NC Math III Standards

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). These have been indicated throughout the document.

The plus (+) standards are standards that are appropriate for ALL students in this course and not exclusive to Honors level courses. They are foundational standards to prepare students for success in 4th level mathematics courses.

NC Math III Standards

Number	Algebra	Function	Geometry	Statistics & Probability
N.RN.3	A.SSE.1a★	F.IF.2	G.CO.1	S.ID.4★
	A.SSE.1b★	F.IF.4★	G.CO.9	
N.Q.1★	A.SSE.2	F.IF.5★	G.CO.10	S.IC.1★
N.Q.2★	A.SSE.3b★	F.IF.7c★	G.CO.11	S.IC.3★
N.Q.3★	A.SSE.4	F.IF.7e★	G.CO.12	S.IC.4★
		F.IF.8a		S.IC.5★
N.CN.1	A.APR.1	F.IF.9	G.SRT.2	S.IC.6★
N.CN.2	A.APR.2		G.SRT.3	
N.CN.7	A.APR.3	F.BF.1a★	G.SRT.4	S.MD.6(+)★
N.CN.9(+)	A.APR.4	F.BF.1b★	G.SRT.5	S.MD.7(+)★
	A.APR.6	F.BF.2★		
	A.APR.7(+)	F.BF.3	G.C.1	
		F.BF.4a	G.C.2	
	A.CED.1★		G.C.3	
	A.CED.2★	F.LE.3★	G.C.5	
	A.CED.3★	F.LE.4★		
	A.CED.4★		G.GPE.1	
		F.TF.1	G.GPE.2	
	A.REI.1	F.TF.2		
	A.REI.2a	F.TF.5★	G.MG.3★	
	A.REI.2b	F.TF.8		
	A.REI.4			
	A.REI.10			
	A.REI.11★			

Standard	Cluster: Use properties of rational and irrational numbers
N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	**Students know and justify that when** • adding or multiplying two rational numbers the result is a rational number. • adding a rational number and an irrational number the result is irrational. • multiplying of a nonzero rational number and an irrational number the result is irrational. ***Example:*** Explain why the number 2π must be irrational, given that π is irrational. ***Sample Response:*** *If 2π were rational, then half of 2π would also be rational, so π would have to be rational as well.* **Note:** Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction and the relationship between multiplication and addition.

Standard	Cluster: Reason quantitatively and use units to solve problems
N.Q.1★ Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included throughout Math I, II and III. Units are a way for students to understand and make sense of problems. **Use units as a way to understand problems and to guide the solution of multi-step problems** • Students use the units of a problem to identify what the problem is asking. They recognize the information units provide about the quantities in context and use units as a tool to help solve multi-step problems. Students analyze units to determine which operations to use when solving a problem. ***For example***, given the speed in *mph* and time traveled in *hours*, what is the distance traveled? From looking at the units, we can determine that we must multiply *mph* times *hours* to get an answer expressed in miles: $\left(\frac{mi}{hr}\right)(hr) = mi$ ***(Note that knowledge of the distance formula is not required to determine the need to multiply in this case.)*** ***Another example***, the length of a spring increases 2 cm for every 4 oz. of weight attached. Determine how much the spring will increase if 10 *oz.* are attached: $\left(\frac{2cm}{4oz}\right)(10oz) = 5cm$. This can be extended into a multi-step problem when asked for the length of a 6 cm spring after 10 *oz.* are attached: $\left(\frac{2cm}{4oz}\right)(10oz) + 6cm = 11cm$. **Choose and interpret units consistently in formulas** • Students choose the units that accurately describe what is being measured. Students understand the familiar measurements such as length (unit), area (unit squares) and volume (unit cubes). They use the structure of formulas and the context to interpret units less familiar. ***For example***, if $density = \frac{mass\ in\ grams}{volume\ in\ mL}$ then the unit for density is $\frac{grams}{mL}$. **Choose and interpret the scale and the origin in graphs and data displays** • When given a graph or data display, students read and interpret the scale and origin. When creating a graph or data display, students choose a scale that is appropriate for viewing the features of a graph or data display. Students understand that using larger values for the tick marks on the scale effectively "zooms out" from the graph and choosing smaller values "zooms in." Students also understand that the viewing window does not necessarily show the *x*- or *y*-axis, but the apparent axes are parallel to the *x*- and *y*-axes. Hence, the intersection of the apparent axes in the viewing window may not be the origin. They are also aware that apparent intercepts may not correspond to the actual *x*- or *y*-intercepts of the graph of a function.

N.Q.2 ★ Define appropriate quantities for the purpose of descriptive modeling.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students define appropriate quantities for the purpose of describing a mathematical model in context. Example: Explain how the unit cm, cm^2, and cm^3 are related. Describe situations where each would be an appropriate unit of measure.
N.Q.3 ★ Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III throughout all three courses. **Students understand the tool used determines the level of accuracy that can be reported for a measurement.** ***Example(s):*** • When using a ruler, one can legitimately report accuracy to the nearest division. If a ruler has centimeter divisions, then when measuring the length of a pencil the reported length must be to the nearest centimeter, or • In situations where units constant a whole value, as the case with people. An answer of 1.5 people would reflect a level of accuracy to the nearest whole based on the fact that the limitation is based on the context. **Students use the measurements provided within a problem to determine the level of accuracy.** ***Example:*** If lengths of a rectangle are given to the nearest tenth of a centimeter then calculated measurements should be reported to no more than the nearest tenth. **Students recognize the effect of rounding calculations throughout the process of solving problems and complete calculations with the highest degree of accuracy possible, reserving rounding until reporting the final quantity.**

Standard	Cluster: Perform arithmetic operations with complex numbers

N.CN.1

Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

Students will review the structure of the complex number system realizing that every number is a complex number that can be written in the form $a + bi$ where a and b are real numbers. If $a = 0$, then the number is a pure imaginary number however when $b = 0$ the number is a real number.

The square root of a negative number is a complex number.

Example:

	Problem	Solution	***bi*** Form
1.	$\sqrt{-36}$	$\sqrt{-36} = \sqrt{-1} \cdot \sqrt{36} = 6i$	$6i$
2.	$2\sqrt{-49}$	$2\sqrt{-49} = 2\sqrt{-1} \cdot \sqrt{49} = 2 \cdot 7i = 14i$	$14i$
3.	$-3\sqrt{-10}$	$-3\sqrt{-10} = -3\sqrt{-1} \cdot \sqrt{10} = -3 \cdot i \cdot \sqrt{10} = -3i\sqrt{10}$	$-3i\sqrt{10}$
4.	$5\sqrt{-8}$	$5\sqrt{-8} = 5\sqrt{-1} \cdot \sqrt{8} = 5 \cdot i \cdot 2\sqrt{2} = 10i\sqrt{2}$	$10i\sqrt{2}$

Example: Explore the powers of i and apply a pattern to simplfy i^{126}.

Connect to N.CN.7 when solving quadratic equations with real and complex solutions.

N.CN.2

Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Students recognize the relationships between different number sets and their properties.

The complex number system possesses the same basic properties as the real number system: that addition and multiplication are commutative and associative; the existence of additive identity and multiplicative identity; the existence of an additive inverse for every complex number and the existence of multiplicative inverse or reciprocal for every non- zero complex number; and the distributive property of multiplication over the addition.

An awareness of the properties minimizes students' rote memorization and links the rules for manipulations with the complex number system to the rules for manipulations with binomials with real coefficients of the form $a + bx$.

The commutative, associative, and distributive properties hold true when adding, subtracting, and multiplying complex numbers.

Example: Simplify the following expression $(3 - 2i)(-7 + 4i)$. Justify each step using the properties to support your argument.

Example: Ohms' Law related the voltage E, current I, and resistence R, in an electrical circuit: $E = IR$. Respectively, these quantities are measured in volts, amperes, and ohms.

a. Find the voltage in an electrical circuit with current $(2 + 4i)$ amperes and resistance $(5 - 4i)$ ohms.
b. Find the necessary resistance value to produce a voltage that is not complex.

Standard	Cluster: Use complex numbers in polynomial identities and equations.
N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.	**Students solve quadratic equations with real coefficients that have solutions of the form $a + bi$ and $a - bi$.** They determine when a quadratic equation in standard form, $ax^2 + bx + c = 0$, has complex roots by looking at a graph of $f(x) = ax^2 + bx + c$ or by calculating the discriminant. ***Example:*** Use the quadratic formula to write quadratic equations with the following solutions. a. One real number solution b. Soultions that are complex numbers in the form $a + bi$, $a \neq 0$ and $b \neq 0$ c. Solutions that are imaginary numbers bi ***Example:*** Given the quadratic equation $ax^2 + bx + c = 0$ that has a solution of $2 + 3i$, determine possible values for a, b,and c. Are there other combinations possible? Explain. Connect to A.REI.4b – recognizing that a negative discrimininant indicates a complex solution. Make the connection to previous work on identifying the number and types of solutions possible in a quadratic equation.
N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	**Students know The Fundamental Theorem of Algebra which states that *every polynomial function of positive degree n has exactly n complex zeros (counting multiplicities).*** Thus a linear equation has 1 complex solution, a quadratic has two complex solutions, a cubic has three complex solutions, and so on. The zeroes do not have to be unique. For instance $(x-3)^2 = 0$ has zeroes at $x = 3$ and $x = 3$. This is considered to have a double root or a multiplicity of two. **Example:** Solve $4x^2 - 12x + 13 = 0$ and discuss how the Fundamental Theorem of Algebra applies in this situation.

Standard	Cluster: Interpret the structure of expressions
A.SSE.1 ★ Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. In previous courses, students studied linear, exponential, quadratic and other polynomial expressions. In Math III, focus on writing the factor in an exponential expression in different ways. ***Example:*** If 500 mg of medicine enters a hospital's patient's bloodstream at noon and decays exponentially the amount remaining active in the patient's blood t hours later can be expressed by $500(e^{ln.85})^t$. Determine the decay rate and explain what it means about the remaining medication in the bloodstream. ***Example:*** What information related to symmetry is revealed by writing the quadratic formula as $x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$?
A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	This standard is included in Math I, II and III. In Math I and II, students factored quadratics. In Math III, extend factoring to include strategies for rewriting more complicated expressions. Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms. Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely. ***Example:*** Factor $x^3 - 2x^2 - 35x$ ***Example:*** Rewrite $m^{2x} + m^x - 6$ into an equivalent form. ***Example:*** Factor $x^3 - 8$

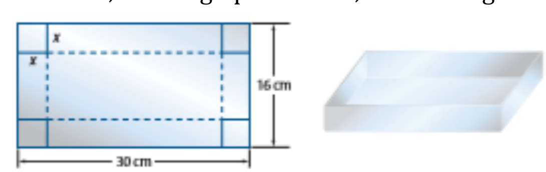
Standard	Cluster: Write expressions in equivalent forms to solve problems.
A.SSE.3 ★ Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students use completing the square to rewrite a quadratic expression in the form $y = a(x - h)^2 + k$ to identify the vertex of the parabola (h, k) and explain its meaning in context.** ***Example:*** The quadratic expression $-x^2 - 24x + 55$ models the height of a ball thrown vertically, Identify the vertex-form of the expression, determine the vertex from the rewritten form, and interpret its meaning in this context. *Note: This implies a thorough understanding of when it is appropriate to use vertex form; thus students should use this strategy when looking for the maximum or minimum value.*
A.SSE.4 ★ Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students understand that a geometric series is the sum of terms in a geometric sequence and can be used to solve real-world problems.** The sum of a finite geometric series with common ratio not equal to 1 can be written as the simple formula $S_n = \frac{a(1-r^n)}{1-r}$ where r is the common ratio, a is the initial value, and n is the number of terms in the series. **Develop the formula for the sum of a finite geometric series when the ratio is not 1.** $S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{?}$ $\underline{-rS_n = ar + ar^2 + ar^3 + \cdots + ar^{n-1} + ar^n}$ $S_n - rS_n = a - ar^n$ $S_n(1 - r) = a(1 - r^n)$ $S_n = \frac{a(1 - r^n)}{1 - r}$ **Use the formula to solve real world problems.** ***Example:*** An amount of $100 was deposited in a savings account on January 1st each of the years 2010, 2011, 2012, and so on to 2019, with annual yield of 7%. What will be the balance in the savings account on January 1, 2020?

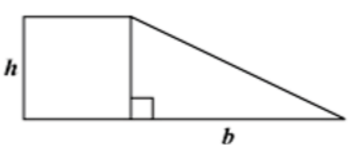
Standard	Cluster: Perform arithmetic operations on polynomials
A.APR.1 Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	This standard is included in Math I, II and III. Throughout all three courses, students operate with polynomials. In Math III, extend student work with adding, subtracting and multiplying polynomials. Focus on polynomials with degree ≥3. **The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials.** In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, polynomial, factor, and term. ***Example:*** Simplify *a.* $(6x^4 + 5x^3 - 7x + 5) - (6x^4 + 5x)$ *b.* $(x + 5)^4$ *c.* $(x^5 + 7)(-2x^2 + 6x - 1)$ A set of numbers is closed under an operation if the result obtained when an operation is performed on any two numbers in the set is also a member of the set. In order to understand that polynomials are closed under addition, subtraction and multiplication, students can compare these ideas with the analogous claims for integers: The sum, difference or product of any two integers is an integer, but the quotient of two integers is not always an integer. Students explore the result of dividing factorable polynomials and simplifying through inspection. Students recognize that some quotients are not polynomials and develop an understanding of closure. ***Example:*** Simplify $\frac{4x^3+6x^2+14x}{2x}$ and determine if the quotient is a polynomial. Are polynomials closed under division? Explain.
A.APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	***The Remainder Theorem*** states that if a polynomial $p(x)$ is divided by any factor $(x - c)$, *which does not have to be a factor of the polynomial*, the remainder is the same as if you evaluate the polynomial for c (meaning to evaluate $p(c)$). If the remainder $p(c) = 0$ then $(x - c)$ is a factor of $p(x)$. ***Example:*** Let $p(x) = x^3 - x^4 + 8x^2 - 9x + 30$. Evaluate $p(-2)$. What does the solution tell you about the factors of $p(x)$? ***Example:*** Consider the polynomial function: $P(x) = x^4 - 3x^3 + ax^2 - 6x + 14$, where a is an unknown real number. If $(x - 2)$ is a factor of this polynomial, what is the value of a?

Standard	Cluster: Understand the relationship between zeros and factors of polynomials.
A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	Students identify the multiplicity of the zeroes of a factored polynomial and explain how the multiplicity of the zeroes provides a clue as to how the graph will behave when it approaches and leaves the x-intercept. Sketch a rough graph using the zeroes of a polynomial and other easily identifiable points such as the y-intercept. ***Example:*** Factor the expression $x^3 + 4x^2 - 64x - 256$ and explain how your answer can be used to solve the equation $x^3 + 4x^2 - 64x - 256 = 0$. Explain why the solutions to this equation are the same as the x-intercepts of the graph of the function $f(x) = x^3 + 4x^2 - 64x - 256$. ***Example:*** For a certain polynomial function, $x = 3$ is a zero with multiplicity two, $x = 1$ is a zero with multiplicity three, and $x = -3$ is a zero with multiplicity one. Write a possible equation for this function and sketch its graph.
A.APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples*	**Prove polynomial identities algebraically by showing steps and providing reasons or explanation.** Polynomial identities should include but are not limited to: • The product of the sum and difference of two terms, • The difference of two squares, • The sum and difference of two cubes, • The square of a binomial **Students prove polynomial identities by showing steps and providing reasons and describing relationships** (e.g. determine $81^2 - 80^2$ by applying differences of squares which leads to $(81 + 80)(81 - 80) = 161$. Illustrate how polynomial identities are used to determine numerical relationships; such as $25^2 = (20 + 5)^2 = 20^2 + 2 \bullet 20 \bullet 5 + 5^2$. ***Example:*** Explain why $x^2 - y^2 = (x - y)(x + y)$ for any two numbers x and y. ***Example:*** Verify the identity $(x - y)^2 = x^2 - 2xy + y^2$ by replacing ***y*** with **–*y*** in the identity $(x + y)^2 = x^2 + 2xy + y^2$. ***Example:*** Show that the pattern shown below represents an identity. Explain. $2^2 - 1^2 = 3$ $3^2 - 2^2 = 5$ $4^2 - 3^2 = 7$ $5^2 - 4^2 = 9$ ***Solution:*** $(n + 1)^2 - n^2 = 2n + 1$ for any whole number n.

Standard	Cluster: Rewrite rational expressions.
A.APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	**Students define rational expressions and determine the best method of simplifying a given rational expression.** Define rational expressions. Rewrite rational expressions, $\frac{a(x)}{b(x)}$, in the form $q(x)+\frac{r(x)}{b(x)}$ by using inspection (factoring) or long division. The polynomial $q(x)$ is called the quotient and the polynomial $r(x)$ is called the remainder. ***Example:*** Express $\frac{-x^2+4x+87}{x+1}$ in the form $q(x)+\frac{r(x)}{b(x)}$. ***Example:*** Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form. **Students determine the best method of simplifying a given rational expression.** ***Example (using inspection):*** $\frac{6x^3+15x^2+12x}{3x}$, $\frac{x^2+9x+14}{x+7}$ ***Example (long division):*** $\frac{x^4+3x}{x^2-4}$, $\frac{x^3+7x^2+13x+6}{x+4}$ *Note: The use of synthetic division may be introduced as a method but students should recognize its limitations (division by a linear term). When students use methods that have not been developed conceptually, they often create misconceptions and make procedural mistakes due to a lack of understanding as to why the method is valid. They also lack the understanding to modify or adapt the method when faced with new and unfamiliar situations. Suggested viewing Synthetic Division: How to understand It by not doing it. http://www.youtube.com/watch?v=V-Q6jBYn3Oc* Connect to A.APR.1 and recognize the need for a new system for when dividing polynomials; analogous to how rational numbers are developed from dividing integers.

A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	Rational expressions are closed under addition, subtraction, multiplication and division by a nonzero rational expression. A set of numbers is said to be closed for a specific mathematical operation if the result obtained when an operation is performed on any two numbers in the set, is itself a member of the set. ***Example:*** Simplify each of the following a. $\frac{4x+13}{x-3}+\frac{x+2}{2x+6}$ c. $\left(\frac{2x+4}{x^2-6x}\right)\left(\frac{x^2-36}{4x+8}\right)$ b. $\frac{3x+7}{x-2}-\frac{3x+15}{2x-4}$ d. $\left(\frac{x^2-4}{x^2+2x-5}\right)\div\left(\frac{x+2}{x^2+2x-5}\right)$ ***Example:*** A rectangle has an area of $\frac{x^2+x-2}{x^3}$ square feet and a height of $\frac{x^2}{x-1}$ feet. Express the width of the rectangle as a rational expression in terms of x.

Standard	Cluster: Create equations that describe numbers or relationships.
A.CED.1 ★ Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. In previous courses, students studied linear, exponential, quadratic, and inverse variation (a simple rational) expression/equations. In Math III, continue working with the aforementioned equation types making a connection to A.REI.4a when completing the square to put a quadratic equation into vertex form. ***Example:*** Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother. a. How many games would Chase have to win in a row in order to have a 75% winning record? b. How many games would Chase have to win in a row in order to have a 90% winning record? c. Is Chase able to reach a 100% winning record? Explain why or why not. d. Suppose that after reaching a winning record of 90% in part (b), Chase had a losing streak. How many games in a row would Chase have to lose in order to drop down to a winning record below 55% again? http://www.illustrativemathematics.org/illustrations/702 ***Example:*** If the world population at the beginning of 2008 was 6.7 billion and growing at a rate of 1.16% each year, in what year will the population be double? *Note: Students can use tables and graphs to solve; however, the intent in Math III is to use more algebraic methods for solving. Students may create the equation* $6.7(1.0116)^t = 13.4$ *then connect to F.LE.4. Students will need to rewrite bases into base 10, 2, or e by recognizing that* $1.0116 = 10^{\log 1.0116} = e^{\ln 1.0116}$
A.CED.2 ★ Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students create equations in two variables and graph them on coordinate axes. There are no limitations as to the type of equations in Math III; to include but not limited to polynomial, inverse variation, rational and trigonometric equations. ***Example:*** A company is manufacturing an open-top rectangular box. They have 30 cm by16 cm sheets of material. The bins are made by cutting squares the same size from each corner of a sheet, bending up the sides, and sealing the corners. Create an equation relating the volume V of the box to the length of the corner cut out x. Graph the equation and identify the dimensions of the box that will have the maximum volume. Explain. 

A.CED.3 ★ Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. In Math I, the focus was on linear equations/inequalities. In Math II, the standard is extended to linear and quadratic relationships and linear and inverse variation relationships. In Math III, extend to more complex systems, which would include linear programming. ***Example:*** Sara's doctor tells her she needs between 400 and 800 milligrams of folate per day, with part coming from her diet and part coming from a multi-vitamin. Each multi-vitamin contains 50 mg of folate, and because of the inclusion of other vitamins and minerals, she can only take a maximum of 8 tablets per day. a. What are the possible combinations of n, number of vitamin tablets taken, and a, the amount of dietary folate, which will give Sarah exactly the minimum of 400 mg of folate each day? Express your answers in a table. b. What are the possible combinations of vitamin tablets and dietary folate, which give the maximum of 800 mg of folate each day? c. Now use your tables from parts (a) and (b) to express your answers as a system of three inequalities. Create a graph of a versus n, and compare your graph to your tables from parts (a) and (b). d. Suppose instead of a multi-vitamin, Sara is given a powdered folate supplement she can add to water. She can drink any amount of the supplement she wants per day, as long as she does not exceed 400 mg per day. What are the possible combinations of folate she can ingest from her diet and from the powder? Graph your solution set. How does this graph compare to the graph from part (a)? *http://www.illustrativemathematics.org/illustrations/1351* *Note: This task requires students consider the reasonableness of their solutions within the given context. For instance, in part (a), students need to be able to see that although the solution region for their system is shaded, only whole numbers of vitamins make sense for Sara to take each day. Alternatively, a reasonable discussion about the plausibility of having fractions of multivitamins might be of some benefit; though should ultimately culminate with at least the distinction between the two interpretations.*
A.CED.4 ★ Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. In Math III extend from simple rational to more complex formulas. ***Example:*** Motion can be described by the formula $s = ut + ½at^2$, where t = time elapsed, u = initial velocity, a = acceleration, and s = distance traveled. a. Why might the equation need to be rewritten in terms of a? b. Rewrite the equation in terms of a. ***Example:*** The figure to the right is made up of a square with height, h units, and a right triangle with height, h units, and base length, b units. The area of the figure is 80 square units. Write an equation that solves for the height, h, in terms of b. Show all work necessary to justify your answer. 

Standard	Cluster: Understand solving equations as a process of reasoning and explain the reasoning.
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	This standard is included in Math I, II and III. In Math III, students should extend their knowledge of solving quadratic equations to include those that cannot be factored and to radical equations. **Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.** • When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method. ***Example:*** Explain each step in solving the quadratic equation $x^2 + 10x = -7$ $x^2 + 10x = -7$ $x^2 - 10x + 25 = 18$ $(x-5)^2 = 18$ $x - 5 = \pm 3\sqrt{2}$ $x = 5 \pm 3\sqrt{2}$ In Math III, focus on explaining steps for justifying equivalency of expressions. Connect with A.APR.4 when students prove polynomial identities. ***Example:*** Prove $(x^3 - y^3) = (x-y)(x^2 + xy + y^2)$. Justify each step. ***Example:*** Explain the steps involved in solving each of the following: a. $5(2^t) + 20 = 60$ $5(2^t) = 40$ $(2^t) = 8$ $2^t = 2^3$ $t = 3$ b. $8(1.5)^x = 200$ $(1.5)^x = 25$ $(10^{log 1.5})^x = 10^{\log 25}$ $\mathrm{x} \cdot \log 1.5 = \log 25$ $x = \frac{\log 25}{\log 1.5} \approx 7.94$ c. $2000(1.06)^x = 10000$ $(1.06)^x = 5$ $e^{\mathrm{x} \cdot \ln 1.06} = e^{\ln 5}$ $x \cdot \ln 1.06 = \ln 5$ $x = \frac{\ln 5}{\ln 1.06}$ ≈ 27.62

	Example: The rational equation has been solved using two different methods. **Method 1** $$\frac{1}{x-8}-1=\frac{7}{x-8}$$ $$\frac{1}{x-8}-1\left(\frac{x-8}{x-8}\right)=\frac{7}{x-8}$$ $$\frac{1}{x-8}+\left(\frac{-x+8}{x-8}\right)=\frac{7}{x-8}$$ $$\frac{-x+9}{x-8}=\frac{7}{x-8}$$ $$-x+9=7$$ $$x=2$$ **Method 2** $$\frac{1}{x-8}-1=\frac{7}{x-8}$$ $$(x-8)\left(\frac{1}{x-8}-1\right)=\left(\frac{7}{x-8}\right)(x-8)$$ $$1-(x-8)=7$$ $$9-x=7$$ $$x=2$$ a. Justify each step using a property of equality. b. How do the solution methods compare? c. Which method would you most likely use? Justify why. Connecting this standard to A. SSE.3b and A.REI.4 will increase students' understanding of the derivation of the quadratic formula.
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	This standard is included in Math II and III. In Math III, focus on solving simple rational and radical equations. ***Example:*** Solve $5-\sqrt{-(x-4)}=2$ for x. ***Example:*** Mary solved $x=\sqrt{2-x}$ for x and got $x=-2$ and $x=1$. Evaluate her solutions and determine if she is correct. Explain your reasoning. **Give examples showing how extraneous solutions may arise.** • Students are proficient with solving simple rational and radical equations that have no extraneous solutions before moving on to equations that result in quadratics and possible solutions that need to be eliminated. It is very important that students are able to reason how and why extraneous solutions arise. The square root symbol (like all even roots) is defined to be the positive square root, so a positive root can never be equal to a negative number. Squaring both sides of the equation will make that discrepancy disappear; the square of a positive number is positive but so is the square of a negative number, so we'll end up with a solution to the new equation even though there was no solution to the original equation.

Example: When raising both sides of an equation to a power, we sometimes obtain an equation, which has more solutions than the original one. (Sometimes the extra solutions are called extraneous solutions.)
Which of the following equations result in extraneous solutions when you raise both sides to the indicated power? Explain.

a. $\sqrt{x} = 5$, Square both sides
b. $\sqrt{x} = -5$, Square both sides
c. $\sqrt[3]{x} = 5$, Cube both sides
d. $\sqrt[3]{x} = -5$, Cube both sides

> This isn't the case with odd roots - a cube root of a positive number is positive, and a cube root of a negative number is negative. When we cube both sides of the last equation, the negative remains, and we end up with a true solution to the equation.

Example: Create a square root equation that when solved algebraically introduces an extraneous solution.
Show the algebraic steps you would follow to look for a solution, and indicate where the extraneous solution arises.

Sample solution: $\sqrt{2x+1} + 7 = 2$

$$\sqrt{2x+1} + 7 = 2$$
$$\left(\sqrt{2x+1}\right)^2 = (-5)^2$$
$$2x + 1 = 25$$
$$2x = 24$$
$$x = 12$$

> This is where the extraneous solution comes in. The square root can't be negative, but by squaring both sides, we're losing that information.

Example: Solve $\frac{3}{x-3} = \frac{x}{x-3} - \frac{3}{2}$. Can x have a value of 3? Explain your reasoning.

Example: Solve $\frac{2x-8}{x-4} = 4$

A.REI.4

Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g. for $x^2 = 49$), taking square roots, completing the square, the

Students use the method of completing the square to rewrite a quadratic expression (A.SSE.3b). They extend the method to a quadratic equation. Some students may set the quadratic equal to zero, rewrite into vertex form $a\,(x-p)^2 + k = 0$, and then begin solving to get the equation into the form $(x-p)^2 = q$ where $q = \frac{-k}{a}$. Other students may adapt the method (i.e. not having to start with the quadratic equal to 0) to get the equation into the same form.

Example: Solve $-2x^2 - 16x = 20$

Students who writes vertex form first

$$-2x^2 - 16x - 20 = 0$$
$$-2(x^2 - 8x) - 20 = 0$$
$$-2(x^2 - 8x + 16) - 20 - 32 = 0$$
$$-2(x-4)^2 - 52 = 0$$
$$-2(x-4)^2 = 52$$
$$(x-4)^2 = 26$$
$$x - 4 = \pm\sqrt{26}$$
$$x = 4 \pm \sqrt{26}$$

Student who adapts method

$$-2(x^2 - 8x) = 20$$
$$-2(x^2 - 8x + 16) = 20 + 32$$
$$-2(x-4)^2 = 52$$
$$(x-4)^2 = 26$$
$$x - 4 = \pm\sqrt{26}$$
$$x = 4 \pm \sqrt{26}$$

quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Example: Solve $ax^2 + bx + c = 0$ for x.

A.REI.4b
Student solved quadratics in Math II. In Math III, extend to quadratics with non-real solutions. Students recognize when the quadratic formula gives complex solutions and are able to write then as $a \pm bi$. Students relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.

Value of Discriminant	Nature of Roots	Nature of Graph
$b^2 - 4ac = 0$	1 real root	Intersects x-axis once
$b^2 - 4ac > 0$	2 real roots	Intersects x-axis twice
$b^2 - 4ac < 0$	2 complex solutions	Does not intersect x-axis

Example: Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have? Find all solutions of the equation.

Example: What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?

Standard	Cluster: Represent and solve equations and inequalities graphically.

A.REI.10

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

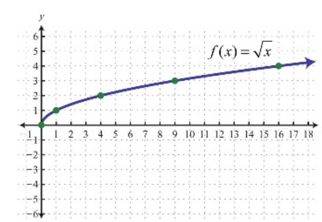
This standard is included in Math I, II and III. In previous courses, students have verified the graph of an equation as the set of all solutions of the equation for linear, exponential, quadratic, inverse variation, and power equations. In Math III, extend this work to include polynomial, rational, and trigonometric equations.

Students explain and verify that every point (x, y) on the graph of an equation represents values x and y that make the equation true.

Example: If $y = 4x^4 - 6x^3 + x - 10$, which of the following is a solution? Select all that apply. Explain.

a. (2, 8)
b. (4, –6)
c. (5, 1745)
d. (–2, 100)

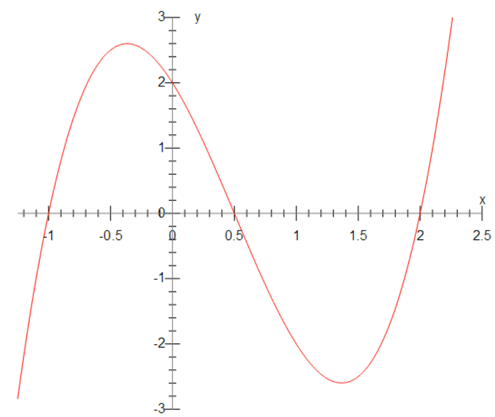
Students can explain and verify that every point (x, y) on the graph of an equation represents values x and y that make the equation true.



Example: The graph of $f(x) = \sqrt{x}$ is shown on the right. Does the graph extend into quadrants II, III, or IV? Explain.

Standard	Cluster: Represent and solve equations and inequalities graphically.
A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	This standard is included in Math I, II and III. In previous courses, students studied linear, exponential and quadratic functions. In Math III, include polynomial, rational, absolute value and logarithmic. **Visual examples of radical and rational equations explores the solution as the intersection of two functions and provides evidence to discuss how extraneous solutions do not fit the model.** ***Example:*** Graph the following system and approximate solutions for $f(x) = g(x)$. $f(x) = \frac{x+4}{2-x}$ and $g(x) = x^3 - 6x^2 + 3x + 10$ ***Example:*** Let $f(x) = \log x$ and $g(x) = \ln x$. Determine solution(s) for $f(x) = g(x)$. Explain what the solution(s) mean in terms of the functions given. ***Example:*** Use technology to solve $e^{2x} + 3x = 15$, treating each side of the statement as two separate equations. *Note: From the standard, we build that $f(x) = g(x)$ where $f(x) = y_1$ and $g(x) = y_2$* *Given that $y_1 = e^{2x} + 3x$ and $y_2 = 15$, find the value of x by identifying the intersection of the two graphs.*

Standard	Cluster: Understand the concept of a function and use function notation.
F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	This standard is included in Math I, II and III. Throughout all three courses, students use function notation, evaluate functions, and interpret statements that use function notation. In Math III, students should use function notation regardless of their familiarity with the function family. Students determine output values from given inputs and interpret statements that use function notation. ***Example:*** The function $h(x) = x^4 - 10x^3 + 32x^2 - 38x + 25$ models the height (in meters) of a roller coaster section where x is the horizontal distance in meters from the start of the section ($0 \leq x \leq 5$). Determine each of the following and explain what it means in terms of the context. a. $h(1)$ b. $h(x) = 10$ ***Example:*** If $f(x) = \frac{3x^2+2x-1}{x-2}$ what is $f(0)$? Determine when $f(x) = 32$. ***Example:*** The function $p(x) = \frac{-25x^2+875x-4750}{750-25x}$ is the profit per ticket as a function of the ticket price, x. Evaluate each of the following and interpret what the statement means. a. $p(10)$ b. $p(20)$ ***Example:*** Radioactive iodine is a dangerous by-product of nuclear explosions, but it decays rather rapidly. Suppose that the function $R(t) = 6(10^{-0.038})^t$ gives the amount in a test sample remaining t days after an experiment begins. Determine $R(3) - R(2)$ and explain what it means in terms of the context.

Standard	Cluster: Interpret functions that arise in applications in terms of the context.
F.IF.4★ For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students interpret the key features of graphs and tables for a variety of different functions. In Math III, extend to more complex functions represented by graphs and tables and focus on interpreting key features of all function types. Also, include periodicity. Students understand periodicity as motion that is repeated in equal intervals of time. **When given a table or graph of a function that models a real-life situation, explain the meaning of the characteristics of the table or graph in the context of the problem.** • Key features include intercepts, intervals of increase/decrease, positive/negative, relative maximum/minimums, symmetries, and end behavior. *Example:* For the function on the right, label and describe the key features. Include intercepts, relative max/min, intervals of increase/decrease, and end behavior.  *Example:* Number of customers at a coffee shop vary throughout the day. The coffee shop opens at 5:00am and number of customers increase slowly at first and increase more and more until reaching a maximum number of customers for the morning at 8:00 am. Number of customers slowly decrease until 9:30 when they drop significantly and then remain steady until 11:00 am when the lunch crowd begins to show. Similar to the morning, the number of customers increase slowly and then begin to increase more and more. The maximum customers is less at lunch than breakfast and is largest at 12:20pm. The smallest number of customers since opening occurs at 2:00 pm. There is a third spike in customers around 5:00 pm and then a late night crowd around 9:00 pm before closing at 10:00 pm. Sketch a graph that would model the number of customers at the coffee shop during the day.

Interpret key features of graphs and tables in terms of quantities

Example: Over a year, the length of the day (the number of hours from sunrise to sunset) changes every day. The table below shows the length of day every 30 days from 12/31/97 to 3/26/99 for Boston Massachusetts.

Data on length of day

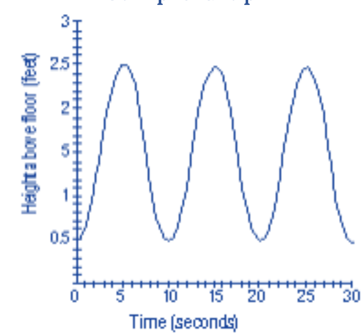
Date	12/31	1/30	3/1	3/31	4/30	5/30	6/29	7/29	8/28	9/27	10/27	11/26	12/26	1/25	2/24	3/26
Day Number	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450
Length (hours)	9.1	9.9	11.2	12.7	14.0	15.0	15.3	14.6	13.3	11.9	10.6	9.5	9.1	9.7	11.0	12.4

During what part of the year do the days get longer? Support your claim using information provided from the table.

Sketch graphs showing key features of polynomial, rational, and trigonometric functions given a verbal description of the relationship.

Example: Jumper horses on carousels move up and down as the carousel spins. Suppose that the back hooves of such a horse are six inches above the floor at their lowest point and two-and-one-half feet above the floor at their highest point. Draw a graph that could represent the height of the back hooves of this carousel horse during a half-minute portion of a carousel ride.

Sample Graph



Connect to F–IF. 7 in graphing functions expressed symbolically highlighting key features.

Standard	Cluster: Interpret functions that arise in applications in terms of the context.
F-IF.5★ Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. In previous courses, students studied linear, exponential, quadratic, right triangle trigonometry and inverse variation functions. In Math III, continue to relate the domain of a function, where applicable, to the aforementioned function types and any other function types addressed in all 3 courses, which include but are not limited to polynomial, rational and trigonometric functions. ***Example:*** The function $p(x) = \frac{-25x^2+875x-4750}{750-25x}$ is the profit per ticket as a function of the ticket price, x. Define a reasonable domain for the function. Explain your reasoning.

Standard	Cluster: Analyze functions using different representations.
F.IF.7★ Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. In Math III, students graph polynomial function, exponential functions and logarithmic functions, in addition to other function types learned in previous courses. ***Example:*** Graph $g(x) = x^3 + 5x^2 + 2x - 8$. a. Identify zeroes b. Discuss the end behavior c. In what intervals is the function increasing? Decreasing? **Students explore the end behavior of a polynomial and develop ideas about the impact of the leading coefficient on the output values as the input values increase.** ***Example:*** Many computer applications use very complex mathematical algorithms. The faster the algorithm, the more smoothly the programs run. The running time of an algorithm depends on the total number of steps needed to complete the algorithm. For image processing, the running time of an algorithm increases as the size of the image increases. For an n-by-n image, algorithm 1 has running time given by $p(n) = n^3 + 3n + 1$ and algorithm 2 has running time given by $q(n) = 15n^2 + 5n + 4$ (measured in nanoseconds, or 10^{-9} seconds). a. Compute the running time for both algorithms for images of size 10-by-10 pixels and 100-by-100 pixels. b. Graph both running time polynomials in an appropriate window (or several windows if necessary). c. Which algorithm is more efficient? Explain your reasoning. http://www.illustrativemathematics.org/illustrations/1539 ***Example:*** Graph $f(x) = 10^x$ and $g(x) = \log x$. Compare the key features of intercepts and end behavior. Discuss how they are related. **Graph trigonometric functions, showing period, midline, and amplitude.** ***Example:*** Graph $y = 3\ cos\ (x) - 5$ identifying the period, midline, and amplitude. Connect to A.APR.3 and F.IF.4

F.IF.8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

This standard is included in Math I, II and III. Throughout all three courses, students rewrite expressions in different ways to reveal properties. In Math III, students use factoring and/or completing the square as a process to show zeroes, extreme values, and symmetry of the graph. Students should also interpret these features in terms of a context.

Example: A company's profit (in thousands of dollars) from an item depends on the price of the item. Three different expressions for the profit at a price of p dollars follow:

$$-2p^2 + 24p - 54$$
$$-2(p-3)(p-9)$$
$$-2(p-6)^2 + 18$$

a. How could you convince someone that the three expressions are equivalent?

b. Which form is most useful for finding:
   i. The break-even prices? What are those prices, and how do you know?
   ii. The profit when the price is 0? What is that profit, and what does it tell about the business situation?
   iii. The price that will yield maximum profit? What is that price, and how do you know?

Connect this standard to A.SSE.2 and A.SSE.3c in terms of the types of factoring problems expected.

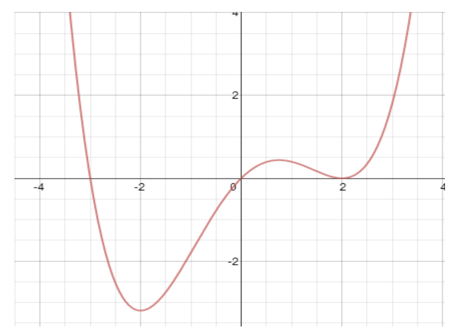
F.IF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

This standard is included in Math I, II and III. Throughout all three courses, students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

Examples: If $f(x) = -(x+7)^2(x-2)$ and $g(x)$ is represented on the graph.

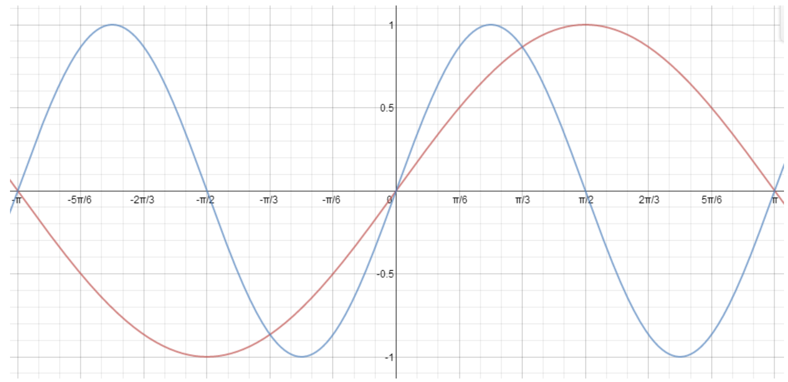
a. What is the difference between the zero with the least value of $f(x)$ and the zero with the least value of $g(x)$?
b. Which has the largest relative maximum?
c. Describe their end behaviors. Why are they different? What can be said about each function?



Standard	Cluster: Build a function that models a relationship between two quantities.
F.BF.1 ★ Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. In previous courses, students studied linear, exponential, quadratic functions. Up until this point, students have only used informal notation for the recursive process. **Determine an explicit expression, a recursive process, or steps for calculation from a context.** ***Example:*** Suppose you deposit \$100 in a savings account that pays 4% interest, compounded annually. At the end of each year you deposit an additional \$50. Write a recursive function that models the amount of money in the account for any year. **Combine standard function types using arithmetic operations.** ***Example:*** A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time.
F.BF.2 ★ Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I and III. In previous courses, students studied arithmetic and geometric sequences along with the explicit formula. Students wrote recursive rules using an informal notation such as Now-Next. In Math III, focus on formalizing the notation. ***Example:*** Given the sequence defined by the function $a_{n+1} = a_n + 12$ with $a_0 = 4$. Write an explicit function rule. ***Example:*** Given the sequence defined by the function $a_{n+1} = \frac{3}{4}a_n$ with $a_0 = 424$. Write an explicit function rule.

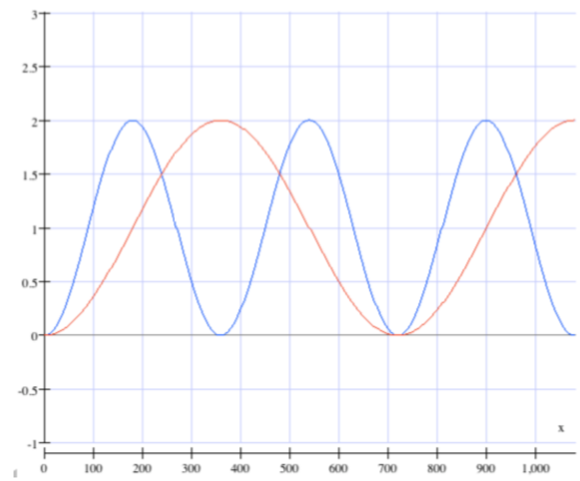
Standard	Cluster: Build new functions from existing functions.
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	This standard is included in Math I, II and III. In previous courses, students studied horizontal and vertical translations and $k \cdot f(x)$. In Math III, include $f\ (kx)$. Include functions: linear, exponential, polynomial, and trigonometric.

Example: Compare the graphs of $y = sin\ x$ and $y = sin\ 2x$. What effect does the factor of 2 on the input value have on the graph? Explain why this occurs.



Example: The graphs represent the height of two Ferris Wheels at the county fair.

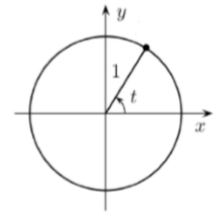
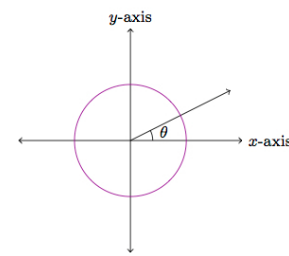
a. Explain how the speed is different for the two Ferris Wheels.
b. Given $f(x)$ is the faster of the two Ferris Wheels. Write the function rule that transforms the faster Ferris Wheel to model the slower of the two Ferris Wheels.



Example: Determine if each of the functions $f(x) = sin\ x$ and $g(x) = x^5 + 6x^4 - 9x^2 + 7$ are even, odd, or neither. Justify your response.

F.BF.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	Students solve a function for the dependent variable and write the inverse of a function by interchanging the values of the dependent and independent variable. They connect the concept of the inverse to the affect on the graph and the input-output pairs. Students find inverse functions for linear and exponential functions. Also, include simple situations where the domain of the functions must be restricted in order for the inverse to be a function, such as $f(x) = x^2, x \leq 0$. ***Example:*** Graph the inverse of $f(x) = -\frac{3}{2}x - 3$. How does $f^{-1}(x)$ relate to $f(x)$? ***Example:*** Find the inverse of the function $g(x) = 2^x$ and demonstrate it is the inverse using input – output pairs. ***Example:*** Let $h(x) = x^3$. Find the inverse function. ***Example:*** If $f(x) = \sin x$, find and graph of $f^{-1}(x)$. a. Is $f^{-1}(x)$ a function? Explain your reasoning. b. How could the domain be restricted so that the inverse would be a function?

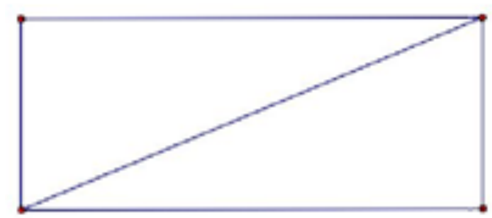
Standard	Cluster: Construct and compare linear, quadratic, and exponential models and solve problems.
F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	This standard is included in Math I and III. In previous courses, students studied linear, exponential, and quadratic models. ***Example:*** Contrast the growth of $f(x) = x^3$ and $g(x) = 3^x$.
F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	Students recognize how to rewrite values using bases 2, 10, or e. Students use calculators to approximate answers. ***Example:*** Solve $200e^{0.04t} = 450$.

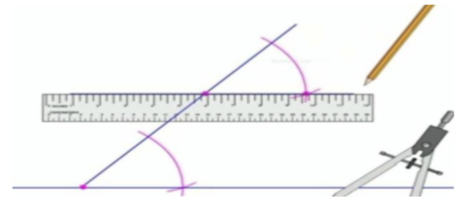
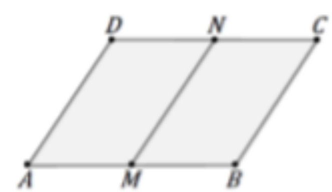
Standard	Cluster: Extend the domain of trigonometric functions using the unit circle
F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	Students know that if the length of an arc subtended by an angle is the same length as the radius of the circle, then the measure of the angle is 1 radian. Students should also determine the radian measures of angles subtended around the circle ***Example:*** What is the radian measure of the angle t in the diagram to the right?  ***Example:*** The minute hand on the clock at the City Hall clock in Stratford measures 2.2 meters from the tip to the axle. a. Through what angle does the minute hand pass between 7:07 a.m. and 7:43 a.m.? b. What distance does the tip of the minute hand travel during this period? Connect to N.Q.1 by converting between radians and degrees. Connect to N.Q.3 by using units appropriate for the problem.
F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Students understand that one complete rotation around the unit circle, starting at (0,1), restricts the domain of trigonometric functions to $0 \leq \theta \leq 2\pi$. As more rotations are considered, the domain extends to all real numbers since the radian measure of any angle is a real number and there is no limit to the number of times one can travel around the unit circle.  ***Example:*** a. Explain why $sin\,(-\theta) = -sin\theta$ and $cos\,(-\theta) = cos\theta$. Do these equations hold for any angle θ? Explain. b. Explain why $sin\,(2\pi + \theta) = sin\,\theta$ and $cos\,(2\pi + \theta) = cos\theta$. Do these equations hold for any angle θ? Explain. http://www.illustrativemathematics.org/illustrations/1704

Standard	Cluster: Model periodic phenomena with trigonometric functions
F.TF.5★ Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students choose the most appropriate trigonometric function to model a given situation dependent on the height and speed of the situation occurring.** ***Example:*** The temperature of a chemical reaction oscillates between a low of 20°C and a high of 120°C.The temperature is at its lowest point when $t = 0$ and completes one cycle over a 6-hour period. a. Sketch the temperature, T, against the elapsed time, t, over a 12-hour period. b. Find the period, amplitude, and the midline of the graph you drew in part a. c. Write a function to represent the relationship between time and temperature. d. What will the temperature of the reaction be 14 hours after it began? e. At what point during a 24-hour day will the reaction have a temperature of 60°C?

Standard	Cluster: Prove and apply trigonometric identities
F.TF.8 Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin\ (\theta)$, $cos\ (\theta)$, or $tan\ (\theta)$ given $sin\ (\theta)$, $cos\ (\theta)$, or $tan\ (\theta)$ and the quadrant of the angle.	**Students prove $\boldsymbol{sin^2(\theta) + cos^2(\theta) = 1}$.** In the unit circle, the cosine is the x-value, while the sine is the y-value. Since the hypotenuse is always 1, the Pythagorean relationship $sin^2(\theta) + cos^2(\theta) = 1$ is always true. **Students use $\boldsymbol{sin^2(\theta) + cos^2(\theta) = 1}$ to find $\boldsymbol{sin\ (\theta)}$, $\boldsymbol{cos\ (\theta)}$, or $\boldsymbol{tan\ (\theta)}$ given $\boldsymbol{sin\ (\theta)}$, $\boldsymbol{cos\ (\theta)}$, or $\boldsymbol{tan\ (\theta)}$ and the quadrant of the angle.** ***Example:*** Given $\cos\theta = \frac{\sqrt{3}}{2}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$ find $sin\ (\theta)$ and $tan\ (\theta)$.

Standard	Cluster: Experiment with transformations in the plane
G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	This standard is included in Math I and III. In Math III, include the distance around a circular arc. ***Example:*** Suppose that the wheels on a mountain bike have a radius of 33 cm. How far does the bike travel in 2 hours if traveling at 80 revolutions per minute?

Standard	Cluster: Prove geometric theorems
G.CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	In 8th grade, students experimented with the properties of angles and lines. In Math III, focus on *proving* the properties; not just knowing and using them. Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. ***Example:*** Prove that any point equidistant from the endpoints of a line segment lies on the perpendicular bisector of the line. ***Example:*** A carpenter is framing a wall and wants to make sure his the edges of his wall are parallel. He is using a cross-brace as show in the diagram.  a. What are some different ways that he could verify that the edges are parallel? b. Write a formal argument to show that these sides are parallel? c. Pair up with another student who created a different argument than yours, and critique their reasoning. Did you modify your diagram as a result of the collaboration? How? Why?

	Example: The diagram below depicts the construction of a parallel line, above the ruler. The steps in the construction result in a line through the given point that is parallel to the given line. Which statement below justifies why the constructed line is parallel to the given line?  a. When two lines are each perpendicular to a third line, the lines are parallel. b. When two lines are each parallel to a third line, the lines are parallel. c. When two lines are intersected by a transversal and alternate interior angles are congruent, the lines are parallel. d. When two lines are intersected by a transversal and corresponding angles are congruent, the lines are parallel. Note: The focus of G.CO.9-11 is on students proving geometric properties; not the particular content items students are proving.
G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	This standard is included in Math II and III. In Math II, students focused on proofs that involved measures of interior angles of triangles and triangle mid-segments. In Math III, students prove that the base angles of isosceles triangles are congruent and the medians of a triangle meet at a point, called the centroid. However, more should be provided for students to become comfortable with analyzing and proving theorems about triangles. ***Example:*** Given an equilateral triangle whose side lengths are each n units; find the area of the triangle. ***Example:*** The task Seven Circles can be found on the Illustrative Mathematics site http://www.illustrativemathematics.org/illustrations/707 Note: The focus of G.CO.9-11 is on students proving geometric properties; not the particular content items students are proving.
G.CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	Extend the standard G.CO.8 from Math II and use the triangle congruency criteria to determine all of the properties of parallelograms and special parallelograms. ***Example:*** Suppose that ABCD is a parallelogram, and that M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$, respectively. Prove that MN =AD, and that the line $\overleftrightarrow{MN}$ is parallel to $\overleftrightarrow{AD}$.  ***Example:*** Lesson "Properties of Quadrilaterals" from Inside Mathematics http://www.insidemathematics.org/index.php/classroom-video-visits/public-lessons-properties-of-quadrilaterals. Note: The focus of G.CO.9-11 is on students proving geometric properties; not the particular content items students are proving.

Standard	Cluster: Make geometric constructions

G.CO.12

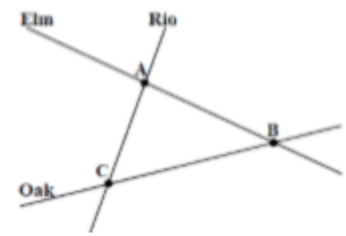
Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Students use various tools to perform the geometric constructions. Students formalize and explain how these constructions result in the desired objects.
The standard includes the following:

- Copying a segment
- Copying an angle
- Bisecting a segment
- Bisecting an angle
- Constructing perpendicular lines, including the perpendicular bisector of a line segment
- Constructing a line parallel to a given line through a point not on the line.

Example: You have been asked to place a warehouse so that it is an equal distance from the three roads indicated on the following map. Find this location and show your work.

a. How would you fold your paper to physically construct this point?
b. Explain why the above construction works and, in particular, why you only needed to make two creases.



Standard	Cluster: Understand similarity in terms of similarity transformations

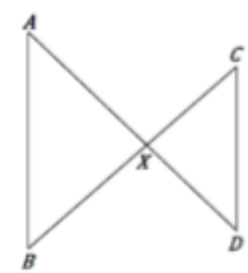
G.SRT.2

Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Students use the idea of dilation transformations to develop the definition of similarity. They understand that a similarity transformation is a rigid motion followed by a dilation. Students demonstrate that in a pair of similar triangles, corresponding angles are congruent (angle measure is preserved) and corresponding sides are proportional. They determine that two figures are similar by verifying that angle measure is preserved and corresponding sides are proportional.

Example: In the picture to the right, line segments AD and BC intersect at X. Line segments AB and CD are drawn, forming two triangles ΔAXB and ΔCXD.
In each part a-d below, some additional *assumptions* about the picture are given. For each assumption:

I. Determine whether the given assumptions are enough to prove that the two triangles are similar. If so, what is the correct correspondence of vertices. If not, explain why not.
II. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one variable to the other.



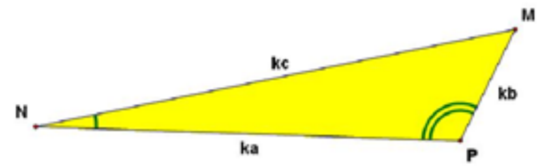
a. The lengths of AX and AD satisfy the equation $2AX = 3XD$.
b. The lengths AX, BX, CX, and DX satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$
c. Lines AB and CD are parallel.
d. $\angle XAB$ is congruent to angle $\angle XCD$.

G.SRT.3

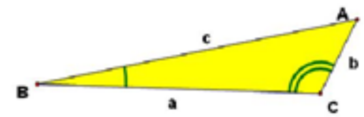
Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Students can use the theorem that the angle sum of a triangle is 180° and verify that the AA criterion is equivalent to the AAA criterion.

Given two triangles for which AA holds, students use rigid motions to map a vertex of one triangle onto the corresponding vertex of the other in such a way that their corresponding sides are in line. Then show that dilation will complete the mapping of one triangle onto the other.



Example: Given that ΔMNP is a dialation of ΔABC with scale factor k, use properties of dilations to show that the AA criterion is sufficient to prove similarity.



Example: Similar Triangles (https://www.illustrativemathematics.org/content-standards/HSG/SRT/A/3/tasks/1422)

Standard	Cluster: Prove theorems involving similarity

G.SRT.4

Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

Use AA, SAS, and SSS similarity theorems to prove triangles are similar. Use triangle similarity to prove other theorems about triangles:

- Prove a line parallel to one side of a triangle divides the other two proportionally, and it's converse
- Prove the Pythagorean Theorem using triangle similarity

Example: How does the Pythagorean Theorem support the case for triangle similarity?

View the video below and create a visual proving the Pythagorean Theorem using similarity.
http://www.youtube.com/watch?v=LrS5_l-gk94

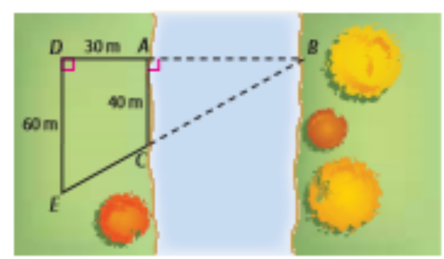
Example: Prove that if two triangles are similar, then the ratio of corresponding altitudes is equal to the ratio of corresponding sides.

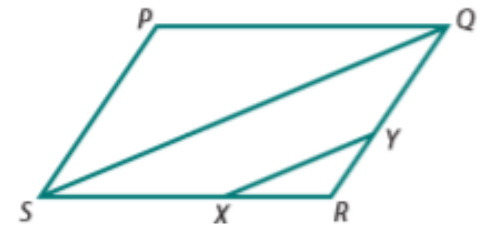
G.SRT.5

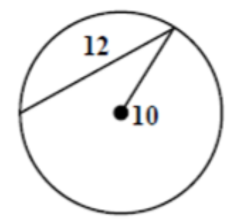
Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

The similarity postulates include SSS, SAS, and AA. The congruence postulates include SSS, SAS, ASA, AAS, and H-L. Students apply triangle congruence and triangle similarity to solve problem situations (e.g., indirect measurement, missing side(s)/angle measure(s), side splitting).

Example: Calculate the distance across the river, AB.



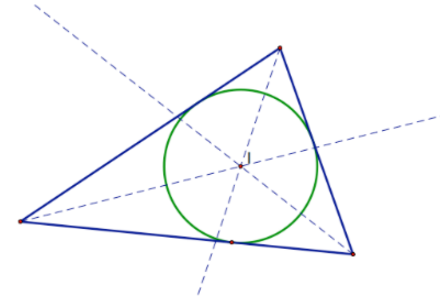
	Example: In the diagram, quadrilateral PQRS is a parallelogram, SQ is a diagonal, and SQ || XY. a. Prove that ΔXYR~ΔSQR. b. Prove that ΔXYR~ΔQSP. 

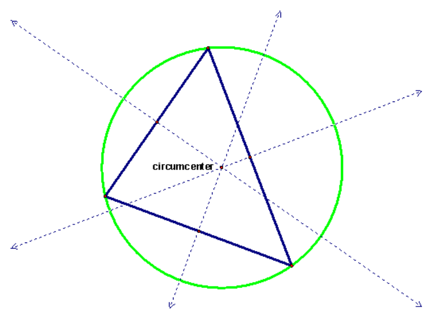
Standard	**Cluster: Understand and apply theorems about circles**
G.C.1 Prove that all circles are similar.	**Students use the fact that the ratio of diameter to circumference is the same for all circles; prove that all circles are similar.** **Students use any two circles in a plane and show that they are related by dilation.** ***Example:*** Show that the two given circles are similar by stating the necessary transformations from C to D. C: centerpoint at (2, 3) with a radius of 5 D: centerpoint at (-1, 4) with a radius of 10
G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	**Students can:** • Identify central angles, inscribed angles, circumscribed angles, diameters, radii, chords, and tangents. • Describe the relationship between a central angle and the arc it intercepts. • Describe the relationship between an inscribed angle and the arc it intercepts. • Describe the relationship between a circumscribed angle and the arcs it intercepts. • Recognize that an inscribed angle whose sides intersect the endpoints of the diameter of a circle is a right angle. • Recognize that the radius of a circle is perpendicular to the tangent where the radius intersects the circle. ***Example:*** Given the circle to the right with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.  ***Example:*** Right Triangles Inscribed by Circles I (https://www.illustrativemathematics.org/content-standards/HSG/C/A/2/tasks/1091) ***Example:*** Right Triangles Inscribed by Circles II (https://www.illustrativemathematics.org/content-standards/HSG/C/A/2/tasks/1093)

G.C.3

Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Students construct the inscribed circle whose center is the point of intersection of the angle bisectors (the incenter).

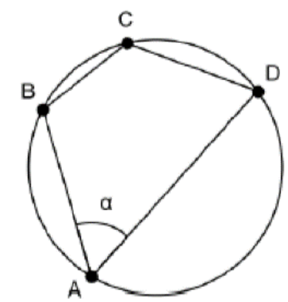




Students construct the circumscribed circle whose center is the point of intersection of the perpendicular bisectors of each side of the triangle (the circumcenter).

Students prove properties of angles for a quadrilateral inscribed in a circle.

Example: Given the inscribed quadrilateral below prove that m∠ B is supplementary to m∠D.



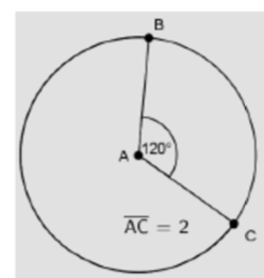
Standard	Cluster: Find the arc lengths and areas of sectors of circles

G.C.5

Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius

All circles are similar (G.C.1). Sectors with the same central angle have arc lengths that are proportional to the radius. The radian measure of the angle is the constant of proportionality.



Example: Find the area of the sectors. What general formula can you develop based on this information?

Example: Find the area of a sector with an arc length of 40 cm and a radius of 12 cm.

Standard	Cluster: Translate between the geometric description and the equation for a conic section
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	This standard is included in Math II and III. In Math III, complete the square to find the center and radius of a circle given by an equation. ***Example:*** Find the center and radius of the circle $4x^2 + 4y^2 - 4x + 2y - 1 = 0$. ***Example:*** Explaining the Equation of a Circle (https://www.illustrativemathematics.org/content-standards/HSG/GPE/A/1/tasks/1425) Connect to A.SSE.3 where students choose and produce equivalent forms of an expression to reveal and explain the properties of the quantities represented in an expression.
G.GPE.2 Derive the equation of a parabola given a focus and directrix.	Students have used parabolas to represent *y* as a function of *x*. This standard introduces the parabola as a geometric figure that is the set of all points an equal distance from a fixed point (focus) and a fixed line (directrix). Students derive the equation of a parabola given the focus and directrix. Students may derive the equation by starting with a horizontal directrix and a focus on the *y*-axis, and use the distance formula to obtain an equation of the resulting parabola in terms of *y* and x^2. Next, they use a vertical directrix and a focus on the *x*-axis to obtain an equation of a parabola in terms of *x* and y^2. Make generalizations in which the focus may be any point, but the directrix is still either horizontal or vertical. Students may use the generalizations in their future work. Allow sufficient time for students to become familiar with new vocabulary and notation. ***Example:*** Write and graph an equation for a parabola whose focus is at (2, 3) and with a directrix at $y = 1$. ***Example:*** A parabola has focus (–2, 1) and directrix $y = -3$. Determine whether or not the point (2, 1) is part of the parabola. Justify your answer. ***Example:*** Given the equation $20(y - 5) = (x + 3)^2$, find the focus, vertex and directrix.

Standard	Cluster: Apply geometric concepts in modeling situations

G.MG.3★

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is included in Math II and III.

Example: Janine is planning on creating a water-based centerpiece for each of the 30 tables at her wedding reception. She has already purchased a cylindrical vase for each table.

- The radius of the vases is 6 cm and the height is 28 cm.
- She intends to fill them half way with water and then add a variety of colored marbles until the waterline is approximately three-quarters of the way up the cylinder.
- She can buy bags of 100 marbles in 2 different sizes, with radii of 9mm or 12 mm. A bag of 9 mm marbles costs $3, and a bag of 12 mm marbles costs $4.

a. If Janine only bought 9 mm marbles how much would she spend on marbles for the whole reception? What if Janine only bought 12 mm marbles? (Note: 1 cm^3 = 1 mL)

b. Janine wants to spend at most d dollars on marbles. Write a system of equalities and/or inequalities that she can use to determine how many marbles of each type she can buy.

c. Based on your answer to part b. How many bags of each size marble should Janine buy if she has $180 and wants to buy as many small marbles as possible?

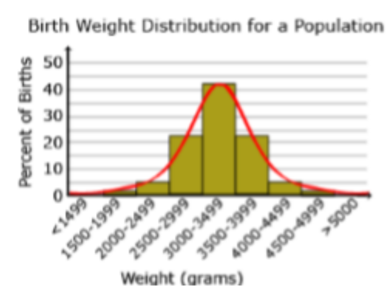
Standard	Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.4★

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

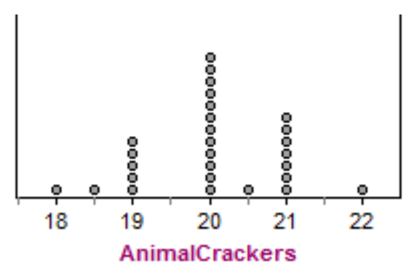
Students use the normal distribution to make estimates of frequencies (which can be expressed as probabilities). They recognize that only some data are well described by a normal distribution. They use the 68-95-99.7 rule to estimate the percent of a normal population that falls within 1, 2, or 3 standard deviations of the mean.



Example: The histogram gives the birth weight of a population of 100 chimpanzees. The curve shows how the weights are normally distributed about the mean, 3250 grams. Estimate the percent of baby chimps weighing 3000–3999 grams.

Example: Scores on a history test have a mean of 80 with standard deviation of 6. How many standard deviations from the mean is the student that scores a 90.

Standard	Cluster: Understand and evaluate random processes underlying statistical experiments
S.IC.1 ★ Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Define populations, population parameter, random sample, and inference.** • A *population* consists of everything or everyone being studied in an inference procedure. It is rare to be able to perform a census of every individual member of the population. Due to constraints of resources it is nearly impossible to perform a measurement on every subject in a population. • A *parameter* is a value, usually unknown (and which therefore has to be estimated), used to represent a certain population characteristic. • *Inferential statistics* considers a subset of the population. This subset is called a statistical sample often including members of a population selected in a random process. The measurements of the individuals in the sample tell us about corresponding measurements in the population. **Students demonstrate an understanding of the different kinds of sampling methods.** ***Example:*** From a class containing 12 girls and 10 boys, three students are to be selected to serve on a school advisory panel. Here are four different methods of making the selection. i. Select the first three names on the class roll. ii. Select the first three students who volunteer. iii. Place the names of the 22 students in a hat, mix them thoroughly, and select three names from the mix. iv. Select the first three students who show up for class tomorrow. Which is the best sampling method, among these four, if you want the school panel to represent a fair and representative view of the opinions of your class? Explain the weaknesses of the three you did not select as the best.

Standard	Cluster: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC.3★ Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students understand the different methods of data collection, specifically the difference between an observational study and a controlled experiment, and know the appropriate use for each.** • *Observational study* – a researcher collects information about a population by measuring a variable of interest, but does not impose a treatment on the subjects. (I.e. examining the health effects of smoking) • *Experiment* – an investigator imposes a change or treatments on one or more group(s), often called treatment group(s). A comparative experiment is where a control group is given a placebo to compare the reaction(s) between the treatment group(s) and the control group. **Students understand the role that randomization plays in eliminating bias from collected data.** ***Example:*** Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as "strict". They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students. a. Describe the parameter of interest and a statistic the students could use to estimate the parameter. b. Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning. c. The students quickly realized that, as there is no definition of "strict", they could not simply ask a student, "Are your parents or guardians strict?" Write three questions that could provide objective data related to strictness. d. Describe an appropriate method for obtaining a sample of 100 students, based on your answer in part (a) above.
S.IC.4★ Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students estimate a sample mean or sample proportion given data from a sample survey. Estimate the population value.** ***Example:*** The label on a Barnum's Animal Cracker box claims that there are 2 servings per box and a serving size is 8 crackers. The graph displays the number of animal crackers found in a sample of 28 boxes. Use the data from the 28 samples to estimate the average number of crackers in a box with a margin of error. Explain your reasoning or show your work.  ***Example:*** Margin of Error for Estimating a Population Mean (https://www.illustrativemathematics.org/content-standards/HSS/IC/B/4/tasks/1956)

S.IC.5★

Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Example: Sal purchased two types of plant fertilizer and conducted an experiment to see which fertilizer would be best to use in his greenhouse. He planted 20 seedlings and used Fertilizer A on ten of them and Fertilizer B on the other ten. He measured the height of each plant after two weeks. Use the data below to determine which fertilizer Sal should use.

Fertlizer A	23.4	30.1	28.5	26.3	32.0	29.6	26.8	25.2	27.5	30.8
Fertlizer B	19.8	25.7	29.0	23.2	27.8	31.1	26.5	24.7	21.3	25.6

a. Use the data to generate simulated treatment results by randomly selecting ten plant heights from the twenty plant heights listed.

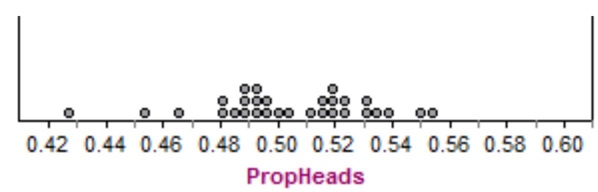
b. Calculate the average plant height for each treatment of ten plants.

c. Find the difference between consecutive pairs of treatment averages and compare. Does your simulated data provide evidence that the average plant heights using Fertlizer A and Fertilizer B is significant?

Example: "Are Starbucks customers more likely to be female?" To answer the question, students decide to randomly select 30-minute increments of time throughout the week and have an observer record the gender of every tenth customer who enters the Starbucks store. At the end of the week, they had collected data on 260 customers, 154 females and 106 males. This data seems to suggest more females visited Starbucks during this time than males.

To determine if these results are statistically significant, students investigated if they could get this proportion of females just by chance if the population of customers is truly 50% females and 50% males. Students simulated samples of 260 customers that are 50-50 females to males by flipping a coin 260 then recording the proportion of heads to represent the number of women in a random sample of 260 customers (e.g., 0.50 means that 130 of the 260 flips were heads). Their results are displayed in the graph at the right.

Use the distribution to determine if the class's data is statistically significant enough to conclude that Starbucks customers are more likely to be female.



S.IC.6★ Evaluate reports based on data.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math II and III. ***Example:*** Read the article below from NPR.org then answer the following questions. Kids and Screen Time: What Does the Research Say? By Juana Summers August 28, 2014 Kids are spending more time than ever in front of screens, and it may be inhibiting their ability to recognize emotions, according to new research out of the University of California, Los Angeles. The study, published in the journal *Computers in Human Behavior*, found that sixth-graders who went five days without exposure to technology were significantly better at reading human emotions than kids who had regular access to phones, televisions and computers. The UCLA researchers studied two groups of sixth-graders from a Southern California public school. One group was sent to the Pali Institute, an outdoor education camp in Running Springs, Calif., where the kids had no access to electronic devices. For the other group, it was life as usual. At the beginning and end of the five-day study period, both groups of kids were shown images of nearly 50 faces and asked to identify the feelings being modeled. Researchers found that the students who went to camp scored significantly higher when it came to reading facial emotions or other nonverbal cues than the students who continued to have access to their media devices. "We were pleased to get an effect after five days," says Patricia Greenfield, a senior author of the study and a distinguished professor of psychology at UCLA. "We found that the kids who had been to camp without any screens but with lots of those opportunities and necessities for interacting with other people in person improved significantly more." If the study were to be expanded, Greenfield says, she'd like to test the students at camp a third time — when they've been back at home with smartphones and tablets in their hands for five days. "It might mean they would lose those skills if they weren't maintaining continual face-to-face interaction," she says. a. Was this an experiment or an observational study? b. What can you conclude? c. Are there any limitations or concerns with this statistical study?

Standard	Cluster: Use probability to evaluate outcomes of decisions.
S.MD.6 (+) ★ Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students recognize that to make fair decisions each possible outcome must be equally likely.** Example: A class of 25 students wants to choose 3 students at random to serve as class ambassadors to the student council. Any set of 3 students should have an equal chance of being chosen. Which of the following strategies will result in a fair decision? a. The students line up alphabetically, and each one in succession flips a fair coin. The first three students to flip heads serve as ambassadors. b. Each student draws a card from a well-shuffled deck of 52 cards. The teacher shuffles a second deck of cards, spreads them out, and draws cards one by one until he matches the cards of three of the students.
S.MD.7 (+) ★ Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students apply probability concepts to analyze decisions and strategies related to real-world situations.** ***Example:*** A pharmaceutical company is testing the effectiveness of a new drug for asthma patients. Out of 160 volunteers who suffer from asthma, 80 are given the drug, and 80 are given a placebo, which has no active ingredients. After 2 weeks, the volunteers are asked if they noticed improvement in their asthma symptoms. The results of the survey are shown in the contingency table at the right. a. What is the probability that a volunteer reported noticeable improvement in symptoms given that he received the test drug? b. What is the probability that a volunteer received the placebo given that he did not report a noticeable improvement in symptoms? c. The pharmaceutical company decides to produce and distribute this drug. The drug is marketed as an effective way to improve the symptoms of asthma. Based on the results of the test, did the company make a good decision? Explain.

	Improved	Did not Improve	Totals
Received the Drug	67	13	80
Received the Placebo	24	56	80
Totals	91	69	160

References

This document includes examples, illustrations and references from the following websites:

YouTube: www.youtube.com

Illustrative Mathematics: www.illustrativemathematics.org

Inside Mathematics: www.insidemathematics.org

NPR: www.npr.org