Limits

A function f(x) has a limit as x approaches c **if and only** if the right-hand and left-hand limits at c exist and are equal.

$$\lim_{x \to c} f(x) = L \leftrightarrow \lim_{x \to c^-} f(x) = L \ \ and \ \lim_{x \to c^+} f(x) = L$$

Consider

$$\lim_{x \to 2}(x^2 - 5x)$$

Read this as either

“**The limit as** *x* **goes to two,** of *x* squared minus five *x*.”

or

“**The limit** of *x* squared minus five *x*, **as** *x* **goes to two**.”

For the following limit theorems, assume:

$$\lim_{x \to a} f(x) = L \quad \text{and} \quad \lim_{x \to a} g(x) = M$$

$$\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L \pm M$$

$$\lim_{x \to a}[f(x)g(x)] = [\lim_{x \to a} f(x)] \cdot \left[\lim_{x \to a} g(x)\right] = L \cdot M$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}$$

$$\lim_{x \to a}(k \cdot f(x)) = k \lim_{x \to a} f(x) = k\,L$$

(where k is a constant)

To find $\lim_{x \to a} f(x)$

- If f(x) is a polynomial, simply evaluate f(a).
- If f(x) is not a polynomial (such as a rational expression), try to evaluate f(a) unless it gives some indeterminate form such as:
  - Division by zero
  - Undefined
  - ∞/∞, 0/0, etc.

  In these cases try to algebraically eliminate the difficulty before substituting in the *a* value.

If it is not possible to eliminate the indeterminate form, the limit is either

∞, $-\infty$, or DNE

Example 1) find $\lim_{x \to -2} x^2 - 4x + 1$

You can do this by plugging in.

$$(-2)^2 - 4(-2) + 1$$

$$\lim_{x \to -2} f(x) = 13$$

Example 2) find $\lim_{x \to -2} \dfrac{2x-6}{x-2}$

You can also do this by plugging in.

$$\frac{2(-2)-6}{-2-2}$$

$$\lim_{x \to -2} f(x) = 2.5$$

Example 3) find $\lim_{x \to -2} \boxed{\frac{x^2 - 2x - 8}{x^2 - 4}}$

Plug in and you get $\frac{0}{0}$ - no good

So attempt to factor and cancel

$$\frac{(x-4)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} = \frac{x-4}{x-2}$$

$$\lim_{x \to -2} \frac{-2-4}{-2-2} = 1.5$$

Example 4: Find $\lim_{x \to 2} \frac{x^4 - 16}{x^2 - 4}$

$$\frac{\cancel{(x^2-4)}(x^2+4)}{\cancel{x^2-4}}$$

$$\lim_{x \to 2} x^2+4 = 8$$

Example 5: Find $\lim_{x \to 4} \frac{5x - 20}{x^2 - 16}$

$$\frac{5\cancel{(x-4)}}{(x+4)\cancel{(x-4)}}$$

$$\lim_{x \to 4} \frac{5}{x+4} = 5/8$$

Example 6: Find $\lim_{x \to 3}\left(\frac{x^2 + 2x - 15}{x - 3}\right)$

L T R

Example 7: Find $$\lim_{x\to 1}\frac{x-1}{x^2-4x+3}$$

$$\frac{\cancel{x-1}}{\cancel{(x-1)}(x-3)}$$

$$\lim_{x\to 1}\frac{1}{x-3} = -\frac{1}{2}$$

Homework: Complete #1-21 on the Handout