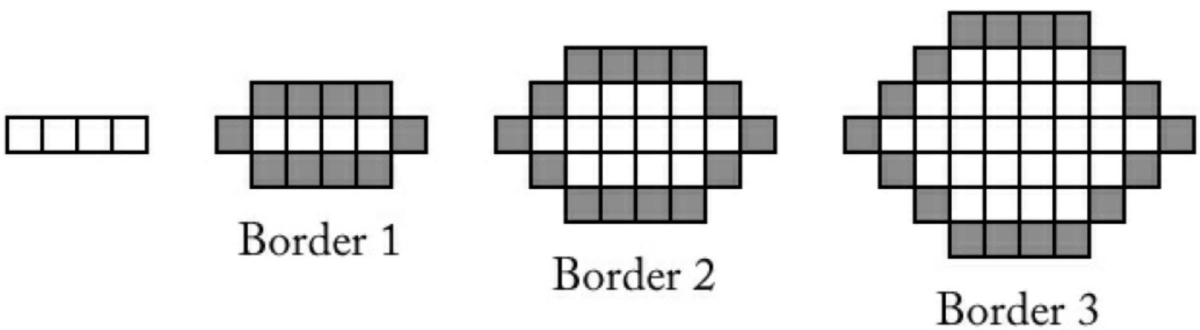
11. A man's age today is three years less than four times the age of his oldest daughter. Let a represent the daughter's age. Write an expression to represent the man's age.

 ____

Real Mathematics:
Application

Kitchen Floor Tiles

Fred has some colored kitchen floor tiles and wants to choose a pattern using them to make a border around white tiles. He generates patterns by starting with a row of four white tiles. He surrounds these four tiles with a border of colored tiles (Border 1). The design continues as shown below:



Fred writes the expression $4(b-1)+\ 10$ for the number of tiles in each border, where b is the border number, $b \geq 1$.

a. Explain why Fred's expression is correct.

b. Emma wants to start with five tiles in a row. She reasons, “Fred started with four tiles and his expression was $4(b-1)+\ 10$. So if I start with five tiles, the expression will be $5(b-1)+10$. Is Emma’s statement correct? Explain your reasoning.

c. If Emma starts with a row of n tiles, what should the expression be?

HOMEWORK

Homework #___1_____

Identify the terms of each expression. The first one is done for you.

1. $6b + 4 + 3c$

   6*b*, 4, 3*c*

2. $7 + 5p + 4r + 6s$

   ____

3. $7.3w + 2.8v + 1.4$

   ____

4. $12m + 16n + 5p + 16$

   ____

Identify the coefficients in each expression. The first one is done for you.

5. $2f - 6g + 3h - 5$

   2, –6, 3

6. $4a + 3b + 6 - 6c$

   ____

7. $4m + 2n - 7p + 5q$

   ____

8. $3w - 4x - 6y + 9z$

   ____

Interpret the meaning of individual terms and expressions.

9. Yolanda is buying supplies for school. She buys n packages of pencils at \$1.40 per package and m pads of paper at \$1.20 each. What does each term in the expression $1.4n + 1.2m$ represent? What does the entire expression represent? ______________________________

10. Chris buys p pairs of pants and 4 more shirts than pairs of pants. Shirts cost \$18 each and pair of pants cost \$25 each. What does each term in the expression $25p + 18(p + 4)$ represent? What does the entire expression represent? ______________________________

Write an expression for each situation. The first one is done for you.

11. The Blue Team scored two more than five times the number of points, p, scored by the Red Team.

    $5p + 2$

12. The Green Team scored seven fewer points, p, than the Orange Team scored.

    ____

13. The Red Team scored three more points, p, than the Brown Team scored.

    ____

14. The Yellow Team scored five times the number of points, p, scored by the Blue Team plus six.

    ____

HOMEWORK

Understanding Polynomial Expressions

A polynomial expression is a monomial or a sum of monomials. You simplify them by combining like terms.

A monomial is an expression consisting of a number, variable, or product of numbers and variables that have whole number exponents. Terms of an expression are parts of the expression separated by plus signs. (Remember that x – y can be written as x + (– y).) A monomial cannot have more than one term, and it cannot have a variable in its denominator. Here are some examples of monomials and expressions that are not monomials.

Monomials					Not Monomials				
4	x	$-4xy$	$0.25x^3$	$\frac{xy}{4}$	$4+x$	$x-1$	$0.7x^{-2}$	$0.25x^{-1}$	$\frac{y}{x^3}$

GUIDED NOTES

Use the following process to determine if $5ab^2$ is a monomial.

(A) $5ab^2$ has ________ term(s), so it ________ be a monomial.

(B) Does $5ab^2$ have a denominator?

____

(C) If possible, split it into a product of numbers and variables.

$5ab^2 = 5 \cdot \square \cdot \square$

(D) List the numbers and variables in the product.

Numbers: $\square$ Variables: $\square$

(E) Check the exponent of each variable. Complete the following table.

Variable	Exponent
a	
b	

(F) The exponents of the variables in $5ab^2$ are all ________________.
Therefore, $5ab^2$ _____ a monomial.

(G) Is $\frac{5}{k^2}$ a monomial?

____

Questions To Ponder

Is x^0 a monomial? Justify your answer in two ways.

SELF CHECK Complete the table below.

Term	Is this a monomial?	Explain your reasoning.
$5ab^2$	yes	$5ab^2$ is the product of a number, 5, and the variables *a* and *b*.
x^2		
$\sqrt{y}$	no	
2^2		
$\frac{5}{k^2}$	no	
$5x + 7$		
$x^2 + 4ab$		
$\frac{k^2}{4}$		

A polynomial can be a monomial or the sum of monomials. Polynomials are classified by the number of terms they contain. A monomial has one term, a **binomial** has two terms, and a **trinomial** has three terms. $3x^2 - 10$, for example, is a binomial.

The terms of a polynomial may be written in any order, but when a polynomial contains only one variable there is a standard form in which it can be written.

The standard form of a polynomial containing only one variable is written with the terms in order of decreasing degree. The first term will have the greatest degree, the next term will have the next greatest degree, and so on, until the final term, which will have the lowest degree.

Example! **Write each polynomial in standard form.**

$20x - 4x^3 + 1 - 2x^2$

Find the degree of each term and then arrange them in descending order of their degree.

$$\text{Degree:}\quad \underbrace{20x}_{1}\ \underbrace{-4x^3}_{3}\ \underbrace{+\,1}_{0}\ \underbrace{-2x^2}_{2} \;=\; \underbrace{-4x^3}_{3}\ \underbrace{-2x^2}_{2}\ \underbrace{+\,20x}_{1}\ \underbrace{+\,1}_{0}$$

The standard form is $-4x^3 - 2x^2 + 20x + 1$. The leading coefficient is -4.

SELF CHECK

Identify each expression as a monomial, a binomial, a trinomial, or none of the above. Write in standard form.

1. $4w^2$

   monomial

2. $9x^3 + 2x$

   ____

3. $35b^2$

   ____

4. $4p - 5p^2 + 11$

   ____

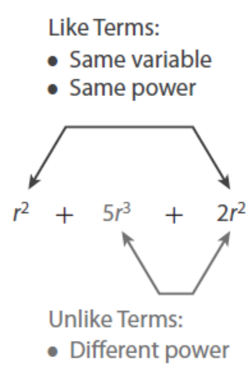
5. $12 + 3x^2 - x$

   ____

6. $3m + 1$

   ____

Polynomials are simplified by combining like terms. Like terms are monomials that have the same variables raised to the same powers. Unlike terms have different powers.



Example! **Identify like terms and combine them using the Distributive Property. Simplify.**

$r^2 + 5r^3 + 2r^2$	
$(r^2 + 2r^2) + 5r^3$	Identify like terms by grouping them together in parentheses.
$r^2(1 + 2) + 5r^3$	Combine using the Distributive Property.
$3r^2 + 5r^3$	Simplify.

SELF CHECK **Simplify each expression and write in standard form.**

7. $6n^2 + 3n - n^2$

$5n^2 + 3n$

8. $5c^2 + 2c - 4c$

____

9. $3b - 1 - 2b - 8$

____

10. $7a^2 - 9a^3 - 3a^2 - 4a$

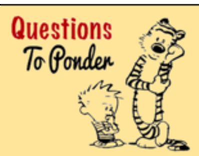
____

11. $5x^2 + 15x - 10x - 9x^2$

____

12. $3p + 8p^2 - 2p - 6 + 5p^2$

____



Prove by counterexample that the sum of monomials is not necessarily a monomial.

Re-Teaching and Independent Practice

Example!

Polynomials have special names based on the number of terms.

POLYNOMIALS				
No. of Terms	**1**	**2**	**3**	**4** or more
Name	**Mono**mial	**Bi**nomial	**Tri**nomial	**Poly**nomial

Identify each polynomial. Write in standard form.

1. $7m^3n$
2. $4y + y^2 + 7$
3. $5x - x^2$

Example!

You can simplify polynomials by combining like terms.

The following are like terms: $4y$ and $7y$ $8x^2$ and $2x^2$ $7m^5$ and m^5

same variables raised to same power

The following are **not** like terms: $3x^2$ and $3x$ 47 and $7y$ $8m$ and m^5

same variable, different exponent

one with variable, one constant

same variable but different power

Examples

Add $3x^2 + 4x + 5x^2 + 6x$.

$3x^2 + 5x^2 + \underline{\underline{4x}} + \underline{\underline{6x}}$ *Identify and rearrange like terms so they are together.*

$8x^2 + 10x$ *Combine like terms.*

Simplify each expression.

4. $2y^2 + 3y + 7y + y^2$ ________________
5. $8m^2 + 3m - 4m^2$ ________________
6. $12x + 10x^2 + 8x$ ________________

7. $6n^3 - n^2 + 3n^4 + 5n^2$

____

8. $c^3 + c^2 + 2c - 3c^3 - c^2 - 4c$

____

9. $11b^2 + 3b - 1 - 2b^2 - 2b - 8$

____

10. $b^3 + 9 - 3b^3 - 4a^4$

____

11. $9x + 5x^2 + 15x - 10x$

____

12. $3p^2 + 8p^3 - 2p^2 + 2p + 5p^3$

____

We've Got to Operate

Previously, you have learned about linear functions, which are first degree polynomial functions that can be written in the form $f(x) = a_1x^1 + a_0$ where a_1 is the slope of the line and a_0 is the y-intercept (Recall: $y = mx + b$, here m is replaced by a_1 and b is replaced by a_0.) Quadratic functions are 2nd degree polynomial functions and can be expressed as $f(x) = a_2x^2 + a_1x^1 + a_0$.

These are just two examples of polynomial functions; there are countless others. A polynomial is a mathematical expression involving a sum of nonnegative integer powers in one or more variables multiplied by coefficients. The standard form of a polynomial in one variable with constant coefficients is written as

$$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ...a_2x^2 + a_1x + a_0$$

where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers.

1. In the form $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ...a_2x^2 + a_1x + a_0$, what does the x^n, x^{n-1}, x^{n-2} indicate about the standard form of a polynomial?

2. What are whole numbers?

3. What are real numbers?

4. Decide whether each function below is a polynomial. If it is, write the function in standard form. If it is not, explain why.

   a. $f(x) = 2x^3 + 5x^2 + 4x + 8$

   b. $f(x) = 2x^2 + x^{-1}$

   c. $f(x) = 5 - x + 7x^3 - x^2$

   d. $f(x) = \frac{2}{3}x^2 - x^4 + 5 + 8x$

   e. $f(x) = 2\sqrt{x}$

   f. $f(x) = \frac{1}{3x^2} + \frac{6}{x} - 2$

HOMEWORK

Homework #___1_____

Identify each expression as a monomial, a binomial, a trinomial, or none of the above.

1. $6b^2 - 7$

____

2. $x^2y - 9x^4y^2 + 3xy$

____

3. $35r^3s$

____

4. $3p + \frac{2p}{q} - 5q$

____

5. $4ab^5 + 2ab - 3a^4b^3$

____

6. $st + t^{0.5}$

____

Simplify each expression. The first one is done for you.

7. $6n^2 + 3n - n^2$

$5n^2 + 3n$

8. $5c^3 + 2c - 4c$

____

9. $3b - 1 - 2b - 8$

____

10. $7a^4 - 9a^3 - 3a^4 - 4a$

____

11. $5x^2 + 15x - 10x - 9x^2$

____

12. $3p + 8p^2 - 2p - 6 + 5p^2$

____

THE BIG IDEA

Adding and Subtracting Polynomial Expressions

You have added numbers and variables, which is the same as adding polynomials of degree 0 and degree 1. Adding polynomials of higher degree is similar, but there are more possible like terms to consider.

Example!

To add and subtract polynomials, combine like terms. You can do so vertically or horizontally. If it is a subtraction problem, rewrite as a sum by adding the opposite, This is also referred to as distributing the -1 to each term.

$(5x+2)-(-2x^2-3x+4)$

$(5x+2)+(2x^2+3x-4)$ Rewrite subtraction as addition of the opposite.

$$\begin{array}{r} 0x^2+5x+2 \\ +\ 2x^2+3x-4 \\ \hline 2x^2+8x-2 \end{array}$$

Use the vertical method. Write $0x^2$ as a placeholder.

Combine like terms.

or

$(2q^2-q-8)-(2q^2+q-4)$

$=(2q^2-q-8)+(-2q^2-q+4)$ Rewrite subtraction as addition of the opposite.

$=(2q^2-2q^2)+(-q-q)+(-8+4)$ Group like terms together.

$=-2q-4$ Simplify.

GUIDED NOTES

You can find similarities between adding polynomials horizontally and vertically.

Add $4x+3x^2+4$ and $6x^2+6-2x$.

How is grouping like terms similar to aligning like terms?

Adding Horizontally $\longrightarrow$ Adding Vertically

Group like terms. | Align like terms.

$(3x^2+6x^2)+(4x-2x)+(4+6)$

$3x^2+4x+4$

$6x^2-2x+6$

Combine like terms. $\longrightarrow$ Combine like terms.

$(3+6)x^2+(4-2)x+(4+6)$

$$\begin{array}{r} 3x^2+4x+4 \\ \underline{+\ 6x^2-2x+6} \end{array}$$

Simplify. → Simplify.

$9x^2 + 2x + 10$ $9x^2 + 2x + 10$

What do you have to do to simplify differences of polynomials? What properties do you use to accomplish this?

SELF CHECK

Add or subtract the polynomials horizontally or vertically.

1. $5x^3 + 2x + 1 - 3x^3 + 6$

____

2. $x - 3x^5 + 2x^4 - 5x^5 - x$

____

3. $(2x^2 + 10x + 4) + (7x^2 + 6x - 2)$

____

4. $(x^3 - 6) + (9 - 2x^2 + x^3)$

____

5. $(6x^4 + 8x - 2) - (2x^4 + 6x)$

____

6. $(3x^2 - 9x) - (x + 2x^3 - 4)$

____

A set has the **closure property** under a particular operation if the result of the operation is always an element in the set. If a set has the closure property under a particular operation, then we say that the set is "***closed under the operation***."

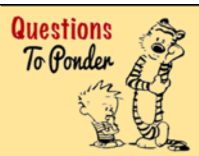
It is much easier to understand a property by looking at examples than it is by simply talking about it in an abstract way, so let's move on to looking at examples so that you can see exactly what we are talking about when we say that a set has the **closure property**.

a. The set of integers is closed under the operations of addition and subtraction because the sum of any two integers is always another integer.

Example: $\underset{\text{integer}}{-37} + \underset{\text{integer}}{8} = \underset{\text{integer}}{-29}$ $\qquad$ $\underset{\text{integer}}{24} - \underset{\text{integer}}{5} = \underset{\text{integer}}{19}$

b. The set of integers is not closed under the operation of division because when you divide one integer by another, you don't **always** get another integer as the answer.

Example: $\underset{\text{integer}}{27} \div \underset{\text{integer}}{-3} = \underset{\text{integer}}{-9}$ $\qquad$ $\underset{\text{integer}}{3} \div \underset{\text{integer}}{27} = \underset{\text{not an integer}}{\frac{1}{9}}$



Go back and look at all of your answers in you Self-Check, in which you added and subtracted polynomials. Do you think that polynomial addition and subtraction is closed? Why or why not?

Re-Teaching and Independent Practice

Example!

You can add polynomials by combining **like terms.**

These are examples of **like terms**: $4y$ and $7y$ $8x^2$ and $2x^2$ m^5 and $7m^5$

These are **like terms** because they have the same variables and same exponent.

These are not like terms: $3x^2$ and $3x$ $4y$ and 7 $8m$ and $8n$

$3x^2$ and $3x$	$4y$ and 7	$8m$ and $8n$
same variable but different exponent	one with a variable, one is a constant	different variables

Add $(5y^2 + 7y + 2) + (4y^2 + y + 8)$.

$(5y^2 + 7y + 2) + (4y^2 + y + 8)$ *Identify like terms.*

$(5y^2 + 4y^2) + (7y + y) + (2 + 8)$ *Rearrange terms so that like terms are together.*

$9y^2 + 8y + 10$ *Combine like terms.*

Add $(5y^2 + 7y + 2) + (4y^2 + y + 8)$.

$(5y^2 + 7y + 2) + (4y^2 + y + 8)$ *Identify like terms.*

$(5y^2 + 4y^2) + (7y + y) + (2 + 8)$ *Rearrange terms so that like terms are together.*

$9y^2 + 8y + 10$ *Combine like terms.*

Add.

1. $(6x^2 + 3x) + (2x^2 + 6x)$ ____________________
2. $(m^2 - 10m + 5) + (8m + 2)$ ____________________
3. $(6x^3 + 5x) + (4x^3 + x^2 - 2x + 9)$ ____________________
4. $(2y^5 - 6y^3 + 1) + (y^5 + 8y^4 - 2y^3 - 1)$ ____________________

Example!

To subtract polynomials, you must remember to add the opposites.

Find the opposite of ($5m^3 - m + 4$).

$(5m^3 - m + 4)$	
$-(5m^3 - m + 4)$	*Write the opposite of the polynomial.*
$-5m^3 + m - 4$	*Write the opposite of each term in the polynomial.*

Subtract ($4x^3 + x^2 + 7$) – ($2x^3$).

$(4x^3 + x^2 + 7) + (-2x^3)$	*Rewrite subtraction as addition of the opposite.*
$(4x^3 + x^2 + 7) + (-2x^3)$	*Identify like terms.*
$(4x^3 - 2x^3) + x^2 + 7$	*Rearrange terms so that like terms are together.*
$2x^3 + x^2 + 7$	*Combine like terms.*

Subtract ($6y^4 + 3y^2 - 7$) – ($2y^4 - y^2 + 5$).

$(6y^4 + 3y^2 - 7) + (-2y^4 + y^2 - 5)$	*Rewrite subtraction as addition of the opposite.*
$(6y^4 + 3y^2 - 7) + (-2y^4 + y^2 - 5)$	*Identify like terms.*
$(6y^4 - 2y^4) + (3y^2 + y^2) + (-7 - 5)$	*Rearrange terms so that like terms are together.*
$4y^4 + 4y^2 - 12$	*Combine like terms.*

Subtract.

5. $(9x^3 - 5x) - (3x)$

____

6. $(6t^4 + 3) - (-2t^4 + 2)$

____

7. $(2x^3 + 4x - 2) - (4x^3 - 6)$

____

8. $(t^3 - 2t) - (t^2 + 2t + 6)$

____

9. $(4c^5 + 8c^2 - 2c - 2) - (c^3 - 2c + 5)$

____

Application #1

1. What algebraic expression must be subtracted from the sum of $y^2 + 5y - 1$ and $3y^2 - 2y + 4$ to give $2y^2 + 7y - 2$ as the result?

   ____

2. What algebraic expression must be added to the sum of $3x^2 + 4x + 8$ and $2x^2 - 6x + 3$ to give $9x^2 - 2x - 5$ as the result?

   ____

Give an example for each statement.

3. .The difference of two binomials is a binomial.

   ____

4. The sum of two binomials is a monomial.

   ____

Solve.

5. Ned, Tony, Matt, and Juan are playing basketball. Ned scored $2p + 3$ points, Tony scored 3 more points than Ned, Matt scored twice as many points as Tony, and Juan scored 8 fewer points than Ned. Write an expression that represents the total number of points scored by all four boys.

   ____

6. The sum of the squares of the first n positive integers is $\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$. The sum of the cubes of the first n positive integers is $\frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$. Write an expression for the sum of the squares and cubes of the first n positive integers. Then find the sum of the first 10 squares and cubes.

   ____

7. Mr. Watford owns two car dealerships. His profit from the first can be modeled with the polynomial $c^3 - c^2 + 2c - 100$, where c is the number of cars he sells. Mr. Watford's profit from his second dealership can be modeled with the polynomial $c^2 - 4c - 300$.

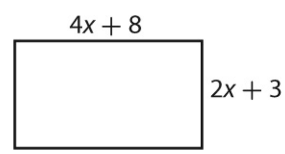
   a. Write a polynomial to represent the difference of the profit at his first dealership and the profit at his second dealership.

   ____

   b. If Mr. Watford sells 45 cars in his first dealership and 300 cars in his second, what is the difference in profit between the two dealerships?

   ____

8. Vincent is going to frame the rectangular picture with dimensions shown. The frame will be $x + 1$ inches wide. Find the perimeter of the frame.



HOMEWORK

Homework #___1_____

The first one is done for you.

1. $8p + 6$
   $-(4p + 2)$

   $\mathbf{4p + 4}$ ____________

2. $9y^2 - 6y + 3$
   $-(5y^2 - 3y + 2)$

   ____

3. $5z^3 + 8z^2 + 5$
   $-(2z^3 + 3z^2 - 2)$

   ____

4. $12k + 3$
   $+ 4k + 2$

   ____

5. $6s^3 + 9s + 10$
   $+ 3s^3 + 4s - 10$

   ____

6. $15a^4 + 6a^2 + a$
   $+ 6a^4 - 2a^2 + a$

   ____

7. $(5x^3 + 14) - (2x^3 - 1)$

   ____

8. $(15g^2 + 6g - 3) - (10g^2 + 2g + 2)$

   ____

9. $(12p^5 + 8) + (8p^5 + 6)$

   ____

10. $(11b^2 + 3b - 1) + (2b^2 + 2b + 8)$

   ____

Solve. The first problem is started for you.

11. Rebecca is building a pen for her rabbits against the side of her house. The polynomial $4n + 8$ represents the length and the polynomial $2n + 6$ represents the width.

    a. What polynomial represents the perimeter of the entire pen?

    $\mathbf{(4n + 8) + (4n + 8) + (2n + 6) + (2n + 6)} =$

    ____

    b. What polynomial represents the perimeter of the pen NOT including the side of the house?

    ____

12. The polynomial $35p + 300$ represents the number of men enrolled in a college and $25p + 100$ represents the number of women enrolled in the same college. What polynomial shows the difference between the number of men and women enrolled in the college?

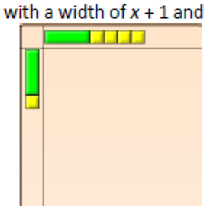
    ____

Multiplying Polynomials

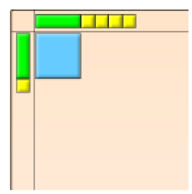
Algebra tiles can be used to model the multiplication. In this lesson, we will model multiplication of polynomials briefly to show a concrete model before representing polynomial multiplication with an area model and algebraically.

Example! Find the product (x + 1)(x + 4).

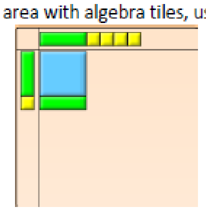
a. Build your model. Make a rectangle with a width of $x + 1$ and a length of $x + 4$.

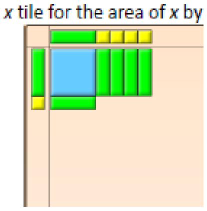


b. Fill in the area with algebra tiles. The area of an x by x is an x^2 tile.

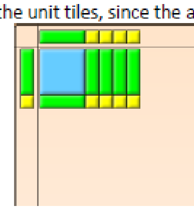


c. Continue to fill in the area with algebra tiles, using an x tile for the area of x by 1.





d. Fill the area of the empty space with the unit tiles, since the area of 1 by 1 is 1.

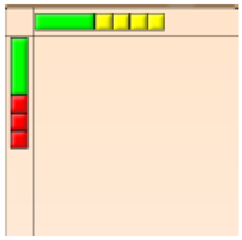


e. To find the product of $(x + 1)(x + 4)$, add all of the algebra tiled area. For this problem, there is one x^2 tile, 5 x tiles, and 4 unit tiles. Therefore, $(x + 1)(x + 4) = x^2 + 5x + 4$.

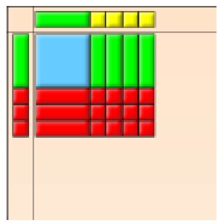
GUIDED NOTES

Find the product $(x - 3)(x + 4)$. Since one of the binomials contains subtraction, we will need to use the negative algebra tiles to represent these values.

a. Build your model. Make a rectangle with a width of x - 3 and a length of x + 4. The -3 units are represented using the red unit tiles.



b. Fill in the area with algebra tiles. Keep in mind that the product of a negative and a positive is a negative value. Be sure to use the appropriate tiles.



c. Use your algebra tiles to determine the product of (x - 3)(x + 4). *(Hint: If your answer is not a trinomial, can you do anything to simply your polynomial?)*

d. How do the algebra tile models in problem #1 and problem #2 differ?

SELF CHECK

Use algebra tiles to find each of the following products.

1. $(x+2)(x+3)$

2. $(x+4)(x-3)$

3. $(x-3)^2$

4. $(2x-3)(x-2)$

Hector noticed that instead of using algebra tiles, he can just draw an area model to multiply binomials. Do you think this method will work? Explain why or why not.

	x	$+8$
x	x^2	$8x$
$+5$	$5x$	40

Example!

You have seen the geometric area model used in previous examples to demonstrate the multiplication of polynomial expressions. Physical models are not always available. However, an area mode may be used in a similar fashion to identify each partial product as we multiply polynomial expressions

For example, fill in the table to identify the partial products of $(x+2)(x+5)$. Then, write the product of $(x+2)(x+5)$ in standard form.

	x + 5	
x +		
2		

SELF CHECK

Use the area model to multiply the polynomials.

1. $(x + 5)(x + 6)$

2. $(a - 7)(a - 3)$

3. $(d + 8)(d - 4)$

4. $(2x - 3)(x + 4)$

5. $(5b + 1)(b - 2)$

6. $(3p - 2)(2p + 3)$

Questions To Ponder

What is the rectangle modeling?

	5 +	x
2 +	10	$2x$
x	$5x$	x^2

Example!

To multiply polynomials algebraically, you use the distributive property.

$(x+5)(x+2)$

$$
\begin{aligned}
(x+5)(x+2) &= x(x+2) + 5(x+2) && \text{Distribute.}\\
&= x(x+2) + 5(x+2) && \text{Redistribute and simplify.}\\
&= x(x) + x(2) + 5(x) + 5(2) &&\\
&= x^2 + 2x + 5x + 10 &&\\
&= x^2 + 7x + 10 &&
\end{aligned}
$$

Questions To Ponder

Show the connection between the area model below and multiplying using the distributive property.

	x	$+8$
x	x^2	$8x$
$+5$	$5x$	40

How are the similar?

SELF CHECK

Multiply the polynomials.

7. $(5k - 9)(2k - 4)$

8. $(2m - 5)(3m + 8)$

9. $(4 + 7g)(5 - 8g)$

10. $(r + 2s)(r - 6s)$

11. $(3 - 2v)(2 - 5v)$

12. $(5 + h)(5 - h)$

GUIDED NOTES

Remember, a set has the **closure property** under a particular operation if the result of the operation is always an element in the set. If a set has the closure property under a particular operation, then we say that the set is "***closed under the operation***."

It is much easier to understand a property by looking at examples than it is by simply talking about it in an abstract way, so let's move on to looking at examples so that you can see exactly what we are talking about when we say that a set has the **closure property**.

a. The set of integers is closed under the operations of addition and subtraction because the sum of any two integers is always another integer.

Example: $\underset{\text{integer}}{-37} + \underset{\text{integer}}{8} = \underset{\text{integer}}{-29}$ $\underset{\text{integer}}{24} - \underset{\text{integer}}{5} = \underset{\text{integer}}{19}$

b. The set of integers is not closed under the operation of division because when you divide one integer by another, you don't **always** get another integer as the answer.

Example: $\underset{\text{integer}}{27} \div \underset{\text{integer}}{-3} = \underset{\text{integer}}{-9}$ $\underset{\text{integer}}{3} \div \underset{\text{integer}}{27} = \underset{\text{not an integer}}{\frac{1}{9}}$

c. Now, go back and look at all of your answers to problems in the Self Check, in which you multiplied polynomials. Do you think that polynomial multiplication is closed? Why or why not?

Re-Teaching and Independent Practice

Example!

Use the Distributive Property to multiply binomial and polynomial expressions.

Examples

Multiply $(x + 3)(x - 7)$.

$(\mathbf{x} + \mathbf{3})(x - 7)$

$\mathbf{x}(x - 7) + \mathbf{3}(x - 7)$	*Distribute.*
$(x)x - (x)7 + (3)x - (3)7$	*Distribute again.*
$x^2 - \underline{7x} + \underline{3x} - 21$	*Multiply.*
$x^2 - 4x - 21$	*Combine like terms.*

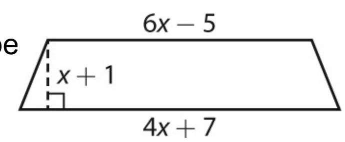
Multiply $(x + 5)(x^2 + 3x + 4)$.

$(\mathbf{x} + \mathbf{5})(x^2 + 3x + 4)$

$\mathbf{x}(x^2 + 3x + 4) + \mathbf{5}(x^2 + 3x + 4)$	*Distribute.*
$(x)x^2 + (x)3x + (x)4 + (5)x^2 + (5)3x + (5)4$	*Distribute again.*
$x^3 + \underline{3x^2} + \underline{4x} + \underline{5x^2} + \underline{15x} + 20$	*Multiply.*
$x^3 + 8x^2 + 19x + 20$	*Combine like terms.*

1. $(y + 3)(y - 3)$ ____________________
2. $(z - 5)^2$ ____________________
3. $(3q + 7)(3q - 7)$ ____________________
4. $(4w + 9)^2$ ____________________
5. $(3a - 4)^2$ ____________________
6. $(5q - 8r)(5q + 8r)$ ____________________
7. $(x + 4)(x^2 + 3x + 5)$ ____________________
8. $(3m + 4)(m^2 - 3m + 5)$ ____________________
9. $(2x - 5)(4x^2 - 3x + 1)$ ____________________

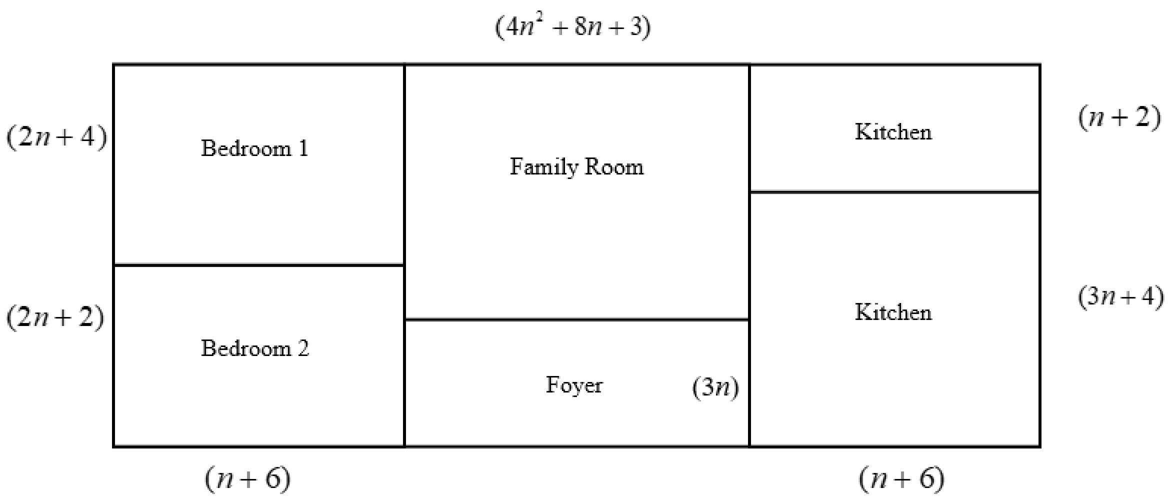
10. Write a polynomial expression that represents the area of the trape $\left(A = \frac{1}{2}h\left(b_1 + b_2\right)\right)$



____

12. Kayla worked $3x + 6$ hours this week. She earns $x - 2$ dollars per hour. Write a polynomial expression that represents the amount Kayla earned this week. Then calculate her pay for the week if $x = 11$.

Floor Plan



Use the above diagram, answer the following questions. To receive credit for this task you must **<u>show all work on separate paper</u>**.

1. What is the length of the North side of the building?

2. What is the width of the West side of the building?

3. What is the perimeter of the building?

4. Find the area of the foyer.

5. Find the area of the kitchen.

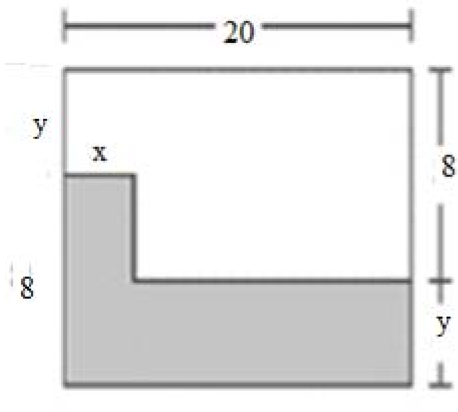
6. Find the perimeter of bedroom 1.

7. If the width of the building is 30 feet, find the value of *n*.

8. If the ceiling in the bedrooms are 8 feet high, find the volume of bedroom 2.

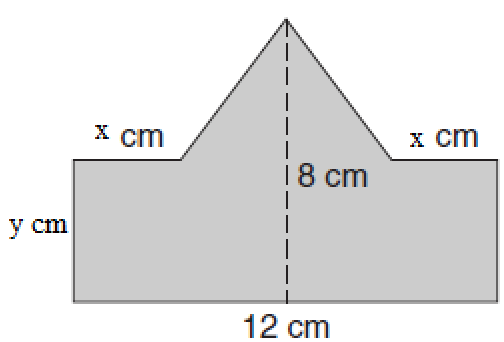
Application 2

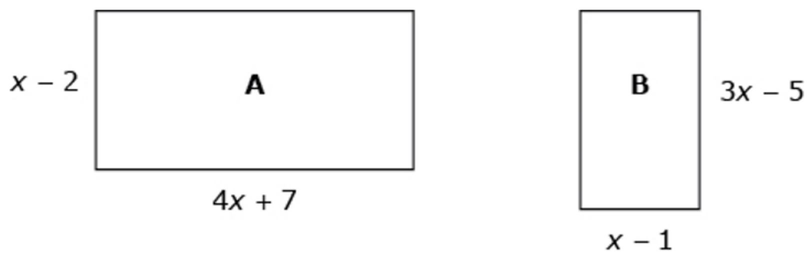
Problem A: This rectangle shows the floor plan of an office. The shaded part of the plan is an area that is getting new tile. Write an algebraic expression that represents the area of the office that is getting new tile.



Problem B: Tyler and Susan each have a box that is the shape of a cube. The edges of Taylor's box are each x cm in length. The edges of Susan's box are 4 cm longer than on Tyler's cube. What binomial expression represents the volume of Susan's box?

Problem C: Find the area, including units, of the shape below.





Problem D: Write an expression in simplest form that represents the difference in the areas, in square centimeters, of rectangle A and rectangle B.

Problem E:

A garden plans to make changes to a square garden that has a measure of 7 feet on each side. The gardener plans to increase the length by *x* feet and decrease the width by x feet. He calculates the area, in square feet, of the new garden to be $49 - x^2$.

Part A Explain what each term in the expression represents.

Part B Prove that the expression is correct. Show or explain your work.

Part C If the gardener decides to double the changes to the garden, how would that change his expression? Show your work or explain your answer

Homework #___1______

1. Which expression is equivalent to $2x+6(x-3)$?

   A $8x-3$

   B $3x+3$

   C $8x-18$

2. Find the product of $(t+8)$ and $(t-7)$.

3. Multiply $11x+3$ by 4.

4. Which expression is **not** equivalent to $(x+4)(x-3)$?

   A $x^2+7x-12$

   B $x^2-3x+4x-12$

   C $x(x-3)+4(x-3)$

5. Multiply $(x+2)(x+3)$. What is the product?

6. What is the product of $5(5x^2+2x-4)$?

7. Multiply $(y+6)(y+6)$.

8. Which product results in a difference of squares?

   A $(z-9)(z+9)$

   B $(z+4)(z+4)$

   C $(z-8)(z-8)$

9. A rectangular television screen has an area of w^2+2w square inches. If the width of the television screen is w inches, what is the length of the television?

10. Celeste has a garden that has a length of $15x$ and a width of $3x+5$ feet.

    a. Write a polynomial that represents the perimeter of the garden.

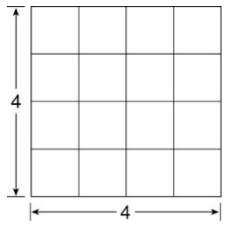
    b. Write a polynomial that represents the area of the garden by multiplying $15x(3x+5)$.

    c. Find the area of the garden if $x=3$ ft.

THE BIG IDEA

Simplifying Radicals

Recall what you learned in eighth grade, a **square root** produces a given number when multiplied by itself. The large square shown below is 4 squares long on each side and has 16 squares. 4 times 4 equals 16. 4 is the **square root** of 16. A number written with a root symbol is called a **radical**.



The 4×4 square can be described with symbols and with words.

Symbols	**Symbols**		**Words**
$4 \bullet 4 = 16$	$4^2 = 16$	$\longrightarrow$	Four squared equals sixteen.

This sign represents square root: $\sqrt{}$ - also called a radicand
$\sqrt{16} = 4$ $\longrightarrow$ Read "The square root of 16 equals 4."
$\sqrt{25} = 5$ $\longrightarrow$ Read "The square root of 25 equals 5."

Compare the symbols for "squared" and "square root."

$4^2 = 16$ and $\sqrt{16} = 4$
$5^2 = 25$ and $\sqrt{25} = 5$

Since $4^2 = 16\ and\ (-4)^2 = 16$, the square roots of 16 are 4 and -4. Thus every positive real number has two square roots, one positive and one negative. The positive square root given by $\sqrt{a}$ and the negative square root by $-\sqrt{a}$. These can be combined as $\pm\sqrt{a}$. It is understood, however, that the square root (radical) symbol denotes only the positive root, called the "principal square root".

The square root of a perfect square is a **rational number**. A rational number is a number that can be written as a ratio in the form $\frac{a}{b}$, where *a* and *b* are integers and b is not 0. 5 from the example above is equal to $\frac{5}{1}$.

Irrational number are numbers that are not rational. In other words, they cannot be written the form $\frac{a}{b}$, where *a* and *b* are integers and b is not 0. Square roots of integers that are not perfect squares are irrational. Other special numbers, like π, are also irrational.

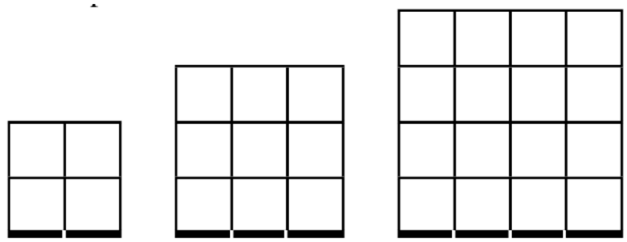
WE SAY THAT A SQUARE ROOT RADICAL is simplified, or in its simplest form, when the radicand has no square factors.

GUIDED NOTES

In your scientific calculator, when you input $\sqrt{20}$ and simplify by hitting enter, the result is $2\sqrt{5}$. How do we get this result? The activity below explores simplifying radicals with a geometric model.

Given a square of area *n*, the length of the side of that square is $\sqrt{n}$.

Complete the information below if each square represents 1 square unit.



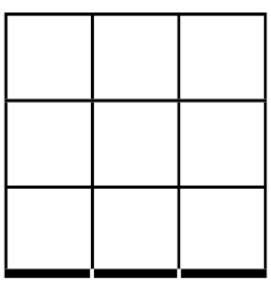
Area = ________ ________ ________

Side Length = ________ ________ ________

But what if the area is not a square number? For example, consider $\sqrt{18}$.

Estimate the value of $\sqrt{18}$? What reasoning did you use to come up with your estimate?

Suppose the area of the large square below is 18. What is the area of each small square? Represent the length of each side of the small square.



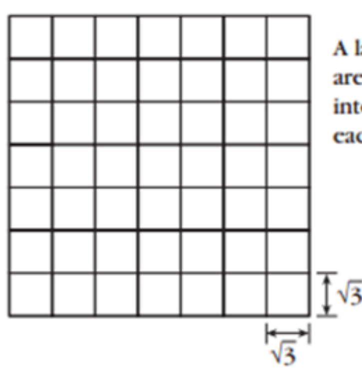
Area (small square): ________

Side length (small square): $\sqrt{}$ ________

How many of the side lengths of the small square does it take to make up the side length of the larger square?

What is another way to write $\sqrt{18}$?

This is the basic idea for writing square roots in *simple radical form.* For example, consider $\sqrt{147}$. Since 147 = 3 ·49, and since 49 is a square number, we can divide a square of area 147 units ^{2} into 49 squares, each of area 3 units ^{2}:



A large square with area = 147 divided into 49 small squares each with area = 3

You will notice that the side of the larger square is $\sqrt{147} = 7\sqrt{3}$

Describe a general strategy for simplifying radicals.

Applying the same strategy, what is another way to represent each of the radicals?

$\sqrt{32}$ $\sqrt{50}$ $\sqrt{72}$

Why is it helpful to be familiar with perfect squares to simplify radical?

Properties of Radicals

Property	Symbols	Example
Product Property of Radicals	For $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.	$\sqrt{36} = \sqrt{9 \cdot 4}$ $= \sqrt{9} \cdot \sqrt{4}$ $= 3 \cdot 2$ $= 6$
Quotient Property of Radicals	For $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.	$-\sqrt{0.16} = -\sqrt{\frac{16}{100}}$ $= -\frac{\sqrt{16}}{\sqrt{100}}$ $= -\frac{4}{10}$ $= -0.4$

Simplify $\sqrt{48}$

1. Find the largest perfect square factor (the largest perfect square that divides into 48 with no remainder).
   $\sqrt{48} = \sqrt{16 \cdot 3}$ *16 is the largest perfect square factor*
2. Give each factor a radical sign.
$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3}$$
3. Simpify the "perfect square" radical.
$$\sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$$
4. Answer:
   $\sqrt{48} = 4\sqrt{3}$

Don't worry if you don't pick the LARGEST perfect sqyare to start. You can still get the correct answer, but you will have to repeat the process.

$$\sqrt{48} = \sqrt{4 \cdot 12} = \sqrt{4} \cdot \sqrt{12} = 2\sqrt{12}$$

12 still has a perfect square

$$2\sqrt{12} = 2\sqrt{4 \cdot 3} = 2 \cdot 2\sqrt{3} = 4\sqrt{3}$$

GUIDED NOTES

Simplify each radical.

A. $\sqrt{12}$

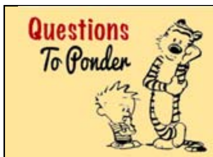
B. $\sqrt{\frac{8}{25}}$

C. $\sqrt{0.27}$

SELF CHECK

Write the following in simple radical form.

1. $\sqrt{45}$
2. $\sqrt{24}$
3. $\sqrt{23}$
4. $\sqrt{75}$
5. $\sqrt{98}$



Does 0 have any square roots? Why or why not?

Re-Teaching and Independent Practice

Example!

Simplify $\sqrt{75}$

Factor out the largest perfect square.

$\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3}$

The square root of 25 is 5.

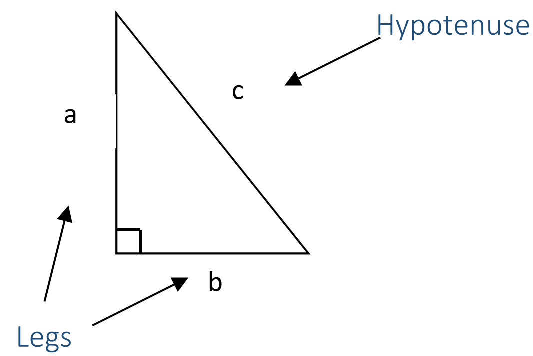
$\sqrt{75} = 5\sqrt{3}$

Simplify.

1. $\sqrt{62}$	2. $\sqrt{140}$
3. $\sqrt{50}$	4. $\sqrt{192}$
5. $\sqrt{200}$	6. $\sqrt{\frac{144}{9}}$
7. $\sqrt{216}$	8. $\sqrt{\frac{45}{4}}$
9. $\sqrt{28}$	10. $\sqrt{125}$
11. $\sqrt{24}$	12. $\sqrt{\frac{80}{16}}$

Pythagorean Theorem Revisited

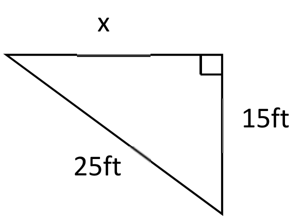
In any ***RIGHT*** triangle, the sum of the squares of the lengths of the two **legs** is equal to the square of the length of the **hypotenuse**.



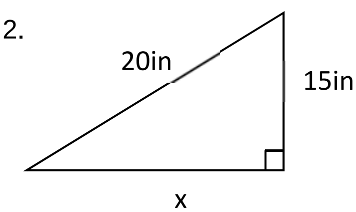
$$a^2 + b^2 = c^2$$

Find the length of the missing side. Keep answer in simplest radical form.

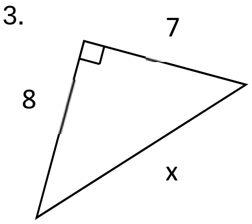
1.



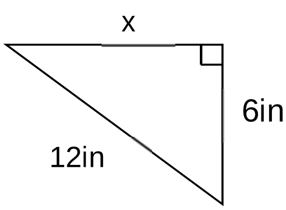
2.



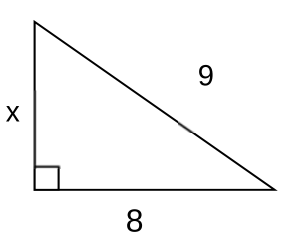
3.



4. Solve for the unknown side in this right triangle. Put your answer in simplest radical form.



5. Solve for the unknown side in this right triangle. Put your answer in simplest radical form.



HOMEWORK

Homework #___1______

$\sqrt{25}$ is read "the square root of 25".

$\sqrt{25} = 5$ because $5^2 = 25$ $\sqrt{36} = 6$ because ____ = ____ $\sqrt{100} =$ _____ $\sqrt{49} =$ _____

In the expression $\sqrt{a}$, the $\sqrt{}$ is called the radical and *a* is called the radicand.

Simplify (Simplifying Perfect Squares):

1. $\sqrt{4}$ 2. $\sqrt{16}$ 3. $-\sqrt{100}$

Simplify (Simplifying Radicals that are not Perfect Squares):

1. $\sqrt{20} = \sqrt{4} \bullet \sqrt{5} = 2\sqrt{5}$ 2. $\sqrt{27} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$ 3. $\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$

4. $\sqrt{45} = \sqrt{}\sqrt{} =$ __$\sqrt{}$ 5. $\sqrt{12} = \sqrt{}\sqrt{} =$ _______ 6. $\sqrt{50} =$

Simplify:

1. $\sqrt{18}$ 2. $\sqrt{125}$ 3. $\sqrt{72}$ 4. $\sqrt{180}$

5. $\sqrt{400}$ 6. $\sqrt{50}$ 7. $\sqrt{32}$ 8. $\sqrt{64}$

9. $\sqrt{28}$ 10. $\sqrt{27}$ 11. $\sqrt{63}$ 12. $\sqrt{98}$

Operations with Radicals

The square root of a perfect square is a **rational number**. A rational number is a number that can be written as a ratio in the form $\frac{a}{b}$, where *a* and *b* are integers and b is not 0. 5 from the example above is equal to $\frac{5}{1}$.

Examples: –0.5, 0, 7, $\frac{3}{2}$, $0.2\overline{6}$

Irrational number are numbers that are not rational. In other words, they cannot be written the form $\frac{a}{b}$, where *a* and *b* are integers and b is not 0. Square roots of integers that are not perfect squares are irrational.

Examples: $\sqrt{3}$, π. $\frac{\sqrt{5}}{2}$

The sum, product, or difference of two rational numbers is always a rational number. The quotient of two rational numbers is always rational when the divisor is not zero.

This lesson will explore operations with irrational square roots and define if the solution is rational or irrational.

Example!

Two radical expressions that have the same index and the same radicand are called ***like radicals***. To add or subtract like radicals, you can use the Distributive Property.

Addition Property of Radicals
$a\sqrt{b} \pm c\sqrt{b} = (a \pm c)\sqrt{b}\,, where\ b\ \geq 0.$

$\sqrt{8} + \sqrt{2}$	
$\sqrt{4 \cdot 2} + \sqrt{2}$	*Factor out the perfect squares.*
$\sqrt{4} \cdot \sqrt{2} + \sqrt{2}$	*Use the product property of radicals.*
$2\sqrt{2} + \sqrt{2}$	*Compute the square roots.*
$(2 + 1)\sqrt{2}$	*Use the distributive property.*
$3\sqrt{2}$	*Add.*

GUIDED NOTES

When the radicals do not have like radicals, we must simplify to see if like radicals can be formed.

$$\sqrt{3} + \sqrt{27}$$

NOTE:

The $\sqrt{3}$ is simplified already, but the $\sqrt{27}$ must still be simplified.

SELF CHECK

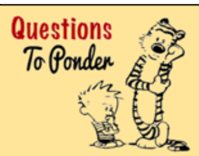
Solve. Write the following in simple radical form.

1. $7\sqrt{6} + 2\sqrt{6}$

2. $2\sqrt{3} + 4\sqrt{12}$

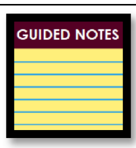
3. $3\sqrt{5} - 2\sqrt{45}$

4. $4\sqrt{2} - 3\sqrt{50}$



Is the sum of $\sqrt{3}$ and $2\sqrt{9}$ rational or irrational?

Is the sum of $\sqrt{16}$ and $2\sqrt{9}$ rational or irrational?



$\sqrt{9} \bullet \sqrt{9} =$ ________ = _____ $\sqrt{6} \bullet \sqrt{6} =$ ________ = ____ $\sqrt{10} \bullet \sqrt{10} =$ ________ = ____

Notice how when we multiply the same square root by itself, the answer becomes the radicand (WITHOUT THE RADICAL SIGN)!

$\left(\sqrt{7}\right)^2 = \sqrt{7} \bullet \sqrt{7} =$ _________ = _____

When we add or subtract radicals they must have the same radicand. This is NOT necessarily true for multiplying!

Example 1: $\sqrt{12} \bullet \sqrt{3} =$ ____________ = _______

Example 2: $\sqrt{2} \bullet \sqrt{32} =$ ___________ = _______

Sometimes when we multiply we do not get a perfect square. In that case, we must simplify our answer!

Example 3: $\sqrt{15} \bullet \sqrt{3} =$ _________ **Example 4**: $\sqrt{2} \bullet \sqrt{22} =$ __________

Multiplication Property of Radicals
$\left(a\sqrt{b}\right)\left(c\sqrt{b}\right) = ac\sqrt{bd}$, *where* $b \geq 0$ *and* $d \geq 0$

One more thing we must deal with when multiplying radicals is coefficients. To find the product of two radical expressions:

1) Multiply what is outside the radical of one radical by what is outside the other radical. (the coeffiecents)
2) Multiply the radicand of one radical by the radicand of the other radical.

3) Then simplify your answer if needed.

Example! Multiplying Radicals

$2\sqrt{5}\cdot 3\sqrt{8}$	
$2\cdot 3\ \cdot\sqrt{5\cdot 8}$	*Multiply the coeffiecents. Multiply the radicals*
$6\sqrt{40}$	*Multiply.*
$6\sqrt{4\cdot 10}$	*Smiplify. Factor out the perfect square.*
$6\ \cdot\sqrt{4}\cdot\sqrt{10}$	*Use the product propery of square roots.*
$6\ \cdot 2\cdot\sqrt{10}$	*Compute the square roots.*
$12\sqrt{10}$	*Multiply*

SELF CHECK Multiply the radicals. Make sure have all answers in simplest form!

1. $\sqrt{4}\bullet\sqrt{4}$
2. $\left(\sqrt{3}\right)^2$
3. $\sqrt{10}\bullet\sqrt{8}$
4. $3\sqrt{2}\bullet 9\sqrt{20}$
5. $-\sqrt{12}\bullet 6\sqrt{5}$
6. $\left(\sqrt{16}\right)^2$
7. $\sqrt{18}\bullet\sqrt{2}$
8. $2\sqrt{16}\bullet 3\sqrt{4}$
9. $\sqrt{26}\bullet\sqrt{26}$

Which product is irrational? Explain your choice.

A. $\sqrt{2} \cdot \sqrt{50}$
B. $\sqrt{64} \cdot \sqrt{4}$
C. $\sqrt{9} \cdot \sqrt{49}$
D. $\sqrt{10} \cdot \sqrt{8}$

Look back at your Self Check and Examples. The product of a nonzero rational number and an irrational number is always irrational. Is the product of two irrationals always irrational? Use counterexamples to justify your response.

Re-Teaching and Independent Practice

Example! Simplify completely.

$(6+2\sqrt{6})(3+\sqrt{2})$

$6(3+\sqrt{2})+2\sqrt{6}(3+\sqrt{2})$ *Distributive*

$(6\cdot 3)+(6\cdot\sqrt{2})+(2\sqrt{6}\cdot 3)+(2\sqrt{6}\cdot\sqrt{2})$ *Distribute*

$18+6\sqrt{2}+6\sqrt{6}+2\sqrt{12}$ *Multiply*

$18+6\sqrt{2}+6\sqrt{6}+2\cdot\sqrt{4\cdot 3}$ *Find perfect squares*

$18+6\sqrt{2}+6\sqrt{6}+2\cdot 2\cdot\sqrt{3}$ *Simplify*

$18+6\sqrt{2}+6\sqrt{6}+4\sqrt{3}$

Simplify completely. If the expression cannot be simplified, write "cannot be simplified".

1. $13\sqrt{19}+14\sqrt{19}$

2. $21\sqrt{21}-4\sqrt{21}$

3. $15\sqrt{13}-4\sqrt{7}+9\sqrt{13}$

4. $\sqrt{2}-\sqrt{8}$

5. $\sqrt{18}+\sqrt{12}$

6. $10\sqrt{63}-2\sqrt{28}+\sqrt{7}$

7. $4+6\sqrt{3}$

8. $22\sqrt{5}+9\sqrt{75}-25\sqrt{5}$

9. $5\sqrt{13}\bullet 2\sqrt{6}$

10. $4\sqrt{2}\bullet 5\sqrt{2}$

11. $(\sqrt{3})(2\sqrt{5})(6\sqrt{10})$

12. $\sqrt{2} \bullet \sqrt{8} - \sqrt{3} \bullet \sqrt{9}$

13. $(5\sqrt{2})(9\sqrt{10}) - 4\sqrt{5}$

14. $\sqrt{7}(2 - \sqrt{2})$

15. $\sqrt{3}(3\sqrt{6} + 2\sqrt{3})$

16. $6\sqrt{5}(3\sqrt{2} + 4\sqrt{10})$

17. $7(9 - 4\sqrt{13})$

18. $(2 + \sqrt{5})(4 - \sqrt{3})$

19. $(6 + 2\sqrt{6})(3 + \sqrt{2})$

20. $(10 + \sqrt{5})(10 - \sqrt{5})$

Is it Rational?

For each of the numbers below, decide whether it is rational or irrational. Explain your reasoning in detail. *This is the Pre-Assessment for the Formative Assessment Lesson*

5	
$\frac{5}{7}$	
0.575	
$\sqrt{5}$	
$5+\sqrt{7}$	
$\frac{\sqrt{10}}{2}$	
$5.75....$	
$(5+\sqrt{5})(5-\sqrt{5})$	
$(7+\sqrt{5})(5-\sqrt{5})$	

Rational or Irrational?

This is the Post-Assessment for the Formative Assessment Lesson Statements about Rational Numbers

1. Circle any of these numbers that are rational:

$\frac{\sqrt{3}}{27}$ $3-27$ $\sqrt{3}+\sqrt{27}$ $\frac{\sqrt{3}}{\sqrt{27}}$ $(\sqrt{27}+3)(\sqrt{27}-3)$

2.
   a. Fubara says, "The sum of two irrational numbers is always irrational" Show that Fubara is incorrect.

   b. Nancy says, "The product of two irrational numbers is always irrational"
   Show that Nancy is incorrect.

3. Complete the table. Make sure to explain your answer.

Statement	True or false	Explanation
If you divide one irrational number by another, the result is always irrational.		
If you divide a rational number by an irrational number, the result is always irrational.		
If the radius of a circle is irrational, the area must be irrational.		

Algebra Unit #1 Concept#4

Lesson Name: A1.U1.C4.G.05.**HW.RadicalOperations**

Write each expression in simplest radical form. State whether each result is rational or irrational.

1. $4\sqrt{27} + 6\sqrt{12}$

2. $(2\sqrt{10})(5\sqrt{3})$

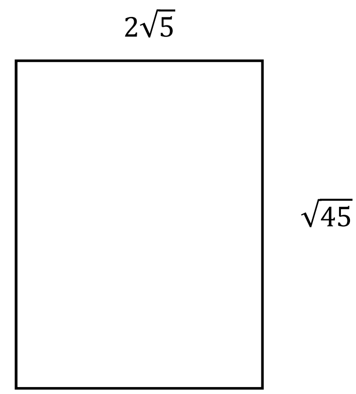
3. $(4\sqrt{12})(5\sqrt{18})$

4. $3\sqrt{36} - 5\sqrt{16} + 4$

5. $3\sqrt{8} + 2\sqrt{6}$

6. $\sqrt{2}(\sqrt{2} + 3\sqrt{6})$

7-8. Determine the perimeter and area of the rectangle.



Unit 2

Reasoning with Linear Equations and Inequalities

Algebra 1

Unit 2: Reasoning with Linear Equations and Inequalities

Concept 1: Creating and Solving Linear Equations

Lesson A: Equations in 1 Variable	(A1.U2.C1.A.____.equations)
Lesson B: Inequalities in 1 Variable	(A1.U2.C1.B.____.inequalities)
Lesson C: Rearranging Formulas (Literal Equations)	(A1.U2.C1.C.____.literal)
Lesson D: Review of Slope and Graphing Linear Equations	(A1.U2.C1.D.____.slopegraph)
Lesson E:Writing and Graphing Linear Equations	(A1.U2.C1.E.____.writingequations)
Lesson F: Systems of Equations	(A1.U2.C1.F.____.systemsofeq)
Lesson G: Graphing Linear Inequalities and Systems of Linear Inequalities	(A1.U2.C1.G.____.systemsofineq)

Concept 2: Function Overview with Notation

Lesson H: Functions and Function Notation	(A1.U2.C2.H.___.functions)

Concept 3: Linear Functions as Sequences

Lesson I: Arithmetic Sequences	(A1.U21.C3.I.____.ArithmeticSequences)

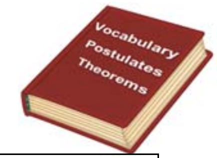
A1.U2.A.01.**Unit Vocabulary**

Term	Definition	Notation	Diagram/Visual
Equation			
Commutative Property			
Associative Property			
Distributive Property			
Addition Property of Equality			
Subtraction Property of Equality			
Multiplication Property of Equality			
Division Property of Equality			

A1.U2.A.01.**Unit Vocabulary**

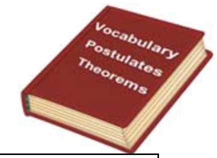
Symmetric Property			
Substitution Property			
Inequality			
Solution Set			
Literal Equation			
Linear Function			
Slope			
Average Rate of Change			
Constant Rate of Change			

A1.U2.A.01.**Unit Vocabulary**



Ordered Pairs			
X-Intercept			
Y-Intercept			
Arithmetic Sequence			
Continuous			
Discrete			
Domain			
End Behaviors			
Explicit Formula			

A1.U2.A.01.**Unit Vocabulary**



Interval Notation			
Linear Model			
Parameter			
Range			
Recursive Formula			
Substitution			

Creating and Solving Linear Equations

Create equations in one variable and use them to solve problems.
A linear equation in one variable can be solved by applying properties to transform the equation to an equivalent equation where the variable is equal to a value. When transforming equations, you should be able to identify the property that is applied.

Example! Properties of Numbers and Equality

When you apply the properties of equality to an equation, the solution to the equation does not change. Equations that have the same solutions are equivalent equations.
The **addition property of equality** states that the same quantity can be added to each side of an equation.
The **subtraction property of equality** states that the same quantity can be subtracted from each side of an equation.
The **multiplication property of equality** sates that each side of an equation can be multiplied by the same quantity.
The **division property of equality** states that each side of an equation can be divided by the same quantity.
The **distributive property** states that the product of a number and a sum is equal to the sum of the products of the number and each addend.

SELF CHECK

Provide the justification to the step to solve 2x – 5 = 13 and check the solution.

1. $2x - 5 = 13$

   $2x - 5 + 5 = 13 + 5$ ______________________

   $2x = 18$ Simplify

   $\frac{2x}{2} = \frac{18}{2}$ ______________________

   $X = 9$ Simplify

2. Solve $5x + 2 = 3(2x - 1)$ using justifications.

3. Solve $7 - 9x = 5(x + 3) + 2x$ using justifications.

4. Solve $x + 9 = \frac{5x}{2}$ using justifications.

Questions To Ponder

1. Suppose you want to solve the equation $2a + b = 2a$, where a and b are nonzero real numbers. Describe the solution to this equation. Justify your descriptions.

Example!

Creating and Solving Equations with Real World Problems

You can use what you know about writing expressions to write an equation that represents a real-world situation. When you create an equation to model a real world problem your equation may involve the distributive property.

Suppose Cory and his friend Walter go a movie. Each of their tickets costs the same amount, and they share a frozen yogurt that costs $5.50. The total amount they spend is $19.90. How can you write an equation that describes the situation?

A. **Identify the important information. (Total cost 19.90 for both, and $5.50 for shared yogurt)**
B. **Write a verbal description. (let x be the cost of the movie ticket)**
C. **Write an equation. (2x + 5.50 = 19.90)**

$2x + 5.50 = 19.90$	Given
$2x + 5.50 - 5.50 = 19.90 - 5.50$	Subtraction prop of equality
$2x = 14.40$	Simplify
$\frac{2x}{2} = \frac{14.40}{2}$	Division prop of equality
$X = 7.20$	Simplify

SELF CHECK

Write an equation for each description.

1. The length of a rectangle is twice its width. The perimeter of a rectangle is 126 ft.
2. Community Gym charges a $50 membership fee and a $55 monthly fee. Workout Gym charges a $200 membership fee and a $45 monthly fee. After how many months will the total amount of money paid to both gyms be the same? What will the amount be?
3. One month, Ruby worked 6 hours more than Isaac, and Mary worked 4 times as many hours as Ruby. Together they worked 126 hours. Find the number of hours each person worked.
4. Ten times the sum of half a number and 6 is 8.

Questions To Ponder

Explain the Error. Kevin and Brittany write an equation to represent the following relationship, and both students solve their equation. Who found the correct equation and solution? Why is the other person incorrect?

5 times the difference of a number and 20 is the same as half the sum of 4 more than 4 times a number.

Kevin	Brittany
$5(x - 20) = \frac{1}{2}(4x + 4)$	**$5(20 - x) = \frac{1}{2}(4x + 4)$**
$5x - 100 = 2x + 2$	**$100 - 5x = 2x + 2$**
$3x - 100 = 2$	$100 - 7x = 2$
$3x = 102$	$-7x = -98$
$X = 34$	$x = 14$

Independent Practice

Re-Teaching and Independent Practice

Example!

What is the solution of $5x - 8 = 32$?

$5x - 8 + 8 = 32 + 8$	To get the variable term alone on the left side, add 8 to each side.
$5x = 40$	Simplify.
$\frac{5x}{5} = \frac{40}{5}$	Divide each side by 5 since *x* is being multiplied by 5 on the left side. This isolates *x*.
$x = 8$	Simplify.
Check $5x - 8 = 32$	Check your solution in the original equation.
$5(8) - 8 = 32$	Substitute 8 for *x*.
$32 = 32$ ✓	Simplify.

Solve the Equation and Justify Your Answer (Explanation or Properties)

1. $4x + 13 = 21$	<u>Reasoning</u>	2. $3y + 2 = 11$	<u>Reasoning</u>
3. $3(2x-5) = 12$	<u>Reasoning</u>	4. $\frac{1}{4}x - 8 = 2$	<u>Reasoning</u>

5. $\frac{3}{4}(2x + 6) = 9$	<u>Reasoning</u>	6. 4x -6 = 2x +10	<u>Reasoning</u>
7. 3(23-2b) +3b =48	<u>Reasoning</u>	8. 3x +1 = 2x - 4	<u>Reasoning</u>
9. $6x + 5 = 10 + 5x$	<u>Reasoning</u>	10. $3x + 5x + 4 - x + 7 = 88$	

11. A large box of cereal contains 5 more ounces than a small box. A large and small box combined contain 27ounces of cereal. Find the number of ounces of cereal in each box.

12. Jimmy, Andrew, and Richard collect comic books. Jimmy has five more comic books than Andrew, and Richard has four times as many comic books as Jimmy. Together they have 145 comic books. Find the number of comic books each person has.

Lesson Name - Application

Lucy's Linear Equations and Inequalities (Practice Task)

Name________________________________ **Date**__________________

Lucy has been assigned the following linear equations and inequality word problems. Help her solve each problem below by using a five step plan.

- Drawing a Sketch (if necessary)
- Defining a Variable
- Setting up an equation or inequality
- Solve the equation or inequality
- Make sure you answer the question

1. The sum of 38 and twice a number is 124. Find the number.

2. Find three consecutive integers whose sum is 171.

3. Find four consecutive even integers whose sum is 244.

4. Alex has twice as much money as Jennifer. Jennifer has $6 less than Shannon. Together they have $54. How much money does each have?

HOMEWORK

Homework #____1_____

1. $5x + 2 = 3(2x - 1)$	Reasoning	2. $x + 9 = \frac{5x}{2}$	Reasoning
3. $7 - 9x = 5(x+3) + 2x$	Reasoning	4. $3x + 1 = 7x - 11$	Reasoning
5. $x + 3(x + 1) = 2x + 7$	Reasoning	6. $5x - 2(3x+2) = 5(x-2)$	Reasoning

7. Brad is one year less than twice as old as Naomi. Their combined age is 29.

8. Jessica bought 12 rose plants and 10 lily plants for her garden. Each rose plant costs $5 more than each lily plant. She spent a total of $148. How much does lily cost? Rose?

Solving and Graphing Inequalities

GUIDED NOTES

An inequality is a statement that compares two expressions that are not strictly equal by using inequality symbols. You can solve inequalities in the same way you can solve equations by following the properties of inequalities. However, there are two fundamental differences between handling inequalities and equations. The first and most obvious difference is when dividing or multiplying both sides of the inequality by a negative number, the direction of the inequality changes (reverses). The second distinction is equations have just ONE solution while inequalities have infinitely many solutions.

Example!

When you need to use an inequality to solve a word problem, you will probably encounter one of the phrases that follows. In the table each phrase is stated along with its mathematical translations into symbols.

Inequality symbols			
<	>	≤	≥
•less than •fewer than	•greater than •more than •exceeds	•less than or equal to •no more than •at most	•greater than or equal to •no less than •at least

- You must be at least 48 inches tall to ride this ride
  $h \geq 48$
- Twelve minus a number is no more than the sum of double the number and 16
  $12 - x \leq 2x + 16$

When solving inequalities with one variable, follow the process for solving equations remembering the two fundamental differences.

$100x - 200 > 50x - 75$
$50x > 125$
$X > 2.5$

$8 - 5x < 12 - x$
$8 - 4x < 12$
$-4x < 4$
$x > -1$

SELF CHECK

1. $3 < -5n + 2n$
2. $9 \geq -2m + 2 - 3$
3. $167 < 6 + 7(2 - 7R)$
4. $-5n - 6n \geq -7(5n - 6) - 6n$
5. Robert is 5 years younger than Antonio. Together, the two boys are at least 37 years old if you combine their ages. What are Antonio's possible ages?

Questions To Ponder

How could you explain to another student why it is necessary to reverse the symbol when multiplying/dividing by a negative number using the initial statement of $6 > 5$?

Independent Practice

Re-Teaching and Independent Practice

Solve: $-3y + 5 > 23$

$\underline{-5} \quad \underline{-5}$ • Subtract 5 from each side.

$\dfrac{-3y}{-3} > \dfrac{18}{-3}$ • Divide each side by negative 3.

$y < -6$ • Reverse the inequality sign.

• Graph the solution.

Open circle, line to the left.

Solve the Linear Inequalities:

1. $3x - 1 \geq 11$	2. $-6b + 22 < -26$
3. $6n - 11 < 17 - 8n$	4. $4(s+3) - 7s \geq 5s - 13$
5. $3 - 4y \leq 11$	6. $2(x + 5) < 8(x-3)$
7. $-6x - 3 \leq -4x + 1$	8. $4m + 12 > 3(2m - 3)$

9. Quail Run Elementary school is having a fall carnival. Admission into the carnival is $3 and each game inside the carnival costs $.25. Write an inequality that represents the possible number of games that can be played having $10. What is the maximum number of games that can be played?	10. Rigsby charges a $75 flat fee for birthday party rental and $6.25 for each person. Jazlyn has no more that $250 to spend on the party. Write an inequality that represents Jazlyn's situation. How many people can Jazlyn invite to her party without exceeding her limit?
11. Mr. Burley's class is putting on an American Idol Talent show to raise money. It cost $650 to rent the banquet hall that they are going to use. If they charge $15 for each ticket, how many tickets do they need to sell in order to raise at least $1000?	12. Daryl has $45. Google Play streams cost $1.25 each. How many songs can he stream and still have $13 left to spend?

Lesson Name - Application

1. Lucy has been assigned the following linear equations and inequality word problems. Help her solve each problem below by using a five step plan.

- Drawing a Sketch(if necessary)
- Defining a Variable
- Setting up an equation or inequality
- Solve the equation or inequality
- Make sure you answer the question

a. The length of a rectangle is 4 cm more than the width and the perimeter is at least 48 cm. What are the smallest possible dimensions for the rectangle?

b. There are three exams in a marking period. A student received grades of 75 and 81 on the first two exams. What grade must the student earn on the last exam to get an average of no less than 80 for the marking period?

2. The local amusement park sells summer memberships for $50 each. Normal admission to the park costs $25; admission for members costs $15.

a. If Darren wants to spend no more than $100 on trips to the amusement park this summer, how many visits can he make if he buys a membership with part of that money?

b. How many visits can he make if he pay normal admission instead?

c. If he increases his budget to $160, how many visits can he make as a member?

d. How many can he make as a non-member with the increased budget of $160?

Homework #____1_____
Solve the Linear Inequalities

1. $5x + 2(x+1) \geq 23$	2. $6y + 2(2y + 3) > 16$
3. $3z - 15z - 30 > 54$	4. $12b - 3b + 5 \geq -31$
5. $3x - (x - 7) \leq 18$	6. $4m + 3 < 7m - 2$
7. $12 + 3(2m - 3) > 48$	8. $-7c - 8 \leq 8c + 10$
9. Jason is saving up to buy a new Iphone that costs \$1200. So far, he saved \$350. He would like to buy the camera 5 weeks from now. What is the equation used to represent how much he must save every week to have enough money to purchase the phone?	10. Monica had \$750 in a saving account at the beginning of the summer. She wants to have at least \$225 in the account by the end of the summer. She withdraws \$35 each week for food, clothes, and movie tickets.

Rearranging Formulas/Literal Equations

GUIDED NOTES

A literal equation is an equation where variables represent known values. Literal equations allow use to represent things like distance, time, interest, and slope as variables in an equation. Using variables instead of words is a real time-saver!

- You can "rewrite" a literal equation to isolate any one of the variables using inverse operations. This is called *solving for a variable*.

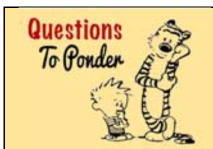
Solving for a Variable
Step 1 Locate the variable you are asked to solve for in the equation.
Step 2 Identify the operations on this variable and the order in which they are applied.
Step 3 Use inverse operations to "undo" operations and isolate the variable.

Example! Guided Practice

Solve $x + y = 15$ for x	Solve $D = \frac{m}{V}$ for V
Solve $y = mx+b$ for x	Solve $3x + 5y = 15$ for y

SELF CHECK

Solve: $QP = X$ for Q	Solve: $P = 2(L + W)$ for W
Solve: $y = 2x + 5$ for x	Solve: $3x + 6y = 18$ for y



Is it possible to solve an equation for any of the given variables? Why or why not?

Re-Teaching and Independent Practice

Independent Practice

Solve $Q = \dfrac{c+d}{2}$ for d

$$Q = \frac{c+d}{2}$$

$$2(Q) = \frac{2}{1}\left(\frac{c+d}{2}\right)$$

$$2Q = \frac{\cancel{2}}{1}\left(\frac{c+d}{\cancel{2}}\right)$$

$$2Q = c + d$$

$$2Q - c = c + d - c$$

$$\mathbf{2Q - c = d}$$

The variable I need to isolate is currently inside a fraction:

1) I need to get rid of the denominator. To do this, I'll multiply through by the denominator's value of 2.
2) On the left-hand side, I'll just do the simple multiplication.
3) On the right-hand side, to help me keep things straight, I'll convert the 2 into its fractional form of 2/1.

Solve for the indicated variable in the parenthesis:

1. P = IRT (T)	2. 3x + 6y = 12 (x)
3. $\frac{3}{4}x + 6y = 12$ (y)	4. $A = \frac{r}{2L}$ (L)
5. 3x – 4y = 36 (y)	6. 12x – 24y = 60 (x)

Independent Practice

7. $A = \frac{a+b+c}{3}$ (c)	8. $a^2 + b^2 = c^2$ (b)
9. P = 2L + 2 w (L)	10. $V = \frac{4}{3}\pi r^3$ (r)

Literal Equations - Application

1. You can use the formula $a = \frac{h}{n}$ to find the batting average, a, of a batter who has $\mathbf{h}$ hits in n times at bat. Solve the formula for h. If a batter has a batting average of .290 and has been at bat 300 times, how many hits does the batter have?

2. Bricklayers use the formula $n = 7lh$ to estimate the number n of bricks needed to build a wall of length l and height h, where both length and height are in feet. Solve the formula for . Estimate the height of a wall 28 feet long that requires 1568 bricks.

3. The final score, *f*, a player receives during each level of a game is given by the equation $f = \frac{p-a}{d}$ where *p* is the number of points earned, *a* is the average expected score, and *d* is the difficulty of the level.

Part A

Two friends are playing this game at the same level of difficulty. Ryan earns 140 points and receives a final score of 14. Tony earns 125 points and receives a final score of 8. What is the difficulty of the game at this level? Explain your answer.

Part B

Rene decides to play the same game at a difficulty of 3.8. If she earns 124 points and a final score of 5, what is the average expected score of the game? Show your work.

Homework #____1_____
Solve the variable in the Literal Equation

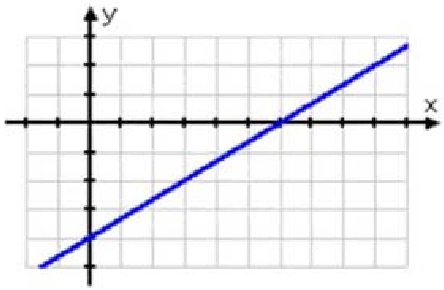
1. $a^2 + b^2 = c^2$ solve for a	2. $V = \pi r^2 h$ solve for r
3. 3x + 6y = 12 solve for x	4. $\frac{x+y}{3} = 5$ solve for x
5. -6x = 12y - 24 solve for y	6. $r^2 + h^2 = l^2$ solve for h
7. 3k – 15m = 30 solve for m	8. $\frac{3+y}{2m} = 18$ solve for m
9. $x = \frac{yz}{6}$ solve for z	10. 12x – 4y = 20 solve for y

Slope – 4 Types

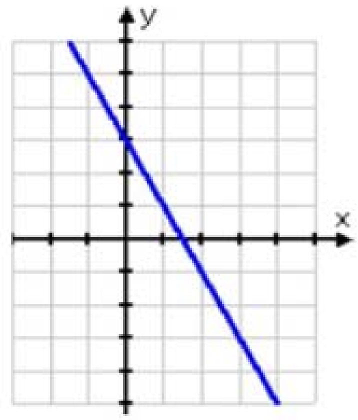
GUIDED NOTES

Slope: the constant rate of change of the rise (vertical change) to the run (horizontal change). Slope is usually represented by the variable m.

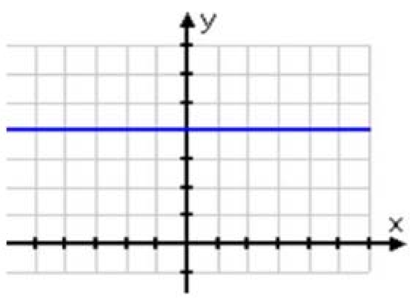
4 Types of Slope:



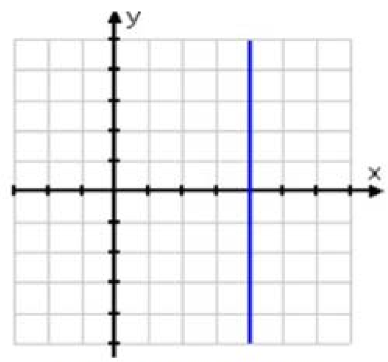
Positive Slope



Negative Slope



Zero Slope



Undefined Slope

Slope – From a Graph

GUIDED NOTES

To find slope from a graph remember $slope = \frac{rise}{run}$

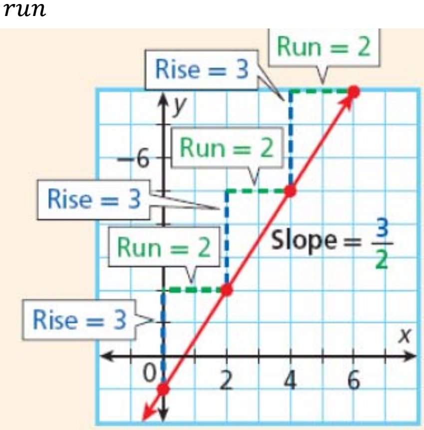
The **rise** is the difference in the ***y*-values** of two points on a line.

The **run** is the difference in the ***x*-values** of two points on a line.

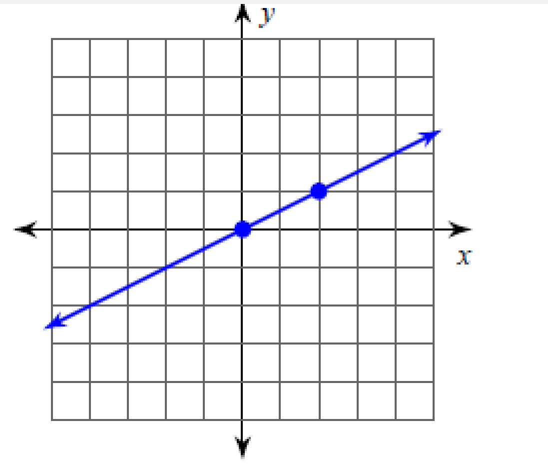
The **slope** of a line is the ratio of rise to run for any two points on the line.

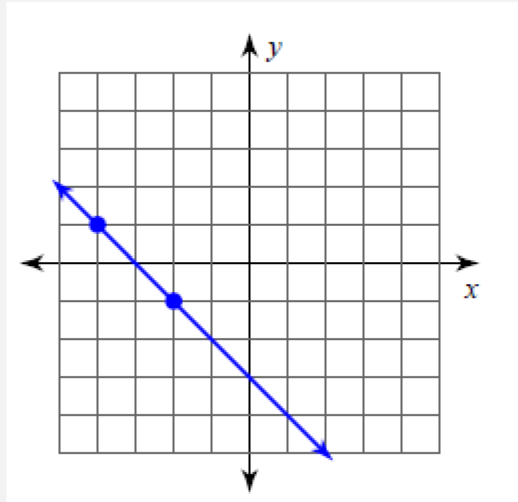
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

(Remember that *y* is the **dependent variable** and *x* is the **independent variable**.)

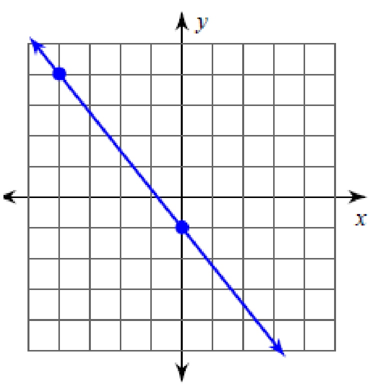


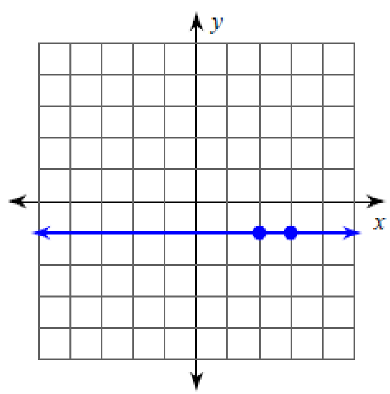
Example! Find the slope of each graph below.





SELF CHECK





Slope from an Equation

GUIDED NOTES

If the equation in given in slope intercept form y = mx+b the slope is the coefficient of the variable x.

For the equation: 5 the slope is 3

If the equation is given in standard form Ax + By = C then rearrange to equation to put it in slope intercept form and then identify the slope.

$$2x + 6y = 12$$
$$\underline{-2x} \quad \underline{-2x}$$
$$6y = -2x + 12$$
$$\frac{6y}{6} = \frac{-2x + 12}{6}$$
$$y = -\frac{1}{3}x + 2$$

In this equation the slope is $-\frac{1}{3}$

Example!

Identify the slope

1. $y = -5x + 4$
2. $2x + 5y = 10$

SELF CHECK

Identify the slope for the following equations:

1. $y = \frac{3}{5}x + 5$
2. $4x + 3y = 12$

Slope from Two-Points

GUIDED NOTES

To find the slope from two points use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example!

Find the slope:

1. $(19, -2), (11, 10)$

2. $(17, 3), (20, 3)$

SELF CHECK

Find the slope:

1. $(6, -10), (-15, 15)$

2. $(-19, 12), (-19, 1)$

Slope from a Table

GUIDED NOTES

To find slope from a table you can:

1. Find the slope by finding $\frac{change\ in\ y}{change\ in\ x}$

2. Pick two points and use the slope formula to find the slope

Example!

	x	y	
	1	−3	
1	2	2	5
1	3	7	5
2	5	17	10

1. *Determine the change between each consecutive x value*
2. *Determine the change between each consecutive y value*
3. *Write corresponding ratios that show* $\frac{change\ in\ y}{change\ in\ x}$
4. *Simplify the ratios to determine if they are equivlaent.*

In this problem the ratios $\frac{5}{1}, \frac{5}{1}, \frac{10}{2}$ are all equivalent to $\frac{5}{1}$, therefore the slope is 5.

SELF CHECK

Find the slope from the following tables:

x	y
2	−5
4	6
7	20
11	40

x	y
5	5
7	3
9	1
11	−1

X and Y Intercepts of Linear Functions

GUIDED NOTES

An intercept is a point at which a line, curve, or surface intersects an axis.

- The y-intercept is the point where the value of the x-coordinate is zero
- The x-intercept is the point where the value of the y-coordinate is zero

Remember to ALWAYS write your intercepts as ORDERED PAIRS.

Intercepts from Slope-Intercept Equations

GUIDED NOTES

When an equation is written in slope-intercept form $y = mx + b$:

- The y intercept is the constant term b in the equation.
- The x intercept is found by plugging 0 in for y and solving the equation for x.

Example!

For the equation below identify the x and y intercepts:

$y = 4x - 2$

The y intercept is (0,-2)

The x intercept is found using the steps below:

$y = 4x - 2$

$0 = 4x - 2$
$+2 \qquad\quad +2$
$2 = 4x$
$\frac{2}{4} = \frac{4x}{4}$

$\frac{1}{2} = x$ The x intercept is $(\frac{1}{2}, 0)$

SELF CHECK

Find the x and y intercepts for equation:
$y = -2x - 8$

Intercepts from Standard Form Equations

GUIDED NOTES

When an equation is written in Standard form $Ax + By = C$

- The y intercept is found by plugging 0 in for x and solving for y
- The x intercept is found by plugging 0 in for y and solving the equation for x.

Example!

EXAMPLE: Find the x- and y-intercept for this equation: 3x + 4y = 12

<u>STEP 1</u> (find the x-int)

3x + 4y = 12

3x + 4(0) =12 (substitute 0 for "y" and solve for x)

3x + 0 = 12

3x = 12

x = 4

The "x-int" is: (4, 0)

<u>STEP 2</u> (find the y-int)

3x + 4y = 12

3(0) + 4y =12 (substitute 0 for "x" and solve for y)

0 + 4y = 12

4y = 12

y = 3

The "y-int" is: (0, 3)

SELF CHECK

Find the x and y intercepts for the following equation: $9x - 3y = 18$

Graphing Linear Functions – Make a Table

GUIDED NOTES

One Option for graphing Linear Functions is to make a table of values and graph the points.

1. Draw the table
2. Choose 3- 5 x-values
3. Plug x-values into the equation to get y-values
4. Plot and connect points on a graph

Example!

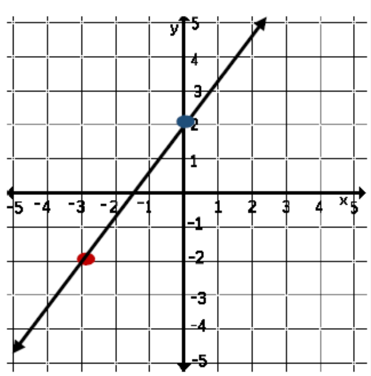
$\mathbf{y = \frac{4}{3}x + 2}$

$y = \frac{4}{3}(-3) + 2$
$y = -2$

$y = \frac{4}{3}(0) + 2$
$y = 2$

$y = \frac{4}{3}(3) + 2$
$y = 6$

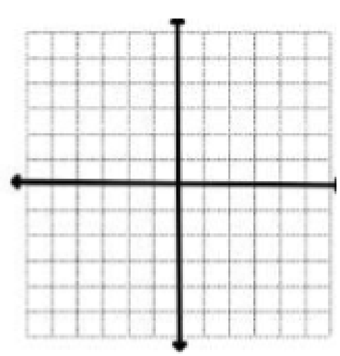
x	y
-3	-2
0	2
3	6



SELF CHECK

Steps:
1. Draw the table
2. Choose 5 x-values
3. Plug x-values into the equation to get y-values
4. Plot and connect points on a graph

x	-2x + 5	y



GUIDED NOTES

Graphing Using Slope and Y intercept

Step 1: Identify the y-intercept (b) and plot the point (0, b)

Step 2: Use the slope (m) to find a second point: $\frac{rise}{run}$ (Put a whole number over 1 to make it a fraction: 2 = $\left(\frac{2}{1}\right)$

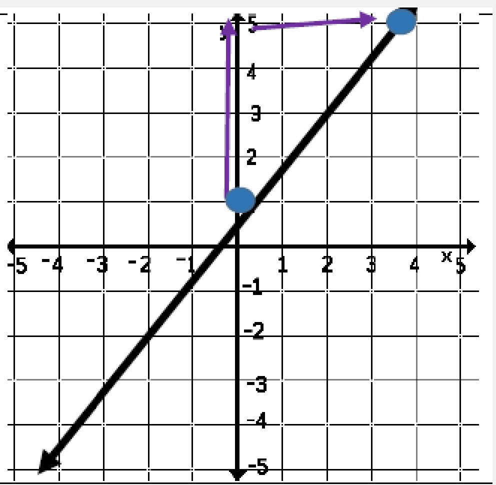
Step 3: Connect the points

Example!

$y = \frac{4}{3}x + 1$

b= 1

$m = \frac{4}{3}$

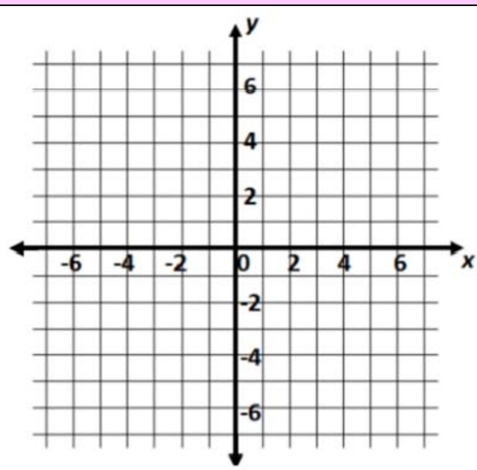


SELF CHECK

y = -3x – 2

m = ________________

b =



Graphing Using Intercepts

Step 1: Find y-intercept	**Step 2**: Find x-intercept
• Let x = 0 • Substitute 0 for x; solve for y. • Graph the point on the y-axis.	• Let y = 0 • Substitute 0 for y; solve for x. • Graph the point on the x-axis.

Step 3: Connect the dots.

Example!

2x - 2y = 8

S1) Let x= 0

$2(0) - 2y = 8$

$-2y = 8$

$\frac{-2y}{-2} = \frac{8}{-2}$

$y = -4$

Ordered pair: (0, -4)

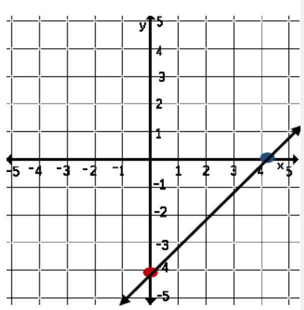
S2) Let y = 0

$2x - 2(0) = 8$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

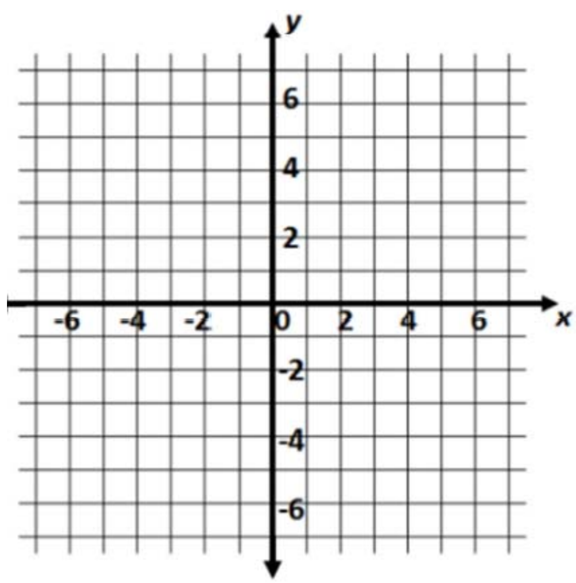
Ordered pair: (4, 0)



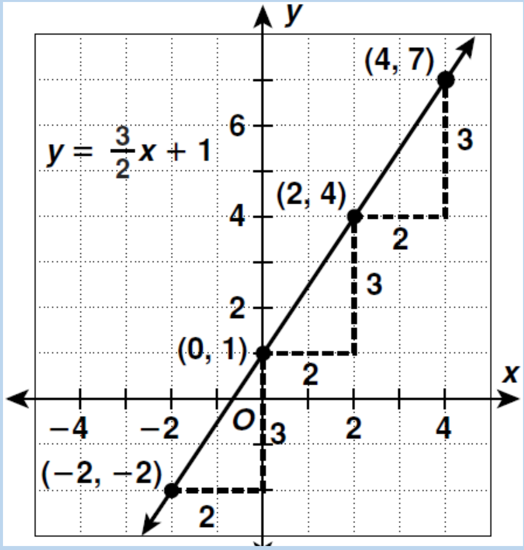
SELF CHECK

Graph the equation using intercepts

$3x + 2y = -8$



Rise over Run:



Graphing using a Table:

$y = 3x - 4$

+1 +1 +1 +1

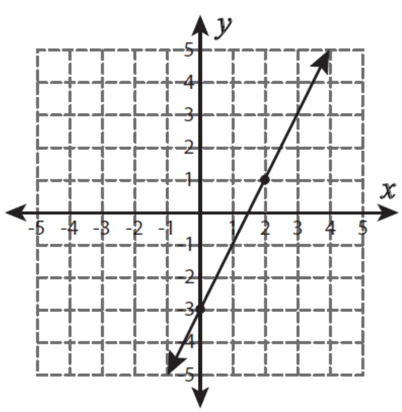
x	−2	−1	0	1	2
y	−10	−7	−4	−1	2

+3 +3 +3 +3

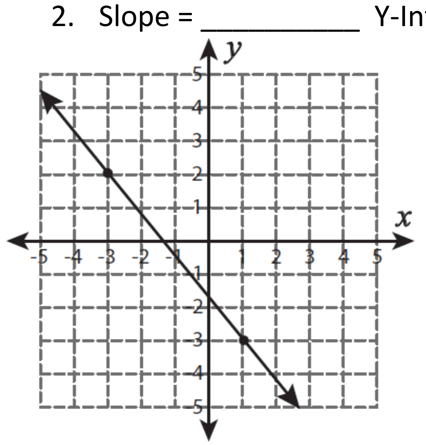
Since a constant change in the *x*-value corresponds to a constant change in the *y*-value, $y = 3x - 4$ is a linear equation.

Identifying the y-intercept and slope (Rate of Change):

1. Slope = __________ Y-Intercept: ___________

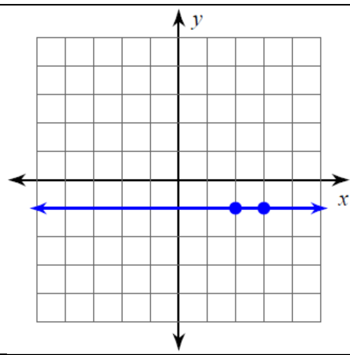


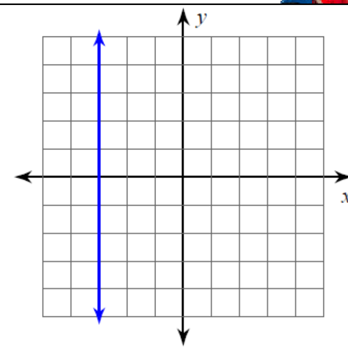
2. Slope = __________ Y-Intercept: ___________



3. Slope = __________ Y-Intercept: _____________

4. Slope = ________ Y-Intercept: __________





5. Slope = __________ Y-Intercept: _____________

X	Y
5	5
7	3
9	1
11	-1
13	-3
15	-5

6. Slope = __________ Y-Intercept: _____________

X	Y
0	-7
3	-3
6	1
9	5
12	9
15	13

7. Slope =__________ Y-Intercept: _____________

X	Y
11	-3
7	-5
3	-7
-1	-9
-5	-11
-9	-13

8. Slope =__________ Y-Intercept: _____________

X	Y
0	-7
3	-3
6	1
9	5
12	9
15	13

9. Slope = __________ Y-Intercept: _____________

X	Y
0	-10
2	-5
4	0
6	5
8	10
10	15

10. Slope = ___________ Y-Intercept: ____________

X	Y
-12	-9
-9	-5
-6	-1
-3	3
0	7
3	11

Re-Teaching and Independent Practice

Example!

Graphing Using Intercept Form:

Graph $3x + 2y = 6$. Label the points where the line crosses the axis.

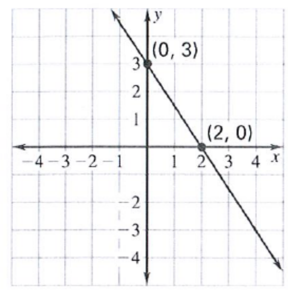
Solution

STEP 1 **Find** the intercepts.

$3x + 2y = 6$	$3x + 2y = 6$
$3x + 2(0) = 6$	$3(0) + 2y = 6$
$x = 2 \longleftarrow$ x-intercept	$y = 3 \longleftarrow$ y-intercept

STEP 2 **Plot** the points that correspond to the intercepts. The x-intercept is 2, so plot and label the point (2, 0). The y-intercept is 3, so plot and label the point (0, 3).

STEP 3 **Connect** the points by drawing a line through them.



Graphing using Slope-Intercept Form (Y= mx + b) :

Write $2x + 6y = 12$ in slope-intercept form. Then graph the line.

Step 1: Solve for y.

$2x + 6y = 12$ *Subtract 2x from both sides.*

$\underline{-2x} \quad \underline{-2x}$

$6y = -2x + 12$

$\dfrac{6y}{6} = \dfrac{-2x + 12}{6}$ *Divide both sides by 6.*

$y = -\dfrac{1}{3}x + 2$ *Simplify.*

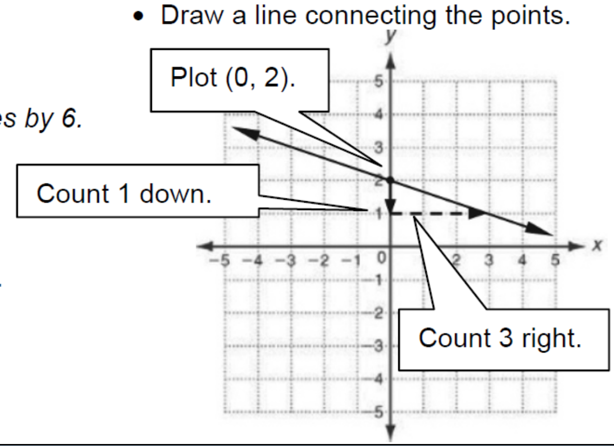
Step 2: Find the slope and y-intercept.

slope: $m = -\dfrac{1}{3} = \dfrac{-1}{3}$

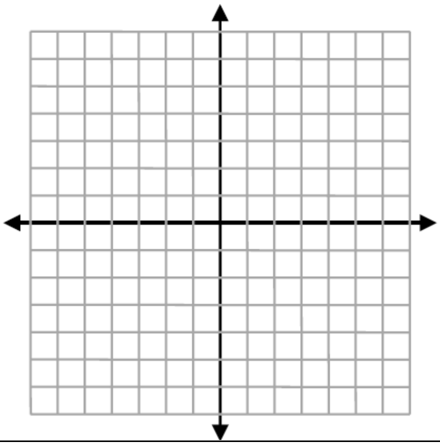
y-intercept: $b = 2$

Step 3: Graph the line.

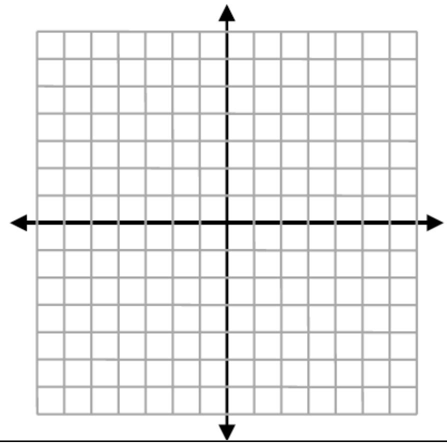
- Plot (0, 2).
- Then count 1 **down** (because the rise is **negative**) and 3 **right** (because the run is **positive**) and plot another point.
- Draw a line connecting the points.



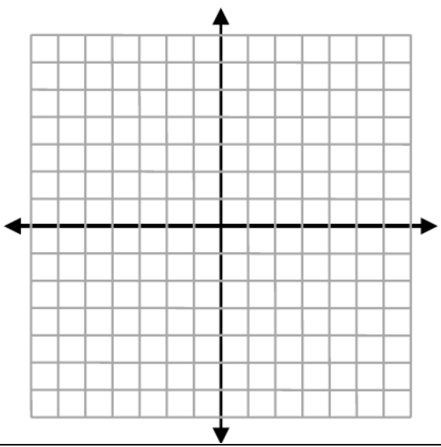
1. Graph the equation using the Intercepts:
   $2x + 4y = 10$



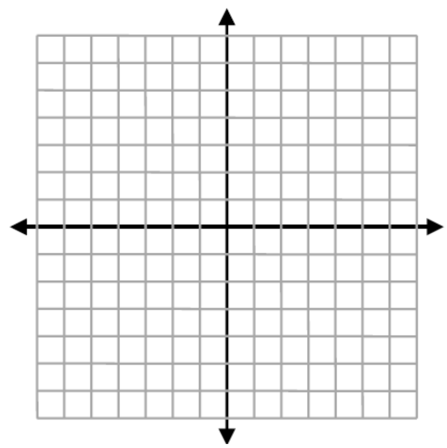
2. Graph the equation using the Intercepts:
   $-4x + 3y = 24$



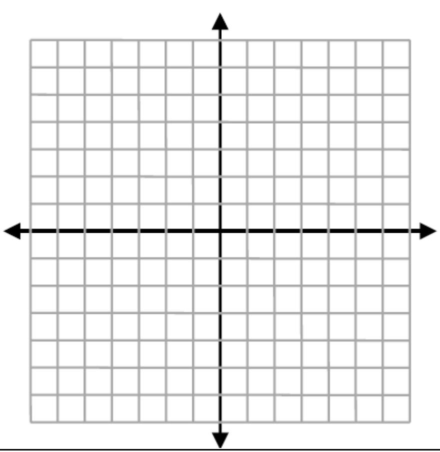
3. Graph the equation using the intercepts:
   $5x – y = 15$



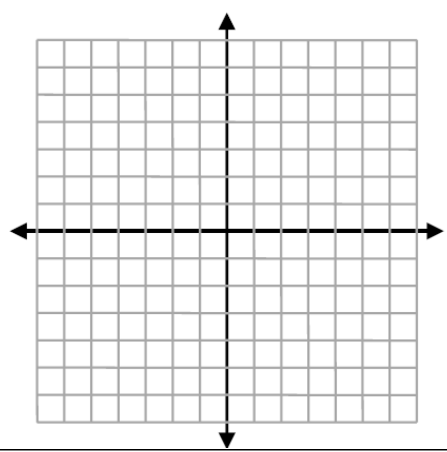
4. Graph the equation using the intercepts:
   $-2x + 3y = 9$



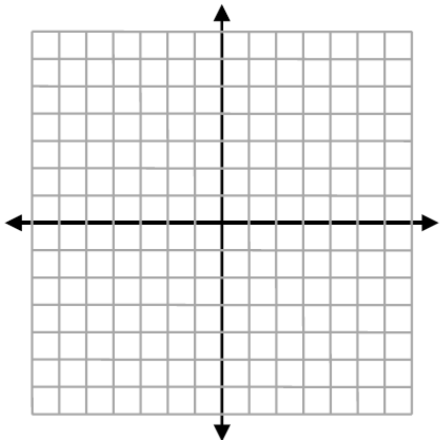
5. Graph the equation using the intercepts:
   $15x -5y = 10$



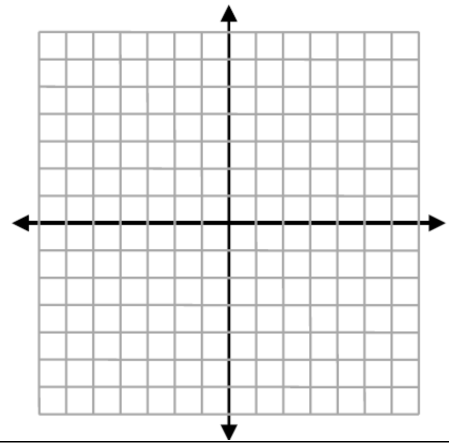
6. Graph the equation using slope intercept form:
   $-x – 6y = 18$



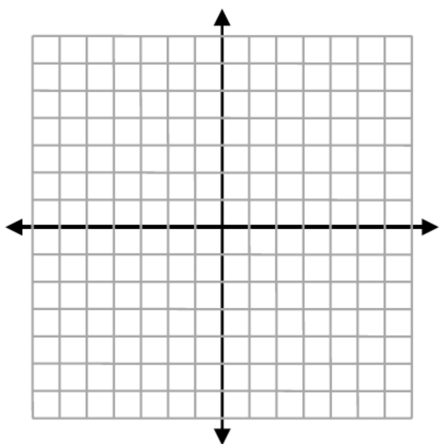
7. Graph the equation using slope intercept form:
   $3x + 5y = 0$



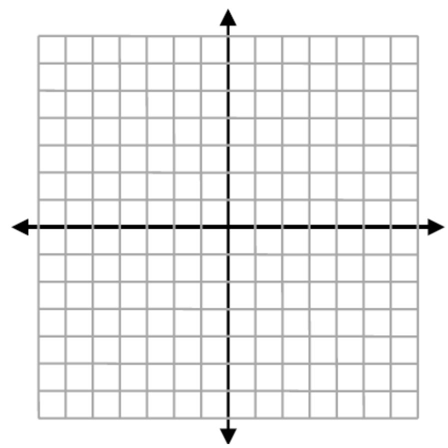
8. Graph the equation using slope intercept form:
   $-3x - 6y = -12$



9. Graph the equation using slope intercept form:
   $12x - 6y = 36$



10. Graph the equation using slope intercept form:
   $2y - 6x = -20$



Independent Practice

Re-Teaching and Independent Practice

Example!

Find the slope (rate of change) using two points:

Find the slope of the line shown.

Solution

Let $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (3, 3)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Write formula for slope.

$= \frac{3 - 0}{3 - (-2)}$ Substitute.

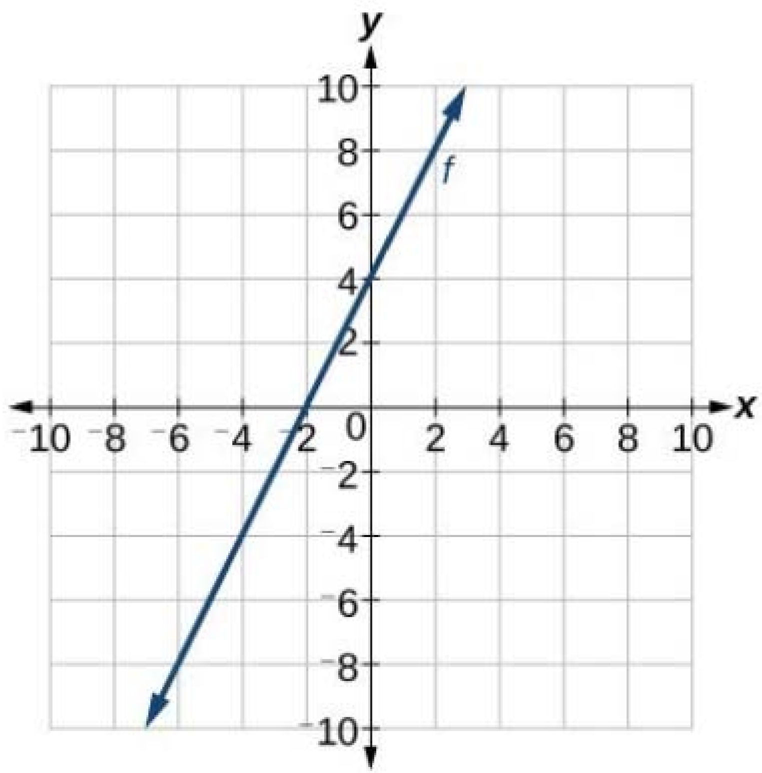
$= \frac{3}{5}$ Simplify.

1. Find slope of the line using two points: (-10, 5) & (0, 8)	2. Find slope of the line using two points: (8, -6) & (10, 6)
3. Find slope of the line using two points: (4, 0) & (6, 10)	4. Find slope of the line using two points: (12, -16) & (6, 8)
5. Find slope of the line using two points: (-3, 5) & (0, 7)	6. Find slope of the line using two points: (13, 8) & (-9, 6)

7. Find slope of the line using two points: (3, 5) & (-6, 0)	8. Find slope of the line using two points: (-3, -6) & (-12, 4)
9. Find slope of the line using two points: (12, -6) & (3, 6)	10. Find slope of the line using two points: (2, -6) & (-5, 8)

Lesson Name - Application

1. William and Thomas use different triangles to determine the slope of the line shown below.



William: William started at (0, 4) and drew a right triangle going up 4 units and right 2 units.

Thomas: Thomas started at (-4, -4) and drew a right triangle going up 12 units and right 6 units.

a. Draw and label both triangles on the graph above.

b. Describe how the two triangles are related.

____

____

____

c. Find the slope of the line using William's triangle and Thomas' triangle.
Show your work.

William's slope: __________

Thomas' slope: __________

d. Justify how the triangles relate to the slope of the line. Why can you find the slope using any two points on the line?

____

____

____

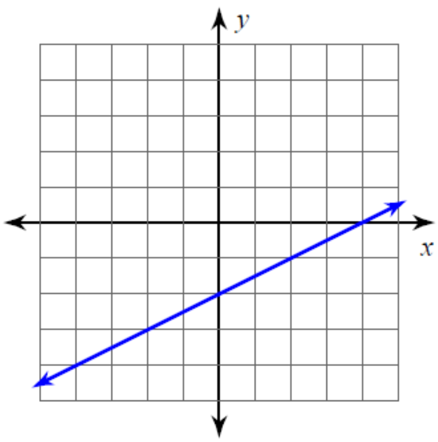
____

HOMEWORK

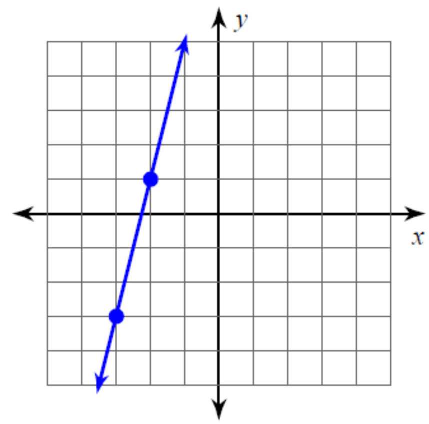
Homework #____1_____

Identifying the y-intercept and slope (Rate of Change):

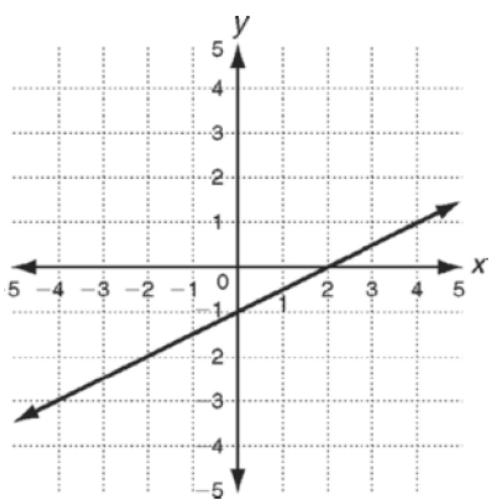
1. Slope: ________ Y-Intercept: ___________



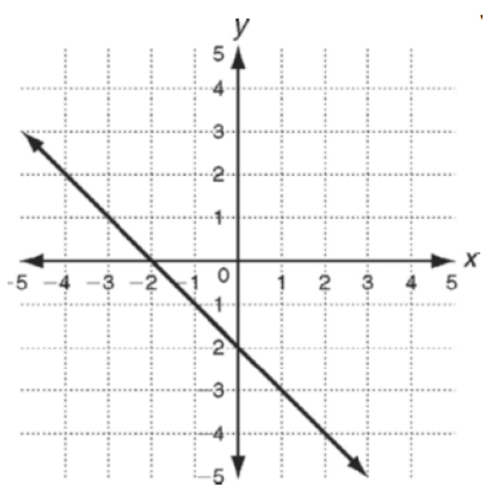
2. Slope: ______ Y – Intercept: __________



3. Slope: _______ Y-Intercept: ___________



4. Slope: _______ Y-Intercept: ___________



5. Slope: ________ Y-Intercept: __________

X	Y
0	-8
3	-4
6	0
9	4
12	8
15	12

6. Slope: ________ Y-Intercept: __________

X	Y
-7	0
-3	2
1	4
5	6
9	8
13	10

7. Slope: __________ Y-Intercept: __________

X	Y
-4	0
-3	2
-2	4
-1	6
0	8
1	10

8. Slope: ________Y-Intercept: ____________

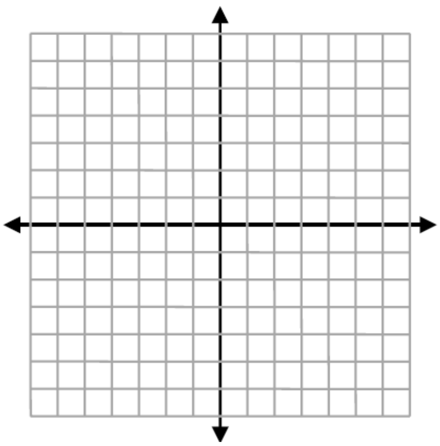
X	Y
12	-9
9	-5
6	-1
3	3
0	7
-3	11

HOMEWORK

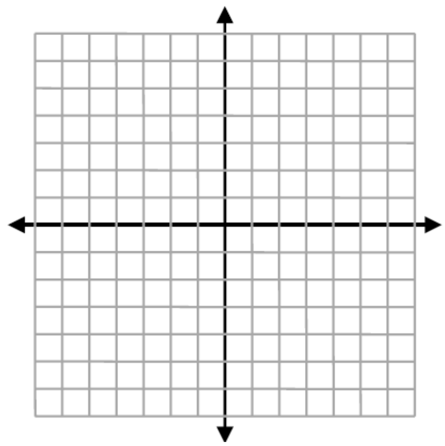
Homework #____2_____

Identifying the y-intercept and slope (Rate of Change):

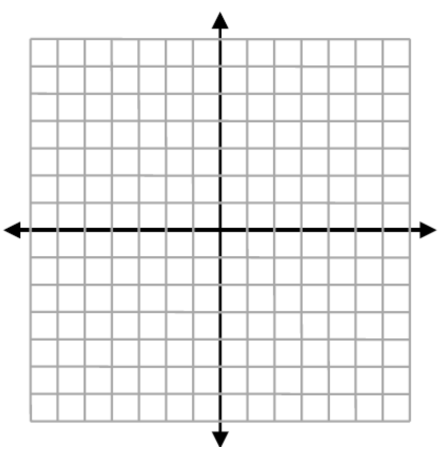
1. Graph the equation using the Intercepts:
   $-4x - 6y = 24$



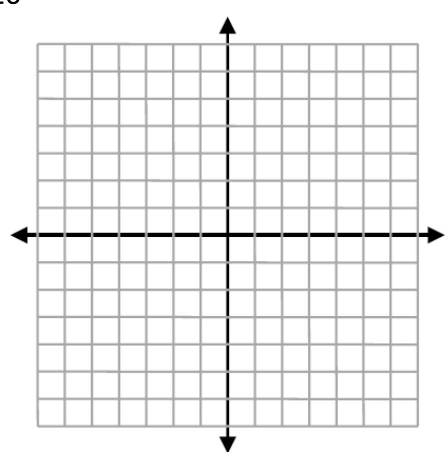
2. Graph the equation using the Intercepts:
   $2x - 6y = 18$



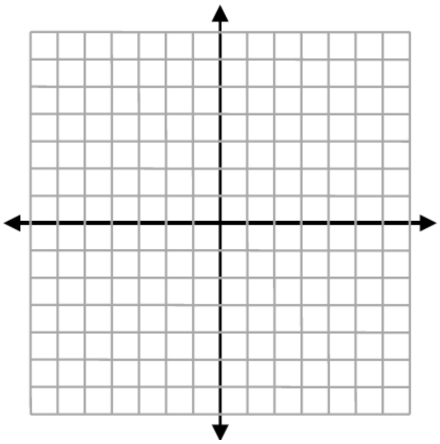
3. Graph the equation using the Intercepts:
   $8x - 16y = 32$



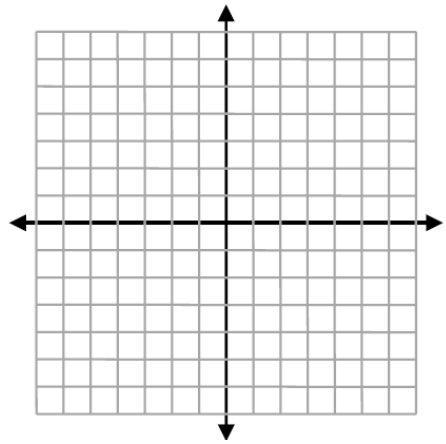
4. Graph the equation using the Intercepts:
   $-7x - 4y = 20$



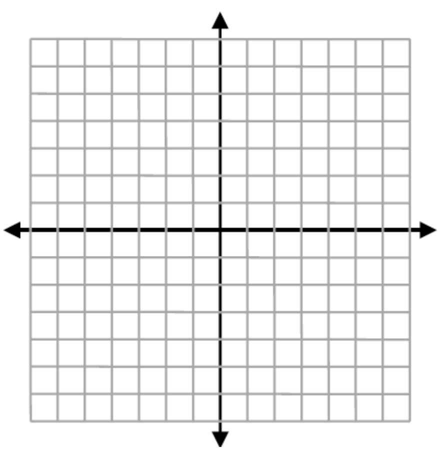
5. Graph the equation using slope intercept form:
   $-7x - 4y = 20$



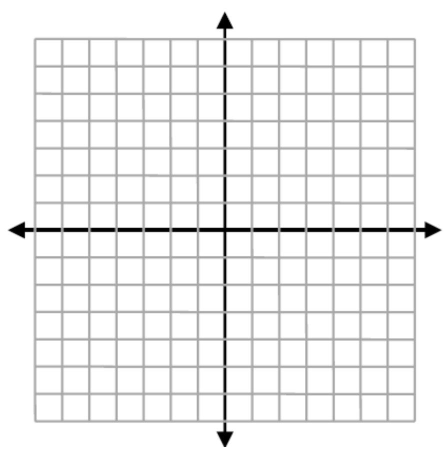
6. Graph the equation using slope intercept form:
   $-4x + 7y = 21$



7. Graph the equation using slope intercept form:
   $2x + 5y = 20$



8. Graph the equation using slope intercept form:
   $3x + 9y = 27$



Homework #____3_____ Find the slope using two coordinates (2 Points):	
1. Find slope of the line using two points: (10, 7) & (2, 5)	2. Find slope of the line using two points: (-4, 6) & (-3, -12)
3. Find slope of the line using two points: (25, -17) & (48, 56)	4. Find slope of the line using two points: (x, y) & (k, h)
5. Find slope of the line using two points: (-7, - 4) & (10, 20)	6. Find slope of the line using two points: (21, 12) & (-11, 22)
7. Find slope of the line using two points: (36, 28) & (-12, 25)	8. Find slope of the line using two points: (-26, 32) & (42, 57)

Creating Linear Equations in 2 Variables

This standard of creating equations in two or more variables to represent relationships between quantities, and graph the equations on the coordinate axes with labels and scales has two significant components. The first is translating word problems into equations with two or more variables and the second is creating graphs of equations on the coordinate axes, which incorporates multiple skills such as visual perception, interpreting data, and synthesizing information.
From the prior lesson, students learned to graph lines from tables, ordered pairs, slope intercept form, and standard form. In this lesson, we will focus on writing the equation from real-world problems by incorporated two variables.

Example!

You can write the equation for a linear model in the same way you would write the slope-intercept equation of a line. The *y-intercept* of a linear model is the quantity that does not depend on *x*. The slope is the quantity that *changes* at a **constant rate** as *x* changes. The change must be at a **constant rate** in order for the equation to be a linear model. **(Slope-Intercept form: y = mx + b)**

Example 1: A machine salesperson earns a base salary of $40,000 plus a commission of $300 for every machine he sells. Write an equation that shows the total amount of income the salesperson earns, if he sells x machines in a year.
The variable x stands for the number of machines that he sells.
The y-intercept is: 40000 because that amount does not change with his sales
The slope is: 300 because he receives $300 for every machine that he sells
Total amount of income: y = 300x + 40000.

Example 2 At a school play, children's tickets cost $3 each and adult tickets cost $7 each. The total amount of money earned from ticket sales equals $210. Write a linear model that relates the number of children's tickets sold to the number of adult tickets sold.

Let x = the number of children's tickets sold and y = the number of adult tickets sold

The amount of money earned from children's tickets is: 3x

The amount of money earned from adult tickets is: 7y

The total amount of money earned from ticket sales is: 3x + 7y

Model: 3x + 7y = 210

Example 3. David compares the sizes and costs of photo books offered at an online store. The table below shows the cost for each size photo book.

Book Size	Base Price	Cost for each additional page
7 in by 9 in	$20	$1.00
8 in by 11 in	$25	$1.00
12 in by 12 in	$45	$1.50

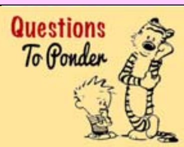
Write an equation to represent the relationship between *the cost, y, in dollars*, and *the number of pages, x,* for each book size. Be sure to place each equation next to the appropriate book size. Assume *that x* is at least 20 pages. What is the cost of a 12-in. by 12-in. book with 28 pages? How many pages are in an 8-in. by 11-in. book that costs $49

In this example, students should recognize that *the base price* is the *starting point (y intercept)* of the equation and that *the cost for each additional page* is the *rate of increase (slope value).*
For a book that is 12 x 12 with 28 pages, *y represents the cost of the book*, and *x is the number of pages beyond 20*, so the resulting equation is *y = 1.5x + 45*. To calculate how many pages are in an 8 x 11 book that costs $49, the resulting equation is *49 = x + 25,* so there are 24 pages in the 8 x 11 book that costs $49.

SELF CHECK

Practice: Model each situation using the variables indicated.

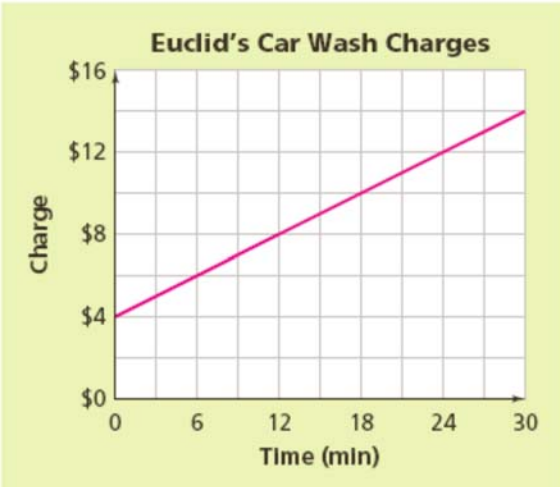
1. Mr. Thompson is on a diet. He currently weighs 260 pounds. He loses 4 pounds per month. Write a linear model that represents Mr. Thompson’s weight after m months.
2. The population of Bay Village is 35,000 today. Every year the population of Bay Village increases by 750 people. Write a linear model that represents the population of Bay Village x years from today.
3. Amery has x books that weigh 2 pounds each and y books that weigh 3 books each. The total weight of his books is 60 pounds. Write a linear model that relates the number of 2 pound books to the number of 3 pound books Amery has.
4. Ben walks at a rate of 3 miles per hour. He runs at a rate of 6 miles per hour. In one week, the combined distance that he walks and runs is 210 miles. Write a linear model that relates the number of hours that Ben walks to the number of hours Ben runs. Use variables w and r.
5. A salesperson receives a base salary of $35,000 and a commission of 10% of the total sales for the year. Write a linear model that shows the salesperson’s total income based on total sales of k dollars.



Suppose the equation $x + y = \$100.00$ represents how much Rory can spend on Christmas presents for his parents, where x represents how much money he can spend for his mom’s Christmas present and y represents how much money he can spend on his dad’s Christmas present. He will spend all of his money.

How many “solutions” are there to this equation? Discuss what this would mean, in the context of Rory’s parents. Discuss how x and y related to one another, too – if x increases, what happens to y?

1. Euclid's Car Wash displays its charges as a graph. Write an equation for the charge plan at Euclid's. Describe what the variables and numbers in your equation tell you about the situation.



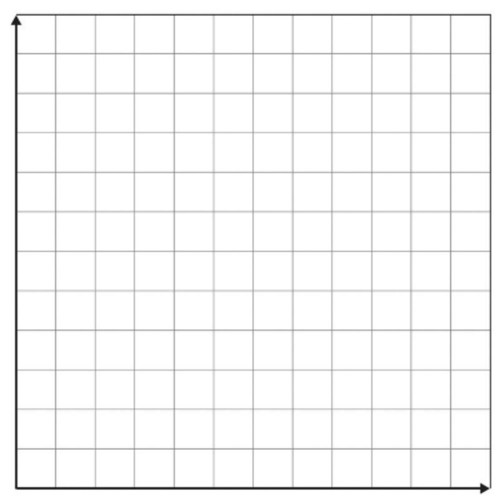
Equation: ______________________

Describe: ____

2. A television production company charges a basic fee of $4000 and then $2000 per hour when filming a commercial.

a. Write an equation in slope-intercept form relating the basic fee and per-hour charge.

b. Graph your equation.



c. Find the production costs if 4 hours of filming were needed.

3. Use the function in the table at the right.

a. Identify the dependent and independent variables.

b. Write a rule to describe the function.

Water Used for Laundry

1 load	34 gallons
2 loads	68 gallons
3 loads	102 gallons
4 loads	136 gallons

c. How many gallons of water would you use for 7 loads of laundry?

d. In one month, you used 442 gallons of water for laundry. How many loads did you wash?

4. Max sells lemonade for $2 per cup and candy for $1.50 per bar. He earns $425 selling lemonade and candy. Write a linear model that relates the number of cups of lemonade he sold (c) to the number of bars of candy (b) he sold.

5. Kim currently has $25 and decides to save $10 per week from her weekly babysitting job. Jenny currently has $160 and decides to spend $5 per week on entertainment.

1. Write an equation that describes Kim's savings.

2. Write an equation that describes Jenny's spending.

3. Draw a sketch on the graph and label axes

4. How long will it take Jenny to run out of money at this rate if she has no additional income?

5. How long will it take Kim to have $150?

6. What does it mean when the two graphs intersect?

7. When will Kim and Jenny have the same amount of money?

8. Who has more money after 10 weeks

Lesson Name – Cara's Candles Revisited

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

Cara started filling out the following table to help determine whether these two candles would ever reach the same height at the same time if allowed to burn the same length of time. Finish the table for Cara. Use the data in the table to determine what time the two candles will be at the same height.

Also, she wants to know what height the two candles would be at that time. If it is not possible, she wants to know why it could not happen and what would need to be true in order for them to be able to reach the same height. To help Cara understand what you are doing, justify your results. You will explain your thinking using the table and create a graphical representation of the situation.

Time (hours)	16 cm candle height (cm)	12 cm candle height (cm)
0	16	12
1	13.5	10.5
2		
3		
4		
5		
6		
7		

1. Complete the table, and use it to write an equation for the height of each candle in terms of the number of hours it has burned. Be sure to include any constraints for the equation.

2. Create a graphical representation of your data, taking into account natural restrictions on domain, range, etc.

3. Cara has another candle that is 15 cm tall. How fast must it burn in order to also be 6 cm tall after 4 hours? Explain your thinking.

4. If Cara had a candle that burned 3 cm every hour, how tall would it need to be to also reach the same height as the other three candles after 4 hours? Explain your thinking.

1. For babysitting, Nicole charges a flat fee of $3, plus $5 per hour. Write an equation for the cost, C, after h hours of babysitting. What do you think the slope and the yintercept represent? How much money will she make if she baby-sits 5 hours?

2. Casey has a small business making dessert baskets. She estimates that her fixed weekly costs for rent and electricity are $200. The ingredients for one dessert basket cost $2.50. If Casey made 40 baskets this past week, what were her total weekly costs? Her total costs for the week before were $562.50. How many dessert baskets did she make the week before?

3. A water tank already contains 55 gallons of water when Baxter begins to fill it. Water flows into the tank at a rate of 8 gallons per minute. Write a linear equation to model this situation. Find the volume of water in the tank 25 minutes after Baxter begins filling the tank.

4. A caterer charges $120 to cater a party for 15 people and $200 for 25 people. Assume that the cost, y, is a linear function of the number of x people. Write an equation in slope-intercept form for this function. What does the slope represent? How much would a party for 40 people cost?

5. When purchased, the value of a machine was $56,000. The machine's value depreciates linearly, with a scrap value of $2,000 after 9 years.
a. Write an equation for the value of the machine, y, in dollars, in terms of the number of years since its purchase, x.
b. Use the equation found in part a to find the value of the machine 3 years after purchase.
c. In the context of this application, what are the meanings of the y-intercept and slope?

Systems of Equations

Two or more linear equations involving the same variables form a **system of equations**. A solution of the system of equations is an ordered pair of numbers that satisfies both equations. You can solve a system of Equations in a variety of ways. You can solve by graphing, substitution and elimination.

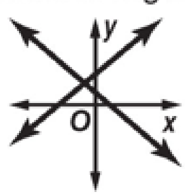
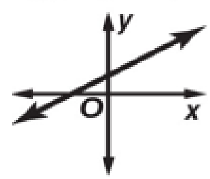
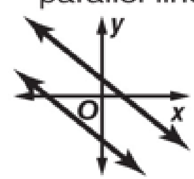
Solving Systems of Equations by Graphing

GUIDED NOTES

When solving a system of equations by graphing you are interested in the point where the two graphs intersect. The point where the graphs intersect is the solution of the system of equations.

There are also special cases:

- the lines are parallel and therefore never intersect.
- the lines are coinciding which basically means they are exactly the same.

Graph of a System	intersecting lines 	same line 	parallel lines 
Number of Solutions			

STEP 1: Solve both equations for y. In other words, put the equation in slope-intercept form.

STEP 2: Using the slope and y intercept graph both lines on the same coordinate plane.

STEP 3: Find the intersection point if it occurs. This point is the solution to the system.

What would have to be true about the graphs in order for the lines to be parallel?

What would have to be true about the graphs in order for the lines to be coinciding?

Example! Guided Practice

Slove the system of equations by graphing.

$3x + y = 9$

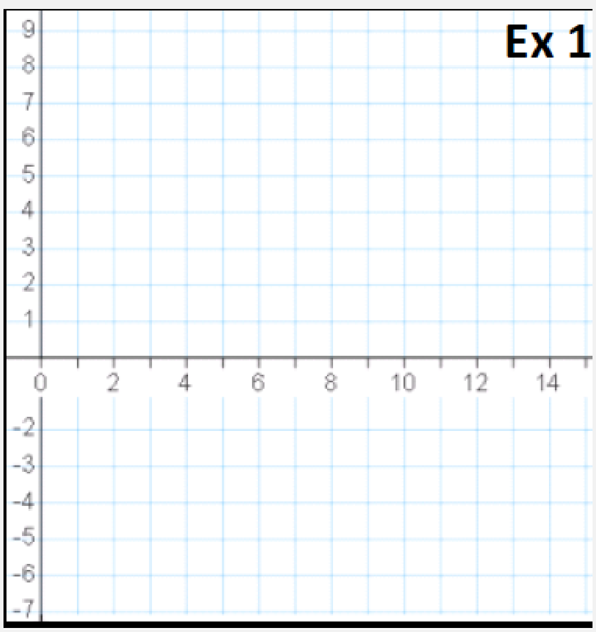
$y = -x + 1$

Step 1: Solve both equations for y.

Which equation do we need to rewrite?____________________

Step 2: Using the slope and y-intercept graph both lines on the coordinate plane.

Step 3: Find the point of intersection if it occurs. _____________



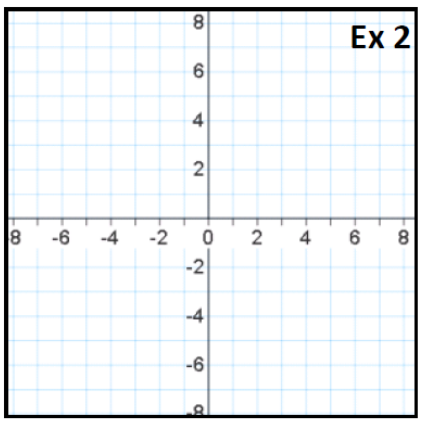
SELF CHECK

Slove the system of equations by graphing.
$-x + 2y = -4$
$y = -2x + 3$

Step 1: Solve both equations for y.
Which equation do we need to rewrite?____________________

Step 2: Using the slope and y-intercept graph both lines on the coordinate plane.

Step 3: Find the point of intersection if it occurs. ____________



WORD PROBLEM:
Suppose you and your friends form a band. You want to record a demo. Studio A rents for \$100 plus \$50 per hour. Studio B rents for \$50 plus \$75 per hour.

a. Write an equation to represent the cost of each studio.

Studio A: ______________________________

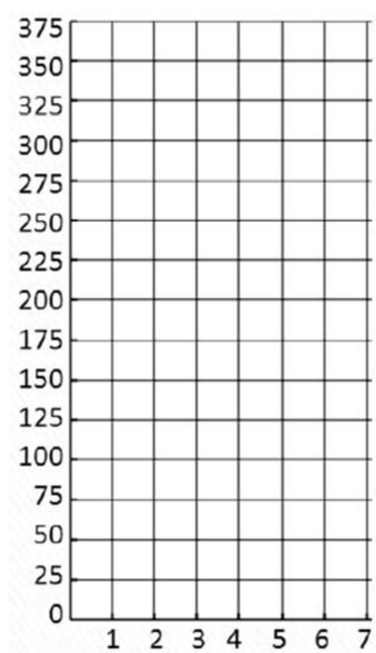
Studio B: ______________________________

b. Solve the system by graphing.

Solution: ___________________

c. Explain what the solution of the system means in terms of renting a studio. ____

____



Solving Systems of Equations by Substitution

Steps for solving systems using SUBSTITUTION:

Step 1: Isolate one of the variables.

Step 2: Substitute the expression from Step 1 into the OTHER equation. • The resulting equation should have only one variable, not both x and y.

Step 3: Solve the new equation. • This will give you one of the coordinates.

Step 4: Substitute the result from Step 3 into either of the original equations.

Step 5: Solve for the other coordinate.

Step 6: Write the solution as an ordered pair. (x, y)

Example!

$$y = 2x - 1$$
$$3x + 2y = 26$$

Step 1: Equation **a** already has y isolated.

Step 2:
$$3x + 2y = 26$$
$$3x + 2(2x - 1) = 26$$

Step 3:
$$3x + 4x - 2 = 26$$
$$7x - 2 = 26$$
$$\underline{+2 \quad +2}$$
$$\frac{7x}{7} = \frac{28}{7}$$
$$\mathbf{x = 4}$$

Step 4: $y = 2(\mathbf{4}) - 1$

Step 5: $\mathbf{y = 7}$

Step 6: **(4, 7)**

Example! Guided Practice

Solve the system of equations by substitution

a) $-4x + y = 6$
b) $-5x - y = 21$

SELF CHECK

$y=2x+4$ $y=x-3$	$2x + y = 5$ $-3x + 2y = 17$

Solving Systems of Equations by Elimination

Steps for solving systems using ELIMINATION:

- STEP 1: Line up the x's and y's.
- STEP 2: Look to see if one variable has opposite coefficients.
  - Yes, move to Step 3.
  - No, multiply one or both equations by a constant (LCM) in order to make the coefficients of the x or y terms opposites.
- STEP 3: Add the equations together to eliminate one of the variables.
- STEP 4: Solve for the remaining variable.
- STEP 5: Substitute the value you found into one of the original equations to solve for the other variable.
- STEP 6: Write your answer as an ordered pair.

Example!

Example 1:

⦿ STEP 1: Line up the x's and y's. ⦿ STEP 2: Look to see if one variable has opposite coefficients. ⦿ Yes, move to Step 3. ⦿ No, ⦿ STEP 3: Add the equations together to eliminate one of the variables. ⦿ STEP 4: Solve for the remaining variable. ⦿ STEP 5: Substitute the value you found into one of the original equations to solve for the other variable. ⦿ STEP 6: Write your answer as an ordered pair.	⦿ STEP 1: $2x+5y=17$ ⦿ STEP 2: $6x-5y=-9$ ○ Yes ⦿ STEP 3: $\frac{8x}{8} = \frac{8}{8}$ (y cancels) ⦿ STEP 4: $x = 1$ ⦿ STEP 5: $2(1) + 5y = 17$ $2 + 5y = 17$ $-2 \quad -2$ $\frac{5y}{5} = \frac{15}{5}$ $y = 3$ ⦿ STEP 6: (1, 3)

Example!

⦿ STEP 1: Line up the x's and y's. ⦿ STEP 2: Look to see if one variable has opposite coefficients. ⦿ Yes ⦿ No, multiply one or both equations by a constant (LCM) to make the coefficients of the x or y terms opposites. ⦿ STEP 3: Add the equations together to eliminate one of the variables. ⦿ STEP 4: Solve for the remaining variable. ⦿ STEP 5: Substitute the value you found into one of the original equations to solve for the other variable. ⦿ STEP 6: Write your answer as an ordered pair.	⦿ STEP 1: $-2x+15y=-32$ ⦿ STEP 2: $3\left[7x-5y=17\right]$ No, Mult bottom by 3 ⦿ STEP 3: (y cancels) ⦿ STEP 4: ⦿ STEP 5: $-2(1) + 15y = -32$ $-2 + 15y = -32$ $+2 \quad +2$ $\frac{15y}{15} = \frac{30}{15}$ $y = 2$ ⦿ STEP 6: (1, 2)	$-2x + 15y = -32$ $21x - 15y = 51$ $\frac{19x}{19} = \frac{19}{19}$ $x = 1$

Example!

⦿ STEP 1: Line up the x's and y's.
⦿ STEP 2: Look to see if one variable has opposite coefficients.
  ⦿ Yes
  ⦿ No, multiply one or both equations by a constant (LCM) to make the coefficients of the x or y terms <u>opposites</u>.
⦿ STEP 3: Add the equations together to eliminate one of the variables.
⦿ STEP 4: Solve for the remaining variable.
⦿ STEP 5: Substitute the value you found into one of the original equations to solve for the other variable.
⦿ STEP 6: Write your answer as an ordered pair.
	⦿ STEP 1: 3 $[3x+2y=1]$
	⦿ STEP 2: -2 $[4x+3y=-2]$
	No, Mult top by 3 & bottom by -2
	⦿ STEP 3: (y cancels)
	⦿ STEP 4:
	⦿ STEP 5: $3(7) + 2y = 1$
	$21 + 2y = 1$
	$-21 \qquad -21$
	$\frac{2y}{2} = \frac{-20}{2}$
	$y = -10$
	⦿ STEP 6: (7, -10)
		$9x + 6y = 3$
		$-8x - 6y = 4$
		
		$x \qquad = 7$

Example! Guided Practice

Solve the system using elimination

$$3x - 2y = -2$$

$$4x - 3y = -4$$

SELF CHECK

$4x + 3y = 6$ $-x - 3y = 3$	$2x + y = 5$ $-3x + 2y = 17$	$-3x - 5y = -7$ $-4x - 3y = -2$

Questions To Ponder

All 3 methods of solving systems of equations will work for any system you are given. How do you decide which method is the best for any given problem?

Solving Systems of Equations Special Cases	
If you eliminate all variables and the resulting statement is true then there are infinitely many solutions $2x + 2y = 8$ $2x + 2y = 8$ $0x + 0y = 0$ $0 = 0$	If you eliminate all variables and the resulting statement is false then there is no solution. $2x + 2y = 8$ $2x + 2y = 10$ $0x + 0y = 2$ $0 = 2$

Example! Guided Practice

$4x + 16y = 16$ $-2x - 8y = -6$	$2x + 4y = 20$ $3x + 6y = 30$

SELF CHECK

$3x + 5y = 1$ $-6x - 10y = 14$	$3x + 6y = 0$ $x + 2y = 0$

Solving Systems of Equations Word Problems

Follow the 5 step process for solving systems of equations word problems.
Step 1: Define your variables.

Step 2: Write two equations.

Step 3: Choose your method for solving.

Step 4: Solve.

Step 5: Write your solution in context**.**

Example! Guided Practice

You are selling tickets for a high school basketball game. Student tickets cost $3 and general admission tickets cost $5. You sell 350 tickets and collect $1450. How many of each type of ticket did you sell?

Define Variables:

Write two equations:

Choose your method and solve:

Write your solution in context**.**

Example! Guided Practice

At an Italian bistro, the costs of 2 plates of spaghetti and 1 salad is $27.50. The cost for 4 plates of spaghetti and 3 salads is $59.50. Find the cost of a plate of spaghetti and a salad.

Define Variables:

Write two equations:

Choose your method and solve:

Write your solution in context**.**

SELF CHECK

Peggy walks at a rate of 2 miles per hour and jogs at a rate of 4 miles per hour. She walked and jogged 3.4 miles in 1.2 hours. For how long did Peggy jog and for how long did she walk?

SELF CHECK

The Smiths are cleaning out their basement. They realize that they have so much trash that they need to rent a dumpster. Dumpers, Inc. rents a dumpster for $100 plus $15 per hour. Garbage R Us only charges $50 but they charge $25 per hour. After how many hours will the costs be the same? What is that cost?

What's the pfunny Word? System of Equations Activity

Solve the following systems of equations. Then write your answers in the first column on the right. Next, translate each number into a letter, using 1 = A, 2 = B, 3 = C, etc.

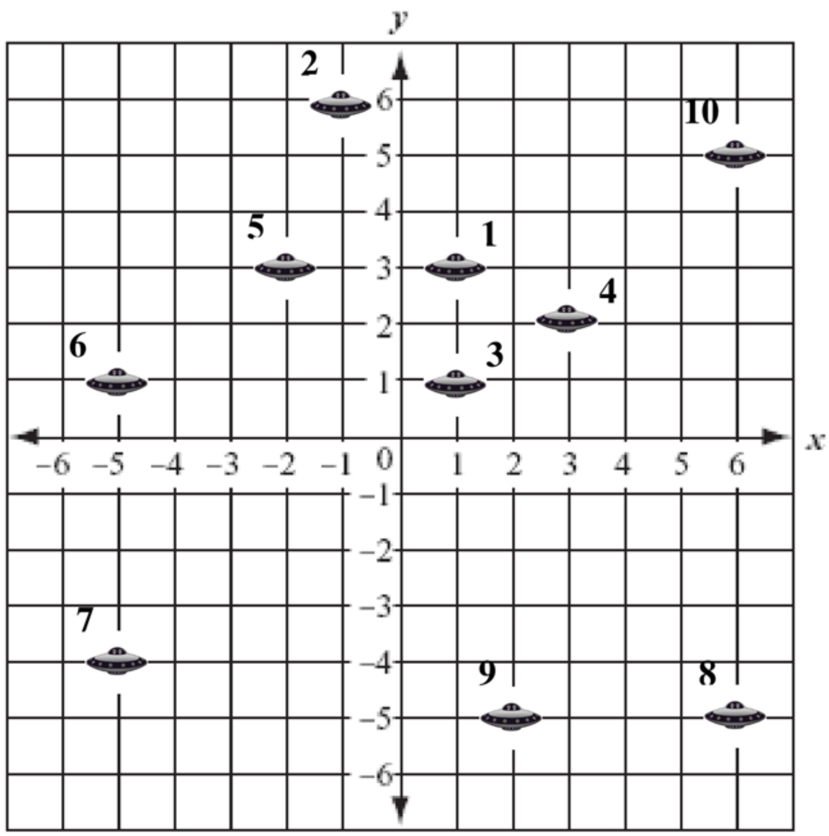
System:	Solution:	Letter:
2x + y = 43	x = _______	_______
x + y = 28	y = _______	_______
x - y = 8	x = _______	_______
x + y = 24	y = _______	_______
3x+2y=27	x = _______	_______
x+y=13	y = _______	_______
3x-2y = 7	x = _______	_______
-x + y = 4	y = _______	_______
7x + 3y = 92	x = _______	_______
2x + y = 27	y = _______	_______
x + 3y = 73	x = _______	_______
x + y = 35	y = _______	_______
5x - 2y = 7	x = _______	_______
3x - y = 8	y = _______	_______

The Word is:

Algebra Name____________________________Period___

Systems of Equations TARGET PRACTICE

Match each system to the target UFO that it locks on to by solving by graphing.
It may help to graph each system with a different color.



UFO	Shot	UFO	Shot
	y = x + 6 y = -x - 4		y = 3x y = -x + 4
	y = -x + 5 y = -2x + 4		y = $\frac{1}{2}$x + 4 y = -2x - 1
	y = $-\frac{1}{5}x-5$ y = $\frac{2}{5}x-2$		y - x = -7 y = -3x + 1
	y = x - 1 y = -x + 5		y = x y = 2x - 1
	x = 6 y - x = -1		y = -5 y = -$\frac{1}{2}$x - 2

Solve the linear system using substitution

1. $x = y - 5$ $4x - y = 4$	2. $y = 4x - 3$ $2y - 3x = 4$	3. $y = 4$ $y = 2x - 8$
4. $y = 5x$ $x + y = 30$	5. $2x + 5y = 26$ $y = x/4$	6. $b - 3a = 1$ $b = 4a$
7. $4p + 2s = 10$ $P - 5s = 8$	8. $5c - 2d = 0$ $6d = 1 + c$	9. $3s - t = 1$ $3s = 9$

NAME ____

Objective: Use Systems of Equations to Solve Real-World Problems

POPULATION

1) In 1990, the population of the Midwest was about 60 million. During the 1990s, the population of this area increased an average of about 0.4 million per year.

The population of the West was about 53 million in 1990. The population of this area increased an average of about 1 million per year during the 1990s.

Assume that the rate of growth of these areas remains the same. Estimate when the population of the West would be equal to the population of the Midwest.

SPORTS

2) At the end of the 2000 baseball season, the New York Yankees and the Cincinnati Reds had won a total of 31 World Series. The Yankees had won 5.2 times as many World Series as the Reds. How many World Series did each team win?

PARKS

3) A youth group and their leaders visited Mammoth Cave. Two adults and 5 students in one van paid $77 for the Grand Avenue Tour of the cave. Two adults and 7 students in a second van paid $95 for the same tour. Find the adult price and the student price of the tour.

BIRD CLUB

4) A birding club holds an annual photography contest among its members. After a set time limit in a particular park, contestants receive 4 points for photos of songbirds and 20 points for photos of birds of prey. Last year's winner had a total of 200 points from 38 photos of individual birds. How many of each type of bird did the winner photograph?

Algebra Unit # 2 Concept# 2F

Lesson Name: A1.U2.C1.F.03.**classworkwordprob.systemsofeq**

Independent Practice

FOOTBALL

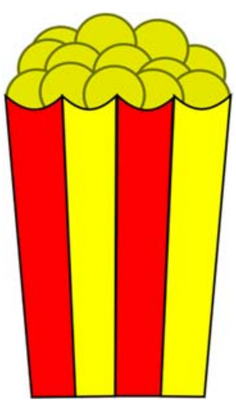
5) During the National Football League's 1999 season, Troy Aikman, the quarterback for the Dallas Cowboys, earned $0.467 million more than Deion Sanders, the Cowboys cornerback. Together they cost the Cowboys $12.867 million. How much did each player make?

BUSINESS

6) The owners of the River View Restaurant have hired enough servers to handle 17 tables of customers, and the fire marshal has approved the restaurant for a limit of 56 customers. How many two-seat tables and how many four-seat tables should the owners purchase?

CELL PHONES

7) The price of a cellular telephone plan is based on peak and nonpeak service.
Kelsey used 45 peak minutes and 50 nonpeak minutes and was charged $27.75.
That same month, Mitch used 70 peak minutes and 30 nonpeak minutes for a total charge of $36. What are the rates per minute for peak and nonpeak time?



MOVIE THEATER

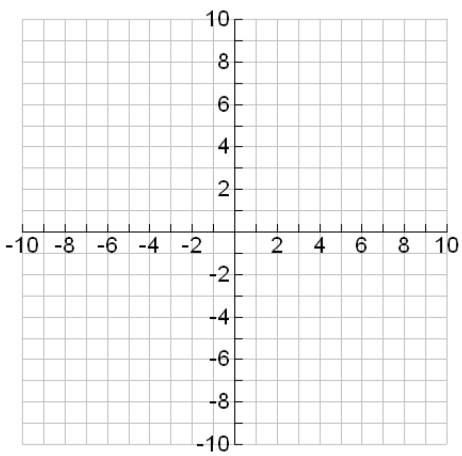
8) The manager of a movie theater found that Saturday's sales were $3675. He knew that a total of 650 tickets were sold Saturday. Adult tickets cost $7.50, and children's tickets cost $4.50. How many of each kind of ticket were sold?

Lesson Name - Application

You are given the following system of two equations: $\begin{cases} 2x + 4y = 32 \\ 3x - 4y = -2 \end{cases}$

1. What are some ways to prove that the ordered pair (6, 5) is a solution?

   a. Prove that (6, 5) is a solution to the system by graphing the system.



   b. Prove that (6, 5) is a solution to the system by substituting in for both equations.

   c. Add both equations together. What happens to the variable *y* in the new equation?

   d. Can you solve the new equation for *x*? What is the value of *x*? Does this answer agree with the original solution?

   e. How could you use the value of *x* to find the value of *y* from one of the original equations? Show your work below.

The method you have just used is called the Elimination Method for solving a systems of equations. When using the Elimination Method, one of the original variables is eliminated from the system by adding the two equations together. Use the Elimination Method to solve the following systems of equations.

2. $\begin{cases} -3x+2y=-6 \\ 5x-2y=18 \end{cases}$

3. $\begin{cases} -5x+7y=11 \\ 5x+3y=19 \end{cases}$

4. Multiply both sides of the equation **x + 2y = 16** by the constant '7'. Show your work.

   7(x + 2y) = 7(16)

   ____________________New Equation

   a. Does the new equation still have a solution of (6, 5)? Justify your answer.

   b. Why do you think the solution to the equation never changed when you multiplied by the 7?

5. Did it have to be a 7 that we multiplied by in order for (6, 5) to be a solution?

   a. Multiply **x + 2y = 16** by three other numbers and see if (6, 5) is still a solution. Pick your own constants.

      i. ______________________

      ii. ______________________

      iii. ______________________

   b. Did it have to be the first equation **x + 2y = 16** that we multiplied by the constant for (6, 5) to be a solution? Multiply **3x – 4y = -2** by 7? Is (6, 5) still a solution?

   c. Multiply **3x – 4y = -2** by three other numbers and see if (6, 5) is still a solution.

i. ______________________

ii. ______________________

iii. ______________________

6. Summarize your findings from this activity so far. Consider the following questions:

   a. What is the solution to a system of equations and how can you prove it is the solution?

   b. Does the solution change when you multiply one of the equations by a constant?

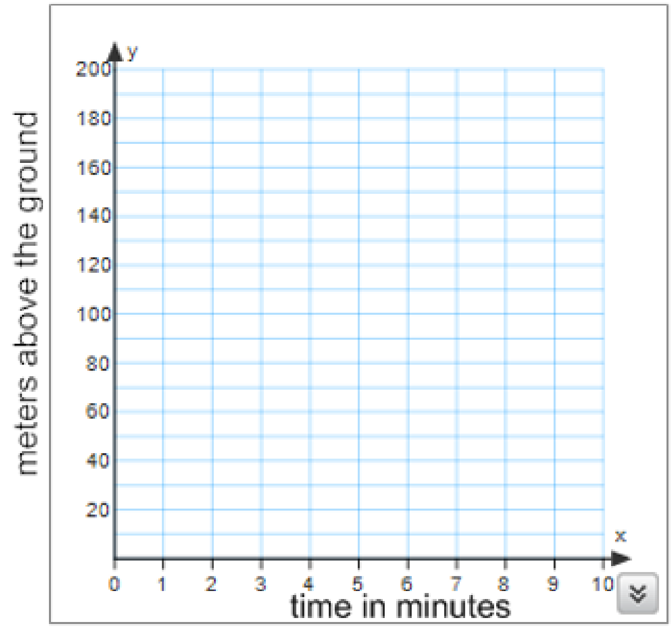
   c. Does the value of the constant you multiply by matter?

   d. Does it matter which equation you multiply by the constant?

Hot Air Balloons

1. Two balloons are in the air. Balloon 1 is 10 meters above the ground and rising at 15 meters per minute. Balloon 2 is 150 meters above the ground and descending at 20 meters per minutes.

a. Write an equation for each balloon below. Use *t* to represent time and *m* to represent meters above the ground.



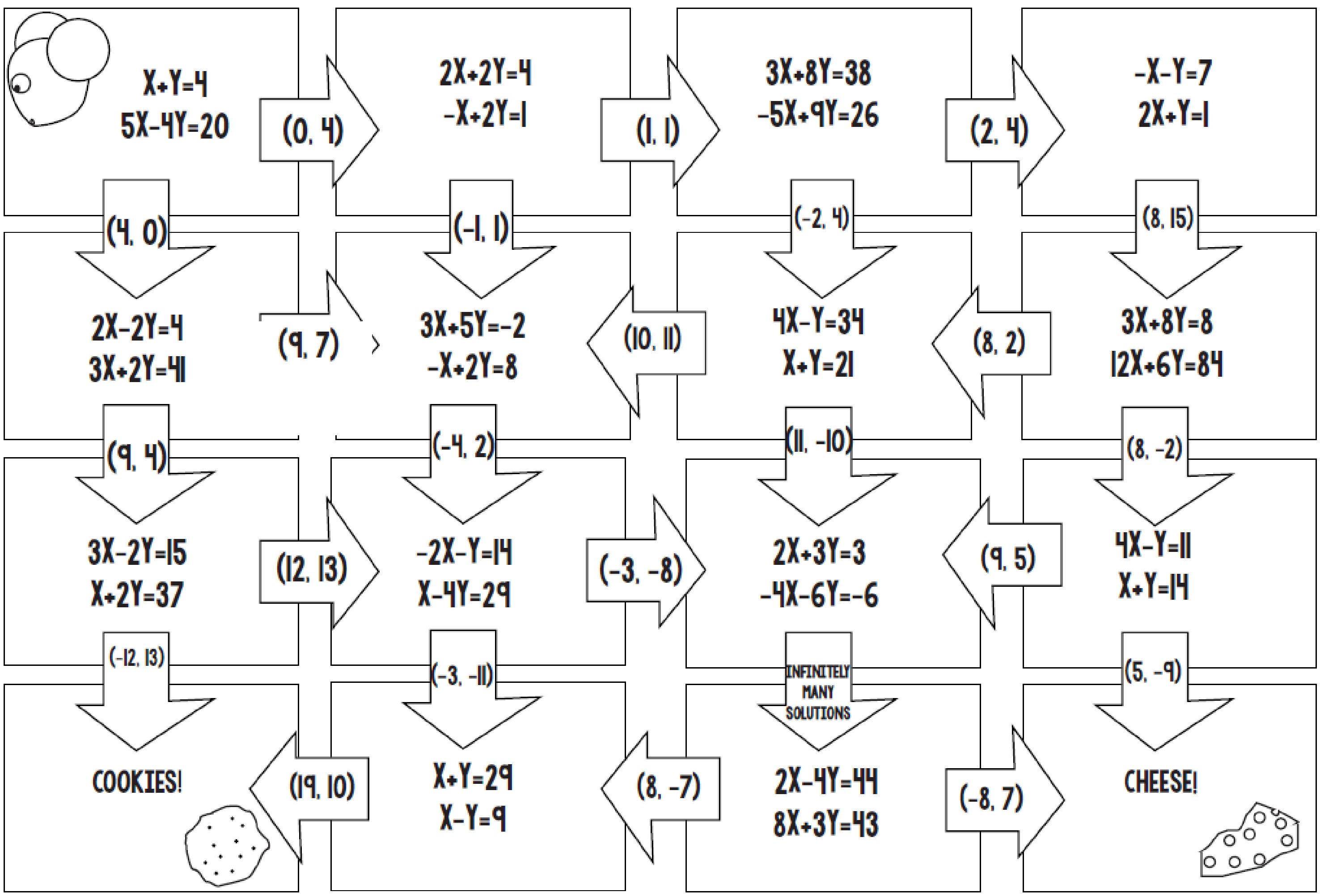
b. Graph above equations then estimate the time when the balloons will be at the same height.

c. Use substation or elimination to find the exact time when the balloons will be the same height.

HOMEWORK

Homework #___2_____
Solve System of Equations (Elimination):

DIRECTIONS: HELP THE MOUSE FIND HIS FAVORITE FOOD! START AT THE START BOX. SOLVE EACH SYSTEM OF EQUATIONS. FOLLOW THE ARROW WITH THE CORRECT ANSWER. COLOR IN YOUR PATH AS YOU GO.

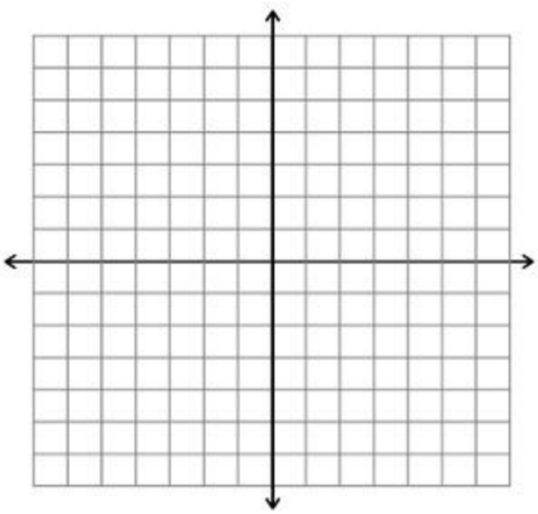


HOMEWORK

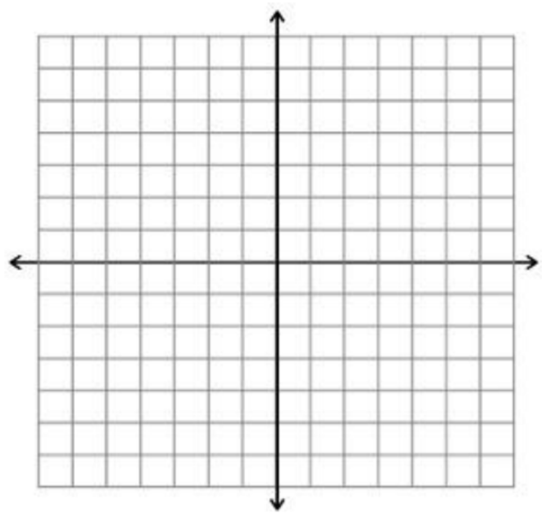
Homework #___1_____

Solve System of Equations (Graphically):

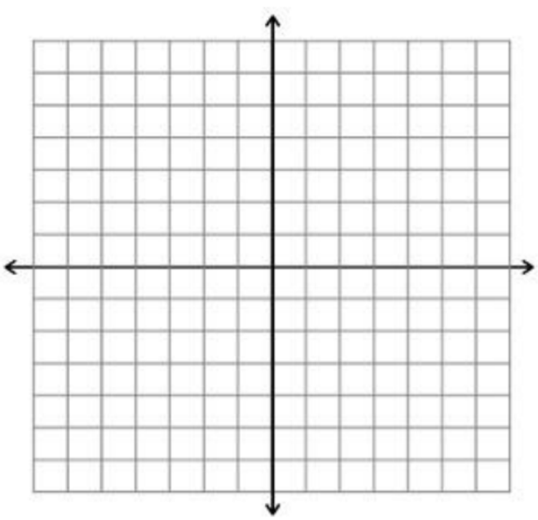
1. $x + y = -2$
   $3y = 2x + 9$



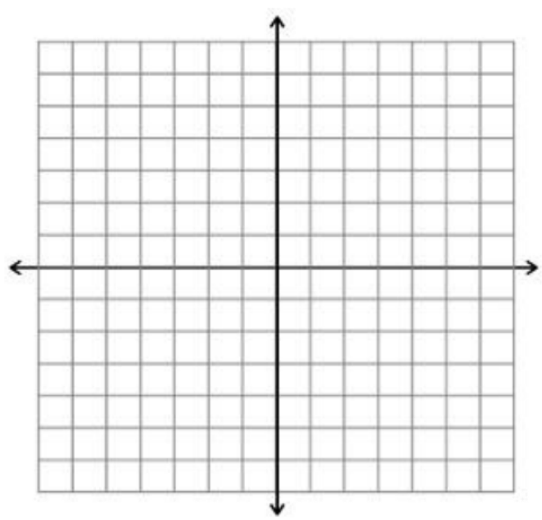
2. $x + y = 3$
   $-2x + y = -6$



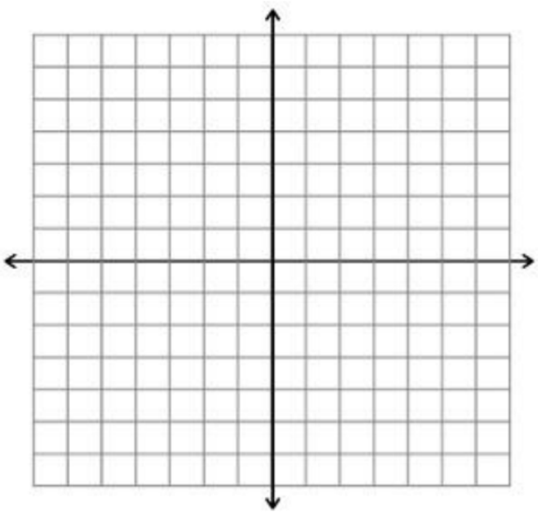
3. $2x - 3y = 9$
   $-4x + 3y = -15$



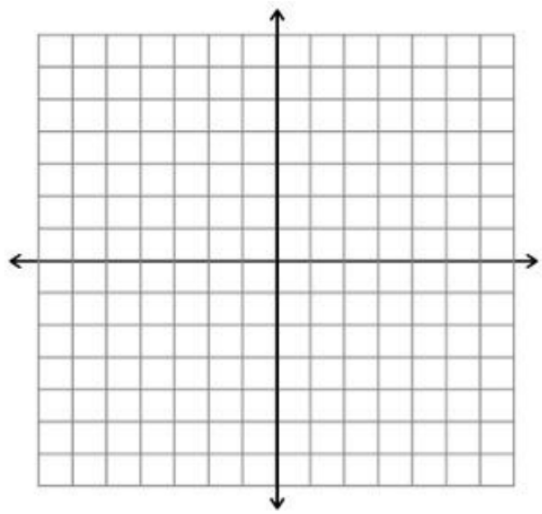
4. $-2x + 4y = 12$
   $x - 2y = 6$



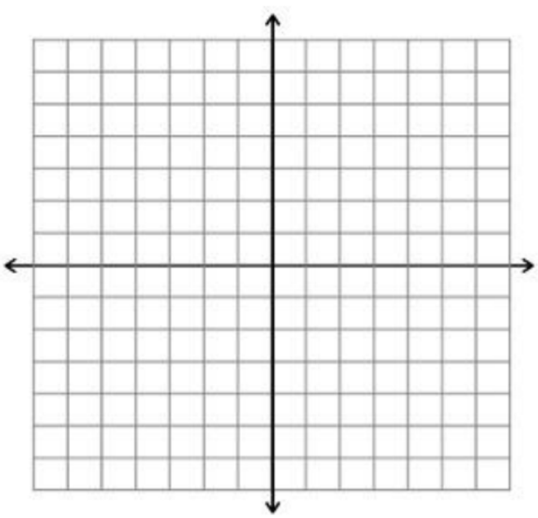
5. $2x + y = 1$
   $3x - 2y = 8$



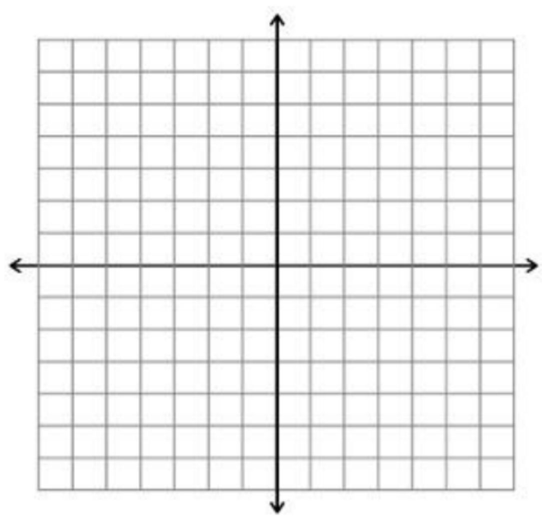
6. $x + y = -6$
   $4x + y = 3$



7. $5x - 7y = -42$
   $4x + 7y = -21$



8. $3x - y = -4$
   $4x + 3y = -27$



HOMEWORK

Homework #___3_____

Solve System of Equations (Substitution):

1. $y = -3x + 2$ $x - y = 2$	2. $y = x - 4$ $2x + y = 5$
3. $y = 3x - 11$ $y = 3x - 13$	4. $x = -2y + 4$ $2x + 4y = 8$
5. $-7x + 7y = -21$ $4x - 2y = 22$	6. $6x - 7y = -8$ $-x - 4y = -9$
7. $5x - 7y = -28$ $2x - 2y = -12$	8. $2x + 4y = 7$ $X + 2y = 3$
9. $9x - 9y = 0$ $6x - 2y = -20$	10. $0.3x - 0.6y = 1$ $-3x + 6y = -10$

Homework #___4_____

Solve System of Equations with Word Problems:

1. A large pizza at Marco's Pizzeria costs \$6.80 plus \$0.90 for each topping. The cost of a large cheese pizza at Uncle Maddio's Pizza is \$7.30 plus \$0.65 for each topping. How many toppings need to be added to a large cheese pizza from Marco's Pizzeria and Uncle Maddio's Pizza in order for the pizzas to cost the same, not including tax?

2. At a restaurant the cost for a breakfast taco and a small glass of milk is \$2.10. The cost for 2 tacos and 3 small glasses of milk is \$5.15. Which pair of equations can be used to determine t, the cost of a taco, and m, the cost of a small glass of milk? What is the cost of taco and glass of milk?

3. Rylan and Nova went to the candy store. Rylan bought 5 pieces of fudge and 3 pieces of bubble gum for a total of \$5.70. Nova bought 2 pieces of fudge and 10 pieces of bubble gum for a total of \$3.60. Use the system of equations to determine the cost of 1 piece of fudge, f, and 1 piece of bubble gum, g?

4. The Walt Disney is having their annual senior day in May. This year the senior class at Houston County High School and the senior class at Warner Robins High School both planned trips there. The senior class at HOCO rented and filled 8 vans and 4 buses with 240 students. WRHS rented and filled 4 vans and 1 bus with 80 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

HOMEWORK

5. Ed and Barb are selling flower bulbs for a school fundraiser. Customers can buy bags of windflower bulbs and bags of daffodil bulbs. Barb sold 10 bags of windflower bulbs and 12 bags of daffodil bulbs for a total of $380. Ed sold 6 bags of windflower bulbs and 8 bags of daffodil bulbs for a total of $244. What is the cost each of one bag of windflower bulbs and one bag of daffodil bulbs?

6. You sell tickets for admission to your school play and collect a total of $104. Admission prices are $6 for adults and $4 for children. You sold 21 tickets. How many adult tickets and how many children tickets did you sell?

7. You bought the meat for Saturday's cookout. A case of ribs cost $51.50 and a case of chicken cost $15. You bought a total of 9 cases of meat and you spent $281. How many cases of ribs meat did you buy?

8. Rent-A-Car rents compact cars for a fixed amount per day plus a fixed amount for each mile driven. Benito rented a car for 6 days, drove it 550 miles, and spent $337. Lisa rented the same car for 3 days, drove it 350 miles, and spend $185. What is the charge per day and the charge per mile for the compact car?

Graphing Inequalities and Systems of Inequalities

GUIDED NOTES

The graph of an **inequality in two variables** is the set of points that represents all solutions to the **inequality**. A **linear inequality** divides the coordinate plane into **two** halves by a boundary line where one half represents the solutions of the **inequality**. The boundary line is dashed for > and < and solid for ≤ and ≥.

A **system of linear inequalities in two variables** consists of at least two linear inequalities in the same variables. The solution of a linear inequality is the ordered pair that is **a solution to all inequalities in the system** and the graph of the linear inequality is the graph of all solutions of the system.

When graphing linear inequalities and systems of inequalities, you following the same process as graphing linear functions with the exception of shading the region that will supply the solutions.

- A linear inequality describes a region of a coordinate plane called a *half plane*.
  - *ALL* points in the region are *solutions*
- A solution to a linear inequality is *any* ordered pair that makes the inequality *true*.
- A system of linear inequalities is a set of two or more linear inequalities containing two or more variables.
- The solutions of a system of linear inequalities consists of *all the ordered pairs* that satisfy *all* the linear inequalities in the system.

Graphing Linear Inequalities

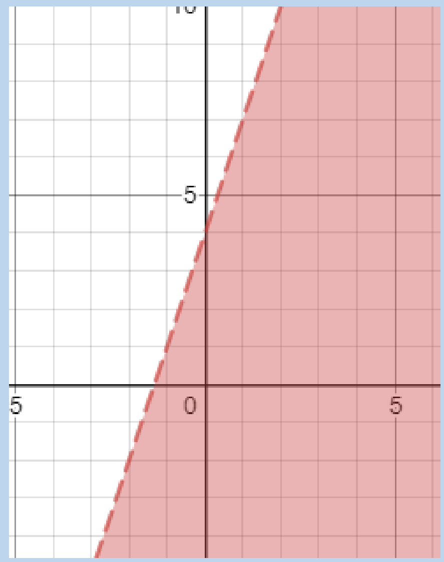
Linear Inequalities	Shade ABOVE	Shade BELOW
DASHED	>	<
SOLID	≥	≤

Graphing Linear Inequality

Graph the solutions of each linear inequality. Check your answer.

A $y < 3x + 4$

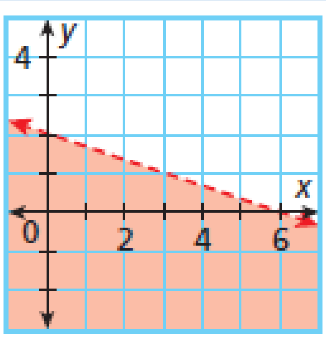
1) Put in slope-intercept form
2) Plot the y-intercept
3) Use the slope (rise over run) to graph two more points
4) Connect with a dashed or solid line
5) Shade above or below the line



Writing an Inequality

- To write an inequality from a graph,
  1) Identify the y-intercept
  2) Identify the slope
  3) Determine the inequality symbol

A



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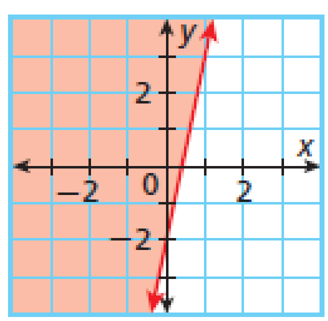

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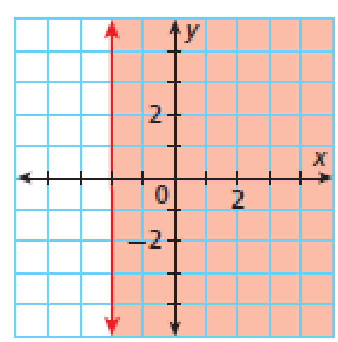
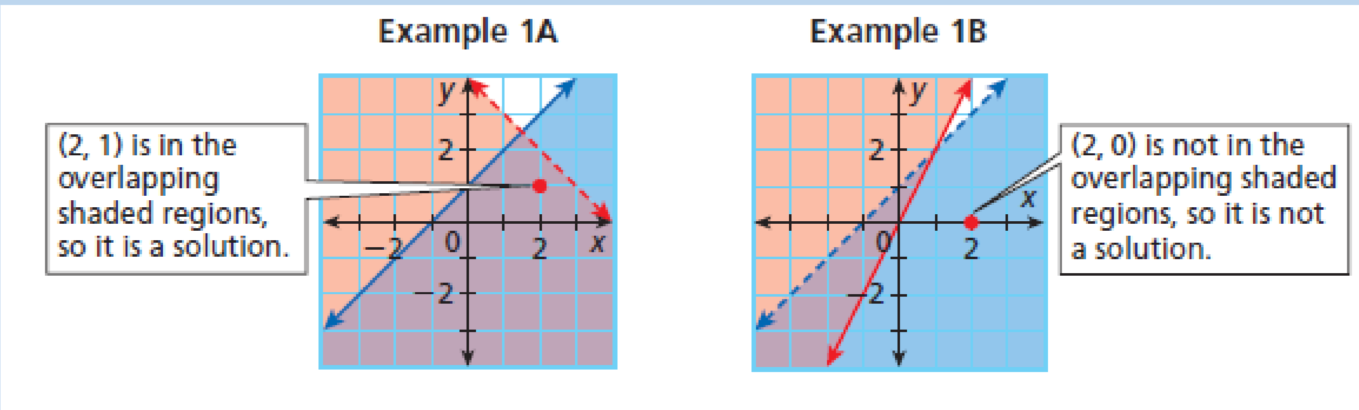


A) $y < -1/3x + 2$ B) $y \geq 5x - 2$ C) $x \geq -2$

Example : Identifying Solutions

Tell whether the ordered pair is a solution of the given system.

A $(2, 1); \begin{cases} y < -x + 4 \\ y \le x + 1 \end{cases}$ **B** $(2, 0); \begin{cases} y \ge 2x \\ y < x + 1 \end{cases}$



Graphing System of Inequalities

1) Put both equations in slope-intercept form
2) Graph the first inequality
    a) Plot the y-intercept and use the slope to create two more points
    b) Make a dashed or solid line
    c) Shade above or below the line with horizontal lines
3) Graph the second inequality
    a) Plot the y-intercept and use the slope to create two more points
    b)Make a dashed or solid line
    c)Shade above or below the line with vertical lines
4) The overlapping reason they both share is the half-plane solution. All the ordered pairs in this reason are true.

Example

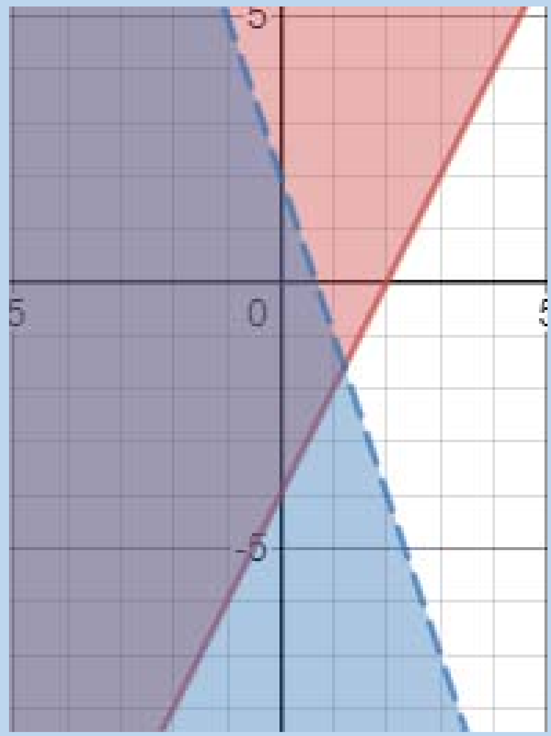
Example!

Graph the following linear system of inequalities

$$y \geq 2x - 4$$

$$y < -3x + 2$$

The solution to this system of inequalities is the region where the solutions to each inequality overlap. This is the region above or to the left of the solid red line and below or to the left of the dashed blue line. The solutions are in the purple region.



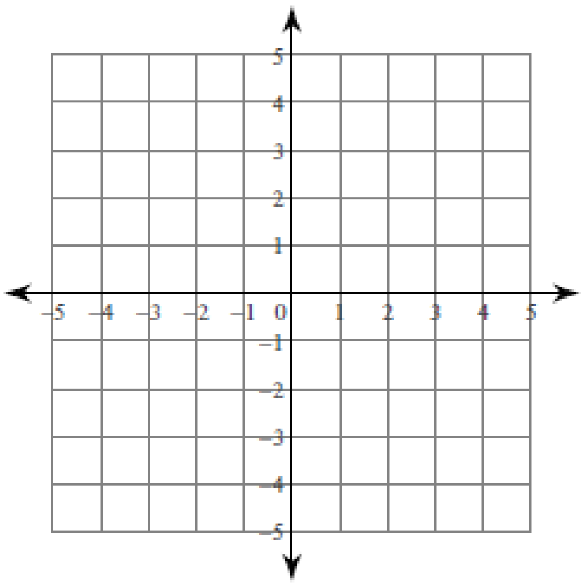
Tell whether the ordered pair is a solution of the given system.

1a. $(0, 1);\begin{cases} y < -3x + 2 \\ y \geq x - 1 \end{cases}$

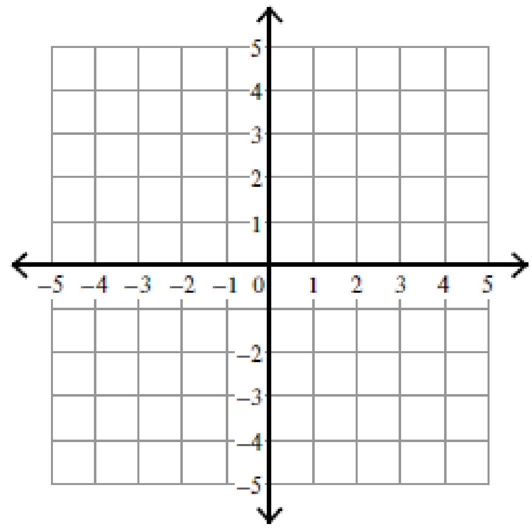
1b. $(0, 0);\begin{cases} y > -x + 1 \\ y > x - 1 \end{cases}$

SELF CHECK

2) $y \geq -5x + 3$
$y > -2$



3) $y \leq \frac{1}{2}x + 2$
$y > 3x - 3$



Questions To Ponder

1. If you know how to graph one linear inequality, how do you think you will graph a system of linear inequalities?

2. If you know the solution to a linear inequality is a region that makes the inequality true, then what will the solution of a system look like?

3. How can you check that you shaded the correct areas?

4. How do you decide which test points to use?

Independent Practice

Example!

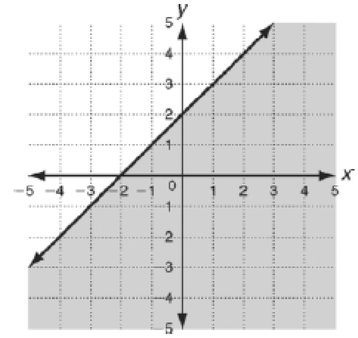
Graphing System of Inequalities:

To use graphs to find the solution to a system of inequalities:

1. Draw the graph of the boundary for the first inequality. Remember to use a solid line for $\le$ or $\ge$ and a dashed line for $<$ or $>$.
2. Shade the region above or below the boundary line that is a solution of the inequality.
3. Draw the graph of the boundary for the second inequality.
4. Shade the region above or below the boundary line that is a solution of the inequality using a different pattern.
5. The region where the shadings overlap is the solution region.

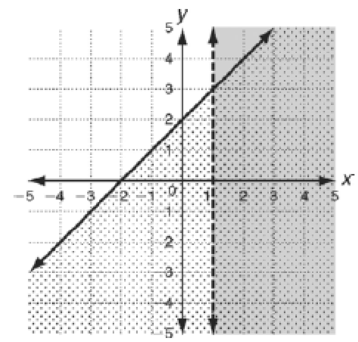
Graph $\begin{cases} y \le x + 2 \\ x > 1 \end{cases}$ Graph $y \le x + 2$.

Graph $y = x + 2$.
Use a solid line for the boundary.
Shade the region below the line.



On the same plane, graph $x > 1$.

Graph $x = 1$.
Use a dashed line for the boundary.
Shade the region to the right of the line.



Check: Test a point in the solution region in both inequalities.
Try (2, 2).

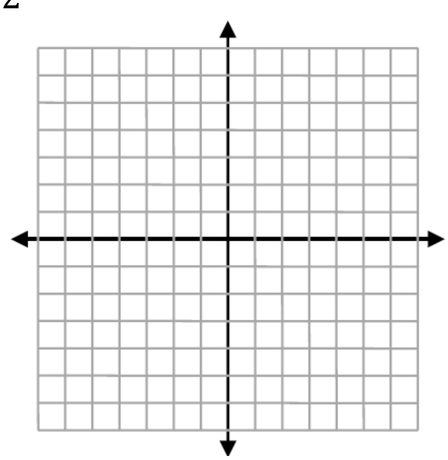
$y \le x + 2$ $\quad$ $x > 1$

$2 \overset{?}{\le} 2 + 2$ $\quad$ $2 > 1 \checkmark$

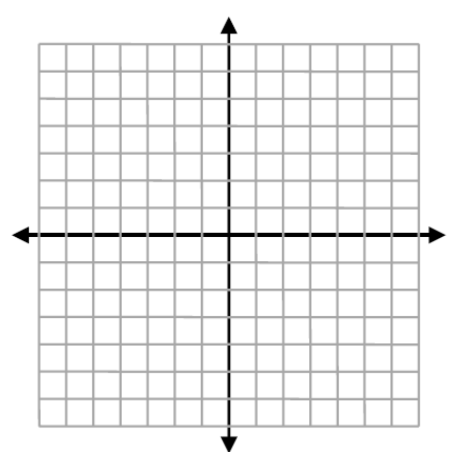
$2 \le 4 \checkmark$

Directions: Graph the system of inequalities

1. $6x + 4y > 12$

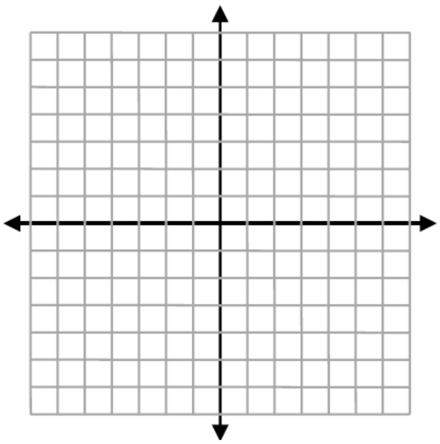


2. $y \le -3x + 5$

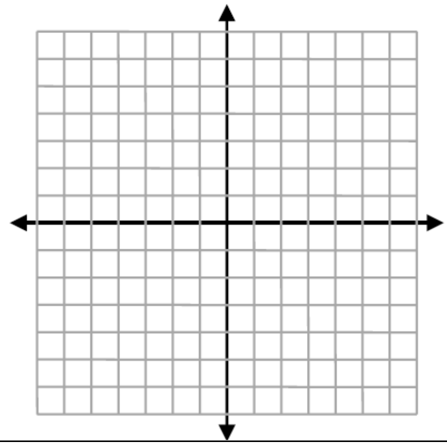


Independent Practice

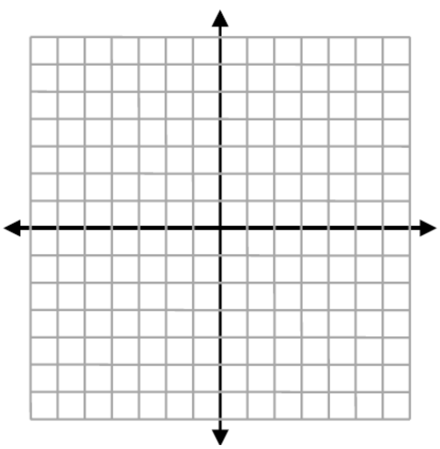
3. $y \leq x + 4$ $\qquad$ $y \geq -2x$



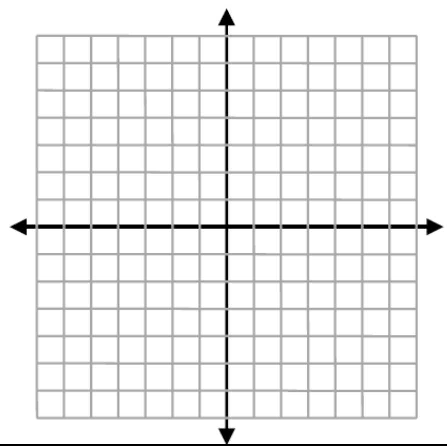
4. $y \leq \frac{1}{2}x + 1$ $\qquad$ $x + y < 3$



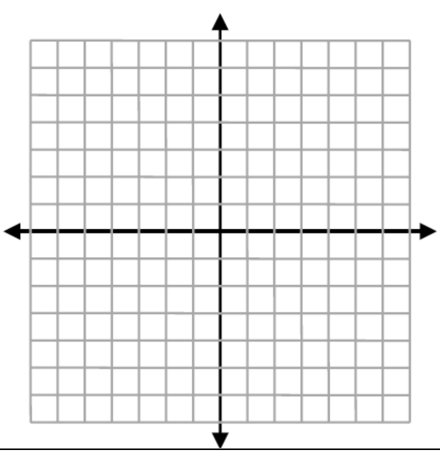
5. $2x - y < 2$ $\qquad$ $2x - y \geq -3$



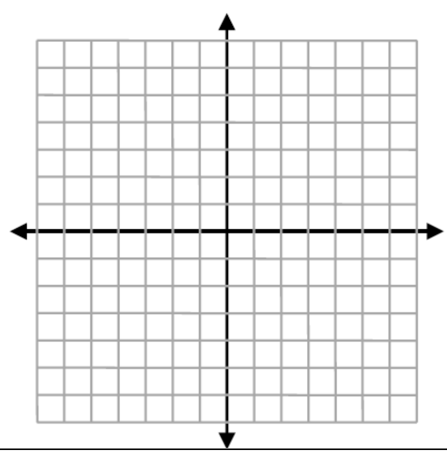
6. $y < x + 4$ $\qquad$ $y > x - 2$



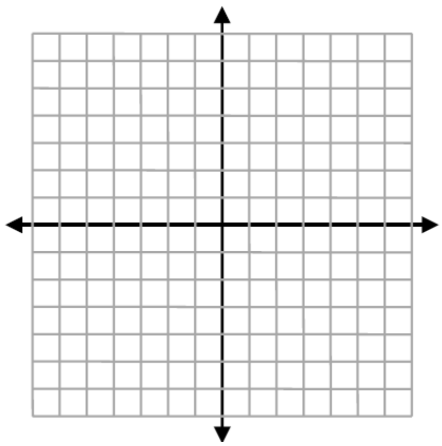
7. $x > -4$ $\qquad$ $y \geq 2$



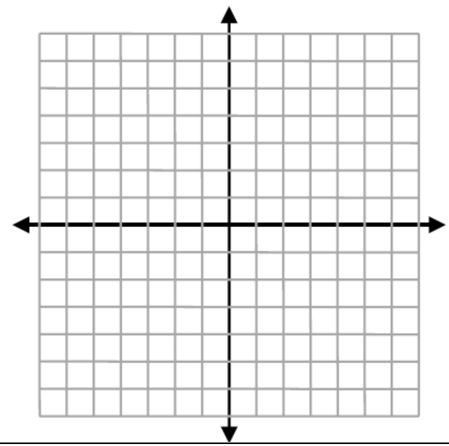
8. $3x - 2y \leq 4$ $\qquad$ $x + 3y \leq 6$



9. $-y \leq 3x + 4$ $\quad$ $-3x + 3y \leq -9$



10. $x + y \geq -3$ $\quad$ $2x + 2y \leq -2$



Lesson Name - Application

Summer Job System of Inequalities (Practice Task)

Name_________________________________ **Date**__________________

In order to raise money, you are planning to work during the summer babysitting and cleaning houses. You earn \$10 per hour while babysitting and \$20 per hour while cleaning houses. You need to earn at least \$1000 during the summer.

1. Write an expression to represent the amount of money earned while babysitting. Be sure to choose a variable to represent the number of hours spent babysitting.

2. Write an expression to represent the amount of money earned while cleaning houses.

3. Write a mathematical model (inequality) representing the total amount of money earned over the summer from babysitting and cleaning houses.

4. Graph the mathematical model. Graph the hours babysitting on the *x*–axis and the hours cleaning houses on the *y*–axis.

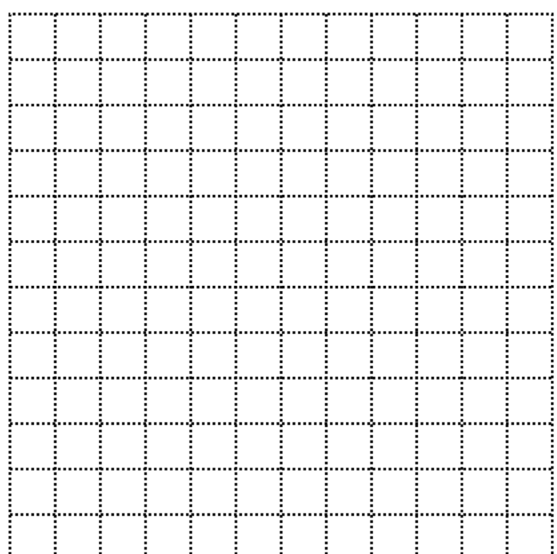
5. Use the graph to answer the following:
   a. Why does the graph only fall in the 1st Quadrant?

   b. Is it acceptable to earn exactly $1000? What are some possible combinations of outcomes that equal exactly $1000? Where do all of the outcomes that total $1000 lie on the graph?

   c. Is it acceptable to earn more than $1000? What are some possible combinations of outcomes that total more than $1000? Where do all of these outcomes fall on the graph?

   d. Is it acceptable to work 10 hours babysitting and 10 hours cleaning houses? Why or why not? Where does the combination of 10 hours babysitting and 10 hours cleaning houses fall on the graph? Are combinations that fall in this area a solution to the mathematical model? Why or why not?

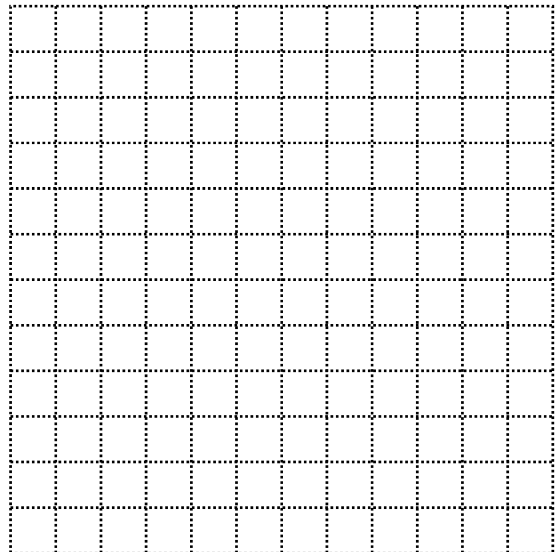
6. How would the model change if you could only earn more than $1000? Write a new model to represent needing to earn more than $1000. How would this change the graph of the model? Would the line still be part of the solution? How would you change the line to show this? Graph the new model.



You plan to use part of the money you earned from your summer job to buy jeans and shirts for school. Jeans cost $40 per pair and shirts are $20 each. You want to spend less than $400 of your money on these items.

7. Write a mathematical model representing the amount of money spent on jeans and shirts.

8. Graph the mathematical model.
   Graph the number of jeans on the *x*–axis and shirts on the *y*–axis.



   a. Why does the graph only fall in the 1st Quadrant?

   b. Is it acceptable to spend less than $400?
      What are some possible combinations of outcomes that total less than $400?

      Where do all of these outcomes fall on the graph?

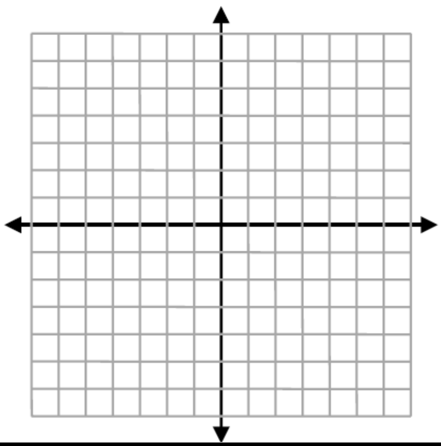
   c. Is it acceptable to spend exactly $400? How does the graph show this?

   d. Is it acceptable to spend more than $400? Where do all of the combinations that total more than $400 fall on the graph?

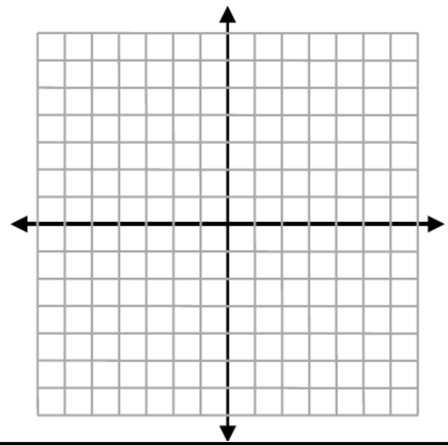
Homework # 4

Solve System of Inequalities:

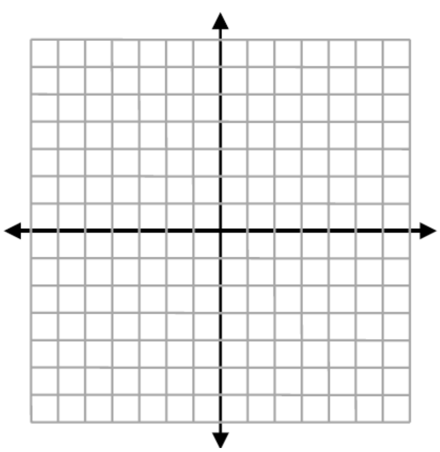
1. $x + 2y < 12$



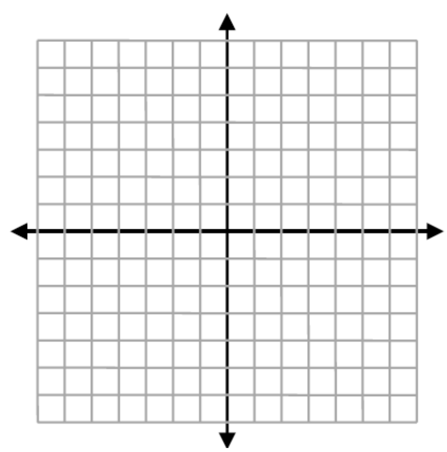
2. $y \geq \frac{-2}{3}x + 6$



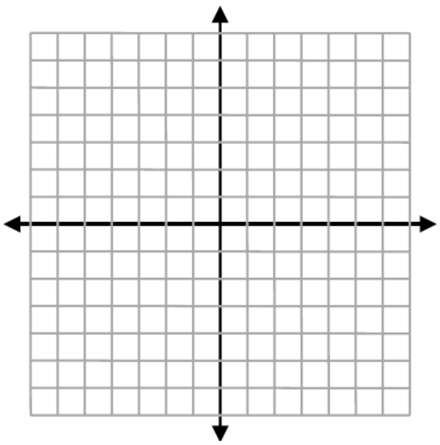
3. $2x + 7y < 21$ $\quad$ $-3x - 6y \geq -18$



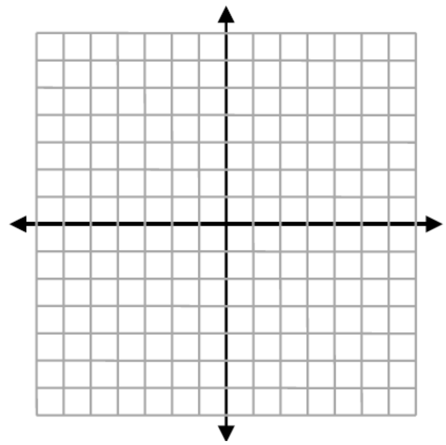
4. $4y > -3x - 8$ $\quad$ $x + 2y < 6$



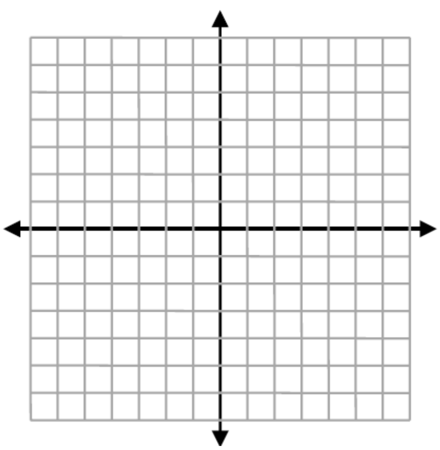
5. $5x + 10y > 25$ $3x + 6y > 15$



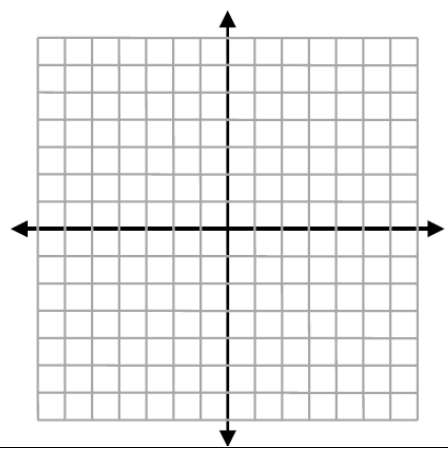
6. $y \geq -4$ $7x - 2y < 14$



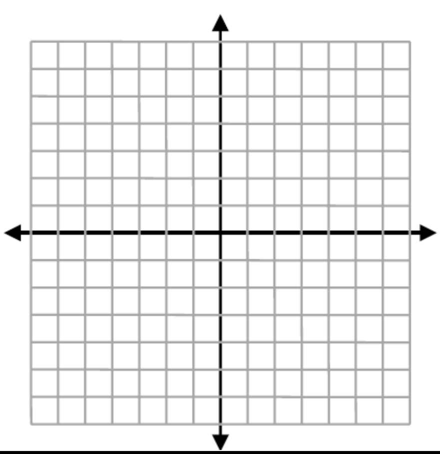
7. $x < -4$ $y \geq 2$



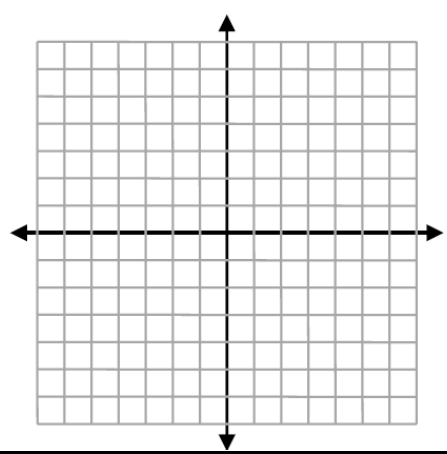
8. $y < 3x - 4$ $y \geq 2x + 3$



9. $-7x + 3y > -15$ $y > x$

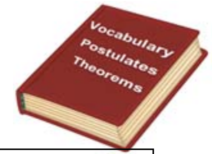


10. $y \leq -3x$ $x < 3$



Term	Definition	Notation	Diagram/Visual
Equation			
Commutative Property			
Associative Property			
Distributive Property			
Addition Property of Equality			
Subtraction Property of Equality			
Multiplication Property of Equality			
Division Property of Equality			

Symmetric Property			
Substitution Property			
Inequality			
Solution Set			
Literal Equation			
Linear Equation			
Slope			
Rate of Change			
Constant Rate of Change			



Ordered Pairs			
X-Intercept			
Y-Intercept			

Independent Practice

Math Models **Name**____________________________________
Worksheet 4.1 Relations and Functions

Relations Expressed as Ordered Pairs

Determine if the following relations are functions. Then state the domain and range.

1. {(1, -2), (-2, 0), (-1, 2), (1, 3)}

Function: ______________

Domain: ______________

Range: ______________

2. {(1, 1), (2, 2), (3, 5), (4, 10), (5, 15)}

Function: ______________

Domain: ______________

Range: ______________

3. $\left\{\left(17,\frac{15}{4}\right),\left(\frac{15}{4},17\right),\left(15,\frac{17}{4}\right),\left(\frac{17}{4},15\right)\right\}$

Function: ______________

Domain: ______________

Range: ______________

4. $\left\{\left(-3,\frac{2}{5}\right),\left(-3,\frac{3}{5}\right),\left(\frac{3}{2},-5\right),\left(5,\frac{2}{5}\right)\right\}$

Function: ______________

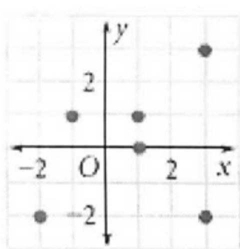
Domain: ______________

Range: ______________

Relations Expressed as Graphing

Write each of the following as a relation, state the domain and range, then determine if it is a function.

5.



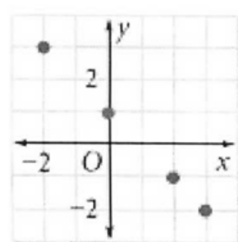
Relation: ______________

Domain: ______________

Range: ______________

Function: ______________

6.



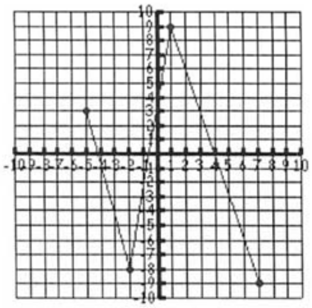
Relation: ______________

Domain: ______________

Range: ______________

Function: ______________

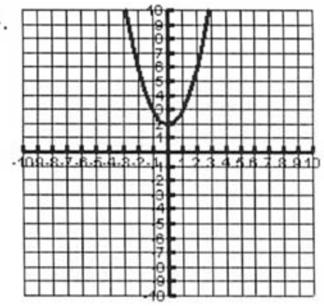
Determine if the graph is a function, then state the domain and range.

13. 

Domain: ________________

Range: __________________

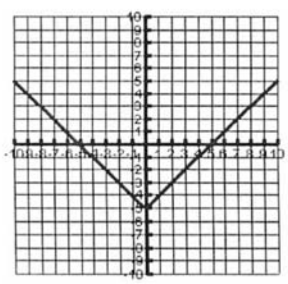
Function: _______________

14. 

Domain: ________________

Range: __________________

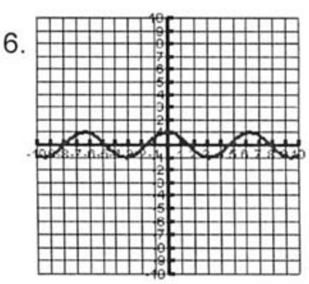
Function: _______________

15. 

Domain: ________________

Range: __________________

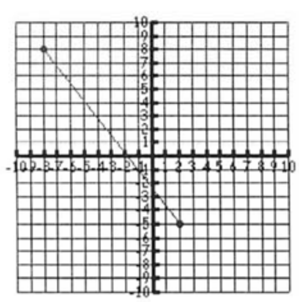
Function: _______________

16. 

Domain: ________________

Range: __________________

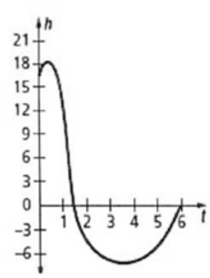
Function: _______________

17. 

Domain: ________________

Range: __________________

Function: _______________

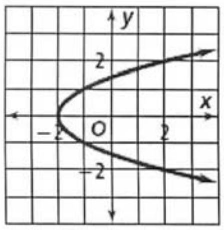
18. 

Domain: ________________

Range: __________________

Function: _______________

19.

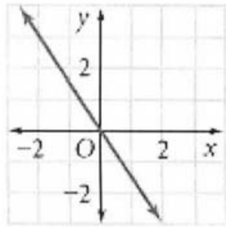


D: ________________

R: ________________

F: ________________

20.

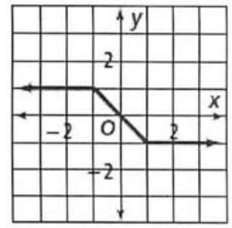


D: ________________

R: ________________

F: ________________

21.

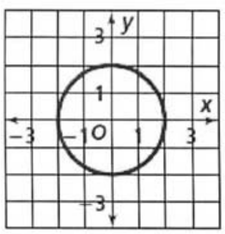


D: ________________

R: ________________

F: ________________

22.

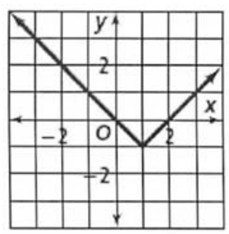


D: ________________

R: ________________

F: ________________

23.

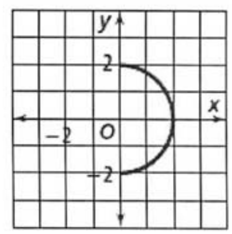


D: ________________

R: ________________

F: ________________

24.

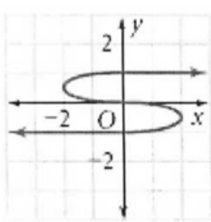


D: ________________

R: ________________

F: ________________

25.



Domain: ________________

Range: ________________

Function: ________________

What is a relation?

A **relation** between two variables *x* and *y* is a set of ordered pairs

An **ordered pair** consist of a x and y-coordinate

A relation may be viewed as ordered pairs, mapping design, table, equation, or written in sentences

The x-values are often called inputs, **domain**, and independent variable.

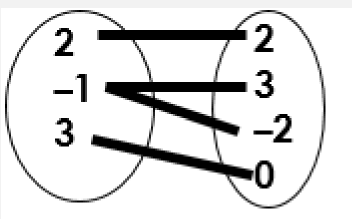
The y-values are often called outputs, **range**, and dependent variable.

Example! Guided Practice

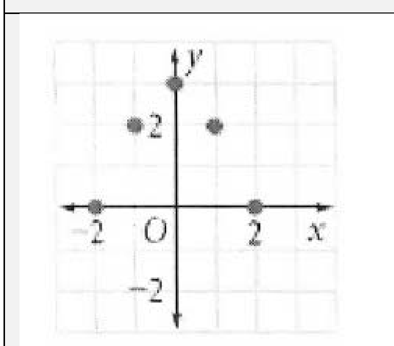
For each of the relations below identify the domain and range.

$$\{(0,-5),(1,-4),(2,-3),(3,-2),(4,-1),(5,0)\}$$

Domain: ________________________ Range: _________________________________



Domain: ________________________ Range: _________________________________



Domain: ________________________ Range: _________________________________

SELF CHECK

$$\{(2, 3), (-1, 5), (0, -1), (3, 5), (5, 0)\}$$

Domain: __________________________________ Range: ____________________________________

Is it a Function?

A function is a relation in which every input is paired with exactly one output.

Focus on the ***x*-coordinates**, when given a relation:

If the set of ordered pairs ALL of the x-coordinates are different, it **IS A** function
If the set of ordered pairs if ANY of the ***x*-coordinates** repeat then it is **<u>NOT</u>** a function.

Y-coordinates have no bearing in determining functions

Example! Guided Practice

$$\{(0,-5),(1,-4),(2,-3),(3,-2),(4,-1),(5,0)\}$$

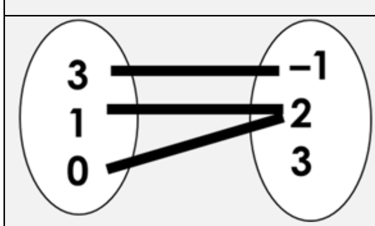
Function: YES or NO Reasoning: ____

Domain: ____________________ Range: ____________________________

$$\{(-1,-7),(1,0),(2,-3),(0,-8),(0,5),(-2,-1)\}$$

Function: YES or NO Reasoning: ____

Domain: ____________________ Range: ____________________________

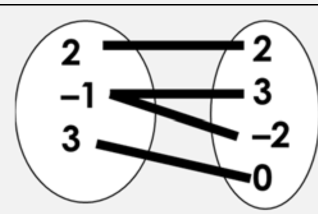


Function: YES or NO

Reasoning: __________________

Domain: ____________________

Range: ____________________________

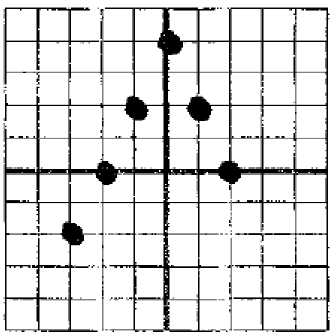


Function: YES or NO

Reasoning: __________________

Domain: ____________________

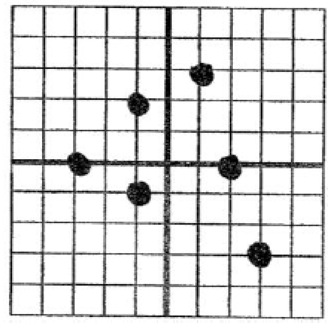
Range: ____________________________



Domain: ______________________________

Range: ______________________________

Function: yes no



Domain: ______________________________

Range: ______________________________

Function: yes no

SELF CHECK

For each table identify the domain, range and determine if the relation represents a function.

x	y
1	−3
6	−2
9	−1
1	3

x	y
0	2
3	1
3	−1
5	3

x	y
−4	−4
−1	−4
0	−4
3	−4

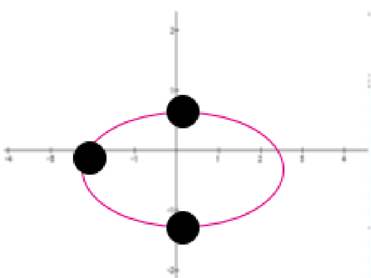
Questions To Ponder

Can you describe a graph that would not represent a function?

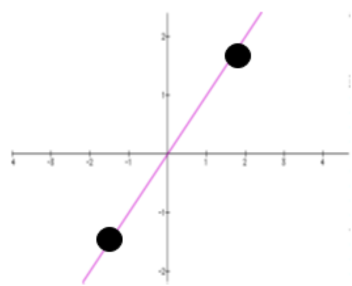
Graphs of Functions

Examine graphs of functions and non-functions, recognizing that if a **vertical line passes through at least two points in the graph**, then *y* (or the quantity on the vertical axis) is **not a function** of x (or the quantity on the horizontal axis).

Example!



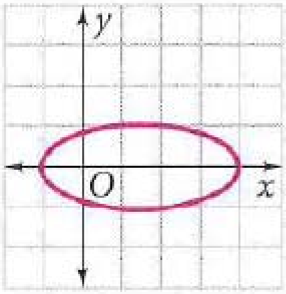
Does this graph represent a function?

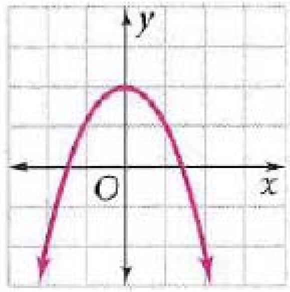


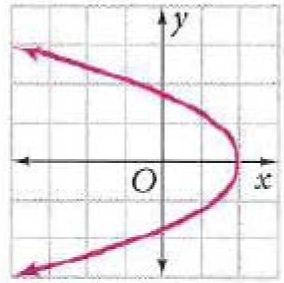
Does this graph represent a function?

SELF CHECK

Which of the graphs below represent functions.


y
O
x


y
O
x


y
O
x

Using function notation

FUNCTION NOTATION

The equation y = 9 – 4x represents a function. This is the notation that you are used to seeing.

You can use the letter *f* to name this function and then use **function notation** to express it. Just replace y with *f(x)*. (Note: In function notation, the parentheses do *not* mean multiplication.)

So y = 9 – 4x, written in function notation is f(x) = 9 – 4x

You read *f*(*x*) as ***f of x*** which means "the output value of the function *f* for the input value *x*."

Evaluating functions using function notation

Find *f(2), mean* what output do you get when you input 2?

f(2) = 9 – 4(2) Notice we substituted 2 for x

When you input 2 into function f the output is 1.
f(2) = 1

What is the value of x when f(x) = -7?

f(x) = 9 – 4x substitute -5 for f(x)

-7 = 9 – 4x Solve for x
-16 = -4x
x = 4

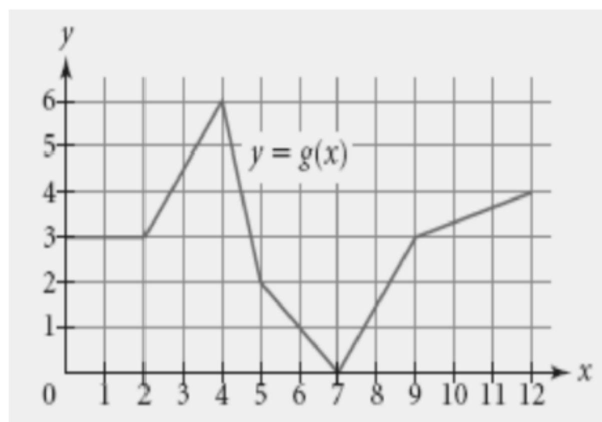
Example! Guided Practice

Evaluate the function $f(x) = 2x - 5$ for the following input values.	For the function $f(x) = 2x - 5$ find the value of x if the value of f(x) is:
$f(-2) =$ ________	$f(x) = 13$ x = ____________
$f(0) =$ ________	$f(x) = -7$ x = ____________
$f(3) =$ ________	$f(x) = 8$ x = ____________

SELF CHECK	
Evaluate each function $w(x)=4x+5$; Find $w(-8)$ $g(n)=4n-5$; Find $g(6)$	If $h(x)=8x+10$ Find the value of x if $h(x)=12$

Application

Not all functions are expressed as equations. Here is a graph of a function *g*. The equation is not given, but you can still use function notation to express the outputs for various inputs.



Examples:

1. *g(0) =*____ 2. *g(4) =*____ 3. *g(6) =*____

4. Can you find *x*-values for which *g(x)* = 3? ______

5. f(x)=6, what is x? ________

6. f(x)=0, what is x? ________

Lesson Name –Functioning Well

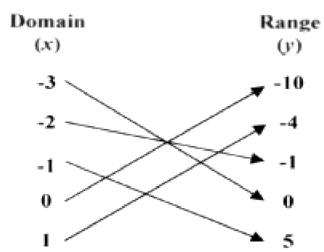
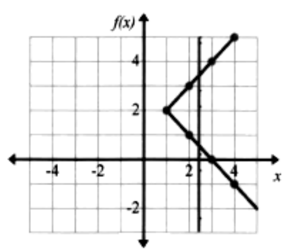
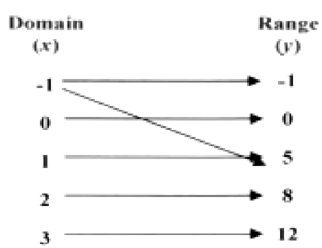
<u>Functioning Well (Practice Task)</u>

Name__________________________________ **Date**__________________

Consider the definition of a function (A function is a *rule* that assigns each element of set A to a *unique* element of set B. It may be represented as a set of ordered pairs such that no two ordered pairs have the same first member, i.e. each element of a set of inputs (the domain) is associated with a unique element of another set of outputs (the range)).

Part I – Function or Not

Determine whether or not each of the following is a function or not. Write "function" or "not a function" and explain why or why not.

	Relation	**Answer and Explanation**
1.		
2.		
3.		
4.	(x, y) = (student's name, student's shirt color) 	

Part II – Function Notation

Suppose a restaurant has to figure the number of pounds of fresh fish to buy given the number of customers expected for the day. Let $p = f(E)$ where p is the pounds of fish needed and E is the expected number of customers.

5. What would the expressions $f(E + 15)$ and $f(E) + 15$ mean?

6. The restaurant figured out how many pounds of fish needed and bought 2 extra pounds just in case. Use function notation to show the relationship between domain and range in this context.

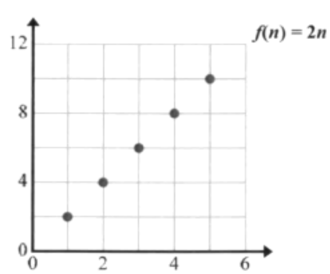
7. On the day before a holiday when the fish markets are closed, the restaurant bought enough fish for two nights. Using function notation, illustrate how the relationship changed.

8. The owner of the restaurant planned to host his 2 fish-loving parents in addition to his expected customers for dinner at the restaurant. Illustrate using function notation

Part III – Graphs are Functions

Write each of the points using function notation.

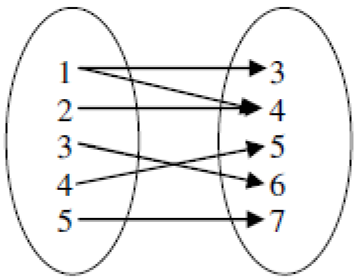
9.



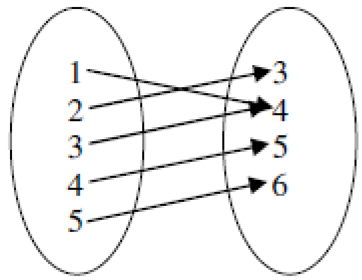
1. Describe the difference between a relation and a function.

2. Does the mapping diagram represent a function? Explain

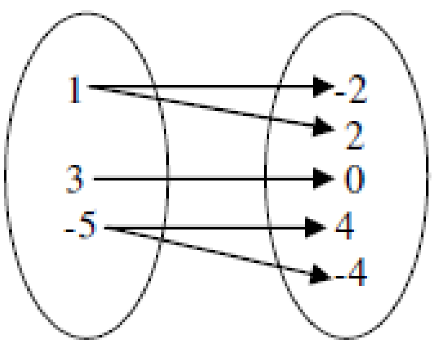
A.

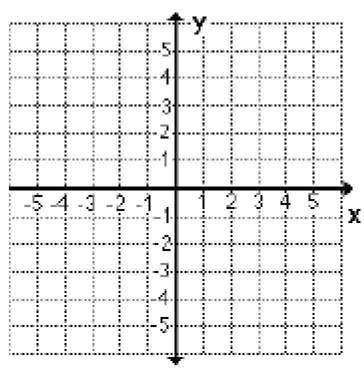


B.



3. Sketch a graph of the relation. Is the relation a function?

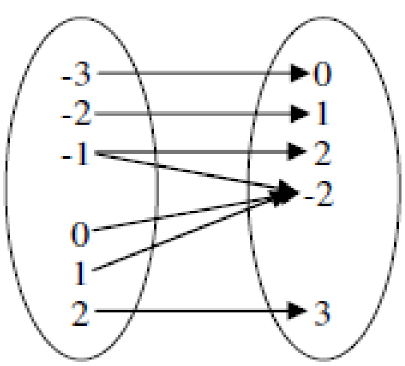


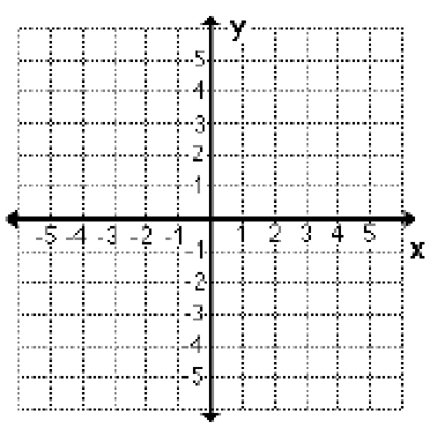


Domain:

Range:

Function





Domain:

Range:

Function:

4. Determine whether the relation is a function. If it is not a function, circle the ordered pairs that cause it not to be a function.

A. Yes No {(-2, 2) , (0, 5) , (1, 6) , (1, 7) , (2, -1) , (3, 2)}

B. Yes No {(0,1) , (2, -1) , (3, 2) , (4, 2) , (5, 3) , (-5,1)}

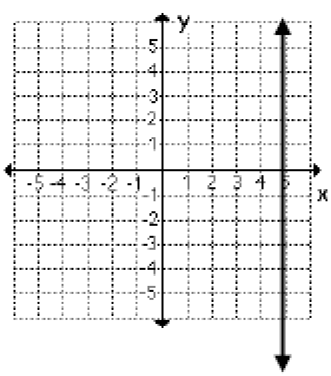
C. Yes No {(0, -5) , (1, 3) , (2, 2) , (0, 4) , (-5, 6) , (3, 4)}

5. If the domain of $f(x) = -x^2$ is integer values of x such that $-3 \le x < 0$, find the range.

6. Which of the following graphs represent functions? Circle your answers. If it is a function, state the domain and range.
If the graph is not included, make a table and gra ph the function by hand.

A.

$x = 5$

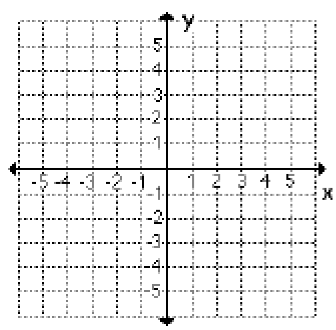


Domain:

Range:

B

$y = x^2 - 2x + 1$

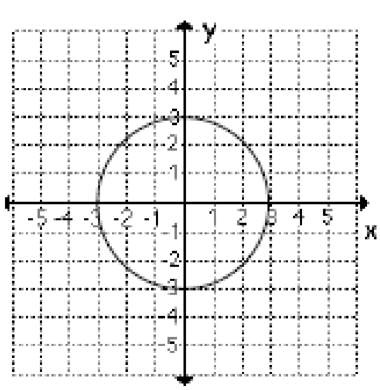


Domain:

Range:

$x^2 + y^2 = 9$

C.

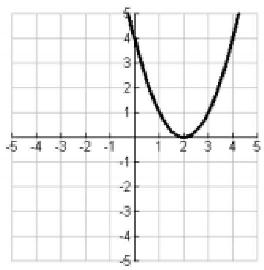


Domain:

Range:

Name____ **Date**________________________, state whether it is a function or not

1.



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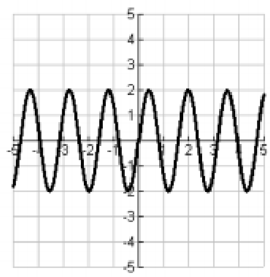

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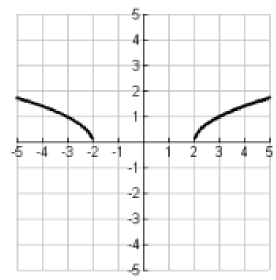

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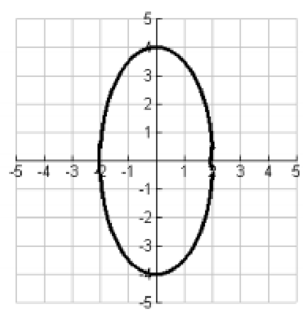
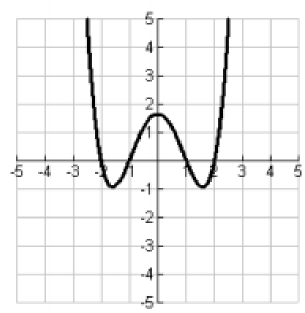
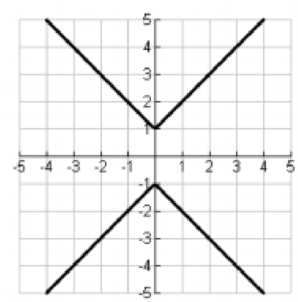


5.



6.



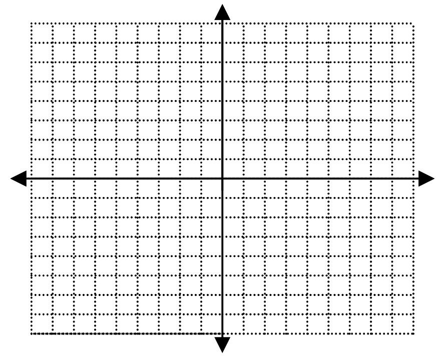
7. Evaluate the following expressions given the functions below:

$g(x) = -3x + 1$ $f(x) = x^2 + 7$ $h(x) = \frac{12}{x}$

a. $g(10) =$ **b.** $f(3) =$ **c.** $h(-2) =$

d. Find x if $g(x) = 16$ **e.** Find x if $h(x) = -2$ **f.** Find x if $f(x) = 23$

8. Translate the following statements into coordinate points, then plot them!



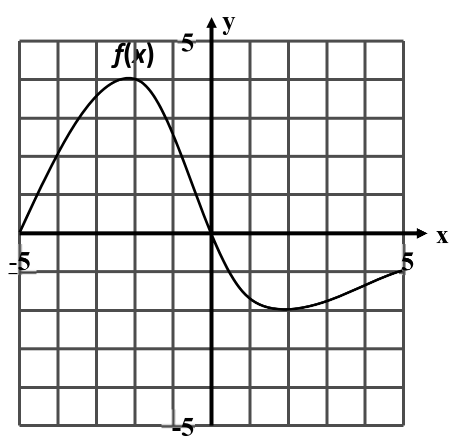
a. $f(-1) = 1$

b. $f(2) = 7$

c. $f(1) = -1$

d. $f(3) = 0$

9. Given this graph of the function f(x):



Find:

a. $f(-4)$ = **b.** $f(0)$ = **c.** $f(3)$ = **d.** $f(-5)$ =

e. x when $f(x) = 2$ **f.** x when $f(x) = 0$

10. Joe had a summer job that pays $7.00 an hour and he worked between 15 and 35 hours every week. His weekly salary can be modeled by the equation: S = 7h, where S is his weekly salary and h is the number of hours he worked in a week.

a) Describe the independent variable for this problem.

b) Describe the domain and range for this problem using appropriate notation.

Domain: Range:

c) What does the statement f(20)=140 mean in context of this problem?

Arithmetic Sequences

A sequence is an ordered list of numbers. This year you will learn about two special types of sequences.

One of these special sequences is called an Arithmetic Sequence are sequences that follow a pattern based on a **common difference.** The common difference is a **constant value** that is added to a previous term. This tells us how much more a term increases beyond the previous term.

The common difference is designated by the variable d.

Each value in a sequence is known as a term.

The first term is usually represented by the variable a_1, for each subsequent term the subscript changes to match the term number.

You may see other variables such as t_1 *or* s_1 it all means the same thing the problem is just using a different variable.

Example!

Determine if the sequence is Arithmetic, if it is state the common difference.

2, 6, 10, 14, … Arithmetic: Yes or No d = __________

15, 13, 11, 9, 7, … Arithmetic: Yes or No d = __________

3, 7, 11, 16, 20, … Arithmetic: Yes or No d = __________

Write the next 5 terms of the arithmetic sequence using the given 1st term and common difference.

$a_1 = 5$ d = 3 5, _______, ________, __________, __________, _________

$t_1 = 19$ d = -4 19, _____, _________, ________, __________, __________

SELF CHECK

7, 10, 13, 16, … Arithmetic: Yes or No d = ________

-5, -9, -13, -17, … Arithmetic: Yes or No d = ________

16, 19, 23, 27 … Arithmetic: Yes or No d = ______

Write the next 4 terms of the arithmetic sequence using the given 1st term and common difference.

$t_1 = 6$ d = -2

6, _______, ________, __________, _________

Two Important Formulas for Arithmetic Sequences

Recursive Formula	**Explicit Formula**
$a_1 = value\ of\ 1st\ term$ $a_n = a_{n-1} + d$ This can be used to find the next term. You must know the previous term use this rule.	$a_n = a_1 + (n-1)d$ The explicit formula can be used to find any term in a sequence.
$a_n =$ the nth term in the sequence $a_1 =$ the first term n is the number of the term d is the common difference	To write a recursive or explicit rule for a given sequence substitute values into the appropriate formula. Then simplify if necessary.

Example! Writing Explicit and Recursive Rules for a given sequence

Write the explicit and recursive formula for each sequence
2, 4, 6, 8, 10, … 1st term = 2 d = 2

Write the Recursive Rule for the sequence:	**Write the Explicit Rule for this sequence:**
General Formula: $a_1 = value\ of\ 1st\ term$ $a_n = a_{n-1} + d$ Becomes this after substituting: $\boldsymbol{a_1 = 2}$ $\boldsymbol{a_n = a_{n-1} + 2}$	General Formula: $a_n = a_1 + (n-1)d$ Becomes this after substituting: $a_n = 2 + (n-1)2$ Then simplify using distributive property. $a_n = 2 + 2n - 2$ Combine Like Terms: $\boldsymbol{a_n = 2n}$

Example! Guided Practice

Write the explicit and recursive formula for each sequence
7, 11, 15, 19… 1^{st} term = _______ d = ___________

Write the Recursive Rule for the sequence:	Write the Explicit Rule for this sequence:

Write the explicit and recursive formula for each sequence
2, -4, -10, -16, … 1^{st} term = _______ d = ___________

Write the Recursive Rule for the sequence:	Write the Explicit Rule for this sequence:

SELF CHECK

Write the explicit and recursive formula for each sequence
-3, 1, 5, 9, … 1st term = _______ d = ___________

Write the Recursive Rule for the sequence:	Write the Explicit Rule for this sequence:

Example! Using the explicit rule to find a specific term in a sequence.

You can write an explicit rule for an arithmetic sequence and then use it to find any term in the sequence.

Write an explicit rule for the sequence below. Then use the explicit rule to find the 35^{th} term.
8,13, 18, 23

Write the explicit Rule:	Use the explicit rule to find the 35^{th} term
$a_1 = 8 \quad d = 5$ $a_n = a_1 + (n-1)d$ $a_n = 8 + (n-1)5$ $a_n = 8 + 5n - 5$ $\boldsymbol{a_n = 5n + 3}$	$\boldsymbol{a_n = 5n + 3}$ $\boldsymbol{a_{35} = 5(35) + 3}$ $\boldsymbol{a_{35} = 5(35) + 3}$ $\boldsymbol{a_{35} = 178}$

Example! Using the explicit rule to find a specific term in a sequence. Guided Practice

You can write an explicit rule for an arithmetic sequence and then use it to find any term in the sequence.

Write an explicit rule for the sequence below. Then use the explicit rule to find the 47^{th} term.
19, 15, 11, 7, …

Write the explicit Rule:	Use the explicit rule to find the 47^{th} term

SELF CHECK

Write an explicit rule for the sequence below. Then use the explicit rule to find the 16^{th} term.

-12, -15, -18, -21, …

Questions To Ponder

Could you have used the recursive rule to find the 16^{th} term in the self-check above?

Example! Recursive to Explicit

If you are given a recursive you can use it to write the explicit rule.

Use the recursive rule to write the explicit rule.	*Guided Example:*
$a_1 = -4$ $a_n = a_{n-1} + 5$ Steps: $a_n = a_1 + (n-1)d$ *Substitute a_1 and d* $a_n = -4 + (n-1)5$ *Distribute* $a_n = -4 + 5n - 5$ *Combine like terms* $a_n = 5n - 9$ *Explicit Rule*	Use the recursive rule to write the explicit rule. $a_1 = 2$ $a_n = a_{n-1} - 4$

SELF CHECK

Use the recursive rule to write the explicit rule.

$a_1 = 7$

$a_n = a_{n-1} - 6$

Questions To Ponder

How does the negative *d* change the process of converting from recursive to explicit?

Example! Explicit to Recursive

If you are given an explicit rule you can use it to write the recursive rule.

Use the explicit rule below to write the corresponding recursive rule. $a_n = -5n + 4$ **Step 1 Determine the common difference.** The common differnce is the coefficient of the variable so $d = -5$ **Step 2 Determine $\boldsymbol{a_1}$** Use the process learned previously to find the 1st term $a_1 = -5(1) + 4$ $a_1 = -1$ **Step 3 Write Recursive Rule** $a_1 = -1$ $a_n = a_{n-1} - 1$	*Guided Example:* Use the explicit rule below to write the corresponding recursive rule $a_n = 4n - 7$

SELF CHECK

Use the explicit rule below to write the corresponding recursive rule.

$a_n = -3n - 6$

Questions To Ponder

Can you think of any advantages to converting an explicit rule to a recursive rule?

Arithmetic Sequences Applications – Guided Example

Suppose you have saved $75 towards the purchase of a new iPad. You plan to save at least $12 from mowing your neighbor's yard each week. In all, what is the minimum amount of money you will have in 26 weeks?

SELF CHECK

1. Edgar is getting better at math. On his first quiz he scored 57 points, then he scores 61 and 65 on his next two quizzes. If his scores continued to increase at the same rate, what will be his score on his 9th quiz? Show all work.

a. Write an explicit formula for the sequence. Explain where you found the numbers you are putting in the formula.

b. Identify the value of n and explain where you found it. Use the explicit formula to solve the problem.

Write your final answer as a sentence

Independent Practice

Re-Teaching and Independent Practice

Example!

Arithmetic Sequences (Recursive & Explicit):

Recursive	Explicit
Steps: 1. Determine the first term (a_1) and common difference (d) 2. Substitute d into: $a_{n+1} = a_n + d$ And state the first term $a_1 =$ What it's used for: Describing the pattern and finding the next few terms Example: Write the recursive formula for the following sequence: $-20, -5, 10, 25$ 1. $a_1 = -20$ and $d = 15$ 2. $a_{n+1} = a_n + 15$ $a_1 = -20$ Step 2 is the entire recursive formula. You must have both parts.	Steps: 1. Determine the first term (a_1) and common difference (d) 2. Substitute into: $a_n = a_1 + d(n-1)$ What it's used for: Finding any term as long as you know the term number (n) Example: Write the explicit formula for the following sequence: $-20, -5, 10, 25$ 1. $a_1 = -20$ and $d = 15$ 2. $a_n = -20 + 15(n-1)$ Step 2 is the explicit formula. If you know the term number, you can substitute that for n to determine the term value.

1. -4, -6, -8, -10, …..	2. 19, 13, 7, 1, …..
Identify: a_1 = ________ d= ________	Identify: a_1 = ________ d= ________
Write the recursive equation:	Write the recursive equation:
Write the explicit equation:	Write the explicit equation:
Evaluate the function to determine the 9th term:	Evaluate the function to determine the 31st term:

Independent Practice

3. -25, 75, 175, 275, 375, ...

Identify: a_1 = ________ d= ________

Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 11th term:

4. -14, -11, -8, -5, -2, ...

Identify: a_1 = ________ d= ________

Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 14th term:

5. 12, 17, 22, 27, 32, ...

Identify: a_1 = ________ d= ________

Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 20th term:

6. 24, 31, 38, 45, 52, ...

Identify: a_1 = ________ d= ________

Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 28th term:

7. 104, 116, 128, 140, 152, ...

Identify: a_1 = ________ d= ________

Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 7th term:

8. -34, -25, -16, -7, 2, ...

Identify: a_1 = ________ d= ________

Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 12th term:

9. There are 20 rows of seats on a concert hall: 25 seats are in the 1st row, 27 seats on the 2nd row, 29 seats on the 3rd row, and so on. What would be number of seats on 16th row?

10. During a science experiment, Kyle counted the number of bacteria present in a petri dish after every minute. Assuming the pattern continues, how many bacteria will there be after 20 minutes?

Min.	1	2	3	4
Bact.	2	6	10	14

COMMUNITY SERVICE, SEQUENCES AND FUNCTIONS

Performance Task: Community Service, Sequences, and Functions

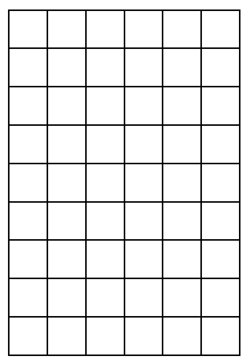
Name________________________________ **Date**__________________

Larry, Moe, and Curly spend their free time doing community service projects. They would like to get more people involved. They began by observing the number of people who show up to the town cleanup activities each day. The data from their observations is recorded in the given table for the Great Four Day Cleanup.

x	*y*
1	5
2	27
3	49
4	71

1. Give a verbal description of what the domain and range presented in the table represents.

2. Sketch the data on the grid below.



3. Determine the type of function modeled in the graph above and describe key features of the graph.

4. Based on the pattern in the data collected, what recursive process could Larry, Curly, and Moe write?

5. Write a linear equation to model the function.

6. How would Larry, Curly, and Moe use the explicit formula to predict the number of people who would help if the cleanup campaign went on for 7 days?

Excited about the growing number of people participating in community service, Larry, Curly, and Moe decide to have a fundraiser to plant flowers and trees in the parks that were cleaned during the Great Four Day cleanup. It will cost them $5,000 to plant the trees and flowers. They decide to sell some of the delicious pies that Moe bakes with his sisters. For every 100 pies sold, it costs Moe and his sisters $20.00 for supplies and ingredients to bake the pies. Larry, Curly, and Moe decide to sell the pies for $5.00 each.

7. Complete the following table to find the total number of pies sold and the amount of money the trio collects.

a. On Day 1, each customer buys the same number of pies as his customer number. In other words the first customer buys 1 pie, the second customer buys 2 pies. Fill in the table showing the number of pies and the amount collected on Day 1. Then calculate the total number of pies sold and dollars collected.

b. Write a recursive and explicit formula for the pies sold on Day 1. Explain your thinking.

Customer Number	***Number of Pies Sold***	***Amount Collected***
1	***1***	***$5***
2	***2***	***$10***
total		

c. On Day 2, the first customer buys 1 pie, the second customer buys 2 pies, the third customer buys 4 pies, the fourth customer buys 8 pies, and so on**.** Complete table based on the pattern established. Then calculate the total number of pies sold and dollars collected.

d. Write a recursive and explicit formula for the pies sold on Day 2. Explain your thinking.

Customer Number	***Number of Pies Sold***	***Amount Collected***
1	***1***	***$5***
2	***2***	***$10***
total		

HOMEWORK

Homework #___1_____

Arithmetic Sequence:

1. 7, 5, 3, 1, -1 , -3, ….. Identify: a_1 = ________ d= ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 8th term:	2. -3, 2, 7, 12, 17, ….. Identify: a_1 = ________ d= ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 7th term:
3. -21, -16, -11, -6, -1, ….. Identify: a_1 = ________ d= ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 43th term:	4. 15, 17, 19, 21, ….. Identify: a_1 = ________ d= ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 33th term:
5. 19, -1, -21, -41, -61, ….. Identify: a_1 = ________ d= ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 9th term:	6. 7, 14, 21, 28, ….. Identify: a_1 = ________ d= ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 9th term:

HOMEWORK

7. -18, -10, -2, 6, …..

Identify: a_1 = ________ d= ________

Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 9th term:

8. 8, 3, -2, -7, -12, …..

Identify: a_1 = ________ d= ________

Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 9th term:

9. Edgar is getting better at math. On his first quiz he scored 57 points, then he scores 61 and 65 on his next two quizzes. If his scores continued to increase at the same rate, what will be his score on his 9th quiz? Show all work.

Identify: a_1: ________ d: ____________

Write the recursive equation:

Write the explicit equation:

10. A house worth $350,000 when purchased was worth $335,000 after the first year and $320,000 after the second year. If the economy does not pick up and this trend continues, what will be the value of the house after 6 years.

Identify: a_1 = ________ d= ________

Write the recursive equation:

Write the explicit equation:

Unit 3

Modeling and Analyzing Exponential Functions

Algebra 1

Unit 3: Modeling and Analyzing Exponential Functions

Geometric Sequences

GUIDED NOTES

We have learned that **arithmetic sequences** are sequences that follow a pattern based on **common differences**. The number added to each term is a **constant**, fixed amount. 1,3,5,7,9… d= 2.

Geometric sequences are built on the same concept, but their patterns are based on a **common ratio**.1,2,4,8,16,….. r = 2.

The fixed amount multiplied from one term to the next is called the **common ratio** (r). Geometric sequences increases (decreases) at a much faster rate than the arithmetic sequences. To find the **common ratio**, we divide a term by its previous term (or as how much is it multiplying by to get to the next term).

For example: 2, 6, 18, 54…. Common ratio, r =3
2, -4, 8, -13…. Common ratio, r = -2
20,10, 5, 2.5…Common ratio, r = ½

Example! *Is the given sequence geometric? If yes, determine the common ratio*

1. 5, 15, 45, 135... Geometric: Yes or No r = ________
2. 15, 30, 45, 60... Geometric: Yes or No r = ________
3. 6, -24, 96, -384… Geometric: Yes or No r = ________
4. 6, 3, 3/2, ¾…. Geometric: Yes or No r = ________

Write the next 5 terms of the geometric sequence using the given 1st term and common ratio.

5. $a_1 = 100$ r = ½ 100, ______, ______, ______, ______, ______
6. $t_1 = 5$ r = -2 5, ______, ______, ______, ______, ________

SELF CHECK *Is the given sequence geometric? If yes, determine the common ratio*

1. 4, -1, -6, -11, … Geometric Y or N r = ____
2. 12, 36, 108, … Geometric Y or N r = ____
3. -1/3, 1/9, -1/27.. Geometric Y or N r = ____

Write the next 4 terms of the geometric sequence using the first term and common ratio.

$t_1 = 81$ r = 1/3

81, ______, ______, ________, ________

Two Important Formulas for Geometric Sequences

Recursive formulas are written to describe what to do to each term to get the next one.

Notation:
a_n is the general nth term you are trying to define.
a_1 is the first term of the sequence
a_{n-1} is "the previous term"
r is the common ratio

Formula: a_1 value of first term

$$a_n = r * a_{n-1}$$

Explicit Formula are written to be able to find a specific term.

Notation:
a_n is the general nth term you are trying to define.
a_1 is the first term of the sequence
n is the term number
r is the common ratio

Formula: $a_n = a_1 * r^{n-1}$

Writing Explicit and Recursive Rules for a given sequence

Write the explicit and recursive formula for each sequence

6, 24, 96, 384, 1536, … 1st term = 6 r = 4

Write the **Explicit Rule** for this sequence:

General Formula: $a_n = a_1 * r^{n-1}$

Given sequence becomes this after substituting: $a_n = 6 * 4^{n-1}$

Write the **Recursive Rule** for this sequence:

General Formula: a_1 value of 1st term

Given sequence becomes this after substituting:
$a_1 = 6$
$a_n = 4 * a_{n-1}$

Guided Practice

Write the explicit and recursive formula for each sequence

120, 60, 30, 15…. 1st term = ____ r = ____

Write the **Explicit Rule** for this sequence:	Write the **Recursive Rule** for this sequence:
Write the explicit and recursive formula for the sequence: 2, 6, 18, 54….. 1^{st} term= _______ r = _______	
Write the **Explicit Rule** for this sequence:	Write the **Recursive Rule** for this sequence:

SELF CHECK *Write the explicit and recursive formula for each sequence*

1, 5, 25, 125…. 1^{st} term=_______ r=________

Explicit Rule	Recursive Rule

Example! *Using the explicit rule to find a specific term in a sequence.*

You can write an explicit rule for a geometric sequence and then use it to find any term in the sequence.

Find a_8 for 4, 8, 16….

1. Write rule = $4(2)^{n-1}$

2. Substitute 8 for n and solve = $4(2)^7 = 4(128) = 512$

SELF CHECK Write the explicit rule for the geometric sequence and solve.

1. Find a_6 for 12, 6, 3….. 2. Find a_{10} for 1, -6, 36…….. 3. Find a_5 for 3, ¾, 3/16……

1.If you know the second term and the common ratio of a geometric sequence, can you write an explicit rule for the sequence? If so, explain how.

2. How can you write the explicit rule for a geometric sequence if you know the recursive rule for the sequence?

Re-Teaching and Independent Practice

Example!

Geometric Sequences (Recursive & Explicit):

To determine whether a sequence is a **geometric sequence**, check for a **common ratio, *r*** ($r \neq 1$).

$-2, 6, -18, 54, -162, \ldots$

Find the ratios of pairs of terms to decide whether the sequence is geometric.

Ratios: $\frac{6}{-2} = -3 \qquad \frac{-18}{6} = -3 \qquad \frac{54}{-18} = -3 \qquad \frac{-162}{54} = -3$

The common ratio is –3. The sequence is geometric.

If you know the first term of a geometric sequence, a_1, and the common ratio, r, then you can find the nth term, a_n, using the following rule.

$$a_n = a_1 r^{n-1}$$

Find the 10th term of the geometric sequence 3, 12, 48, 192, 768, …

Step 1 Find the common ratio, r.

$r = \frac{12}{3} = 4$

Step 2 Identify the first term, a_1.

$a_1 = 3$

Step 3 Use the formula with $r = 3$ to find the 10th term, a_{10}.

$a_n = a_1 r^{n-1}$	*Write the rule.*
$a_{10} = a_1 r^{10-1}$	*Substitute $n = 10$.*
$a_{10} = 3(4)^9$	*Substitute $a_1 = 3$ and $r = 4$.*
$a_{10} = 3(262{,}144) = 786{,}432$	*Simplify.*

The 10th term of the sequence is 786,432.

Each rule represents a geometric sequence. If the given rule is recursive, write it as an explicit rule. If the rule is explicit, write it as a recursive rule. Assume that $f(1)$ is the first term of the sequence. Write the first 4 terms of the sequence.

$f(1) = \frac{1}{4},\ f(n) = f(n-1) \bullet 2$ for $n \geq 2$

Step 1. $f(n) = \frac{1}{4} \bullet 2^{n-1}$

Step 2. $\frac{1}{4}, \frac{1}{2}, 1, 2, \ldots$

$f(n) = 3 \bullet (2)^{n-1}$

Step 1. $f(1) = 3,\ f(n) = f(n-1) \bullet 2$ for $n \geq 2$

Step 2. 3, 6, 12, 24, …

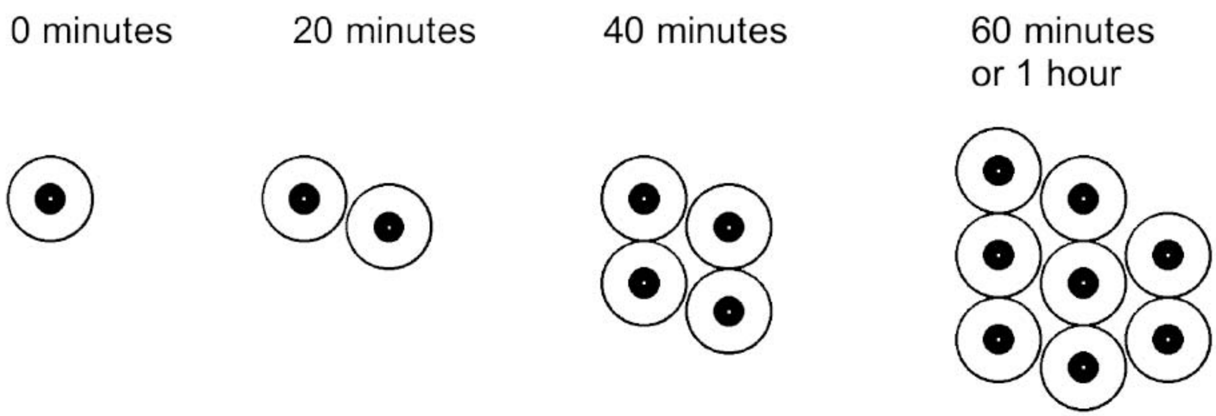
Independent Practice

1. -4, 12, -36, -108, ….. Identify: a_1 = ________ r= ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 9^{th} term:	2. 0.03, 0.12, 0.48, 1.92, ….. Identify: a_1 = ________ r = ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 12^{th} term:
3. 2, 16, 128, 1024, ….. Identify: a_1 = ________ r = ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 8^{th} term:	4. 0.5, 1, 2, 4, ….. Identify: a_1 = ________ r = ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 14^{th} term:
5. 81, -27, 9, -3, 1, ….. Identify: a_1 = ________ r = ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 16^{th} term:	6. 54, 18, 6, 2, $\frac{2}{3}$….. Identify: a_1 = ________ r = ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 11^{th} term:

7. 8, 20, 50, 125, ... Identify: a_1 = ________ r = ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 22th term:	8. -6, 12, -24, 48, -96, ... Identify: a_1 = ________ r = ________ Write the recursive equation: Write the explicit equation: Evaluate the function to determine the 18th term:
9. Suppose you drop a tennis ball from a height of 15 feet. After the ball hits the floor, it rebounds to 85% of its previous height. How high will the ball rebound after its third bounce? Round to the nearest tenth.	10. A biologist studying ants started on day 1 with a population of 1500 ants. On day 2, there were 3000 ants, and on day 3, there were 6000. The increase in an ant population can be represented geometric sequence. Write the explicit equation for the ant population and determine the ant population on day 6.

MULTIPLYING CELLS

Name__________________________________ **Date**__________________



- Mrs. Lucas's class has a 2-hour science lab.
- She gives each student a dish with one cell in it.
- She tells the class that in 20 minutes the cell will divide into two cells, and each 20 minutes after that **each** cell in the dish will divide into two cells.

1.

Complete the second row in this table to show how the number of cells increases during the lab.

Time (minutes)	**0**	**20**	**40**	**60**	**80**	**100**	**120**
Number of cells	1	2	4				
Number of cells as a power of 2	2^0	2^1					

2. Olan says that the numbers of cells can be written in the form 2^n.
Complete the third row in the table to show how the number of cells can be written in this form.

3. Linda says that the number of cells after 3 hours will be 2^7 (= 2x2x2x2x2x2x2)

   Is she correct? _______________________

   If not, then what is the correct number? _____________________

   Explain how you figured it out.

   ____

   ____

   ____

4. How many cells will be in the dish after 5 hours? ___________________________

   Give your answer as a normal number, not as a power of 2.

   Show how you figured it out.

5. How long will it take for the number of cells to reach at least 100,000?
Give your answer to the nearest 20 minutes. ______________________
Show how you figured it out.

HOMEWORK

Homework #___1_____

Geometric Sequence:

1. 81, 27, 9, 3, 1, …..

Identify: a_1 = _________ r = _________
Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 8th term:

2. 10, 1, 0.1, 0.01, 0.001, …..

Identify: a_1 = _________ r = _________
Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 10th term:

3. 1, 4, 16, 64, …..

Identify: a_1 = _________ r = _________
Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 6th term:

4. 2, 10, 50, 250 …..

Identify: a_1 = _________ r = _________
Write the recursive equation:

Write the explicit equation:

Evaluate the function to determine the 7th term:

5. Write an explicit rule for

$$a_3 = \frac{1}{48} \text{ and } a_4 = \frac{1}{192}$$

6. Write the recursive rule for $f(n)= 11(2)^{n-1}$

7. Write the explicit rule for $f(1)= 2.5$; $f(n) = f_{n-1} * 3.5$

8. Write an explicit rule for the following:
$a_1 = 90$ and $a_2 = 360$

Building a Function that Models a Relationship Between Two Quantities

GUIDED NOTES

Sometimes the data for a function is presented as a sequence that can be modeled **exponentially**. For a sequence to fit an **exponential model**, the ratio of successive terms must be constant.

Exponential functions are built using powers. A power is the combination of a base with an exponent. For example, in the power 5^3, the base is 5 and the exponent is 3. An exponential function gets its name from the fact that the **variable x** is in the **exponent.**

Exponential functions are of the form $f(x) = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$. In an exponential function, the base, b, is a constant and a is the coefficient.

An exponential function can be described using a ***function rule***, which represents an output value, or element of the range, in terms of an input value, or element of the domain. An exponential function rule can be written in ***function notation***. Here is an example of a function rule and its notation.

$y = 2^x$	y is the output and x is the input.
$f(x) = 2^x$	Read as "f of x."
$f(2) = 2^2$	"f of 2," the value of the function at $x = 2$, is the output when 2 is the input.

Be careful—do not confuse the parentheses used in notation with multiplication.

The elements of the domain and the values obtained by substituting them into the function rule form the coordinates of the points that lie on the graph of the function.

Properties associated with exponential functions are:

- 'b' is the value of the common ratio. Within the function, as the x-value increases by 1, the y-value is multiplied by the common ratio.
- If $b > 1$ then the curve will represent **exponential growth**.
- If $0 < b < 1$ then the curve will represent **exponential decay**.
- Every exponential function of the form $y = ab^x$ will pass through the point $(0,a)$. a will always be the **y-intercept** of the function, or its value at time 0.
- Every exponential function of the form $y = ab^x$ will have a domain and range:

$$\text{Domain} = \{x \mid x \in R\} \text{ and Range} = \{y \mid y > 0,\ y \in R\}$$

Solving Exponential Equations

GUIDED NOTES

In Algebra I you will solve exponential equations by rewriting each side of the equation using the same base.

If we consider the problem $2^{x+6} = 32$, the base of the exponent is 2 and we need to decide if we can rewrite the number 32 using only the number 2. In this case it is possible to write the number 32 using only 2's, 32 = 2·2·2·2·2 =2^5.

So $2^{x+6} = 32$, is the same as $2^{x+6} = 2^5$

We can now solve the equation using the Equality of Bases Property

If B^M=B^N, then M = N.

Using the Equality of Bases Property we can solve the equation

$x + 6 = 5$
$\underline{\ \ -6 \ \ -6}$

Which means that x = -1

Example! Guided Practice

Use the Equality of Bases Property to solve the equations

$6^{3x} = 6^6$	$3^{3x} = 9$	$2^{x+7} = 32$

SELF CHECK

Use the Equality of Bases Property to solve the equations

$3^{x-5} = 3^{10}$	$2^{7x} = 16$	$5^{2x+5} = 125$

Evaluating Exponential Functions

Remember that an exponential function is a function in the form of $f(x) = b^x$ for a fixed base b, where b > 0 and b ≠ 1.

We can evaluate exponential functions by substituting values for x or $f(x)$ similar to what we did when evaluation linear functions.

Example!

Given the function $f(x) = 10(3)^x$ Evaluate the function for $f(3)$ $f(x) = 10(3)^x$ $f(3) = 10(3)^3$ $f(3) = 10(27)$ $f(3) = 270$	Given the function $f(x) = 10(3)^x$ Find the value of x if $f(x) = 810$ $f(x) = 10(3)^x$ $810 = 10(3)^x$ Divide by 10 $81 = (3)^x$ Write 81 as a power with a base of 3 $3^4 = 3^x$ Rewrite using Equality of Bases Property 4 = x

SELF CHECK

Given the function $f(x) = 2(5)^{x+1}$ Evaluate the function for $f(3)$	Given the function $f(x) = 2(5)^{x+1}$ Find the value of x if $f(x) = 51$

Re-Teaching and Independent Practice

Example!

Exponential Equations and Function Notation:

You can solve an equation with a variable exponent by writing both sides with the same base.

Example

Solve $25^3 = 5^x$.

$(5^2)^3 = 5^x$	Write 25 as a power of 5.
$5^6 = 5^x$	Use the properties of exponents to simplify.
$6 = x$	Since the bases are equal, the exponents are equal.

Find the value of the variable {f(x)} in notation form:

$f(x) = 2^x + 6$ when $f(x) = 14$	**Step #1: $f(x) = y$**
$14 = 2^x + 6$	**Step #2: substitute the value of f(x) in the equation**
$14 – 6 = 2^x + 6 - 6$	**Step #3: subtract 6 from both sides of the equation**
$8 = 2^x$	**Step #4: Equality of Bases Property**
$2^3 = 2^x$	**Step #5: Use the properties of exponents to simplify.**
$3 = x$	**Step #6: Since the bases are equal, the exponents are equal**

Evaluate the exponential equation and notations:

1. $f(x) = 3^x$; when f(6)	2. $m(x) = 4(2)^x$; when m(3)
3. $g(x) = 7^x$; when g(4)	4. $f(x) = 5^{x+2}$; when f(7)
5. $b(x) = -5(3)^x - 8$; when b(2)	6. $k(x) = 9^x + 5$; when k(3)
7. $m(x) = 6^x$; find the value of x when m(x) = 216	8. $d(x) = 10(2)^x$; find the value of x when d(x) = 160

Independent Practice

9. $t(x) = 8(3)^x - 4$; find the value of x when $t(x) = 644$	10. $h(x) = -4(2)^{x+5} + 47$; find the value of x when $h(x) = -209$
11. $j(x) = 4(7)^x + 5$; find the value of x when $j(x) = 1377$	12. $f(x) = 3(2)^{x-3} + 3$; find the value of x when $f(x) = 9$

Evaluating Exponential Functions

A population of 6000 bacteria is growing at a rate of 16% each day. This situation can be modeled by the function $f(x) = 6000(1.16)^x$, where x is the number of days and $f(x)$ represents the number of bacteria.

a) About how many bacteria are present after 10 days. Round to the nearest whole number.

A population of 500 elk is released in a wildlife preserve. Each year, the population grows by 6.4%. Let x stand for the number of years since the release, and let f(x) stand for the elk population.

This situation can be modeled by the function $f(x) = 500(1.064)^x$

a) After 5 years, how many elk are there?

b) How many years will it take for the elk population to exceed 800 elk?

Homework #___1_____

Exponential Equations and Notations:

1. $k(x) = 7^x$; when k(3)	2. $f(x) = -2(4)^x$; when f(7)
3. $m(x) = 5^x$; when m(6)	4. $g(x) = 3^{x+2}$; when g(4)
5. $h(x) = 2(8)^x - 54$; when h(3)	6. $t(x) = 10(4)^{x-4} + 13$; when t(11)
7. $d(x) = 2(3)^x$; find the value of x when $d(x) = 54$	8. $n(x) = 8^x$; find the value of x when $n(x) = 4096$
9. $f(x) = -(4)^x + 7$; find the value of x when $f(x) = -57$	10. $k(x) = 3(3)^{x+2} - 5$; find the value of x when $f(x) = 238$
11. $p(x) = 2(8)^{x-4} + 32$; find the value of x when $p(x) = 160$	12. $w(x) = 12(13)^{x-3} + 22$; find the value of x when $w(x) = 2050$

Exponential functions are very useful in life, especially in the worlds of business and science. If you've ever earned interest in the bank (or even if you haven't), you've probably heard of "compounding", "appreciation", or "depreciation"; these have to do with exponential functions. Just remember when exponential functions are involved, **functions are increasing or decreasing very quickly** (multiplied by a fixed number). That's why it's really good to start saving your money early in life and let it grow with time.

An **exponential function** is a function whose successive output values are related by a constant ratio. An exponential function can be represented by an equation of the form $f(x) = ab^x$, where *a, b*, and *x* are real numbers, $a \neq 0$, $b > 0$, and $b \neq 1$. The constant ratio is the base *b*. **a** is the initial or starting value **when x = 0,** and ***b*** is the common ratio also called **the "growth" or "decay" factor**.
Remember that exponential functions are named that because of the **"x"** in their exponents!

When *b*>1, we have **exponential growth** (the function is getting larger), and when 0<*b*<1, we have **exponential decay** (the function is getting smaller). This makes sense, since when you multiply a fraction (less than 1) many times by itself, it gets smaller, since the denominator gets larger. When evaluating exponential functions, you will need to use the properties of exponents, including zero and negative exponents.

Example! Writing Exponential Functions from Verbal Descriptions

You can write an equation for an exponential function $f(x) = ab^x$ by finding or calculating the values of *a* and *b*. The value of *a* is the value of the function when *x* = 0 and *b* is the common ratio of successive function values.

Example 1: An initial population of 750 endangered turtles triples each year. What will the population of turtles be after 5 years?

To write the exponential function for this situation students should recognize- a= 750, b=3 and x = 5. $F(x) = 750(3)^5$ which results in 182250 turtles.

Example 2: You have $5 in a jar. Each week the amount doubles. How much will you have after 10 weeks?
a= ________, b=_________, x = ________ Write the function f(x) =________________.

Example! Exponential Growth and Decay

Exponential growth occurs when a quantity increases by the same rate *r* in each period *t*. When this happens, the value of the quantity at any given time can be calculated as a function of the rate and the original amount.

Exponential Growth

An exponential growth function has the form $y = a(1 + r)^t$, where $a > 0$.

y represents the final amount.

a represents the original amount.

r represents the rate of growth expressed as a decimal.

t represents time.

Example 1: The original value of a painting is $9,000 and the value increases by 7% each year. Write an exponential growth function to model this situation. Then find the painting's value in 15 years.

Step 1: Write the exponential growth function for this situation.

$$y = a(1 + r)^t$$
$$= 9000\ (1 + 0.07)^t$$
$$= 9000(1.07)^t$$

Step 2: Find the value in 15 years.

$$y = 9000(1.07)^{15}$$
$$\approx 24{,}831.28$$ value of the painting in 15 years.

Exponential decay occurs when a quantity decreases by the same rate r in each time period t. Just like exponential growth, the value of the quantity at any given time can be calculated by using the rate and the original amount.

Exponential Decay

An exponential decay function has the form $y = a(1 - r)^t$, where $a > 0$.

- y represents the final amount.
- a represents the original amount.
- r represents the rate of decay as a decimal.
- t represents time.

Notice an important difference between exponential growth functions and exponential decay functions. For exponential growth, the value inside the parentheses will be greater than 1 because r is added to 1. For exponential decay, the value inside the parentheses will be less than 1 because r is subtracted from 1.

Example 2: The fish population in a local stream is decreasing at a rate of 3% per year. The original population was 48,000. Write an exponential decay function to model this situation. Then find the population after 7 years.

Step 1: Write the exponential decay function for this situation.

$$y = a\,(1 - r)^t$$

$$= 48{,}000\,(1 - 0.03)^t$$

$$= 48{,}000(0.97)^t$$

Step 2: Find the population in 7 years.

$$y = 48{,}000(0.97)^7$$

$\approx$ 38,783 fishes after 7 years.

Write an exponential function to model each situation. Find each amount after the specified time

SELF CHECK

1. A city of 2,950,000 people has a 2.5% annual decrease in population. Determine the population after 5 years.
2. A used car was purchased for $12,329 this year. Each year the car's value decreases 8.5%. What will the car be worth in ten years?
3. A certain type of bacterium, given a favorable growth medium, doubles in population every 6.5 hours. Given that there were approximately 100 bacteria to start with, how many bacteria will there be in a day and a half?
4. Amy bought a diamond ring for $6,000. If the value of the ring increases at a constant rate of 3.83% per year, how much will the ring be worth in twenty-one years?
5. A population of 90 frogs increases at an annual rate of 22%. How many frogs will there be in 5 years?

1. A student was asked to find the value of a $2500 item after 4 years. The item was depreciating at a rate of 20% per year. What is wrong with the student's work? $2500(0.2)^4$

2. The value of a certain car can be modeled by the function $y = 18000(0.76)^t$, where *t* is time in years. Will the value of the function ever by 0?

Exponential Half-life

A common application of exponential decay is *half-life.* The <u>half-life</u> of a substance is the time it takes for one-half of the substance to decay into another substance.

Half-life

$A = P(0.5)^t$

A represents the final amount.

P represents the original amount.

t represents the number of half-lives in a given time period.

Example 3: Astatine-218 has a half-life of 2 seconds. Find the amount left from a 500 gram sample of astatine-218 after 10 seconds.

Step 1: Find *t*, the number of half-lives in the given time period.

$\frac{10s}{2s}$ = 5 Divide the time period by the half-life. The value of t is 5.

Step 2: A = P $(0.5)^t$
= 500 $(0.5)^5$
= 15.625 There are 15.625 grams of Astatine-218 remaining after 10 seconds.

Example 4: Cesium-137 has a half-life of 30 years. Find the amount of cesium-137 left from a 100 milligram sample after 180 years.

Step 1: $\frac{180\ years}{30\ years}$ = 6 value of t

Step 2: A = 100 $(0.5)^6$

= 1.5626 There are 1.5625 milligrams of Cesium-137 remaining after 180 years

SELF CHECK Write and solve an exponential function to model the half-life.

1. Iodine-131 has a half-life of about 8 days. Find the amount left from a 30 gram sample of iodine-131 after 40 days.

2. Bismuth-210 has a half-life of 5 days. Find the amount of bismuth-210 left from a 100 gram sample after 5 weeks (Hint: Change 5 weeks to days).

3. Actinium-226 has a half-life of 29 hours. If 100mg of actinium-226 disintegrates over a period of 58 hours, how many mg of actinium-226 will remain?

Example! Compound interest

A common application of exponential growth is *compound interest.* Recall that simple interest is earned or paid only on the principal. Compound interest is interest earned or paid on *both* the principal and previously earned interest.

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A represents the balance after t years.

P represents the principal, or original amount.

r represents the annual interest rate expressed as a decimal.

n represents the number of times interest is compounded per year.

t represents time in years.

Reading Math

For compound interest

- *annually* means "once per year" ($n = 1$).
- *quarterly* means "4 times per year" ($n = 4$).
- *monthly* means "12 times per year" ($n = 12$).

Example 5: Write a compound interest function to model each situation. Then find the balance after the given number of years. $1200 invested at a rate of 2% compounded quarterly; 3 yrs.

Step 1: Write the compound interest function for this situation.

$$A = P\left[1 + \frac{r}{n}\right]^{nt}$$
$$= 1200\left[1 + \frac{0.02}{4}\right]^{4t}$$
$$= 1200[1.005]^{4t}$$

Step 2: Find the balance after 3 years.

$$A = 1200[1.005]^{12}$$
$$\approx 1274.01$$ The balance after 3 years.

SELF CHECK Write and solve an exponential function to model the compound interest

1. $15,000 invested at a rate of 4.8% compounded monthly; 2 years.
2. $4000 invested at a rate of 3% compounded annually; 8 years
3. Find the value of $2,000 invested in a C.D. that earned an annual percentage rate of 4.25%, compounded quarterly, for 5 years.
4. Find the value of $8,000 invested in a savings account that earned 2% annual interest if compounded semi-annually (twice a year), for 9 years.

Over a long period of time, does the initial deposit or the interest rate have a greater effect on the amount of money in an account that has interest compounded yearly? Explain your reasoning.

Re-Teaching and Independent Practice

Example!

Writing Exponential Functions from Context:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- **A** represents the amount of money after a certain amount of time
- **P** represents the principle or the amount of money you start with
- **r** represents the interest rate and is always represented as a decimal
- **t** represents the amount of time in years
- **n** is the number of times interest is compounded in one year, for example:

if interest is compounded annually then n = 1
if interest is compounded quarterly then n = 4
if interest is compounded monthly then n = 12

Suppose Karen has $1000 that she invests in an account that pays 3.5% interest compounded quarterly. How much money does Karen have at the end of 5 years?
Let's look at our formula and see how many values for the variables we are given in the problem.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

The $1000 is the amount being invested or **P**. The interest rate is 3.5% which must be changed into a decimal and becomes r = 0.035. The interest is compounded quarterly, or four times per years, which tells us that n = 4. The money will stay in the account for 5 years so t = 5. We have values for four of the variables. We can use this information to solve for A.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 1000\left(1 + \frac{.035}{4}\right)^{(4)(5)}$$

$$A = 1190.34$$

Write the exponential equation:

1. A population of mice has a growth factor of 3. After 1 month, there are 36 mice. After 2 months, there are 108 mice.	2. Maya's grandfather opened a savings account for her when she was born. He opened the account with $100 and did not add or take out any money after that. The money in the account grows at a rate of 4% per year.

3. You deposit \$1600 in a bank account. Find the balance after 3 years, account pays 2.5% annual interest compounded monthly.	4. The amount of money, *A*, accrued at the end of *n* years when a certain amount, *P*, is invested at a compound annual rate, *r*, is given by $A = P(1 + r)^n$. If a person invests \$250 in an account that pays 10% interest compounded annually, find the balance after 15 years.
5. Shawn Carter bought a Basquiat's "Charles The First" painting in 2002 for 4 million dollars. The painting appreciated 7.5% each year. What is the current value for the painting in 2019?	6. Find the value of \$1000 deposited for 8 years in an account paying 8% annual interest compounded semi-annually.
7. How much money must be deposited now in an account paying 8% annual interest, compounded quarterly, to have a balance of \$1000 after 10 years?	8. The foundation of your house has about 1,200 termites. The termites grow at a rate of about 2.4% per day. How long until the number of termites doubles?
9. The value of a textbook is \$120 and decreases at a rate of 12% per year. Write a function to model the situation, and then find the value of the textbook after 9 years.	10. The population of a town is 4200 and increasing at a rate of 3% per year. What is the population in 7 years?
11. In 2010, the population of Warner Robins is 18,548 people and decreases by 2.5% each year. What will the population be in 2025?	12. Kelly plans to put her graduation money into an account and leave it there for 4 years while she goes to college. She receives \$2550 in graduation money that she puts it into an account that earns 4.25% interest compounded quarterly. How much will be in Kelly's account at the end of four years?

Real Mathematics:
Application

PENNY- TOSS EXPERIMENT

Performance Task: Penny- Toss Experiment

Name________________________________ **Date**_________________

Step 1: In your group place all 50 pennies in your cup.

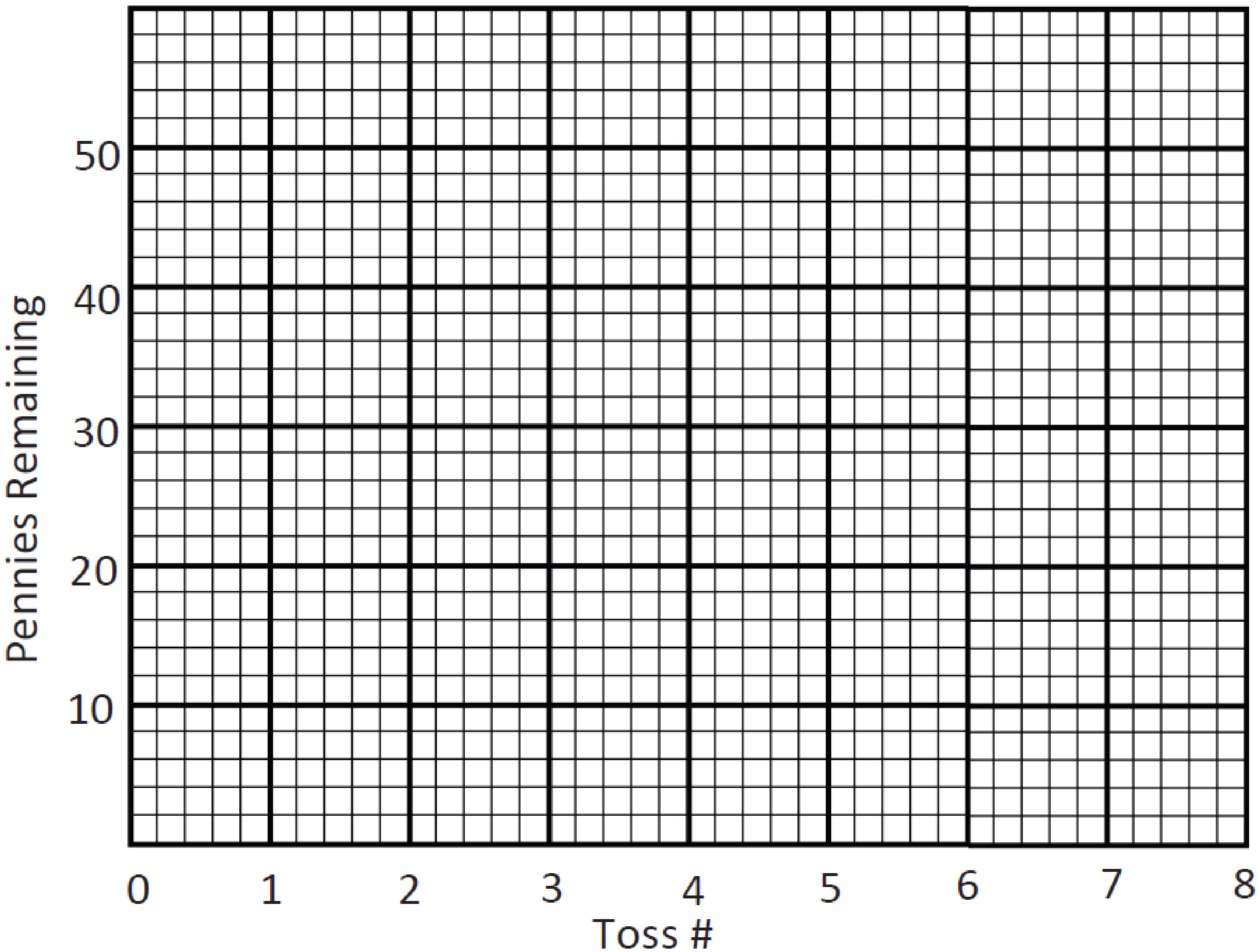
Step 2: Shake them around really well and pour them out onto the table.

Step 3: Remove all pennies that were tails and set them aside, count and record the number of remaining pennies in the table below and place them back in your cup.

Step 4: Repeat steps 2 and 3 until no pennies remain.

Toss #	Pennies Remaining
0	50
1	
2	
3	
4	
5	
6	
7	
8	

Step 5: Plot your experimental results below.



HOW LONG DOES IT TAKE?

How Long Does It Take? (Constructing Task)

Name________________________________ Date___________________

Before sending astronauts to investigate the new planet of Exponential, NASA decided to run a number of tests on the astronauts.

1. A specific multi-vitamin is eliminated from an adult male's bloodstream at a rate of about 20% per hour. The vitamin reaches peak level in the bloodstream of 300 milligrams.

- How much of the vitamin remains 2 hours after the peak level? 5 hours after the peak level? Make a table of values to record your answers. Write expressions for how you obtain your answers.

a. Using your work from (a), write expressions for each computed value using the original numbers in the problem (300 and 20%). For example, after 2 hours, the amount of vitamin left is [300 * (1 - .2)]* (1 - .2).

b. Using part (b), write a function for the vitamin level with respect to the number of hours after the peak level, *x*.

c. How would use the function you wrote in (c) to answer the questions in (a)? Use your function and check to see that you get the same answers as you did in part (a).

d. After how many hours will there be less than 10 mg of the vitamin remaining in the bloodstream? Explain how you would determine this answer using both a table feature and a graph.

e. Write an equation that you could solve to determine when the vitamin concentration is exactly 10 mg. Could you use the table feature to solve this equation? Could you use the graph feature? How could you use the intersection feature of your calculator? Solve the problem using one of these methods.

f. How would you solve the equation you wrote in (f) algebraically? What is the first step?

2. **Extension** - A can of Instant Energy, a 16-ounce energy drink, contains 80 mg of caffeine. Suppose the caffeine in the bloodstream peaks at 80 mg. If ½ of the caffeine has been eliminated from the bloodstream after 5 hours (the half-life of caffeine in the bloodstream is 5 hours), complete the following:

a. How much caffeine will remain in the bloodstream after 5 hours? 10 hours? 1 hour? 2 hours? Make a table to organize your answers. Explain how you came up with your answers. (Make a conjecture. You can return to your answers later to make any corrections.).

HOMEWORK

Homework #___1_____ Writing Exponential Functions from Context:	
1. The population of a school is 800 students and is increasing at a rate of 3.5 % per year. What will the population of the school in 6 years?	2. The value of a company's equipment is $25,000 and decreases at a rate of 15% per year. What will the value of the equipment be in 8 years?
3. In 1978, Kendal invested $20,000 at a percentage of 5.2% compounded semi-annually. What is the value of investment after 30 years?	4. Jaylan bought a Dodge Hellcat for $84,200. He finance the car loan for 4 years at 3.25% compounded annually. What will Jaylan have paid for your car after 4 years?
5. If, at the end of two years, a savings account has a balance of $1,172.75. The interest rate 6% is compounded monthly at 3 years.	6. An initial deposit of $5,000 is made into a savings account that compounds 7.1% interest annually. How much is in the account at the end of five years?
7. An initial population of 750 endangered turtles triples each year. What will the population of turtles be after 5 years?	8. A house is purchased for $150,000 in 2002. The value of the house depreciates at a rate of 7%. How much is the house worth in 2013?
9. The number of birds in a forest is decreasing by 3% every year. Originally there were 5,400 birds. How many birds will there be in 9 years? Round your answer to the nearest whole number.	10. The population of a city is increasing at a rate of 4% each year. In 2000, there were 236,000 people in the city. Write an exponential growth function to model this situation. Then find the population in 2009.

Exponential Functions is an equation involving exponential functions of a variable. For the exponential equation $y = ab^x$, what will the common ratio be between output values for consecutive integer input values?

Exponential functions are also represented by coefficient, base and exponent.

Coefficient is the initial value. Base is the rate of exponential function. Exponent is the power or index of the base. $y = ab^x$

Exponential function is simply a function, you can transform the parent graph of an exponential function in the same way as any other function:

Remember that exponential functions are named that because of the "x" in their exponents!

Exponential functions are written $y=ab^x$, $b>0$. "*b*" is called the base of the exponential function, since it's the number that is multiplied by itself "x" times (and it's not an exponential function when $b=1$). *b* is also called the "growth" or "decay" factor.

Exponential Transformation & Shifts:

Example! Exponential Transformation

Exponential transformation: The equation $\boldsymbol{f(x) = (a)b^{x-h} + k}$ is the translation function that helps us understand how changing values impacts the resulting graph.

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h tells us about horizontal movement.

If **h is positive**.... Shift left by **(h)** units If **h is negative**... Shift right by **(h)** units

---

a tells us about stretching, reflecting, and compressing.

- If **a < 0**... reflects over the x-axis
- If **a > 1**... stretches by a scale factor of (a)
- If **0 < a < 1**... compressing by scale factor of (a)

---

k tells us about vertical movement.

If **k is positive**…shift up by **(k)** units	If **k is negative**…shift down by **(k)** units

Example 1: Identify the transformation of the graph from the parent function $f(x)= a(b)^x$

$f(x) = 4(5)^{x+3} - 4$

Step 1: Identify the values: $a = 4$, $h = -3$; $k = -4$

Step 2: Identify the transformation:

- the graph stretch by a scale factor of 4
- the graph shift left 4 units
- the graph shift down 4 units

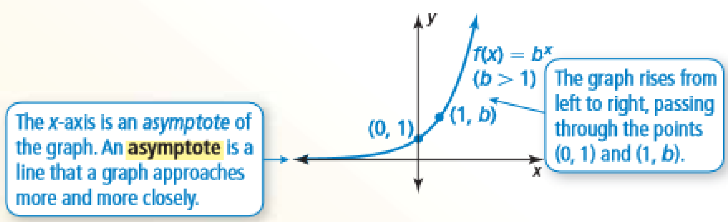
SELF CHECK Compare & identify the transformation of each exponential equation: $f(x)= 4^x$

1. $f(x) = 4^x - 5$
2. $f(x)= 4^{x+3}$
3. $f(x) = -2(4)^x+3$

<u>Exponential growth & decay Function:</u> An exponential function has the form $y = ab^x$, where $a \neq 0$ and the base b is a positive real number other than 1. If $a > 0$ and $b > 1$, then $y = ab^x$ is an exponential growth function, and b is called the growth factor. The simplest type of exponential growth function has the form $y = b^x$.

Parent Function for Exponential Growth Functions

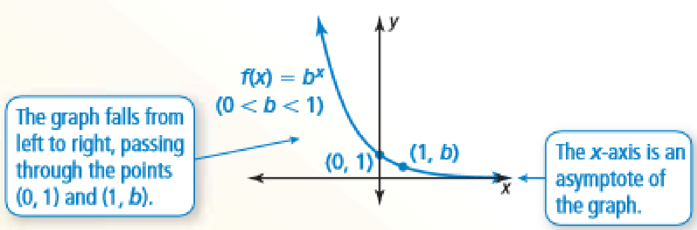
The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base b. The graph shows the general shape of an exponential growth function.



The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The graph shows the general shape of an exponential decay function.



The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Notice: Asymptote is a line that a graph does not cross. (asymptote is the k value)

Example 2: Determine whether growth or decay. Identify the Domain, Range & Asymptotes

$f(x) = 4^x - 8$

Step 1: Find the b value and constant (k)

- the equation is a growth because b>1; (b = 4; 4>1)

Step 2: Identify the domain, range & asymptote

- Domain is all real numbers

- Range is $y > -8$

SELF CHECK

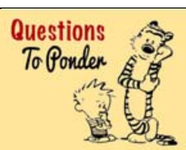
Determine whether the exponential function is growth or decay. Identify the domain, range & asymptotes.

1. $f(x) = -3(5)^x + 2$

2. $f(x) = 2\left(\frac{2}{3}\right)^x - 4$

3. $g(x) = 7(3)^x$

4. $h(x) = 8(.5)^x$



How can you determine the characteristic of a function without graphing or creating a table? Domain, Range, Intercepts, Max. or Min.

For each of the following exponential functions, identify whether it is a growth or decay, and the transformations.

Exponential Function Growth or Decay Transformations?

1. $y = \left(\frac{2}{3}\right)^{x} + 2$

2. $y = (2)^{x+2} + 5$

3. $y = -(0.83)^{x-1} - 1$

4. $y = -5(2)^{x+3} - 9$

5. $y = -4^{x} - 6$

Write the function for each graph described below.

6. the graph of $f(x) = 2^x$, reflected across the x axis. ________________________________

7. the graph of $f(x) = 1/3^x$, translated up 5 units.____________________________________

8. the graph of $f(x) = 3^x$, vertically stretched by 5, left 2, and down 3.__________________

9. the graph of $f(x) = 1/2^x$, stretched horizontally by a factor of 3, shifted up 4. ______________

10. the graph of $f(x) = 2^x$, right 3, reflected over x, and down 5.

State the domain, range, and asymptote for each function.

11. $f(x) = 3^x + 1$

D:

R:

A:

12. $f(x) = 1/2^{x-1}$

D:

R:

A:

13. $f(x) = -2^x - 3$

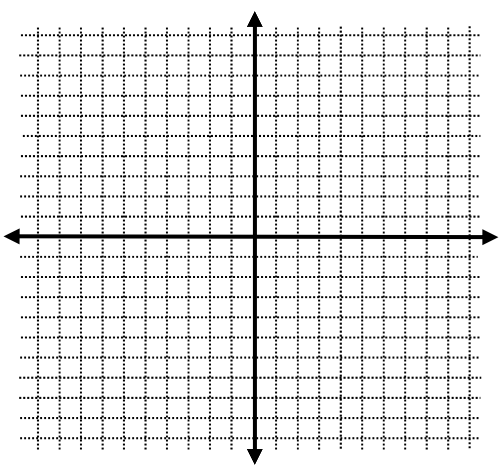
D:

R:

A:

Graph the equation

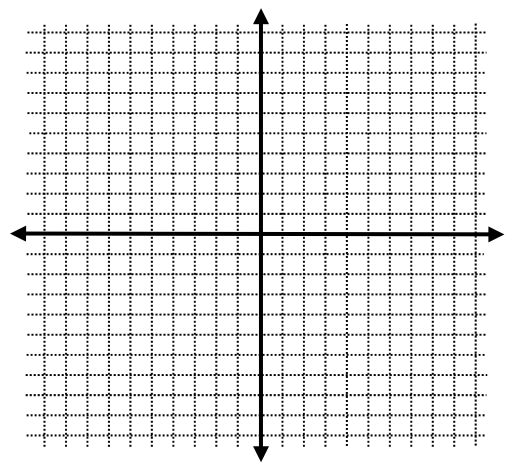
14. $f(x) = 2^x + 3$



x	-2	-1	0	1	2	3
y						

Describe the transformation

15. $f(x) = 2^{x+3}$



x	-2	-1	0	1	2	3
y						

Describe the transformation

Investigating Graphs of Exponential Functions

INVESTIGATING GRAPHS OF EXPONENTIAL EQUATIONS Name___________

1. Graph the following equation on graphing paper using your TABLE feature of your calculator.

   $y = 2^x$

   a. This graph is called exponential growth. How can you describe its shape?

   b. What is the lowest y value you see? Draw a horizontal line at y = 0. This line is called an asymptote. Your graph will get closer and closer to it, but never reach it.

2. Graph the following equation on graphing paper using your calculator.

   $y = -2^x$

   a. What happened to your graph? This is called a **reflection**.

   b. Do you have an asymptote now? Explain.

3. Graph each of the following equations using your calculator. Describe how the graph compares to the graph of $y = 2^x$ for each.

   a. $y = 2^x + 3$

   b. $y = 2^x - 2$

   ***The graphs above have **vertical translations**. Explain what that means.

   c. $y = 2^{x+2}$

   d. $y = 2^{x-2}$

   ***The graphs above have **horizontal translations**. Explain what that means.

4. Build a table of values and make a graph for each of these equations.

a) $y = \mathbf{2^{x-1} + 2}$

Where is the asymptote? Write the equation.

Describe the translations.

x	y
0	
1	
2	
3	
4	

b) $y = \mathbf{2^{x+1} - 2}$

Where is the asymptote? Write the equation.

Describe the translations.

x	y
-4	
-3	
-2	
-1	
0	

For the following functions, determine if they are linear or exponential, and the transformations. Growth/decay for exponent functions.

1. $y = 23(4)^x + 1$ Linear or Exponential If exponential, growth or decay?
Transformations?

2. $y = -7(0.83)^{x-1} + 3$ Linear or Exponential If exponential, growth or decay?
Transformations?

3. $y = ½ (1/5)^{x-5} - 1$ Linear or Exponential If exponential, growth or decay?
Transformations?

4. $f(x) = -3(x+2) – 4$ Linear or Exponential If exponential, growth or decay?
Transformations?

5. $f(x) = -8x + 23$ Linear or Exponential If exponential, growth or decay?
Transformations?

Complete the chart

Transformation	**Equation**	**Description**
_________________ _________________	$f(x) = b^{x+k}$	- Shifts the graph $f(x) = b^x$ to the left k units if k>0 - Shifts the graphs $f(x) = b^x$ to the right k units if c<0
Vertical Stretching or Shrinking	______________	- Stretches the graph of $f(x) = b^x$ if k>1 - Shrinks the graph of $f(x) = b^x$ if 0< k <1
Reflecting	$f(x) = - b^x$ $f(x) = b^{-x}$	-___________________________ -___________________________
___________________ __________________	$f(x) = b^x + k$	- Shifts the graph of $f(x) = b^x$ upward k units if k>0 - Shifts the graph $f(x) = b^x$ downward k units if k<0

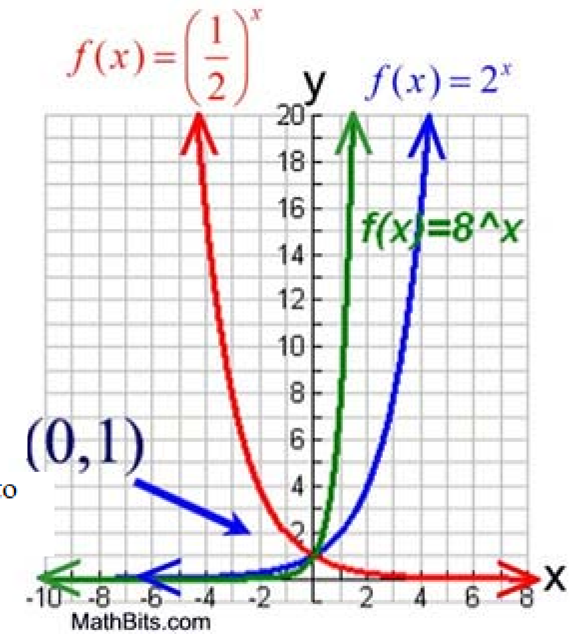
Key Characteristics of Exponential Functions

GUIDED NOTES

Characteristics of Exponential Functions

The graphs of functions of the form $y = b^x$ have certain characteristics in common.

- graph crosses the y-axis at $(0,1)$
- when $b > 1$, the graph increases
- when $0 < b < 1$, the graph decreases
- the domain is all real numbers
- the range is all positive real numbers (never zero)
- graph passes the vertical line test for functions
- graph is **asymptotic** to the x-axis - gets very, very close to the x-axis but, in this case, does not touch it or cross it.



The Intercepts of Exponentials

By examining the nature of the exponential graph, we have seen that the parent function will stay above the x-axis, unless acted upon by a transformation.

• The **parent function**, $y = b^x$, will always have a ***y*-intercept of one**, occurring at the ordered pair of (0,1). Algebraically speaking, when $x = 0$, we have $y = b^0$ which is always equal to 1.

There is **no *x*-intercept** with the parent function since it is asymptotic to the x-axis (approaches the x-axis but does not touch or cross it).

• The **transformed parent function of the form *y* = *ab*ˣ,** will always have a ***y*-intercept of *a***, occurring at the ordered pair of $(0, a)$. Again, algebraically speaking, when $x = 0$, we have $y = ab^0$ which is always equal to $a \cdot 1$ or a. Note that the value of a may be positive or negative.

Like the parent function, this transformation will be asymptotic to the x-axis, and will have **no *x*-intercept.**

• If the **transformed parent function includes a vertical or horizontal shift**, all bets are off. The horizontal shift will affect the *y*-intercept and the vertical shift will affect the possibility of an *x*-intercept. In this situation, you will need to examine the graph carefully to determine what is happening.

The End Behavior of Exponentials

The end behavior of an exponential graph also depends upon whether you are dealing with the parent function or with one of its transformations.

• The **end behavior of the parent function** is consistent.

- if $b > 1$ (*increasing* function), the left side of the graph approaches a *y*-value of 0, and the right side approaches positive infinity.

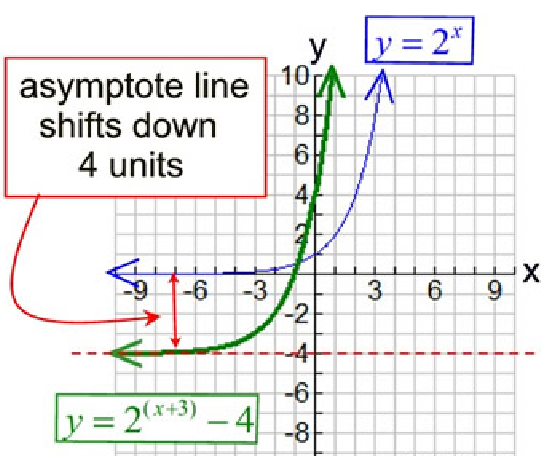
when $b > 1$:
as $x \to \infty, f(x) \to \infty$
as $x \to -\infty, f(x) \to 0$

- if $0 < b < 1$ (*decreasing* function), the right side of the graph approaches a *y*-value of 0, and the left side approaches positive infinity.

when $0 < b < 1$:
as $x \to \infty, f(x) \to 0$
as $x \to -\infty, f(x) \to \infty$

• The **end behavior of a transformed parent function** is not always consistent, but is dependent upon the nature of the transformation.

Example! Consider the following example.



For the transformed equation $y = 2^{(x+3)} - 4$, the vertical shift of -4 will push the asymptote line down four units. Thus the **end behavior** will be:

when $k = -4$:
as $x \to \infty, f(x) \to \infty$
as $x \to -\infty, f(x) \to -4$

The ***y*-intercept**, where *x* = 0, is 4.
$y = 2^{(0+3)} - 4 = 8 - 4 = 4$

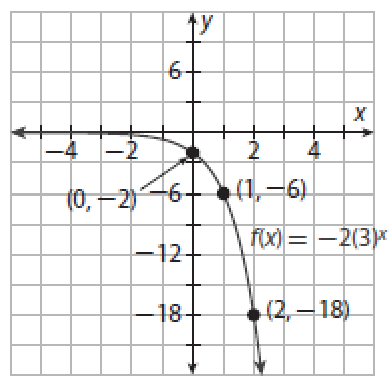
The ***x*-intercept,** where *y* = 0, is -1.
$0 = 2^{(x+3)} - 4$
$4 = 2^{(x+3)}$
$2^2 = 2^{(x+3)}$
$x = -1$

Example 2: Graph each exponential function. After graphing, identify *a* and *b*, the *y*-intercept, and the end behavior of the graph. Use end behavior to discuss the behavior of the graph.

A. $f(x) = -2(3)^x$

Choose several values of *x* and generate ordered pairs.

x	$f(x) = -2(3)^x$
−1	−0.7
0	−2
1	−6
2	−18



Graph the ordered pairs and connect them with a smooth curve.

$a = -2$

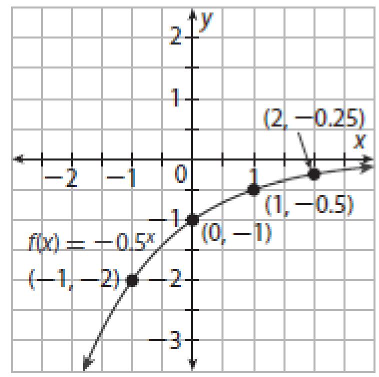
$b = 3$

y-intercept: (0, -2)

End Behavior: As $x \to \infty$, $y \to -\infty$ and as $x \to -\infty$, $y \to 0$.

B. $f(x) = -0.5^x$ Choose several values of *x* and generate ordered pairs.

x	$f(x) = -0.5^x$
−1	−2
0	−1
1	−0.5
2	−0.25



Graph the ordered pairs and connect them with a smooth curve.

a = -1

b = 0.5

y-intercept: (0, -1)

End Behavior: As $x \to \infty$, $y \to 0$ and as $x \to -\infty$, $y \to -\infty$.

SELF CHECK

Now you try:

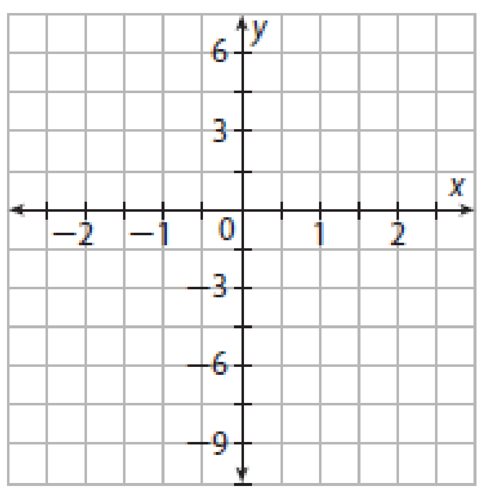
1. $f(x) = -3\,(0.4)^x$

Choose several values of *x* and generate ordered pairs. Graph the ordered pairs and connect them with a smooth curve.

x	$f(x) = -3(0.4)^x$
−1	
0	
1	
2	

a = *b* = *y*-intercept: (,)

End Behavior: As $x \to \infty$, $y \to$ _________ and as $x \to -\infty$, $y \to$ _________ .



State a, b, and the y-intercept then graph the function and describe the end behavior of the graphs.

2. $f(x) = 3\,(3)^x$

3. $f(x) = 5\,(0.6)^x$

4. $f(x) = -6(0.7)^x$

5. $f(x) = -4(3)^x$

1. Assume that the domain of the function $f(x) = 3(2)^x$ is the set of all real numbers. What is the range of the function? What are the end behavoriors?

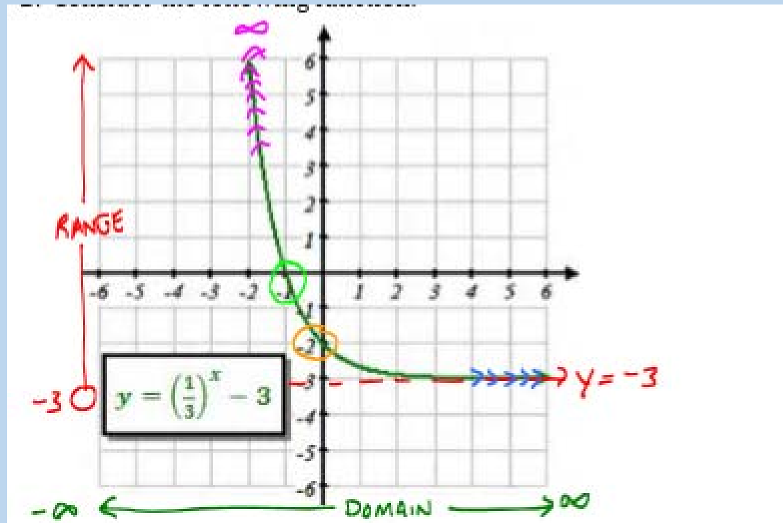
2. Using the graph of an exponential function, how can *b* be found?
3. Respond to each statement by answering sometimes, always, or never.
a. The graph of $y = ab^x$ is a straight line.
b. The *y*-intercept of $y = ab^x$ is at *a*.
c. When *a* is negative, the graph of $y = ab^x$ is contained entirely in quadrants III and IV.
d. When *b* is a fraction, the graph of $y = ab^x$ is contained entirely in quadrants I and II.

Re-Teaching and Independent Practice

Example!

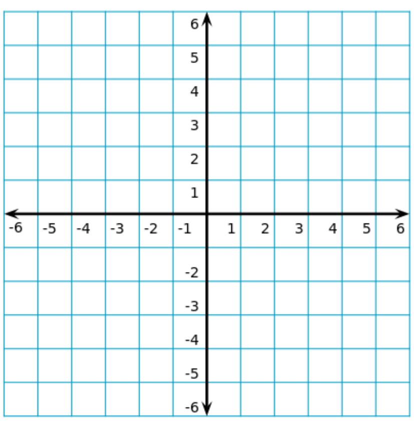
Graphing and interpret key features of exponential functions:

- **Domain: all reall numbers**
- **Range: y> -3**
- **X-Intercepts: (-1,0)**
- **Y-Intercepts: (0,-2)**
- **Asymptotes: y = -3**
- **End Behavioral: as x→∞, f(x) →-3**
  As x→- ∞, f(x) →∞
- **Interval of decrease:(- ∞,∞)**
- **Interval of increase: n/a**



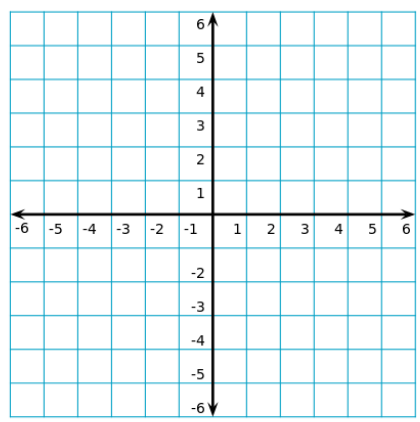
Graph the exponential function & Identify characteristics:

1. $f(x) = 3^x + 4$



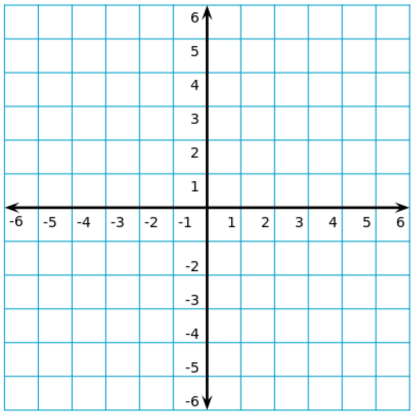
Domain:
Range:
Intercepts:
End Behavior:
Asymptote:
Interval of Increase/Decrease:

2. $f(x) = 2^{x+1} – 3$



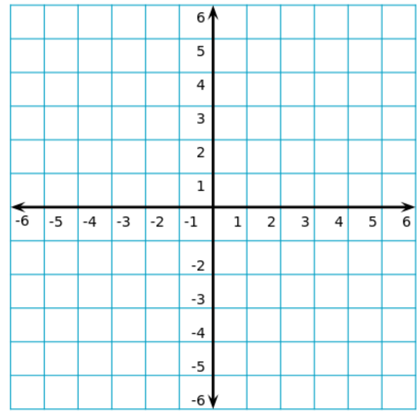
Domain:
Range:
Intercepts:
End Behavior:
Asymptote:
Interval of Increase/Decrease:

3. $f(x) = -4(0.5)^x$



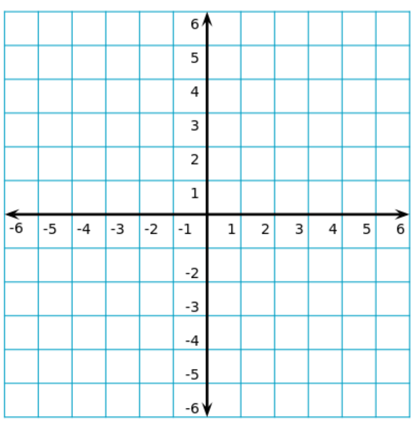
Domain:
Range:
Intercepts:
End Behavior:
Asymptote:
Interval of Increase/Decrease:

4. $f(x) = 3^{x+4} - 2$



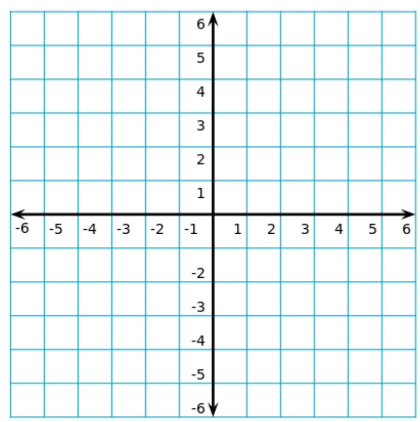
Domain:
Range:
Intercepts:
End Behavior:
Asymptote:
Interval of Increase/Decrease:

5. $f(x) = -0.5(2)^{x-2} + 2$



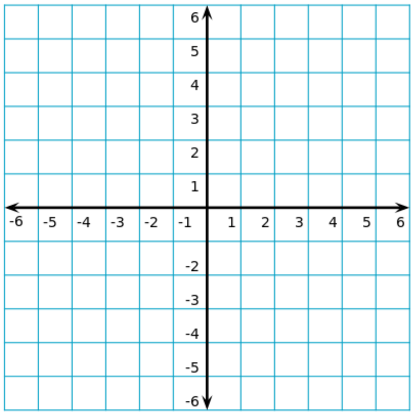
Domain:
Range:
Intercepts:
End Behavior:
Asymptote:
Interval of Increase/Decrease”

6. $f(x) = 2(0.5)^{x+3} - 3$



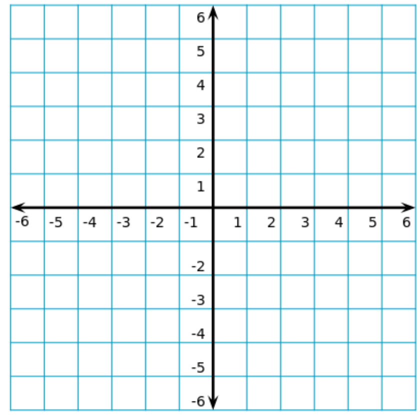
Domain:
Range:
Intercepts:
End Behavior:
Asymptote:
Interval of Increase/Decrease:

7. $f(x) = -(4)^x$



Domain:
Range:
Intercepts:
End Behavior:
Asymptote:
Interval of Increase/Decrease:

8. $f(x) = -2(2)^x + 1$



Domain:
Range:
Intercepts:
End Behavior:
Asymptote:
Interval of Increase/Decrease:

Compare / Contrast: Exponential Functions

Show similarities and differences between the two exponent functions: What things are being compared? How are they similar? How are they different?

$f(x) = (1/2)^x + 3$ $f(x) = 2^x + 3$

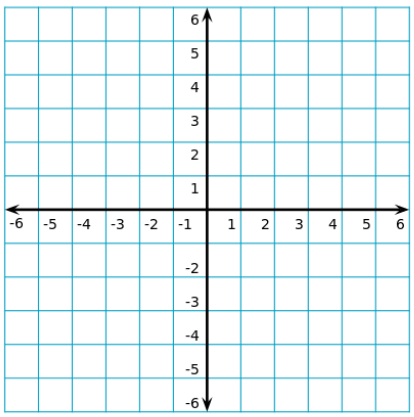
Attribute	Function 1	Function 2
Rate of change		
Domain & Range		
Intercepts		
Asymptotes		
End Behavior		

HOMEWORK

Homework #___1_____

Graphing and Interpreting key features of exponential functions:

1. $f(x) = (0.5)^x + 3$



Domain___________ Range_____________

Asymptote___________

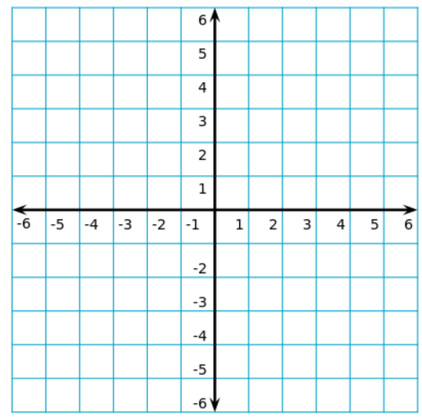
Increasing or Decreasing (circle one): ______________

X-intercept __________ Y-intercept ___________

End Behavior: x→ ______, f(x) → ______

x→ ______, f(x) → ______

2. $f(x) = -5^x$



Domain___________ Range_____________

Asymptote___________

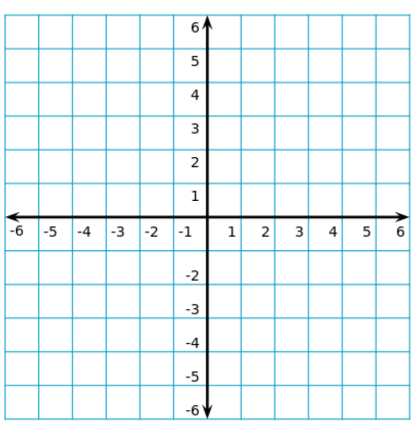
Increasing or Decreasing (circle one): ____________

X-intercept __________ Y-intercept ___________

End Behavior: x→ ______, f(x) → ______

x→ ______, f(x) → ______

3. $f(x) = -(3)^{x+1} - 2$



Domain___________ Range_____________

Asymptote___________

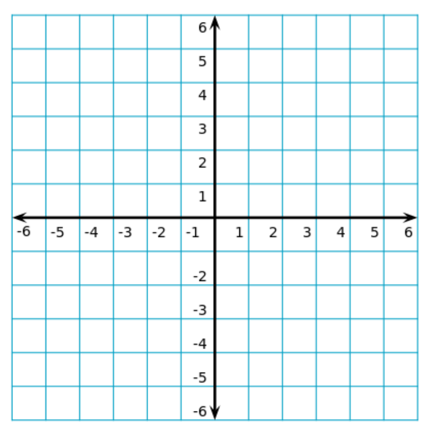
Increasing or Decreasing (circle one): ____________

X-intercept __________ Y-intercept ___________

End Behavior: x→ ______, f(x) → ______

x→ ______, f(x) → ______

4. $f(x) = -5^x + 2$



Domain___________ Range_____________

Asymptote___________

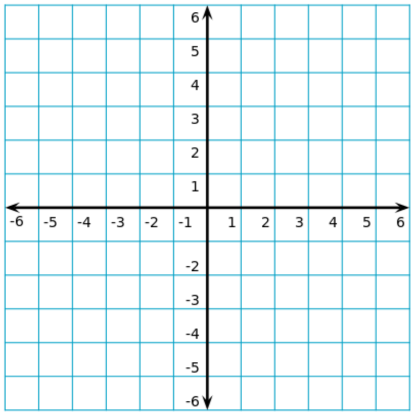
Increasing or Decreasing (circle one): ____________

X-intercept __________ Y-intercept ___________

End Behavior: x→ ______, f(x) → ______

x→ ______, f(x) → ______

5. $f(x) = 3^x$



Domain____________ Range______________

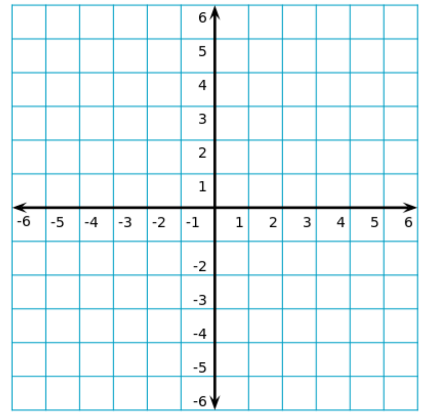
Asymptote____________

Increasing or Decreasing (circle one): _____________

X-intercept ___________ Y-intercept ____________

End Behavior: x→ _______, f(x) → _______
x→ _______, f(x) → _______

6. $f(x) = 2(.25)^x$



Domain____________ Range______________

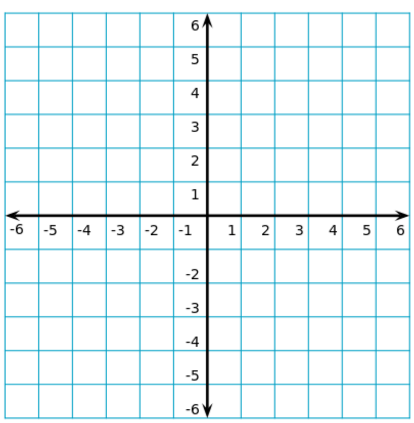
Asymptote____________

Increasing or Decreasing (circle one): _____________

X-intercept ___________ Y-intercept ____________

End Behavior: x→ _______, f(x) → _______
x→ _______, f(x) → _______

7. $f(x) = .75(2)^x$



Domain____________ Range______________

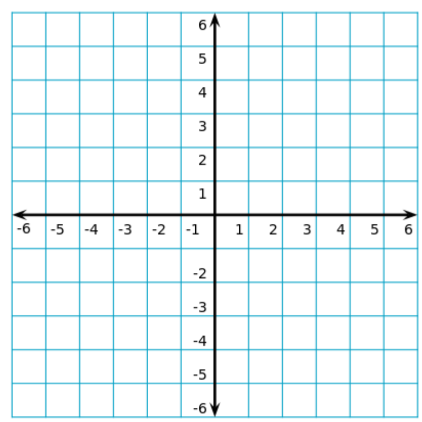
Asymptote____________

Increasing or Decreasing (circle one): _____________

X-intercept ___________ Y-intercept ____________

End Behavior: x→ _______, f(x) → _______
x→ _______, f(x) → _______

8. $f(x) = 2(5)^x$



Domain____________ Range______________

Asymptote____________

Increasing or Decreasing (circle one): _____________

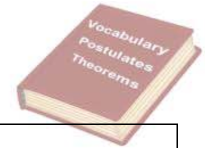
X-intercept ___________ Y-intercept ____________

End Behavior: x→ _______, f(x) → _______
x→ _______, f(x) → _______

A1.U3.**Vocabulary**

Term	Definition	Notation	Diagram/Visual
Average Rate of Change			
Continuous			
Discrete			
Domain			
End Behaviors			
Explicit Expression			
Exponential Function			
Exponential Model			

A1.U3.Vocabulary



Geometric Sequence			
Horizontal Translation			
Interval Notation			
Ordered Pair			
Parameter			
Range			
Recursive Formula			
Reflection			
Vertical Translation			
X-Intercept			

Y-Intercept			

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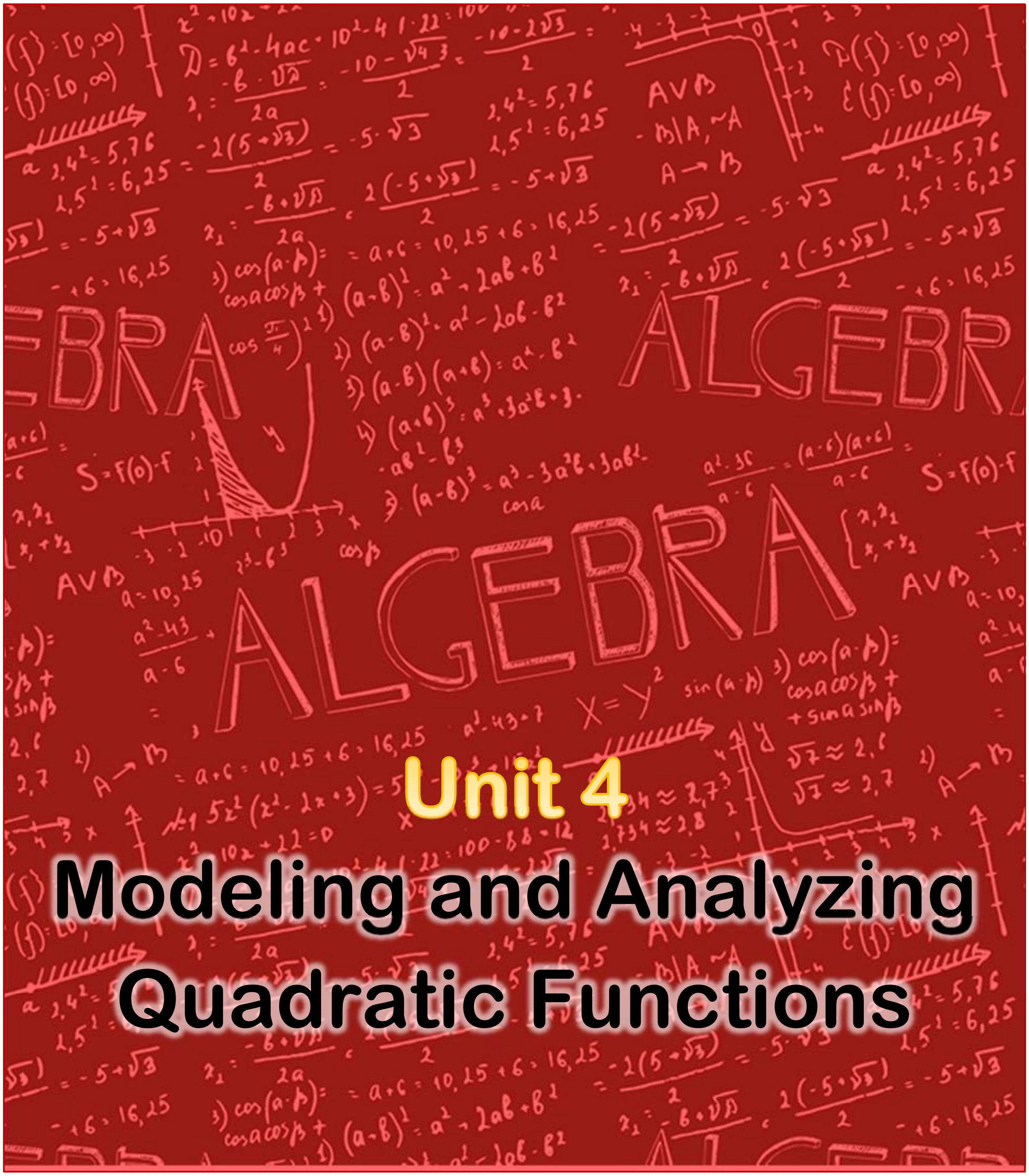


Unit 4

Modeling and Analyzing Quadratic Functions

Algebra 1

Unit 4: Modeling and Analyzing Quadratic Functions

Concept 1: Analyzing Quadratic Functions through Graphing

Lesson A: Quadratic Graph Features	(A1.U4.C1.A.____.QuadraticGraphFeatures)
Lesson B: Quadratic Transformations	(A1.U4.C1.B.____.QuadraticTransformations)
Lesson C: Graphing Quadratic Functions	(A1.U4.C1.C.____.QGraphing)
Lesson D: Converting Between Standard, Factored, and Vertex Forms	(A1.U4.C1.D.____.ConvertingForms)
Lesson E: Real-World Situations That Are Quadratic	(A1.U4.C1.E.____.QuadraticApplications)

Concept 2: Using Factors to Solve Quadratic Equations

Lesson F: Factoring	(A1.U4.C2.F.____.QFactoring)
Lesson G: Factoring to Solve Quadratic Equations	(A1.U4.C2.G.____.FactorToSolveQ)
Lesson H: Special Trinomials	(A1.U4.C2.H.____.SpecialTrinomials)

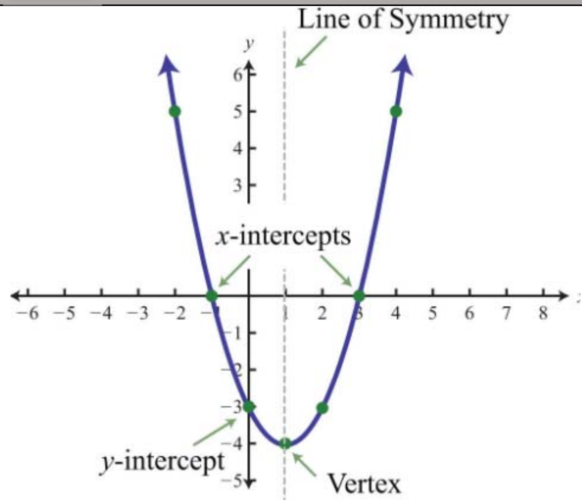
Concept 3: Using Other Methods to Solve Quadratic Equations

Lesson I: Choosing Method (Graph, Complete the Square, Factor, Quadratic Formula) to Solve Quadratic Equations	(A1.U4.C1.I.____.Solve4Methods)

Term	Definition	Notation	Diagram/Visual
Complete Factorization over the Integers			
Completing the Square			
Difference between two squares			
Discriminant of the Quadratic Equation			
Horizontal shift			
Perfect Square Trinomial			
Quadratic Equation			
Quadratic Function			

Root			
Standard form of a Quadratic Function			
Vertex			
Vertex form			
Zero Property			

Graphs of Quadratic Functions – Key features



The graph of a quadratic function is a U-shaped curve called a **parabola**.

One important feature of the graph is that it has an extreme point, called the **vertex**.

If the parabola opens up, the vertex represents the **lowest point** on the graph, or the **minimum value** of the quadratic function.

If the parabola opens down, the vertex represents the **highest point** on the graph, or the **maximum value**.

In either case, the vertex is a **turning point** on the graph. The graph is also **symmetric** with a **vertical** line drawn through the vertex, called the **axis of symmetry**.

The axis of symmetry is written as an equation of a vertical line, x = _____ (the x value of the vertex).

The ***y*-intercept** is the point at which the parabola crosses the *y*-axis.

The ***x*-intercepts** are the point(s) at which the parabola crosses the *x*-axis.

If they exist, the *x*-intercepts represent the **solutions**, **zeros,** or **roots**, of the quadratic function. The zeros or roots can be found when *y* = 0.

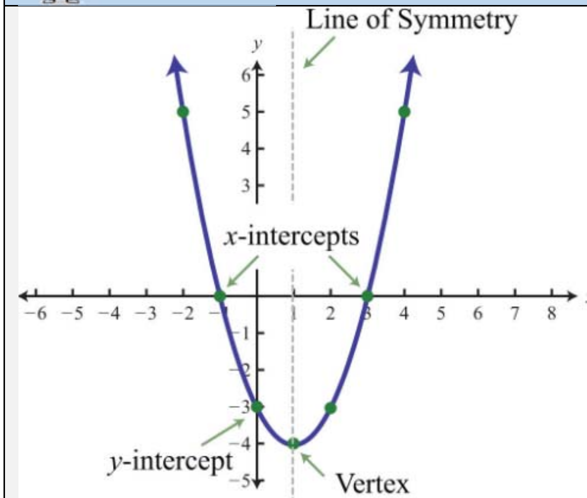
The **domain** for all quadratic functions is all real numbers

The **range** is dependent upon where the vertex is. For example if the vertex of a function is a minimum point, (-5, 3), the range of the function starts at the y value of the vertex, 3, and increases to positive infinity. In interval notation, the range is [3, ∞).

Questions To Ponder

Which key feature(s) described above are the same for exponential functions and linear functions?

Example!



Identify the key features of the graph to the left.

Vertex: (1, -4), since the graph opens upward the vertex is a minimum point.

The axis of symmetry is x = 1

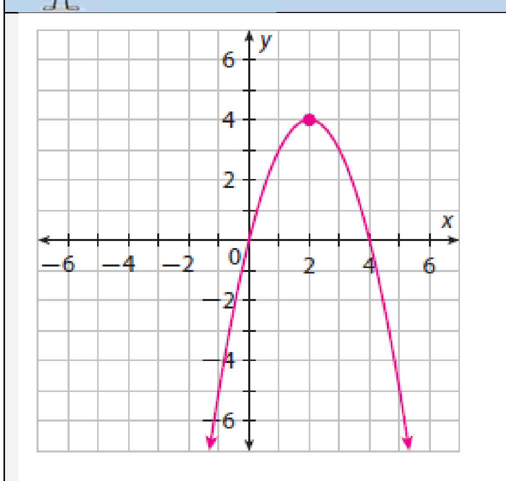
The y-intercept is (0,-3)

The x intercepts are (-1,0) and (3,0)

The domain is all real numbers or (∞, -∞) using interval notation.

The range is [-4,∞)

Example! Guided Practice



Identify the key features of the graph to the left.

Vertex:

Equation of the Axis of symmetry:

y-intercept:

x intercept(s):

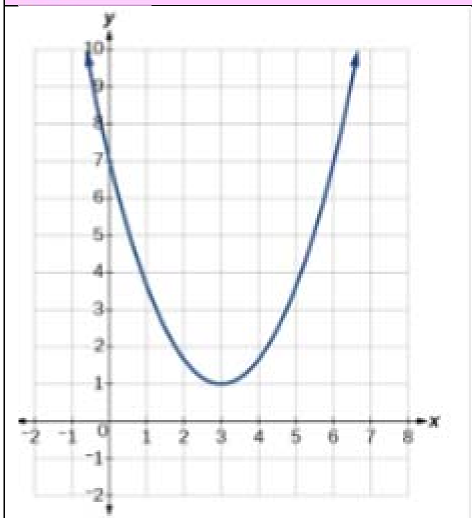
Domain:

Range:

Questions To Ponder

What are the key differences between the two graphs above?

SELF CHECK



vertex coordinates

maximum value or minimum value

equation to the axis of symmetry

y-intercept of the parabola

x-intercept of the parabola

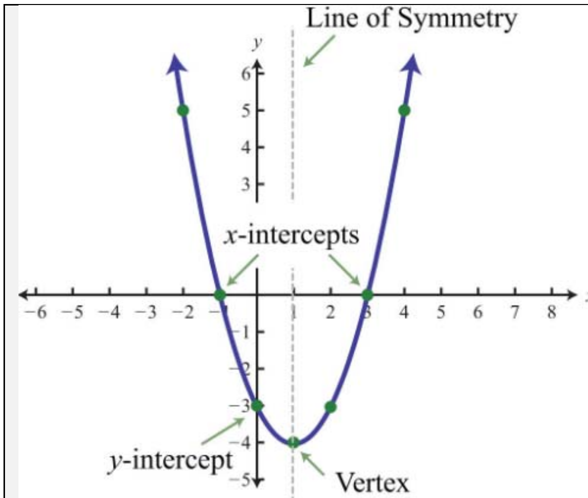
domain in interval notation

range in interval notation

Graphs of Quadratic Functions – End Behavior

The **end behavior** of a function is the behavior of the graph of $f(x)$ as x approaches positive infinity or negative infinity. With parabolas the end behavior is the same for each side of the graph.

Example!



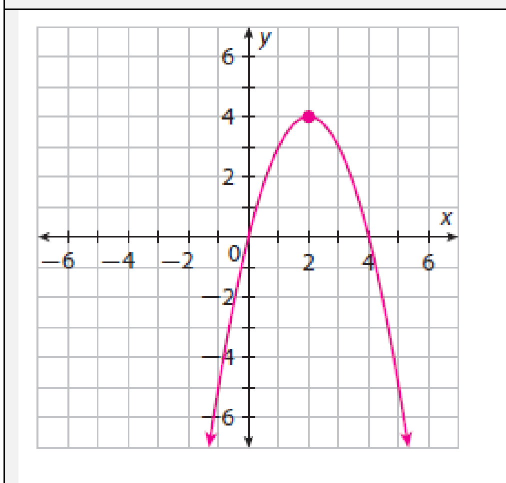
Look at the Left side of the graph is it increasing or decreasing? It is increasing. So as the x's are decreasing the value of the function is increasing.

Look at the right side of the graph is is also increasing. So as the x's are increasing the value of the function is increasing.

The end behavior for this function is as follows:

$x \to \infty$ as $f(x) \to \infty$

$x \to -\infty$ as $f(x) \to \infty$



Look at the Left side of the graph is it increasing or decreasing? It is decreasing. So as the x's are decreasing the value of the function is decreasing.

Look at the right side of the graph is is also increasing. So as the x's are increasing the value of the function is decreasing.

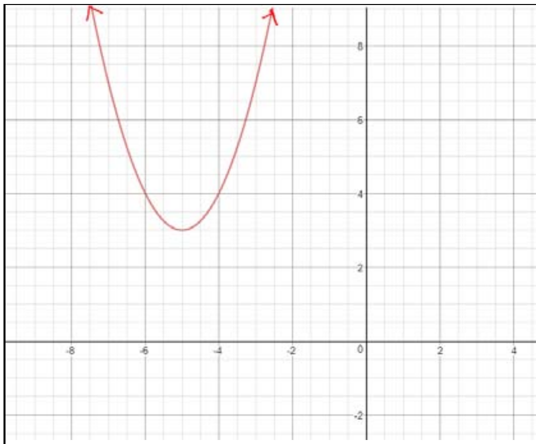
The end behavior for this function is as follows:

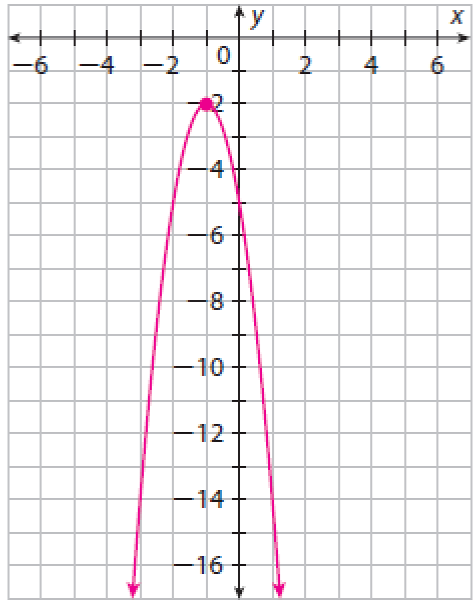
$x \to \infty$ as $f(x) \to -\infty$

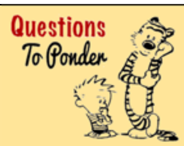
$x \to -\infty$ as $f(x) \to -\infty$

SELF CHECK

Describe the end behavior for each of the parabolas below.



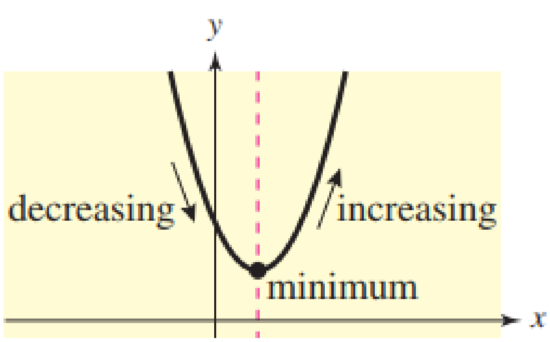


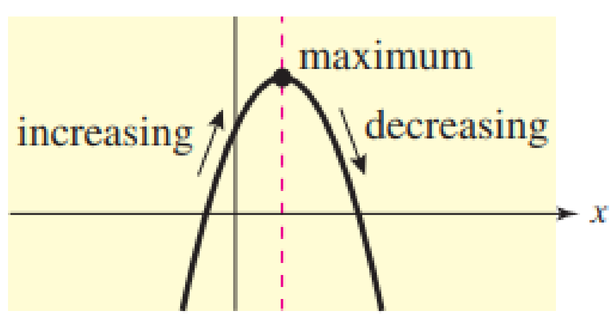


How is the end behavior for a quadratic function different from the end behavior of a linear or exponential graph?

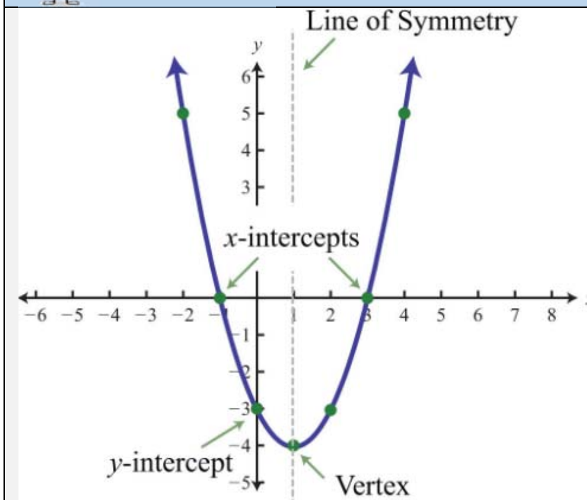
Graphs of Quadratic Functions – Intervals of Increase and Decrease

The vertex is the point at which the function changes direction. We use the x-value of the vertex to indicate the interval in which the function is increasing AND decreasing. Intervals of increase and decrease must be written in interval notation.





Example!



Think about the graph to the left, imagine that you are standing on a very small value of x, based on what you know about graphs is the value of the function (the y value) very large or very small? The y values are of course very large.

As you move on the **x axis** towards the vertex what is happening to the y values? They are decreasing, they continue decreasing until they reach the vertex.

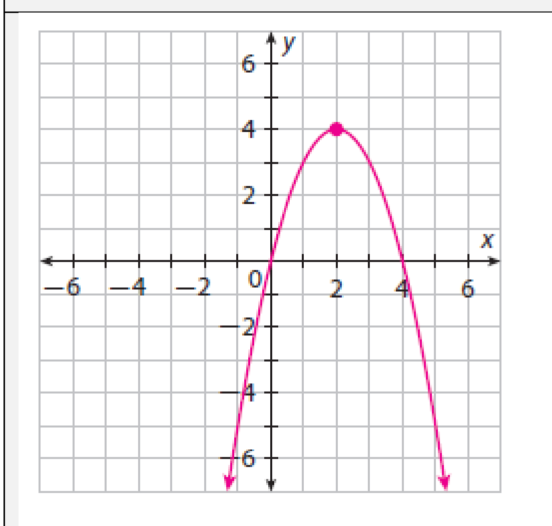
For this graph the **INTERVAL of DECREASE** is from -∞ to the x value of the vertex which is 1. In interval notation this is written as $(-\infty,1)$.

This is an **INTERVAL of DECREASE** because the value of the function, the y-values are **DECREASING**.

Remember that the vertex is a turning point, so once the graph reaches the vertex it changes direction. In other words this graph begins INCREASING once it passes the vertex. It will continue to increase towards infinity.

The **INTERVAL of INCREASE** is from 1 to ∞. In interval notation this is written as $(1, \infty)$

This is an **INTERVAL of INCREASE** because the value of the function, the y-values are **INCREASING.**



Think about the graph to the left, imagine that you are standing on a very small value of x, based on what you know about graphs is the value of the function (the y value) very large or very small? The y values are of course very small.

As you move on the **x axis** towards the vertex what is happening to the y values? They are increasing, they continue increasing until they reach the vertex.

For this graph the **INTERVAL of INCREASE** is from -∞ to the x value of the vertex which is 2. In interval notation this is written as $(-\infty,2)$.

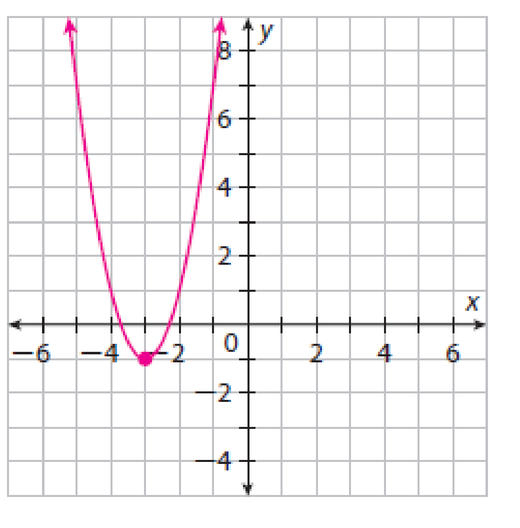
This is an **INTERVAL of INCREASE** because the value of the function, the y-values are **INCREASING**.

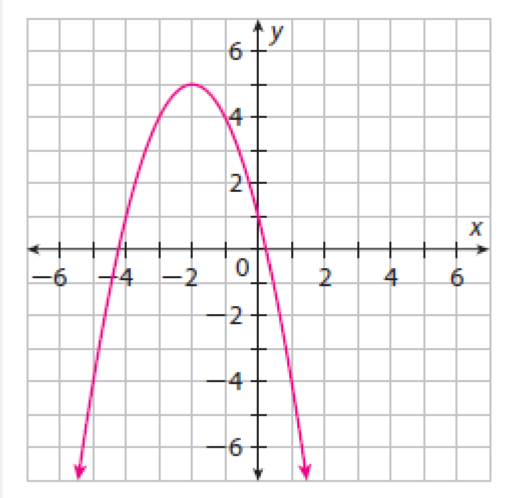
Remember that the vertex is a turning point, so once the graph reaches the vertex it changes direction. In other words this graph begins **DECREASING** once it passes the vertex. It will continue to increase towards negative infinity.

	The **INTERVAL of INCREASE** is from 2 to -∞. In interval notation this is written as (2, -∞) This is an **INTERVAL of DECREASE** because the value of the function, the y-values are **DECREASING.**

Example! GUIDED PRACTICE

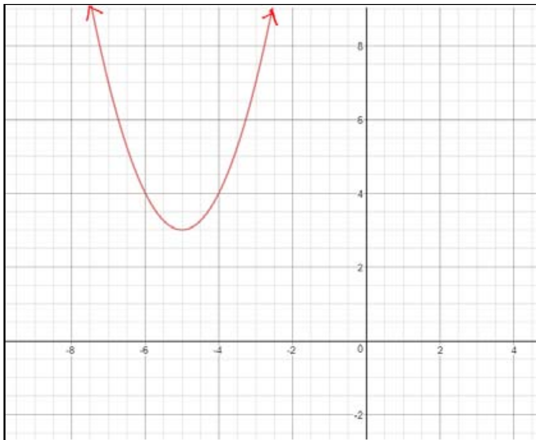
Describe the intervals of increase and decrease for each of the graphs below.

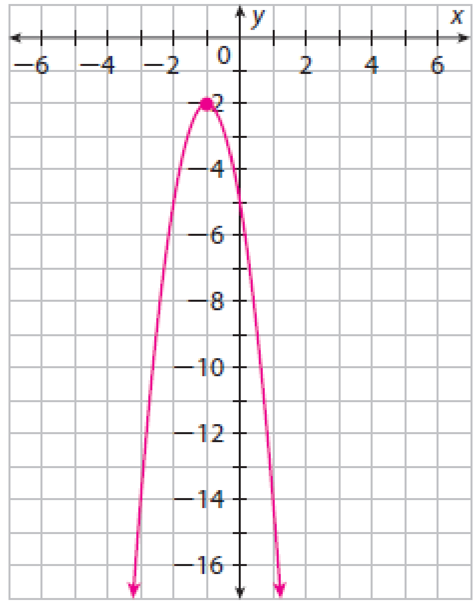




SELF CHECK

Identify the intervals of increase and decrease for the parabolas below.

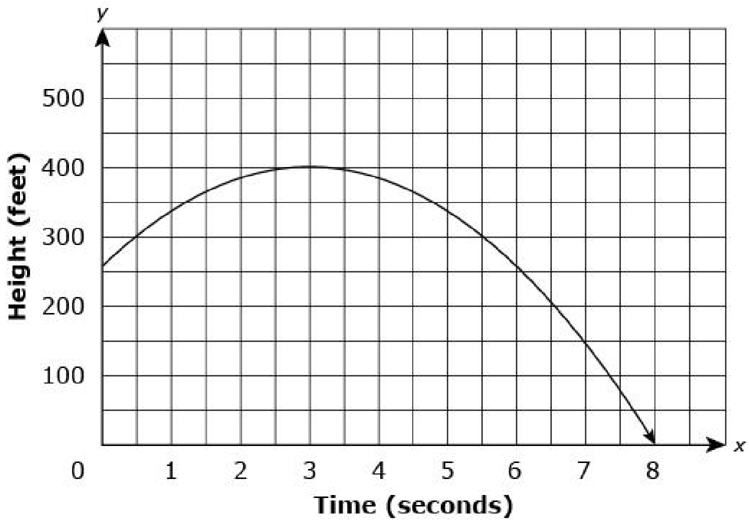




When you are writing the intervals of increase and decrease why do you use only parentheses instead of brackets?

Key Features

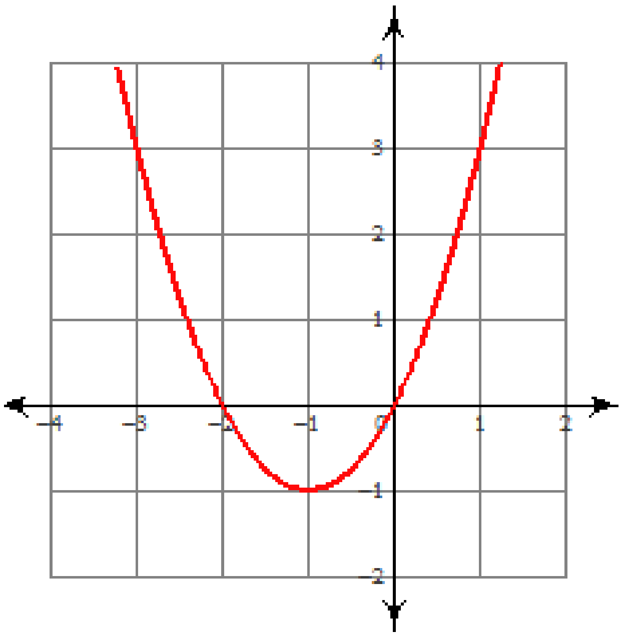
From the rooftop of a 256-foot tall building, a baseball is thrown upward. The baseball starts to descend to the ground after reaching a maximum height above ground. The graph on the coordinate plane best models the function between *y*, the height of the baseball, in feet, above the ground after *x* seconds. The vertex of the parabola is (3,400)



Part A: What is an appropriate domain for this graph? Explain

Part B: What is an appropriate range for this graph? Explain

$y = x^2 + 2x$



Vertex Coordinates: ________________

Is this vertex a maximum or minimum? Explain

Equation of the axis of symmetry: _______________

y-intercept: ____________

x-intercept: ____________

Domain: __________

Range: __________

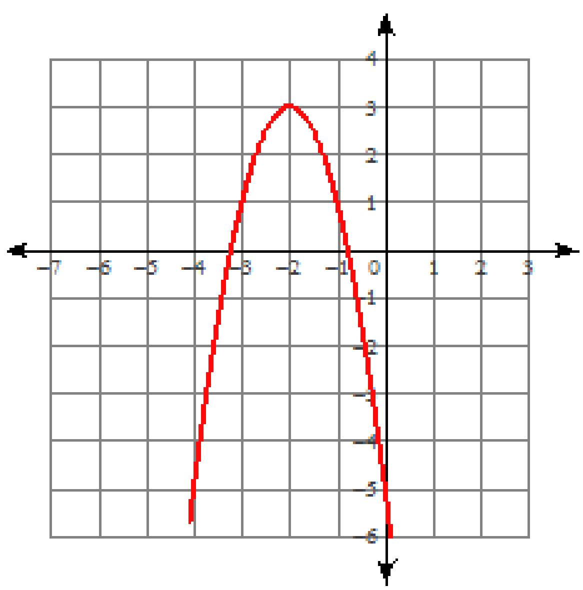
Interval of Increase: _________________

Interval of Decrease: _______________

End Behavior: ___________________

____

$y = -2x^2 - 8x - 5$



Vertex Coordinates: ________________

Is this vertex a maximum or minimum? Explain

Equation of the axis of symmetry: _______________

y-intercept: ____________

x-intercept: ____________

Domain: __________

Range: __________

Interval of Increase: _________________

Interval of Decrease: _______________

End Behavior: ___________________

____

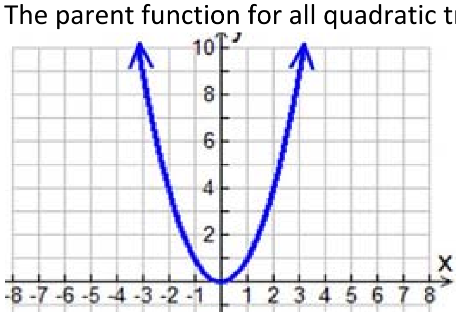
Vertex Form and Transformations of Parabolas

Just like we did with exponential functions we can also describe the transformations of quadratics using the parameters a, h, and k.

When describing transformations we will be working with the vertex form of the quadratic equation. The vertex form is written as follows: $f(x) = a(x-h)^2 + k$

- **a is the vertical stretch or compression**
  - $a > 1$ stretch
  - $0 < a < 1$ compression
  - \- reflection
- **h is the horizontal shift left or right**
  - h lies to our eyes!
  - x – h; shift right
  - x + h; shift left
- **k is the vertical shift up and down**
  - \+ k; shift up
  - \- k; shift down

The parent function for all quadratic transformations is $f(x) = x^2$



The vertex here is (0, 0).

The vertex of the parabola is (h ,k)

Identify the vertex and the axis of symmetry for the quadratic functions below.

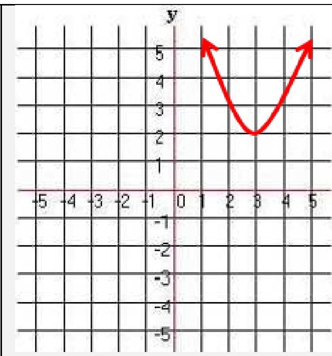
$f(x) = 2(x+2)^2 - 3$

$f(x) = 2(x-2)^2 + 3$

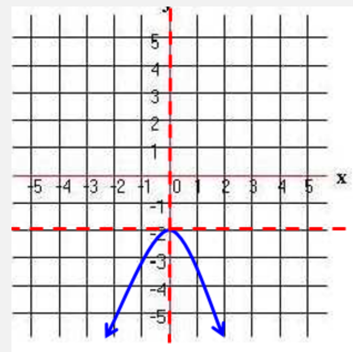
$f(x) = 2(x-2)^2 + 3$

$f(x) = 2(x+2)^2 + 3$

Example!

$f(x) = (x-3)^2 + 2$ a = 1 h = 3 k = 2	Describe the transformations that have occurred. • Horizontal shift right by 3 units • Vertical shift up by 2 units **The vertex is (3,2)** It opens upward since a is positive. So this vertex is a minimum point.	

Example!

$f(x) = -x^2 - 2$ a = -1 h = 0 k = -2	Describe the transformations that have occurred. • Reflection over the x-axis • Vertical shift down by 2 units • **The vertex is (0,-2)** It opens downward since a is negative. So this vertex is a maximum point.	

Example! Guided Practice

For each equation describe the transformation(s), identify the vertex and determine if the vertex represents a maximum or minimum.

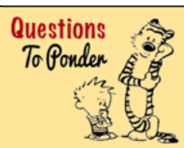
$f(x) = -\frac{1}{4}(x-4)^2 + 3$	$g(x) = \frac{4}{3}(x+4)^2 - 3$	$h(x) = -9(x+3)^2 - 6$

Write the equation using the given transformations. Identify the vertex and determine if the vertex represents a maximum or minimum.

The graph of $y = x^2$ is reflected across the x axis then translated 6 units upward and 7 units to the left.	The graph of $y = x^2$ is vertically stretched by a factor of -3, a horizontal shift right 2 units, and translated up 6 units.

SELF CHECK

For each equation describe the transformation(s), identify the vertex and determine if the vertex represents a maximum or minimum.	
$f(x) = -3(x-4)^2 - 3$	$f(x) = \frac{2}{3}(x+4)^2 + 3$
Write the equation using the given transformations. Identify the vertex and determine if the vertex represents a maximum or minimum.	
The graph of $y = x^2$ is compressed vertically by a factor of $\frac{1}{5}$, a horizontal shift left 4 units, and translated down 1 unit.	



Given the equation $f(x) = -2(x+5)^2 - 2$

Why does this graph move to the left?

1. Write the vertex form of a quadratic function.

2. Being specific, name 3 ways that a parabola changes with different types of "a" values.

3. What does changing the "h" variable do to the graph of a quadratic function?

4. What does changing the "k" variable do to the graph of a quadratic function?

5. Describe the transformations:

   a. $y = (x-2)^2$

   b. $f(x) = (x+2)^2$

   c. $y = (x-1)^2 + 1$

   d. $y = (x+1)^2 - 1$

   e. $f(x) = 2(x-2)^2 + 3$

   f. $y = -2(x+2)^2 - 4$

   g. $y = -\frac{1}{3}(x-3)^2 + 5$

   h. $f(x) = \frac{2}{5}(x+6)^2 + 5$

6. Use the given descriptions to write a quadratic function in vertex form.

   a. The function $f(x) = x^2$ is shifted right 3 units and down 12 units.

   b. The function $f(x) = x^2$ is stretched vertically by a factor of 2, then shifted left 6 units and up 4 units.

   c. The function $f(x) = x^2$ is compressed vertically by a factor of $\frac{1}{4}$, then shifted left 2 units and down 3 units.

   d. The function $f(x) = x^2$ is compressed vertically by a factor of $\frac{2}{3}$, reflected over the x axis, then shifted left 5 units and up 3 units.

Transformations of Quadratic Functions – Application

Quadratic Fanatic and the Case of the Foolish Function

Adapted From Jody Haynes, Fayette County School System

"Quadratic Fanatic, we need your help!" declared the voice on the answering machine. While out helping solve the town's problems, a crime had occurred at the Function Factory, and now it was up to the Quadratic Fanatic to straighten things out.

When he arrived at the factory, Quadratic Fanatic was given three different groups of suspects, each group representing a different shift. He was told that an employee from each shift had worked together to commit the crime.

The employees from the first shift were all quadratic functions in vertex form. "They are always acting kind of shifty," said the manager. The list of suspects from the first shift is below. For each suspect, list the transformational characteristics of the function.

Function	Vertical Shift	Horizontal Shift	Vertical Stretch/Shrink	Reflected?
A. $f(x) = -\frac{1}{2}(x-3)^2 - 4$				
B. $f(x) = 2(x-4)^2 + 3$				
C. $f(x) = 2(x+4)^2 - 3$				
D. $f(x) = -\frac{1}{2}(x-4)^2 + 3$				
E. $f(x) = -2(x+4)^2 + 3$				
F. $f(x) = -4(x-3)^2 - 2$				
G. $f(x) = 3(x+4)^2 - 2$				

According to a several witnesses, the following information about the suspect was gathered:

"He was shifted up three." Which of the employees above could be suspects? ________________________________

"His axis of symmetry was $x = -4$." Which of the employees above could be suspects? __________________________

"He had a vertical stretch of 2." Which of the employees above could be suspects? ____________________________

"I could see his reflection." Which of the employees above could be suspects? ________________________________

Based on the above information, which employee is guilty? Explain how you know.

Describe how the following equations transformed from $y = x^2$

1. $y = x^2 - 5$

2. $y = (x-5)^2$

3. $y = (x+4)^2 - 6$

4. $y = 3x^2$

5. $y = \frac{1}{3}(x+1)^2$

6. $y = -2(x-3)^2 + 7$

Write the quadratic equations under the specific transformations from $y = x^2$

9. Shift 1 unit to the right and 5 units down

10. Shift 4 units to the left and 9 units up

11. Vertical stretch by 2, then reflect about the *x*-axis the shift 8 units left and 7 units down.

12. Vertically compress by a factor of $\frac{4}{5}$ then reflect about the *x*-axis and then shift up 10 units and right 4 units.

Graphing Quadratics

Quadratic functions are functions where your input/independent variable x is raised to the second power.

As we discussed in a previous lesson this causes the function to behave differently than a linear function or an exponential function. The graph of a quadratic function is called a _________________. Here are three forms of quadratics that we will learn to graph with in this lesson.

$f(x) = ax^2 + bx + c$	$f(x) = a(x-h)^2 + k$	$f(x) = a(x-x_1)(x-x_2)$
______________________	______________________	____________________

- Notice that all three forms above start with the 'a' term. If that term is positive (a), the graph opens ___________. If that term is negative (-a), the graph opens down.
- All quadratic equations have a ______________, which is the turning point of the graph. Quadratic graphs are also symmetrical across the axis of symmetry, which runs through the vertex.

Describing Transformations

After graphing your Quadratic functions you can also describe the transformations that have occurred.

The "a" value tells you if there is a vertical stretch or compression. If "a" is negative then there is also a reflection about the x-axis.

The "h" value, which is the x value of the vertex tells you if the function moved left or right.

The "k" value, which is the y value of the vertex tells you if the function moved up or down.

Graphing in STANDARD FORM

The standard form of a quadratic function is $f(x) = ax^2 + bx + c$. The y intercept is the constant term c in the function. So this is one point that we can use to graph our function.

First we need to pull out the vertex from the function. We do that with the vertex formula, which tells us the x-value of the vertex. You then substitute the x-value you get into the original function to get the y-value. Lastly, pick x-values above and below the vertex to plug in and get more points.

The vertex formula is $x = \frac{-b}{2a}$

Example! Guided Practice

Identify the y intercept of the functions below. Then use the vertex formula to determine the vertex of the parabola.

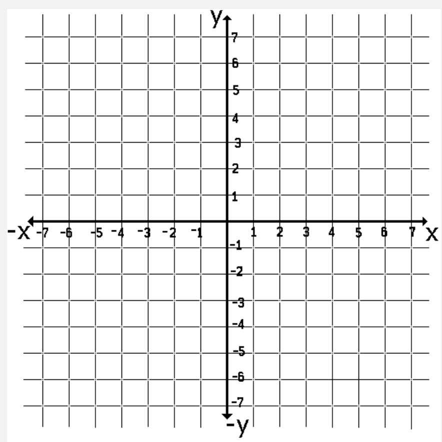
$f(x) = 2x^2 + 6x - 3$

$f(x) = 3x^2 + 9x + 5$

Example! Guided Practice

Graph the function: **f(x) = 4x² – 8x + 1**

x	f(x)



First Identify the y intercept: _________. Go ahead and plot that point on your coordinate plane above.

Then use the vertex formula to find the vertex. Plot that point on your coordinate plane. Plot this point on the coordinate plane then put the x and y values in the middle of the table above.

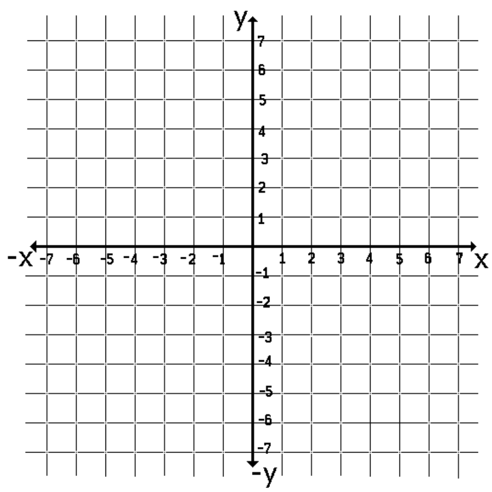
Pick 2 x-values that are above the vertex and 2 x-values below the vertex. Substitute into the function. Record your resulting y values in the chart above then plot the points to make the graph.

Describe all transformations:

SELF CHECK

Graph the function: $f(x) = 2x^2 + 8x - 2$

x	f(x)



Describe all transformations:

Questions To Ponder

Why is it important to make sure you find the vertex when graphing a quadratic function in standard form?

Graphing in Vertex Form

When a quadratic function is in vertex form, the vertex is easy to recognize.

$f(x) = a(x - h)^2 + k$ The vertex is always the values of (h, k)

To graph in vertex form, put the vertex in the table and pick x values above and below to get more points.

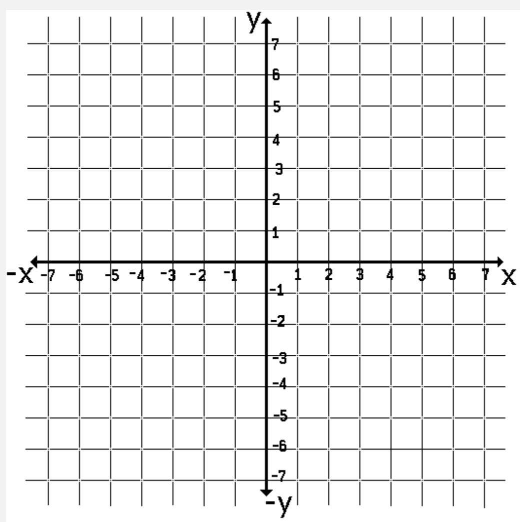
Quick Review: Find the vertex for each function below.

$f(x) = 2(x - 2)^2 + 4$ Vertex: $f(x) = -4(x + 3)^2 - 5$ Vertex:

Example! Guided Practice

Graph the function: $\mathbf{f(x) = 2(x - 3)^2 - 4}$

x	f(x)



Identify the vertex: ___________. Put this value in the middle of the table, then graph it on your coordinate plane.

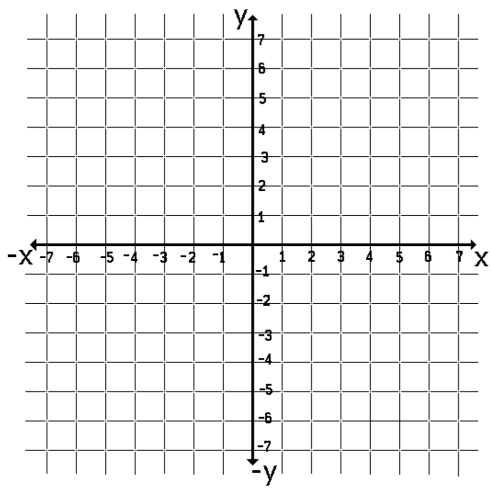
Pick 2 values above vertex and 2 values below the vertex and generate more points. Plot the points and make the graph.

Describe all transformations:

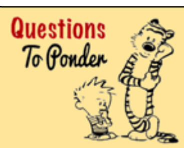
SELF CHECK

Graph the function: $f(x) = -2(x+4)^2 - 1$

x	f(x)



Describe all transformations:



If given a quadratic function in vertex form is it possible to determine if the graph opens up or down just by looking at the function, before you graph it? Explain.

Graphing in Factored Form

The factored form of a quadratic equation look like this $f(x) = a(x-p)(x-q)$ In this form, the equation has been factored out. The q and the p represent the x intercepts of the quadratic function. You will learn more about factoring and finding solutions to quadratics later in the unit but this will be a small preview of what is to come.

To find the x intercepts you will need to set each factor equal to zero and solve the resulting equations.

For example if the function is $f(x) = 2(x-4)(x-6)$ you find the x intercepts by setting up the following equations:

$(x-4) = 0$ and $(x-6) = 0$

You should be able to tell that the x intercepts are $(4,0)$ and $(6,0)$

To find the y intercept just plug 0 in everywhere you see x in the original function. It may or may not be helpful in graphing.
$2(0-4)(0-6) = 2(-4)(-6) = 48$ So the y intercept is $(0,48)$

Finding the Vertex
To find x value of the vertex you add the x intercepts and divide by 2

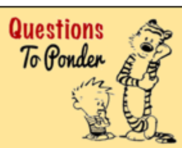
$$x = \frac{4+6}{2} = \frac{10}{2} = 5$$

Then plug 5 in for x to get the y value of the vertex.

$2(5-4)(5-6) = 2(1)(-1) = -2$

The vertex is $(5,-2)$

You can use these points to create your graph.

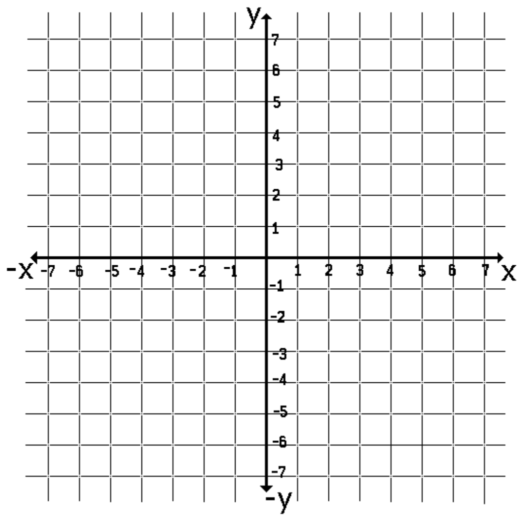


Is the graph in the example above going to open upwards or downwards? Explain.

Example! Guided Practice

Graph the function**: f(x) = (x + 3)(x – 2)**

x	f(x)



Step 1: Identify x intercepts. Plot on the coordinate plane and enter in the table.

Step 2 Find y Intercept. Plot on the coordinate plane if it will fit. Enter it in the table.

Step 3 Find the vertex. Plot on the coordinate plane and enter the values in the table.

Describe all transformations:

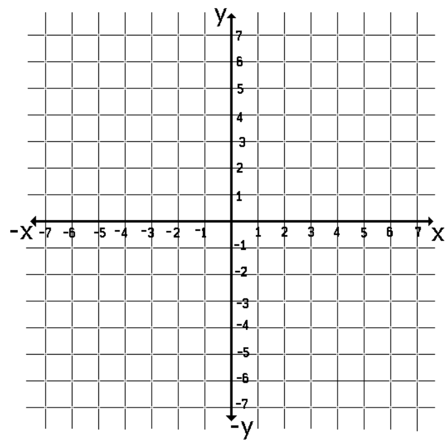
Questions To Ponder

How could you use the Axis of Symmetry to help you create your graph?

SELF CHECK

Graph the function**: f(x) = (x + 1)(x – 4)**

x	f(x)



Describe all transformations:

Graph each function using the method discussed in the lesson. After you have graphed the function describe all transformations that have occurred.

1. $f(x) = -3x^2$

Y-intercept:__________

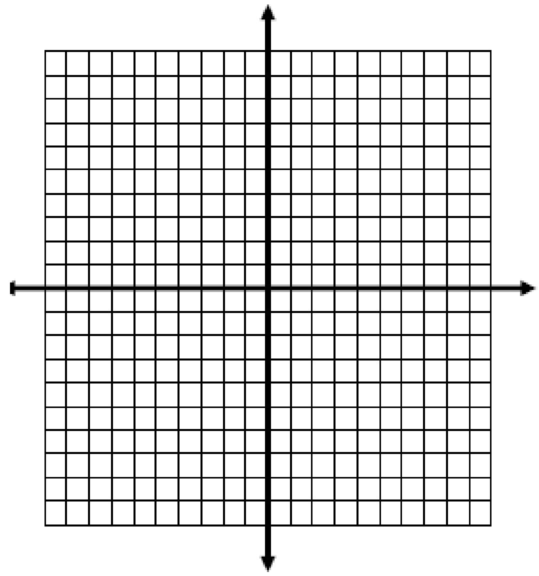
Vertex: __________

Maximum or Minimum

Axis of Symmetry: __________

Transformations:

x	f(x)



1. $f(x) = 3x^2 - 6x + 4$

Y-intercept:__________

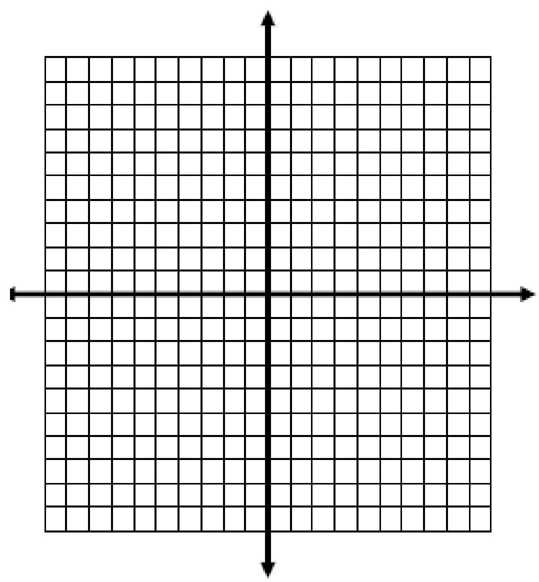
Vertex: __________

Maximum or Minimum

Axis of Symmetry: __________

Transformations:

x	f(x)



1. $f(x) = 3(x-4)^2 - 6$

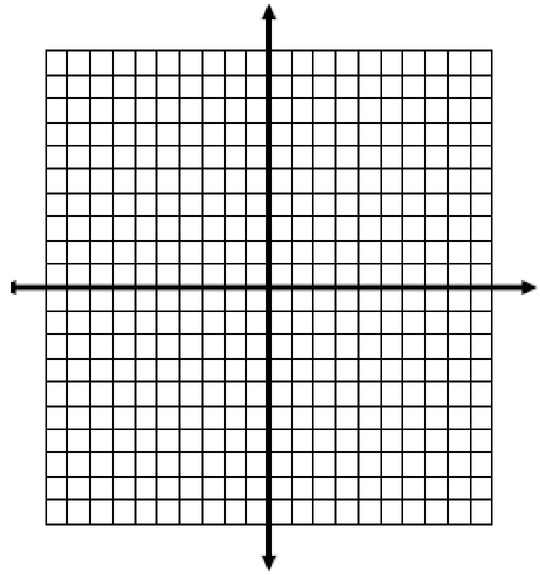
Vertex: ____________

Maximum or Minimum

Axis of Symmetry: ___________

Transformations:

x	f(x)



1. $f(x) = -\frac{1}{2}(x+2)^2 + 3$

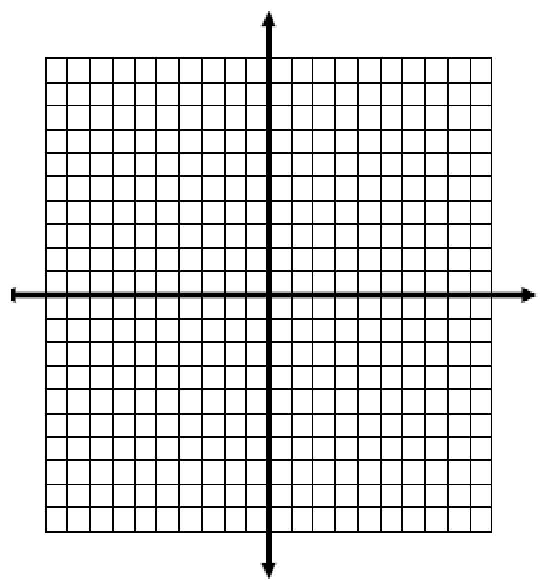
Vertex: ____________

Maximum or Minimum

Axis of Symmetry: ___________

Transformations:

x	f(x)



1. $f(x) =$(x-1)(x+3)

x-intercepts:___________

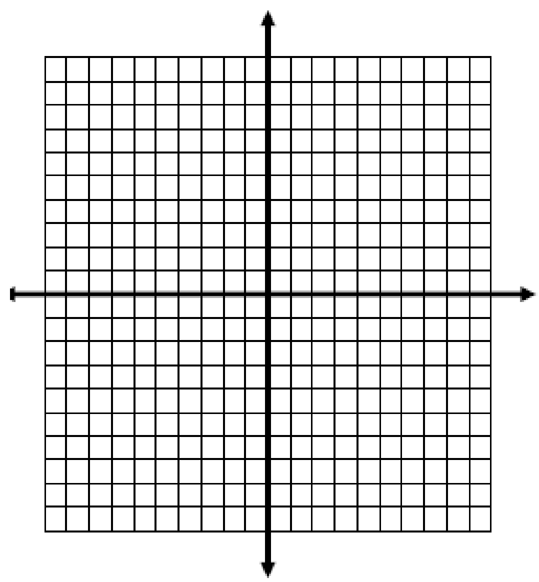
Y-intercept: ___________

Vertex: ______________

Maximum or Minimum

Transformations:

x	f(x)



1. $f(x) = -(x+2)(x-4)$

x-intercepts:___________

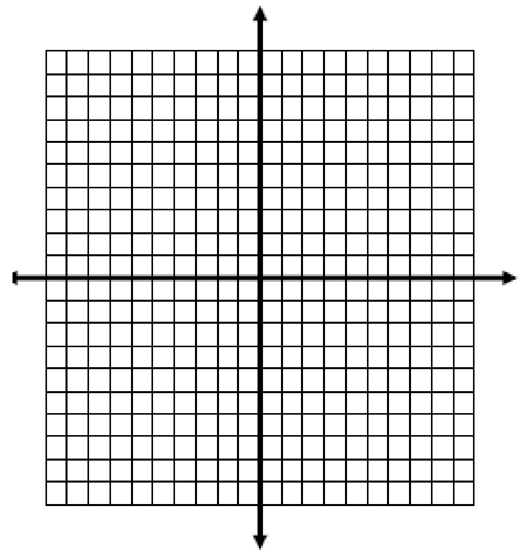
Y-intercept: ___________

Vertex: ______________

Maximum or Minimum

Transformations:

x	f(x)



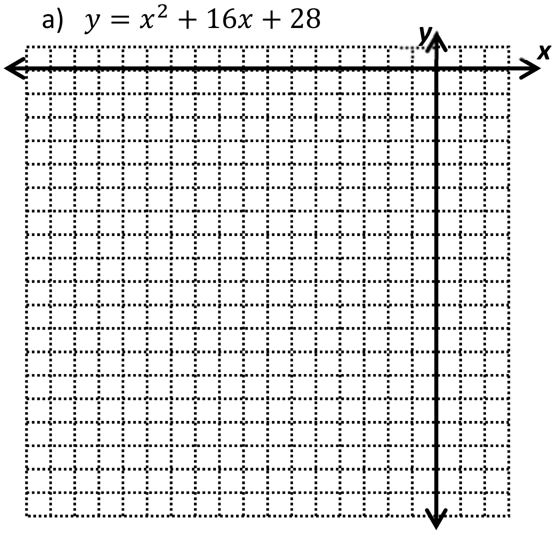
Graphing Quadratics

Characteristics of Quadratic Functions (Performance Task)

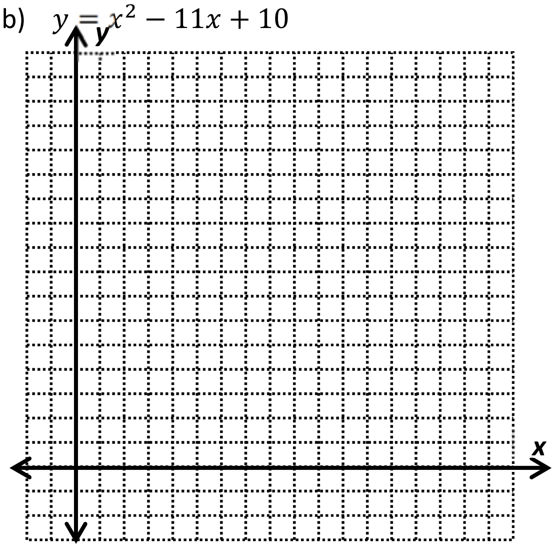
Name______________________________ Date____________

Complete a table, graph, and investigate the following functions.

a) $y = x^2 + 16x + 28$



b) $y = x^2 - 11x + 10$

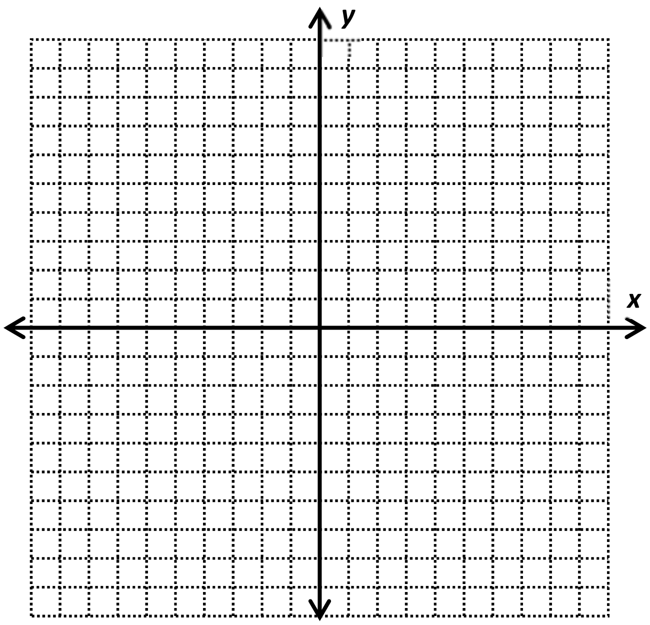


State the following…

a)	b)
Domain:	Domain:
Range:	Range:
Zeros:	Zeros:
Y–Intercept:	*Y*–Intercept:
Interval of Increase:	Interval of Increase:
Interval of Decrease:	Interval of Decrease:
Maximum:	Maximum:
Minimum:	Minimum:
End Behavior:	End Behavior:Transformations:
Transformation:	

Complete a table, graph, and investigate the following functions.

c) $y = x^2 - 5x + 6$



State the following

Domain:

Range:

Zeros:

Y–Intercept:

Interval of Increase:

Interval of Decrease:

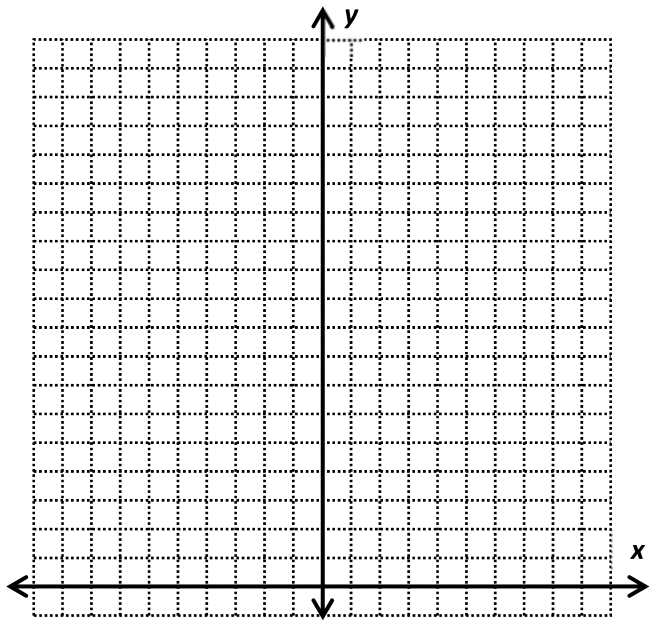
Maximum:

Minimum:

End Behavior:

Transformations:

d) $y = 5x^2 - 10x + 20$



Domain:

Range:

Zeros:

Y–Intercept:

Interval of Increase:

Interval of Decrease:

Maximum:

Minimum:

End Behavior:

Transformations:

1. $f(x) = -x^2 - 2x - 1$

Y-intercept:___________

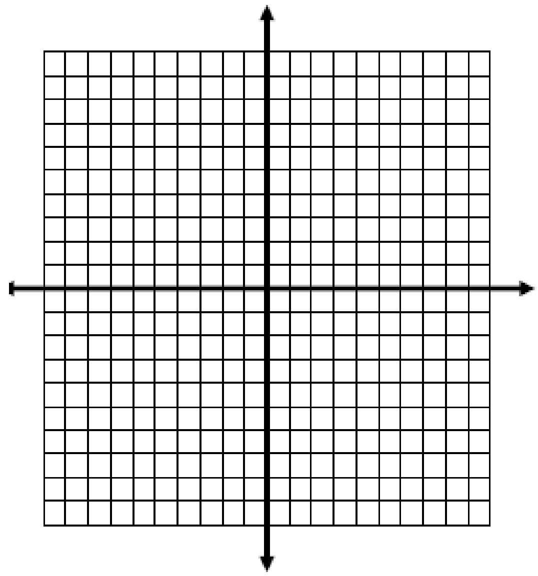
Vertex: ____________

Maximum or Minimum

Axis of Symmetry: ___________

Transformations:

x	f(x)



2. $f(x) = -\frac{1}{2}(x+6)^2 - 4$

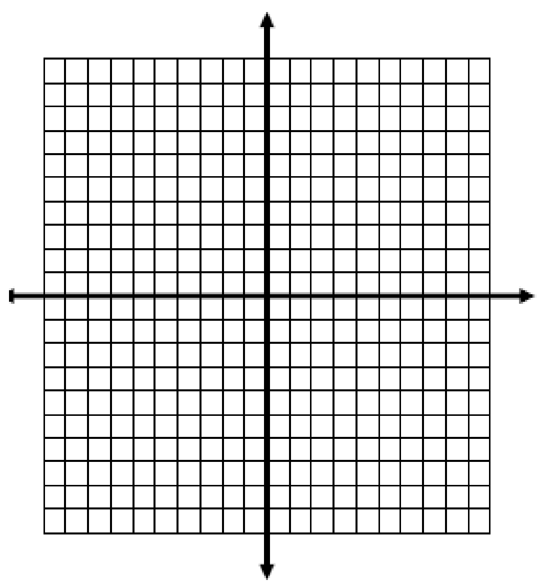
Vertex: ____________

Maximum or Minimum

Axis of Symmetry: ___________

Transformations:

x	f(x)



3. $f(x) = -2(x+1)(x+3)$

x-intercepts:___________

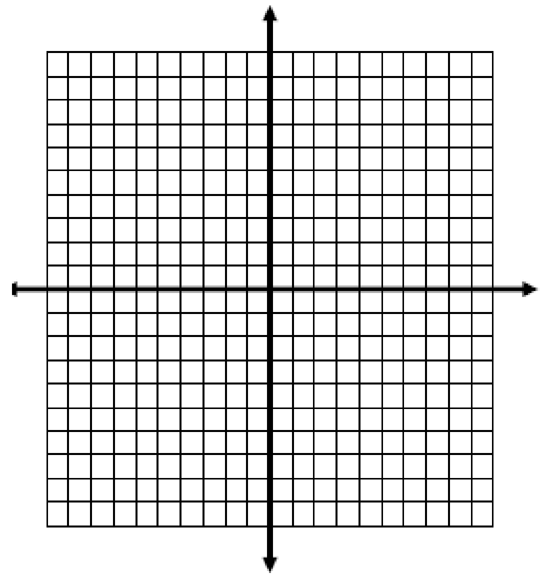
Y-intercept: ___________

Vertex: _______________

Maximum or Minimum

Transformations:

x	f(x)



Converting Vertex Form to Standard Form

Sometimes you are given quadratic functions in vertex form $f(x) = a(x-h)^2 + k$, this is a great form for graphing, but not really efficient for solving.

The standard form of a quadratic equation is $f(x) = ax^2 + bx + c$, where a, b, and c are coefficients and $f(x)$ and x are variables.

It is **easier to solve a quadratic equation** when it is in **standard form** because you compute the solution with a, b, and c.

Example!

Convert Vertex Form to Standard Form

$$f(x) = a(x-h)^2 + k \;\; to \;\; f(x) = ax^2 + bx + c$$

Example 1:

$\mathbf{y = 5(x + 2)^2 - 9}$	
$y = 5(x + 2)(x + 2) - 9$	⇐ Rewrite $(x + 2)^2$
$y = 5(x^2 + 4x + 4) - 9$	⇐ Simplify $(x + 2)(x + 2)$
$y = 5x^2 + 20x + 20 - 9$	⇐ Distribute the 5
$\mathbf{y = 5x^2 + 20x + 11}$	⇐ Combine Like Terms

Example 2:

$\mathbf{y = -3(x - 4)^2 + 7}$	
$y = -3(x - 4)(x - 4) + 7$	⇐ Rewrite $(x - 4)^2$
$y = -3(x^2 - 8x + 16) + 7$	⇐ Simplify $(x - 4)(x - 4)$
$y = -3x^2 + 24x - 48 + 7$	⇐ Distribute the -3
$\mathbf{y = -3x^2 + 24x - 41}$	⇐ Combine Like Terms

SELF CHECK

$f(x) = 8(x + 1)^2 - 7$	$f(x) = (x - 5)^2 + 19$	$Y = -(x + 3)^2 + 6$

Converting Standard Form to Vertex Form

Once a quadratic equation is in **vertex form** it becomes easier to draw an accurate parabola. But how do you convert a quadratic equation from standard form to vertex form? Follow these steps:

STEP 1: Make sure that one side of the equation is equal to 0.
STEP 2: Find the *a* value. The *a* value in vertex form is the same as the *a* value in standard form.
STEP 3: Find the *h* value. In order to find the *h* value of a quadratic equation use the formula $-\frac{b}{2a}$, where *a* and *b* are the values from the standard form.
STEP 4: Find the *k* value. In order to find the *k* value of a quadratic equation, substitute your *h* value into your quadratic equation and solve.
STEP 5: Place all of your values into the vertex form of a quadratic equation.

Remember that the h value is the x of the vertex and the k value is the y of the vertex

Example!

Convert Standard Form to Vertex Form

$$f(x) = ax^2 + bx + c \ \ to \ \ f(x) = a(x-h)^2 + k$$

Ex 1: $x^2 - 2x = -4$

STEP 1: Add 4 to both sides of the equation to make it $x^2 - 2x + 4 = 0$
STEP 2: In the standard form, *a* = 1. Therefore, in vertex form *a* = 1 as well.
STEP 3: *h* = $-\frac{b}{2a}$ when *b* is -2, and *a* is 1. Therefore *h* = $-\frac{(-2)}{2(1)} = \frac{2}{2} = 1$.
STEP 4: To find the value of *k*, substitute 1 into the equation $x^2 - 2x + 4$. $(1)^2 - 2(1) + 4$, $1 - 2 + 4$ = 3.
STEP 5: The vertex form of the standard quadratic equation $x^2 - 2x + 4 = 0$ is

$$\boldsymbol{f(x) = (x-1)^2 + 3}$$

Guided Practice: $4x^2 + 16x + 2 = 0$

SELF CHECK

$x^2 + 2x - 2 = 0$	$2x^2 - 12x + 25 = 0$	$x^2 - 4x = -7$

Questions To Ponder

Why is it important to be able to switch between the different forms of a quadratic function?

Convert the following quadratics from vertex form to standard form.

1) $y = -(x - 1)^2 - 1$

2) $y = 2(x - 2)^2 - 3$

3) $y = (x + 4)^2 + 4$

Convert the following quadratics from standard form to vertex form.

4) $y = x^2 - 8x + 15$

5) $y = x^2 - 4x$

6) $y = x^2 + 8x + 18$

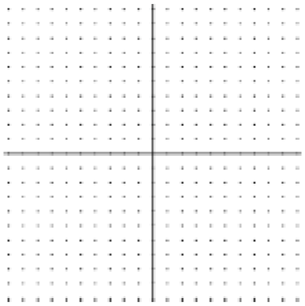
7) $y = x^2 + 4x + 3$

8) $y = x^2 - 2x + 5$

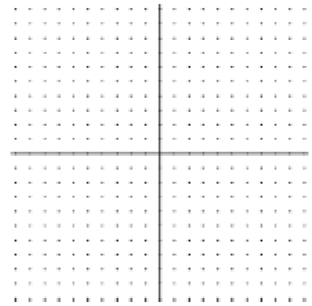
9) $y = x^2 - 8x + 17$

Convert the following quadratics from standard form to vertex form, then graph them.

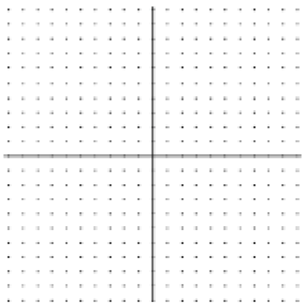
10) $y = x^2 - 6x + 7$



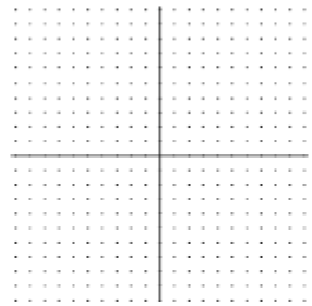
11) $y = x^2 + 6x + 5$



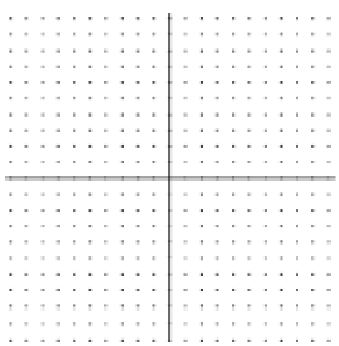
12) $y = -x^2 + 4x - 1$



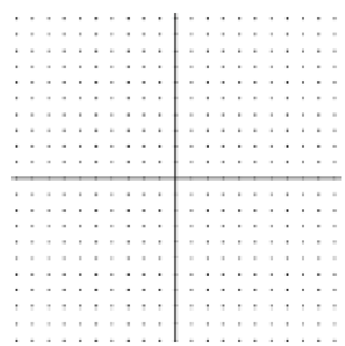
13) $y = -x^2 - 6x - 7$



14) $y = 2x^2 - 8x + 9$



15) $y = -x^2 - 6x - 10$



Ball from a Tower

A ball is thrown upward from a tower. The function indicating the height of the ball over time, *t, is*
$f(t) = -16t^2 + 32t + 6{,}361$. Rewrite the function to show the time and the height at which the ball reaches its maximum height.

Explain why your function shows the maximum height and the time it takes to reach it.

Converting from Vertex Form to Standard Form

Multiply out the binomial, distribute (if needed), & combine like terms.

1. $f(x) = (x-1)^2 + 8$

2. $f(x) = 2(x+3)^2 - 5$

3. $f(x) = -(x-4)^2 + 3$

4. $f(x) = 2(x+1)^2 - 2$

Converting from Standard Form to Vertex Form

Find the Vertex *Method:*

- ★ Identify a, b, & c.
- ★ Find the line of symmetry or "h" by using $x = \frac{-b}{2a}$
- ★ Find the y value of the vertex, or "k" by substituting "x" into the equation.
- ★ Go get "a" (it stays the same).
- ★ Write the equation in vertex form using your found values of *a*, *h*, and *k*.

  $f(x) = \mathbf{a}(x - \mathbf{h})^2 + \mathbf{k}$

5. $f(x) = x^2 + 8x + 1$

6. $f(x) = x^2 + 10x + 20$

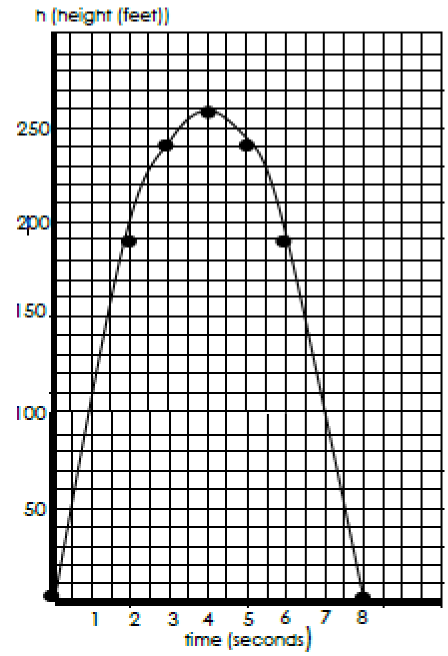
7. $f(x)=3x^2-6x+5$	8. $f(x)=-2x^2-16x-32$

Algebra 1 Unit # 4 Concept# 1E

Lesson Name: A1.U4.C1.E.03.**Classwork.QuadraticApplications**

Independent Practice

1 Using the graph at the right, It shows the **height** *h* in feet of a small rocket *t* **seconds** after it is launched. The path of the rocket is given by the equation:



$h = -16t^2 + 128t$.

1. How long is the rocket in the air? __________

2. What is the greatest height the rocket reaches? _____

3. About how high is the rocket after 1 second? ________

4. After 2 seconds,

about how high is the rocket?__________

is the rocket going up or going down? _________

5. After 6 seconds,

about how high is the rocket? ________

is the rocket going up or going down? _________

6. Do you think the rocket is traveling faster from 0 to 1 second or from 3 to 4 seconds? Explain your answer.

7. Using the equation, find the **exact** value of the height of the rocket at 2 seconds.

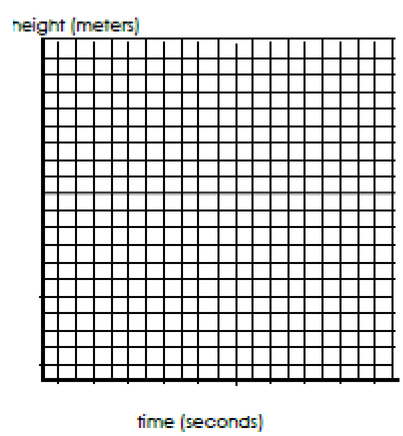
8. What is the domain of the graph?

9. What is the range of the graph?

10. Express the interval over which the graph is increasing.

11. Express the interval over which the graph is decreasing

2. A ball is thrown in the air. The path of the ball is represented by the equation $h = -t^2 + 8t$. Graph the equation over the interval $0 \le t \le 8$ on the accompanying grid.

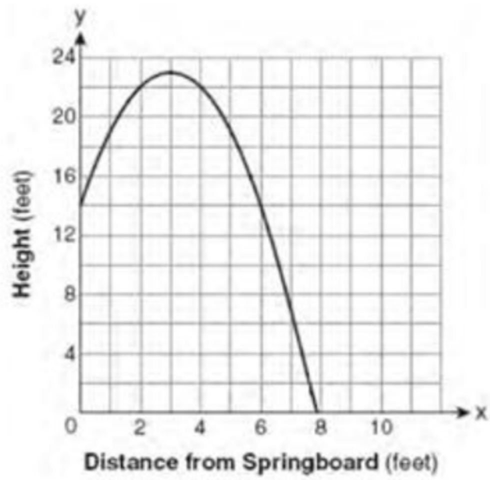


a) What is the maximum height of the ball?___________

b) What is the amount of time that the ball is

above 7 meters? __________________

3. A swim team member performs a dive from a 14-foot high springboard. The parabola below shows the path of her dive.



a) What is the axis of symmetry? __________

b) Find f(6) __________________________

4. After *t* seconds, a ball tossed in the air from the ground level reaches a height of *h* feet given

by the function : $h(t) = 144t - 16t^2$.

a. What is the height of the ball after 3 seconds? ____________________

b. What is the maximum height the ball will reach? ____________________

c. After how many seconds will the ball hit the ground before rebound?_____________________

5. A rock is thrown from the top of a tall building. The distance, in feet, between the rock and the ground t seconds after it is thrown is given by $d(t) = -16t^2 - 4t + 382$. How long after the rock is thrown is it 370 feet from the ground?

6. A rocket carrying fireworks is launched from a hill 80 feet above a lake. The rocket will fall into the lake after exploding at its maximum height. The rocket's height above the surface of the lake is given by the

Function: $h(t) = -16t^2 + 64t + 80$.

a. What is the height of the rocket after 1.5 seconds? ___________________

b. What is the maximum height reached by the rocket? _________________

c. After how many seconds after it is launched will the rocket hit the lake?________________

HOMEWORK

1. Jason jumped off of a cliff into the ocean in Acapulco while vacationing with some friends. His height as a function of time could be modeled by the function $h(t)= -16t^2+16t +480$, where t is the time in seconds and h is the height in feet.

a. How long did it take for Jason to reach his maximum height?____________

b. What was the highest point that Jason reached?_______________

c. What was Jason's initial height?____________

2. If a toy rocket is launched vertically upward from ground level with an initial velocity of 128 feet per second, then its height h, after t seconds is given by the equation

$h(t) = -16t^2 + 128t$ (air resistance is neglected)

a. How long will it take the rocket to hit its maximum height?___________

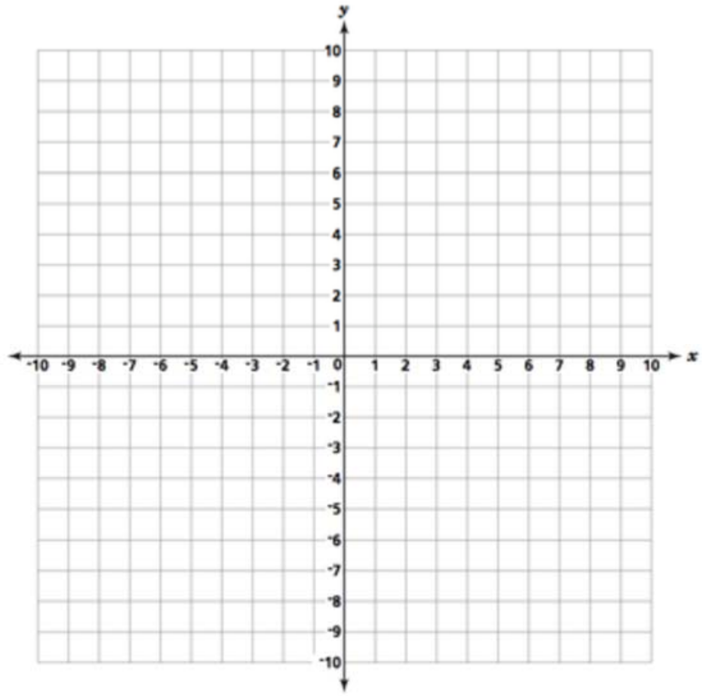
b. What is the maximum height?___________

c. How long did it take for the rocket to reach the ground?_____________

3. The following equation represents the path of a donut hole being thrown by Mr. London where x represents the time (in seconds) the donut is in the air and y represents the height (in feet) of the donut.

$f(x)= -x^2+ 4x - 2$

a. Graph the equation to show the path of the donut hole, show at least three points.



b. At what time does the donut reach its maximum height?_______________

c. What is the maximum height of the donut?________________

d. Rewrite the equation in vertex form.________________

4. You are trying to dunk a basketball. You need to jump 2.5 ft in the air to dunk the ball. The height that your feet are above the ground is given by the function $h(t)= -16t^2 + 12t$. What is the maximum height your feet will be above the ground? Will you be able to dunk the basketball?

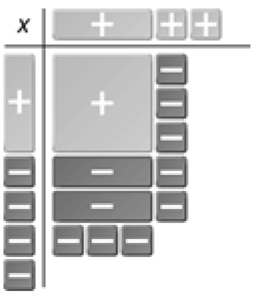
Factoring Quadratic Trinomials

When beginning this concept, it might be helpful to ensure that students can recall the terms factors, and greatest common factor (GCF).

Factors of a number are the numbers that divide the original number evenly and the GCF is the largest common factor of 2 or more numbers.

Additionally, a quick review of multiplying binomials that results in a trinomial in standard form might be useful for students to make the connection between the trinomials and the process of factoring.

From our prior lessons using algebra tiles, students should be able to describe the binomials used in the multiplication problem below.



Name the two binomials shown by the algebra tiles?

(X + 2)(x – 4) which resulted in $x^2 - 2x - 8$.

When you are asked to "factor" an expression, you are being asked to write the expression as the product (multiplication) of other expressions. In factoring, you are given the answer to a multiplication and asked to find the numbers or expressions that were multiplied to get the answer. The first step in every factoring problem is to determine and factor out the GCF.

Let's start with factoring a monomial from a polynomial:

Process:

1. Find the greatest common factor of all terms of the polynomial.
2. To clearly see what is happening, list each term as the product of the GCF and the other factor.
3. Use the distributive property to factor out the GCF.

Example 1: Factor: $2x^4 + 6x^3 - 4x^2$.

Solution:

1. GCF = $2x^2$
2. $2x^2 \cdot x^2 + 2x^2 \cdot 3x - 2x^2 \cdot 2$
3. $2x^2(x^2 + 3x - 2)$ ANSWER

You can always tell if your factoring is correct!!!

Check your factored answer by multiplying through your answer using the distributive property. The result should be the expression with which you started.

$$2x^2(x^2 + 3x - 2) = 2x^4 + 6x^3 - 4x^2$$

Example 2:

Next we will factor quadratic trinomials of the form $ax^2 + bx + c$, a=1

In plain English: If, the leading coefficient (a) is 1, you need to find "the factors that multiply to give the last term (c), and add to give the middle term's coefficient (b).

Factoring $x^2 + bx + c$				
WORDS	**EXAMPLE**			
To factor a quadratic trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b.	To factor $x^2 + 9x + 18$, look for factors of 18 whose sum is 9.			
	Factors of 18	Sum		
	1 and 18	19	x	
If no such Integers exist, the trinomial is not factorable.	2 and 9	11	x	
	3 and 6	9	✓	$x^2 + 9x + 18 = (x + 3)(x + 6)$

If c is positive, the constant terms of the factors have the same sign.
If c is negative, the one constant term of the factors is positive and one is negative.
Table Method:
Step 1: Look at $x^2 + 11x +30$. Find the values of b and c. b =11 and c = 30
Step 2: Find the factors of c, 30, and have the sum of b, 11.

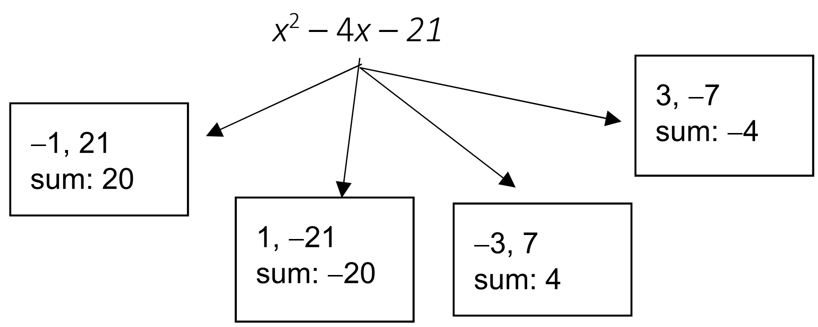
Factors of 30	Sum of Factors
1 and 30	1 + 30 = 31
2 and 15	2 + 15 = 17
3 and 10	3 + 10 = 13
5 and 6	5 + 6 = 11

Step 3: Use this factor pair to factor the trinomial. $x^2 + 11x + 30 = (x + 5)(x + 6)$.

Guess and Check

Factor $x^2 - 4x - 21$

First notice that, *c*, the product term is negative. This will result if one term is positive and one negative. The *b* value in this case is also negative. Students will have to use the rules of integers to determine which factor is positive and which is negative when writing the factored form of the trinomial.



By using guess and check, students can factor the above trinomial as (x + 3)(x -7).

Guided Practice: For the following problems, use the guess and check method to find the factored form of each trinomial.

1. $x^2 + x - 6$ Factors of -6 are [-1,6], [6, -1], [2, -3], [-2, 3]
Sum of 1 ✔
Factor terms are (x – 2)(x + 3)

2. $x^2 - 8x + 15$ Guess and Check

3. $x^2 - x - 12$ Guess and Check

4. $x^2 - 11x + 24$ Guess and Check

5. $x^2 + 10x + 21$ Guess and Check

1. When c is negative, one factor is ___________________ and one factor is ______________________________.

2. When c is positive, both factors are ______________________________ or both factors are ______________________________.

SELF CHECK

____ 3.

_______________When b is negative and c is positive, both factors are

______________________________.

4.To factor a trinomial, find two factors whose ______________________________ is equal to b and whose ______________________________ is equal to c.

Questions To Ponder

When factoring a trinomial of the form $x^2 + bx + c$, where **c is negative**, one binomial factor contains a positive factor of c and one contains a negative factor of c. How do you know which factor of c should be positive and which should be negative?

Example!

Guided Practice :

Next we will factor quadratic trinomials of the form $ax^2 + bx + c$, a>1

Write the factored form of the quadratic expression

$$2b^2 + 17b + 21$$

Factors of $2b^2$: 2b, b

Factor pairs of 21: 1, 21
3, 7

Factors of $2b^2$

Factor pairs of 21

(__2b__ + __7__) (__b__ + __3__)

Use any combination of pairs to make your first guess

7b

Find the inside product

6b

Find the outside product

7b + 6b = 13b ≠ 17b

Since the sum of the inside and outside products is not the linear term of the quadratic expression, we chose the wrong pairs or the wrong order of factors.
Next, we will try another combination of factors.

$2b^2 + 17b + 21$

Factors of $2b^2$ | Factor pairs of 21

(2b + 3) (b + 7)

3b

Find the inside product

14b

Find the outside product

Factors of $2b^2$: 2b, b

Factor pairs of 21: 1, 21
3, 7

Use any combination of pairs to make your first guess

3b + 14b = 17b

Since the sum of the inside and outside products IS the linear term of the quadratic expression, we have the correct pairs of factors. The factored form of the quadratic expression is $(2b + 3)(b + 7)$.

Tips for factoring:

1. Check the inside and outside products mentally so you do not get frustrated by a lot of writing.
2. For expressions with a negative constant, be sure to list factor pairs with both +/- and -/+ combinations.
3. If the sign of the linear term is wrong but the number is correct, switch the signs of the factors.
4. Do not erase your incorrect combinations of factors because it is easy to write the same thing again without making progress. Simply, draw a line through the combinations that do not work.
5. More tips from class discussion: ____
____.
6. ____
____.
7. ____
____.

Example! We are ready to factor a quadratic expression with a negative constant.
Factor $5p^2 - p - 18$.

Factors of $5p^2$ | Factor pairs of -18

(________ + ________) (________ + ________)

Find the inside product

Find the outside product

Factors of $5p^2$:

Factor pairs of -18:

Use any combination of pairs to make your first guess

______ + ______ = ______

Is this equal to the linear term?
If not, try another combination.

(_______ + ________) (________ + ________)

Find the inside product

Find the outside product

Use any combination of pairs to make your next guess

______ + ______ = ______

Is this equal to the linear term?
If not, try another combination.

(_______ + ________) (________ + ________)

Find the inside product

Find the outside product

Use any combination of pairs to make your next guess

______ + ______ = ______

Is this equal to the linear term?
If not, try another combination.

(_______ + ________) (________ + ________)

Find the inside product

Find the outside product

Use any combination of pairs to make your next guess

______ + ______ = ______

Is this equal to the linear term?

Questions To Ponder

In the previous example, the leading coefficient was prime. What changes when the leading coefficient is composite?

SELF CHECK

Use the Distributive property to check factoring for the following problems in this lesson. Try the following:

1. $2x^2 + 11x + 12$

2. $10x^2 - 31x + 15$

3. $2x^2 - 13x + 21$

4. $8x^2 + 26x + 15$

Factoring Trinomials Practice

1. $k^2+18k+81$	2. $v^2-13v+30$
3. $3v^2-5v-28$	4. $7x^2+52x+60$
5. $-3x^2+31x-70$	6. $20v^2-104v+20$
7. $9r^2+87r+54$	8. $-25a^2+185a-70$
9. $2x^2+25x+63$	10. $-5k^2+48k+20$
11. $9p^2-39p-30$	12. $2p^2-11v-63$

Factoring Trinomials Investigation

Name:________________________________ Date:________________ Factoring Trinomials Investigation

1. Use Algebra Tiles to model → $x^2 + 5x + 6$

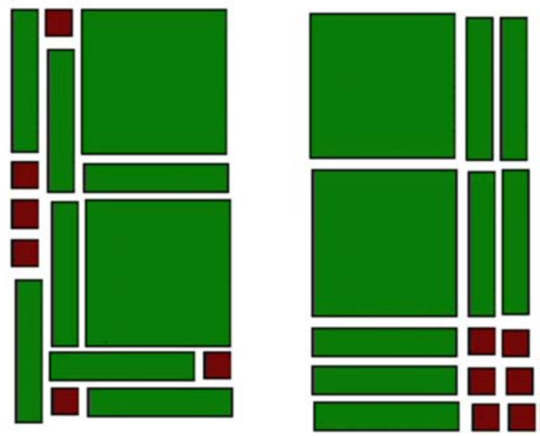
Example

a. Your model must form a perfect rectangle

b. The big squares cannot touch the little squares *(See the example model on the right)* →

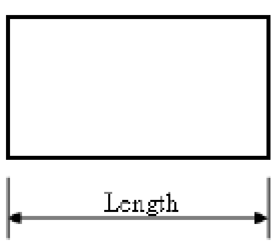
c. The little squares must be together.

d. Only sides of the same length can touch each other



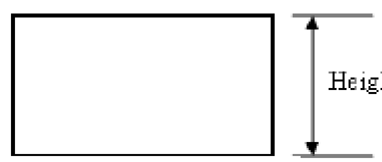
NO! **YES!**

2. What is the length of the polynomial?
(Hint – The length of the rectangle will be the binomial expression on the **bottom** of the rectangle)



The length of the rectangle is __________.

3. What is the height of the polynomial?
(Hint – The height of the rectangle will be the binomial expression on the **right** side of the rectangle)



The height of the rectangle is__________.

4. Rewrite the length and height of the rectangle below. Multiply the two binomials together.

5. What happened when you multiplied the two binomials together?

6. Repeat the process with the trinomial: $x^2 + 6x + 8$.

a. What is the length?

b. What is the height?

c. Multiply the length and height together. What happened?

d. What are the factors of this trinomial? How do you know?

7. Use your Algebra Tiles to find the factors of the following trinomial expressions below:

Trinomial	**Factors**	**Check Your Answer**
1. $x^2+7x + 12$		
2. $x^2+6x + 5$		
3. $x^2+8x + 16$		
4. $x^2+2x + 1$		
5. $x^2+ 9x +$ **20**		
6. $x^2+10x + 25$		
7. $x^2+30x + 200$		

8. What can we conclude about the factors of a trinomial?

9. Use the Algebra Tiles to model $\mathbf{x^2+ 6x + 9}$**.** Next, use more Algebra Tiles to model $\mathbf{x^2+ 6x + 8}$ beside it. Then, write down the factors of both trinomials below.

$\mathbf{x^2+ 6x + 9}$	$\mathbf{x^2+ 6x + 8}$
Factors →	**Factors →**

a. How are the two trinomials different? How are they the same? How did the different constant values affect the shape of the rectangle? How did it affect the factors?

What Happened When the Boarding House Blew Up?

Factor each trinomial below. Find one of the factors in **each** column of binomials. Notice the letter next to one factor and the number next to the other. Write the letter in the box at the bottom of the page that contains the matching number.

1. $3x^2 + 7x + 2$
2. $2x^2 + 5x + 3$
3. $3x^2 - 16x + 5$
4. $7x^2 - 9x + 2$
5. $6u^2 + 5u + 1$
6. $8u^2 - 9u + 1$
7. $10u^2 + 17u + 3$
8. $9u^2 - 9u + 2$
9. $5u^2 + 11u + 6$

5 $(5u + 3)$	Y $(3u - 2)$
3 $(x - 1)$	E $(x - 5)$
8 $(3x + 1)$	G $(8u - 1)$
14 $(3u - 1)$	O $(7x - 2)$
6 $(2u + 3)$	R $(5u + 1)$
15 $(x + 1)$	W $(x + 2)$
9 $(5u + 6)$	L $(7x + 2)$
7 $(2u + 1)$	I $(2x + 3)$
11 $(3x - 1)$	E $(u + 1)$
17 $(u - 1)$	S $(3u + 1)$

10. $3n^2 + 2n - 1$
11. $5n^2 - 4n - 1$
12. $2n^2 + 5n - 3$
13. $7n^2 - 13n - 2$
14. $3t^2 + 14t - 5$
15. $4t^2 - 11t + 7$
16. $6t^2 + 5t - 1$
17. $3t^2 - 20t - 7$

12 $(3t - 1)$	N $(n + 3)$
5 $(n - 1)$	R $(t - 1)$
4 $(3t + 1)$	P $(2t + 1)$
10 $(n - 2)$	O $(n + 1)$
13 $(t + 1)$	F $(t + 5)$
2 $(3n - 1)$	E $(5n + 1)$
16 $(2n - 1)$	M $(t - 7)$
4 $(3t - 7)$	R $(7n + 1)$
1 $(4t - 7)$	L $(6t - 1)$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

OBJECTIVE 3-o: To factor trinomials of the form $ax^2 + bx + c$, where a is a positive integer greater than 1.

Quadratic equation in one variable is the equation with standard form $ax^2+bx+c=0$.

- a, b and c are some numbers and x is variable. Note, that a can't be zero.

In essence, quadratic equation is nothing more than quadratic polynomial ("quad" means square) on the left hand side, and zero on the right hand side.

a quadratic equation:

- the leading coefficient must contain an x^2 term
- must NOT contain terms with degrees higher than x^2 eg. x^3, x^4 etc

Examples of NON-quadratic Equations

- $bx - 6 = 0$ is NOT a quadratic equation because there is no x^2 term.
- $x^3 - x^2 - 5 = 0$ is NOT a quadratic equation because there is an x^3 term (not allowed in quadratic equations).

Signs Patterns:

Equation	Template
$x^2 + bx + c$	$(+)(+)$
$x^2 + bx - c$	$(+)(-)$
$x^2 - bx + c$	$(-)(-)$
$x^2 - bx - c$	$(+)(-)$

	-C	-C
+C	+	+
-C	-	-

Example! Factor Quadratic Equation with coefficient equal to 1:

An equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$ is called a quadratic equation. Its related quadratic function is $y = ax^2 + bx + c$.

Steps to Factoring Quadratic:

- To solve a quadratic equation by factoring,
- Put all terms on one side of the equal sign, leaving zero on the other side.
- Factor.
- Set each factor equal to zero.
- Solve each of these equations.
- Check by inserting your answer in the original equation.

Example 1: $X^2 + 8x = -15$

Step 1: Put all terms on one side of the equal sign, leaving zero on the other side:

$X^2 + 8x + 15 = -15 + 15$

$X^2 + 8x + 15 = 0$

Step 2: Find the factors the constant (c) to give you the sum of the middle term (bx).

Factors of c: 1 & 15; 3 & 5; -3 & -5; -1 & -15

$1 + 15 \neq 8$ $\qquad$ $-1 + (-15) \neq 8$

$3 + 5 = 8$ $\qquad$ $-3 + (-5) \neq 8$

Step 3: Set each factor equal to zero:

$(x + 3) = 0$ $\qquad$ $(x + 5) = 0$

$X + 3 - 3 = 0 - 3$ $\qquad$ $x + 5 - 5 = 0 - 5$

$X = -3$ $\qquad$ $x = -5$

Step 4: Check the solution

$x^2 + 8x = -15$

$(-3)^2 + 8(-3) = -15$ $\qquad$ $(-5)^2 + 8(-5) = -15$

$9 - 24 = -15$ $\qquad$ $25 - 40 = -15$

$-15 = -15$ $\qquad$ $-15 = -15$

SELF CHECK

Factor Quadratic Equation with coefficient equal to 1:

1. $x^2 + 4x = -4$

2. $x^2 - 12x + 32 = 0$

3. $x^2 - 7x = -12$

4. $x^2 + 3x - 18 = 0$

Factoring quadratic with Greatest Common Factor:

The **greatest common factor** (GCF) for a polynomial is the largest monomial that is a factor of (divides) each term of the polynomial.

Note: The GCF must be a factor of **EVERY** term in the polynomial.

Steps to factor quadratic with common factors:

- Find the largest number common to every coefficient or number.
- Find the GCF of each variable.
  - It will always be the variable raised to the smallest exponent.
- Find the terms that the GCF would be multiplied by to equal the original polynomial.

It looks like the distributive property when in factored form...GCF (terms).

Example 2: $4x^2 - 8x - 60 = 0$

Step 1: Find the GCF factor of the equation

4, 8 & 60 all multiples of 4
$4(x^2 - 2x - 15) = 0$

Step 2: Find the factors the constant (c) to give you the sum of the middle term (bx).

-15 & 1, 15 & -1; -5 & 3; 5 & -3
$4(x + 3)(x - 5) = 0$

Step 3: Set each factor equal to zero:

$x + 3 = 0$	$x - 5 = 0$
$x + 3 - 3 = 0 - 3$	$x - 5 + 5 = 0 + 5$
$x = -3$	$x = 5$

Step 4: Check the solution

$4x^2 - 8x - 60 = 0$

$4(-3)^2 - 8(-3) - 60 = 0$	$4(5)^2 - 8(5) - 60 = 0$
$36 + 24 - 60 = 0$	$100 - 40 - 60 = 0$
$0 = 0$	$0 = 0$

Factor Quadratic Equation with Greatest Common Factor:

SELF CHECK

1. $3v^2 - 27v - 30 = 0$
2. $6m^2 + 12m = 144$
3. $5b^2 + 45b = 0$
4. $2x^2 + 28x + 96 = 0$

Questions To Ponder

1. $7x^2 - 49x = -84$
2. $2x^2 + 18x + 36 = 0$

Example! Non-factorable Quadratic Equations:

Sometimes it is impossible to factor a polynomial into the product of two polynomials having integer coefficients. Such polynomials are said to be **non factorable over the integers.**

For example, $x^2 + 3x + 7$ is non factorable over the integers because there are no integers whose product is 7 and whose sum or difference is 3.

The trial method sometimes can be used to factor trinomials of the form $ax^2 + bx + c$, which do not have a leading coefficient of 1.

Example 3: $4x^2 + 12x + 20 = 0$

Step 1: Find the Greatest Common Factors:

4, 12 & 20 all are multiples of 4

GCF is 4

$4(x^2 + 3x + 5) = 0$

Step 2: Find the factors the constant (c) to give you the sum of the middle term (bx).

$x^2 + 3x + 5 = 0$ - Multiply (a)(c) = b

$1 + 5 \neq 3$ - factors of 5: 1,5

$6 \neq 3$

The equation is not factorable because there are factors of 5 which can add to give the middle term.

Factor the quadratic equations:

SELF CHECK

1. $9r^2 – 7r -10 = 0$

2. $x^2 + 11x = - 7$

3. $x^2 + 7x + 16 = 0$

Example! Factoring quadratic with leading coefficient ≠ 1:

$$3x^2 - 23x - 8 = 0$$

Step 1: Multiply the lead coefficient by the constant.
We will refer to this as the "product".

$3x^2 - 23x - 8$

Product: $3(-8) = -24$ (Don't forget to take the sign in front of each term.)

Step 2: Identify the linear term (middle term).
We will refer to this as the "sum".

$3x^2 - 23x - 8$

Sum: -23 (Don't forget to take the sign in front of each term.)

Step 3: Find two numbers that match the product and the sum.
(Two numbers that we multiply to get -24 and add to get -23)

- First think of all the factors of –24
  (-12 and 2) or (12 and -2)
  (-8 and 3) or (8 and -3)
  (-6 and 4) or (6 and -4)
  (-24 and 1) or (24 and -1)
- Which set has a product of -24 and sum of -23?
  (-24 and 1) $(-24)(1) = \mathbf{-24}$ and $(-24)+1 = \mathbf{-23}$
- **Our Magic Numbers are: -24 and 1**

Step 4: Replace the middle term with the sum of the two magic numbers.

$3x^2 - 23x - 8$

$3x^2 - 24x + 1x - 8$

Notice how both of these expressions are equal. $-24x + 1x$ is still equal to the original $-23x$.

Step 5: Factor using the grouping method.

$3x^2 - 24x$ $+ 1x - 8$	Divide into 2 groups
$3x(x - 8)$ $1(x - 8)$	Factor out the GCF.
	**Remember if there is no GCF, factor out a 1.
$(x-8)(3x+1)$	Factor out the GCF of (x-8)

Step 6: **Final Factors: (3x+1) (x-8)**

Step 7: Set each factor to zero

$3x+1 = 0$	$x - 8 = 0$
$3x + 1 - 1 = 0 - 1$	$x - 8 + 8 = 0 + 8$
$3x = -1$	$x = 8$
$X = \frac{-1}{3}$	

Example 5: $6x^2 + 5x - 6 = 0$

Step 1: Multiple leading coefficient and the constant.

Product 6 (-6) = -36

Step 2: Identify the middle term

- Factors of -35 which equal to 5
- -1 , 36; -2, 18; -3, 12; -4, 9; -6, 6

-4 + 9 = 5

Step 3: Replace the middle term with the sum of the numbers

$6x^2 + 9x - 4x - 6 = 0$

Step 4: Factor using the grouping method:

$(6x^2 + 9x) + (-4x - 6) = 0$

$(3x - 2)(2x + 3) = 0$

$3x - 2 = 0$	$2x + 3 = 0$
$3x - 2 + 2 = 0 + 2$	$2x + 3 - 3 = 0 - 3$
$3x = 2$	$2x = -3$
$x = \frac{2}{3}$	$x = \frac{-3}{2}$

Factor and solve quadratic equations:

SELF CHECK

1. $9k^2 - 11k + 2 = 0$

2. $2k^2 + 17k = -21$

3. $2n^2 + 15n + 7 = 0$

Factor quadratic equations with coefficient not equal to 1:

1. $8y^2 - 10y - 3 = 0$

2. $5n^2 + 30n + 40 = 0$

Solving Quadratic Equations

In the previous lessons, you discovered which characteristics of quadratic functions can be found in the vertex forms and standard forms of the equation. You also recognized that the zeros of a quadratic function cannot be found in either vertex or standard forms. In this lesson, you will learn to find the factored form of a quadratic function and use it to find the zeros of the quadratic function.

GUIDED NOTES

When you know the ____________________ of a quadratic equation, you can apply the Zero Product Property to find the zeros of the function.

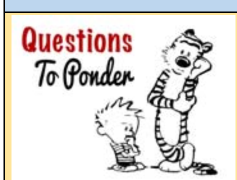
Zero Product Property: If ________ $= 0$, then ____ $= 0$ or ____ $= 0$.

Note: We will wait until the next page to learn how to convert to this form.

Use the Zero Product Property to solve $(2x+5)(x-4)=0$

The factors of the quadratic equation are $2x+5$ and $x-4$. When they are multiplied together, their product is zero. So, we are ready to apply to Zero Product Property to solve the equation.

$(2x+5)(x-4)=0$			Given
$2x+5=0$	or	$x-4=0$	Zero Product Property
$x=-\frac{5}{2}$	or	$x=4$	Solve each linear equation (show the steps if you need to)



The order of operations tells us that we should divide before we subtract. Why is the first equation solved by subtracting then dividing?

SELF CHECK

Use the Zero Product Property to solve the following equations.

$(x+1)(x+9)=0$ *$(x+2)(3x-1)=0$* *$x(x-6)=0$*

Questions To Ponder

Do you know how to write the factored form of a quadratic equation with zeros of 5 and 12?.

You are ready to combine factoring with the Zero Product Property to find the zeros of quadratic functions. Zeros sometimes called roots are where the quadratic function will cross the x-axis.

Example!

Find the zeros of the quadratic functions.

$y=3x^2-5x+2$	Given
$0 = 3x^2-5x+2$	Substitute 0 for y to find where the graph crosses the x axis.
$0=(3x-2)(x-1)$	Factor
$0= 3x-2$ or $0= x-1$	Zero Product Property
$X = \frac{2}{3}$ or $x = 1$	Solve each linear equation

$(\frac{2}{3}, 0)$ and $(1,0)$ are the zeros

Questions To Ponder

1. Can you graph the function in the previous example? What are the easiest characteristics to identify? Which characteristics are the minimum you must find to be able to sketch the graph?

2. Explain how you can know there are never more than two solutions to a quadratic equation, based on what you know about the graph of a quadratic function.

Example!

Now we can solve a quadratic equation.

$7x^2 = 32x + 60$	Given
______ + ______ + ______ = 0	Put the equation in standard form
	List the factors of the quadratic term:
	List the factors of the constant:
(____ + ____) (____ + ____) = 0	Make your first guess for the factors.
_____ + _____ = _____	Are the factors correct? If not, try another one.
(____ + ____) (____ + ____) = 0	Make your next guess for the factors.

_____ + _____ = _____ Are the factors correct? If not, try another one.

(____ + ____) (____ + ____) = 0 Make your next guess for the factors.
_____ + _____ = _____ Are the factors correct? If not, try another one.

The factored form is
(7x + ____)(____ - 6) = 0

_________ = 0 or _________ = 0 Zero Product Property
x = _____ or x = _____ Solve each linear equation

Since you can use the Distributive Property to check factoring, you are able to check your work for problems in this lesson. Try the following:

SELF CHECK

1. $2x^2 + 11x + 12$

2. $10x^2 - 31x + 15$

3. Find the zeros of the function. $3x^2 - 2x - 5$

4. Solve the quadratic equation. $2 = 4x^2 - 7x$

1. $f(x) = x^2 + 3x + 3$ is a quadratic function that does Not factor. What ideas do you have for finding its zeros?

2. If none of the factor pairs for *a* and *c* result in the correct value for *b*, what do you know about the quadratic?

3. What happens if you do not remove the common factor from the coefficients before trying to factor the quadratic equation?

Independent Practice

Re-Teaching and Independent Practice

Factoring Quadratic with coefficient ≠ 1 and Greatest Common Factors:

Use the Zero Product Property to solve each equation. The first one is done for you.

1. $2x^2 - 5x + 2 = 0$

   $(2x - 1)(x - 2) = 0$

   $2x - 1 = 0$ or $x - 2 = 0$

   $2x = 1$ or $x = 2$

   $x = \frac{1}{2}$ or $x = \underline{2}$

2. $3x^2 + 12x - 15 = 0$

   $3(x^2 +$ ___ $x -$ ___$) = 0$

   3(_______)(_______) = 0

   (______) = 0 or (______) = 0

   $x =$ ____ or $x =$ _____

GCF Reteaching Quadratic:

$3d^2 - d = 0$

Step 1. Factor.

$d(3d - 1) = 0$ Factor out the greatest common factor, d.

Step 2. Use the zero product property to solve.

According to the zero product property, if $d(3d - 1) = 0$, then d must be 0 or $3d - 1$ must be 0. Write the two equations and solve for d.

$d = 0$	or	$3d - 1 = 0$
$d = 0$		$3d = 1$
		$d = 1/3$

The solutions are $d = 0$ and $d = 1/3$.

Evaluate the quadratic equations with coefficient ≠ 1:

1. $9x^2 - 3x = 0$	2. $4p^2 - 6p = 0$
3. $4x^2 + 18x - 6 = 0$	4. $3x^2 = -5x - 2$

5. $8x^2 - 25x + 3 = 0$	6. $6x^2 + 19x = -3$
7. $6x^2 - 17x - 3 = 0$	8. $3x^2 - 24x + 48 = 0$
9. $x^2 - x - 30 = 0$	10. $4x^2 = 12x - 40$

Independent Practice

Re-Teaching and Independent Practice

Factoring Quadratic with coefficient = 1:

To find the factors for a trinomial in the form $x^2 + bx + c$, answer these 2 questions.

1. What numbers have a product equal to c?
2. What numbers have a sum equal to b?

Find numbers for which the answer to both is yes.

Factor $x^2 + 5x + 6$.

What numbers have a product equal to c, 6?

1 and 6   –1 and –6   2 and 3   –2 and –3

What numbers have a sum equal to b, 5?

1 and 6   –1 and –6   2 and 3   –2 and –3

The factors of $x^2 + 5x + 6$ are $(x + 2)$ and $(x + 3)$.

Solve the trinomial by setting it equal to 0. Factor and use the Zero Product Property to solve.

Example

Solve $x^2 + 5x + 6 = 0$.

$x^2 + 5x + 6 = 0$

$(x + 2)(x + 3) = 0$ *Factor $x^2 + 5x + 6$.*

$x + 2 = 0$ or $x + 3 = 0$ *Set each factor equal to 0.*

$x = -2$ or $x = -3$ *Solve each equation for x.*

Evaluate the quadratic equations with coefficient = 1 (non-factorable):

1. $x^2 - 3x - 4 = 0$	2. $x^2 = -2x + 15$
3. $x^2 - 18x + 81 = 0$	4. $x^2 = -4x + 21$

5. $x^2 - 7x - 44 = 0$	6. $x^2 = 5x - 6$
7. $x^2 - 14x + 33 = 0$	8. $x^2 - 18x + 81 = 0$
9. $x^2 - x - 20 = 0$	10. $x^2 = -19x - 18$

Transformations of Quadratic Functions – Application

Quadratic Fanatic and the Case of the Foolish Function

Adapted From Jody Haynes, Fayette County School System

"Quadratic Fanatic, we need your help!" declared the voice on the answering machine. While out helping solve the town's problems, a crime had occurred at the Function Factory, and now it was up to the Quadratic Fanatic to straighten things out.

When he arrived at the factory, Quadratic Fanatic was given three different groups of suspects, each group representing a different shift. He was told that an employee from each shift had worked together to commit the crime.

The employees from the second shift were all quadratic functions in standard form. "They always follow standard procedure," said the manager. The list of suspects from the second shift is below. For each suspect, factor and find its solutions.

Function	Factors	First Solution	Second Solution
H. $g(x) = 3x^2 - 10x + 3$			
I. $g(x) = 3x^2 - 21x + 30$			
J. $g(x) = 2x^2 - 2x - 4$			
K. $g(x) = x^2 - x - 12$			
L. $g(x) = x^2 + 3x - 18$			
M. $g(x) = x^2 - 12x + 35$			
N. $g(x) = 5(x-4)^2 - 125$ Hint: Solve by Square Root Method!			

According to a several witnesses, the following information about the suspect was gathered:

"Both solutions were integers." Which of the employees above could be suspects? ____________________________

"One of the solutions was negative." Which of the employees above could be suspects? ________________________

"One of the solutions was two." Which of the employees above could be suspects? ____________________________

"One of the solutions was negative one." Which of the employees above could be suspects? _____________________

Based on the above information, which employee is guilty? Explain how you know.

A1.U4.C2.G.04.**Task.FactorToSolveQ**

Part I: Factoring Expressions

Factor each problem using the method presented.

1. Greatest Common Factor

Example: $12a^3 + 15a =$
$3a\left(4a^2 + 5\right)$

a. $24x^2 - 8x$

b. $10x^2 + 35x$

c. $12x^2 - 9x + 15$

d. $9m^2 - 4n + 12$

e. $3n^3 - 12n^2 - 30n$

2. Factoring Polynomials Using Algebra Tiles

a)	2x	-3
2x	$4x^2$	-6x
+5	10x	-15

$4x^2 + 4x - 15 = (2x-3)(2x+5)$

b)		

$9x^2 + 12x + 4 = (\quad)(\quad)$

c)		

$14x^2 + 39x + 10 = (\quad)(\quad)$

d)		

$10x^2 + 19x - 15 = (\quad)(\quad)$

e)		

$6x^2 + 11x - 10 = (\quad)(\quad)$

f)		

$10x^2 - 17x + 3 = (\quad)(\quad)$

g)		

$6x^2 + 25x - 9 = (\quad)(\quad)$

h)		

$20x^2 + 21x - 5 = (\quad)(\quad)$

i)		

$30x^2 - 13x - 10 = (\quad)(\quad)$

HOMEWORK

Homework #___2_____

Using Factors to Solve Quadratic Equations:

1. $24b^2 - b = 10b - 1$	2. $5a^2 - 20a + 20 = 0$
3. $4m^2 + 16m = 48$	4. $3x^2 = 9x$
5. $3p^2 + 12p - 15 = 0$	6. $5q^2 + 6q = -5q - 2$
7. $2r^2 - 21 = -11r$	8. $w^2 - 2w = 8$
9. $4x^2 + 16x = 48$	10. $5z^2 - 2z + 20 = 20z$

HOMEWORK

Homework #___1_____

Using Factors to Solve Quadratic Equations:

1. $y^2 + 3y + 2 = 0$	2. $a^2 - a - 20 = 0$
3. $m^2 - 7m = 8$	4. $d^2 + 9d = -20$
5. $h^2 + h - 2 = 0$	6. $y^2 + 4y - 2 = 0$
7. $t^2 + 8t - 9 = 0$	8. $w^2 - 2w = 8$
9. $x^2 - 2x + 7 = 0$	10. $d^2 - 10d + 16 = 0$

Factoring Quadratic with Special Properties:

When factoring there are a few special products that, if we can recognize them, can help us factor polynomials. The first is one we have seen before. When multiplying special products we found that a sum and a difference could multiply to a difference of squares. Here we will use this special product to help us factor:

Perfect square trinomials of the form: $a^2 + 2ab + b^2$

A difference of squares: $a^2 - b^2$

Difference of Squares: $a^2 - b^2 = (a + b)(a - b)$

Perfect Square Trinomials: $a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Difference of Squares:

Special case 1: Difference of Squares Difference of squares is a special case of factoring, which follows a specific pattern. Firstly, it is important to be able to recognize a difference of squares.

For an algebraic expression to be a difference of squares the first and last terms must be perfect squares. The two perfect squares must be subtracted.

Perfect square – a number whose square root is a whole number and/or a variable with an even exponent.

Numbers that are perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, ...

Variables are perfect squares if they have an even exponent (e.g. 2, 4, 6, 8, etc.)

Examples of Difference of Squares NOT Examples of Difference of Squares

$4x^2 - 9$ $16y^4 - 25$ $169p^8 - 1$

$4x^2 + 9$ $16y^5 - 25$ $168p^8 - 1$

If you can recognize a difference of squares, the factoring can be done in one line according to the pattern below. Factoring a difference of squares: $a^2 - c^2 = (a + c)(a - c)$

Notice that the first term inside both brackets is the square root of a^2; the second term inside both brackets is the square root of c^2; the operation sign inside one bracket is + and inside the other bracket is −.

Example 1: Factor. $25x^2 – 121 = 0$.

Step 1: Try to common factor first. There are no common factors of 25x2 and 121.

Step 2: Recognize that this is a difference of squares since $25x^2$ and 121 are both perfect squares AND these two perfect squares are being subtracted.

Step 3: Factor according to $a^2 - c^2 = (a + c)(a - c)$.

$$25x^2 - 121 = (\sqrt{25x^2} + \sqrt{121})(\sqrt{25x^2} - \sqrt{121})$$

$$= (5x + 11)(5x - 11)$$

Step 4: Set each factor to equal zero

$5x + 11 = 0$
$5x + 11 - 11 = 0 - 11$
$5x = -11$
$X = \frac{-11}{5}$

$5x - 11 = 0$
$5x - 11 + 11 = 0 + 11$
$5x = 11$
$x = \frac{11}{5}$

Step 5: To double check the answer, expand $(5x + 11)(5x - 11)$. The result of expanding should be $25x^2 - 121$.

$$(5x + 11)(5x - 11)$$

$$= 25x^2 - 55x + 55x - 121 = 25x^2 - 121 + 0x = 25x^2 - 121$$

Factor Quadratic Equation with Difference of Squares:

1. $x^2 - 4 = 0$

2. $x^2 - 169 = 0$

3. $49x^4 - 9 = 0$

SELF CHECK **4.** $36x^2 = 81$

A difference of squares can sometimes be "disguised" when it is multiplied by a common factor. This is why it's important to always try to common factor an algebraic expression first.

Factoring quadratic with Common Factor:
Example 2:
Factor. $98x^3 - 2x$.

Step 1: Common factor the expression first.

- 98 and 2 are both divisible by 2
- x^3 and x are both divisible by x
- Thus, the common factor of $98x^3$ and $2x$ is $2x$.
- $98x^3 - 2x = 2x(49x^2 - 1)$

Notice that the algebraic expression inside the brackets, $49x^2 - 1$, is a difference of squares.

Step 2: Factor the difference of squares according to the pattern.

$$49x^2 - 1 = (\sqrt{49x^2} + \sqrt{1})(\sqrt{49x^2} - \sqrt{1})$$

$$= (7x + 1)(7x - 1)$$

Step 3: Write the final answer by putting all of the factors together.

$$98x^3 - 2x = 2x(7x + 1)(7x - 1)$$

Step 4: To double check the answer, expand $2x(7x + 1)(7x - 1)$.
The result of the expanding should be $98x^3 - 2x$

$$2x(7x + 1)(7x - 1)$$

Multiply 2x by each term in the first bracket, 7x + 1. (Note: 2x does NOT get multiplied by each bracket since you would be multiplying 2x twice).

$$= (14x^2 + 2x)(7x - 1)$$

Use FOIL Method to multiply binomial by a binomial.

$= 98x^3 - 2x + 14x^2 - 14x^2$ Collect like terms.
$= 98x^3 - 2x + 0x^2$ Simplify $0x^2$.
$= 98x^3 - 2x$

Example!

Perfect Square Trinomial:

Perfect square trinomial is another special case of factoring, which follows a specific pattern. Firstly, it is important to be able to recognize a perfect square trinomial.

For an algebraic expression to be a perfect square trinomial **the first and last terms must be perfect squares. The middle term has to equal to twice the square root of the first term times the square root of the last term.**

Examples of Perfect Square Trinomials	NOT Examples of Perfect Square Trinomials	
$x^2 + 12x + 36$	$x^2 + 12x \boxed{-36}$	-36 is NOT a perfect square
$16y^2 - 40y + 25$	$16y^2 \boxed{-41y} + 25$	$-41y$ does NOT equal to $2(\sqrt{16y^2})(\sqrt{25})$
$169p^2 + 26p - 1$	$\boxed{168p^2} + 26p - 1$	$168p^2$ is NOT a perfect square
$4x^2 + 12x + 9$	$4x^2 + 12x \boxed{+8}$	8 is NOT a perfect square

If you can recognize a perfect square trinomial, the factoring can be done in one line according to the pattern below.

Factoring a perfect square trinomial:

$$a^2 + 2ac + c^2 = (a + c)(a + c) = (a + c)^2$$

$$a^2 - 2ac + c^2 = (a - c)(a - c) = (a - c)^2$$

Notice that the first term inside both brackets is the square root of a^2; the second term inside both brackets is the square root of c^2; the operation sign (+ or −) inside the brackets is the same as the operation sign in front of 2ac.

Example 2: Factor. $36x^2 - 132x + 121 = 0$

Step 1: Find the GCF factor of the equation

There are no common factors of $36x^2$, $132x$ & 121.

Step 2: Recognize that this is a perfect square trinomial since $36x^2$ and 121 are both perfect squares AND the middle term, 132x, is equal to

$2(\sqrt{36x^2})(\sqrt{121}) = 2(6x)(11)$

Step 3: Factor according to $a^2 - 2ac + c^2 = (a - c)^2$:

$$\mathbf{36x^2} - 132x + \mathbf{121} = (\sqrt{36x^2} - \sqrt{121})^2 = (\mathbf{6x} - \mathbf{11})^2$$

$$\mathbf{a^2} \quad - 2ac \; + \; \mathbf{c^2} \;=\; (\sqrt{a^2} - \sqrt{c^2}\,)^2 \;=\; (\mathbf{a} - \mathbf{c})^2$$

Step 4: $(6x - 11)^2 = 0$

$6x - 11 = 0$

$6x - 11 + 11 = 0 + 11$

$6x = 11$

$x = \dfrac{11}{6}$

Step 5: To double check the answer, expand $(6x - 11)^2$. The result of expanding should be $36x^2 - 132x + 121$.

$$(6x - 11)^2 = (6x - 11)(6x - 11)$$

$= 36x^2 - 66x - 66x + 121$
$= 36x^2 - 132x + 121$

SELF CHECK

Factor Quadratic Equation with Perfect Trinomial:

1. $9v^2 + 6v + 1 = 0$

2. $36m^2 + 144m + 144 = 0$

3. $36b^2 - 60b + 25 = 0$

4. $169x^2 - 182x + 49 = 0$

Special Cases

Special case 1: Difference of Squares

Difference of squares is a special case of factoring, which follows a specific pattern. Firstly, it is important to be able to recognize a difference of squares.

For an algebraic expression to be a difference of squares **the first and last terms must be perfect squares. The two perfect squares must be subtracted.**

Perfect square – a number whose square root is a whole number and/or a variable with an even exponent.

Numbers that are perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, …

Variables are perfect squares if they have an ***even*** exponent (e.g. 2, 4, 6, 8, etc.)

Examples of Difference of Squares			NOT Examples of Difference of Squares		
$4x^2 – 9$	$16y^4 – 25$	$169p^8 − 1$	$4x^2 \boxed{+} 9$	$16y^{\boxed{5}} – 25$	$\boxed{168}p^8 – 1$

If you can recognize a difference of squares, the factoring can be done in one line according to the pattern below.

Factoring a difference of squares:

$$a^2 − c^2 = (a + c)\ (a − c)$$

Notice that the first term inside both brackets is the square root of a^2; the second term inside both brackets is the square root of c^2; the operation sign inside one bracket is + and inside the other bracket is −.

Example:

Factor. $25x^2 – 121$.

Step 1: Try to common factor first. There are no common factors of $25x^2$ and 121.

Step 2: Recognize that this is a difference of squares since $25x^2$ and 121 are both perfect squares AND these two perfect squares are being subtracted.

Step 3: Factor according to $a^2 − c^2 = (a + c)\ (a − c)$.

$$25x^2 – 121 = (\sqrt{25x^2} + \sqrt{121})\ (\sqrt{25x^2} – \sqrt{121})$$
$$= (5x + 11)(5x – 11)$$

Step 4 (Optional): To double check the answer, expand $(5x + 11)(5x - 11)$. The result of expanding should be $25x^2 - 121$.

$$(5x + 11)(5x - 11)$$

$$= 25x^2 - 55x + 55x - 121$$

$$= 25x^2 - 121 + 0x$$

$$= 25x^2 - 121$$

Example! Factor Quadratic Equation with coefficient equal to 1:

A difference of squares can sometimes be "disguised" when it is multiplied by a common factor. This is why it's important to **always try to common factor an algebraic expression first**.

Example:

Factor. $98x^3 - 2x$.

Step 1: Common factor the expression first.

98 and 2 are both divisible by 2

x^3 and x are both divisible by x

Thus, the common factor of $98x^3$ and 2x is 2x.

$98x^3 - 2x = 2x(49x^2 - 1)$

Notice that the algebraic expression inside the brackets, $49x^2 - 1$, is a difference of squares.

Step 2: Factor the difference of squares according to the pattern.

$49x^2 - 1 = (\sqrt{49x^2} + \sqrt{1})(\sqrt{49x^2} - \sqrt{1})$

$= (7x + 1)(7x - 1)$

Step 3: Write the final answer by putting all of the factors together.

$98x^3 - 2x = 2x(7x + 1)(7x - 1)$

Step 4 (Optional): To double check the answer, expand $2x(7x + 1)(7x - 1)$. The result of expanding should be $98x^3 - 2x$.

$2x(7x + 1)(7x - 1)$ Multiply 2x by each term in the first bracket, 7x + 1. (Note: 2x does NOT get multiplied by *each* bracket since you would be multiplying 2x twice).

$= (14x^2 + 2x)(7x - 1)$ Use FOIL method to multiply binomial by a binomial.

$= 98x^3 - 2x + 14x^2 - 14x^2$ Collect like terms.

$= 98x^3 - 2x + 0x^2$ Simplify $0x^2$.

$= 98x^3 - 2x$

Example 1:

Step 1:

Factor Quadratic Equation with coefficient equal to 1:

SELF CHECK

Example!

Special case 2: Perfect Square Trinomial

Perfect square trinomial is another special case of factoring, which follows a specific pattern. Firstly, it is important to be able to recognize a perfect square trinomial.

For an algebraic expression to be a perfect square trinomial **the first and last terms must be perfect squares. The middle term has to equal to twice the square root of the first term times the square root of the last term.**

Examples of Perfect Square Trinomials	NOT Examples of Perfect Square Trinomials	
$x^2 + 12x + 36$	$x^2 + 12x \boxed{-36}$	-36 is NOT a perfect square
$16y^2 - 40y + 25$	$16y^2 \boxed{-41y} + 25$	$-41y$ does NOT equal to $2(\sqrt{16y^2})(\sqrt{25})$
$169p^2 + 26p - 1$	$\boxed{168p^2} + 26p - 1$	$168p^2$ is NOT a perfect square
$4x^2 + 12x + 9$	$4x^2 + 12x \boxed{+8}$	8 is NOT a perfect square

If you can recognize a perfect square trinomial, the factoring can be done in one line according to the pattern below.

Factoring a perfect square trinomial:

$$a^2 + 2ac + c^2 = (a + c)(a + c) = (a + c)^2$$

$$a^2 - 2ac + c^2 = (a - c)(a - c) = (a - c)^2$$

Example 2:

Step 1:

SELF CHECK

Factor Quadratic Equation with Greatest Common Factor:

1. $3v^2$ –

1\. $7x^2$ -

Non-factorable Quadratic Equations:

Example:

Factor. $36x^2 - 132x + 121$.

Step 1: Try to common factor first. There are no common factors of $36x^2$, $132x$ and 121.

Step 2: Recognize that this is a perfect square trinomial since $36x^2$ and 121 are both perfect squares AND the middle term, $132x$, is equal to $2(\sqrt{36x^2})(\sqrt{121}) = 2(6x)(11)$

Step 3: Factor according to $a^2 - 2ac + c^2 = (a - c)^2$

$$36x^2 - 132x + 121 = (\sqrt{36x^2} - \sqrt{121})^2 = (6x - 11)^2$$

$$a^2 \quad - 2ac \; + \; c^2 \; = \; (\sqrt{a^2} - \sqrt{c^2}\,)^2 \quad = (a - c)^2$$

Step 4 (Optional): To double check the answer, expand $(6x - 11)^2$. The result of expanding should be $36x^2 - 132x + 121$.

$$(6x - 11)^2 = (6x - 11)(6x - 11)$$

$$= 36x^2 - 66x - 66x + 121$$

$$= 36x^2 - 132x + 121$$

A perfect square trinomial can sometimes be "disguised" when it is multiplied by a common factor. This is why it's important to **always try to common factor an algebraic expression first**.

Example:

Factor. $-18x^3 - 96x^2 - 128x$

Step 1: Common factor the expression first.

$-18x^3$, $-96x^2$ and $-128x$ are all divisible by $-2x$.

Thus, the common factor is −2x.

$-18x^3 - 96x^2 - 128x = -2x(9x^2 + 48x + 64)$

Notice that the algebraic expression inside the brackets, $9x^2 + 48x + 64$, is a perfect square trinomial.

Step 2: Factor the perfect square trinomial according to $a^2 + 2ac + c^2 = (a + c)^2$

$$9x^2 + 48x + 64 = (\sqrt{9x^2} + \sqrt{64})^2 = (3x + 8)^2$$

$$a^2 + 2ac + c^2 = (\sqrt{a^2} + \sqrt{c^2})^2 = (a + c)^2$$

Step 3: Write the final answer by putting all the factors together.

$-18x^3 - 96x^2 - 128x = -2x(3x + 8)^2$

From Step 1. From Step 2.

Step 4 (Optional): To double check the answer, expand $-2x(3x + 8)^2$. The result of expanding should be $-18x^3 - 96x^2 - 128x$.

$-2x(3x + 8)^2 = -2x(3x + 8)(3x + 8)$

$= (-6x^2 - 16x)(3x + 8)$

$= -18x^3 - 48x^2 - 48x^2 - 128x$

$= -18x^3 - 96x^2 - 128x$

Example 3:
Step 2:

SELF CHECK Factor the quadratic equations:

Example! Factoring quadratic with leading coefficient ≠ 1:

Example 5:

SELF CHECK Factor and solve quadratic equations:

1. $9k^2$

Factor quadratic equations with coefficient not equal to 1:

Re-Teaching and Independent Practice

Factoring Quadratic with Special Cases:

1. Rearrange the terms to equal zero.	4. Let $a = \sqrt{\text{first term}}$. Let $b = \sqrt{\text{last term}}$. If whole numbers, continue.
2. Combine like terms.	5. If there are two terms with a minus sign between, use difference of squares.
3. Look for common factors.	6. If there are three terms, and the middle term equals $2ab$, use perfect squares.

Problem 1: $\mathbf{12x^2 - 2 = 25}$

Rearrange: $12x^2 - 2 - 25 = 0$

Combine: $12x^2 - 27 = 0$

Factors: $3(4x^2 - 9) = 0$

Find a and b: $a = \sqrt{4x^2} = 2x$ $b = \sqrt{9} = 3$; here, a and b are whole numbers.

There are two terms with a minus sign between them. So use the difference of squares. The difference of squares is $3(2x - 3)(2x + 3)$.

Problem 2: $\mathbf{50x^2 + 120x + 100 = 2 - 20x}$

Rearrange: $50x^2 + 120x + 100 - 2 + 20x = 0$

Combine: $50x^2 + 140x + 98 = 0$

Factors: $2(25x^2 + 70x + 49) = 0$

Find a and b: $a = \sqrt{25x^2} = 5x$ $b = \sqrt{49} = 7$; here, a and b are whole numbers.

There are three terms, and $2ab = 70x$. So use perfect squares. $2(5x + 7)(5x + 7)$

Evaluate the quadratic equations with special cases:

1. $16x^2 - 9 = 0$	2. $4m^2 - 25 = 0$

3. $4x^2 - 4x + 1 = 0$	4. $x^2 = 49$
5. $49d^2 - 56d + 16 = 0$	6. $x^2 = 5x - 6$
7. $10p^3 - 1960p = 0$	8. $200m^4 + 800m^3 + 8m^2 = 0$
9. $81v^4 - 900v^2 = 0$	10. $3 + 6b + 3b^2 = 0$

HOMEWORK

Homework #___1_____

Solve Quadratic Equations with Special Cases:

1. $48b^2 = 108$	2. $72a^2 + 8 = 48x$
3. $5m^2 – 20 = 0$	4. $64x^2 = 9$
5. $25p^2 - 10p + 1 = 0$	6. $q^2 - 8q = -16$
7. $16r^2 + 9 = -24r$	8. $9w^2 + 30w = -25$
9. $4x^2 - 12x +9 = 0$	10. $25z^2 -20z + 4 = 0$

Solving Quadratics by Square Root, Completing the Square, and Quadratic Formula – Guided Notes

Solve by TAKING SQUARE ROOT

SQUARE ROOT PRINCIPLE
If $x^2 = k$, then $x = \pm\sqrt{k}$

When should I solve using this method?

A) When the equation is in standard form with no "*bx*" term: $y = ax^2 + c$
B) When the equation is in vertex form: $y = a(x - h)^2 + k$

How do I use this method?

Step 1: Isolate the perfect square.
Step 2: Take the square root of each side. Don't forget the ±.
Step 3: Simplify the radical.
Step 4: Separate the ± into to equations and solve for x.

Example! **Solve each equation by taking the square root.**

1. $2x^2 - 5 = 95$

Step 1: Isolate the perfect square.
To isolate x^2 we need to add 5 then divide by 2.
$2x^2 = 100$
$x^2 = 50$

Step 2: Take the square root of each side. Don't forget ±.
$\sqrt{x^2} = \pm\sqrt{50}$

Step 3: Simplify the radical.
$x = \pm 5\sqrt{2}$

Step 4: Separate the ± into to equations and solve for x.
$x = 5\sqrt{2}$ and $x = -5\sqrt{2}$

*****Since x is already isolated, there is no need to continue solving for x.*****

2. $3(x + 7)^2 - 2 = 22$

Step 1: Isolate the perfect square.
To isolate $(x + 7)^2$ add 2 then divide by 3.
$3(x + 7)^2 = 24$
$(x + 7)^2 = 8$

Step 2: Take the square root of each side. Don't forget ±.
$\sqrt{(x + 7)^2} = \pm\sqrt{8}$

Step 3: Simplify the radical.
$x + 7 = \pm 2\sqrt{2}$

Step 4: Separate the ± into to equations and solve for x.
$x + 7 = 2\sqrt{2}$ and $x + 7 = -2\sqrt{2}$
$x = -7 + 2\sqrt{2}$ and $x = -7 - 2\sqrt{2}$

*****It is important to understand that there are two solutions but the answer can be written in a condensed form as*** $x = -7 \pm 2\sqrt{2}$.******

SELF CHECK **Solve each equation by taking the square root.**

1. $9x^2 = 16$

2. $4(x - 6)^2 - 35 = 14$

Solve by COMPLETING THE SQUARE

GUIDED NOTES

Completing the Square

WORDS	NUMBERS	ALGEBRA
To complete the square of $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$. • Divide b by 2. • Then square the result.	$x^2 + 6x + \blacksquare$ $x^2 + 6x + \left(\frac{6}{2}\right)^2$ $x^2 + 6x + 9$ $(x + 3)^2$	$x^2 + bx + \blacksquare$ $x^2 + bx + \left(\frac{b}{2}\right)^2$ $\left(x + \frac{b}{2}\right)^2$

When should I solve using this method? A) When the equation is not factorable in standard form.

How do I use this method?

Step 1: Get all the x terms on the left and the constant on the right.
Step 2: Divide each term by a (the coefficient of x.)
Step 3: Complete the square: Divide b by 2. Then square the result.
To keep the equation balanced add $\left(\frac{b}{2}\right)^2$ to both sides.
Step 4: Write the trinomial as a binomial in the factored form: $\left(x + \frac{b}{2}\right)^2$
Step 5: Take the square root of each side and solve for x.
Follow the square root method steps from the previous section.

Example! **Solve each equation by completing the square.**

1. $x^2 + 32 = 12x$

Step 1: Get all the x terms on the left and the constant on the right.
$x^2 - 12x = -32$

Step 2: Divide each term by a (the coefficient of x.)
Since a = 1, we can skip step 2.

Step 3: Complete the square: Divide b by 2. Then square the result.
To keep the equation balanced add $\left(\frac{b}{2}\right)^2$ to both sides.

$x^2 - 12x$ _____ $= -32$ _____
$x^2 - 12x \underline{+36} = -32 \underline{+36}$

$\frac{-12}{2} = -6$
$(-6)^2 = 36$

Step 4: Write the trinomial on the left in the factored form: $\left(x + \frac{b}{2}\right)^2$ and simplify on the right.
$(x-6)^2 = 4$

Step 5: Take the square root of each side and solve for x.
$\sqrt{(x-6)^2} = \pm\sqrt{4}$
$x - 6 = \pm 2$
$x - 6 = 2$ and $x - 6 = -2$
$x = 8$ and $x = 4$

2. $2x^2 = 54 - 8x$

Step 1: Get all the x terms on the left and the constant on the right.
$2x^2 + 8x = 54$

Step 2: Divide each term by a (the coefficient of x.)
$\frac{2x^2}{2} + \frac{8x}{2} = \frac{54}{2}$
$x^2 + 4x = 27$

Step 3: Complete the square: Divide b by 2. Then square the result.
To keep the equation balanced add $\left(\frac{b}{2}\right)^2$ to both sides.

$x^2 + 4x$ _____ $= 27$ _____
$x^2 + 4x \underline{+4} = 27 \underline{+4}$

$\frac{4}{2} = 2$
$(2)^2 = 4$

Step 4: Write the trinomial on the left in the factored form: $\left(x + \frac{b}{2}\right)^2$ and simplify on the right.
$(x+2)^2 = 31$

Step 5: Take the square root of each side and solve for x.
$\sqrt{(x+2)^2} = \pm\sqrt{31}$
$x + 2 = \pm\sqrt{31}$
$x + 2 = \sqrt{31}$ and $x + 2 = -\sqrt{31}$
$x = -2 + \sqrt{31}$ and $x = -2 - \sqrt{31}$

***It is important to understand that there are two solutions but the answer can be written in a condensed form as** $x = -2 \pm \sqrt{31}$.**

SELF CHECK **Solve each equation by completing the square.**

1. $x^2 - 2x - 5 = 0$

2. $2x^2 - 12x + 16 = 0$

Tina tried to solve the quadratic equation x² + 5x - 2 = 0 by completing the square, but she made a mistake.
In which line of her work, did she make the mistake? <u>Explain</u> (in complete sentences) and correct her error.

$x^2 + 5x - 2 = 0$

[Line 1] $\Rightarrow x^2 + 5x = 2$

[Line 2] $\Rightarrow x^2 + 5x + 6.25 = 2 + 6.25$

[Line 3] $\Rightarrow (x + 6.25)^2 = 8.25$

[Line 4] $\Rightarrow (x + 6.25) = \pm\sqrt{8.25}$

[Line 5] $\Rightarrow x = -6.25 \pm\sqrt{8.25}$

[Line 6] $\Rightarrow x = -3.38$ or -9.12 to 2 decimal places

Solve by QUADRATIC FORMULA

Quadratic Formula

$$x = \frac{-(b) \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$$

When should I solve using this method? When the equation is in standard form: $y = ax^2 + bx + c$

How do I use this method?
Step 1: Get the equation in standard form: $ax^2 + bx + c = 0$
Step 2: Identify the value of *a*, *b*, and *c*.
Step 3: Plug the values into the formula.
Step 4: Simplify the expression. If the radical simplifies to a rational number, separate the ± then simplify

Example! **Solve each equation using the quadratic formula.**

1. $10x = 2 - x^2$

Step 1: Get the equation in standard form: $ax^2 + bx + c = 0$

$x^2 + 10x - 2 = 0$

Step 2: Identify the value of *a*, *b*, and *c*.
$a = 1 , b = 10 , c = -2$

Step 3: Plug the values into the formula.

$$x = \frac{-(10) \pm \sqrt{(10)^2 - 4(1)(-2)}}{2(1)}$$

Step 4: Simplify the expression.

$$x = \frac{-(10) \pm \sqrt{(10)^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{-10 \pm \sqrt{100 + 8}}{2}$$
$$= \frac{-10 \pm \sqrt{108}}{2}$$
$$= \frac{-10 \pm 6\sqrt{3}}{2}$$

2. $3x^2 = 2x + 1$

Step 1: Get the equation in standard form: $ax^2 + bx + c = 0$

$3x^2 - 2x - 1 = 0$

Step 2: Identify the value of *a*, *b*, and *c*.
$a = 3 , b = -2 , c = -1$

Step 3: Plug the values into the formula.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)}$$

Step 4: Simplify the expression.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)}$$
$$= \frac{2 \pm \sqrt{4 + 12}}{6}$$
$$= \frac{2 \pm \sqrt{16}}{6}$$

Solving Quadratics by Square Root, Completing the Square, and Quadratic Formula – Practice

Solve each equation by taking the square root.

1. $-7x^2 = -448$

2. $4x^2 + 1 = 325$

3. $(x-4)^2 = 16$

4. $4(6x-1)^2 - 5 = 223$

Solve each equation by taking the square root.

5. $x^2 - 2x - 15 = 0$

6. $2x^2 + 8x - 7 = -2$

7. $x^2 + 4x = 3$

8. $x^2 - 7x = 18$

Solve each equation using the quadratic formula.

9. $x^2 - 2x - 15 = 0$

10. $2x^2 + 8x - 7 = -2$

11. $x^2 + 4x = 3$

12. $x^2 - 7x = 18$

Solving Quadratics by Square Root, Completing the Square, and Quadratic Formula – Application

Completing the Square & Deriving the Quadratic Formula (Spotlight Task)

Section 1: Area models for multiplication

1. If the sides of a rectangle have lengths $x + 3$ and $x + 5$, what is an expression for the area of the rectangle? Draw the rectangle, label its sides, and indicate each part of the area.

2. For each of the following, draw a rectangle with side lengths corresponding to the factors given. Label the sides and the area of the rectangle:
   a. $(x + 3)(x + 4)$
   b. $(x + 1)(x + 7)$
   c. $(x – 2)(x + 5)$
   d. $(2x + 1)(x + 3)$

Section 2: Factoring by thinking about area and linear quantities

For each of the following, draw a rectangle with the indicated area. Find appropriate factors to label the sides of the rectangle.

1. $x^2 + 3x + 2$
2. $x^2 + 5x + 4$
3. $x^2 + 7x + 6$
4. $x^2 + 5x + 6$
5. $x^2 + 6x + 8$
6. $x^2 + 8x + 12$

Section 3: Completing the square

1. What number can you fill in the following blank so that $x^2 + 6x +$ ____ will have two equal factors?

   What are the factors?

   Draw the area and label the sides.

   What shape do you have?

2. What number can you fill in the following blank so that $x^2 + 8x +$ ____ will have two equal factors?

   What are the factors?

   Draw the area and label the sides.

   What shape do you have?

3. What number can you fill in the following blank so that $x^2 + 4x +$ ____ will have two equal factors?

   What are the factors?

   Draw the area and label the sides.

   What shape do you have?

4. What would you have to add to $x^2 + 10x$ in order to make a square?

   What could you add to $x^2 + 20x$ to make a square?

   What about $x^2 + 50x$?

   What if you had $x^2 + bx$?

Section 4: Solving equations by completing the square

1. Solve $x^2 = 9$ without factoring.

   How many solutions do you have?

   What are your solutions?

2. Use the same method as in question 5 to solve $(x + 1)^2 = 9$.

   How many solutions do you have?

   What are your solutions?

3. In general, we can solve any equation of this form $(x + h)^2 = k$ by taking the square root of both sides and then solving the two equations that we get. Solve each of the following:

a. $(x + 3)^2 = 16$

c. $(x - 3)^2 = 4$

b. $(x + 2)^2 = 5$

d. $(x - 4)^2 = 3$

4. Now, if we notice that we have the right combination of numbers, we can actually solve other equations by first putting them into this, using what we noticed in questions 1 – 4. Notice that if we have $x^2 + 6x + 9 = 25$, the left side is a square, that is, $x^2 + 6x + 9 = (x + 3)^2$. So, we can rewrite $x^2 + 6x + 9 = 25$ as $(x + 3)^2 = 25$, and then solve it just like we did in question 7. (What do you get?)

5. Sometimes, though, the problem is not written quite in the right form. That's okay. We can apply what we already know about solving equations to write it in the right form, and then we can solve it. This is called completing the square. Let's say we have $x^2 + 6x = 7$. The left side of this equation is not a square, but we know what to add to it. If we add 9 to both sides of the equation, we get $x^2 + 6x + 9 = 16$. Now we can solve it just like the ones above. What is the solution?

6. Try these:

a. $x^2 + 10x = -9$

d. $x^2 + 6x - 7 = 0$

b. $x^2 + 8x = 20$

e. $2x^2 + 8x = -6$

c. $x^2 + 2x = 5$

Section 5: Deriving the quadratic formula by completing the square

If you can complete the square for a general quadratic equation, you will derive a formula you can use to solve any quadratic equation. Start with $ax^2 + bx + c = 0$, and follow the steps you used in Section 4.

Solving Quadratics by Square Root, Completing the Square, and Quadratic Formula – Homework

For each of the following quadratic equations, choose a method for solving. Be strategic with your choice of method. You must use each method once.

1. **Circle One Method:** **Factoring** **Square Root** **Completing the Square** **Quadratic Formula**

   $2x^2 - 8x + 7 = 0$

2. **Circle One Method:** **Factoring** **Square Root** **Completing the Square** **Quadratic Formula**

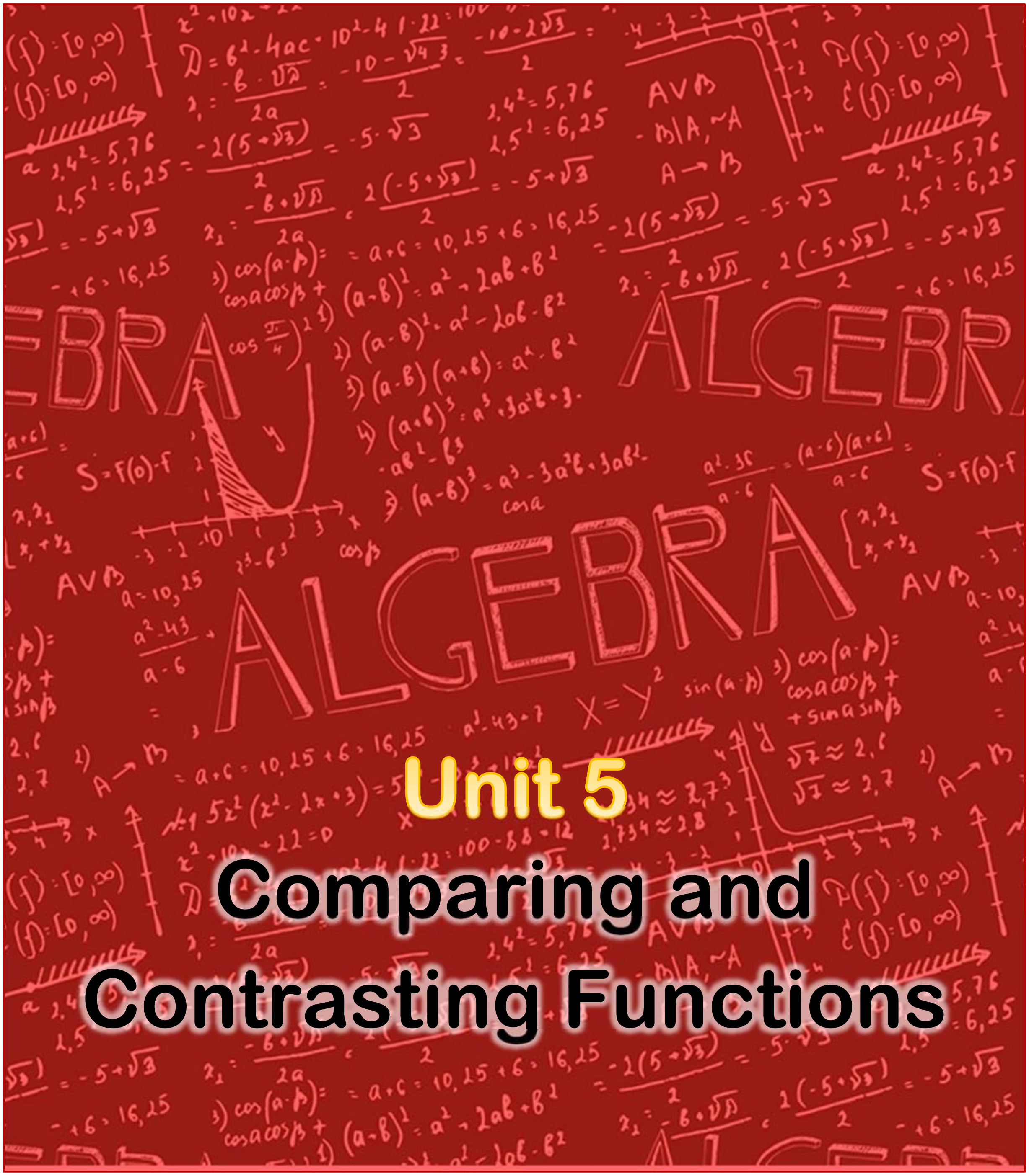
   $5x^2 + 11x + 2 = 0$

3. **Circle One Method:** **Factoring** **Square Root** **Completing the Square** **Quadratic Formula**

$x^2 - 10x + 26 = 8$

4. **Circle One Method:** **Factoring** **Square Root** **Completing the Square** **Quadratic Formula**

$9(2x - 3)^2 + 8 = 449$



Unit 5

Comparing and Contrasting Functions

Algebra 1

Unit 5: Comparing and Contrasting Functions

Concept 1: Distinguishing Between Linear, Exponential, and Quadratic Functions

Lesson A: Comparing Linear, Exponential, and Quadratic Functions	(A1.U5.C1.A.____.ComparingLEQFunctions)

Concept 2: Graphing Linear, Exponential, and Quadratic Functions

Lesson B: Graphing Linear, Exponential, and Quadratic Functions and Average Rate of Change	(A1.U5.C2.B.___. .AvgRateOfChangeLEQ)

Term	Definition	Notation	Diagram/Visual
Average Rate of Change			
Constant Rate of Change			
Continuous			
Discrete			
Domain			
End Behaviors			
Even Function			
Interval Notation			

Odd Function			
Range			

Distinguishing between LEQ Functions

In general, we are doing a review of the three functions that we have covered this year, linear, exponential and quadratic. Additionally, this unit will compare the average rates of change for the functions, and identify odd and even functions.

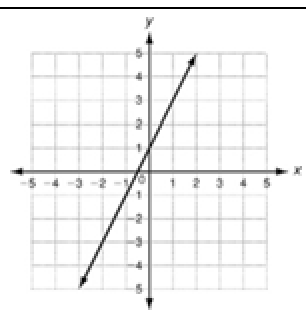
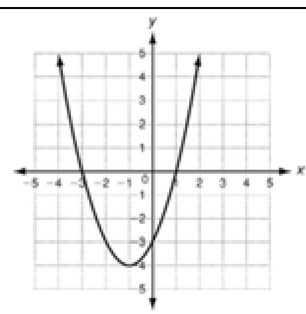
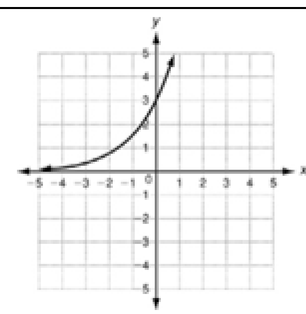
How can you determine whether a given data set is best modeled by a linear, quadratic, or exponential function? You can (a) look at the graph of the function and consider its shape and its end behavior; (b) use a table of data and determine first differences, second differences, and ratios of consecutive function values.

Linear functions are used to model phenomena that increase or decrease at a constant rate. These types of functions are polynomial functions with a highest exponent of one on the variable. The graphs of these functions are in the shape of a line : $\boldsymbol{f(x) = mx + b}$.
Exponential functions are functions that have the variable in the exponent. They increase or decrease slowly then quickly or quickly then slowly : $\boldsymbol{f(x) = ab^x}$.
A Quadratic function is a polynomial function with a highest exponent of two. These types of functions are used to model phenomena that increase and hit a maximum then decrease, or decrease and hit a minimum then increase. Their graphs look like a U or an upside down U : $\boldsymbol{f(x) = ax^2 + bx + c}$.

Guided Notes

Example!

As we compare the LEQ functions, we can examine the graphs, the equations, or apply how the table values change. Below is a comparison of the LEQ functions from a pattern, graph or equation view.

Model	Linear	Quadratic	Exponential
Pattern (y-values)	constant first differences	constant second differences	constant ratios
Graph			
Equation	$y = mx + b$	$y = ax^2 + bx + c$	$y = ab^x$

Are the following Linear, Quadratic or Exponential?
1. y = 6x + 3 ______________________
2. $y = 7x^2 + 5x - 2$ ______________________

3. $9x + 3 = y$ ______________________

4. $4^{2x} = 8 + y$ ___________________

Below are models of LEQ tables and how to determine the type of function based on the change in y-values.

Linear functions have constant 1st differences.

x	*y*
2	5
4	2
6	−1
8	−4

−3 −3 −3

Quadratic functions have constant 2nd differences.

x	*y*
1	−8
2	−5
3	0
4	7

+3 +5 +7

+2 +2

Exponential functions have a constant ratio.

x	*y*
0	−2
1	−8
2	−32
3	−128

×4 ×4 ×4

Remember!

When the independent variable changes by a constant amount,

- linear functions have constant first differences.
- quadratic functions have constant second differences.
- exponential functions have a constant ratio.

Determine which table is linear, exponential or quadratic?

A.

x	y
1	5
2	9
3	13
4	17
5	21

B.

x	y
1	0
2	-1
3	0
4	3
5	8

C.

x	y
1	3
2	9
3	27
4	81
5	243

________________ ___________________ _________________

KARATE: The table shows the number of children enrolled in a beginner's karate class for four consecutive years. Determine which model best represents the data. Then write a function that models that data.

Time (years)	0	1	2	3	4
Number Enrolled	8	11	14	17	20

Type of function _________________________

Write the function _______________________

WILDLIFE: The table shows the growth of prairie dogs in a colony over the years. Determine which model best represents the data. Then write a function that models the data.

Time (years)	0	1	2	3	4
Prarie Dogs	4	8	16	32	64

Type of function ____________________

Write the function ___________________

SELF CHECK

Determine the type of function from the tables.

1.

x	y
0	6
1	12
2	24
3	48

____

2.

x	y
0	10
1	18
2	28
3	40

____

3.

x	y
3	4
6	−2
9	−8
12	−14

____

4.

x	2	4	6	8	10
y	-11	-9	-7	-5	-3

____

1. What are two things that differentiate a data set that is better modeled by a quadratic function from a data set this better modeled by a linear function?

2. What do function tables tell you that graphs don't?

1. Match each graph with its description.

_______ I. An exponential function that is always increasing.

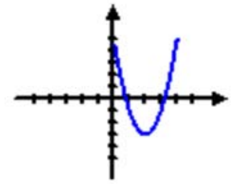
_______ II. An exponential function that is always decreasing.

_______ III. A quadratic function with a local maximum.

_______ IV. A quadratic function with a local minimum.

_______ V. A linear function that is always increasing.

_______ VI. A linear function that is always decreasing.

a.


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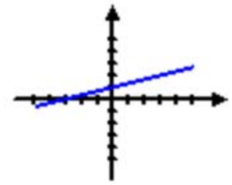

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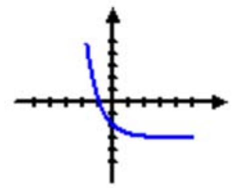

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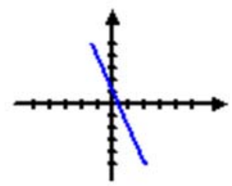

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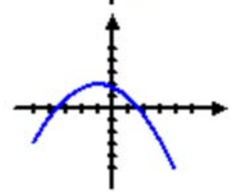
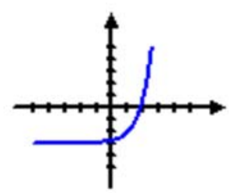


f.


2. Which is the only function that might have end behavior such that as ***x*** approaches infinity, ***f(x)*** approaches 4?

A. Linear Function B. Quadratic Function C. Exponential Function

3. Which is the only function below that might have end behavior such that:

As $x \to -\infty$, $f(x) \to \infty$ As $x \to \infty$, $f(x) \to \infty$

A. Linear Function B. Quadratic Function C. Exponential Function

4. Which is the only function below that might have end behavior such that:

As $x \to -\infty$, $f(x) \to -\infty$ As $x \to \infty$, $f(x) \to -\infty$

A. Linear Function B. Quadratic Function C. Exponential Function

Based on the partial set of values given for a function, identify which description best fits the function.

5.

x	0	1	2	3	4
$a(x)$	1	5	9	13	17

Model (circle one):
Linear Growth Quadratic (Local Max) Exponential Growth
Linear Decay Quadratic (Local Min) Exponential Decay

6.

x	1	2	3	4	5
$b(x)$	1	2	1	- 2	- 7

Model (circle one):
Linear Growth Quadratic (Local Max) Exponential Growth
Linear Decay Quadratic (Local Min) Exponential Decay

7.

x	1	2	3	4	5
$e(x)$	65	33	17	9	5

Model (circle one):
Linear Growth Quadratic (Local Max) Exponential Growth
Linear Decay Quadratic (Local Min) Exponential Decay

8.

x	1	2	3	4	5
$f(x)$	9	7	5	3	1

Model (circle one):
Linear Growth Quadratic (Local Max) Exponential Growth
Linear Decay Quadratic (Local Min) Exponential Decay

9.

x	0	1	2	3	4
$d(x)$	3	0	-1	0	3

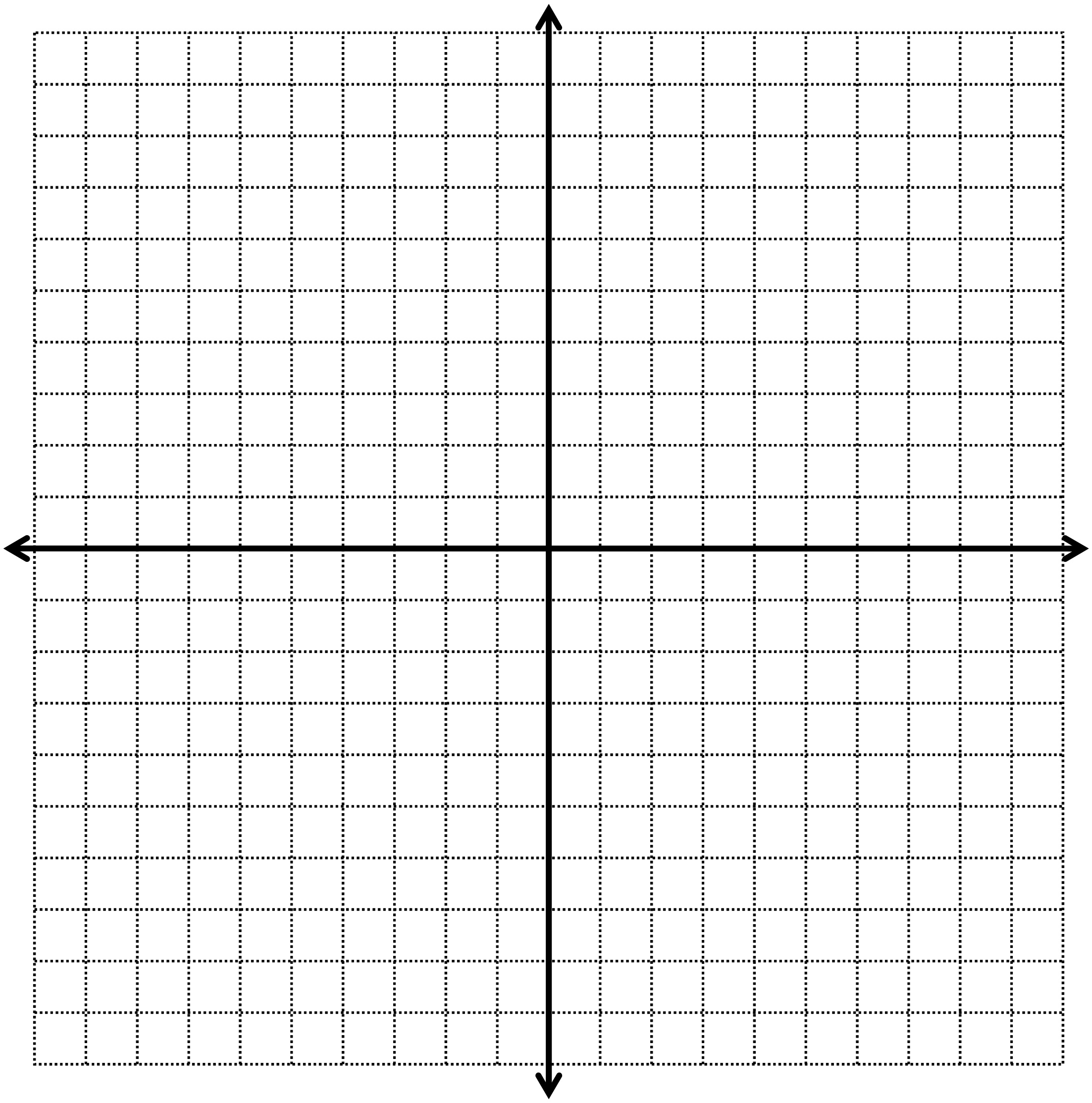
Model (circle one):
Linear Growth Quadratic (Local Max) Exponential Growth
Linear Decay Quadratic (Local Min) Exponential Decay

COMPARING LINEAR, QUADRATIC AND EXPONENTIAL MODELS GRAPHICALLY

1. Complete the tables below.

Linear		**Quadratic**		**Exponential**	
$f(x) = 2x$		$g(x) = x^2$		$h(x) = 2^x$	
x	$f(x)$	x	$g(x)$	x	$h(x)$
–5		**–5**		**–5**	
–4		**–4**		**–4**	
–3		**–3**		**–3**	
–2		**–2**		**–2**	
–1		**–1**		**–1**	
0		**0**		**0**	
1		**1**		**1**	
2		**2**		**2**	
3		**3**		**3**	
4		**4**		**4**	
5		**5**		**5**	

2. Draw and label each graph on the same set of axes.



3. Identify the following features of each function.

(a) Domain and Range

(b) Description of Shape

(c) Any characteristics unique to each function

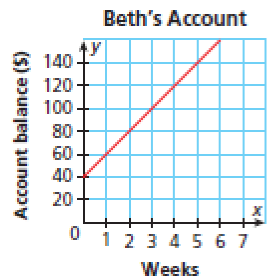
Comparing LEQ Functions Homework

1. Deidre and Beth each deposit money into their checking accounts weekly. Their account information for the past several weeks is shown below.

Deirdre's Account

Weeks	Account Balance ($)
0	60
1	75
2	90
3	105
4	120

Beth's Account



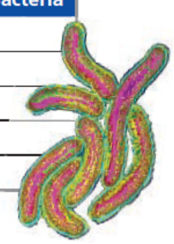
a. Compare the slopes and y-intercepts and interpret those values in context of the situation.

	Deidre	Beth	Interpret and Compare
Slope			
Y-intercept			

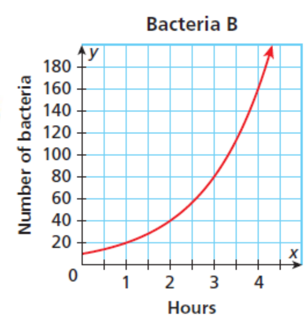
2. A biologist tracked the hourly growth of two different strains of bacteria in the lab. Here data are shown below. Compare the number of bacteria by finding the rate of change (percent of growth). Which bacteria is growing faster?

Bacteria A

Hours	Number of Bacteria
0	5
1	15
2	45
3	135
4	405



Bacteria B



3. Kevin and Darius each hiked a different mountain trail at different rates as shown below. Compare the hikes by finding and interpreting the slopes and y-intercepts in context of the situation.

Kevin's Hike

Time (h)	0	1	2	3
Distance from Camp (mi)	1.5	3.5	5.5	7.5

Darius's Hike

$y = 2.2x + 1$

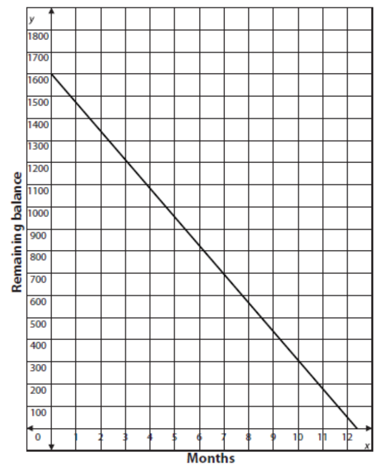
	Kevin	Darius	Interpret and Compare
Slope			
Y-intercept			

4. Maddy and Savannah both had to have repairs done on their cars after they were in a fender bender with each other. The table and graph below show the remaining balance in dollars, f(x), of the cost of car repairs after x months. Who had the higher costs for repair and who is repaying their balance faster?

Maddy:

Months (x)	Remaining balance ($f(x)$)
0	1560
1	1430
2	1300
3	1170

Savannah:

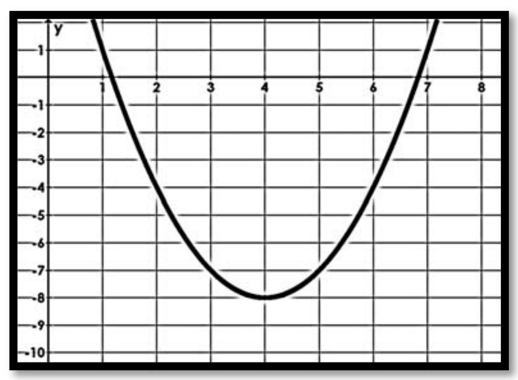


5. Which quadratic function has the lower minimum value? Explain why.

Function A.

X	-4	-3	-2	-1	0	1
y	0	-5	-8	-9	-8	-5

Function B.



6. Which quadratic function has the bigger y-intercept? Explain why.

Function A: $y = -x^2 + 3x + 8$

Function B:

X	-4	-3	-2	-1	0	1
y	9	13	19	13	9	7

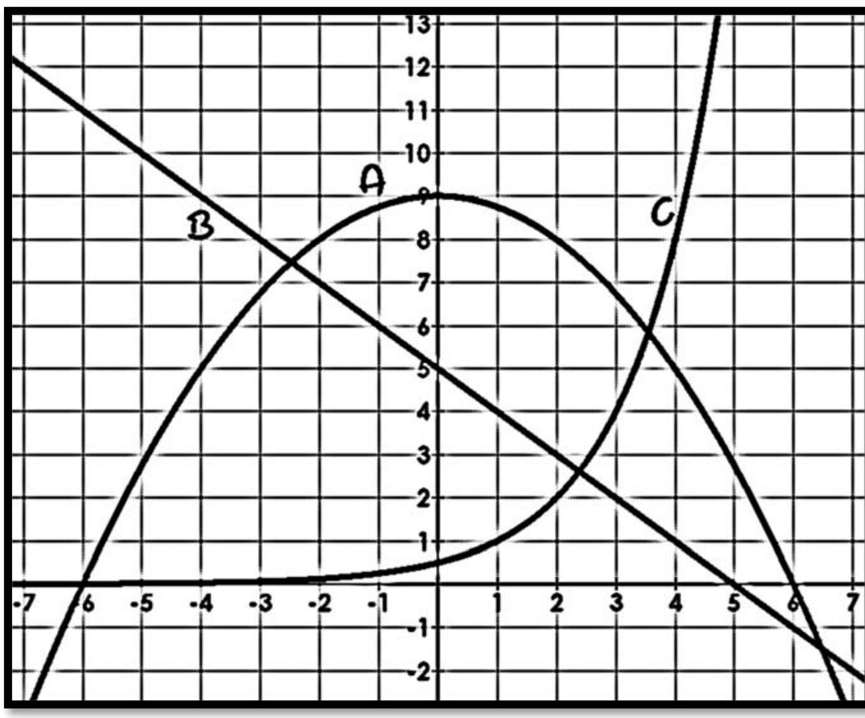
7. For the following equations, determine for what x-values g(x) exceeds f(x).

a. $f(x) = 2x + 3$ and $g(x) = x^2$

b. $f(x) = 3x^2$ and $g(x) = 2^x$

c. $f(x) = 1.9^x$ and $g(x) = 0.8(2)^x$

8. Use the graph below to answer the following questions:



a. List the functions in order from least to greatest for y-intercepts:

b. Which function has the largest x-intercept?

c. List the functions in order from smallest to largest when x = -4.

d. List the functions in order from smallest to largest when x = 0.

e. List the functions in order from smallest to largest when x = 2.

f. List the functions in order from smallest to largest when x = 5.

g. Which graphs has the largest rate of change on the interval [-5, -3]?

h. Which graph has the largest rate of change on the interval [4, 5]?

Average Rate of Change

GUIDED NOTES

Another concept in this unit is to find the Average Rate of Change (AROC) for the three functions.

Average rate of change (**AROC**) is a way to compare functions over a **specific interval**. From our previous lessons, we found that linear functions have a constant rate of change otherwise known as slope. We will use this concept when calculating AROC of nonlinear functions.

If $y = f(x)$, then the average rate of change on the interval [a, b] is:

$$\text{Average rate of change} = \frac{f(b)-f(a)}{b-a} = \frac{y2-y1}{x2-x1} = \text{Slope}$$

Example! Ano

Linear Functions:

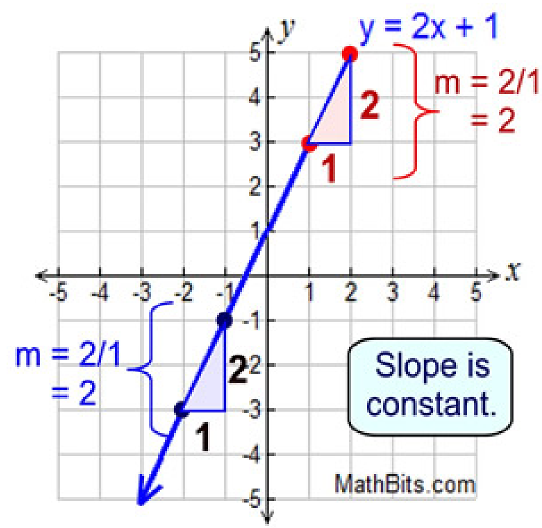
You are already familiar with the concept of "average rate of change".
When working with **straight lines** (*linear functions*) you saw the "average rate of change" to be:

$$\boxed{\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = m = SLOPE}$$

The word "slope" may also be referred to as "gradient", "incline" or "pitch", and be expressed as:

$$\text{Slope} = m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

A special circumstance exists when working with straight lines (*linear functions*), in that the "average rate of change" (the slope) is constant. No matter where you check the slope on a straight line, you will get the same answer.

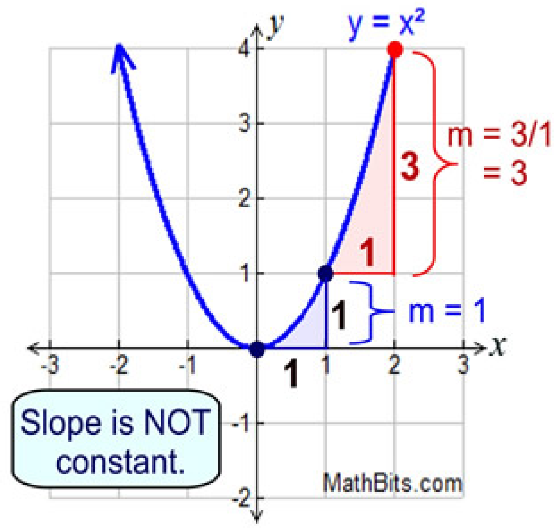


Non-Linear Functions:

When working with *non-linear functions,* the "average rate of change" is not constant.

The process of computing the "average rate of change", however, remains the same as was used with straight lines: two points are chosen, and $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\text{rise}}{\text{run}}$ is computed.

FYI: You will learn in later courses that the "average rate of change" in non-linear functions is actually the slope of the secant line passing through the two chosen points. A secant line cuts a graph in two points.



When you find the "average rate of change" you are finding the rate at which (how fast) the function's *y*-values (output) are changing as compared to the function's *x*-values (input).

When working with functions (of all types), the "average rate of change" is expressed using *function notation.*

Average Rate of Change

For the function $y = f(x)$ between $x = a$ and $x = b$, the

$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(b) - f(a)}{b - a}$$

Example 1: Finding average rate of change from a table.

Function $f(x)$ is shown in the table at the right.
Find the average rate of change over the interval $1 \leq x \leq 3$.

x	$f(x)$
0	1
1	4
2	9
3	16

Solution:
If the interval is $1 \leq x \leq 3$, then you are examining the points (1,4) and (3,16). From the first point, let $a = 1$, and $f(a) = 4$. From the second point, let $b = 3$ and $f(b) = 16$.

Substitute into the formula: $\dfrac{f(b)-f(a)}{b-a} = \dfrac{16-4}{3-1} = \dfrac{12}{2} = \dfrac{6}{1} = 6$

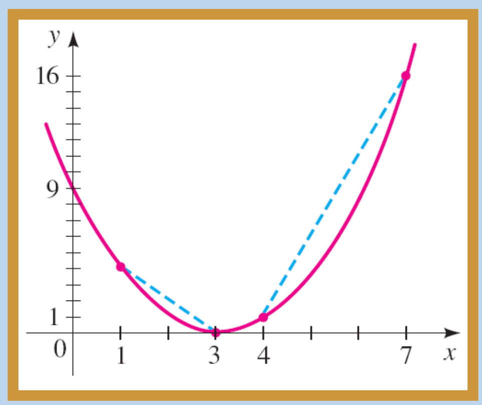
The average rate of change is 6 over 1, or just 6.
The y-values change 6 units every time the x-values change 1 unit, on this interval.

Example 2: Finding average rate of change from a graph.

For the function $f(x) = (x - 3)^2$, whose graph is shown, find the average rate of change between the following points:

(a) $x = 1$ and $x = 3$

(b) $x = 4$ and $x = 7$



Average rate of change (a)

$= \dfrac{f(3) - f(1)}{3 - 1}$

$= \dfrac{(3-3)^2 - (1-3)^2}{3-1}$

$= \dfrac{0 - 4}{2}$

$= -2$

(b)

$= \dfrac{f(7) - f(4)}{7-4}$

$= \dfrac{(7-3)^2 - (4-3)^2}{7-4}$

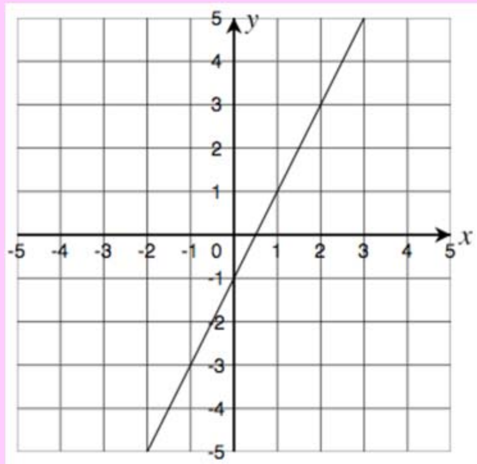
$= \dfrac{16 - 1}{3}$

$= 5$

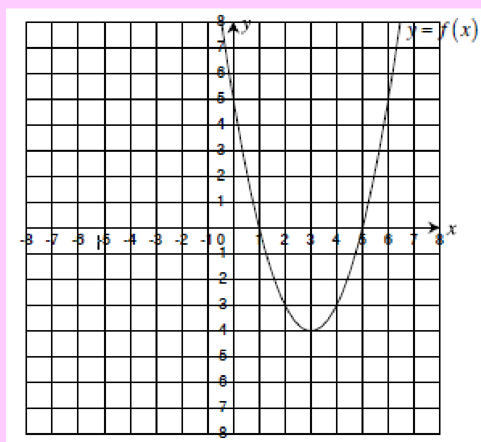
The AROC of example (a) is -2 and AROC of example (b) is 5.

SELF CHECK

1. Consider the linear function $f(x) = 2x - 1$ whose graph is shown.
Find the average rate of change of $f(x) = 2x - 1$ from $x = 0$ to $x = 1$.



2. Consider the quadratic function whose graph is shown. Find the average rate of change from $x = 0$ to $x = 1$

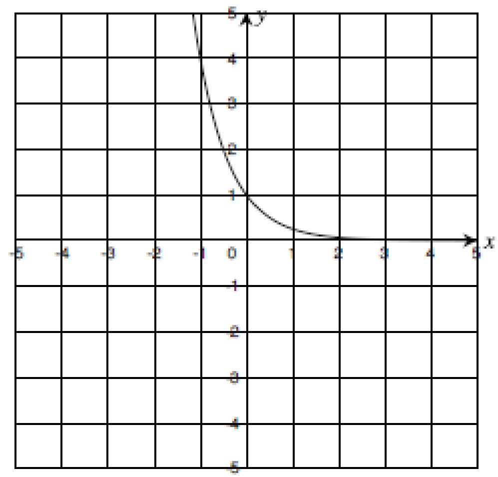


3. Find the average rate of change for the following functions over the given interval. What does the average rate of change tell you about the function on the interval?

x	0	1	2	3	4
$f(x)$	3	6	12	24	48

From $x = 0$ to $x = 3$

a)



From $x=-1$ to $x=1$

b)

4. A bird is traveling feet over time in seconds which is modeled by the equation $f(x)=-3x^2-x+2$, What is the average rate of change at the given interval [0, 3]

Questions To Ponder

1. Is rate of change the same for ALL functions?
2. Should the rate of change be positive/negative for an increasing/decreasing graph?

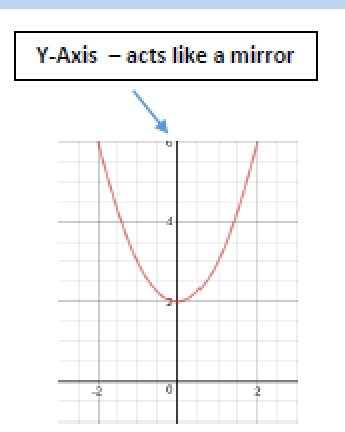
Example!

Even and Odd Functions

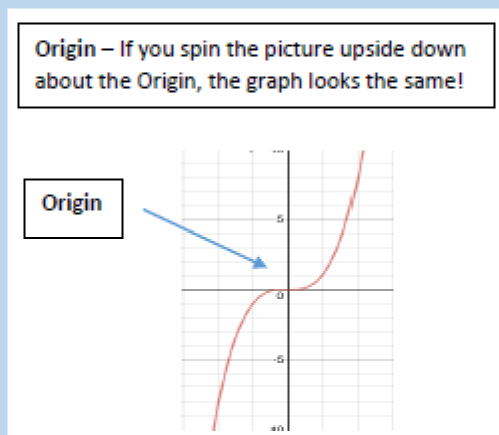
A function can be classified as Even, Odd, or Neither. This classification can be determined graphically or algebraically.

Graphical Interpretation -

Even Functions:
Have a graph
Have a graph
symmetric with
symmetric with
to the Y-Axis.



Odd Functions:
that is
that is
respect
respect
to the Origin.



If we cannot classify a function as **even** or **odd**, then we call it **neither**.

Even functions are symmetric with respect to the ________________. This means we could fold the graph on the y-axis, and it would line up perfectly on both sides! (Check the coordinate (x, y) *should match* $(-x, y)$ ex (1,1) should have a matching coordinate $(-1,1)$ and split the y axis

Odd functions are symmetric with respect to the ______________. This means we can rotate the image 180^o about the origin and it will appear exactly the same! Or check Coordinates (x, y) *should match* $(-x, -y)$ ex (1,1) should have a matching coordinate $(-1, -1)$ and pass through the origin

Algebraic Test:
Substitute $(-x)$ in for x everywhere in the function and analyze the results of $(-x)$, by comparing it to the original function $f(x)$.
Even Function: $\boldsymbol{y = f(x)}$ is ***Even*** when, for each x in the domain of $f(x)$, $f(-x) = f(x)$
Odd Function: $\boldsymbol{y = f(x)}$ is ***Odd*** when, for each x in the domain of $f(x)$, $f(-x) = -f(x)$

Examples:

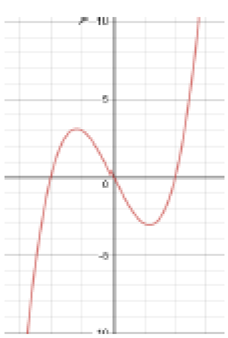
a. $f(x) = x^2 + 4$
$f(-x) = (-x)^2 + 4$
$f(-x) = x^2 + 4$
$f(-x) = f(x)$
Even Function!

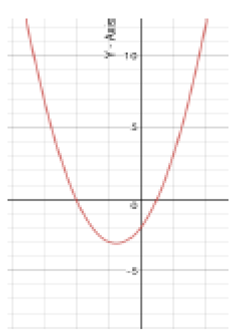
b. $f(x) = x^3 - 2x$
$f(-x) = (-x)^3 - 2(-x)$
$f(-x) = -x^3 + 2x$
$f(-x) = -(x^3 - 2x) = -f(x)$
Odd Function!

c. $f(x) = x^2 - 3x + 4$
$f(x) = (-x)^2 - 3(-x) + 4$
$f(-x) = x^2 + 3x + 4$
$f(-x) \neq f(x) \neq -f(x)$
Neither!

Try these:

A. Graphically determine whether the following functions are Even, Odd, or Neither


1.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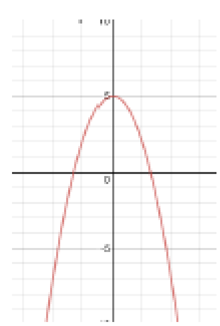
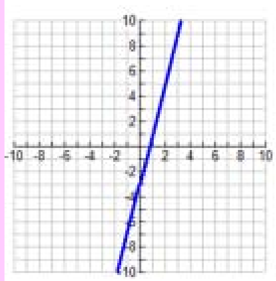
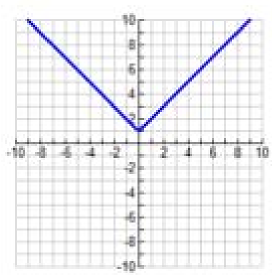

3.

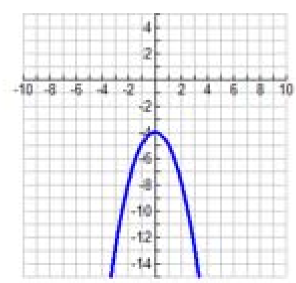
SELF CHECK Determine Algebraically if the functions are even, odd, or neither.

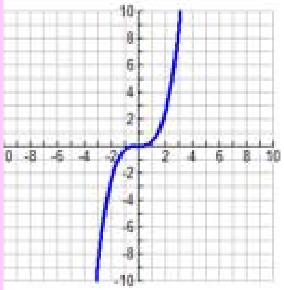
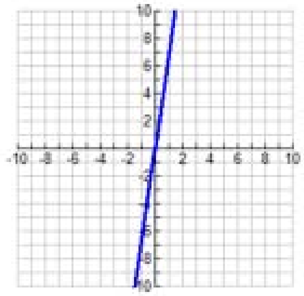
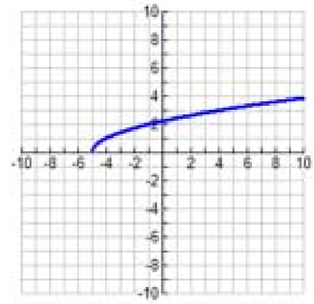
1. $f(x) = x^3 - 6x$ A) Degree___ Possibly______ Answer: Symmetric to:	2. $g(x) = x^4 - 2x^2$ A) Degree___ Possibly______ Answer: Symmetric to:
3. $h(x) = x^2 + 2x + 1$ A) Degree___ Possibly_____ Answer: Symmetric to:	4. $f(x) = x^2 + 6$ A) Degree___ Possibly______ Answer: Symmetric to:
5. $g(x) = 7x^3 - x$ A) Degree___ Possibly______ Answer: Symmetric to:	6. $f(x) = 2x^4 - x^3 - 1$ A) Degree___ Possibly____ Answer: Symmetric to:

Determine graphically using possible symmetry, whether the following functions are even, odd, or neither.



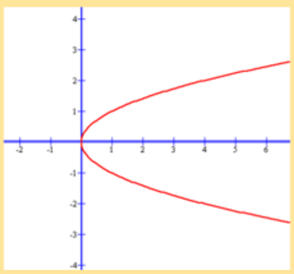




1. If (-4,5) is on a graph, then what other points must also be on the graph if it is symmetric about the a) x-axis b) y-axis c) origin

The graph below has x-axis symmetry. Explain why you think that a graph with x-axis symmetry does NOT relate to even and odd functions.



1. Find the average rate of change from $x = -1$ to $x = 2$ for each of the functions below.

   a. $a(x) = 2x + 3$ b. $b(x) = x^2 - 1$ c. $c(x) = 2^x + 1$

   d. Which function has the greatest average rate of change over the interval $[-1, 2]$?

2. Find the average rate of change on the interval $[2, 5]$ for each of the functions below.

   a. $a(x) = 2x + 1$ b. $b(x) = x^2 + 2$ c. $c(x) = 2^x - 1$

   d. Which function has the greatest average rate of change over the interval $x = 2$ to $x = 5$?

3. In general as $x \to \infty$, which function eventually grows at the fastest rate?

   a. $a(x) = 2x$ b. $b(x) = x^2$ c. $c(x) = 2^x$

4. Find the average rate of change from $x = -1$ to $x = 2$ for each of the continuous functions below based on the partial set of values provided.

a.

x	-1	0	1	2	3
$a(x)$	-3	-2	1	6	13

b.

x	-1	0	1	2	3
$b(x)$	1	3	5	7	9

c.

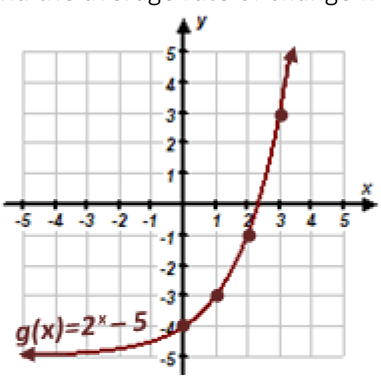
x	-1	0	1	2	3
$c(x)$	-2	-1	1	5	13

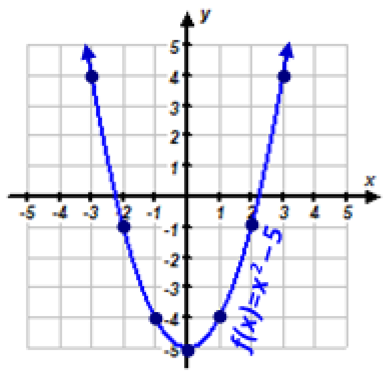
   d. Which function has the greatest average rate of change over the interval $[-1, 2]$?

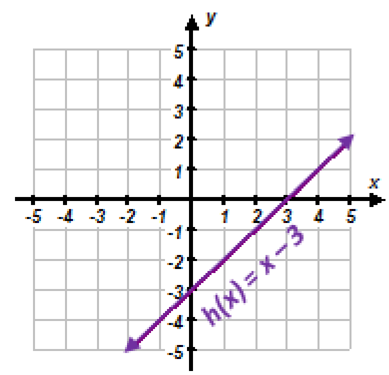
5. Consider the table below that shows a partial set of values of two continuous functions. Based on any interval of x provided in th table which function always has a larger average rate of change?

x	$f(x)$	$g(x)$
-1	-2	-4
0	0	0
1	3	8
2	7	24

6. Find the average rate of change from $x = 1$ to $x = 3$ for each of the functions graphed below.







d. Find an interval of x over which all three graphed functions above have the same average rate of change.

The Tortoise, The Hare, and the Aardvark

In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. The distance from the starting line of the hare is given by the function:

$d = t^2$ (d in meters and t in seconds)

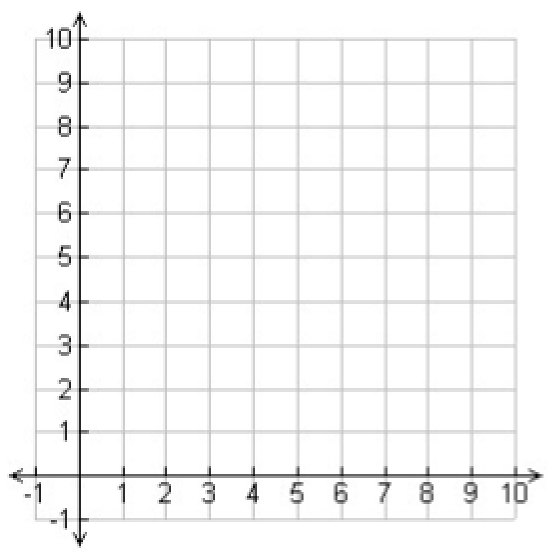
Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

$d = 2^t$ (d in meters and t in seconds)

A little known part of the story is that when the race was about to begin an aardvark came strolling over and decided that he thought he could beat both of them with his consistent trot that can be described by the function:

$d = 2t$ (d in meters and t in seconds)

1. Graph all three functions on the same grid to help see how the race is going to go.



2. Identify the key features of each function of the graph.

3. Fill out the table.

	0	1	2	3	4	5	6	7	8
d= 2^t									
d= t^2									
d=2t									

4. At what time did the aardvark catch up to tortoise?

5. At what time does the hare catch up to the tortoise?

6. If the race course is very long, who wins: the tortoise, the hare, or the aardvark? Why?

7. Is there a point in time when all three are tied? If so, when is this? If not, when are they closest?

8. Is there a point when there is just a 2 way tie? If so, when is this?

9. If the race course were 3 meters long who wins, the tortoise, the hare, or the aardvark? Why?

10. If the race course were 15 meters long who wins, the tortoise, the hare, or the aardvark? Why?

11. Use the properties to explain the speeds of the tortoise and the hare in the following time intervals:

Interval	Tortoise $d = 2^t$	Hare $d = t^2$	Aardvark $d = 2t$
$0 \le x < 1$			
$1 \le x < 2$			
$2 \le x < 4$			
$x \ge 4$			

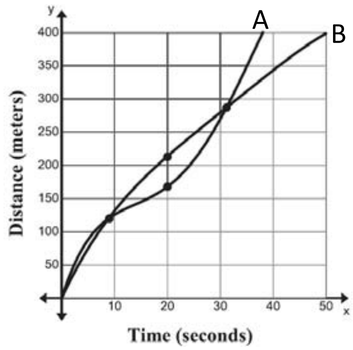
12. What would happen to the equation $d = 2t$ if the hare did not give a 1 meter head start to the tortoise? How does this change any of your previous conclusions?

13. What would happen to the equation $d = 2t$ if the aardvark got the 1 meter head start instead of the tortoise? How does this change any of your previous conclusions?

14. What happens to the equations $d = 2t, = t2, and\ d = 2t$ if the Hare gave both competitors a 1 second head start instead of 1 meter? How does this change any of your previous conclusions?

AROC Homework

Below is the graph and table for 2 runners running the 400 meter hurdles race.



Time	Runner A	Runner B
0	0	0
9	120	120
20	168	213
31	287	287

1. Which runner has a faster average speed for the first 9 seconds?

2. Which runner has a faster average speed from 9 to 20 seconds?

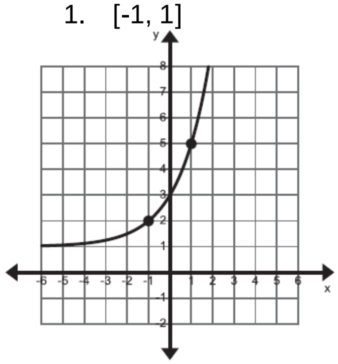
3. Which runner has a faster average speed from 20 to 31 seconds?

4. Which runner has a faster average speed from 9 to 31 seconds?

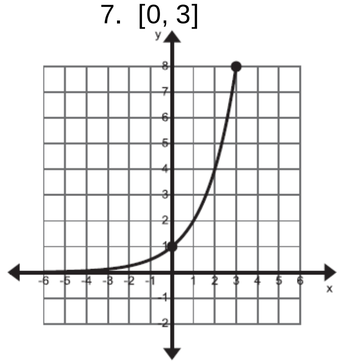
5. Which runner wins the race? How do you know?

Find the average rate of change for each of the following graphs over the given interval.

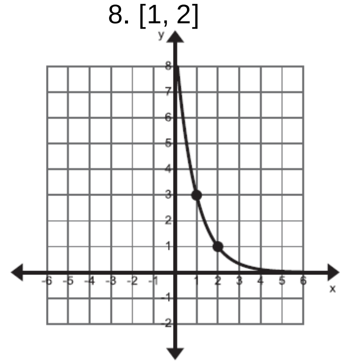
1. [-1, 1]



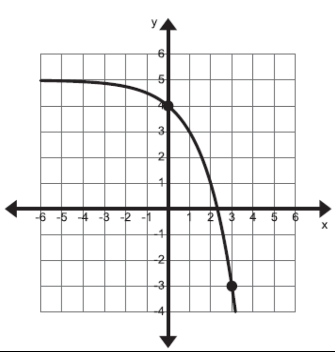
7. [0, 3]



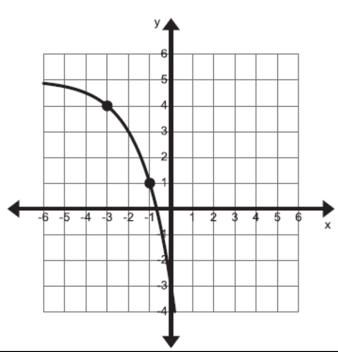
8. [1, 2]



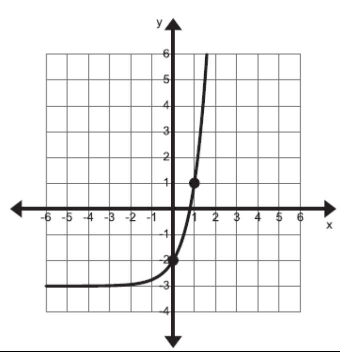
9. [0, 3]



10. [-3, -1]



11. [0, 1]



Suppose 25 flour beetles are left undisturbed in a warehouse bin. The beetle population doubles in size every week. The equation $P(x) = 25 \bullet 2^x$ can be used to determine the number of beetles after x weeks. Complete the table.

Week	Population
0	
1	
2	
3	
4	
5	

12. Calculate the average growth rate between weeks 1 and 3.

13. Calculate the average growth rate for the first five weeks [0, 5].

14. Which average growth rate is higher? Why do you think it is higher?

Find the rate of change of Pete's height from 3 to 5 years.

15.

Time (years)	1	2	3	4	5	6
Height(in.)	27	35	37	42	45	49

For $f(x) = x^2 - 2$, find the rate of change on the interval [-2, 4].

16.

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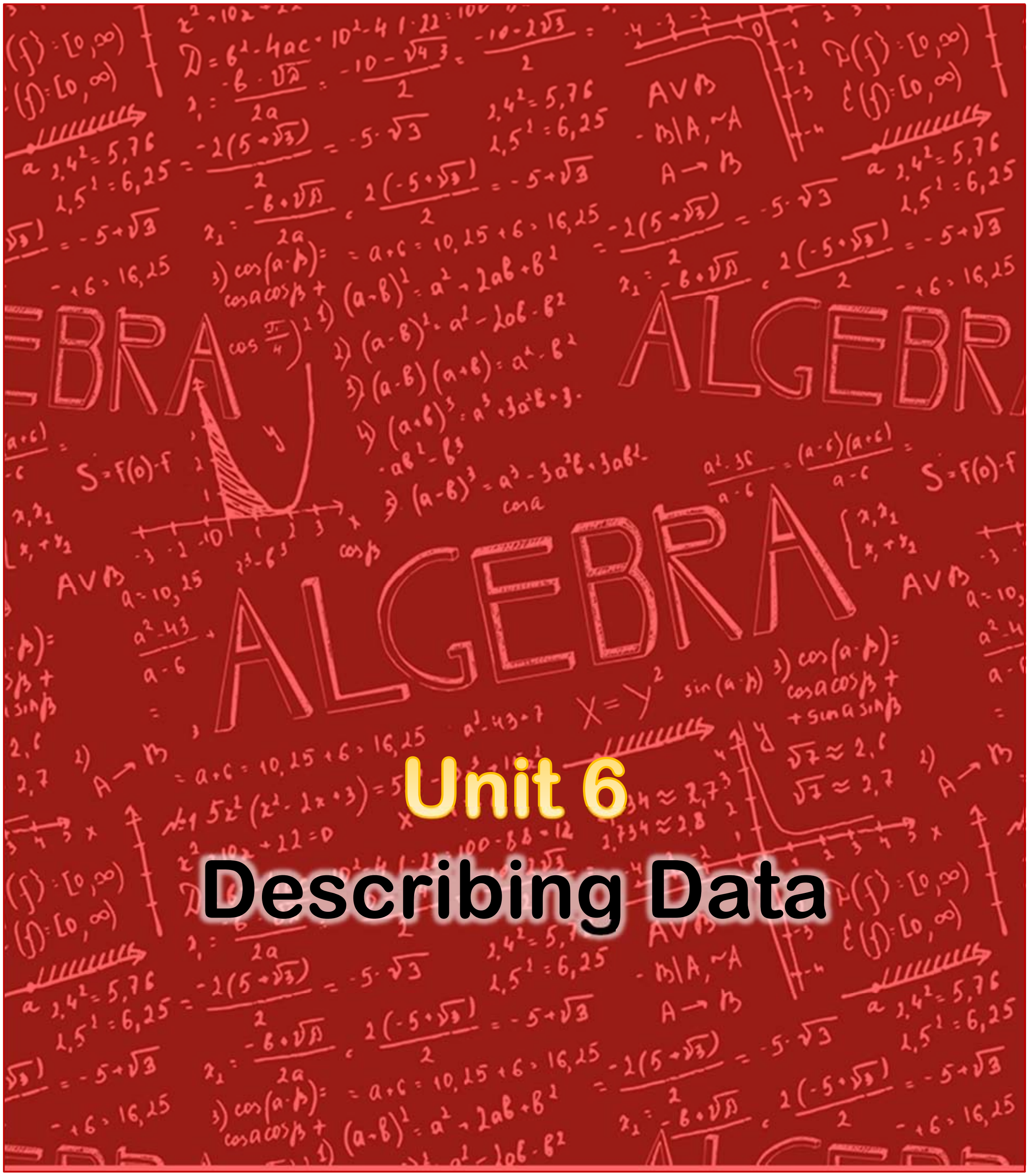


Unit 6

Describing Data

Algebra 1

Unit 6: Describing Data

Term	Definition	Notation	Diagram/Visual
Box Plot	______________ ______________ ______________		
Dot Plots	______________ ______________ ______________		
Histogram	______________ ______________ ______________		
Measure of Center	______________ ______________ ______________		
Measure of Spread	______________ ______________ ______________		
Shape of Distribution	______________ ______________ ______________		
Outliers	______________ ______________ ______________		

Measures of Center and Spread

Recall what you learned in middle school, finding mean and median is finding the Measure of Center of a data set. Measure of Center refers to the summary measures used to describe the most "typical" value in the data set. The **mean**, $\overline{x}$, is the <u>average</u> of the data. The **median** is the <u>middle</u> of the data.

Find the mean and median of the following sets of data:

5, 11, 13, 5, 9

Mean: _________
Median:________

24, 18, 26, 30

Mean:________
Median:_______

Recall what you learned in middle school, finding the range and interquartile range (IQR) is finding the Measure of Spread of a data set. Measure of Spread refers to the variability of the data. If the data is clustered around a single central value, the spread is smaller. The further the observations fall from the center, the great the spread or variability. The **range** is the <u>difference from the lowest and highest values</u>. The **IQR** is the <u>distance</u> <u>between the first and third quartiles of the data set.</u>

Find the range and interquartile range of the following sets of data:

72, 91, 93, 89, 77, 82

Range:_________
IQR:__________

1.2, 0.4, 1.2, 2.4, 1.7, 1.6, 0.9, 1.0

Range:_________
IQR:_________

With that review, Measure of Spread can be found one more additional way. **Mean absolute deviation (MAD)** is the average of the distances between each data and the mean. You find this by adding the distances between each data value and the mean, then dividing by the number of data values.

The ___________ the mean absolute deviation the smaller amount of __________ in the data.

A. lower, variability
B. lower, continuity
C. higher, variability
D. higher, continuity

<u>To find the MAD:</u>

1. Find the mean
2. Subtract each data value from the mean
3. Take the absolute value of each value from step #2.
4. Add up all values from step #3.
5. Divide by the number of data values.

EX. Find the MAD (Mean Absolute Deviation) of the numbers shown below.

80, 76 , 63 , 92 , 47 , 82 and 76.

MAD worksheet

x	$\bar{x}$	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$
80			
76			
63			
92			
47			
82			
76			

SUM: __________

$$MAD = \frac{sum}{n} =$$

The following data represents the oldest family member of each person in April's class:

55, 72, 100, 45, 66, 71, 58, 62

Mean = _____ Median = _______ IQR = ________ MAD = ________

MAD Worksheet

x	$\bar{x}$	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$

SUM: _________

$$MAD = \frac{sum}{n} =$$

Questions To Ponder

Why is the mean absolute deviation, MAD, sometimes used to find the Measure of Spread instead of range and IQR?

Example!

4, 12, 5, 7, 11, 3, 6, and 12

Mean = 7.5 Median = 6.5 IQR = 7 MAD = 3.125

x	$\bar{x}$	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$
4	7.5	-3.5	3.5
12	7.5	4.5	4.5
5	7.5	-2.5	2.5
7	7.5	-0.5	0.5
11	7.5	3.5	3.5
3	7.5	-4.5	4.5
6	7.5	-1.5	1.5
12	7.5	4.5	4.5

$MAD = \frac{sum}{n} = \underline{3.125}$

SUM: 25

SELF CHECK

Find the Mean Absolute Deviation for the following data:

Number of Daily Visitors to a Web Site				
112	145	108	160	122

x	$\bar{x}$	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$

Measures of Center and Spread

Example!

30, 35, 42, 15, 20, 87, and 23

Mean = 36 Median = 30 IQR = 22 MAD = 16.286

x	$\bar{x}$	$x-\bar{x}$	$\lvert x-\bar{x} \rvert$
30	36	-6	6
35	36	-1	1
42	36	6	6
15	36	-21	21
20	36	-16	16
87	36	51	51
23	36	-13	13

$MAD = \frac{sum}{n} =$ 16.286

SUM: 114

Find the mean absolute deviation for the following data sets:

Brad: 70, 76, 78, 80, 90, 94, 94, 98
Jin: 80, 82, 84, 84, 86, 86, 88, 90

1. Find Brad's mean test score.

   85

2. Find Jin's mean test score.

   ____

3. Find Brad's median test score.

   ____

4. Find Jin's median test score.

   ____

5. Find Brad's range.

   ____

6. Find Jin's range.

   ____

7. Find Brad's first and third quartiles.

   ____

8. Find Jin's first and third quartiles.

   ____

9. Find Brad's interquartile range.

   ____

10. Find Jin's interquartile range.

   ____

11. Find the MAD of Brad's tests scores

   ____

12. Find the MAD of Jin's tests scores

   ____

WATER PARKS The table shows the height of waterslides at two different water parks. Find the mean absolute deviation for each set of data. Round to the nearest hundredth. Then write a few sentences comparing their variation.

Height of Waterslides (ft)									
Splash Lagoon					Wild Water Bay				
75	95	80	110	88	120	108	94	135	126

Splash Lagoon:

x	$\bar{x}$	$x-\bar{x}$	$\lvert x-\bar{x} \rvert$

Wild Water Bay:

x	$\bar{x}$	$x-\bar{x}$	$\lvert x-\bar{x} \rvert$

Looking at the Splash Lagoon data:

a) Which measure of central tendency best describes the data and why?

____

____

____

b) Which measure of variability best describes the data and why?

____

____

____

Looking at the Wild Water Bay data:

c) Which measure of central tendency best describes the data and why?

____

____

____

d) Which measure of variability best describes the data and why?

____

____

____

Comparing both of the data sets:

e) What do you notice about the differences in the numbers of Splash Lagoon and Wild Water Bay?

____

____

____

Task- Measures of Center and Spread

Mrs. Ingram has two Math 2 classes. With one class, he lectured and the students took notes. In the other class, the students worked in small groups to solve math problems. After the first test, Mrs. Ingram recorded the student grades to determine if his different styles of teaching might have impacted student learning.

Class 1: 80, 81, 81, 75, 70, 72, 74, 76, 77, 77, 77, 79, 84, 88, 90, 86, 80, 80, 78, 82

Class 2: 70, 90, 88, 89, 86, 86, 86, 86, 84, 82, 77, 79, 84, 84, 84, 86, 87, 88, 88, 88

1. Does either class have outliers? Show your calculations.
   Class 1: Class 2:

2. Calculate the measures of center (mean and median) for each class.

   Class 1:
   Mean _____ Median _____

   Class 2:
   Mean _____ Median _____

3. Calculate the measures of spread (IQR and MAD) for each class.

   Class 1:
   IQR _____ MAD _____

   Class 2:
   IQR _____ MAD _____

x	$\overline{x}$	$x-\overline{x}$	$\mid x-\overline{x} \mid$

x	$\overline{x}$	$x-\overline{x}$	$\mid x-\overline{x} \mid$

4. Which measure of center is more appropriate to use? Explain.
   Class 1: Class 2:

5. Which measure of spread is more appropriate to use? Explain.

Real Mathematics: Application

Class 1: Class 2:

Homework- Measure of Center and Spread

Directions: Find the measures of center and spread for the following data. The data sets below show the price that a homeowner paid, per term, for natural gas during each of the first ten months of 2011 and 2012. Use the data for 1–2.

2011: $1.59, $1.72, $1.71, $1.86, $2.32, $2.54, $2.45, $2.80, $2.38, $2.25
2012: $1.57, $1.61, $1.96, $1.71, $1.98, $2.17, $2.51, $2.44, $2.52, $2.10

1. Find the following for the year 2011.
   a. Mean:__________
   b. Median:_________
   c. Range:__________
   d. Interquartile Range:_________
   e. Mean Absolute Deviation:_________

x	$\bar{x}$	$x-\bar{x}$	$\lvert x-\bar{x} \rvert$

2. Find the following for the year 2012.
   a. Mean:__________
   b. Median:_________
   c. Range:__________
   d. Interquartile Range:_________
   e. Mean Absolute Deviation:_________

x	$\bar{x}$	$x-\bar{x}$	$\lvert x-\bar{x} \rvert$

Creating Graphs and Tables

GUIDED NOTES

Recall what you learned from middle school about creating graphs and tables to display data. There are many different ways to display data. A **dot plot** is a data representation that uses a number line and X's, dots, or other symbols to show frequency.

Create a dot plot for the following data:
Twelve employees at a small company make the following annual salaries (in thousands of dollars):
25, 30, 35, 35, 35, 40, 40, 40, 45, 45, 50, 60

Salary (thousands of dollars)

A **histogram** is a bar graph that is used to display the frequency of data divided into equal intervals. Notice that this is *similar* to a bar graph, but there are some differences. The bars of a histogram must be equal width and should touch but NOT overlap.

Listed are the scores from a golf tournament.

68, 78, 76, 71, 69, 73, 72, 74, 76, 70, 77, 74, 75, 76, 71, 74

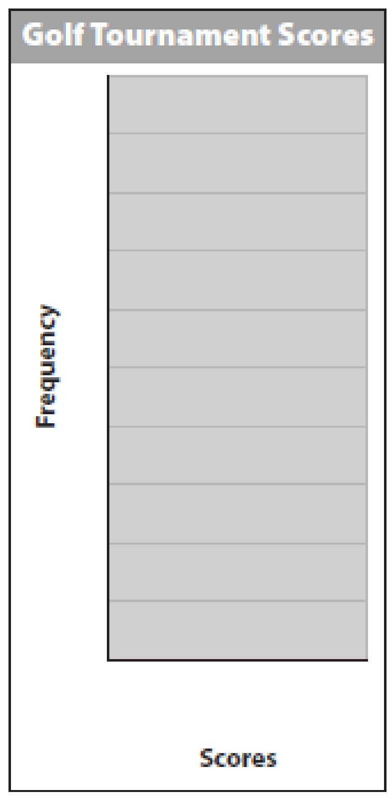
Create a frequency table. The data values range from ☐ to ☐, so use an interval width of 3, and start the first interval at ☐.

Score Interval	Frequency
68–☐	☐
71–☐	☐
74–☐	☐
77–☐	☐

Check that the sum of the frequencies is ☐.

$3 + 4 + 7 + 2 =$ ☐

Use the frequency table to create a histogram.



A **box plot** can be used to show how the values in a data set are distributed by using 5 values: minimum, first quartile, median, third quartile, and maximum.

The numbers of runs scored by a softball team in 20 games are given.

3, 4, 8, 12, 7, 5, 4, 12, 3, 9, 11, 4, 14, 8, 2, 10, 3, 10, 9, 7

#1 Order the data from least to greatest.

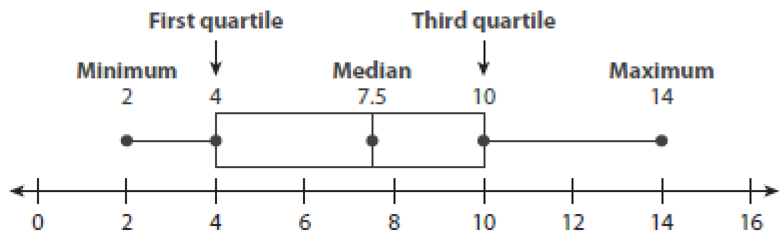
2, 3, 3, 3, 4, 4, 4, 5, 7, 7, 8, 8, 9, 9, 10, 10, 11, 12, 12, 14

#2 Identify the 5 needed values. Those values are the minimum, first quartile, median, third quartile, and maximum.

2, 3, 3, 3, 4, 4, 4, 5, 7, 7, 8, 8, 9, 9, 10, 10, 11, 12, 12, 14

Minimum	Q_1	Median	Q_3	Maximum
2	4	7.5	10	14

#3 Draw a number line and plot a point above each of the 5 needed values. Draw a box whose ends go through the first and third quartiles, and draw a vertical line segment through the median. Draw horizontal line segments from the box to the minimum and maximum.

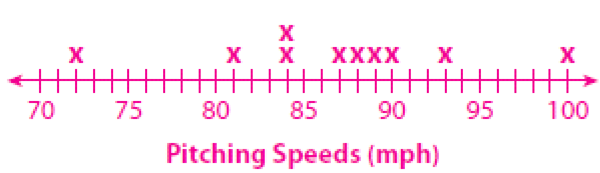


There are some data points that might be too low or too high that can skew the data. We call these data points outliers. An **outlier** is a value in a data set that is much greater or much less than most of the other values in the data set.

How to Identify an Outlier
A data value x is an outlier if $x < Q_1 - 1.5(\text{IQR})$ or if $x > Q_3 + 1.5(\text{IQR})$.

Sports Baseball pitchers on a major league team throw at the following speeds (in miles per hour): 72, 84, 89, 81, 93, 100, 90, 88, 80, 84, and 87.

Create a dot plot using an appropriate scale for the number line. Determine whether the extreme value is an outlier.



$72 \overset{?}{<} Q_1 - 1.5(\text{IQR})$

$72 \overset{?}{<} 81 - 1.5(9)$

$72 < 67.5$ False Therefore, 72 is not an outlier

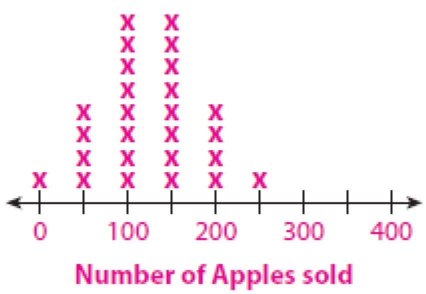
How do we know which graph to use when just looking at the data? Which graph best represents the data?

Example!

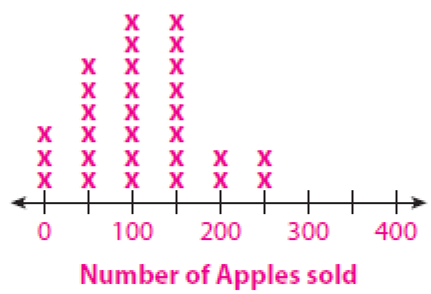
The table shows the number of days, over the course of a month, that specific numbers of apples were sold by competing grocers.

Number of Apples Sold	0	50	100	150	200	250	300
Grocery Store A	1	4	8	8	4	1	0
Grocery Store B	3	6	8	8	2	2	1

Grocery Store A



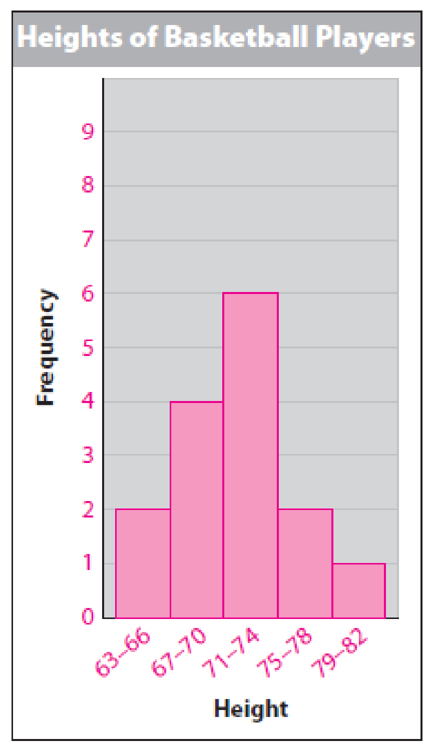
Grocery Store B



Listed are the heights of players, in inches, on a basketball team. Create a frequency table from the data. Then use the frequency table to create a histogram. **Possible table and histogram shown.**

79, 75, 74, 68, 63, 76, 74, 73, 69, 65, 71, 68, 74, 73, 70

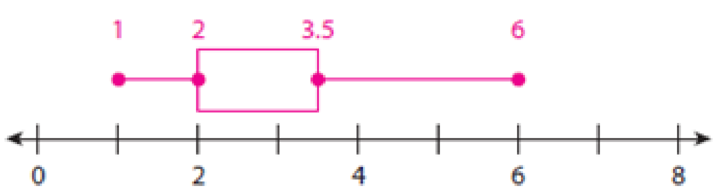
Height Interval	Frequency
63–66	2
67–70	4
71–74	6
75–78	2
79–82	1



The numbers of goals scored by Lisa's soccer team in 13 games are listed below.

2, 3, 4, 1, 1, 3, 4, 2, 6, 2, 2, 3, 2

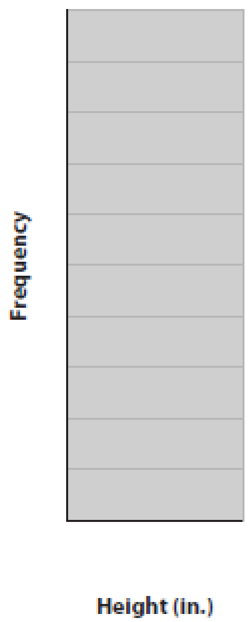
1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 4, 4, 6



Create a histogram for Marci's karate class.

Listed are the heights, in inches, of the students in Marci's karate class.

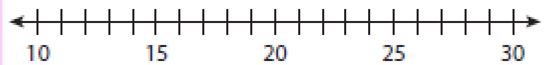
42, 44, 47, 50, 51, 53, 53, 55,
56, 57, 57, 58, 59, 60, 66



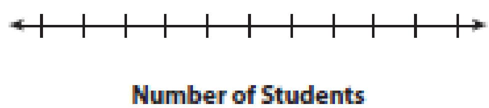
Create a box plot for Kelly's basketball team.

The numbers of baskets Kelly's team scored in 9 games are listed below.

14, 24, 29, 15, 16, 20, 24, 15, 15



Create a dot plot for the number of students in 10 randomly chosen classes in high school:
18, 22, 26, 31, 25, 20, 23, 26, 29, and 30.



For Marci's karate class, if we added the height of 100 in to the class would this data point be considered an outlier? Calculate and explain why or why not.

Independent Practice- Creating Graphs and Tables

The list below shows the number of milligrams of caffeine in certain types of tea.

8	47	19	34	30
10	58	20	39	32
12	4	22	40	92
18	85	26	27	

1. Use intervals to make a frequency table.

2. Use the frequency table you created in #1 to construct a histogram

3. Make two observations about the data based on the histogram you constructed.

____

____

____

4. Calculate if there are any outliers in the data set.

Crunchy Peanut Butter received the following quality ratings from taste testers:

72 68 91 86 59 88 82 95 75 40 78 82 86 96 73

5. Create a box plot using the data above.

6. Make two observations about the data based on the box plot you constructed.

____
____
____

7. Calculate if there are any outliers in the data set.

A survey of "How long does it take you to eat breakfast?" has these results:

Minutes:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
People:	6	2	3	5	2	5	0	0	2	3	7	4	1	8

8. Create a dot plot to represent the data above.

9. Make two observations about the data based on the dot plot you constructed.

Task- Creating Graphs and Tables

Part I

A group of 6th graders selected the color they wanted to wear for a field trip to the local zoo. The results are displayed in the table.

Color	Number of Girls	Number of Boys
Red	8	7
Blue	5	4
Green	2	5
Yellow	2	1

a. Construct a graphical display of the data.

b. Compare your graphical display to a classmate's graphical displays. How are they similar? How are they different?

____
____
____

Part 2

Your class keeps track of the number of hours they spend playing Fortnite each week. Display the data in the dot plot below.

12, 7, 5, 3, 8, 14, 8, 6, 6, 5, 4, 8, 8, 5, 7, 7, 6, 13, 15, 20, 24, 0

Part 3

The teacher asked each student in your class how many sodas they drink in a month. Use the data to create a frequency table, then create a histogram.

10, 12, 15, 35, 40, 15, 55, 60, 25, 20, 20, 30, 25, 35, 40, 35, 10, 30, 15, 30, 35, 40

Part 4

The following data below was collected from a survey of girls answering the question: "How long, in minutes, does it take to do your makeup?" Use the data below and create a box plot.

37, 84, 70, 64, 24, 93, 25, 39, 82, 49, 82

Homework- Creating Graphs and Tables

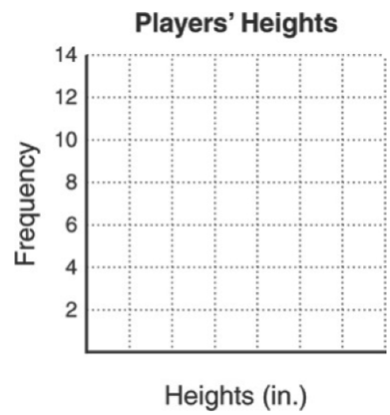
The heights of players at a basketball game are given in the table below. Use the data for 1–4; and 9.

Players' Heights (in.)											
75	78	80	87	72	80	81	83	85	78	76	81
77	78	83	83	78	82	79	80	75	84	82	90

1. Use the data above to make a frequency table. The first one is started for you.

Players' Heights	
Heights (in.)	**Frequency**
72–76	4
77–81	
82–86	
87–91	

2. Use your frequency table to make a histogram for the data.



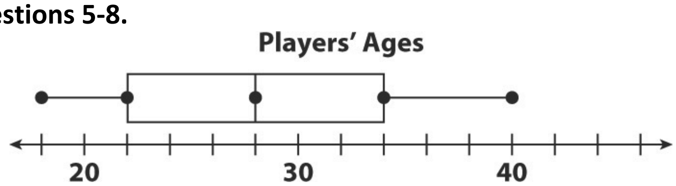
3. Which interval has the most players?

____

4. Describe the shape of the distribution.

____

Use the box plot to answer questions 5-8.



5. Find the median age.

____

6. Find the range.

____

7. Find the interquartile range.

____

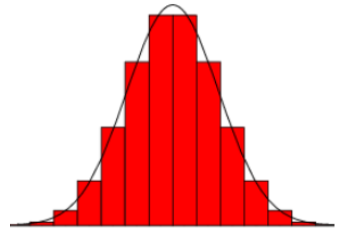
8. Find the age of the oldest player.

____

9. Create a dot plot from the Player's Height data from questions 1-4.

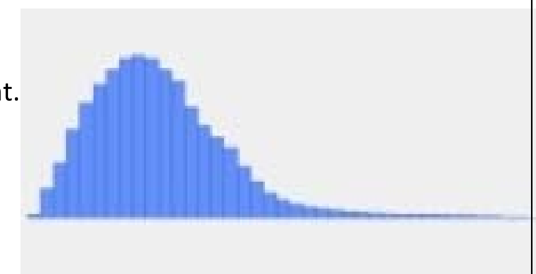
Analyzing Graphs and Tables

Now that we know how to find the measures of center and shape as well as creating all types of graphs, we can now explore how to describe graphs. We can describe data by the shape of the graph; it is easiest to describe the data from a dot plot or histogram.

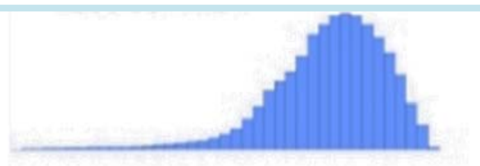


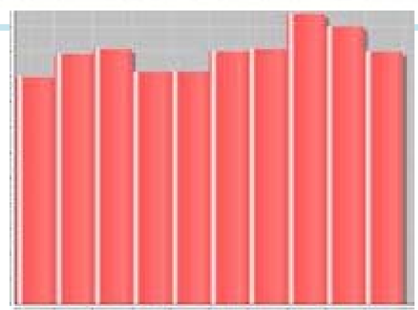
Normal distribution- ___________ distribution with bell-shaped curve.

Skewed right- __________ skew, __________ outlier pulls curve to the right.

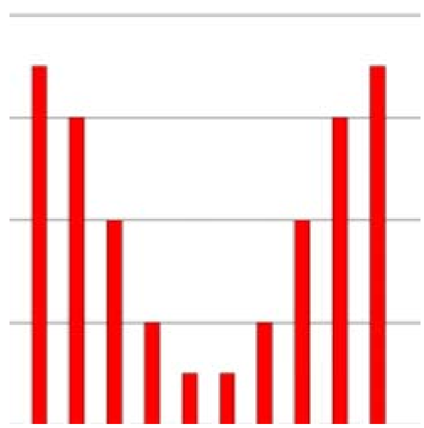


Skewed left- __________ skew, __________ outlier pulls curve to the left.





Uniform distribution- __________ height of all bars or frequency.



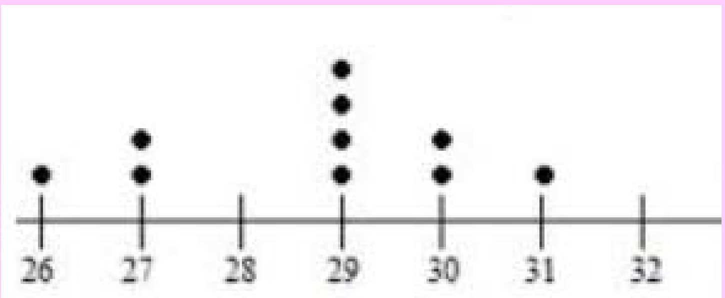
Bi-Modal distribution- distribution with ______ distinct peaks.

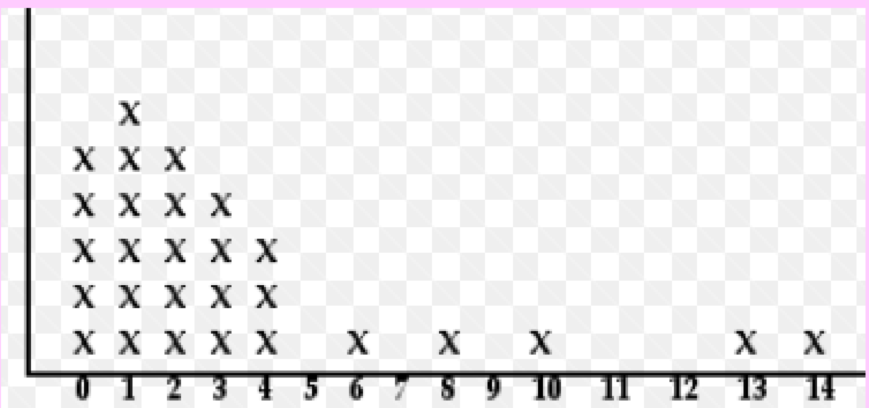
Questions To Ponder

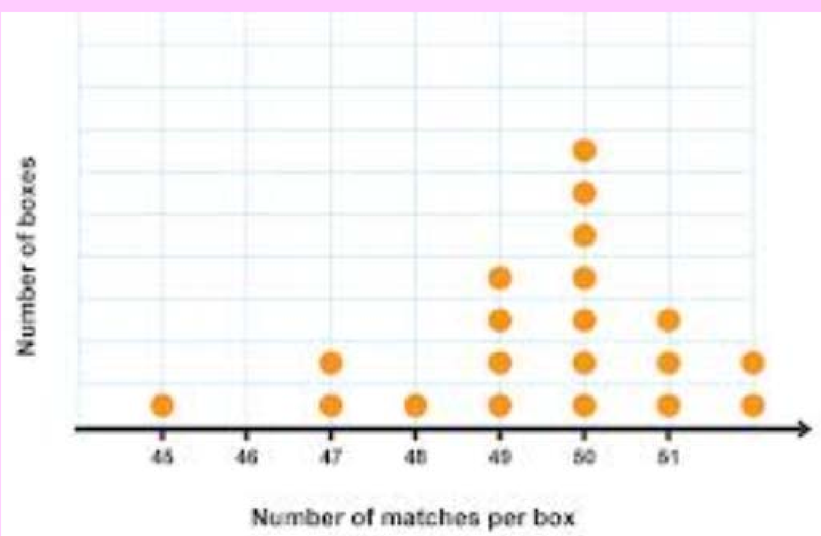
How do outliers affect the shape of a dot plot and/or histogram? Explain in detailed sentences.

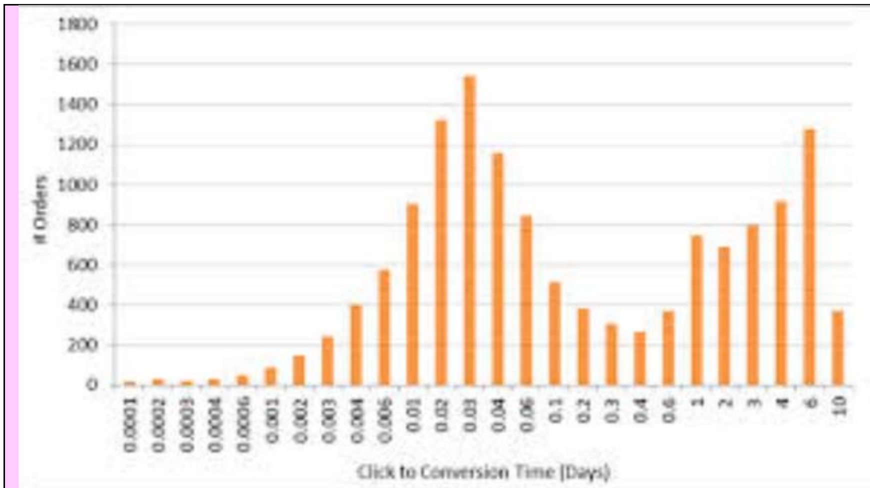
SELF CHECK

Describe each graph by the following: normal distribution, skewed left, skewed right, uniform distribution, and bi-modal distribution.








1800
1600
1400
1200
1000
800
600
400
200
0
Orders
0.0001
0.0002
0.0003
0.0004
0.0006
0.001
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0.003
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0.02
0.03
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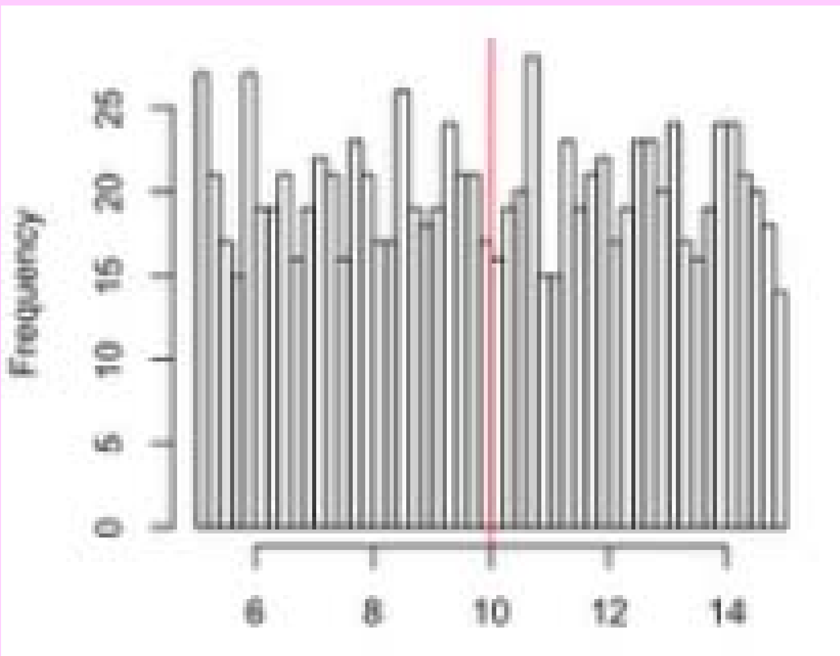
6
10
Click to Conversion Time (Days)


Frequency
0
5
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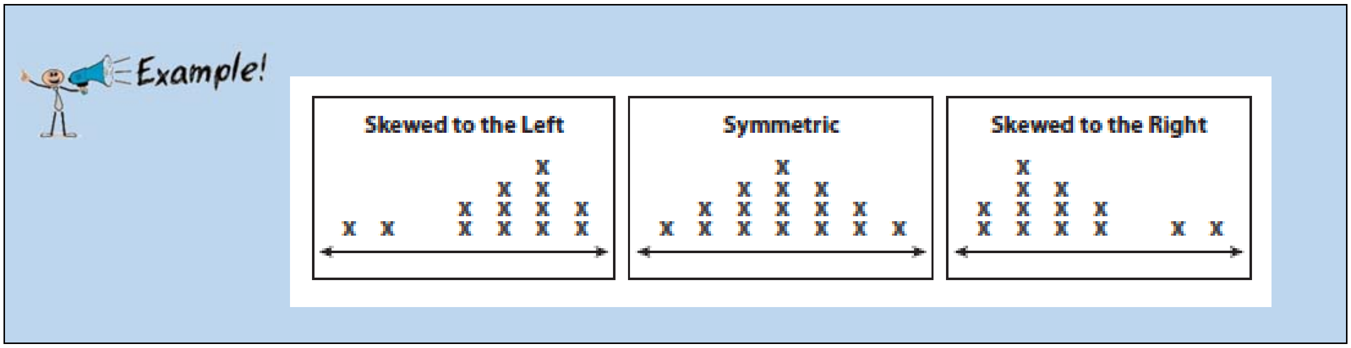
14

Independent Practice- Analyzing Graphs and Tables



Complete a dot plot for the following Grocery Stores for the number of apples sold.

The table shows the number of days, over the course of a month, that specific numbers of apples were sold by competing grocers.

Number of Apples Sold	0	50	100	150	200	250	300
Grocery Store A	1	4	8	8	4	1	0
Grocery Store B	3	6	8	8	2	2	1

Grocery Store A

Grocery Store B

Circle the following adjectives that describes the dot plots.

a. The distribution for grocery store A is: left-skewed/right-skewed/symmetric.

This means that the number of apples sold each day is evenly/unevenly distributed about the mean.

b. The distribution for grocery store B is: left-skewed/right-skewed/symmetric.

This means that the number of apples sold each day is evenly/unevenly distributed about the mean.

Answer the following questions based on the shape of the graph.

c. Will the mean and median in a symmetric distribution always be approximately equal? Explain.

____

____

____

d. Will the mean and median in a skewed distribution always be approximately equal? Explain.

____

____

Task- Analyzing Graphs and Tables

Bob believes he is a basketball star and so does his friend Alan.

Bob's Points per Game	**Alan's Points per Game**
8, 15, 10, 10, 10, 15, 7, 8, 10, 9, 12, 11, 11, 13, 7, 8, 9, 9, 8, 10, 11, 14, 11, 10, 9, 12, 14, 14, 12, 13, 5, 13, 9, 11, 12, 13, 10, 8, 7, 8	1, 3, 0, 2, 4, 5, 7, 7, 8, 10, 4, 4, 3, 2, 5, 6, 6, 6, 8, 8, 10, 11, 11, 10, 12, 12, 5, 6, 8, 9, 10, 15, 10, 12, 11, 11, 6, 7, 7, 8

1. Create dot plots for Bob and Alan's last forty games.

2. Create box plots for Bob and Alan's last forty games.

3. Create histograms of both Bob's and Alan's data.

4. Which graphical representation best displayed Bob's and Alan's data? Why do you think that is the best graphical representation?

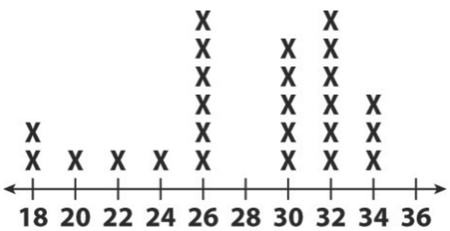
5. Describe each person's data in terms of shape.

6. Are there any outliers for Bob and Alan's last forty games? If so, calculate them. How do outliers affect the shape of their graph?

Homework- Analyzing Graphs and Tables

Identify each dot plot as symmetric, skewed to the left, or skewed to the right. The first one is done for you.

1.



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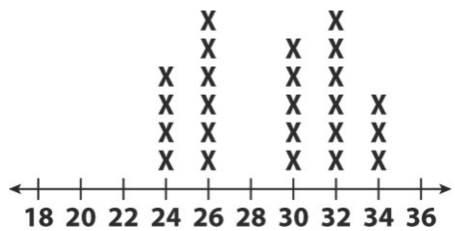
2.



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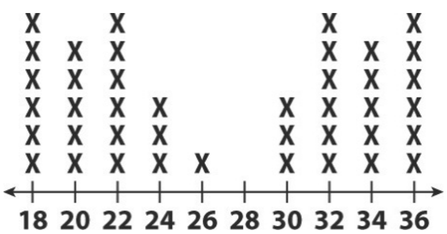
3.



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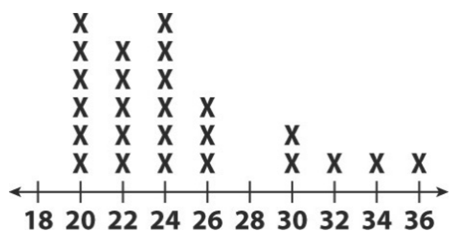
4.



____

The table below shows the scores of ten golfers in a tournament.
Use the data for Problems 5–10. The first one is done for you.

68	69	70	73	74	74	74	75	75	76

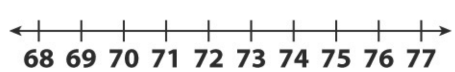
5. Find the mean and median.

   mean: 72.8; median: 74

6. Find the range and interquartile range.

   ____

7. Make a dot plot for the data.



8. Identify the dot plot as symmetric, skewed to the left, or skewed to the right.

   ____

9. Suppose an 11th golfer with a score of 95 is added to the tournament scores. Which of the statistics from Problems 5 and 6 would change?

   ____

10. If a score of 95 were added, would it be an outlier? Explain.

____

Term	Definition	Notation	Diagram/Visual
Bivariate Data	________________ ________________ ________________		
Categorical Variables	________________ ________________ ________________		
Conditional Frequencies	________________ ________________ ________________		
Joint Frequencies	________________ ________________ ________________		
Marginal Frequencies	________________ ________________ ________________		
Quantitative Variables	________________ ________________ ________________		

Creating Two-Way Frequency Tables

Data that can be expressed with numerical measurements are quantitative data. In this lesson you will examine qualitative data, or categorical data, which cannot be expressed using numbers. Data describing animal type, model of car, or favorite song are examples of categorical data.
A frequency table shows how often each item occurs in a set of categorical data. If a data set has two categorical variables, you can list the frequencies of the paired values in a two-way frequency table. A two-way frequency table is a frequency table that displays data collected from one source that belong to two different categories. One category of data is represented by rows, and the other is represented by columns.

GUIDED NOTES

a. A high school's administration asked 100 randomly selected students in the 9th and 10th grades about what fruit they like best. Complete the table.

	Preferred Fruit			
Grade	**Apple**	**Orange**	**Banana**	**Total**
9th	19	12	23	
10th	22	9	15	
Total				

Row totals:

9th: $19 + 12 + 23 = 54$

10th: $22 + 9 + 15 = 46$

Column totals:

Apple: $19 + 22 = 41$

Orange: $12 + 9 = 21$

Banana: $23 + 15 = 38$

Grand total:

Sum of row totals: $54 + 46 = 100$

Sum of column totals: $41 + 21 + 38 = 100$

Both sums should equal the grand total.

b. Jenna asked some randomly selected students whether they preferred dogs, cats, or other pets. She also recorded the gender of each student. The results are shown in the two-way frequency table below. Each entry is the frequency of students who prefer a certain pet and are a certain gender. For instance, 8 girls prefer dogs as pets. Complete the table.

	Preferred Pet			
Gender	**Dog**	**Cat**	**Other**	**Total**
Girl	8	7	1	$\square$
Boy	10	5	9	$\square$
Total	$\square$	$\square$	$\square$	$\square$

Row totals:

Girl: $8 + 7 + 1 = \square$

Boy: $10 + 5 + 9 = \square$

Column totals:

Dog: $8 + 10 = \square$

Cat: $7 + 5 = \square$

Other: $1 + 9 = \square$

Grand total:

Sum of row totals: $16 + \square = \square$

Sum of column totals: $18 + \square + \square = \square$

Both sums should equal the grand total.

c. One hundred students were surveyed about which beverage they chose at lunch. Some of the results are shown in the two-way frequency table below. Complete the table.

	Lunch Beverage			
Gender	**Juice**	**Milk**	**Water**	**Total**
Girl	10		17	
Boy	15	24	21	60
Total				

Find the total number of girls by subtracting: $100 - 60 = \boxed{}$

So, the total number of girls is $\boxed{}$. The number of girls who do not choose milk is $\boxed{} + \boxed{} = \boxed{}$.

Find the number of girls who chose milk by subtracting: $\boxed{} - \boxed{} = \boxed{}$

d. Teresa surveyed 100 students about whether they like pop music or country music. Out of the 100 students surveyed, 42 like only pop, 34 like only country, 15 like both pop and country, and 9 do not like either pop or country. Complete the two-way frequency table.

	Like Pop		
Like Country	**Yes**	**No**	**Total**
Yes			
No			
Total			

e. Luis surveyed 100 students about whether they like soccer. The number of girls and the number of boys completing the survey are equal. 20 girls and 35 boys responded that they do not like soccer. Create a two-way frequency table for this data.

1. When is it appropriate to use a two-way frequency table to represent data?

2. What information is provided by a two-way frequency table, and how is that information organized?

3. To complete a two-way frequency table that is missing information, is it necessary to know the total of all the values in the table? Explain.

EX 1. The results of a survey of 150 students about whether they own an electronic tablet or a laptop are shown in the two-way frequency table.

	Device				
Gender	**Electronic tablet**	**Laptop**	**Both**	**Neither**	**Total**
Girl	15	54		9	88
Boy		35	8	5	
Total					

	Device				
Gender	**Electronic tablet**	**Laptop**	**Both**	**Neither**	**Total**
Girl	15	54	**10**	9	88
Boy	**14**	35	8	5	**62**
Total	**29**	**89**	**18**	**14**	**150**

Complete the table. Do the surveyed students own more laptops or more electronic tablets?

The number of boys is 150 − 88 = 62.

Girls who own both electronic devices: 88 − 9 − 54 − 15 = 10

Boys who own an electronic tablet: 62 − 5 − 8 − 35 = 14; Students own more laptops.

Which group had more people answer the survey, boys or students who own an electronic tablet only? Explain.

Boys: 62 boys answered the survey, which is more than the 29 people who own an electronic tablet only.

SELF CHECK

1. A group of 200 high school students were asked about their use of email and text messages. The results are shown in the two-way frequency table. Complete the table.

	Text Messages		
Email	**Yes**	**No**	**Total**
Yes	72		90
No		45	
Total			

2. Teresa surveyed 100 students about whether they wanted to join the math club or the science club. Thirty-eight students wanted to join the math club only, 34 wanted to join the science club only, 21 wanted to join both math and science clubs, and 7 did not want to join either. Complete the two-way frequency table.

	Science		
Math	**Yes**	**No**	**Total**
Yes		38	
No	34		
Total			

3. A surveyor asked students whether they favored or did not favor a change in the Friday lunch menu at their school.
   - The survey involved 200 students.
   - The number of boys surveyed equaled the number of girls surveyed.
   - Fifty percent of the girls favored the change.
   - The number of boys who did not favor the change was two-thirds of the number of boys who favored the change.

   Complete the two-way frequency table. Explain your reasoning.

	Favor or Disfavor the Change		
Gender	**Yes**	**No**	**Total**
Girls			

Creating Two-Way Frequency Tables

Example!

Complete the two-way frequency table.

Suppose you are given the circled information in the table and instructed to complete the table.

	Eat Cereal for Breakfast		
Gender	**Yes**	**No**	**Total**
Girl	42	12	54
Boy	36	10	46
Total	78	22	100

Find the total number of boys by subtracting: 100 - 54 = 46
Find the number of boys who do eat cereal by subtracting: 46 - 10 = 36
Add to find the total number of students who eat cereal: 42 + 36 = 78
Add to find the total number of students who do not eat cereal: 12 + 10 = 22

<u>Categorical Data and Frequencies:</u> Data that can be expressed with numerical measurements are quantitative data. In this lesson you will examine qualitative data, or categorical data, which cannot be expressed using numbers. Data describing animal type, model of car, or favorite song are examples of categorical data.

1. Circle the categorical data variable. Justify your choice.

temperature weight height color

____

____

2. Identify whether the given data is categorical or quantitative.

large, medium, small ________________

120 ft^2, 130 ft^2, 140 ft^2 ________________

3. A **frequency** table shows how often each item occurs in a set of categorical data. Use the categorical data listed on the left to complete the frequency table.

Ways Students Get to School
bus car walk car car car bus walk walk walk bus bus car bus bus walk bus car bus car

Way	Frequency
bus	8
car	
____	

4. How did you determine the numbers for each category in the frequency column?

____

____

5. What must be true about the sum of the frequencies in a frequency table?

____

____

<u>Constructing Two-Way Frequency Tables:</u> If a data set has two categorical variables, you can list the frequencies of the paired values in a two-way frequency table.

6. Antonio surveyed 60 of his classmates about their participation in school activities and whether they have a part-time job. The results are shown in the two-way frequency table below. Complete the table.

	Activities				
Job	**Clubs Only**	**Sports Only**	**Both**	**Neither**	**Total**
Yes	12	13	16	4	
No	3	5	5	2	
Total					

7. Jen surveyed 100 students about whether they like baseball or basketball. Complete the table.

	Like Basketball		
Like Baseball	**Yes**	**No**	**Total**
Yes	61	13	
No	16	10	
Total			

8. 100 students were asked what fruit they chose at lunch. The two-way frequency table shows some of the results of the survey. Complete the table.

	Lunch Fruit			
Gender	**Apple**	**Pear**	**Banana**	**Total**
Girl		17	11	49
Boy		10	16	
Total				

9. 200 high school teachers were asked whether they prefer to use the chalkboard or projector in class. The two-way frequency table shows some of the results of the survey. Complete the table.

	Preferred Teaching Aid		
Gender	**Chalkboard**	**Projector**	**Total**
Female		56	99
Male	44		
Total	87	113	200

10. You are making a two-way frequency table of 5 fruit preferences among a survey sample of girls and boys. What are the dimensions of the table you would make? How many entries would you need to fill the table with frequencies and totals?

____

____

11. A 3 categories-by-3 categories two-way frequency table has a row with 2 numbers, and no row or column totals. Can you fill the row?

____

____

____

12. **Essential Question Check-In** How can you summarize categorical data for 2 categories?

____

____

Real Mathematics: Application

Creating Two-Way Frequency Tables Performance Task

1. Two hundred students were asked about their favorite sport. Of the 200 students surveyed, 98 were female. Some of the results are shown in the following two-way frequency table.

	Favorite Sport				
Gender	**Football**	**Baseball**	**Basketball**	**Soccer**	**Total**
Female			36	12	
Male	38	19			
Total	64			36	

a. Complete the table.

b. Which sport is the most popular among the students? Which is the least popular? Explain.

c. Which sport is most popular among the females? Which sport is most popular among the males? Explain.

2. **Explain the Error** Find the mistake in completing the two-way frequency table for a survey involving 50 students. Then complete the table correctly.

	Favorite Foreign Language Class			
Gender	**Russian**	**German**	**Italian**	**Total**
Girl	8	8	8	24
Boy	42	9	7	58
Total	50			

Correct table:

	Favorite Foreign Language Class			
Gender	**Russian**	**German**	**Italian**	**Total**
Girl	8	8	8	24
Boy		9	7	
Total				

3. **Justify Reasoning** Charles surveyed 100 boys about their favorite color. Of the 100 boys surveyed, 44 preferred blue, 25 preferred green, and 31 preferred red.

a. Explain why it is not possible to make a two-way frequency table from the given data.

b. Suppose Charles also surveyed some girls. Of the girls surveyed, 30 preferred blue and 43 preferred green. Can Charles make a two-way frequency table now? Can he complete it?

4. **Persevere in Problem Solving** Shown are two different tables about a survey involving students. Each survey had a few questions about musical preferences. All students answered all questions. Complete the tables. What type of music do the students prefer?

	Likes Classical Music		
Gender	**Yes**	**No**	**Total**
Girl	21		
Boy		22	
Total			100

	Likes Blues Music		
Gender	**Yes**	**No**	**Total**
Girl		15	49
Boy		15	
Total			

Creating Two-Way Frequency Tables Homework

1. Circle the letter of each data set that is categorical. Select all that apply.

A. 75°, 79°, 77°, 85°

B. apples, oranges, pears

C. male, female

D. blue, green, red

E. 2 feet, 5 feet, 12 feet

F. classical music, country music

G. 1 centimeter, 3 centimeters, 9 centimeters

2. James surveyed some of his classmates about what vegetable they like best. Complete the table.

	Preferred Vegetable			
Grade	**Carrots**	**Green Beans**	**Celery**	**Total**
9th	30	15	24	
10th	32	9	20	
Total				

3. 200 students were asked to name their favorite science class. The results are shown in the two-way frequency table. Complete the table.

	Favorite Science Class			
Gender	**Biology**	**Chemistry**	**Physics**	**Total**
Girl	42	39	23	104
Boy		45	32	
Total				

Use the table from question 3 to answer questions 4 and 5.

4. How many boys were surveyed? Explain how you found your answer.

5. How many more girls than boys chose biology as their favorite science class? Explain how you found your answer.

Interpret & Analyze Two-Way Frequency Tables

You can extract information about paired categorical variables by reading a two-way frequency table. Entries in the body of the table are called joint frequencies. The cells which contain the sum of the initial counts by row and by column are called marginal frequencies.

To show what portion of a data set each category in a frequency table makes up, you can convert the data to relative frequencies. The relative frequency of a category is the frequency of the category divided by the total of all frequencies. When a two-way frequency table that displays percentages or ratios (called relative frequencies) instead of just frequency counts, the table is referred to as a two-way relative frequency table. These two-way tables can show relative frequencies for the whole table, for rows, or for columns. The relative frequencies may be displayed as a ratio, a decimal, or percent.

GUIDED NOTES

a. Take a look at the vocabulary used to identify cell locations in two-way frequency tables. Entries in the body of the table (the blue cells where the initial counts appear) are called **joint frequencies**. The cells which contain the sum (the orange "Totals" cells) of the initial counts by row and by column are called **marginal frequencies**. Note that the lower right corner cell (the total of all the counts) is not labeled as a marginal frequency.

Joint Frequency

Marginal Frequency

	Sport Utility Vehicle (SUV)	Sports Car	Totals
male	21	39	60
female	135	45	180
Totals	156	84	240

MathBits.com

b. Relative Frequency for WHOLE table:

Whole Table Relative Frequencies -
Divide all cells by 240.

MathBits.com	Sport Utility Vehicle (SUV)	Sports Car	Totals
male	$\frac{21}{240}=0.09$	$\frac{39}{240}=0.16$	$\frac{60}{240}=0.25$
female	$\frac{135}{240}=0.56$	$\frac{45}{240}=0.19$	$\frac{180}{240}=0.75$
Totals	$\frac{156}{240}=0.65$	$\frac{84}{240}=0.35$	$\frac{240}{240}=1.00$

c. When comparing the categorical variables in a two-way frequency table, we need to exam the table by separate categories (rows or columns). When a relative frequency is determined based upon a row or column, it is called a "conditional" relative frequency. To obtain a conditional relative frequency, divide a joint frequency (count inside the table) by a marginal frequency total (outer edge) that represents the condition being investigated. You may also see this term stated as row conditional relative frequency or column conditional relative frequency.
Basically, we are going to look at the women and men separately, based upon how many women were surveyed, and how many men were surveyed.

Conditional Relative Frequency for Rows: If the two-way relative frequency is for rows, the entries in each row of the table are divided by the total for that row (on the right hand side). The ratio of "1", or 100%, occurs in all right hand "total" cells. See image below.

Row Relative Frequencies -
Divide male row by 60.
Divide female row by 180.
Divide Totals row by 240.

Conditional relative frequencies

	Sport Utility Vehicle (SUV)	Sports Car	Totals
male	$\frac{21}{60} = 0.35$	$\frac{39}{60} = 0.65$	$\frac{60}{60} = 1.00$
female	$\frac{135}{180} = 0.75$	$\frac{45}{180} = 0.25$	$\frac{180}{180} = 1.00$
Totals	$\frac{156}{240} = 0.65$	$\frac{84}{240} = 0.35$	$\frac{240}{240} = 1.00$

MathBits.com

Conditional Relative Frequency for Columns: If the two-way relative frequency is for columns, the entries in each column of the table are divided by the total for that column (at the bottom). The ratio of "1", or 100%, occurs in all of the "total" cells at the bottom. See image below.

Column Relative Frequencies -
Divide SUV column by156.
Divide Sports Car column by 84.
Divide Totals column by 240.

Conditional relative frequencies

	Sport Utility Vehicle (SUV)	Sports Car	Totals
male	$\frac{21}{156} = 0.13$	$\frac{39}{84} = 0.46$	$\frac{60}{240} = 0.25$
female	$\frac{135}{156} = 0.87$	$\frac{45}{84} = 0.54$	$\frac{180}{240} = 0.75$
Totals	$\frac{156}{156} = 1.00$	$\frac{84}{84} = 1.00$	$\frac{240}{240} = 1.00$

MathBits.com

d. A variety of questions can be answered by examining a two-way frequency table. Let's look at some possibilities:

Two-way frequency table

	Sport Utility Vehicle (SUV)	Sports Car	Totals
male	21	39	60
female	135	45	180
Totals	156	84	240

MathBits.com

How many people responded to the survey? 240

How many males responded to the survey? 60

How many people chose an SUV? 156

How many females chose a sports car? 45

How many males chose an SUV? 21

Two-way relative frequency table

(whole table)

	Sport Utility Vehicle (SUV)	Sports Car	Totals
male	$\frac{21}{240}=0.09$	$\frac{39}{240}=0.16$	$\frac{60}{240}=0.25$
female	$\frac{135}{240}=0.56$	$\frac{45}{240}=0.19$	$\frac{180}{240}=0.75$
Totals	$\frac{156}{240}=0.65$	$\frac{84}{240}=0.35$	$\frac{240}{240}=1.00$

MathBits.com

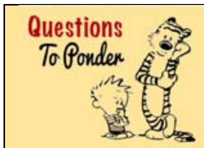
What percentage of the survey takers was female? 75%

What is the relative frequency of males choosing a sports car?

$\frac{39}{240}$ or about 0.16

Was there a higher percentage of males or females choosing an SUV?
higher percentage of females

e. An "association" exists between two categorical variables if the row (or column) conditional relative frequencies are different for the rows (or columns) of the table. The bigger the differences in the conditional relative frequencies, the stronger the association between the variables. If the conditional relative frequencies are nearly equal for all categories, there may be no association between the variables. Such variables are said to be independent.



1. How can you recognize possible associations and trends between two categories of categorical data?

2. Which values in a frequency table are compared to the total by a joint relative frequency? Which values are compared to the total by a marginal relative frequency? Explain, using examples.

3. When determining a conditional relative frequency, how do you know what number to use as the numerator of the fraction?

4. What numbers in a two-way relative frequency table can be used to determine whether a possible association exists? Explain.

5. How can you determine whether one category has an influence over the distribution of the data in a two-way frequency table?

Example!

EX 1. For her survey about sports preferences, Kenesha also recorded the gender of each student. The results are shown in the two-way frequency table for Kenesha's data.

	Preferred Sport			
Gender	**Basketball**	**Football**	**Soccer**	**Total**
Girl	6	12	18	36
Boy	14	20	10	44
Total	20	32	28	80

To find the relative frequencies, divide each frequency by the grand total.

	Preferred Sport			
Gender	**Basketball**	**Football**	**Soccer**	**Total**
Girl	$\frac{6}{80} = 0.075$	$\frac{12}{80} = 0.15$	$\frac{18}{80} = 0.225$	$\frac{36}{80} = 0.45$
Boy	$\frac{14}{80} = 0.175$	$\frac{20}{80} = 0.25$	$\frac{10}{80} = 0.125$	$\frac{44}{80} = 0.55$
Total	$\frac{20}{80} = 0.25$	$\frac{32}{80} = 0.4$	$\frac{28}{80} = 0.35$	$\frac{80}{80} = 1$

The joint relative frequencies tell what percent of all those surveyed are in each category:

- 7.5% are girls who prefer basketball.
- 15% are girls who prefer football.
- 22.5% are girls who prefer soccer.
- 17.5% are boys who prefer basketball.
- 25% are boys who prefer football.
- 12.5% are boys who prefer soccer.

The marginal relative frequencies tell what percent of totals has a given single characteristic:

- 25% prefer basketball.
- 40% prefer football.
- 35% prefer soccer.
- 45% are girls.
- 55% are boys.

EX 2.

Shown at the right is the two-way frequency table about a survey that one school administrator conducted. The goal was to help determine the basis for class projects.

A student needs to use relative frequencies to interpret associations and trends between two categories in two-way frequency tables. How can such frequencies be calculated?

	Response		
Grade	**State Park**	**Train Station**	**Total**
9	64	56	120
10	54	66	120
Total	118	122	240

Marginal Relative Frequency
Compare, using a ratio, a column or row total to the grand total.

Train Station

The train station total is 122. → $\frac{122}{240}$ ✓

Joint Relative Frequency
Compare, using a ratio, a non-total frequency to the grand total.

Grade 9 and Train Station

56 is where Grade 9 and Train Station intersect. → $\frac{56}{240}$ ✓

Conditional Relative Frequency
Compare, using a ratio, a non-total frequency to a row total.
Condition: Grade 9. Use that **row**.

Given **Grade 9,** then Train Station

120 is the row total. → $\frac{56}{\mathbf{120}}$ ✓

Compare, using a ratio, a non-total frequency to a column total.
Condition: Train Station. Use that **column**.

Given **Train Station,** then Grade 9

122 is the column total. → $\frac{56}{\mathbf{122}}$ ✓

For this type of relative frequency, a condition is given first. This determines whether a row or column total is used as the denominator.

SELF CHECK

1. Millie performed a survey of students in the lunch line and recorded which type of fruit each student selected along with the gender of each student. The two-variable frequency data she collected is shown in the table.

	Fruit			
	Apple	**Banana**	**Orange**	**Total**
Girl	16	10	14	40
Boy	25	13	14	52
Total	41	23	28	92

	Fruit			
	Apple	**Banana**	**Orange**	**Total**
Girl	17.4%			
Boy	27.2%			
Total	44.6%			

The joint relative frequencies:

- ______ are girls who selected an apple.
- ______ are girls who selected a banana.
- ______ are girls who selected an orange.
- ______ are boys who selected an apple.
- ______ are boys who selected a banana.
- ______ are boys who selected an orange.

The marginal relative frequencies:

- ______ selected an apple.
- ______ selected a banana.
- ______ selected an orange.
- ______ are girls.
- ______ are boys.

Interpret & Analyze Two-Way Frequency Tables

<u>Two-Way Relative Frequency Tables:</u> Two types of relative frequencies are found in a relative frequency table:

- **A <u>joint relative frequency</u> is found by dividing a frequency that is not in the Total row or the Total column by the grand total. It tells what portion of the total has both of the two specified characteristics.**
- **A <u>marginal relative frequency</u> is found by dividing a row total or a column total by the grand total. It tells what portion of the total has a specified characteristic.**

1. The following frequency data shows the number of states, including the District of Columbia, that favored each party in the presidential popular vote in 1976 and in 2012. Complete the table above with relative frequencies using percents.

	2012 Election		
1976 Election	**Democrat**	**Republican**	**Total**
Democrat	12 =	12 =	24 =
Republican	15 =	12 =	27 =
Total	27 =	24 =	51 =

Use the table from question 1 to answer questions 2 and 3.

2. What percent switched from Democrat in 1976 to Republican in 2012? What type of frequency is this?

3. What percent voted Republican in 1976? What type of frequency is this?

4. In some states, a driver of a vehicle may not use a handheld cell phone while driving. In one state with this law, 250 randomly selected drivers were surveyed to determine the association between drivers who know the law and drivers who obey the law. The results are shown in the table below. Complete the table of two-way relative frequencies using percents.

	Knows the Law		
Obeys the Law	**Yes**	**No**	**Total**
Yes	160 =	45 =	
No	25 =	20 =	
Total			

Use the table from question 4 to answer questions 5 and 6.

5. What is the relative frequency of drivers who know and obey the law?

6. What is the relative frequency of drivers who know the law?

<u>Conditional Relative Frequencies:</u> A conditional relative frequency describes what portion of a group with a given characteristic also has another characteristic. A conditional relative frequency is found by dividing a frequency that is not in the Total row or the Total column by the total for that row or column.

Refer to the election data from Exercises 1 – 3. Answer using percents rounded to the nearest tenth.

7. What is the conditional relative frequency of a state's popular vote being won by the Democrat in 2012, given that it was won by the Democrat in 1976?

8. What is the conditional relative frequency of a state's popular vote being won by the Democrat in 1976, given that it was won by the Democrat in 2012?

Refer to the cell phone law data from Exercises 4 – 6. Answer using percents rounded to the nearest tenth.

9. What percent of drivers obey the law despite not knowing the law?

10. What is the conditional relative frequency of drivers who obey the law, given that they know the law?

<u>Finding Possible Associations:</u> You can analyze two-way frequency tables to locate possible associations or patterns in the data. Use the previously described data to determine whether there are associations between the categories surveyed.

Refer to the election data from Exercises 1 – 3.

11. Is there an association between the party that won the popular vote in a state in 1976 and in 2012?

Refer to the cell phone law data from Exercises 4 – 6.

12. Most drivers who don't know that it is illegal to operate a cell phone while driving obey the law anyway, presumably out of a general concern for safe driving. Does this mean there is no association between knowledge of the cell phone law and obeying the cell phone law?

Analyze & Interpret Two-Way Frequency Tables Performance Task

1. **Multipart Classification** Classify each statement as describing a *joint*, *marginal*, or *conditional* relative frequency.

a. In a study on age and driving safety, 33% of drivers were considered younger and a high accident risk.

b. In a study on age and driving safety, 45% of older drivers were considered a high accident risk.

c. In a study on age and driving safety, 67% of drivers were classified as younger.

d. In a pre-election poll, 67% of the respondents who preferred the incumbent were men.

e. In a pre-election poll, 33% of women preferred the challenger.

f. In a pre-election poll, 16% of respondents were men who preferred the challenger.

A public opinion survey explored the association between age and support for increasing the minimum wage. Although age is a quantitative variable, the researchers create three categories of age groups and treated the age data as categorical. The results are found in the following two-way frequency table.

	For	Against	No Opinion	TOTAL
Ages 21-40	25	20	5	50
Ages 41-60	30	30	15	75
Over 60	50	20	5	75
TOTAL	105	70	25	200

Frequency Count

2. What percentage of those individuals surveyed were in the 21 – 40 age group and for increasing the minimum wage? Show your work for finding this percentage. This percentage is called a joint percentage.

3. For the 21 to 40 age group, what percentage supports increasing the minimum wage? Explain how you arrived at your percentage. This percentage is called a conditional percentage.

4. For the 41 to 60 age group, what percentage supports increasing the minimum wage? Show your work.

5. For the over 60 age group, what percentage supports increasing the minimum wage? Show your work.

6. What percentage of all individuals surveyed favored increasing the minimum wage? Explain how you arrived at your percentage. This percentage is called a marginal percentage.

7. If there is no association between age group and opinion about increasing the minimum wage, we would expect the conditional percentages found in parts 2-4 to all be the same. We observe that the three percentages are different. What numerical value for the conditional percentage of each age group that favor increasing the minimum wage would we expect if there is no association between age group and opinion? Hint: Consider the percentage who favor increasing the minimum wage ignoring the age group classification of the individuals surveyed.

The table below gives the responses of 50 teachers' responses to a survey asking which activity they enjoyed most: dancing, playing/watching sports, or seeing movies. Is there an association between gender of the teacher and type of activity enjoyed the most?

	Dance	**Sports**	**Movies**	**TOTAL**
Female	16	6	8	**30**
Male	2	10	8	**20**
TOTAL	**18**	**16**	**16**	50

8. In exploring whether an association exists between gender and type of activity, we are interested in knowing if gender of the teacher helps predict the type of activity the teacher enjoys the most. Gender is the explanatory variable and type of activity is the response variable.

Finish constructing the table below displaying conditional percentages for type of activity conditioned upon gender.

	Dance	**Sports**	**Movies**	**TOTAL**
Female	16/30 = 53%			**100%**
Male	2/20=10%			**100%**

9. If a teacher is female, what percentage enjoys sports the most

10. If a teacher is male, what percentage enjoys sports the most?

11. Of all the teachers surveyed, what percentage enjoys sports the most?

12. Based on your answers does there appear to be an association between gender of teacher and type of activity enjoyed? Explain using your three percentages above.

13. Comment on how genders differ with respect to type of activity. Comment on any similarities.

Interpret & Analyze Two-Way Frequency Tables Homework

The results of a survey of 45 students and the foreign language they are studying are shown in the two-way frequency table.

	Language			
Gender	**Chinese**	**French**	**Spanish**	**Total**
Girl	2	8	15	25
Boy	4	4	12	20
Total	6	12	27	45

1. Fill in the table of two-way relative frequencies using decimals, rounded to the nearest thousandth. Then use the table to answer questions 2 and 3.

	Language			
Gender	**Chinese**	**French**	**Spanish**	**Total**
Girl				
Boy				
Total				

2. What fraction of the surveyed students are boys taking Spanish?

3. What fraction of the surveyed students are taking Chinese?

4. What fraction of girls are studying French?

5. What fraction of Spanish students are boys?

6. Can you use gender to predict a preference for taking Spanish?

7. Is there an association between gender and a preference for French?

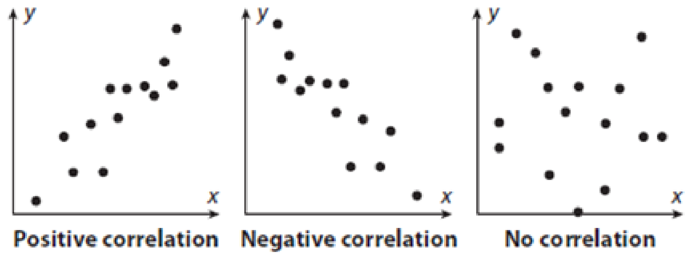
Lesson Name: A1.U6.C3.F.01.**vocab.ScatterPlotBestFit-LEQ**

Term	Definition	Notation	Diagram/Visual
Scatter plot			
Causation			
Correlation			
Correlation Coefficient			
Line of best fit			
Lurking Variables			
Residuals (error)			
Trend			

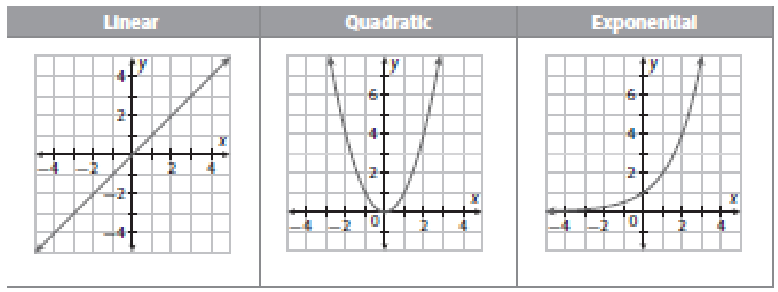
Scatterplots and Trends

Two-variable data is a collection of paired variable values, such as a series of measurements of air temperature at different times of day. One method of visualizing two-variable data is called a **scatterplot**: a graph of points with one variable plotted along each axis. A recognizable patter in the arraignment suggests a mathematical relationship between the variables.

Correlation is a measure of the strength and direction of the relationship between two variables. The correlation is positive if both values tend to increase together, negative if one decreases while the other increases, and we say there is 'no correlation' if the change in the variables appear to be unrelated.

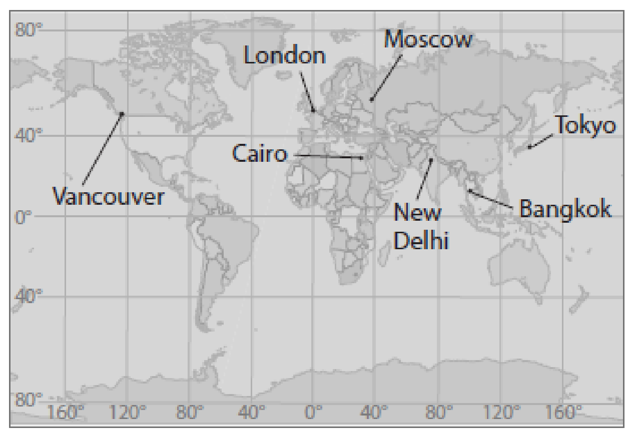


Recall what linear, quadratic, and exponential functions create on a graph. Not all correlations will be linear.



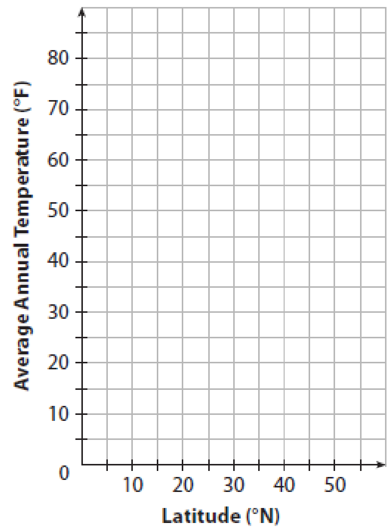
The table below presents two-variable data for seven different cities in the Northern hemisphere.

City	Latitude (°N)	Average Annual Temperature (°F)
Bangkok	13.7	82.6
Cairo	30.1	71.4
London	51.5	51.8
Moscow	55.8	39.4
New Delhi	28.6	77.0
Tokyo	35.7	58.1
Vancouver	49.2	49.6



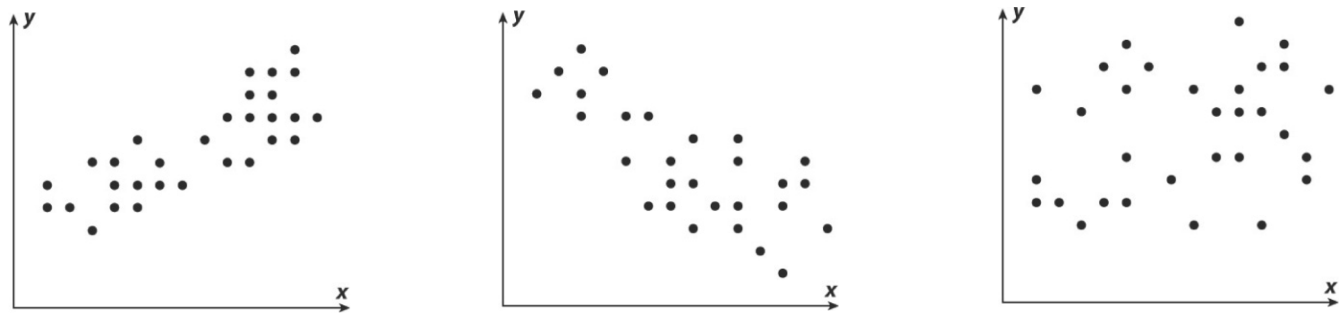
The two variables are ___________ and _______________.

Create a scatterplot from the table above.



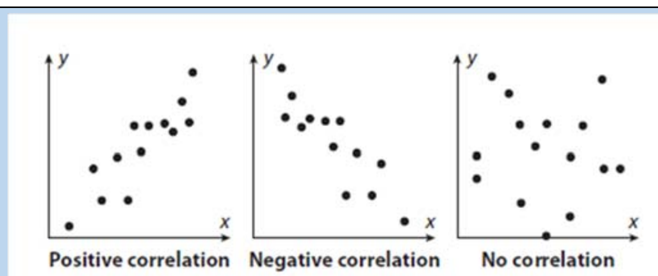
The variables are ________________ correlated.

Write, positive, negative, or none to describe the correlation in each scatterplot.



____________________________ ____________________________ ____________________________

Why are the points in this scatter plot not connected in the same way plots of linear equations are?



SELF CHECK State whether you would expect positive, negative, or no correlation between the two data sets.

1. temperature and ice cream sales: ____
2. a child's age and the time it takes him or her to run a mile:________________________________
3. The age of a car and its odometer reading:______________________
4. The amount of time spent fishing and the amount of bait in the bucket:____________________________
5. The number of passengers in a car and the number of traffic lights on the route:_____________________

GUIDED NOTES

For the linear function, $f(x)$, as x approaches positive infinity, $f(x)$ approaches positive/negative infinity, and as x approaches negative infinity, $f(x)$ approaches positive/negative infinity.

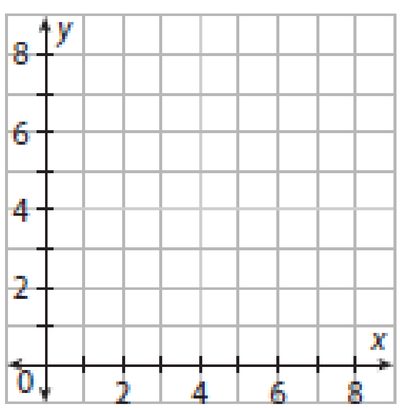
For the quadratic function, $g(x)$, as x approaches positive infinity, $g(x)$ approaches positive/negative infinity, and as x approaches negative infinity, $g(x)$ approaches positive/negative infinity.

For the exponential function, $h(x)$, as x approaches positive infinity, $h(x)$ approaches positive/negative infinity, and as x approaches negative infinity, $h(x)$ approaches positive/negative infinity.

We have been focusing on data sets that have a linear correlation, but we know that not all functions are linear. This applies to scatterplots as well.

From the tables below, plot all of the points and determine which function is the best fit.

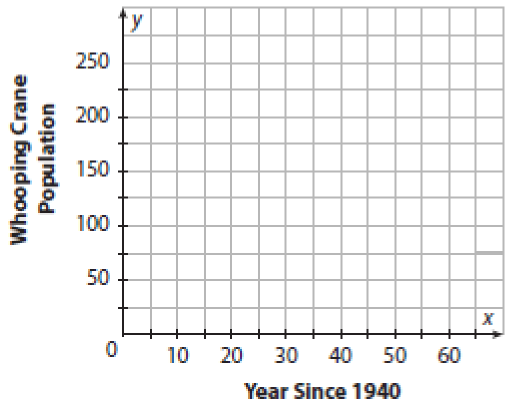
x	f(x)
0	3
1	1.5
2	1
3	1.5
4	3
5	5.5
6	9



The data set appears to best fit an linear/quadratic/exponential function.

The end behavior of this data as x approaches positive/negative infinity, $f(x)$ approaches positive/negative infinity.

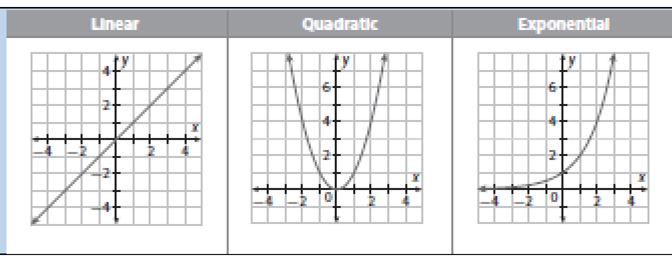
Whooping Crane	
Year	**Population**
1940	22
1950	34
1960	33
1970	56
1980	76
1990	146
2000	177
2010	281



The data set appears to best fit an linear/quadratic/exponential function.

The end behavior of this data as x approaches positive/negative infinity, $f(x)$ approaches positive/negative infinity.

Example!



SELF CHECK Examine the two models that represent aunnual tuition for two colleges.

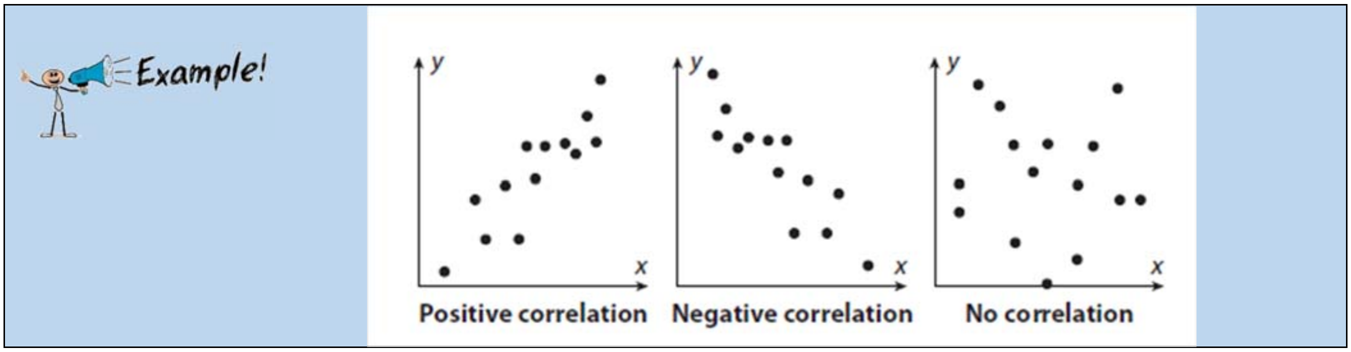
a. Describe each model as linear, quadratic, or exponential.

b. Both models have the same value for year 0. What does this mean?

c. Why do both models have the same value for year 1?

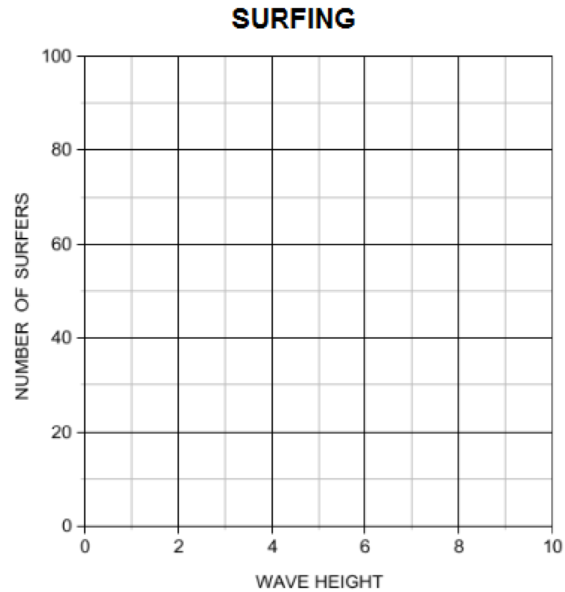
Years after 2004	Tuition at College 1 ($)	Tuition at College 2 ($)
0	2000	2000
1	2200	2200
2	2400	2420
3	2600	2662
4	2800	2928.20

Independent Practice- Scatterplots and Trends



1) The table shows the heights (in feet) of the waves at a beach and the numbers of surfers at the beach.

Wave Height	3	6	5	1	4
Number of Surfers	24	61	56	15	35

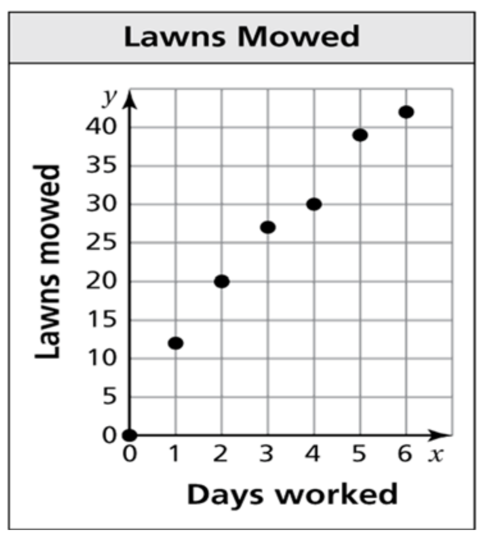


a) Plot the data from the table on the graph.

b) Describe the relationship between the two data sets.

c) How many surfers might be at the beach if the waves were 2 feet high?

2) The scatter plot shows the numbers of lawns mowed by a local lawn care business during one week.



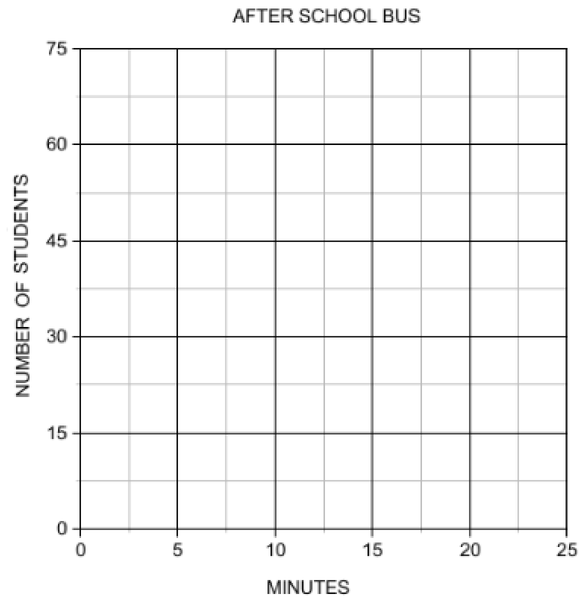
a) How many days does it take to mow 30 lawns?

b) About how many lawns can be mowed in 1 day?

c) Describe the relationship shown by the data.

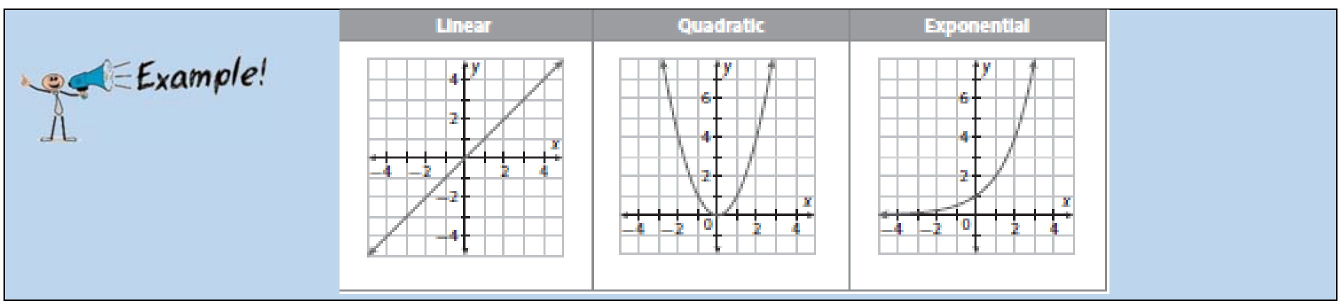
3) The table shows the numbers of students remaining on an after-school bus and the numbers of minutes since leaving the school.

Minutes	0	5	9	15	23	26	32
Number of students	56	45	39	24	17	6	0



a) Plot the data from the table on the graph.

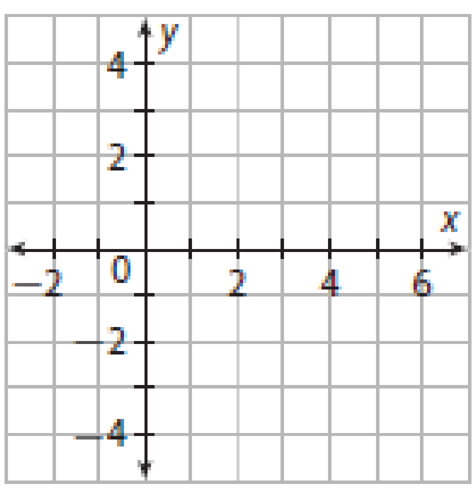
b) Describe the relationship between the two data sets.



Plot the points in the table on the graph. Determine if the correlation is linear, quadratic, or exponential.

1.

x	$f(x)$
0	–0.79
1	2.81
2	4.41
3	4.01
4	1.61
5	–2.79

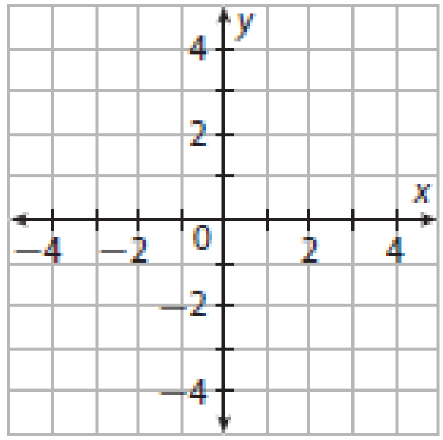


Circle the function that best fits the graph.

Linear/exponential/ quadratic

2.

x	$f(x)$
–2	–4
–1	–3.3
0	–1.2
1	2.3
2	7.2
3	13.5

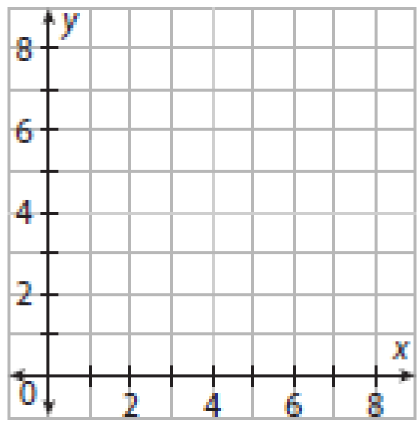


Circle the function that best fits the graph.

Linear/exponential/ quadratic

3.

x	$f(x)$
0	7.6
1	6.25
2	4.6
3	2.65
4	0.4

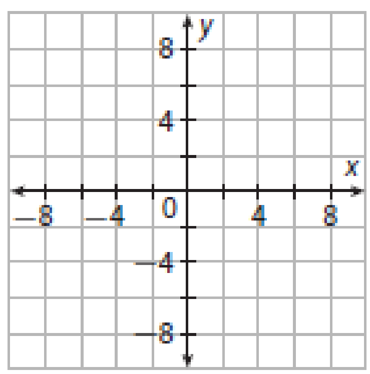


Circle the function that best fits the graph.

Linear/exponential/ quadratic

4.

x	$f(x)$
1	–7.2
2	–3.8
3	0
4	4.2
5	8.8

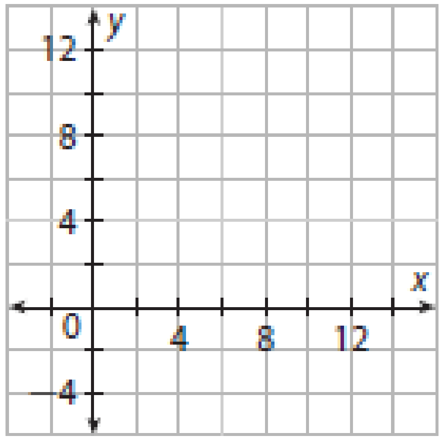


Circle the function that best fits the graph.

Linear/exponential/ quadratic

5.

x	$f(x)$
−2	−5.75
0	1
2	6.25
4	10
6	12.25
8	13
10	12.25
12	10
14	6.25



Circle the function that best fits the graph.

Linear/exponential/quadratic

Task- Scatter plots and Trends

Penny Circle Desmos

https://teacher.desmos.com/activitybuilder/custom/586ab17c2f8cd5bc3bcaf259

Penny Circle

CLASS CODE

30-45 minutes | Application

In this activity, students gather data, build a model, and then use that model to answer the question, "How many pennies fit in a large circle?"

French translation courtesy of Jocelyn Dagenais:
https://teacher.desmos.com/activitybuilder/custom/5a8eef7d68b0820a2138d7b9

Activity Checklist

- [ ] Complete the activity using student preview.
- [ ] Identify your learning targets for the activity.
- [ ] Determine the screens where you'll bring the class together using Teacher Pacing and Pause Class. What will you discuss on those screens?
- [ ] Anticipate screens where students will struggle, then plan your response.
- [ ] Plan a challenge for students who finish the activity quickly and successfully.
- [ ] Make yourself available during the activity to students for individual help and questions when appropriate.
- [ ] Write out your summary of the activity's main ideas. How will you pull student work into that summary? Which parts of the activity can you skip to ensure that summary receives sufficient time?

My Learning Targets:

___

___

___

___

Activity Screens: Teacher Pacing and Pause Class

Use this page to plan your use of Teacher Pacing and Pause Class. Teacher Pacing lets you restrict students to a single screen or a range of screens. Pause Class keeps students from interacting with whatever screens they are currently viewing. Use these two tools to create conversations in your classroom.
Consider these questions as you plan:

- Which screen(s) should everyone work on at the same time? Why?
- Which screen(s) do you want to keep students from seeing until you're ready for the class to see them together? (Perhaps because they reveal answers or require a whole class conversation for introduction.)
- Are there any points in the lesson where you will want to make sure students aren't playing with the screens while you discuss something as a class?

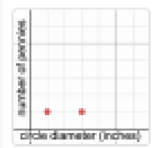
1 Make a prediction.

Watch the video, then predict:

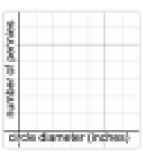
2 Strategy

The dot plot shows your class's "just right" estimates for the number

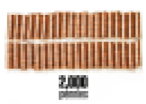
3 Collect some data.
Drag some virtual pennies into

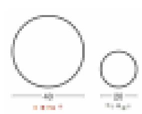
4 Build a model.


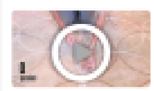
The graph here shows your data

5 Use your model.


The model you built on Screen 4 is
$f(x)$

6 Extension #1

What is the diameter of the smallest
$f(x)$

7 Extension #2

Would you rather have all the

8 Reveal

Let's compare your estimate from earlier in the activity

9 Tables, Graphs, ...
In this activity, you used multiple representations to predict the number of

1 Make a prediction.

Watch the video, then predict:

Watch the video, then predict:

How many pennies will fit in the large circle?

Enter your TOO LOW / JUST RIGHT / TOO HIGH estimates in the table and press "Submit."

Teacher Moves

Emphasize the range of student responses on this screen. It's okay—even desirable—to lack consensus at this stage. The activity will build toward consensus later on.

My Notes:

---

2 Strategy

The dot plot shows your class's "just right" estimates for the number of pennies in the large circle

The dot plot shows your class's "just right" estimates for the number of pennies in the large circle.

On the next screen, we'll look at SMALLER circles to help us make a prediction about the LARGER circle.

My Notes:

3 Collect some data.

Drag some virtual pennies into the virtual

Drag some virtual pennies into the virtual circle. Pack in as many as you can, then record your data in the table.

Do this for at least three different circles.

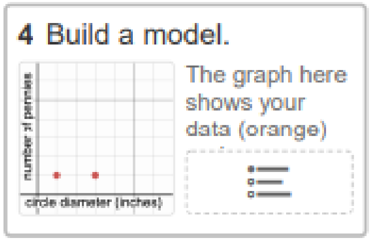
Each time you fill the circle, enter your data in the table.

Teacher Moves

Students should enter at least three ordered pairs, though they can contribute up to nine. (Anything beyond that will be ignored on future screens.)

Be on the lookout for students who reverse the numbers in the table (entering "pennies" under "diameter," and vice-versa). When that occurs, rather than point out the error directly, casually ask students to explain the meaning of the points in the table. If they don't see their mistake, ask what the headers in the table indicate.

My Notes:



The graph here shows your data (orange) and your classmates' data (blue) from the previous screen.

What kind of function will best fit the data?

After you make your selection, drag the red points to fit the curve to the data.

Teacher Moves

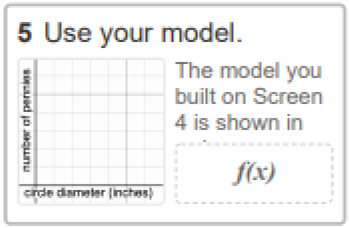
We recommend using teacher pacing on Screens 4-5 so students don't see the answer on Screen 6. Use Screen 5 for a summary conversation where you highlight and connect the different reasons students have given for their model selection.

We anticipate that some of your students will judge the quality of the model based on how well it fits the data. That will likely rule out the linear model but it doesn't offer a great deal of help with the quadratic and exponential models. For a small set of data, both models look reasonable.

So another way to decide on the model is not just to look at the data but to consider where the data CAME from and why the constraints around the data would recommend one model over another. Here we have a DISTANCE measurement on the x-axis and an AREA measurement on the y-axis. We often SQUARE distance to get area, which would recommend a QUADRATIC model.

We encourage you to highlight for your students still another way to select a model which is to see if it makes sense for a wide range of values. In this case, evaluating the quadratic and exponential models for a 0-inch circle gives 0 pennies in the quadratic case and 1 penny in the exponential case, recommending the quadratic model. Both models also give very different answers for the 22-inch circle. Hundreds in the quadratic model. Hundreds of thousands in the exponential model, which will likely be very different from the student's initial predictions.

My Notes:



The model you built on Screen 4 is shown in red.

The equation of your model is EQUATION.

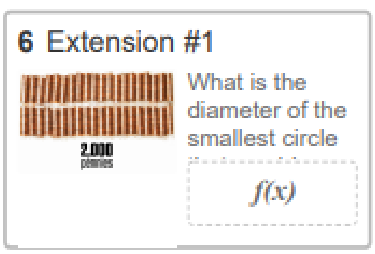
Based on everything you know now, make a final calculation: How many pennies will fit in the 22-inch circle?

Teacher Moves

Invite students to notice that the range of calculations is narrower than the range of estimates from earlier in the activity, if that's true. Math is power, not punishment!

Dashboard Note: ☑ indicates that the student's calculation is within 5% of what their model suggests.

My Notes:



What is the diameter of the smallest circle that would enclose 2,000 pennies? How do you know?

Teacher Moves

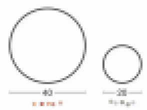
Dashboard Note: ☑ indicates that the student's calculation is within 5% of what their model suggests.

Sample Responses

About 38.2 inches. I plotted the data from earlier in this activity, built a quadratic model, graphed the line $y = 2000$, and found the intersection. https://www.desmos.com/calculator/nihn64i09z

My Notes:

7 Extension #2



Would you rather have all the PENNIES

Would you rather have all the PENNIES that fill a 40-inch diameter circle, or all the NICKELS that fill a 20-inch diameter circle?

Teacher Moves

Encourage students to support their choice with mathematical reasoning (ideally, more sophisticated than "nickels are worth more, so I'll choose them!").

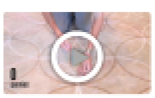
Here are two potential approaches:

1. BUILD A MODEL FOR NICKELS
Students use their existing penny model to find the value of the coins in the 40-inch circle. Then they use real nickels, paper with circles of various sizes to gather data, desmos.com/calculator to build and use a model. (For building the model, they might use regression.)

2. REASON ABOUT RATIOS
Students use a search engine (or Wikipedia) to find the diameters of each coin type. Then they consider the ratio of coin size (i.e., area), coin value, and circle size (again, area). Here's what this second approach might look like:

"A nickel takes up slightly more space (about 1.24 times), but is worth considerably more (5 times). So filling a same-size circle with nickels would be worth $\frac{5}{1.24} = 4.032$ as much. But these circles are not the same size. The half-size radius means the area will be one-quarter the size, meaning that the values will be very, very close (with a slightly higher value for the nickels)."

8 Reveal



Let's compare your estimate from earlier in the activity and your final calculation.

Let's compare your estimate from earlier in the activity and your final calculation.

Teacher Moves

Consider asking one or more follow up questions:

(1) How did your predictions compare to the actual answer?
(2) Which model proved most accurate? Why do you think that's the case?

My Notes:

Real Mathematics: Application

9 Tables, Graphs, and …
In this activity, you used multiple representations to predict the number of pennies

In this activity, you used multiple representations to predict the number of pennies in the large circle.

What are the advantages of each representation?

Teacher Moves

Highlight several student responses for the class. Start with informal math language and reasoning, then move to more formal responses.

Sample Responses

- With **tables**, it's easier to see several specific values at once.
- With **graphs**, it's easier to see the overall shape of the data, and whether a particular curve fits a given data set.
- With **graphs**, it's easier to see whether an original estimate is close (or not close) to the rest of the data and / or the actual answer.
- With **equations**, it's easier to calculate a relatively accurate final prediction.

My Notes:

Summary Notes:

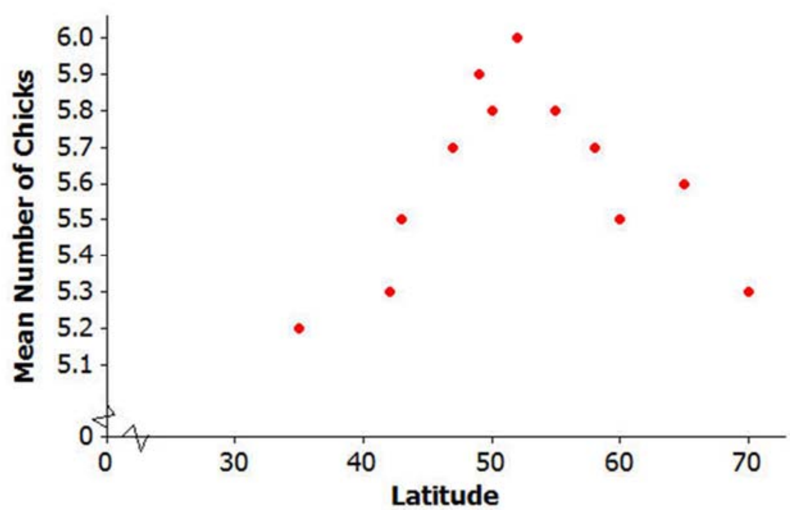
Homework- Scatter Plots and Trends

1. Sketch each graph of a typical linear, exponential, and quadratic function looks like. Describe the functions.

2. How are quadratic and exponential functions different from linear functions?

3. How quadratic functions are different form exponential functions?

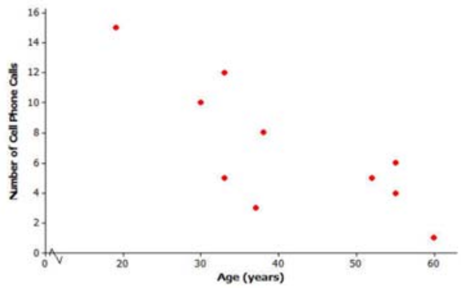
Biologists conducted a study of the nesting behavior of a type of bird called a flycatcher. They examined a large number of nests and recorded the latitude for the location of the nest and the number of chicks in the nest.



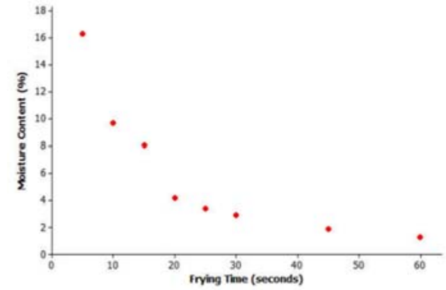
4. What type of model (linear, quadratic, or exponential) would best describe the relationship between latitude and mean number of chicks?

5. Use the following scatter plots to answer the questions a and b.

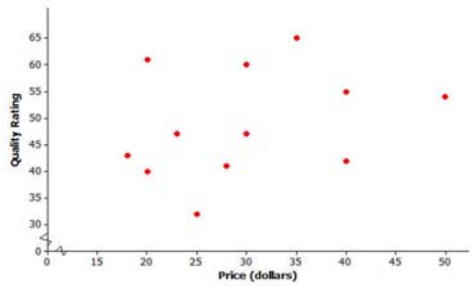
Scatter Plot 1



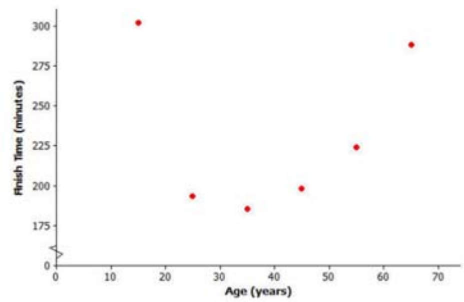
Scatter Plot 2



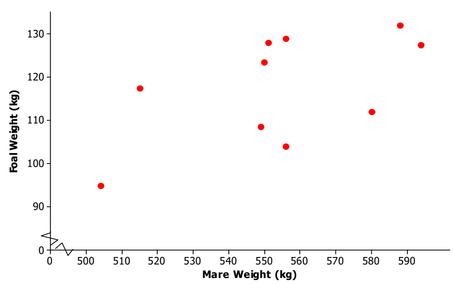
Scatter Plot 3



Scatter Plot 4



Scatter Plot 5



a. Which of the five scatter plots show a better that could be reasonably described by a quadratic curve?

b. Which of the five scatter plots shows a pattern that could reasonably described by an exponential curve?

6. For each of the scatter plots above in question 5, draw the line that best fits that function.

Digging Deeper into Linear Correlation

There are many ways to come up with a line of fit. This lesson addresses a visual method: Using a straight edge, draw the line that the data points appear to be clustered around. It is not important that any of the data points actually touch the line; instead the line should be drawn as straight as possible and should go through the middle of the scattered points. Once a line of fit has been drawn onto the scatter plot, you can choose two points on the line to write an equation for the line. The slope, m, is calculated by finding the change in y over the change in x. Remember slope as "rise over run."

Understand the purpose of each step when using a calculator to find a line of best fit. First input the data, then create a scatter plot, then use the linear regression feature to find the equation for the line of best fit. Next input the equation and graph it.

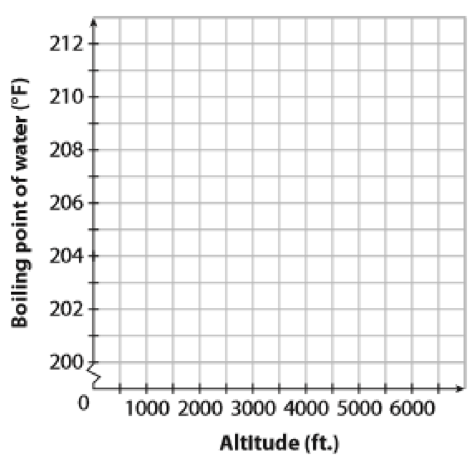
A common error when interpreting paired data is to observe a correlation and conclude that causation has been demonstrated. Causation means that a change in the one variable results directly from changing the other variable. In that case, it is reasonable to expect the data to show correlation. However, the reverse is not true: observing a correlation between variables does not necessarily mean that the change to one variable caused the change in the other. They may both have a common cause related to a variable not included in the data set or even observed (sometimes called lurking variables), or the causation may be the reverse of the conclusion.

GUIDED NOTES

a. Determine a line of fit for the data, and write the equation of the line.

The boiling point of water is lower at higher elevations because of the lower atmospheric pressure. The boiling point of water in some different cities is given in the table.

City	Altitude (feet)	Boiling Point (°F)
Chicago	597	210
Denver	5300	201
Kathmandu	4600	205
Madrid	2188	207
Miami	6	210



A line of fit may go through points (___ , ___) and (___ , ___).

$m = \frac{\square}{\square}$ $b = \square$

The equation is of this line of fit is $y =$ ___ $x +$ ___.

b. What do the slope and y-intercept of the model above represent?

c. Given latitudes and average temperatures in degrees Celsius for several cities, use your calculator to find an equation for the line of best fit. Then interpret the correlation coefficient and use the line of best fit to estimate the average temperature of another city using the given latitude.

City	Latitude	Average Temperature (°F)
Fairbanks, Alaska	64.5°N	30
Moscow, Russia	55.5°N	39
Ghent, Belgium	51.0°N	46
Kiev, Ukraine	50.3°N	49
Prague, Czech Republic	50.0°N	50
Winnipeg, Manitoba	49.5°N	52
Luxembourg	49.4°N	53
Vienna, Austria	48.1°N	56
Bern, Switzerland	46.6°N	59

Estimate the average temperature in degrees Fahrenheit in Bath, England, at 51.4°N.

Enter the data into data lists on your calculator.

Use the Linear Regression feature to find the equation for the line of best fit using the lists of data you entered. Be sure to have the calculator also display values for the correlation coefficient r and r^2.

The correlation coefficient is about ________, which indicates a ____________ correlation. The correlation coefficient indicates that the line of best fit [is/ is not] reliable for estimating temperatures of other locations within the same range of latitudes.

The equation for the line of best fit is $y \approx -$ $\boxed{}$ $x +$ $\boxed{}$.

Use the equation to estimate the average temperature in Bath, England at 51.4°N.

$y \approx -$ $\boxed{}$ $x +$ $\boxed{}$

The average temperature in degrees Fahrenheit in Bath, England, should be around $\boxed{}$ °F.

Graph the line of best fit with the data points in the scatter plot. Then use the TRACE function to find the approximate average temperature in degrees Fahrenheit for a latitude of 51.4°N.

d. Read the description of the experiment, identify the two variables and describe whether changing either variable is likely, doubtful, or unclear to cause a change in the other variable.

A traffic official in a major metropolitan area notices that the more profitable toll bridges into the city are those with the slowest average crossing speeds.

The variables are ________ and ______________.

It is [likely | doubtful | unclear] that increased profit causes slower crossing speed.

It is [likely | doubtful | unclear] that slower crossing speeds cause an increase in profits.

Questions To Ponder

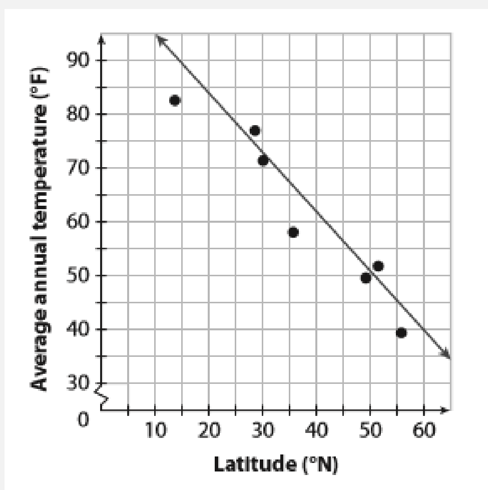
1. The calculator's linear regression feature gives you an equation in the form y = ax + b. What form of linear equation is that? What do a and b represent?

2. How does the line of fit displayed on the calculator differ from lines of fit found in previous exercises? Explain.

3. Can a data set be likely to show causation without showing a strong correlation? Explain.

Example!

EX 1. Determine a line of fit for the data, and write the equation of the line.



A line of fit has been added to the graph. The points (10, 95) and (60, 40) appear to be on the line.

$$m = \frac{40 - 95}{60 - 10} = -1.1$$

$$y = mx + b$$

$$95 = -1.1(10) + b$$

$$106 = b$$

The model is given by the equation

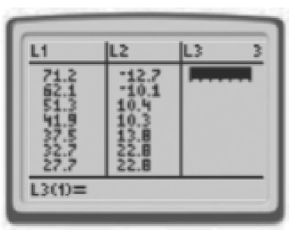
$$y = -1.1x + 106$$

EX 2. Given latitudes and average temperatures in degrees Celsius for several cities, use your calculator to find an equation for the line of best fit. Then interpret the correlation coefficient and use the line of best fit to estimate the average temperature of another city using the given latitude.

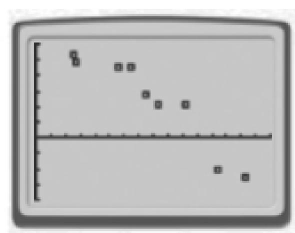
City	Latitude	Average Temperature (°C)
Barrow, Alaska	71.2°N	−12.7
Yakutsk, Russia	62.1°N	−10.1
London, England	51.3°N	10.4
Chicago, Illinois	41.9°N	10.3
San Francisco, California	37.5°N	13.8
Yuma, Arizona	32.7°N	22.8
Tindouf, Algeria	27.7°N	22.8
Dakar, Senegal	14.0°N	24.5
Mangalore, India	12.5°N	27.1

Estimate the average temperature in Vancouver, Canada at 49.1°N.

Enter the data into data lists on your calculator. Enter the latitudes in column **L1** and the average temperatures in column **L2**.

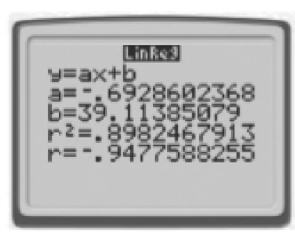


Create a scatter plot of the data.



Use the Linear Regression feature to find the equation for the line of best fit using the lists of data you entered. Be sure to have the calculator also display values for the correlation coefficient r and r^2.

The correlation coefficient is about −0.95, which is very strong. This indicates a strong correlation, so we can rely on the line of fit for estimating average temperatures for other locations within the same range of latitudes.

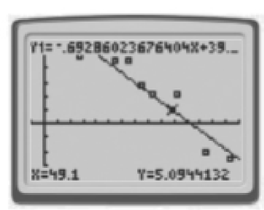


The equation for the line of best fit is $y \approx -0.693x + 39.11$.

Graph the line of best fit with the data points in the scatter plot.

Use the TRACE function to find the approximate average temperature in degrees Celsius for a latitude of 49.1°N.

The average temperature in Vancouver should be around 5°C.



EX 3. Read the description of the experiment, identify the two variables and describe whether changing either variable is likely, doubtful, or unclear to cause a change in the other variable.

The manager of an ice cream shop studies its monthly sales figures and notices a positive correlation between the average air temperature and how much ice cream they sell on any given day.

The two variables are ice cream sales and average air temperatures.

It is likely that warmer air temperatures cause an increase in ice cream sales.

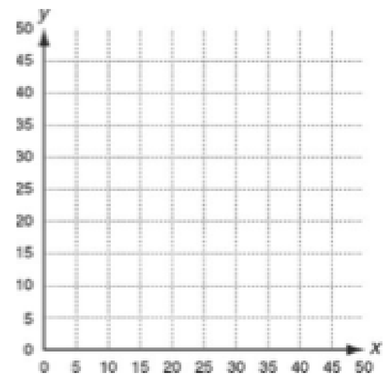
It is doubtful that increased ice cream sales cause an increase in air temperatures.

SELF CHECK

Graph a scatter plot and find the correlation.

1. A biologist in a laboratory comes up with the following data points. Make a scatter plot using the data in the table.

x	2	6	9	14	16	21	25	28
y	3	7	15	33	38	35	40	41



2. Draw a line of fit on the graph and find the equation for the liner model. Estimate the correlation coefficient, *r* (choose 1, 0.5, 0, –0.5, or –1).

3. Use a graphing calculator to find the equation for the line of best fit for the data presented in the table above. Use a graphing calculator to find the correlation coefficient, *r*.

4. Compare the results you found in step 3, using a graphing calculator, to those you found in step 2, estimating. The calculator provides a line of BEST fit, while the line you drew by hand is called a line of fit. Explain the difference.

5. Read each description. Identify the variables in each situation and determine whether it describes a positive or negative correlation. Explain whether the correlation is a result of causation.
   a. A group of biologists is studying the population of wolves and the population of deer in a particular region. The biologists compared the populations each month for 2 years. After analyzing the data, the biologists found that as the population of wolves increases, the population of deer decreases.

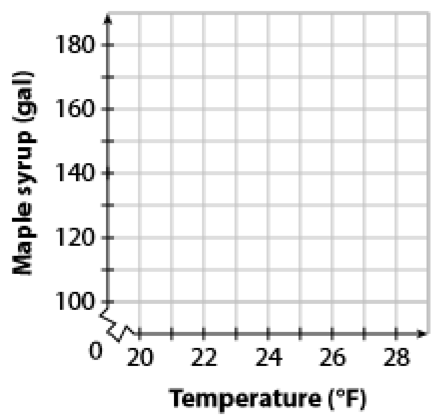
   b. Researchers at an auto insurance company are studying the ages of its policyholders and the number of accidents per 100 policyholders. The researchers compared each year of age from 16 to 65. After analyzing the data, the researchers found that as age increases, the number of accidents per 100 policyholders decreases.

Digging Deeper into Linear Correlation Independent Practice

The owner of a maple syrup farm is studying the average winter temperature in Fahrenheit and the number of gallons of maple syrup produced. The relationship between the temperature and the number of gallons of maple syrup produced for the past 8 years is shown in the table. Use this table to respond to questions 1 – 4.

Temperature (°F)	Number of gallons of maple syrup
24	154
26	128
25	141
22	168
28	104
21	170
24	144
22	160

1. Make a scatter plot of the data and draw a line of fit that passes as close as possible to the plotted points.



2. Find the equation of this line of fit without using a calculator.

3. Use a graphing calculator to find a line of best fit for this data set. What is the equation for the best-fit line?

4. Identify the slope and y-intercept for the line of best fit generated by the graphing calculator and interpret it in the context of the problem.

5. Use the given data and your calculator to find an equation for the line of best fit. Then interpret the correlation coefficient and use the line of best fit to estimate the average temperature of another city using the given latitude.

City	Latitude	Average Temperature (°F)
Anchorage, United States	61.1°N	18
Dublin, Ireland	53.2°N	29
Zurich, Switzerland	47.2°N	34
Florence, Italy	43.5°N	37
Trenton, New Jersey	40.1°N	
Algiers, Algeria	36.5°N	46
El Paso, Texas	31.5°N	49
Dubai, UAE	25.2°N	56
Manila, Philippines	14.4°N	61

For questions 6 & 7: Read each description. Identify the variables in each situation and determine whether it describes a positive or negative correlation. Explain whether the correlation is a result of causation.

6. Educational researchers are investigating the relationship between the number of musical instruments a student plays and a student's grade in math. The researchers conducted a survey asking 110 students the number of musical instruments they play and went to the registrar's office to find the same 110 students' grades in math. The researchers found that students who play a greater number of musical instruments tend to have a greater average grade in math.

7. Researchers are studying the relationship between the median salary of a police officer in a city and the number of violent crimes per 1000 people. The researchers collected the police officers' median salary and the number of violent crimes per 1000 people in 84 cities. After analyzing the data, researchers found that a city with a greater police officers' median salary tends to have a greater number of violent crimes per 1000 people.

Digging Deeper into Linear Correlation Performance Task

<u>Guided Learning Task: TV/Test Grades</u>

1. Students in Ms. Garth's Algebra II class wanted to see if there are correlations between test scores and height and between test scores and time spent watching television. Before the students began collecting data, Ms. Garth asked them to predict what the data would reveal. Answer the following questions that Ms. Garth asked her class.

a. Do you think students' heights will be correlated to their test grades? If you think a correlation will be found, will it be a positive or negative correlation? Will it be a strong or weak correlation?

b. Do you think the average number of hours students watch television per week will be correlated to their test grades? If you think a correlation will be found, will it be a positive or negative correlation? Will it be a strong or weak correlation? Do watching TV and low test grades have a cause and effect relationship?

2. The students then created a table in which they recorded each student's height, average number of hours per week spent watching television (measured over a four-week period), and scores on two tests. Use the actual data collected by the students in Ms. Garth's class, as shown in the table below, to answer the following questions.

Student	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Height (in inches)	60	65	51	76	66	72	59	58	70	67	65	71	58
TV hrs/week (average)	30	12	30	20	10	20	15	12	15	11	16	20	19
Test 1	60	80	65	85	100	78	75	95	75	90	90	80	75
Test 2	70	85	75	85	100	88	85	90	90	90	95	85	85

a. Which pairs of variables seem to have a positive correlation? Explain.

b. Which pairs of variables seem to have a negative correlation? Explain.

c. Which pairs of variables seem to have no correlation? Explain.

3. For each pair of variables listed below, create a scatter plot with the first variable shown on the y-axis and the second variable on the x-axis. Are the two variables correlated positively, correlated negatively, or not correlated? Determine whether each scatter plot suggests a linear trend.

a. Score on test 1 versus hours watching television

b. Height versus hours watching television

c. Score on test 1 versus score on test 2

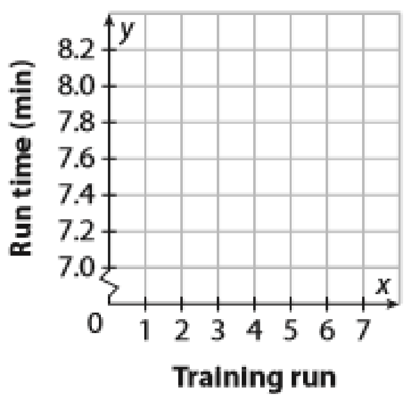
d. Score on test 2 vs. hours watching television

4. Using the statistical functions of your graphing calculator, determine a line of good fit for each scatter plot that suggests a linear trend.

Digging Deeper into Linear Correlation Homework

Training Run	Run Time (min)
1	8.2
2	8.1
3	7.5
4	7.8
5	7.4
6	7.5
7	7.1
8	7.1

1. Make a scatter plot of the data provided in the table above and draw a line of fit that passes as close as possible to the plotted points.



2. Determine a line of fit for the data, and write the equation of your line without using a calculator.

3. Use a graphing calculator to find a line of best fit for this data set. What is the equation for the best-fit line?

4. Identify the slope and y-intercept for the line of best fit generated by the graphing calculator and interpret it in the context of the problem.

5. Use the given data and your calculator to find an equation for the line of best fit. Then interpret the correlation coefficient and use the line of best fit to estimate the average temperature of another city using the given latitude.

City	Latitude	Average Temperature (°F)
Tornio, Finland	65.5°N	28
Riga, Latvia	56.6°N	36
Minsk, Belarus	53.5°N	39
Quebec City, Quebec	46.5°N	45
Turin, Italy	45.0°N	47
Pittsburgh, Pennsylvania	40.3°N	49
Lisbon, Portugal	38.4°N	52
Jerusalem, Israel	31.5°N	
New Orleans, Louisiana	29.6°N	60
Port-au-Prince, Haiti	18.3°N	69

For questions 6 & 7: Read each description. Identify the variables in each situation and determine whether it describes a positive or negative correlation. Explain whether the correlation is a result of causation.

6. The owner of a ski resort is studying the relationship between the amount of snowfall in centimeters during the season and the number of visitors per season. The owner collected information about the amount of snowfall and the number of visitors for the past 30 seasons. After analyzing the data, the owner determined that seasons that have more snowfall tend to have more visitors.

7. Government researchers are studying the relationship between the price of gasoline and the number of miles driven in a month. The researchers documented the monthly average price of gasoline and the number of miles driven for the last 36 months. The researchers found that the months with a higher average price of gasoline tend to have more miles driven.