NC Math I Standards

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). These have been indicated throughout the document.

Last Revised July, 2015

NC Math I Standards

Number	Algebra	Function	Geometry	Statistics & Probability
N.RN.1	A.SSE.1★	F.IF.1	G.CO.1	S.ID.1★
N.RN.2	A.SSE.2	F.IF.2		S.ID.2★
	A.SSE.3★	F.IF.3	G.GPE.4	S.ID.3★
N.Q.1★		F.IF.4★	G.GPE.5	S.ID.5★
N.Q.2★	A.APR.1	F.IF.5★	G.GPE.6	S.ID.6★
N.Q.3★		F.IF.6★	G.GPE.7	S.ID.7★
	A.CED.1★	F.IF.7★		S.ID.8★
	A.CED.2★	F.IF.8	G.GMD.1	S.ID.9★
	A.CED.3★	F.IF.9	G.GMD.3	
	A.CED.4★			
		F.BF.1★		
	A.REI.1	F.BF.2★		
	A.REI.3	F.BF.3		
	A.REI.5			
	A.REI.6	F.LE.1★		
	A.REI.10	F.LE.2★		
	A.REI.11★	F.LE.3★		
	A.REI.12	F.LE.5★		

Standard

Cluster: Extend the properties of exponents to rational exponents.

N.RN.1

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of* 5 *because we want* $\left(5^{1/3}\right)^3 = \left(5^{1/3}\right)^3$ *to hold, so* $\left(5^{1/3}\right)^3$ *must equal* 5.

The meaning of an exponent relates the frequency with which a number is used as a factor. So 5^3 indicates the product where 5 is a factor 3 times. Extend this meaning to a rational exponent, then $125^{1/3}$ indicates one of three equal factors whose product is 125.

Students recognize that a fractional exponent can be expressed as a radical or a root.
For example, an exponent of a $1/3$ is equivalent to a cube root; an exponent of $1/4$ is equivalent to a fourth root.

Students extend the use of the power rule, $(b^n)^m = b^{nm}$ from whole number exponents i.e., $(7^2)^3 = 7^6$ to rational exponents.

They compare examples, such as $\left(7^{1/2}\right)^2 = 7^{1/2*2} = 7^1 = 7$ to $(\sqrt{7})^2 = 7$ to establish a connection between radicals and rational exponents: $7^{1/2} = \sqrt{7}$ and, in general, $b^{1/2} = \sqrt{b}$.

Example: Determine the value of *x*

a. $64^{\frac{1}{2}} = 8^x$
b. $(12^5)^x = 12$

Example: A biology student was studying bacterial growth. The population of bacteria doubled every hour as indicated in the following table:

# of hours of observation	0	1	2	3	4
Number of bacteria cells (thousands)	4	8	16	32	64

How could the student predict the number of bacteria every half hour? every 20 minutes?

Solution: If every hour the number of bacteria cells is being multiplied by a factor of 2 then on the half hour the number of cells is increasing by a factor of $2^{1/2}$. For every 20 minutes, the number of cells is increasing by a factor of $2^{1/3}$.

N.RN.2

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

The standard is included in Math I and Math II. In Math I limit to rational exponents with a numerator of 1.

Example: Simplify $\sqrt[3]{8x^6}$

Example: Simplify $(2^4 3^8 x^2)^{1/2}$

Example: Simplify $\sqrt{32}$
Solution: $\sqrt{32} = \sqrt{4^2 \cdot 2} = \sqrt{4^2} \cdot \sqrt{2} = 4\sqrt{2}$
Note: Students should be able to simplify square roots in Math I. This is a foundational skill for simplifying the solutions generated by using the quadratic formula in Math II.

Standard	Cluster: Reason quantitatively and use units to solve problems
N.Q.1 ★ Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included throughout Math I, II and III. Units are a way for students to understand and make sense of problems. **Use units as a way to understand problems and to guide the solution of multi-step problems** • Students use the units of a problem to identify what the problem is asking. They recognize the information units provide about the quantities in context and use units as a tool to help solve multi-step problems. Students analyze units to determine which operations to use when solving a problem. ***For example***, given the speed in *mph* and time traveled in *hours*, what is the distance traveled? From looking at the units, we can determine that we must multiply *mph* times *hours* to get an answer expressed in miles: $\left(\frac{mi}{hr}\right)(hr) = mi$ ***(Note that knowledge of the distance formula, $d = rt$, is not required to determine the need to multiply in this case.)*** ***Another example***, the length of a spring increases 2 cm for every 4 oz. of weight attached. Determine how much the spring will increase if 10 *oz.* are attached: $\left(\frac{2cm}{4oz}\right)(10oz) = 5cm$. This can be extended into a multi-step problem when asked for the length of a 6 cm spring after 10 *oz.* are attached: $\left(\frac{2cm}{4oz}\right)(10oz) + 6cm = 11cm$. **Choose and interpret units consistently in formulas** • Students choose the units that accurately describe what is being measured. Students understand the familiar measurements such as length (unit), area (unit squares) and volume (unit cubes). They use the structure of formulas and the context to interpret units less familiar. ***For example***, if $density = \frac{mass\ in\ grams}{volume\ in\ mL}$ then the unit for density is $\frac{grams}{mL}$. **Choose and interpret the scale and the origin in graphs and data displays** • When given a graph or data display, students read and interpret the scale and origin. When creating a graph or data display, students choose a scale that is appropriate for viewing the features of a graph or data display. Students understand that using larger values for the tick marks on the scale effectively "zooms out" from the graph and choosing smaller values "zooms in." Students also understand that the viewing window does not necessarily show the *x*- or *y*-axis, but the apparent axes are parallel to the x- and y-axes. Hence, the intersection of the apparent axes in the viewing window may not be the origin. They are also aware that apparent intercepts may not correspond to the actual *x*- or *y*-intercepts of the graph of a function.

N.Q.2 ★ Define appropriate quantities for the purpose of descriptive modeling.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students define the appropriate variables to describe the model and situation represented. ***Example(s):*** Explain how the units cm, cm^2, and cm^3 are related and how they are different. Describe situations where each would be an appropriate unit of measure.
N.Q.3 ★ Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III throughout all three courses. **Students understand the tool used determines the level of accuracy that can be reported for a measurement.** ***Example(s):*** • When using a ruler, one can legitimately report accuracy to the nearest division. If a ruler has centimeter divisions, then when measuring the length of a pencil the reported length must be to the nearest centimeter, or • In situations where units constant a whole value, as the case with people. An answer of 1.5 people would reflect a level of accuracy to the nearest whole based on the fact that the limitation is based on the context. **Students use the measurements provided within a problem to determine the level of accuracy.** ***Example:*** If lengths of a rectangle are given to the nearest tenth of a centimeter then calculated measurements should be reported to no more than the nearest tenth. **Students recognize the effect of rounding calculations throughout the process of solving problems and complete calculations with the highest degree of accuracy possible, reserving rounding until reporting the final quantity.**

Standard	Cluster: Create equations that describe numbers or relationships.
A.SSE.1 ★ Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students interpret expressions that represent quantities. In Math I, the focus is on linear expressions, exponential expressions with integer exponents and quadratic expressions. Throughout all three courses, students: • ***Explain*** the difference between an expression and an equation • ***Use*** appropriate vocabulary for the parts that make up the whole expression • ***Identify*** the different parts of the expression and explain their meaning within the context of a problem • ***Decompose*** expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts ***Note:*** *Students should understand the vocabulary for the parts that make up the whole expression, be able to identify those parts, and interpret their meaning in terms of a context.* **Interpret parts of an expression, such as terms, factors, and coefficients.** • Students recognize that the linear expression $mx + b$ has two terms, m is a coefficient, and b is a constant. • Students extend beyond simplifying an expression and address interpretation of the components in an algebraic expression. ***For example***, a student recognizes that in the expression $2x + 1$, "2" is the coefficient, "2" and "x" are factors, and "1" is a constant, as well as "2x" and "1" being terms of the binomial expression. Also, a student recognizes that in the expression $4(3)^x$, 4 is the coefficient, 3 is the factor, and x is the exponent. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored. ***Example***: The height *(in feet)* of a balloon filled with helium can be expressed by $5 + 6.3s$ where s is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression. ***Example:*** A company uses two different sized trucks to deliver sand. The first truck can transport x cubic yards, and the second y cubic yards. The first truck makes S trips to a job site, while the second makes T trips. What do the following expressions represent in practical terms? (https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/531) and (http://www.mcphersonmath.com/2015/04/pta-series-1-establish-goals-to-focus.html) a. $S + T$ b. $x + y$ c. $xS + yT$ d. $\frac{xS + yT}{S + T}$

	Interpret complicated expressions by viewing one or more of their parts as a single entity. • Students view mx in the expression $mx + b$ as a single quantity. ***Example***: The expression $20(4x) + 500$ represents the cost in dollars of the materials and labor needed to build a square fence with side length x feet around a playground. Interpret the constants and coefficients of the expression in context. ***Example***: A rectangle has a length that is 2 units longer than the width. If the width is increased by 4 units and the length increased by 3 units, write two equivalent expressions for the area of the rectangle. ***Solution****: The area of the rectangle is $(x + 5)(x + 4) = x^2 + 9x + 20$. Students should recognize $(x + 5)$ as the length of the modified rectangle and $(x + 4)$ as the width. Students can also interpret $x^2 + 9x + 20$ as the sum of the three areas (a square with side length x, a rectangle with side lengths 9 and x, and another rectangle with area 20 that have the same total area as the modified rectangle.* ***Example***: Given that income from a concert is the price of a ticket times each person in attendance, consider the equation $I = 4000p - 250p^2$ that represents income from a concert where p is the price per ticket. What expression could represent the number of people in attendance? *Solution: The equivalent factored form, p $(4000 - 250p)$, shows that the income can be interpreted as the price times the number of people in attendance based on the price charged. Students recognize $(4000 - 250p)$ as a single quantity for the number of people in attendance.* ***Example***: The expression $10{,}000(1.055)^n$ is the amount of money in an investment account with interest compounded annually for n years. Determine the initial investment and the annual interest rate. ***Note****: The factor of* 1.055 *can be rewritten as* (1 + 0.055), *revealing the growth rate of* 5.5% *per year.* Connection to **N.Q.1** ***Another example***, the length of a spring increases 2 cm for every 4 oz. of weight attached. Determine how much the spring will increase if 10 *oz.* are attached: $\left(\frac{2cm}{4oz}\right)(10oz) = 5cm$. This can be extended into a multi-step problem when asked for the length of a 6 cm spring after 10 *oz.* are attached: $\left(\frac{2cm}{4oz}\right)(10oz) + 6cm = 11cm$.
A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	This standard is included in Math I, II and III. **Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.** Use factoring techniques such as common factors, grouping, the difference of two squares, or a combination of methods to factor quadratics completely. Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is a factorable quadratic, students should factor the expression further. Students should be prepared to factor quadratics in which the coefficient of the quadratic term is an integer that may or may not be the GCF of the expression.

	Students rewrite algebraic expressions by combining like terms or factoring to reveal equivalent forms of the same expression. Connect to multiplying linear factors in **A-APR.1** and interpreting factors in **A.SSE.1b**. ***Example:*** Rewrite the expression $2(x-1)^2-4$ into an equivalent quadratic expression of the form ax^2+bx+c. ***Example:*** Rewrite the following expressions as the product of at least two factors and as the sum or difference of at least two totals. a. x^2-25 b. $5x^2-15x+10$ c. $3x+6x$

Standard	**Cluster: Write expressions in equivalent forms to solve problems.**
A.SSE.3a★ Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students factor quadratic expressions and find the zeroes of the quadratic function they represent.** Students should be prepared to factor quadratics in which the coefficient of the quadratic term is an integer that may or may not be the GCF of the expression. **Students explain the meaning of the zeroes as they relate to the problem.** Connect to A.SSE.2 ***Example:*** The expression $-4x^2 + 8x + 12$ represents the height of a coconut thrown from a person in a tree to a basket on the ground where *x* is the number of seconds. a. Rewrite the expression to reveal the linear factors. b. Identify the zeroes and intercepts of the expression and interpret what they mean in regards to the context. c. How long is the ball in the air? ***Note:*** *The standard A-REI.4 is in Math 2 (not Math I) and includes solving quadratic equations in one variable. For this standard in Math I, focus on quadratic expressions and how the factors can reveal the zeroes.*

Standard

Cluster: Perform arithmetic operations on polynomials.

A.APR.1

Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

This standard is included in Math I, II and III. Throughout all three courses, students operate with polynomials. In Math I, focus on adding and subtracting polynomials (like terms) and multiplication of linear expressions.

The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, polynomial, factor, and term.

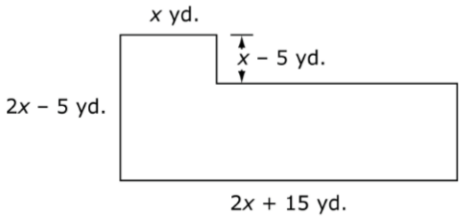
Example: Write at least two equivalent expressions for the area of the circle with a radius of $5x - 2$ kilometers.

Example: Simplify each of the following:

a. $(4x + 3) - (2x + 1)$
b. $(x^2 + 5x - 9) + 2x(4x - 3)$

Example: A town council plans to build a public parking lot. The outline below represents the proposed shape of the parking lot.

a. Write an expression for the area, in square feet, of this proposed parking lot. Explain the reasoning you used to find the expression.



b. The town council has plans to double the area of the parking lot in a few years. They plan to increase the length of the base of the parking lot by p yards, as shown in the diagram below.

Write an expression in terms of *x* to represent the value of *p*, in feet. Explain the reasoning you used to find the value of *p*.

Standard	**Cluster: Create equations that describe numbers or relationships.**
A.CED.1 ★ Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students recognize when a problem can be modeled with an equation or inequality and are able to write the equation or inequality. Students create, select, and use graphical, tabular and/or algebraic representations to solve the problem. In Math I, focus on linear and exponential contextual situations that students can use to create equations and inequalities in one variable and use them to solve problems. ***Example:*** The Tindell household contains three people of different generations. The total of the ages of the three family members is 85. a. Find reasonable ages for the three Tindells. b. Find another reasonable set of ages for them. c. In solving this problem, one student wrote $C + (C + 20) + (C + 56) = 85$ 1. What does C represent in this equation? 2. What do you think the student had in mind when using the numbers 20 and 56? 3. What set of ages do you think the student came up with? ***Example:*** Mary and Jeff both have jobs at a baseball park selling bags of peanuts. They get paid $12 per game and $1.75 for each bag of peanuts they sell. Create equations, that when solved, would answer the following questions: a. How many bags of peanuts does Jeff need to sell to earn $54? b. How much will Mary earn if she sells 70 bags of peanuts at a game? c. How many bags of peanuts does Jeff need to sell to earn at least $68? ***Example:*** Phil purchases a used truck for $11,500. The value of the truck is expected to decrease by 20% each year. When will the truck first be worth less than $1,000? ***Example:*** A scientist has 100 grams of a radioactive substance. Half of it decays every hour. How long until 25 grams remain? Be prepared to share any equations, inequalities, and/or representations used to solve the problem.
A.CED.2 ★ Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students: • Create equations in two variables • Graph equations on coordinate axes with labels and scales clearly labeling the axes defining what the values on the axes represent and the unit of measure. Students also select intervals for the scale that are appropriate for the context and display adequate information about the relationship. • Students interpret the context and choose appropriate minimum and maximum values for a graph.

In Math I, focus on linear, exponential and quadratic contextual situations for students to create equations in two variables. Limit exponential situations to only ones involving integer input values.

Linear equations can be written in a multitude of ways; $y = mx + b$ and $ax + by = c$ are commonly used forms (given that *x* and *y* are the two variables). Students should be flexible in using multiple forms and recognizing from the context, which is appropriate to use in creating the equation.

Example: The FFA had a fundraiser by selling hot dogs for $1.50 and drinks for $2.00. Their total sales were $400.

a. Write an equation to calculate the total of $400 based on the hot dog and drink sales.
b. Graph the relationship between hot dog sales and drink sales.

Example: A spring with an initial length of 25 cm will compress 0.5 cm for each pound applied.

a. Write an equation to model the relationship between the amount of weight applied and the length of the spring.
b. Graph the relationship between pounds and length.
c. What does the graph reveal about limitation on weight?

Quadratic equations can be written in a multitude of ways; $y = ax^2 + bx + c$ and $y = k\ (x + m)(x + n)$ are commonly used forms (given that *x* and *y* are the two variables). Students should be flexible in using multiple forms and recognizing from the context, which is appropriate to use in creating the equation.

Example: The cheerleaders are launching t-shirts into the stands at a football game. They are launching the shirts from a height of 3 feet off the ground at an initial velocity of 36 feet per second. What function rule shows a t-shirt's height h related to the time t? (Use $16t^2$ for the effect of gravity on the height of the t-shirt.)

Example: The local park is designing a new rectangular sandlot. The sandlot is to be twice as long as the original square sandlot and 3 feet less than its current width. What must be true of the original square lot to justify that the new rectangular lot has more area?

Note: *Students can construct the equations for each area and graph each equation. Students should select scales for the length of the original square and the area of the lots suitable for the context.*

Exponential equations can be written in a couple of ways; $y = ab^x$and $y = a\ (1 \pm r)^x$ are the most common forms (given that *x* and *y* are variables). Students should be flexible in using all forms and recognizing from the context which is appropriate to use in creating the equation.

Example: In a woman's professional tennis tournament, the money a player wins depends on her finishing place in the standings. The first-place finisher wins half of $1,500,000 in total prize money. The second-place finisher wins half of what is left; then the third-place finisher wins half of that, and so on.

a. Write a rule to calculate the actual prize money in dollars won by the player finishing in nth place, for any positive integer n.
b. Graph the relationship between the first 10 finishers and the prize money in dollars.
c. What pattern is indicated in the graph? What type of relationship exists between the two variables?

A.CED.3★ Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students: • Recognize when a constraint can be modeled with an equation, inequality or system of equations/inequalities. These constraints may be stated directly or be implied through the context of the given situation. • Create, select, and use graphical, tabular and/or algebraic representations to solve the problem. In Math I, focus on linear equations and inequalities. Linear programming is not expected; students should approach optimization problems using a problem-solving process such as using a table. **Represent constraints by equations or inequalities:** Example: The relation between quantity of chicken and quantity of steak if chicken costs \$1.29/lb and steak costs \$3.49/lb, and you have \$100 to spend on a dinner party where chicken and steak will be served. a. Write a constraint b. Justify your reasoning for writing the constraint as either an equation or an inequality c. Determine two solutions d. Graph the equation or inequality and identify your solutions **Represent constraints by a system of equations or inequalities:** Example: A club is selling hats and jackets as a fundraiser. Their budget is \$1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs \$5 and each jacket costs \$8. a. Write a system of inequalities to represent the situation. b. Graph the inequalities. **Interpret solutions as viable or nonviable options in a modeling context.** a. If the club buys 150 hats and 100 jackets, will the conditions be satisfied? b. What is the maximum number of jackets they can buy and still meet the conditions?
A.CED.4★ Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students solve multi-variable formulas or literal equations for a specific variable. In Math I, limit to formulas that are linear in the variable of interest, or to formulas involving squared or cubed variables. ***Example:*** Solve $V = \frac{4}{3}\pi r^3$ for radius *r*. *Explicitly connect this to the process of solving equations using inverse operations.*

Standard

Cluster: Understand solving equations as a process of reasoning and explain the reasoning

A.REI.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

This standard is included in Math I, II and III. In Math I, students should focus on solving linear equations and be able to extend and apply their reasoning to other types of equations in future courses.

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.

- When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Example: Assuming an equation has a solution; construct a convincing argument that justifies each step in the solution process. Justifications may include the associative, commutative, and division properties, combining like terms, multiplication by 1, etc.

Method 1:

$$\begin{aligned} 5(x+3)-3x &= 55 \\ 5x+15-3x &= 55 \\ 2x+15 &= 55 \\ 2x+15-15 &= 55-15 \\ 2x &= 40 \\ \frac{2x}{2} &= \frac{40}{2} \\ x &= 20 \end{aligned}$$

Method 2:

$$\begin{aligned} 5(x+3)-3x &= 55 \\ \frac{5(x+3)}{5}-\frac{3x}{5} &= \frac{55}{5} \\ x+3-\frac{3}{5}x &= 11 \\ \frac{2}{5}x+3 &= 11 \\ \frac{2}{5}x+3-3 &= 11-3 \\ \frac{2}{5}x &= 8 \\ \frac{5}{2}\left(\frac{2}{5}\right)x &= \frac{5}{2}(8) \\ x &= 20 \end{aligned}$$

Construct a viable argument to justify a solution method.

- Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions.

- Challenge students to justify each step of solving an equation. Transforming $2x-5=7$ to $2x=12$ is possible because 5 = 5, so adding the same quantity to both sides of an equation makes the resulting equation true as well. Each step of solving an equation can be defended, much like providing evidence for steps of a geometric proof.

Standard	**Cluster: Solve equations and inequalities in one variable**
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Solve: • $\frac{7}{3}y - 8 = 111$ • $3x > 9$ • Solve $ax + 7 = 12$ for *x.* • $\frac{3+x}{7} = \frac{x-9}{4}$ • $\frac{2}{3}x + 9 < 18$ • Solve $(y - y_1) = m(x - x_1)$ for *m*.

Standard	**Cluster: Solve systems of equations**
A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	The focus of this standard is to provide mathematics justification for the addition (elimination) method of solving systems of equations ultimately transforming a given system of two equations into a simpler equivalent system that has the same solutions as the original system. Build on student experiences in graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5, which requires students to prove the slope criteria for parallel lines. ***Example(s):*** Use the system $\begin{cases} 2x + y = 13 \\ x + y = 10 \end{cases}$ to explore what happens graphically with different combinations of the linear equations. a. Graph the original system of linear equations. Describe the solution of the system and how it relates to the solutions of each individual equation. (*Note: connect to A-REI.10)* b. Add the two linear equations together and graph the resulting equation. Describe the solutions to the new equation and how they relate to the system's solution. c. Explore what happens with other combinations such as: i. Multiply the first equation by 2 and add to the second equation ii. Multiply the second equation by -2 and add to the first equation iii. Multiply the second equation by -1 and add to the first equation iv. Multiply the first equation by -1 and add to the second equation d. Are there any combinations that are more informative than others?

	e. Make a conjecture about the solution to a system and any combination of the equations. ***Example:*** Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that the two numbers, x and y, satisfy the equations $x + y = 10$ and $x - y = 4$.
A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of linear equations. ***Example:*** José had 4 times as many trading cards as Philippe. After José gave away 50 cards to his little brother and Philippe gave 5 cards to his friend for his birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system. ***Example:*** A restaurant serves a vegetarian and a chicken lunch special each day. Each vegetarian special is the same price. Each chicken special is the same price. However, the price of the vegetarian special is different from the price of the chicken special. • On Thursday, the restaurant collected $467 selling 21 vegetarian specials and 40 chicken specials. • On Friday, the restaurant collected $484 selling 28 vegetarian specials and 36 chicken specials. What is the cost of each lunch special? ***Example:*** Solve the system of equations: $x + y = 11$ and $3x - y = 5$. Use a second method to check your answer.

Standard	**Cluster: Represent and solve equations and inequalities graphically**
A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	This standard is included in Math I, II and III. In Math I, students focus on linear and exponential equations and are able to adapt and apply that learning to other types of equations in future courses. Students can explain and verify that every point (x, y) on the graph of an equation represents all values for x and y that make the equation true. ***Example:*** Which of the following points are on the graph of the equation $-5x + 2y = 20$? How many points are on this graph? Explain. a. (4, 0) b. (0, 10) c. (-1, 7.5) d. (2.3, 5) ***Example:*** Verify that (-1, 60) is a solution to the equation $y = 15(\frac{1}{4})^x$. Explain what this means for the graph of the function.

A.REI.11★ Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students use graphs, tables and algebraic methods (including substitution and elimination for systems) to solve equations and systems of equations. Students understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graphs, or solve a variety of functions. In Math I, focus on linear and exponential functions. ***Example:*** The functions $f(m) = 18 + 0.4m$ and $g(m) = 11.2 + 0.54m$ give the lengths of two different springs in centimeters, as mass is added in grams, m, to each separately. a. Graph each equation on the same set of axes. b. What mass makes the springs the same length? c. What is the length at that mass? d. Write a sentence comparing the two springs. ***Example:*** Solve the following equations by graphing. Give your answer to the nearest tenth. a. $3(2^x) = 6x - 7$ b. $10x + 5 = -x + 8$
A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	In Math I, students graph linear inequalities and systems of linear inequalities. **Graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary for non-inclusive inequalities.** ***Example:*** Graph the following inequalities: $3x - 4y \leq 7$ $y > -2x + 6$ $-9x + 4y \geq 1$ **Students understand that the solutions to a system of inequalities in two-variables are the points that lie in the intersection of the corresponding half-planes.** ***Example:*** Compare the solution to a system of equations to the solutions of a system of inequalities. ***Example:*** Describe the solution set of a system of inequalities. **Graph the solution set to a system of linear inequalities in two variables as the intersection of their corresponding half-planes.**

Example: Graph the solution set for the following system of inequalities:

$$3x + 5y \leq 10$$
$$y > -4$$

Example: Graph the system of linear inequalities below and determine if (3, 2) is a solution to the system.

$$x - 3y > 0$$
$$x + y \leq 2$$
$$x + 3y > -3$$

Connect to A-CED.2 and A-CED.3

Standard

Cluster: Understand the concept of a function and use function notation.

F.IF.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f\ (x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f\ (x)$.

Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of functions should NOT occur at this level. While F.IF.1 is not repeated in any other course, students will apply these concepts throughout their future mathematics courses.

In Math I, draw examples from linear, quadratic, and exponential functions.

Students use the definition of a function to determine whether a relationship is a function given a table, graph or words.

Example: Determine which of the following tables represent a function and explain why.

Table A	
x	*y*
0	1
1	2
2	2
3	4

Table B	
r	*T*
0	0
1	2
1	3
4	5

Given the function $f\ (x)$, students explain that input values are guaranteed to produce unique output values and use the function rule to generate a table or graph. They identify *x* as an element of the domain, the input, and $f\ (x)$ as an element in the range, the output. The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain.

Example: A pack of pencils cost \$0.75. If *n* number of packs are purchased then the total purchase price is represented by the function $t\ (n)\ =\ 0.75n$.

a. Explain why *t* is a function.
b. What is a reasonable domain and range for the function *t*?

Students recognize that the graph of the function, *f*, is the graph of the equation *y* = *f* (*x*) and that (*x*, *f* (*x*)) is a point on the graph of *f*.

F.IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

This standard is included in Math I, II and III. In Math I, students should focus on linear and exponential functions and be able to extend and apply their reasoning to other types of functions in future courses.

Use function notation, evaluate functions for inputs in their domains

Example: Evaluate $f(2)$ for the function $f(x) = 5(x - 3) + 17$.

Example: Evaluate $f(2)$ for the function $f(x) = 1200(1 + .04)^x$

Interpret statements that use function notation in terms of a context

You placed a ham in the oven and, after 45 minutes, you take it out. Let f be the function that assigns to each minute after you placed the yam in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the following to explain what it means in everyday language.

a. $f(0) = 65$
b. $f(5) < f(10)$
c. $f(40) = f(45)$
d. $f(45) > f(60)$

Example: The rule $f(x) = 50(0.85)^x$represents the amount of a drug in milligrams, $f(x)$, which remains in the bloodstream after x hours. Evaluate and interpret each of the following:

a. $f(0)$
b. $f(2) = k \cdot f(1)$. What is the value of k?
c. $f(x) < 6$

F.IF.3

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

A sequence can be described as a function, with the input numbers consisting of a subset of the integers, and the output numbers being the terms of the sequence. The most common subset for the domain of a sequence is the Natural numbers {1, 2, 3, …}; however, there are instances where it is necessary to include {0} or possibly negative integers.

Whereas, some sequences can be expressed explicitly, there are those that are a function of the previous terms. In which case, it is necessary to provide the first few terms to establish the relationship.

Example: The sequence 1, 1, 2, 3, 5, 8, 13, 21,… has an initial term of 1 and subsequent terms. The first 7 terms are displayed. Since each term is unique to its position in the sequence then, by definition, the sequence is a function. Explore the sequence and evaluate each of the following:

a. $f(0)$
b. $f(3)$
c. $f(10)$

Example: A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern.

a. If the theater has 20 rows of seats, how many seats are in the twentieth row?
b. Explain why the sequence is considered a function.
c. What is the domain of the sequence? Explain what the domain represents in context.

Connect to arithmetic and geometric sequences (F-BF.2). Emphasize that arithmetic and geometric sequences are examples of linear and exponential functions, respectively.

Standard

Cluster: Interpret functions that arise in applications in terms of the context.

F.IF.4★

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is included in Math I, II and III. Throughout all three courses, students interpret the key features of a variety of different functions. There are two categories of functions to consider. The first are the "classical" functions usually referred to by families.

In Math I, students focus on linear, exponential, and quadratic functions. *(Connect to F.IF.7)*

There are also functions that are not included into families. For example, plots over time represent functions as do some scatterplots. These are often functions that "tell a story" hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables and graphs.

When given a table or graph of a function that models a real-life situation, explain the meaning of the characteristics of the table or graph in the context of the problem.

The following are examples of "classical" family of functions:

Key features of a linear function are slope and intercepts.

> ***Example:*** The local newspaper charges for advertisements in their community section. A customer has called to ask about the charges. The newspaper gives the first 50 words for free and then charges a fee per word. Use the table at the right to describe how the newspaper charges for the ads. Include all important information.

# of words	Cost to place ad ($)
50	0
60	0.50
70	1
80	1.50
90	2
100	2.5

Key features of a quadratic function are intervals of increase/decrease, positive/negative, maximum/minimum, symmetry, and intercepts.

Example: The table represents the relationship between daily profit for an amusement park and the number of paying visitors.

a. What are the x-intercepts and y-intercepts and explain them in the context of the problem.
b. Identify any maximums or minimums and explain their meaning in the context of the problem.
c. Determine if the graph is symmetrical and identify which shape this pattern of change develops.
d. Describe the intervals of increase and decrease and explain them in the context of the problem.

# paying visitors (thousands)	Daily profit (thousands of $)
0	0
1	5
2	8
3	9
4	8
5	5
6	0

Example: The graph represents the height (in feet) of a rocket as a function of the time (in seconds) since it was launched. Use the graph to answer the following:

a. What is the practical domain for *t* in this context? Why?
b. What is the height of the rocket two seconds after it was launched?
c. What is the maximum value of the function and what does it mean in context?
d. When is the rocket 100 feet above the ground?
e. When is the rocket 250 feet above the ground?
f. Why are there two answers to part e but only one practical answer for part d?
g. What are the intercepts of this function? What do they mean in the context of this problem?
h. What are the intervals of increase and decrease on the practical domain? What do they mean in the context of the problem?

Key features of an exponential function include y-intercept and increasing/decreasing.

Example: Jack planted a mysterious bean just outside his kitchen window. Jack kept a table (shown below) of the plant's growth. He measured the height at 8:00 am each day.

Day	0	1	2	3	4
Height (cm)	2.56	6.4	16	40	100

a. What was the initial height of Jack's plant?
b. How is the height changing each day?
c. If this pattern continues, how tall should Jack's plant be after 8 days?

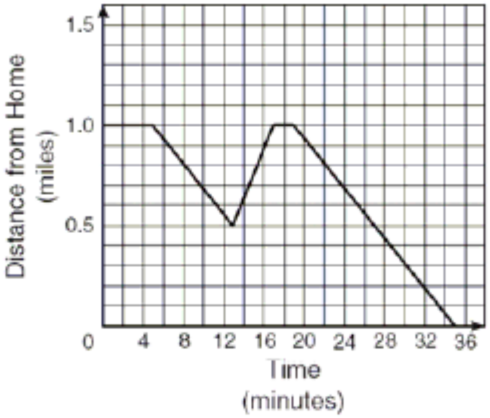
The following are examples of functions that “tell a story”:

There are a number of videos on this site http://graphingstories.com Some are aligned to Math I while others are more appropriate for Math 2 or 3. The following are suggested videos for Math I:

- Water Volume
- Weight
- Bum Height Off Ground
- Air Pressure
- Height of Stack

Example: Marla was at the zoo with her mom. When they stopped to view the lions, Marla ran away from the lion exhibit, stopped, and walked slowly towards the lion exhibit until she was halfway, stood still for a minute then walked away with her mom. Sketch a graph of Marla’s distance from the lions’ exhibit over the period of time when she arrived until she left.

Example: Describe a situation, in detail, that could be represented by the graph at the right.



F.IF.5★

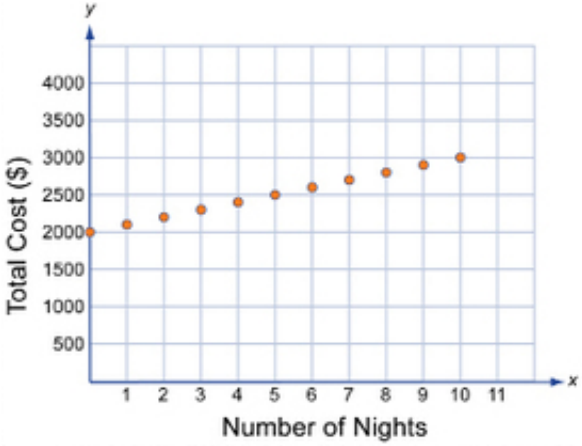
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*[*]

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is included in Math I, II and III. Throughout all three courses, students relate the domain of a function to its graph and, where applicable, the quantitative relationship it describes.

In Math I, focus on linear and exponential.

Example: An all-inclusive resort in Los Cabos, Mexico provides everything for their customers during their stay including food, lodging, and transportation. Use the graph at the right to describe the domain of the total cost function.



Example: Jennifer purchased a cell phone and the plan she decided upon charged her $50 for the phone and $0.10 for each minute she is on the phone. (The wireless carrier rounds up to the half minute.) She has budgeted $100 for her phone bill. What would be the appropriate domain for the cost as a function of the total minutes she used the phone? Describe what the point (10,51) represents in the problem.

Example: Oakland Coliseum, home of the Oakland Raiders, is capable of seating 63,026 fans. For each game, the amount of money that the Raiders' organization brings in as revenue is a function of the number of people, *n*, in attendance. If each ticket costs $30, find the domain of this function.

***Sample Response*:** Let r represent the revenue that the Raider's organization makes, so that $r = f(n)$. Since n represents a number of people, it must be a nonnegative whole number. Therefore, since 63,026 is the maximum number of people who can attend a game, we can describe the domain of *f* as follows: Domain = {n: $0 \leq n \leq$ 63,026 and n is an integer}. (Students should be able to understand and express using this notation.)

The deceptively simple task above asks students to find the domain of a function from a given context. The function is linear and if simply looked at from a formulaic point of view, students might find the formula for the line and say that the domain and range are all real numbers. However, in the context of this problem, this answer does not make sense, as the context requires that all input and output values are non-negative integers, and imposes additional restrictions.

F.IF.6★

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

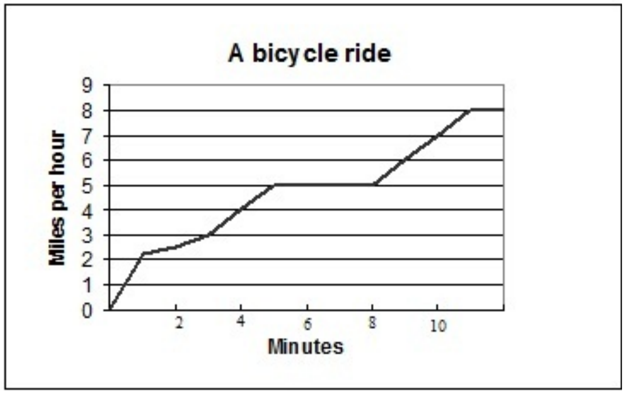
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

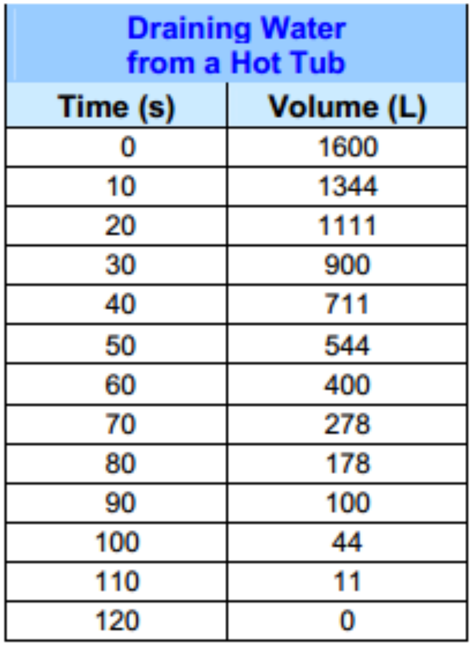
In Math I, focus on the average rate of change for linear and exponential functions whose domain is a subset of the integers when given symbolically.

Students calculate the average rate of change of a function given a graph, table, and/or equation.

- The average rate of change of a function $y = f(x)$ over an interval [a, b] is $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$.

Example: What is the average rate at which this bicycle rider traveled from four to ten minutes of her ride?





Draining Water from a Hot Tub	
Time (s)	Volume (L)
0	1600
10	1344
20	1111
30	900
40	711
50	544
60	400
70	278
80	178
90	100
100	44
110	11
120	0

Example: The plug is pulled in a small hot tub. The table gives the volume of water in the tub from the moment the plug is pulled, until it is empty. What is the average rate of change between:

- 60 seconds and 100 seconds?
- 0 seconds and 120 seconds?
- 70 seconds and 110 seconds?

Standard	**Cluster: Analyze functions using different representations.**
F.IF.7★ Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. F.IF.7e is included in Math I, II and III. In Math I, students graph linear, exponential and quadratic functions expressed symbolically. *(Connect to F.IF.4)* **Students recognize how the form of the linear function provides information about key features such as slope and *y*-intercept. Also, the *x*-intercept can be revealed by determining the input value *x* such that $f(x) = 0$.** ***Example:*** Describe the key features of the graph $f(x) = \frac{-2}{3}x + 8$ and use the key features to create a sketch of the function. **Students recognize how the form of the quadratic function provides information about key features such as intercepts and maximum/minimum values.** ***Example:*** Without using the graphing capabilities of a calculator, sketch the graph of $f(x) = x^2 + 7x + 10$ and identify the x-intercepts, y-intercept, and the maximum or minimum point. *Note: Students can identify the max/min point by using the symmetry of the graph. If the x-intercepts are at* $x = 2$ *and* $x = 5$ *then the minimum point is when* $x = 3.5$*. Using the function, students can use* $x = 3.5$ *as the input and calculate the output to get the maximum point.* **Students recognize how the form of the exponential function provides information about key features such as initial values and rates of growth or decay.** ***Example:*** The function $f(x) = 300(0.70)^x - 25$ models the amount of aspirin left in the bloodstream after *x* hours. Graph the function showing the key features of the graph. Interpret the key features in context of the problem.

F.IF.8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

F.IF.8a is included in Math I, II and III.

Students use the property of exponents to rewrite exponential expressions in different yet equivalent forms. Students use the form to explain the special characteristics of the function.

In Math I, focus on factoring as a process to show zeroes, extreme values, and symmetry of the graph. Students should also interpret these features in terms of a context. At this level, completing the square is *not* expected. Students should be prepared to factor quadratics in which the coefficient of the quadratic term is an integer that may or may not be the GCF of the expression.

Students must use the factors to reveal and explain properties of the function, interpreting them in context. Factoring just to factor does not fully address this standard.

> ***Example:*** The quadratic expression $-5x^2 + 10x + 15$ represents the height of a diver jumping into a pool off a platform. Use the process of factoring to determine key properties of the expression and interpret them in the context of the problem.

Connect to F.IF.7a when students are graphing the functions to show key features and F.IF.4 when they are interpreting the key features given a graph or a table.

Students can determine if an exponential function models growth or decay. Students can also identify and interpret the growth or decay factor.

Students can rewrite an expression in the form $a(b)^{kx}$ as $a(b^k)^x$. They can identify b^k as the growth or decay factor.

Students recognize that when the factor is greater than 1, the function models growth and when the factor is between 0 and 1 the function models decay.

> ***Example:*** The projected population of Delroysville is given by the function $p(t) = 1500(1.08)^{2t}$ where t is the number of years since 2010. You have been selected by the city council to help them plan for future growth. Explain what the function $p(t) = 1500(1.08)^{2t}$ means to the city council members.

> ***Example:*** Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 2 every hour. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of days.

> ***Example:*** The expression $50(0.85)^x$ represents the amount of a drug in milligrams that remains in the bloodstream after x hours.
> a. Describe how the amount of drug in milligrams changes over time.
> b. What would the expression $50(0.85)^{12x}$ represent?
> c. What new or different information is revealed by the changed expression?

F.IF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*

This standard is included in Math I, II and III. Throughout all three courses, students compare properties of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

In Math I, students focus on linear, exponential, and quadratic.

Example: David and Fred each throw a baseball into the air. The velocity of David's ball is given by $v(t) = 50 - 32t$ where v is in feet per second and t is in seconds. The velocity of Fred's ball is given in the table.

t	$v(t)$
0.2	38.6
0.4	32.2
0.6	25.8
0.8	19.4
1.0	13

What is the difference in the initial velocity between the two throws?

Example: Examine the two functions represented below. Compare the x-intercepts and find the difference between the minimum values.

$$f(x) = x^2 + 8x + 15$$

x	-7	-6	-5	-4	-3	-2	-1
$g(x)$	4	1.5	0	-0.5	0	1.5	4

Example: Joe is trying to decide which job would allow him to earn the most money after a few years.

- His first job offer agrees to pay him $50 per week. If he does a good job, they will give him a 2% raise each year.
- His other job offer agrees to pay him $40 per week and if he does a good job, they will give him a 3% raise each year.

Which job would you suggest Joe take? Justify your reasoning.

Standard	**Cluster: Build a function that models a relationship between two quantities.**

F.BF.1★

Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is included in Math I, II and III. Throughout all three courses, students write functions that describe relationships. Students will

- Analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change.
- Specify intervals of increase, decrease, constancy, and relate them to the function's description in words or graphically

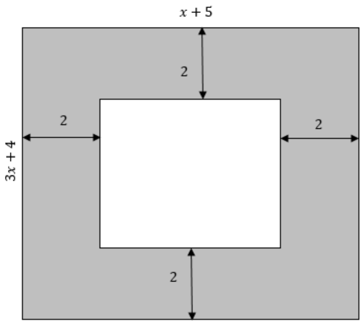
In Math I, focus on linear, exponential, and quadratic.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Example: Provide a table showing how far a car has driven after a given number of minutes, traveling at a uniform speed. Students examine the table by analyzing the outputs to describe a recursive relationship, as well as analyzing input, output pairs to determine an explicit function that represents the distance traveled when the number of minutes is known.

From context, write an explicit expression or describe the calculations needed to model a function between two quantities.

Example: A carpet company uses the template to the right to design rugs. Write an expression for the area of the shaded region as it relates to the variable length *x*.



Determine an explicit expression, a recursive process, or steps for calculation from a context.

Example: A movie theater has to pay 8% in monthly taxes on its profit. The profit from tickets and the concession stand is \$7.75 per person. The theater has expenses of \$15,000 per month. Build a function to represent the amount the theater must pay in taxes each month as a function of the number of people attending the movies.

Example: A single bacterium is placed in a test tube and splits in two after one minute. After two minutes, the resulting two bacteria split in two, creating four bacteria. This process continues.

a. How many bacteria are in the test tube after 5 minutes? 15 minutes?
b. Write a recursive rule to find the number of bacteria in the test tube from one minute to the next.
c. Convert this rule into explicit form. How many bacteria are in the test tube after one hour?

Combine standard function types using arithmetic operations.

At this level, a constant may be combined to linear, quadratic or exponential functions and a linear function may be combined to a linear or quadratic function.

Example: A ball is thrown straight up into the air starting at 2 meters with an initial upward velocity of 20 meters per second. Write a function rule that relates the height *(in meters)* of the ball and the time *(in seconds).* Note that gravity has an effect on the height with the distance fallen represented by $4.9t^2$ where *t* is time in seconds.

Example: A retail store has two options for discounting items to go on clearance.

- Option 1: Decrease the price of the item by 15% each week.
- Option 2: Decrease the price of the item by \$5 each week.

If the cost of an item is \$45, write a function rule for the difference in price between the two options.

Example: Blake has a monthly car payment of \$225. He has estimated an average cost of \$0.32 per mile for gas and maintenance. He plans to budget for the car payment the minimal he needs with an additional 3% of his total budget for incidentals that may occur. Build a function that gives the amount Blake needs to budget as a function of the number of miles driven.

F.BF.2★

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is included in Math I and 3.

In Math I, students use informal recursive notation (such as $NEXT = NOW + 5\ starting\ at$ 3 is intended). NOW – NEXT equations are the first step in the process of formalizing a sequence using function notation. The NOW – NEXT representation allows students to explore and understand the concept of a recursive function before being introduced to symbolic notation in Math 3 such as $A_n = A_n - 1 + 6$ or $f(n) = f(n-1) + 6$. *(Connect to F.IF.3)*

In an ***arithmetic sequence***, each term is obtained from the previous term by adding the same number each time. This number is called the **common difference**. In a ***geometric sequence***, each term is obtained from the previous term by multiplying by a constant amount, called the **common ratio.**

Example: Given the following sequences, write a recursive rule:

a. 2, 4, 6, 8, 10, 12, 14, …
b. 1000, 500, 250, 125, 62.5, …

NOW – NEXT equations are informal recursive equations that show how to calculate the value of the next term in a sequence from the value of the current term.

The ***arithmetic sequence*** represented by $NEXT = NOW \pm c\ starting\ at\ d$ is the recursive form of a linear function. The common difference *c* corresponds to the slope *m* in the explicit form of a linear function, $y = mx + b$. The initial value of the sequence *d* may or may not correspond to the *y*-intercept, *b*. This varies based on the given information.

Example: If the sequence is 37, 32, 27, 22, 17, … it can be written as 37, 37-5, 37-5-5, 37-5-5-5, 37-5-5-5-5, etc., so that students recognize an initial value of 37 and a -5 as the common difference.

The ***geometric sequence*** represented by $NEXT = NOW \times b$ is the recursive form of an exponential function. The common ratio *b* corresponds to the base *b* in the explicit form of an exponential function, $y = ab^x$. The initial value may or may not correspond to the *y*-intercept, *a*. This varies based on the given information.

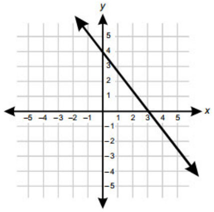
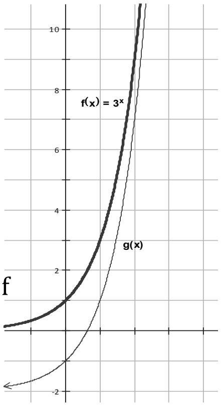
Example: If the sequence is 3, 6, 12, 24,… it can be written as 3,3(2), 3(2)(2), 3(2)(2)(2), etc., so that students recognize an initial value of 3 and 2 is being used as the common ratio.

Note: *Write out terms in a table in an expanded form to help students see what is happening.*

Translate between the two forms

Example: A concert hall has 58 seats in Row 1, 62 seats in Row 2, 66 seats in Row 3, and so on. The concert hall has 34 rows of seats.

a. Write a recursive formula to find the number of seats in each row. How many seats are in row 5?
b. Write the explicit formula to determine which row has 94 seats?

Standard	Cluster: Build new functions from existing functions.
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x+k)$, $k\ f(x)$, $f(kx)$, and $f(x\ +\ k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	This standard is included in Math I, II and III. The standard addresses specific functions and specific effects throughout each course. In Math I, focus on vertical and horizontal translations of linear and exponential functions.  ***Example:*** Describe how $f(x)+3$ compares to $f(x)$ represented in the graph. ***Example:*** Given the graph of $f(x)$ whose x-intercept is 3, find the value of k if $f(x+k)$ resulted in the graph having an x-intercept of -4. ***Example:*** Describe how the graph of $f(x)+k$ compares to $f(x)$ if k positive. If k is negative. ***Example:*** The graph of $f(x)=3^x$ is shown at right. The graph of $g(x)=f(x)+k$. Find the value of k. 

Standard	Cluster: Construct and compare linear, quadratic, and exponential models and solve problems.
F.LE.1★ Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students distinguish between a constant rate of change and a constant percent rate of change.** ***Example:*** Town A adds 10 people per year to its population, and town B grows by 10% each year. In 2006, each town has 145 residents. For each town, determine whether the population growth is linear or exponential. Explain. ***Example:*** Sketch and analyze the graphs of the following two situations. What information can you conclude about the types of growth each type of interest has?

intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

- Lee borrows \$9,000 from his mother to buy a car. His mom charges him 5% interest a year, but she does not compound the interest.
- Lee borrows \$9,000 from a bank to buy a car. The bank charges 5% interest compounded annually.

Students recognize situations where one quantity changes at a constant rate per unit interval relative to another.

Example: A streaming movie service has three monthly plans to rent movies online. Graph the equation of each plan and analyze the change as the number of rentals increase. When is it beneficial to enroll in each of the plans?

Basic Plan: \$3 per movie rental
Watchers Plan: \$7 fee + \$2 per movie with the first two movies included with the fee
Home Theater Plan: \$12 fee + \$1 per movie with the first four movies included with the fee

Note: The intent of this example is not to formally introduce piecewise functions, but to provide a realistic context to compare different rates.

Students recognize situations where one quantity changes as another changes by a constant percent rate.

- When working with symbolic form of the relationship, if the equation can be rewritten in the form $y = a(1 \pm r)^x$, then the relationship is exponential and the constant percent rate per unit interval is r.
- When working with a table or graph, either write the corresponding equation and see if it is exponential or locate at least two pairs of points and calculate the percent rate of change for each set of points. If these percent rates are the same, the function is exponential. If the percent rates are not all the same, the function is not exponential.

Example: A couple wants to buy a house in five years. They need to save a down payment of \$8,000. They deposit \$1,000 in a bank account earning 3.25% interest, compounded quarterly. How long will they need to save in order to meet their goal?

Example: Carbon 14 is a common form of carbon which decays exponentially over time. The half-life of Carbon 14, that is the amount of time it takes for half of any amount of Carbon 14 to decay, is approximately 5730 years. Suppose we have a plant fossil and that the plant, at the time it died, contained 10 micrograms of Carbon 14 (one microgram is equal to one millionth of a gram).

a. Using this information, make a table to calculate how much Carbon 14 remains in the fossilized plant after n number of half-lives.
b. How much carbon remains in the fossilized plant after 2865 years? Explain how you know.
c. When is there one microgram of Carbon 14 remaining in the fossil?

F.LE.2 ★

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

In Math I, students construct linear and exponential functions.

Students use graphs, a verbal description, or two-points (which can be obtained from a table) to construct linear or exponential functions.

Example: Albuquerque boasts one of the longest aerial trams in the world. The tram transports people up to Sandia Peak. The table shows the elevation of the tram at various times during a particular ride.

Minutes into the ride	2	5	9	14
Elevation in feet	7069	7834	8854	10,129

a. Write an equation for a function that models the relationship between the elevation of the tram and the number of minutes into the ride.
b. What was the elevation of the tram at the beginning of the ride?
c. If the ride took 15 minutes, what was the elevation of the tram at the end of the ride?

Example: After a record setting winter storm, there are 10 inches of snow on the ground! Now that the sun is finally out, the snow is melting. At 7 am there were 10 inches and at 12 pm there were 6 inches of snow.
a. Construct a linear function rule to model the amount of snow.
b. Construct an exponential function rule to model the amount of snow.
c. Which model best describes the amount of snow? Provide reasoning for your choice.

Note: In order to write the exponential function as the amount of snow for every hour, connect to F.IF.8b. Students could start with $10(.6)^x$ *where x is the number of 5 hour periods then rewrite it to be* $10(.6)^{(\frac{1}{5}x)} = 10(.6^{\frac{1}{5}})^x \approx 10(.9)^x$ *where x is the number of hours since 7 am.*

Students need to be able to construct exponential functions from a table or two points that may or may not include 0 as an input value.

Also Connect to F.LE.1, F.BF.1, and F.BF.2

F.LE.3 ★

Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

This standard is included in Math I and Math III.

In Math I, focus comparisons between linear, exponential, and quadratic growth.

Students know that the value of the exponential function eventually is greater than the other function types.

> ***Example:*** Kevin and Joseph each decide to invest \$100. Kevin decides to invest in an account that will earn \$5 every month. Joseph decided to invest in an account that will earn 3% interest every month.
>
> a. Whose account will have more money in it after two years?
> b. After how many months will the accounts have the same amount of money in them?
> c. Describe what happens as the money is left in the accounts for longer periods of time.
>
> ***Example:*** Compare the values of the functions $f(x) = 2x$, $f(x) = 2^x$, and $f(x) = x^2$ for $x \geq 0$.

Standard

Cluster: Interpret expressions for functions in terms of the situation they model.

F.LE.5 ★

Interpret the parameters in a linear or exponential function in terms of a context.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Use real-world situations to help students understand how the parameters of linear and exponential functions depend on the context.

> ***Example:*** A plumber who charges \$50 for a house call and \$85 per hour can be expressed as the function $y = 85x + 50$. If the rate were raised to \$90 per hour, how would the function change?
>
> ***Example:*** Lauren keeps records of the distances she travels in a taxi and what it costs:

Distance *d* in miles	**Fare *f* in dollars**
3	8.25
5	12.75
11	26.25

> a. If you graph the ordered pairs (d, f) from the table, they lie on a line. How can this be determined without graphing them?
> b. Show that the linear function in part a. has equation $F = 2.25d + 1.5$.
> c. What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides.

	Example: A function of the form $f(n) = P(1 + r)^n$ is used to model the amount of money in a savings account that earns 8% interest, compounded annually, where n is the number of years since the initial deposit. a. What is the value of *r*? Interpret what *r* means in terms of the savings account? b. What is the meaning of the constant P in terms of the savings account? Explain your reasoning. c. Will n or $f(n)$ ever take on the value 0? Why or why not? ***Example:*** The equation $y = 8{,}000(1.04)^x$ models the rising population of a city with 8,000 residents when the annual growth rate is 4%. a. What would be the effect on the equation if the city's population were 12,000 instead of 8,000? b. What would happen to the population over 25 years if the growth rate were 6% instead of 4%?

Standard	Cluster: Experiment with transformations in the plane.
G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	This standard is in Math I and III. Students recognize the importance of having precise definitions and use the vocabulary to accurately describe figures and relationships among figures. **Students define angles, circles, perpendicular lines, parallel lines, and line segments precisely using the undefined terms.** ***Example:*** Draw an example of each of the following and justify how it meets the definition of the term. • Angle • Circle • Perpendicular lines • Parallel lines • Line segment

Standard	Cluster: Use coordinates to prove simple geometric theorems algebraically.
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* $(1, \sqrt{3})$ *lies on the circle centered at the origin and containing the point* $(0, 2)$.	Students determine the slope of a line and the length of a line segment using a variety of methods including counting intervals and using formulas. Students have prior experience with the slope formula. Students need to develop the distance formula as an application of the Pythagorean Theorem. **Use the concepts of slope and distance to prove that a figure in the coordinate system is a special geometric shape. Students recognize the length of the line segment is the same as the distance between the end points.** ***Example:*** The coordinates for the vertices of quadrilateral MNPQ are $M(3, 0)$, $N(1, 3)$, $P(-2, 1)$, and $Q(0, -2)$. a. Classify quadrilateral MNPQ. b. Identify the properties used to determine your classification c. Use slope and length to provide supporting evidence of the properties

	Example: If quadrilateral ABCD is a rectangle, where $A(1,2)$, $B(6,0)$, $C(10,10)$ and $D(?\,,?\,)$ is unknown. a. Find the coordinates of the fourth vertex. b. Verify that ABCD is a rectangle providing evidence related to the sides and angles. *Conics is not the focus at this level, therefore students do not need to know about circles on the coordinate plane.* Connect to G.GPE.5 to understand how slope can be used to determine parallel and perpendicular lines.
G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	Use the formula for the slope of a line to determine whether two lines are parallel or perpendicular. Two lines are parallel if they have the same slope and two lines are perpendicular if their slopes are opposite reciprocals of each other. In other words, the product of the slopes of lines that are perpendicular is (-1). ***Example:*** Suppose a line k in a coordinate plane has slope $\frac{c}{d}$. a. What is the slope of a line parallel to k? Why must this be the case? b. What is the slope of a line perpendicular to k? Why does this seem reasonable? **Find the equations of lines that are parallel or perpendicular given certain criteria.** ***Example:*** Two points $A(0,-4)$, $B(2,-1)$ determines a line, $\overleftrightarrow{AB}$. a. What is the equation of the line AB? b. What is the equation of the line perpendicular to $\overleftrightarrow{AB}$. passing through the point $(2,-1)$? Connect to F.LE.2.
G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	This standard is in Math I and II. In Math I, students find the midpoint of a line segment. The midpoint partitions the ratio into 1:1 and thus from either direction the point is the same. Given two points on a line, find the point that divides the segment into an equal number of parts. The midpoint is always halfway between the two endpoints. The *x*-coordinate of the midpoint will be the mean of the *x*-coordinates of the endpoints and the *y*-coordinate will be the mean of the *y*-coordinates of the endpoints. ***Example:*** If you are given the midpoint of a segment and one endpoint. Find the other endpoint. a. midpoint: $(6,2)$ endpoint: $(1,3)$ b. midpoint: $(-1,-2)$ endpoint: $(3.5,-7)$ ***Example:*** Jennifer and Jane are best friends. They placed a map of their town on a coordinate grid and found the point at which each of their house lies. If Jennifer's house lies at (9, 7) and Jane's house is at (15, 9) and they wanted to meet in the middle, what are the coordinates of the place they should meet?

G.GPE.7

Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard provides practice with the distance formula and its connection with the Pythagorean Theorem. Use the coordinates of the vertices of a polygon graphed in the coordinate plane and use the distance formula to compute the perimeter and to find lengths necessary to compute the area.

Example: Calculate the area of triangle ABC with altitude CD, given A (-4,-2), B(8,7), C(1, 8) and D(4, 4).

Example: Find the perimeter and area of a rectangle with vertices at C (-1, 1), D(3,4), E(6, 0), F (2, -3). Round your answer to the nearest hundredth.

Standard	Cluster: Explain volume formulas and use them to solve problems.

G.GMD.1

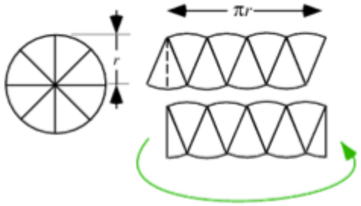
Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

Circumference of a circle:

Students begin with the measure of the diameter of the circle or the radius of the circle. They can use string or pipe cleaner to represent the measurement. Next, students measure the distance around the circle using the measure of the diameter. They discover that there are 3 diameters around the circumference with a small gap remaining. Through discussion, students conjecture that the circumference is the length of the diameter π times. Therefore, the circumference can be written as $C = \pi d$. When measuring the circle using the radius, students discover there are 6 radii around the circumference with a small gap remaining. Students conjecture that the circumference is the length of the radius 2π times. Therefore, the circumference of the circle can also be expressed using $C = 2\pi r$.

Area of a circle:

Students may use dissection arguments for the area of a circle. Dissect portions of the circle like pieces of a pie and arrange the pieces into a figure resembling a parallelogram as indicated below. Reason that the base is half of the circumference and the height is the radius. Students use the formula for the area of a parallelogram to derive the area of the circle.



A_{rect} = Base x Height
Area = ½ $(2\pi r)$ x r
Area = πr x r
Area = πr^2

http://mathworld.wolfram.com/Circle.html

Volume of a cylinder:

Students develop the formula for the volume of a cylinder based on the area of a circle stacked over and over again until the cylinder has the given height. Therefore the formula for the volume of a cylinder is $V = Bh$. This approach is similar to Cavalieri's principle. In Cavalieri's principle, the cross-sections of the cylinder are circles of equal area, which stack to a specific height.

Volume of a pyramid or cone:

For pyramids and cones, the factor $\frac{1}{3}$ will need some explanation. An informal demonstration can be done using a volume relationship set of plastic shapes that permit one to pour liquid or sand from one shape into another. Another way to do this for pyramids is with Geoblocks. The set includes three pyramids with equal bases and altitudes that will stack to form a cube. An algebraic approach involves the formula for the sum of squares ($1^2 + 2^2 + \cdots + n^2$). After the coefficient $\frac{1}{3}$ has been justified for the formula of the volume of the pyramid ($A = \frac{1}{3}Bh$), one can argue that it must also apply to the formula of the volume of the cone by considering a cone to be a pyramid that has a base with infinitely many sides.

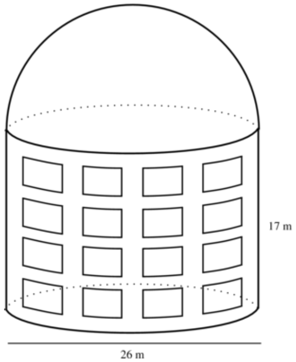
Informal limit arguments are not the intent at this level.

G.GMD.3

Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Example: The Southern African Large Telescope (SALT) is housed in a cylindrical building with a domed roof in the shape of a hemisphere. The height of the building wall is 17 m and the diameter is 26 m. To program the ventilation system for heat, air conditioning, and dehumidifying, the engineers need the amount of air in the building. What is the volume of air in the building?



Formulas for pyramids, cones, and spheres will be given.

Standard	**Cluster: Summarize, represent, and interpret data on a single count or measurement variable.**
S.ID.1 ★ Represent data with plots on the real number line (dot plots, histograms, and box plots).	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **A statistical process is a problem-solving process consisting of four steps:** 1. Formulating a statistical question that anticipates variability and can be answered by data. 2. Designing and implementing a plan that collects appropriate data. 3. Analyzing the data by graphical and/or numerical methods. 4. Interpreting the analysis in the context of the original question. The four-step statistical process was introduced in Grade 6, with the recognition of statistical questions. In middle school students describe center and spread in a data distribution. In Math I, students need to become proficient in the first step of generating meaningful questions and choose a summary statistic appropriate to the characteristic of the data distribution, such as the shape of the distribution or the existence of extreme data points. **Students will graph numerical data on a real number line using dot plots, histograms, and box plots.** • Analyze the strengths and weakness inherent in each type of plot by comparing different plots of the same data. • Describe and give a simple interpretation of a graphical representation of data. **Students should use appropriate tools strategically.** The use of a graphing calculator to represent the data should be utilized. Emphasis must be on analyzing the data display to make decisions, not on making the display itself. ***Example:*** The following data set shows the number of songs downloaded in one week by each student in Mrs. Jones class: 10, 20, 12, 14, 12, 27, 88, 2, 7, 30, 16, 16, 32, 25, 15, 4, 0, 15, 6. Choose and create a plot to represent the data

S.ID.2 ★

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Given two sets of data or two graphs, students

- Identify the similarities and differences in shape, center and spread
- Compare data sets and summarize the similarities and difference between the shape, and measures of center and spreads of the data sets
- Use the correct measure of center and spread to describe a distribution that is symmetric or skewed
- Identify outliers and their effects on data sets

The mean and standard deviation are most commonly used to describe sets of data. However, if the distribution is extremely skewed and/or has outliers, it is best to use the median and the interquartile range to describe the distribution since these measures are not sensitive to outliers.

Example: You are planning to take on a part time job as a waiter at a local restaurant. During your interview, the boss told you that their best waitress, Jenni, made an average of $70 a night in tips last week. However, when you asked Jenni about this, she said that she made an average of only $50 per night last week. She provides you with a copy of her nightly tip amounts from last week. Calculate the mean and the median tip amount.

a. Which value is Jenni's boss using to describe the average tip? Why do you think he chose this value?
b. Which value is Jenni using? Why do you think she chose this value?
c. Which value best describes the typical amount of tips per night? Explain why.

Day	Tip Amount
Sunday	$50
Monday	$45
Wednesday	$48
Friday	$125
Saturday	$85

Example: Delia wanted to find the best type of fertilizer for her tomato plants. She purchased three types of fertilizer and used each on a set of seedlings. After 10 days, she measured the heights (in cm) of each set of seedlings. The data she collected is shown below. Construct box plots to analyze the data. Write a brief description comparing the three types of fertilizer. Which fertilizer do you recommend that Delia use? Explain your answer.

Fertilizer A		
7.1	6.3	1.0
5.0	4.5	5.2
3.2	4.6	2.4
5.5	3.8	1.5
6.2	6.9	2.6

Fertilizer B		
11.0	9.2	5.6
8.4	7.2	12.1
10.5	14.0	15.3
6.3	8.7	11.3
17.0	13.5	14.2

Fertilizer C		
10.5	11.8	15.5
14.7	11.0	10.8
13.9	12.7	9.9
10.3	10.1	15.8
9.5	13.2	9.7

S.ID.3 ★ Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students understand and use the context of the data to explain why its distribution takes on a particular shape (e.g. Why is the data skewed? are there outliers?)** ***Examples:*** Why does the shape of the distribution of incomes for professional athletes tend to be skewed to the right? Why does the shape of the distribution of test scores on a really easy test tend to be skewed to the left? Why does the shape of the distribution of heights of the students at your school tend to be symmetrical? **Students understand that the higher the value of a measure of variability, the more spread out the data set is. Measures of variability are range (100% of data), standard deviation (68-95-99.7% of data), and interquartile range (50% of data).** ***Example:*** On last week's math test, Mrs. Smith's class had an average of 83 points with a standard deviation of 8 points. Mr. Tucker's class had an average of 78 points with a standard deviation of 4 points. Which class was more consistent with their test scores? How do you know? **Students explain the effect of any outliers on the shape, center, and spread of the data sets.** ***Example:*** The heights of Washington High School's basketball players are: 5 ft. 9in, 5 ft. 4in, 5 ft. 7 in, 5ft. 6 in, 5 ft. 5 in, 5 ft. 3 in, and 5 ft. 7 in. A student transfers to Washington High and joins the basketball team. Her height is 6 ft. 10in. a. What is the mean height of the team before the new player transfers in? What is the median height? b. What is the mean height after the new player transfers? What is the median height? c. What affect does her height have on the team's height distribution and stats (center and spread)? d. How many players are taller than the new mean team height? e. Which measure of center most accurately describes the team's average height? Explain.

Standard

Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

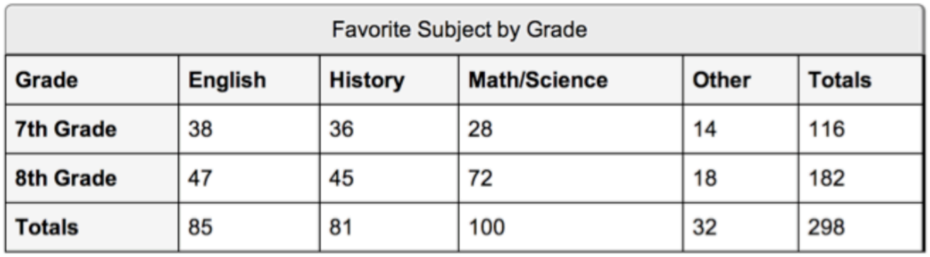
S.ID.5★

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

When students are proficient with analyzing two-way frequency tables, build upon their understanding to develop the vocabulary.

- The table entries are the joint frequencies of how many subjects displayed the respective cross-classified values.
- Row totals and column totals constitute the marginal frequencies.
- Dividing joint or marginal frequencies by the total number of subjects define relative frequencies, respectively.



Favorite Subject by Grade

Grade	English	History	Math/Science	Other	Totals
7th Grade	38	36	28	14	116
8th Grade	47	45	72	18	182
Totals	85	81	100	32	298

Conditional relative frequencies are determined by focusing on a specific row or column of the table. They are particularly useful in determining any associations between the two variables.

Students are flexible in identifying and interpreting the information from a two-way frequency table. They complete calculations to determine frequencies and use those frequencies to describe and compare.

Example: At the NC Zoo 23 interns were asked their preference of where they would like to work. There were three choices: African Region, Aviary, or North American Region. There were 13 who preferred the African Region, 5 of them were male. There were 6 who preferred the Aviary, 2 males and 4 females. A total of 4 preferred the North American Region and only 1 of them was female.

a. Use the information on the NC Zoo Internship to create a two way frequency table.
b. Use the two-way frequency table from the NC Zoo Internship, calculate:
    i. the percentage of males who prefer the African Region
    ii. the percentage of females who prefer the African Region
c. How does the percentage of males who prefer the African Region compare to the percentage of females who prefer the African Region?
d. 15% of the paid employees are male and work in the Aviary. How does that compare to the interns who are male and prefer to work in the Aviary? Explain how you made your comparison.

S.ID.6★

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
b. Informally assess the fit of a function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggests a linear association.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

In Math I, focus on linear and exponential.

Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

Example: In an experiment, 300 pennies were shaken in a cup and poured onto a table. Any penny 'heads up' was removed. The remaining pennies were returned to the cup and the process was repeated. The results of the experiment are shown below.

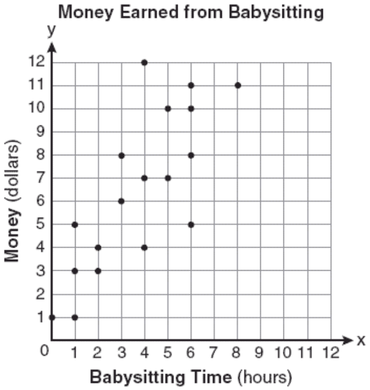
# Of rolls	0	1	2	3	4	5
# Pennies	300	164	100	46	20	8

Write a function rule suggested by the context and determine how well it fits the data.

Note: *The rule suggested by the context is* $300(0.5)^x$ *since the probability of the penny remaining is 50%.*

Example: Which of the following equations best models the (babysitting time, money earned) data?

$y = x$ $\qquad y = \frac{6}{5}x + 2$ $\qquad y = \frac{3}{2}x + 4$ $\qquad y = \frac{1}{4}x + 4$



Informally assess the fit of a function by plotting and analyzing residuals.

Year (0=1990)	Tuition Rate
0	6546
1	6996
2	6996
3	7350
4	7500
5	7978
6	8377
7	8710
8	9110
9	9411
10	9800

A ***residual*** is the difference between the actual y-value and the predicted y-value $(y - \hat{y})$, which is a measure of the error in prediction. (Note: $\hat{y}$ is the symbol for the predicted y-value for a given x-value.) A residual is represented on the graph of the data by the vertical distance between a data point and the graph of the function.

A ***residual plot*** is a graph that shows the residuals on the vertical axis and the independent variable on the horizontal axis. If the points in a residual plot are randomly dispersed around the horizontal axis, a linear regression model is appropriate for the data; otherwise, a non-linear model is more appropriate.

Example: The table to the left displays the annual tuition rates of a state college in the U.S. between 1990 and 2000, inclusively. The linear function $R(t) = 326x + 6440$ has been suggested as a good fit for the data. Use a residual plot to determine the goodness of fit of the function for the data provided in the table.

Fit a linear function for a scatter plot that suggests a linear association.

Example: The data below give number of miles driven and advertised price for 11 used models of a particular car from 2002 to 2006.

a. Use your calculator to make a scatter plot of the data.
b. Use your calculator to find the correlation coefficient for the data above. Describe what the correlation means in regards to the data. (Connect to S.ID.8)
c. Use your calculator to find an appropriate linear function to model the relationship between miles driven and price for these cars.
d. How do you know that this is the best-fit model?

Miles driven (In thousands)	Price (In dollars)
22	17,998
29	16,450
35	14,998
39	13,998
45	14,599
49	14,988
55	13,599
56	14,599
69	11,998
70	14,450
86	10,998

Standard

Cluster: Interpret linear models

S.ID.7 ★

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Example: Data was collected of the weight of a male white laboratory rat for the first 25 weeks after its birth. A scatterplot of the rat's weight (in grams) and the time since birth (in weeks) shows a fairly strong, positive linear relationship. The linear regression equation W=100+40t (where W = weight in grams and t = number of weeks since birth) models the data fairly well.

a. What is the slope of the linear regression equation? Explain what it means in context.
b. What is the y-intercept of the linear regression equation? Explain what it means in context.

S.ID.8★ Compute (using technology) and interpret the correlation coefficient of a linear fit.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. The correlation coefficient, r, is a measure of the strength and direction of a linear relationship between two quantities in a set of data. The magnitude (absolute value) of r indicates how closely the data points fit a linear pattern. If $r = 1$, the points all fall on a line. The closer $\|r\|$ is to 1, the stronger the correlation. The closer $\|r\|$ is to zero, the weaker the correlation. The sign of r indicates the direction of the relationship – positive or negative. *Note: Students often misinterpret the correlation coefficient to mean that a given linear model is a good or bad fit for the data especially since it is often provided when completing a linear regression on the calculator. The correlation coefficient, r, measures the strength and the direction of a linear relationship between two variables. It can be calculated without a linear model. It is independent of the linear model and thus does not describe the fit of the model. Students should use residuals to assess the fit of the function.* ***Example:*** The correlation coefficient of a given data set is 0.97. List three specific things this tells you about the data.
S.ID.9★ Distinguish between correlation and causation.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. Some data leads observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment. ***Example:*** A study found a strong, positive correlation between the number of cars owned and the length of one's life. Larry concludes that owning more cars means you will live longer. Does this seem reasonable? Explain your answer. ***Example:*** Choose two variables that could be correlated because one is the cause of the other; defend and justify selection of variables. Explore the website http://tylervigen.com/ for correlations for discussion regarding causation and association.

References

This document includes examples, illustrations and references from the following websites:

Graphing Stories: http://graphingstories.com

Spurious Correlations: http://tylervigen.com/

Wolfram Mathworld: http://mathworld.wolfram.com/Circle.html

Illustrative Mathematics: https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/531