AP Calculus AB

2020-2021 Pacing

**Textbook Requirement**

Larson, Ron and Edwards, Bruce. Calculus of a Single Variable. 9th ed. Boston: Brooks/Cole, 2010 **CR4**

All students are issued a copy of this text.

**CR4** – Students and teachers have access to a college-level calculus textbook.

**Graphing Calculator Required**

All students are provided a TI-84 Plus Silver Edition on the first day of class. **CR3a**

**CR3a** – Students have access to graphing calculators

**Curricular Requirements**

1a The course is structured around the enduring understandings within Big Idea 1: Limits.

1b The course is structured around the enduring understandings within Big Idea 2: Derivatives.

1c The course is structured around the enduring understandings within Big Idea 3: Integrals and the

 Fundamental Theorem of Calculus.

2a The course provides opportunities for students to reason with definitions and theorems.

2b The course provides opportunities for students to connect concepts and processes.

2c The course provides opportunities for students to implement algebraic/computational processes.

2d The course provides opportunities for students to engage with graphical,analytical,numerical and verbal

 representations and demonstrate connections among them.

2e The course provides opportunities for students to build notational fluency.

2f The course provides opportunities for students to communicate mathematical ideas in words, both orally

 and in writing.

3a Students have access to graphing calculators.

3b Students have opportunities to use calculators to solve problems.

3c Students have opportunities to use graphing calculator to explore and interpret calculus concepts.

4 Students and teachers have access to a college-level calculus textbook.

Semester 1

**BIG IDEA 1: Limits**

**I. Limits CR1a: limits**

1. **Investigate limits**
2. Tables - find limits numerically and graphically
3. Graphs – find limits graphically and numerically
4. Analysis
5. direct substitution
6. factor
7. rationalize
8. special limits
9. Squeeze Theorem

**Activity**: In a do now activity, students must describe how to find a limit for a given function and explain what the limit means in the context of the problem algebraically, numerically, and graphically and present orally to the class. **CR2f**: oral

**CR1a** – The course is structured around the enduring understandings within Big Idea 1: Limits.

**CR2f** – The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

1. **Infinite Limits**
2. Limits that approach
3. Limits as x approaches
4. Emphasize graphical (asymptotes), numerical, and analytical approaches to limits
5. Discuss asymptotes/unbounded behavior in terms of graphical behavior as well as in terms of limits.
6. Compare unbounded growth and rates of change.

**Activity**: Pairs of students are given a function and asked to determine its limit. One finds the limit analytically and the other student graphs the function on the calculator to determine the limit by inspecting the graph. After sharing and explaining their results, they switch assignments for the next problem. This helps visualize ways limits do not exist. **CR3b**

**CR3b** – Students have opportunities to use calculators to solve problems.

1. **Continuity – CR1a**: **continuity**
2. Discuss intuitive approach to continuity – One-Sided Limits
3. Define in terms of limits – Discuss graphical, numerical, and analytical interpretations using a variety of activities and examples.
4. Removeaable and non-removeable discontinuities
5. Intermediate Value Theorem
6. Extreme Value Theorem

**Activity**: Students are given a worksheet with a set of functions listed as equations or graphs. CR2d: analytical and graphical Students discuss the continuity of each function in their groups. **CR2f:** oral Each group creates a function satisfying continuities and discontinuities which may include limits that exist and present to the class.

**CR1a** – The course is structured around the enduring understandings within the Big Idea 1: Limits

**CR2d** – The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

**CR2f** – The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

**BIG IDEA 2: Derivatives**

**Derivatives and Rates of Change CR1b: derivatives**

1. **What is a rate of change?**
2. Average rate of change vs instantaneous rate of change, and how instantaneous rate of change is the limit of the average rate of change.
3. Relate graphically to slope
4. **Finding slopes of tangent lines using two limits**
5. 2.1 The Derivative and Tangent Line Problem: Proving the derivative does not exist at a point c. The alternative form of the derivative is useful in investigating the relationship between differentiability and continuity.

**Activity**: Show a video from Calculus Phobe that displays a graph that draws a secant line and animates the picture to see how the secant line becomes the tangent line as h approaches 0.

1. **Derivative**
2. Discuss the term “derivative” and its connection to limits.
3. Practice calculating derivatives using the definition.
4. Understand what it means to take the derivative at a point. (slope of a curve)
5. Use a table to estimate derivatives.
6. Use a graph to estimate derivatives. 2.2 Basic Differentation Rules and Rates of Change
7. Interpret the meaning of derivatives in word problems – explain orally and in writing the meaning of units in relation to the derivative (like ft/hour). Explore how the derivative at a point can be used to predict. Practice Free Response Questions
8. Local linear approximation.

**Activity**: Group quiz listing functions where students must show the relationship of the function and its derivative expressing each algebraically, numerically, graphically, and verbally in complete sentences. **CR2b** **CR2d CR2f**

**CR1b** – The course is structured around the enduring understandings within Big Idea 2: Derivatives

**CR2b** – The course provides opportunities for students to connect concepts and processes.

**CR2d** – The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrates connections among them.

**CR2f** – The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

1. **Differentiability**
2. What does it mean for a function to be differentiable?
3. Local linearity
4. Differentiability compared to continuity.
5. Piecewise functions
6. Understand differentiability using limits (analytically), graphs, tables (numerically), and explain why functions are not differentiable both orally and in writing.

 **Activity**: Do now worksheet using the two definitions of derivative. Students must explain orally what the limit represents and whether the function is continuous and differentiable.

**Activity**: Students explore the local linearity of functions using their calculators to determine differentiability. **CR3c** They also discuss the continuity of the functions. Discussion leads to conclude that differentiability implies continuity, but continuity does not imply differentiability. **CR2a**

**CR2a** - The course provides opportunities for students to reason with definitions and theorems.

**CR3c** – Students have opportunities to use a graphing calculator to explore and interpret calculus concepts.

**Differentiation Rules and Motion on a Line**

1. **Constant and power rules**

Find derivatives of basic functions and notice patterns

**Activity**: Groups work on a worksheet containing functions. They are to graph the functions and its derivative on the same axis to see the power rule graphically.

1. **Derivatives of trigonometric functions – sine and cosine**

Use their definitions and look at these trigonometric functions graphically and numerically

1. **Product and quotient rules**
2. Develop rules from the limit definition
3. Find the derivatives of the other trigonometric functions using these rules
4. **Higher order derivatives**

Emphasize chain rule problems

1. **Motion on a line**
2. Velocity
3. Speed
4. Acceleration
5. Speeding up and slowing down
6. Displacement vs. total distance
7. Move left/right, changing direction

**Semester 2**

**More Differentiation and Curve Sketching**

1. **Implicit differentia**tion

Stress use of chain rule

1. **Derivatives of inverse functions**
2. Use implicit derivatives to find the derivative of an inverse function
3. Derivatives of an inverse function using a table
4. **Derivatives of Logarithmic functions**

Natural logs and logs of other bases

1. **Derivatives of exponential functions**

Base e and other bases

1. **Curve sketching**
2. Sketch the first derivative given the function
3. Increasing and decreasing behavior using first derivative
4. Concavity and points of inflection using second derivative
5. Sketch f given first and second derivatives
6. Verbalize descriptions of functions using derivatives
7. Compare characteristics of a function and its first and second derivatives
8. Sketch f given f ‘ and f “
9. Translate equations using derivatives into verbal descriptions

**Activity**: Students are given worksheets where they draw f ‘ and f “ given f and vice versa.

**Application of Derivatives**

1. **The Mean Value Theorem** **CR1b**

Discuss graphical interpretation and geometric consequences – Rolle’s Theorem and Intermediate Value Theorem

**Activity**: Students are given a polynomial graph and asked to draw a secant line through two given points. They are asked how many tangent lines can be drawn to the curve that will be parallel to the secant line. The MVT is given, and the hypothesis and conclusion are discussed. The class reinforce the idea that the MVT applies only if certain conditions are met. In the context of real-world problems, is there a time when the first derivative is 0?

**CR1b** – The course is structured around the enduring understanding within Big Idea 2: Derivatives

1. **Related Rates**

Analyze rate of change using a variety of real-world applications

Activity: Tootsie Roll Pop Lab – Students suck on TRP to determine the rate of change of their radius. They calculate the rate of change of the Pops’ volume. Students then use this rate of change to estimate the rate of change of the volume of the Pop when its radius us three-fourths of its original radius. Each student prepares their presentation on chart paper to display their data collection with a table, graph, equation, and paragraph written in complete sentences explaining “How Many Licks?” it takes to reach the center of the TRP. **CR1b**

**CR2b CR2c CR2d CR2f**

**CR2b –** The course provides opportunities for students to connect concepts and processes.

**CR2c** – The course provides opportunities for students to implement algebraic/computational processes.

**CR2d** – The course provides opportunities for students to engage with graphical,numerical,analytical, and verbal representations and demonstrate connections among them.

**CR2f** – The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.

1. **Optimization**

Solve word problems that require students to find extrema

**Activity**: Students study optimization by building a model. They must minimize cost of fencing materials by attaching three free-standing sides of a fence to a wall. They are given fencing costs and total area limitations in square meters. **CR2b CR2c**

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1. **Differentials**

Discuss analytical and graphical meanings of differentials

1. **L’Hospital’s Rule**

The indeterminate form , ; used to evaluate limits

**BIG IDEA 3: Integrals and The Fundamental Theorem of Calculus**

**Integration CR1c**: **Integrals**

 **CR1c** – The course is structured around the enduring understandings within Big Idea 3: Integrals and the Fundamental Theorem of Calculus.

1. **Estimating with finite limits**

Establish area with units of measure – 4.2 Area

Riemann sums and estimate area using left, right, midpoint, and trapezoid sums: Problems include functions shown graphically, algebraically, and numerically.

**Activity**: Students work in groups to compute Riemann sums of functions and tables. using midpoint, left, right, and trapezoidal rules. Class discusses the accumulation of area and each rules accuracy.

1. **The definite integral**
2. Develop and define using a limit of Riemann sums:
3. Integral as a rate of change over an interval as the accumulated change of the quantity over the interval emphasizing units.
4. Basic properties of definite integrals using graphs
5. Calculate definite integrals using calculators

**Activity**: Students approximate a definite integral by calculating the distance covered during a 20-minute drive with a friend or parent. Student must record the car’s odometer reading before they begin. Using the speedometer, they record the car’s speed at one-minute intervals. At the end of the drive, they record the car’s odometer reading. Students graph speed versus time and use integration to approximate the distance traveled.

They compare the distance with the actual mileage on the odometer. Students write a report explaining their data collection, graph the data, interpret the data, and the closeness of their approximation to the odometer reading. **CR1c CR2b CR2c CR2d**

**CR1c –** The course is structured around the enduring understandings within Big Idea 3: Integrals and the Fundamental Theorem of Calculus.

**CR2b** – The course provides opportunities for students to connect concepts and processes.

**CR2c** – The course provides opportunities for students to implement algebraic/computational processes.

**CR2d** – The course provides opportunities for students toengage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.

1. **Antiderivatives and indefinite integrals**
2. Notice patterns and find basic antiderivatives
3. Explore the difference between antiderivatives and definite integrals
4. Explain +C both analytically and graphically
5. **The Fundamental Theorem of Calculus CR1c parts 1 and 2**
6. If f is continuous on [a,b], then the function g defined by g(x) = is an antiderivative of f.

That g ‘ (x) = f(x) for a < x < b. Fundamental Theorem of Calculus **Part II**

1. If f is continuous on [a,b], then where F is any antiderivative of f(x). Fundamental Theorem of Calculus **Part I**
2. Evaluate definite integrals using the Fundamental Theorem
3. Use Fundamental Theorem to analyze functions analytically and graphically

**Activity**: Students work in groups on free-response questions that require them to use the Fundamental Theorem of Calculus. They realize that the rate of change of the area function A(x) = , which is the graph of f. **CR2e**

**CR1c** – The course is structured around the enduring understandings within Big Idea 3: Integrals and the Fundamental Theorem of Calculus.

**CR2e** – The course provides opportunities for students to build notational fluency.

1. **Integration by substitution**
2. Practice using substitution in integrating a variety of functions
3. Discuss the necessity of changing the limits of integration on the integral
4. **Integration by parts**

When to use integration by parts instead of substitution

1. **The average value of a function**

Application problems worked analytically, graphically, verbally, and numerically.

**Activity**: Students are given a worksheet to calculate definite integrals using definitions, tables, and graphs.

**CR2c** Students interpret the integral’s meaning using proper units.

**CR2c** – The course provides opportunities for students to implement algebraic/computational processes.

**Other Applications of Integration**

1. **Differential equations**
2. What is a differential equation?
3. Separation of variables
4. Particular vs. general solutions
5. Slope fields and their relationship to solution curves for differential equations

**Activity**: Students are given a worksheet representing a series of differential equations and asked to draw a slope field to describe its solution curve. They analytically solve the differential equation. **CR2d**  Students are given differential equations they cannot solve. Using slope fields, they can discuss the solution curves.

**CR2d** – The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrates connections among them.

1. **Area**
2. Area under a curve and area between two curves
3. Curves that are a function of x and y

**Activity**: Students work in groups on a worksheet to calculate area between two curves. They can use the integral feature o their calculators.

1. **Volume**
2. Use known cross sections
3. Cross sections formed by revolving regions around axes of solids
4. Discs and washers using functions of x and y
5. Shells

**Activity**: Students build their own 3-D shapes to visualize volume using cross sections.