DeSoto
COUNTY SCHOOLS

Geometry

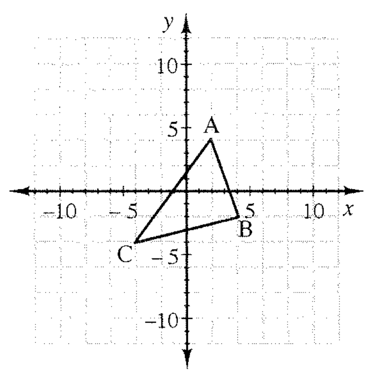
Week 3

So far, students have measured, described, and transformed geometric shapes. In this chapter we focus on comparing geometric shapes. We begin by dilating shapes: enlarging them as one might on a copy machine. When students compare the original and enlarged shapes closely, they discover that the shape of the figure remains exactly the same (this means the angle measures of the enlarged figure are equal to those of the original figure), but the size changes (the lengths of the sides increase). Although the size changes, the lengths of the corresponding sides all have a constant ratio, known as the zoom factor, or ratio of similarity.

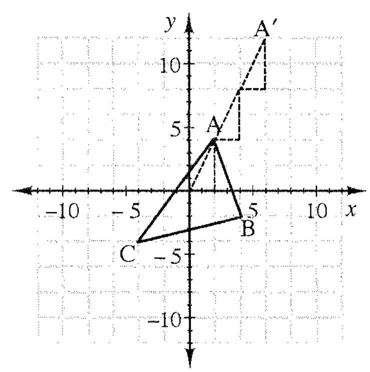
See the Math Notes boxes in Lessons 3.1.1, 3.1.2, 3.1.3, and 3.1.4 for more information about dilations and similar figures.

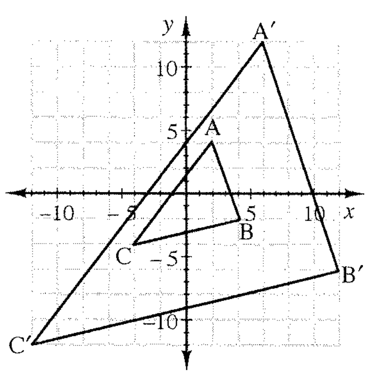
Example 1

Enlarge the figure at right from the origin by a factor of 3.



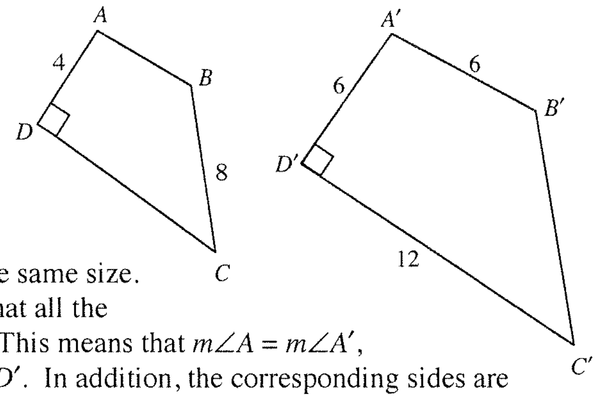
Students used rubber bands to create a dilation (enlargement) of several shapes. We can do this using a grid and slope triangles. Create a right triangle so that the segment from the origin to point A, (2, 4), is the hypotenuse, one leg lies on the positive x-axis, and the other connects point A to the endpoint of the leg at (2, 0). This triangle is called a slope triangle since it represents the slope of the hypotenuse from (0, 0) to vertex A. Add two more slope triangles exactly like this one along the line from (0, 0) to point A as shown in the figure at right. Using three triangles creates an enlargement by a factor of 3 and gives us the new point A′ at (6, 12). Repeat this process for the other two vertices, forming a new slope triangle for each vertex.





This will give us new points B′ at (12, –6) and C′ at (–12, –12). Connecting points A′, B′, and C′, we form a new triangle that is an enlargement of the original triangle by a factor of 3, as shown at left.

Example 2



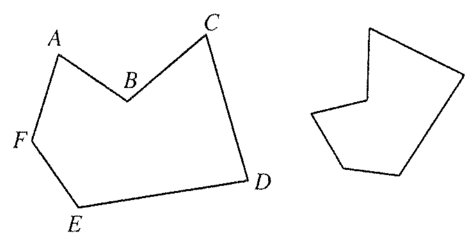
The two quadrilaterals at right are similar. What parts are equal? Can you determine the lengths of any other sides?

Similar figures have the same shape, but not the same size. Since the quadrilaterals are similar, we know that all the corresponding angles have the same measure. This means that $m\angle A = m\angle A'$, $m\angle B = m\angle B'$, $m\angle C = m\angle C'$, and $m\angle D = m\angle D'$. In addition, the corresponding sides are **proportional**, which means the ratio of corresponding sides is a constant. To find the ratio, we need to know the lengths of one pair of corresponding sides. From the picture we see that $\overline{AD}$ corresponds to $\overline{A'D'}$. Since these sides correspond, we can write $\frac{AD}{A'D'} = \frac{4}{6}$.

Therefore, the ratio of similarity is $\frac{4}{6}$, or $\frac{2}{3}$. We can use this value to find the lengths of other sides when we know at least one length of a corresponding pair of sides.

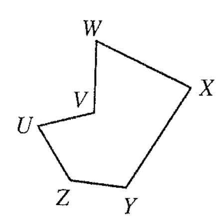
$\frac{AB}{A'B'} = \frac{4}{6}$	$\frac{BC}{B'C'} = \frac{4}{6}$	$\frac{CD}{C'D'} = \frac{4}{6}$
$\frac{AB}{6} = \frac{4}{6}$	$\frac{8}{B'C'} = \frac{4}{6}$	$\frac{CD}{12} = \frac{4}{6}$
$AB = 4$	$4B'C' = 48$	$6CD = 48$
	$B'C' = 12$	$CD = 8$

Example 3



The pair of shapes at right is similar ($ABCDEF \sim UVWXYZ$). Label the second figure correctly to reflect the similarity statement.

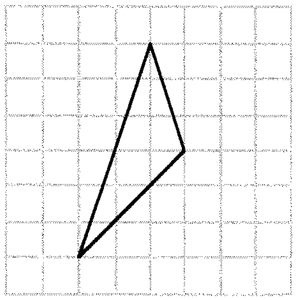
Since similar figures have the same shape, just different sizes, this means that the corresponding angles have equal measure. When we write a similarity statement, we write the letters so that the corresponding angles match up. By the similarity statement, we must have $m\angle A = m\angle U$, $m\angle B = m\angle V$, $m\angle C = m\angle W$, $m\angle D = m\angle X$, $m\angle E = m\angle Y$, and $m\angle F = m\angle Z$.



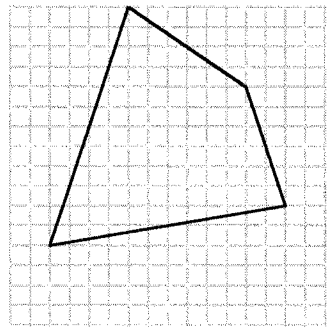
The smaller figure is labeled at right. If it is difficult to tell which original angle corresponds to its enlargement or reduction, try rotating the figures so that they have the same orientation.

Problems

1. Copy the figure below onto graph paper and then enlarge it by a factor of 2.

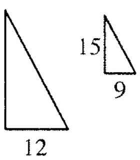


2. Create a figure similar to the one below with a zoom factor of 0.5.

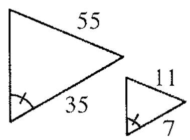


For each pair of similar figures below, find the ratio of similarity, for large:small.

3.



4.



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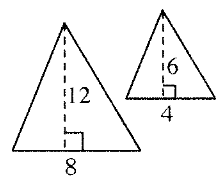

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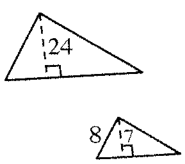

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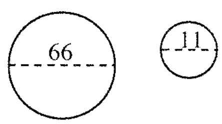

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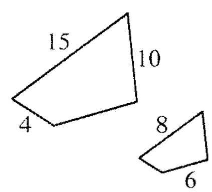
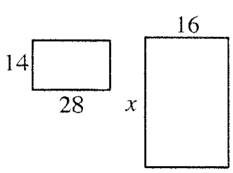


For each pair of similar figures, state the ratio of similarity, then use it to find x.

9.



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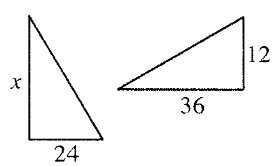

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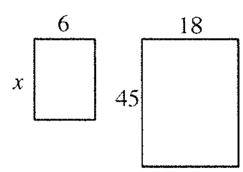

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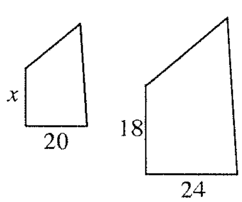

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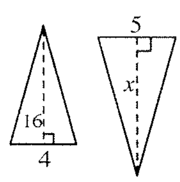

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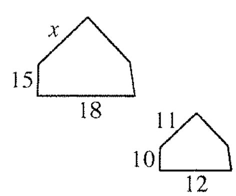


For problems 15 through 20, use the given information and the figure to find each length.

15. JM = 14, MK = 7, JN = 10 Find NL.

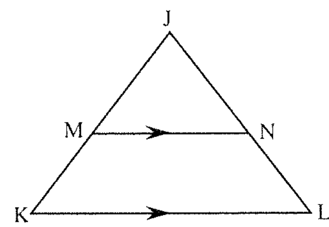
16. MN = 5, JN = 4, JL = 10 Find KL.

17. KL = 10, MK = 2, JM = 6 Find MN.

18. MN = 5, KL = 10, JN = 7 Find JL.

19. JN = 3, NL = 7, JM = 5 Find JK.

20. JK = 37, NL = 7, JM = 30 Find JN.

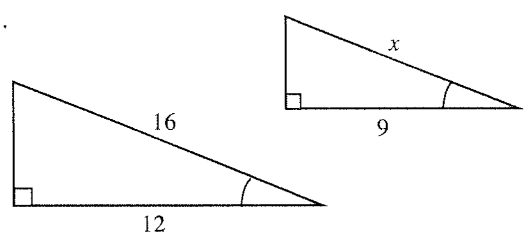


21. Standing 4 feet from a mirror lying on the flat ground, Palmer, whose eye height is 5 feet, 9 inches, can see the reflection of the top of a tree. He measures the mirror to be 24 feet from the tree. How tall is the tree?

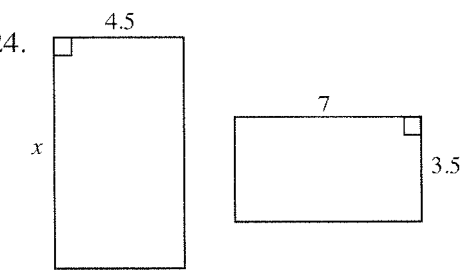
22. The shadow of a statue is 20 feet long, while the shadow of a student is 4 ft long. If the student is 6 ft tall, how tall is the statue?

Each pair of figures below is similar. Use what you know about similarity to solve for x.

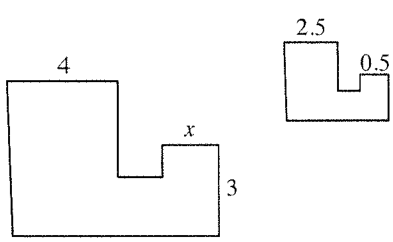
23.



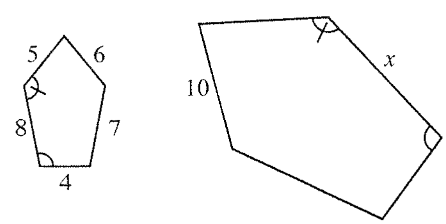
24.



25.

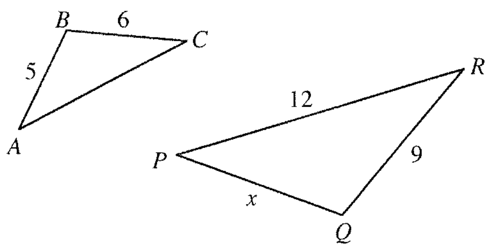


26.

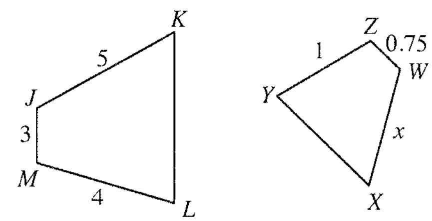


Solve for the missing lengths in the pairs of similar figures below.

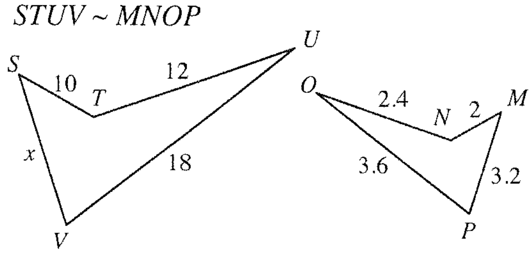
27. $\Delta ABC \sim \Delta PQR$



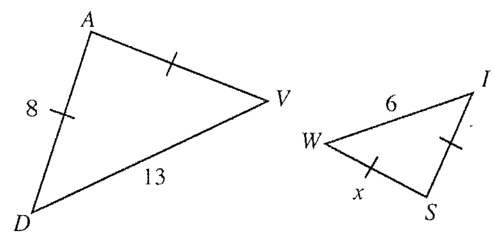
28. $JKLM \sim WXYZ$



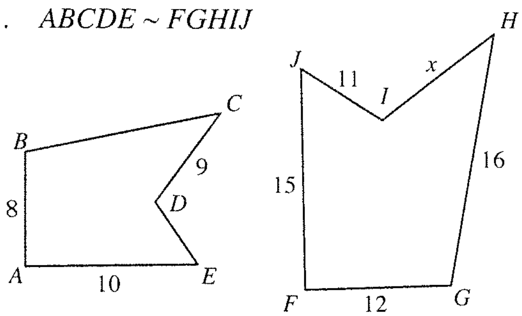
29. $STUV \sim MNOP$



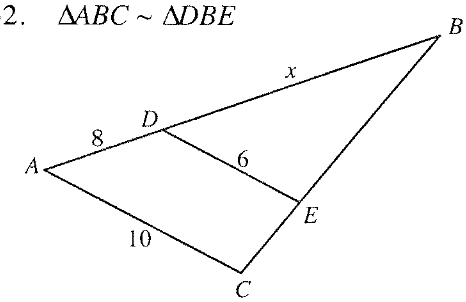
30. $\Delta DAV \sim \Delta ISW$



31. $ABCDE \sim FGHIJ$



32. $\Delta ABC \sim \Delta DBE$



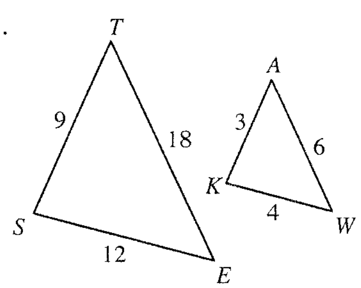
CONDITIONS FOR TRIANGLE SIMILARITY 3.2.1 – 3.2.6

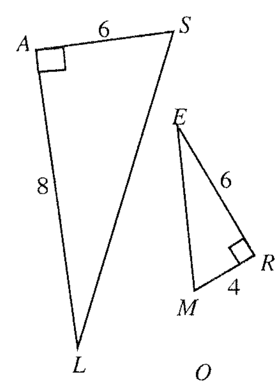
When two figures are related by a series of transformations (including dilations), they are similar. Another way to check for similarity is to measure all the angles and sides of two figures. In this section students develop conditions to shorten the process. These are the **AA Triangle Similarity Condition (AA ~)**, the **SAS Triangle Similarity Condition (SAS ~)**, and the **SSS Triangle Similarity Condition (SSS ~)**. The first condition states that if two pairs of corresponding angles have equal measures, then the triangles are similar. The second condition states that if two pairs of corresponding side lengths have the same ratio, *and* their included angles have the same measure, then the triangles are similar. The third condition states that if all three pairs of corresponding side lengths have the same ratio, then the triangles are similar. Additionally, students found that if similar figures have a ratio of similarity of 1, then the shapes are **congruent**, that is, they have the same size and shape. Students used flowcharts in this section to help organize their information and make logical conclusions about similar triangles. Now students are able to use similar triangles to find side lengths, perimeters, heights, and other measurements.

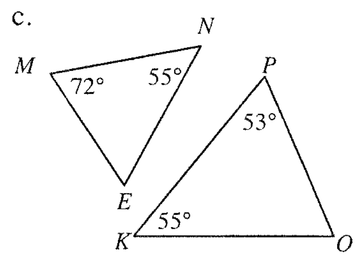
See the Math Notes boxes in Lessons 3.2.1, 3.2.2, 3.2.4, and 3.2.5 for more information about similar triangles, congruent triangles, and writing flowcharts.

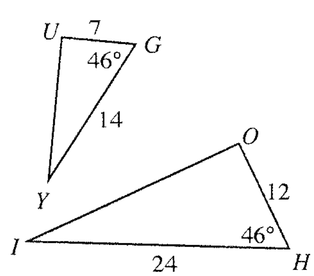
Example 1

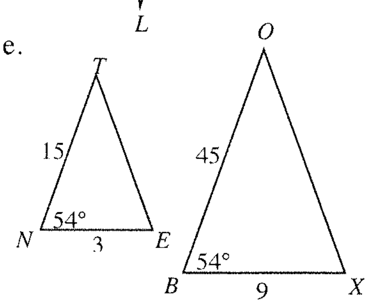
Based on the given information, is each pair of triangles similar? If they are similar, write the similarity statement. Justify your answer completely.

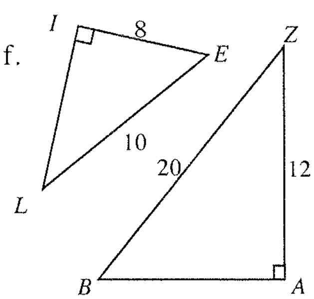
a.


b.


c.


d.


e.


f.


We will use the three similarity conditions to test whether or not the triangles are similar.

In part (a), we have the lengths of the three sides, so it makes sense to check whether the SSS ~ holds true. Write the ratios of the corresponding side lengths and compare them to see if they are the same, as shown at right. Each ratio reduces to 3, so they are equal. Therefore, $\Delta TES \sim \Delta AWK$ by SSS ~.

$$\frac{ST}{KA} \stackrel{?}{=} \frac{TE}{AW} \stackrel{?}{=} \frac{ES}{WK}$$
$$\frac{9}{3} = \frac{18}{6} = \frac{12}{4}$$

The measurements given in part (b) suggest we look at SAS ~. $\angle A$ and $\angle R$ are the included angles. Since they are both right angles, they have equal measures. Now we need to check that the corresponding sides lengths have the same ratio, as shown at right.

$$\frac{LA}{ER} \stackrel{?}{=} \frac{AS}{RM}$$
$$\frac{8}{6} \stackrel{?}{=} \frac{6}{4}$$
$$\frac{4}{3} \neq \frac{3}{2}$$

Although the triangles display the SAS ~ pattern and the included angles have equal measures, the triangles are not similar because the corresponding side lengths do not have the same ratio.

In part (c), we are given the measures of two angles of each triangle, but not corresponding angles. $m\angle K = 55° = m\angle N$ which is one pair of corresponding angles. For AA ~, we need two pairs of equal angles. If we use the fact that the measures of the three angles of a triangle add up to 180°, we can find the measures of $\angle O$ and $\angle E$ as shown at right. Now we see that all pairs of corresponding angles have equal measures, so $\Delta POK \sim \Delta EMN$ by AA ~.

$$m\angle O = 180° - 53° - 55°$$
$$m\angle O = 72°$$

$$m\angle E = 180° - 55° - 72°$$
$$m\angle E = 53°$$

Part (d) shows the SAS~ pattern and we can see that the included angles have equal measures, $m\angle G = m\angle H$. We also need to have the ratio of the corresponding side lengths to be equal. Since the two fractions are equal (the second reduces to the first), the corresponding side lengths have the same ratio. Therefore, $\Delta YUG \sim \Delta IOH$ by SAS ~.

$$\frac{UG}{OH} \stackrel{?}{=} \frac{GY}{HI}$$
$$\frac{7}{12} = \frac{14}{24}$$

In part (e), we see that the included angles have equal measures, $m\angle B = m\angle N$. Since $\frac{45}{15} = \frac{9}{3} = \frac{3}{1}$, the corresponding sides are proportional. Therefore, $\Delta BOX \sim \Delta NTE$ by SAS ~.

In part (f), we only have one pair of angles that are equal (the right angles), but those angles are not between the sides with known lengths. However, we can find the lengths of the third sides by using the Pythagorean Theorem.

$$8^2 + (IL)^2 = 10^2$$
$$64 + (IL)^2 = 100$$
$$(IL)^2 = 36$$
$$IL = 6$$

$$12^2 + (AB)^2 = 20^2$$
$$144 + (AB)^2 = 400$$
$$(AB)^2 = 256$$
$$AB = 16$$

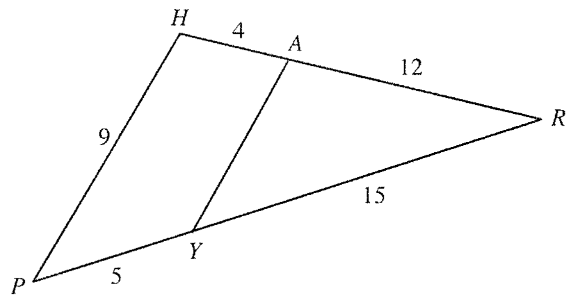
Now that we know all three sides, we can check to see if the triangles are similar by SSS ~. Since the ratios of the corresponding sides are the same, $\Delta ELI \sim \Delta BZA$ by SSS ~.

$$\frac{12}{6} \stackrel{?}{=} \frac{16}{8} \stackrel{?}{=} \frac{20}{10}$$
$$2 = 2 = 2$$

Example 2

In the figure at right, $\overline{AY} \parallel \overline{HP}$. Decide whether or not there are any similar triangles in the figure. Justify your answer with a flowchart.

Can you find the length of $\overline{AY}$? If so, find it. Justify your answer.



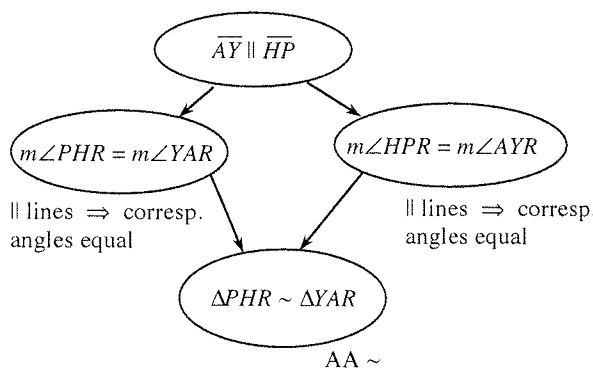
Recalling information we studied in earlier chapters, the parallel lines give us angles with equal measures. In this figure, we have two pairs of corresponding angles with equal measures: $m\angle PHR = m\angle YAR$ and $m\angle HPR = m\angle AYR$. Because two pairs of corresponding angles have equal measures, we can say the triangles are similar: $\Delta PHR \sim \Delta YAR$ by AA ~. Since the triangles are similar, the lengths of corresponding sides are proportional (i.e., have the same ratio). This means we can write the solution at right.

$$\frac{RA}{RH} = \frac{AY}{HP}$$

$$\frac{12}{16} = \frac{AY}{9}$$

$$AY = \frac{9 \cdot 12}{16} = 6.75$$

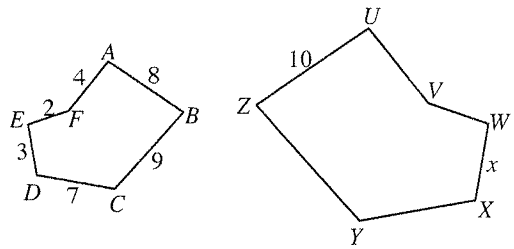
We can justify this result with a flowchart as well. The flowchart at right organizes and states what is written above.



Problems

Each pair of figures below is similar. Write a correct similarity statement and solve for x.

1.



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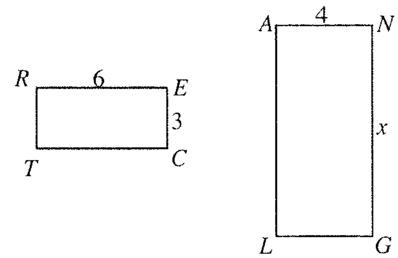

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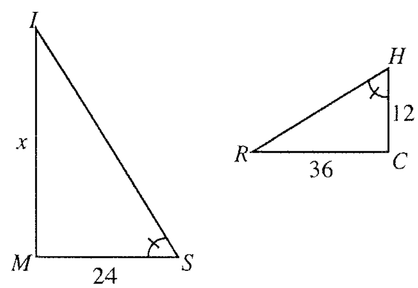

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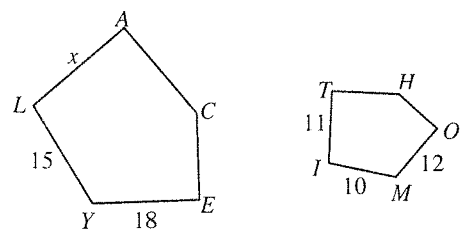
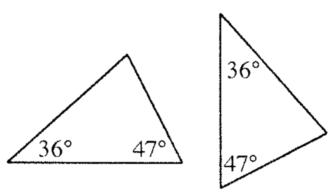


Determine if each pair of triangles is similar. If they are similar, justify your answer.

5.



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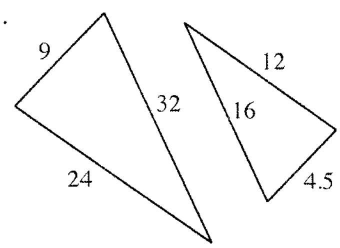

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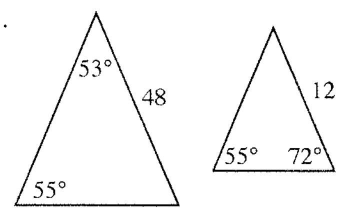

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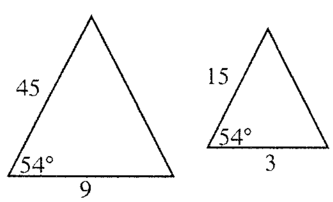
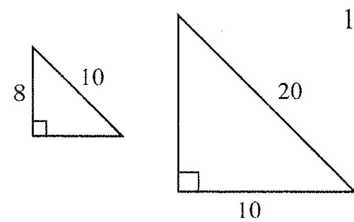


9.



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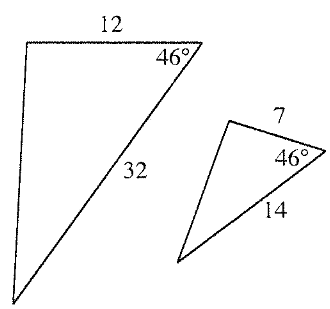
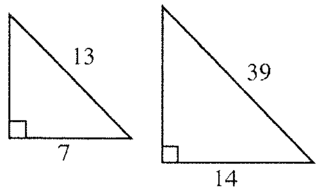
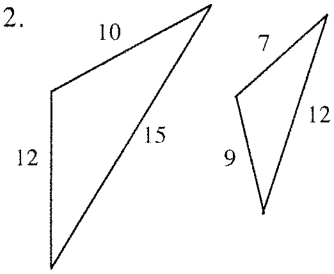
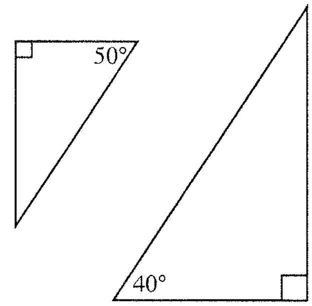
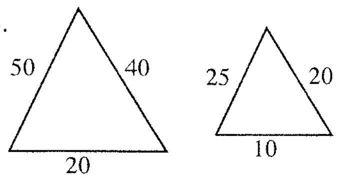
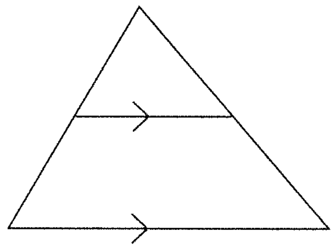


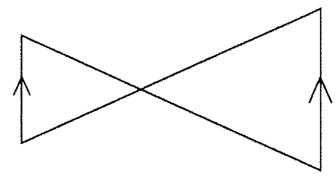
11. 

12. 

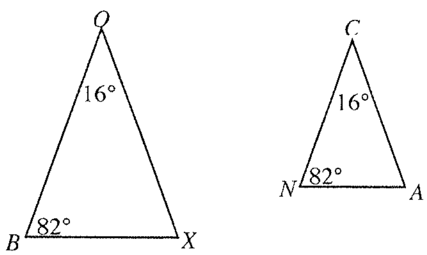
13. 

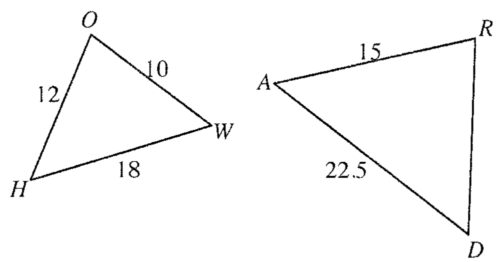
14. 

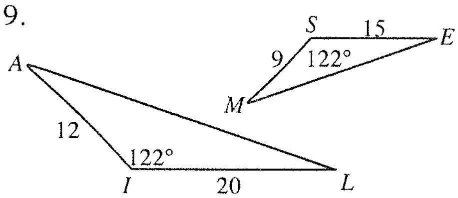
15. 

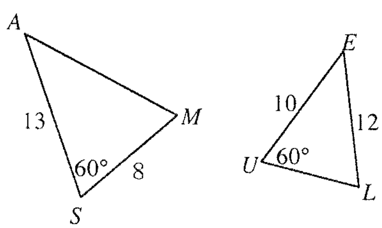
16. 

Decide if each pair of triangles is similar. If they are similar, write a correct similarity statement and justify your answer.

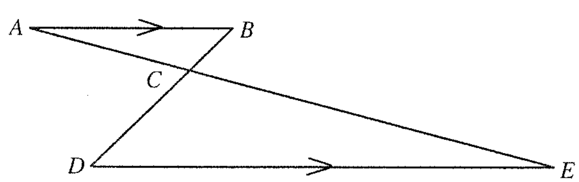
17. 

18. 

19. 

20. 

21. In the figure at right $\overline{AB} \parallel \overline{DE}$. Is ΔABC similar to ΔEDC? Use a flowchart to organize and justify your answer.



22. Standing four feet from a mirror resting on the flat ground, Palmer, whose eye height is 5 feet, 9 inches, can see the reflection of the top of a tree. He measures the mirror to be 24 feet from the tree. How tall is the tree? Draw a picture to help solve the problem.

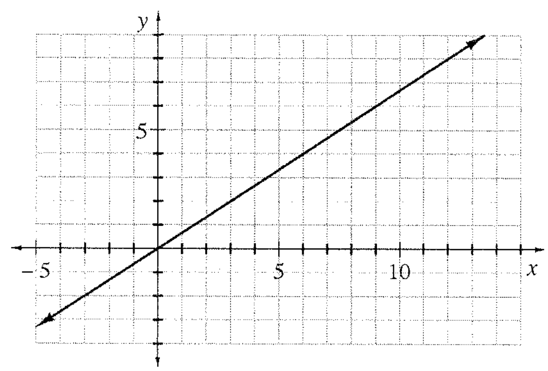
TANGENT – THE SLOPE RATIO (TRIGONOMETRY) 4.1.1 – 4.1.5

In the first section of Chapter 4, students consider different slope triangles for a given line or segment and notice that for each line, the slope remains constant no matter where they draw the slope triangle on that line or how large or small each slope triangle is. All the slope triangles on a given line are similar. These similar slope triangles allow students to write proportions to calculate lengths of sides and angle measures. This constant slope ratio is known as the "**tangent**" (trigonometric) relationship. Using the tangent button on their calculators, students are able to find measurements in application problems.

See the Math Notes boxes in Lessons 4.1.1, 4.1.2, and 4.1.4 for more information about slope angles and the tangent ratio.

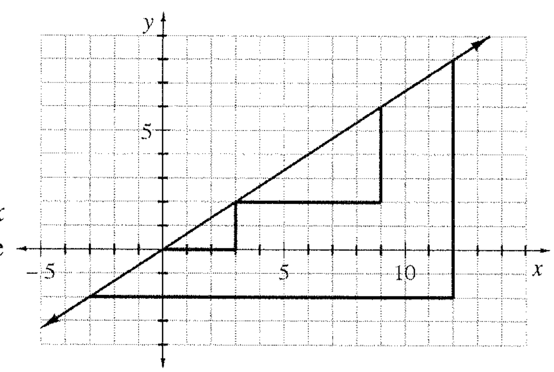
Example 1

The line graphed at right passes through the origin. Draw in three different slope triangles for the line. For each triangle, what is the slope ratio, $\frac{\Delta y}{\Delta x}$? What is true about all three ratios?



Note: Δx (delta x) and Δy (delta y) are read "change in x" and "change in y."

A slope triangle is a right triangle that has its hypotenuse on the line that contains it. This means that the two legs of the right triangle are parallel to the axes: one leg runs vertically, the other horizontally. There are infinitely many slope triangles that we can draw, but it is always easiest if we draw triangles that have their vertices on lattice points (that is, their vertices have integer coordinates). The length of the horizontal leg is Δx and the length of the vertical leg is Δy. At right are three possible slope triangles. For the smallest triangle, $\Delta x = 3$ (the length of the horizontal leg), and $\Delta y = 2$ (the length of the vertical leg). For the smallest triangle we have $\frac{\Delta y}{\Delta x} = \frac{2}{3}$.



In the medium sized triangle, $\Delta x = 6$ and $\Delta y = 4$, which means $\frac{\Delta y}{\Delta x} = \frac{4}{6}$.

Lastly, the lengths on the largest triangle are $\Delta x = 15$ and $\Delta y = 10$, so $\frac{\Delta y}{\Delta x} = \frac{10}{15}$.

If we reduce the ratios to their lowest terms we find that the slope ratios, no matter where we draw the slope triangles for this line, are all equal.

$$\frac{\Delta y}{\Delta x} = \frac{2}{3} = \frac{4}{6} = \frac{10}{15}$$

NAME________________________ DATE____________________ CLASS_____

ACT MATH PREP - GEOMETRY

1. A cube has a volume of 27 in^3. What is the length of each of its edges?

- **A.** 3
- **B.** 6
- **C.** 9
- **D.** 27
- **E.** 81

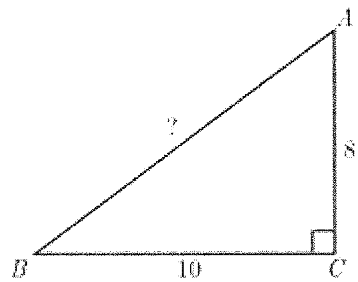
2. A triangle has an area of 24 square inches and a height of 8 inches. What could be the length of its base?

- **A.** 3
- **B.** 4
- **C.** 6
- **D.** 12
- **E.** 48

3. Cylinder *A* and cylinder *B* have a height of 3 inches, but cylinder *B* has a diameter four inches greater than that of cylinder *A*. If the diameter of cylinder *B* is 12 inches, how much greater is the volume of cylinder *B* than cylinder *A* ?

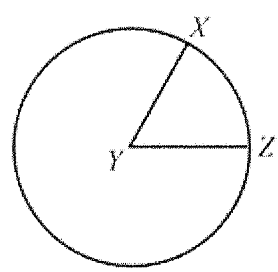
- **A.** 4π
- **B.** 12π
- **C.** 20π
- **D.** 60π
- **E.** 240π

4. In right triangle *ABC* below, $\overline{AC} = 8$ and $\overline{BC} = 10$. What is the length *AB* ?



- **A.** 6
- **B.** $\sqrt{82}$
- **C.** $4\sqrt{5}$
- **D.** $2\sqrt{41}$
- **E.** $4\sqrt{41}$

5. In the figure below, *X* and *Z* lie on the circle *Y*, which has a radius of 6. If the angle *XYZ* is $60°$, what is the area of sector *XYZ* ?



- **A.** 2π
- **B.** 3π
- **C.** 6π
- **D.** 9π
- **E.** 36π

ACT MATH PREP - GEOMETRY

6. A square has a side length of 6 inches. What is the length of its diagonal?

A. $2\sqrt{6}$
B. $6\sqrt{2}$
C. $6\sqrt{3}$
D. 6
E. 12

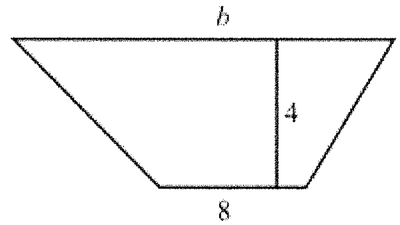
7. What is the slope of the linear equation $3x-2y-4=0$?

A. $\frac{3}{2}$
B. $-\frac{3}{2}$
C. $\frac{2}{3}$
D. $-\frac{2}{3}$
E. 3

8. What is the distance between the points $(-2,3)$ and $(4,-5)$ on the standard (x,y) coordinate plane?

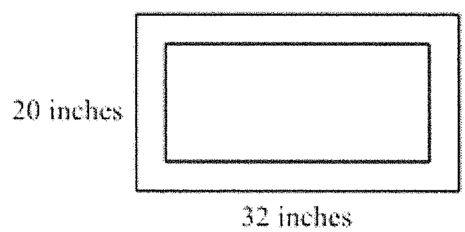
A. 10
B. $\sqrt{10}$
C. 100
D. $\sqrt{106}$
E. $2\sqrt{17}$

9. The area of the trapezoid below is 36 square inches, the altitude is 4 inches, and the length of one base is 8 inches. What is the length, b, of the other base, in inches?



A. 10
B. 12
C. 16
D. 26
E. 32

10. The picture shown below has a uniform frame-width of $\frac{3}{4}$ inches. What is the approximate area, in square inches, of the viewable portion of the picture?



A. 49.00
B. 50.50
C. 564.25
D. 601.56
E. 640.00

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ACT MATH PREP - GEOMETRY

11. Two sides of a triangle measure 5 inches and 6 inches, respectively. Which of the following could be the length of the third side?

A. 10
B. 11
C. 12
D. 13
E. 14

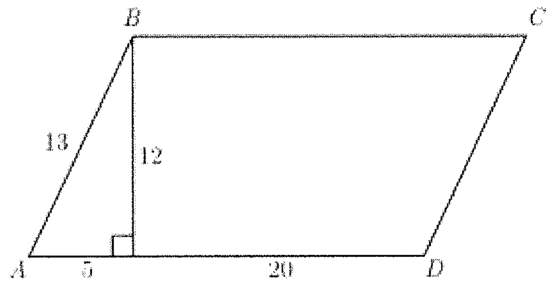
12. A circle in the standard (x, y) coordinate plane is tangent to the x-axis at 5 and tangent to the y-axis at 5. Which of the following is an equation of the circle?

A. $x^2 + y^2 = 5$
B. $x^2 + y^2 = 25$
C. $(x-5)^2 + (y-5)^2 = 5$
D. $(x-5)^2 + (y-5)^2 = 25$
E. $(x+5)^2 + (y+5)^2 = 25$

13. If the volume of a cube is 8, what is the shortest distance from the center of the cube to the base of the cube?

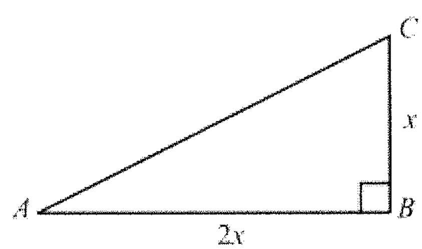
A. 1
B. 2
C. 4
D. $\sqrt{2}$
E. $2\sqrt{2}$

14. What is the area, in square units, of the parallelogram shown below?



A. 150
B. 240
C. 260
D. 300
E. 325

15. In the figure below, ΔABC is a right triangle with legs that measure x and $2x$ inches, respectively. What is the length, in inches, of the hypotenuse?



A. $\sqrt{3}$
B. $\sqrt{5}$
C. x
D. $x\sqrt{3}$
E. $x\sqrt{5}$

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ACT MATH PREP - GEOMETRY

16. A rectangular patio is 12 feet longer than it is wide. Its area is 160 square feet. How long, in feet, is it?

A. 8
B. 10
C. 16
D. 20
E. 28

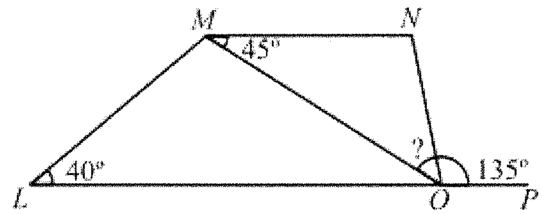
17. The ratio of the radii of two circles is 4:9. What is the ratio of their circumferences?

A. 2 : 3
B. 4 : 6
C. 4 : 9
D. 8 : 18
E. 16 : 81

18. In $\triangle ABC$, $AB \cong BC$ and the measure of $\angle A$ is $42°$. What is the measure of $\angle C$?

A. 21°
B. 42°
C. 48°
D. 84°
E. 138°

19. In the figure below, *LMNO* is a trapezoid, *P* lies on *LO*, and angle measures are as marked. What is the measure of angle *MON*?



A. 40°
B. 45°
C. 50°
D. 90°
E. 95°

20. For what value of *n* would the following system of equations have an infinite number of solutions?

$$4x + 5y = 16$$
$$20x + 25y = 2n$$

A. 8
B. 10
C. 32
D. 40
E. 80