Introduction to Vectors

A **vector** is a quantity that has both magnitude and direction.

The sum of two or more vectors is called the **resultant** of the vectors.

Vectors can be represented algebraically using ordered pairs of real numbers. We can determine the resultant vector using the component method.

Component Method

Component method: Break each vector up into its two perpendicular components. Add all the horizontal components and call the sum R_H (or R_X). Add all the vertical components and call the sum R_V (or R_Y).

Create the final resultant by adding these two vector components.

Example 1: Find the ordered pair that represents the resultant vector for the following:

$\vec{u} = \langle 1, -4 \rangle \qquad \vec{v} = \langle 0, 8 \rangle$

a) $\vec{u} + \vec{v} = \langle 1+0, -4+8 \rangle$

$\langle 1, 4 \rangle$

b) $2\vec{u} - 3\vec{v} = \langle 2-0, -8-24 \rangle$

$\langle 2, -32 \rangle$

Example 2: Find the ordered pair that represents the resultant vector for the following:

$\vec{m} = \langle 5, -7 \rangle$ $\vec{n} = \langle 0, 4 \rangle$ $\vec{p} = \langle -1, 3 \rangle$

a) $\vec{m} + \vec{p} = \langle 4, -4 \rangle$

b) $2\vec{m} + 3\vec{n} - \vec{p} =$

$\langle 11, -5 \rangle$

Determining Magnitude and Direction

We can calculate the magnitude and direction from an ordered pair using the pythagorean theorem and an inverse tan function (2nd tan)

Sometimes, we will have to use a reference angle to adjust the answer given from the calculator depending on the quadrant that our vector is in.

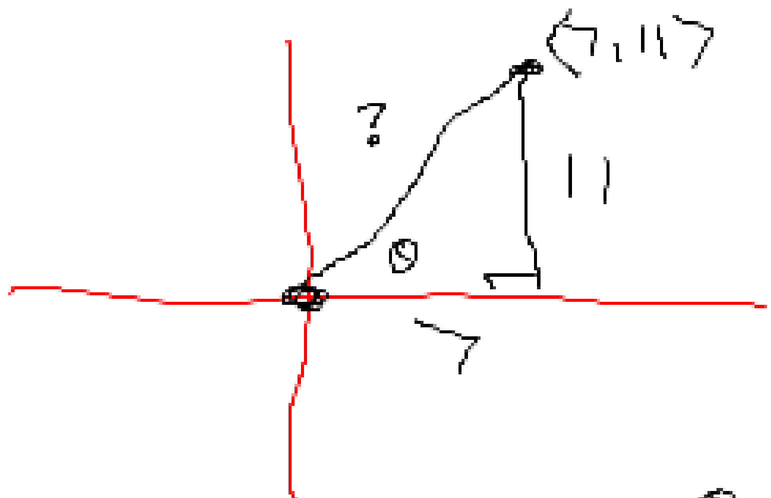
Example 3: Determine the magnitude and direction for the vector represented by the following ordered pair:

$$\vec{a} = \langle 7, 11 \rangle$$

$$7^2 + 11^2 = ?^2$$

$$49 + 121 = ?^2$$

$$\sqrt{170} = \sqrt{?^2}$$

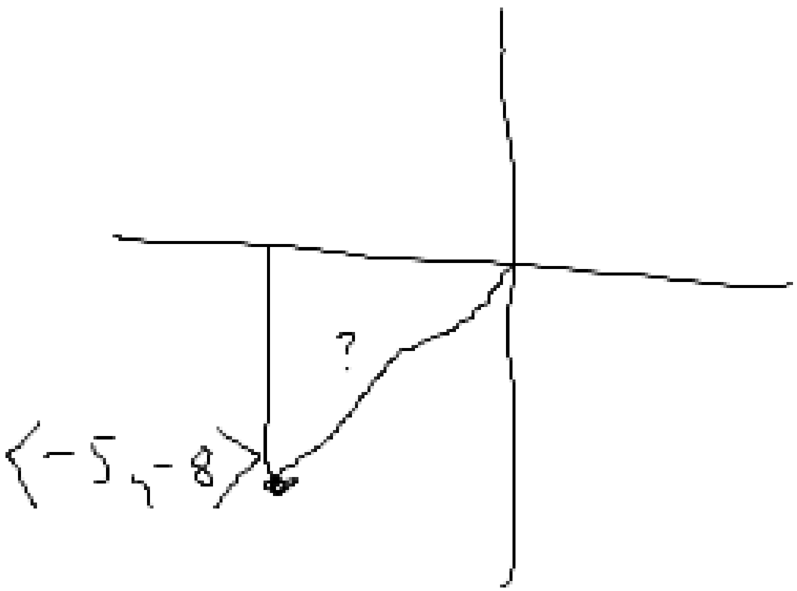


$$\vec{a} = 13.04 \text{ at } \underline{57.5^\circ}$$

$$\tan\theta = 11/7 = 57.5$$

Example 4: Determine the magnitude and direction for the vector represented by the following ordered pair:

$$\vec{w} = \langle -5, -8 \rangle$$



$$(-5)^2 + (-8)^2 = ?^2$$

$$? = \sqrt{89} = 9.43$$

so

$$\vec{w} = \underline{9.43} \text{ at } \underline{238^\circ}$$

$$\tan\theta = \frac{-8}{-5}$$

$$\theta = 58 + 180$$

$$238$$

The same operations can be applied to a Three-Dimensional Vector.

Example 5: Find the ordered ~~pair~~ triple that represents the resultant vector for the following, and then determine the magnitude.

$\vec{u} = \langle 0, 2, 4 \rangle \qquad \vec{v} = \langle 4, 3, -1 \rangle$

a) $\vec{u} + 2\vec{v} = \langle \overset{0+8}{8}, \overset{2+6}{8}, \overset{4-2}{2} \rangle$

$\text{Mag} = \sqrt{8^2 + 8^2 + 2^2} = 11.5$

Example 6: Find the ordered pair that represents the resultant vector for the following, and then determine the magnitude.

$\vec{a} = \langle 2, 1, 4 \rangle$ $\quad$ $\vec{b} = \langle 6, 0, 6 \rangle$

a) $\vec{a} + \vec{b} = \langle 8, 1, 10 \rangle$

$\text{Mag} = \sqrt{8^2 + 1^2 + 10^2} = \sqrt{165}$

≈ 12.8

Lastly, two important concepts concerning vectors are the "dot product" and "cross product".

The dot product is used to determine if two vectors are perpendicular.

The cross product is used to determine a vector perpendicular to both given vectors.

Example 7:

Determine if vector a and vector b are perpendicular.
Then find a vector perpendicular to vectors a and b.

$\vec{a} = \langle -3, 1, 1 \rangle \quad \vec{b} = \langle 2, 8, -2 \rangle$

$\vec{a} \cdot \vec{b} = -3(2) + 1(8) + 1(-2)$

$-6 + 8 - 2 = 0$

If dot product is 0,
then $\vec{a}$ & $\vec{b}$ are perp.

$$\vec{a} \times \vec{b} = \begin{vmatrix} -3 & 1 & 1 \\ 2 & 8 & -2 \end{vmatrix} = \langle -10, 4, -26 \rangle$$

Example 8:

Determine if vector a and vector b are perpendicular. Then find a vector perpendicular to vectors a and b.

$\vec{a} = \langle -4, 1, 0 \rangle$ $\quad$ $\vec{b} = \langle 5, 4, -2 \rangle$

$\vec{a} \cdot \vec{b} = -20 + 4 + 0 = -16$

Not $\perp$

$$\vec{a} \times \vec{b} = \begin{vmatrix} -4 & 1 & 0 \\ 5 & 4 & -2 \end{vmatrix} = \langle -2, 8, -21 \rangle$$

Homework:

p497 #24-34 even (determine ordered pair, magnitude, and direction)

p503 #22-27 (determine ordered triple, and magnitude)

p 509 #11-19, 21-26