Week Two

Algebra II

Desoto County Schools

NAME _____ DATE _____ PERIOD ____

9-3 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Completing the Square

Complete the Square Since few quadratic expressions are perfect square trinomials, the method of **completing the square** can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

Step 1	Find $\frac{b}{2}$.
Step 2	Find $\left(\frac{b}{2}\right)^2$.
Step 3	Add $\left(\frac{b}{2}\right)^2$ to $ax^2 + bx$.

Example **Solve $x^2 + 6x + 3 = 10$ by completing the square.**

$x^2 + 6x + 3 = 10$	Original equation
$x^2 + 6x + 3 - 3 = 10 - 3$	Subtract 3 from each side.
$x^2 + 6x = 7$	Simplify.
$x^2 + 6x + 9 = 7 + 9$	Since $\left(\frac{6}{2}\right)^2 = 9$, add 9 to each side.
$(x + 3)^2 = 16$	Factor $x^2 + 6x + 9$.
$x + 3 = \pm 4$	Take the square root of each side.
$x = -3 \pm 4$	Simplify.

$x = -3 + 4$ or $x = -3 - 4$
$= 1$ $= -7$

The solution set is $\{-7, 1\}$.

Exercises

Solve each equation by completing the square. Round to the nearest tenth if necessary.

1. $t^2 - 4t + 3 = 0$
2. $y^2 + 10y = -9$
3. $y^2 - 8y - 9 = 0$
4. $x^2 - 6x = 16$
5. $p^2 - 4p - 5 = 0$
6. $x^2 - 12x = 9$
7. $c^2 + 8c = 20$
8. $p^2 = 2p + 1$
9. $x^2 + 20x + 11 = -8$
10. $x^2 - 1 = 5x$
11. $a^2 = 22a + 23$
12. $m^2 - 8m = -7$
13. $x^2 + 10x = 24$
14. $a^2 - 18a = 19$
15. $b^2 + 16b = -16$
16. $4x^2 = 24 + 4x$
17. $2m^2 + 4m + 2 = 8$
18. $4k^2 = 40k + 44$

Lesson 9-3

NAME _____ DATE _____ PERIOD ____

9-3 Skills Practice

Solving Quadratic Equations by Completing the Square

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $c^2 - 12c + 36 = 4$

2. $w^2 - 10w + 25 = 16$

3. $b^2 + 16b + 64 = 9$

4. $y^2 + 2y + 1 = 3$

5. $r^2 + 4r + 4 = 7$

6. $a^2 - 8a + 16 = 12$

Find the value of c that makes each trinomial a perfect square.

7. $g^2 + 6g + c$

8. $y^2 + 4y + c$

9. $a^2 - 14a + c$

10. $n^2 - 2n + c$

11. $s^2 - 18s + c$

12. $p^2 + 20p + c$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

13. $x^2 + 4x - 12 = 0$

14. $v^2 - 8v + 15 = 0$

15. $q^2 + 6q = 7$

16. $r^2 - 2r = 15$

17. $m^2 - 14m + 30 = 6$

18. $b^2 + 12b + 21 = 10$

19. $z^2 - 4z + 1 = 0$

20. $y^2 - 6y + 4 = 0$

21. $r^2 - 8r + 10 = 0$

22. $p^2 - 2p = 5$

23. $2a^2 + 20a = -2$

24. $0.5g^2 + 8g = -7$

NAME _____ DATE _____ PERIOD ____

9-4 Study Guide and Intervention (continued)

Solving Quadratic Equations by Using the Quadratic Formula

The Discriminant In the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The discriminant can be used to determine the number of real roots for a quadratic equation.

Case 1: $b^2 - 4ac < 0$	Case 2: $b^2 - 4ac = 0$	Case 3: $b^2 - 4ac > 0$
no real roots	one real root	two real roots

Example **State the value of the discriminant for each equation. Then determine the number of real roots.**

a. $12x^2 + 5x = 4$

Write the equation in standard form.

$12x^2 + 5x = 4$ Original equation

$12x^2 + 5x - 4 = 4 - 4$ Subtract 4 from each side.

$12x^2 + 5x - 4 = 0$ Simplify.

Now find the discriminant.

$b^2 - 4ac = (5)^2 - 4(12)(-4)$

$= 217$

Since the discriminant is positive, the equation has two real roots.

b. $2x^2 + 3x = -4$

$2x^2 + 3x = -4$ Original equation

$2x^2 + 3x + 4 = -4 + 4$ Add 4 to each side.

$2x^2 + 3x + 4 = 0$ Simplify.

$b^2 - 4ac = (3)^2 - 4(2)(4)$

$= -23$

Since the discriminant is negative, the equation has no real roots.

Exercises

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

1. $3x^2 + 2x - 3 = 0$
2. $3n^2 - 7n - 8 = 0$
3. $2d^2 - 10d - 9 = 0$
4. $4x^2 = x + 4$
5. $3x^2 - 13x = 10$
6. $6x^2 - 10x + 10 = 0$
7. $2k^2 - 20 = -k$
8. $6p^2 = -11p - 40$
9. $9 - 18x + 9x^2 = 0$
10. $12x^2 + 9 = -6x$
11. $9a^2 = 81$
12. $16y^2 + 16y + 4 = 0$
13. $8x^2 + 9x = 2$
14. $4a^2 - 4a + 4 = 3$
15. $3b^2 - 18b = -14$

NAME _____ DATE _____ PERIOD _____

9-4 Skills Practice

Solving Quadratic Equations by Using the Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $u^2 - 49 = 0$

2. $n^2 - n - 20 = 0$

3. $s^2 - 5s - 36 = 0$

4. $b^2 + 11b + 30 = 0$

5. $c^2 - 7c = -3$

6. $p^2 + 4p = -1$

7. $a^2 - 9a + 22 = 0$

8. $x^2 + 6x + 3 = 0$

9. $2x^2 + 5x - 7 = 0$

10. $2h^2 - 3h = -1$

11. $2p^2 + 5p + 4 = 0$

12. $2g^2 + 7g = 9$

13. $3t^2 + 2t - 3 = 0$

14. $3x^2 - 7x - 6 = 0$

State the value of the discriminant for each equation. Then determine the number of real roots of the equation.

15. $q^2 + 4q + 3 = 0$

16. $m^2 + 2m + 1 = 0$

17. $a^2 - 4a + 10 = 0$

18. $w^2 - 6w + 7 = 0$

19. $z^2 - 2z - 7 = 0$

20. $y^2 - 10y + 25 = 0$

21. $2d^2 + 5d - 8 = 0$

22. $2s^2 + 6s + 12 = 0$

23. $2u^2 - 4u + 10 = 0$

24. $3h^2 + 7h + 3 = 0$

MORE ON COMPLETING THE SQUARE

Although students can find the vertex of a parabola by averaging the x-intercepts, they also can use the algebraic method known as completing the square. This allows students to go directly from standard (or non-graphing) form to graphing form without the intermediate step of finding the x-intercepts. Completing the square is also used when the equation of a circle is written in an expanded form. When the students first looked at how to complete the square, they used tiles so that they could see how the method works. When they tried to create a square (complete it) by arranging the tiles, there were either too many or missing parts. This visual representation helps students see how to rewrite the equation algebraically.

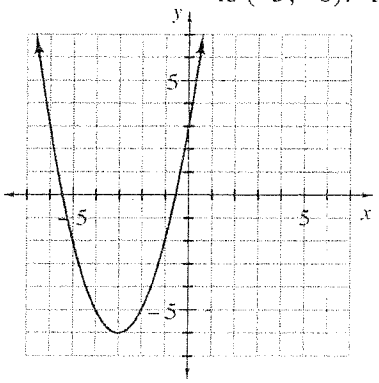
Example 1

The function $f(x) = x^2 + 6x + 3$ is written in standard form. Complete the square to write it in graphing form. Then state the vertex of the parabola and sketch the graph.

The general equation of a parabola in graphing form is $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex. The original equation needs to be changed into a set of parentheses squared, with a constant either added to or subtracted from it. To do this, we must know that $(x - h)^2 = x^2 - 2xh + h^2$. We will use this form of a perfect square to complete the square of the given equation or function.

$$\begin{aligned} f(x) &= x^2 + 6x + 3 \\ &= x^2 + 6x + \square + 3 - \square \\ &= x^2 + 6x + \boxed{9} + 3 - \boxed{9} \\ &= (x + 3)^2 - 6 \end{aligned}$$

The first box holds a space for the number we have to add to complete the square. The second box is to subtract that same number so as not to change the balance of the equation. To determine the missing number, take half the coefficient of x (half of 6), and then square it and place the result in both boxes. With the equation in graphing form, we know the vertex is $(-3, -6)$. The graph is shown below.



Example 2

The equation $x^2 - 8x + y^2 + 16y = 41$ is the equation of a circle. Complete the square to determine the coordinates of its center and the length of the radius.

As with the last example, we will fill in the blanks to create perfect squares. We need to do this twice: once for x, and again for y.

$$x^2 - 8x + y^2 + 16y = 41$$
$$x^2 - 8x + \square - \square + y^2 + 16y + \square - \square = 41$$
$$x^2 - 8x + \boxed{16} - \boxed{16} + y^2 + 16x + \boxed{64} - \boxed{64} = 41$$
$$(x-4)^2 - 16 + (y+8)^2 - 64 = 41$$
$$(x-4)^2 + (y+8)^2 = 41 + 16 + 64$$
$$(x-4)^2 + (y+8)^2 = 121$$

This is a circle with center $(4, -8)$ and a radius of $\sqrt{121} = 11$.

Problems

Write each of the following equations in graphing form. Then state the vertex and the direction the parabola opens.

1. $y = x^2 - 8x + 18$
2. $y = -x^2 - 2x - 7$
3. $y = 3x^2 - 24x + 42$
4. $y = 2x^2 - 6$
5. $y = \frac{1}{2}x^2 - 3x + \frac{1}{2}$
6. $y = x^2 + 18x + 97$

Find the center and radius of each circle.

7. $(x+2)^2 + (y+7)^2 = 25$
8. $3(x-9)^2 + 3(y+1)^2 = 12$
9. $x^2 + 6x + y^2 = 91$
10. $x^2 - 10x + y^2 + 14y = -58$
11. $x^2 + 50x + y^2 - 2y = -602$
12. $x^2 + y^2 - 8x - 16y = 496$

AAT Quadratics Test Review

Name_____

MULTIPLE CHOICE.

Solve the equation by factoring.

1) $9x^2 - 9x = 0$ 1) _____

A) $\{0, 1\}$ B) $\{-1, 0\}$ C) $\{0\}$ D) $\{1, -1\}$

2) $x^2 = x + 12$ 2) _____

A) $\{-3, 4\}$ B) $\{-3, -4\}$ C) $\{3, 4\}$ D) $\{1, 12\}$

3) $-6x - 2 = (3x + 1)^2$ 3) _____

A) $\left\{\frac{1}{3}, 1\right\}$ B) $\left\{-1, -\frac{1}{3}\right\}$ C) $\left\{-\frac{1}{3}\right\}$ D) $\varnothing$

Solve the equation by completing the square.

4) $x^2 + 8x = 5$ 4) _____

A) $\{4 + \sqrt{21}\}$

B) $\{-1 - \sqrt{21}, -1 + \sqrt{21}\}$

C) $\{-4 - \sqrt{21}, -4 + \sqrt{21}\}$

D) $\{-4 - 2\sqrt{21}, -4 + 2\sqrt{21}\}$

Solve the equation by the square root property.

5) $4x^2 + 4 = 20$ 5) _____

A) $\{10\}$ B) $\{-3, 3\}$ C) $\{-2, 2\}$ D) $\{2\}$

6) $3(x - 3)^2 = 18$ 6) _____

A) $\{-9, 3\}$ B) $\{-3 \pm \sqrt{6}\}$ C) $\{3 \pm \sqrt{6}\}$ D) $\{-3, 9\}$

7) $(x - 3)^2 = -16$ 7) _____

A) $\{-3 \pm 4i\}$ B) $\{3i \pm 4\}$ C) $\{3 \pm 4i\}$ D) $\left\{\pm\frac{4i}{3}\right\}$

8) $(5x - 12)^2 = 8$ 8) _____

A) $\{-2\sqrt{5}, 2\sqrt{5}\}$

B) $\left\{\frac{-12 - 2\sqrt{2}}{5}, \frac{-12 + 2\sqrt{2}}{5}\right\}$

C) $\left\{\frac{12 - 2\sqrt{2}}{5}, \frac{12 + 2\sqrt{2}}{5}\right\}$

D) $\left\{\frac{4}{5}, 4\right\}$

Solve the equation by completing the square.

9) $5x^2 - 2x - 4 = 0$ 9) _____

A) $\left\{\frac{5 - \sqrt{21}}{25}, \frac{5 + \sqrt{21}}{25}\right\}$

B) $\left\{-4, \frac{22}{5}\right\}$

C) $\left\{\frac{1 - \sqrt{21}}{5}, \frac{1 + \sqrt{21}}{5}\right\}$

D) $\left\{\frac{-1 - \sqrt{21}}{5}, \frac{-1 + \sqrt{21}}{5}\right\}$

10) $x^2 + 18x + 58 = 0$ 10) _____

A) $\{9 - \sqrt{58}, 9 + \sqrt{58}\}$

B) $\{-9 - \sqrt{23}, -9 + \sqrt{23}\}$

C) $\{-18 + \sqrt{58}\}$

D) $\{9 + \sqrt{23}\}$

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Solving Quadratics MC

Solve the equation using the quadratic formula.

11) $x^2 - 10x + 34 = 0$ 11) _____

A) $\{5 - 3i, 5 + 3i\}$ B) $\{2, 8\}$ C) $\{5 + 3i\}$ D) $\{5 - 9i, 5 + 9i\}$

12) $9x^2 + 7x + 2 = 0$ 12) _____

A) $\left\{\frac{-7 \pm \sqrt{23}}{18}\right\}$ B) $\left\{\frac{7 \pm i\sqrt{23}}{18}\right\}$ C) $\left\{\frac{7 \pm \sqrt{23}}{18}\right\}$ D) $\left\{\frac{-7 \pm i\sqrt{23}}{18}\right\}$

13) $x^2 + 7x - 60 = 0$ 13) _____

A) $\{-12, 1\}$ B) $\{-5, 12\}$ C) $\{12, 5\}$ D) $\{-12, 5\}$

14) $7x^2 + 12x + 3 = 0$ 14) _____

A) $\left\{\frac{-12 - \sqrt{15}}{7}, \frac{-12 + \sqrt{15}}{7}\right\}$ B) $\left\{\frac{-6 - \sqrt{57}}{7}, \frac{-6 + \sqrt{57}}{7}\right\}$

C) $\left\{\frac{-6 - \sqrt{15}}{7}, \frac{-6 + \sqrt{15}}{7}\right\}$ D) $\left\{\frac{-6 - \sqrt{15}}{14}, \frac{-6 + \sqrt{15}}{14}\right\}$

Solve the equation by the method of your choice.

15) $(4x + 6)^2 = 25$ 15) _____

A) $\left\{\frac{1}{4}, \frac{11}{4}\right\}$ B) $\left\{-\frac{1}{4}, 0\right\}$ C) $\left\{\frac{19}{4}\right\}$ D) $\left\{-\frac{11}{4}, -\frac{1}{4}\right\}$

16) $4x^2 - 19x - 5 = 0$ 16) _____

A) $\{-4, 5\}$ B) $\left\{-\frac{1}{4}, 4\right\}$ C) $\left\{-\frac{1}{4}, \frac{1}{19}\right\}$ D) $\left\{-\frac{1}{4}, 5\right\}$

17) $3x^2 + 8x = -1$ 17) _____

A) $\left\{\frac{-4 - \sqrt{13}}{3}, \frac{-4 + \sqrt{13}}{3}\right\}$ B) $\left\{\frac{-4 - \sqrt{13}}{6}, \frac{-4 + \sqrt{13}}{6}\right\}$

C) $\left\{\frac{-6 - \sqrt{42}}{6}, \frac{-6 + \sqrt{42}}{6}\right\}$ D) $\left\{\frac{-12 - \sqrt{30}}{6}, \frac{-12 + \sqrt{30}}{6}\right\}$

18) $3x^2 = -12x - 7$ 18) _____

A) $\left\{\frac{-6 - \sqrt{57}}{3}, \frac{-6 + \sqrt{57}}{3}\right\}$ B) $\left\{\frac{-6 - \sqrt{15}}{6}, \frac{-6 + \sqrt{15}}{6}\right\}$

C) $\left\{\frac{-6 - \sqrt{15}}{3}, \frac{-6 + \sqrt{15}}{3}\right\}$ D) $\left\{\frac{-12 - \sqrt{15}}{3}, \frac{-12 + \sqrt{15}}{3}\right\}$

19) $6x^2 + 10x + 1 = 0$ 19) _____

A) $\left\{\frac{-5 - \sqrt{31}}{6}, \frac{-5 + \sqrt{31}}{6}\right\}$ B) $\left\{\frac{-5 - \sqrt{19}}{6}, \frac{-5 + \sqrt{19}}{6}\right\}$

C) $\left\{\frac{-10 - \sqrt{19}}{6}, \frac{-10 + \sqrt{19}}{6}\right\}$ D) $\left\{\frac{-5 - \sqrt{19}}{12}, \frac{-5 + \sqrt{19}}{12}\right\}$

20) $3x^2 = 45$ 20) _____

A) {16} B) $\{-\sqrt{15}, \sqrt{15}\}$ C) {-15, 15} D) {22.5}

21) $5x^2 - 10 = 0$ 21) _____

A) $\{-\sqrt{2}, \sqrt{2}\}$ B) $\left\{-\frac{\sqrt{10}}{5}, \frac{\sqrt{10}}{5}\right\}$ C) $\{\sqrt{2}\}$ D) $\{-\sqrt{10}, \sqrt{10}\}$

22) $x^2 + 18x + 62 = 0$ 22) _____

A) $\{9 + \sqrt{19}\}$ B) $\{9 - \sqrt{62}, 9 + \sqrt{62}\}$

C) $\{-18 + \sqrt{62}\}$ D) $\{-9 - \sqrt{19}, -9 + \sqrt{19}\}$

23) $4x^2 - 56x + 260 = 0$ 23) _____

A) {7 + 4i} B) {3, 11} C) {7 - 16i, 7 + 16i} D) {7 - 4i, 7 + 4i}

24) $(3x + 5)^2 = 7$ 24) _____

A) $\left\{\frac{-5 \pm \sqrt{7}}{3}\right\}$ B) $\left\{-4, \frac{2}{3}\right\}$ C) $\left\{\frac{5 \pm \sqrt{7}}{3}\right\}$ D) $\left\{\frac{\sqrt{7} \pm 5}{3}\right\}$

25) $(x + 2)(x - 1) = 2$ 25) _____

A) $\left\{\frac{1 \pm \sqrt{17}}{2}\right\}$ B) $\left\{\frac{-1 \pm \sqrt{17}}{2}\right\}$ C) $\left\{\frac{1 \pm i\sqrt{17}}{2}\right\}$ D) $\left\{\frac{-1 \pm i\sqrt{17}}{2}\right\}$

Solve the problem.

26) The formula $N = 3x^2 + 4x + 2$ represents the number of households N, in thousands, in a certain city that have a computer x years after 1990. According to the formula, in what year were there 134 thousand households with computers in this city? 26) _____

A) 1995 B) 1994 C) 1996 D) 1997

27) A square sheet of paper measures 43 centimeters on each side. What is the length of the diagonal of this paper? 27) _____

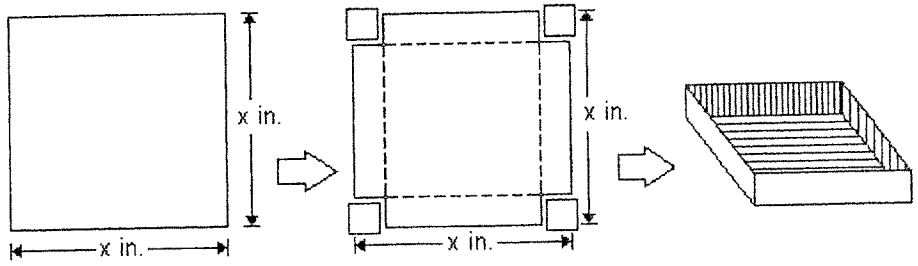
A) 3698 cm B) 86 cm C) 43 cm D) $43\sqrt{2}$ cm

28) The length of a rectangular storage room is 3 feet longer than its width. If the area of the room is 180 square feet, find its dimensions. 28) _____

A) 11 feet by 16 feet B) 13 feet by 16 feet

C) 11 feet by 14 feet D) 12 feet by 15 feet

29) Suppose that an open box is to be made from a square sheet of cardboard by cutting out 4-inch squares from each corner as shown and then folding along the dotted lines. If the box is to have a volume of 100 cubic inches, find the original dimensions of the sheet of cardboard. 29) _____



A) 13 in. by 13 in.
B) 10 in. by 10 in.
C) 5 in. by 5 in.
D) $\sqrt{5}$ in. by $2\sqrt{5}$ in.

30) A machine produces open boxes using square sheets of plastic. The machine cuts equal-sized squares measuring 4 inches on a side from each corner of the sheet, and then shapes the plastic into an open box by turning up the sides. If each box must have a volume of 1600 cubic inches, find the length of one side of the open box. 30) _____

A) 20 in.
B) 19 in.
C) 28 in.
D) 24 in.

NAME_____ DATE_____ CLASS_____

ACT MATH SKILLS PREP – FACTORING

1. Which of the following is a polynomial factor of x^2-5x+4?

   A. $x-1$
   B. $x+1$
   C. $x-5$
   D. $x+5$
   E. $x+4$

2. Which of the following is a factor of the polynomial $x^2-5x-24$?

   A. $(x-6)$
   B. $(x+6)$
   C. $(x-8)$
   D. $(x-2)$
   E. $(x-12)$

3. Which of the following is a factor of the expression $(3z^2+11z-4)$?

   A. $(z-1)$
   B. $(z-4)$
   C. $(z+4)$
   D. $(3z-4)$
   E. $(3z+1)$

4. Which of the following is NOT a factor of the polynomial x^4-81?

   A. (x^2-9)
   B. (x^2+9)
   C. $(x-81)$
   D. $(x+9)$
   E. $(x-9)$

5. Which of the following is a factored form of the expression $12x^2+5x-2$?

   A. $(4x-1)(3x+2)$
   B. $(4x-1)(3x-2)$
   C. $(4x+1)(3x+2)$
   D. $(3x-1)(4x+2)$
   E. $(4x-3)(3x+1)$

6. Which of the following is equivalent to the expression $(n+2)(n-3)+4$?

   A. $(n+6)(n+1)$
   B. $(n+8)(n-12)$
   C. $(n+2)(n+1)$
   D. $(n-2)(n-1)$
   E. $(n-2)(n+1)$

NAME_____ DATE_____ CLASS_____

ACT MATH SKILLS PREP – FACTORING Solutions

1. The correct answer is **A**.

   The expression x^2-5x+4 can be factored as $(x-4)(x-1)$.

2. The correct answer is **C**.

   The expression $x^2-5x-24$ can be factored as $(x-8)(x+3)$.

3. The correct answer is **C**.

   The expression $3z^2+11z-4$ can be factored as $(z+4)(3z-1)$.

4. The correct answer is **C**.

   $x^4-81=(x^2-9)(x^2+9)$

   $=(x+3)(x-3)(x^2+9)$

5. The correct answer is **A**.

   Expanding, $(4x-1)(3x+2)$ is equivalent to $12x^2+5x-2$.

6. The correct answer is **E**.

   $(n+2)(n-3)+4=n^2-n-6-4$

   $=n^2-n-2=(n-2)(n+1)$