

You must show all work to receive credit.

For problems 1-3, use the Fundamental Theorem of Calculus to evaluate the integral (3pts each).

$$1. \int_2^7 x^2 dx = \left. \frac{1}{3} x^3 \right|_2^7 = \frac{1}{3}(7)^3 - \frac{1}{3}(2)^3 = 335/3$$

$$2. \int_{-\pi}^{\pi} (5 \cos x + 4 \sin x) dx = \left. 5 \sin x - 4 \cos x \right|_{-\pi}^{\pi} = 0$$

$$3. \int_0^3 3x^2 - 2x + 1 dx = \left. x^3 - x^2 + x \right|_0^3 = 21$$

For Problems 4-6, suppose that  $\int_3^7 f(x) dx = 8$ ,  $\int_3^{11} f(x) dx = 27$ , and  $\int_3^{11} g(x) dx = 36$  (3pts each).

$$4. \text{ Find } \int_7^3 f(x) dx. \quad -8$$

$$5. \text{ Find } \int_7^{11} f(x) dx. \quad 27 - 8 = 19$$

$$6. \text{ Find } \int_3^{11} [f(x) - g(x)] dx. \quad 27 - 36 = -9$$

For Problems 7-8, let  $f(x) = x^2 + 3$  on  $[1,5]$ . (3pts each).

7. Using a left hand Riemann sum with 4 equal subintervals, approximate  $F(x)$ . Does

this Riemann sum overestimate or underestimate  $F(x)$ ? Give a reason for your

answer.  $1 \cdot f(1) + 1 \cdot f(2) + 1 \cdot f(3) + 1 \cdot f(4) = 1 \cdot 4 + 1 \cdot 7 + 1 \cdot 12 + 1 \cdot 19 = 42$  under since  $f(x)$  is strictly increasing & we used a LHS

8. Evaluate  $F(x)$  exactly using the fundamental theorem of calculus.  $\int_1^5 x^2 + 3 dx = 53/3$

For questions 9-10, find the derivative. (3pts each).

$$9. \text{ Find } A'(x) \text{ where } A(x) = \int_3^x \sin(t^3) dt \quad A'(x) = \sin(x^3) \cdot 1$$

$$10. \text{ Find } G'(x) \text{ where } G(x) = \int_{-2}^{\sin(x)} (t^3) dt \quad G'(x) = (\sin x)^3 \cdot \cos x$$

For questions 11-15, evaluate the integral. (4pts each).

$$11. \int \frac{x^4 - 3x^2 + 1}{x^2} dx$$

$$\int x^2 - 3 + x^{-2} dx = \frac{1}{3}x^3 - 3x - x^{-1} + C$$

$$12. \int x(x^3 - 4x) dx$$

$$\int x^4 - 4x^2 = \frac{1}{5}x^5 - \frac{4}{3}x^3 + C$$

$$13. \int -4 \tan^3 x \cdot \sec^2 x dx$$

$$\begin{aligned} u &= \tan x \\ du &= \sec^2 x dx \end{aligned} \quad \int -4 u^3 du = -4 \cdot \frac{1}{4} (\tan x)^4 + C$$

$$14. \frac{1}{2} \int 2x \cdot \sqrt[4]{x^2 - 6} dt$$

$$\begin{aligned} u &= x^2 - 6 \\ du &= 2x dx \end{aligned} \quad \frac{1}{2} \int u^{1/4} du = \frac{1}{2} \cdot \frac{4}{5} (x^2 - 6)^{5/4} + C$$

$$2/5 (x^2 - 6)^{5/4} + C$$

$$15. \int_0^4 x^3 - 3x^2 + 2 dx$$

$$\left[ \frac{1}{4}x^4 - x^3 + 2x \right]_0^4 = \left[ \frac{1}{4}(4)^4 - (4)^3 + 2(4) \right] - [0] = 8$$

16. Find  $f(x)$  where  $f''(x) = x^3 - x^2$  with  $f'(2) = -1$ , and  $f(1) = 3$ . (6pts)

$$\int x^3 - x^2 dx = \frac{1}{4}x^4 - \frac{1}{3}x^3 + C \quad \int \frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{7}{3} dx$$

$$\frac{1}{4}(2)^4 - \frac{1}{3}(2)^3 + C = -1 \quad \frac{1}{20}x^5 - \frac{1}{12}x^4 - \frac{7}{3}x + C = 3$$

$$\frac{1}{20}x^5 - \frac{1}{12}x^4 - \frac{7}{3}x + C = 3$$

Also, **LOOK** at the lab we did concerning Riemann Sums.  $\frac{1}{20}(1)^5 - \frac{1}{12}(1)^4 - \frac{7}{3}(1) + C = 3$

$$\text{So } f'(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{7}{3}$$

$$f(x) = \frac{1}{20}x^5 - \frac{1}{12}x^4 - \frac{7}{3}x + \frac{16}{30}$$