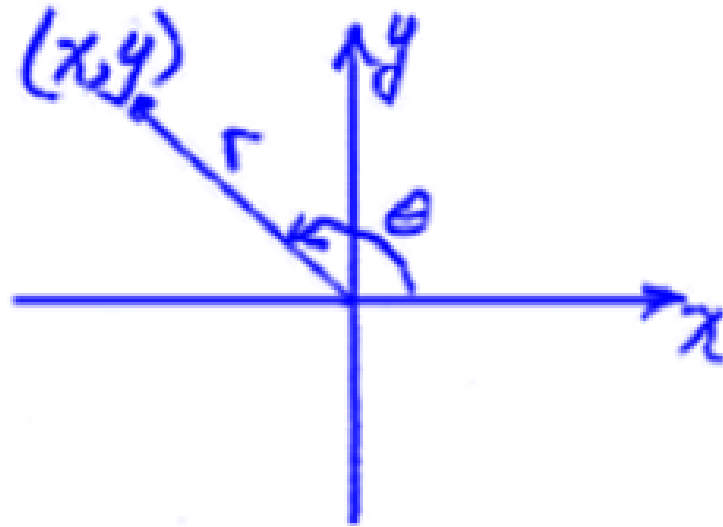


We have two definitions of the sine (abbreviated sin), cosine (abbreviated cos), and tangent (abbreviated tan) of an angle (we will call our angle by the Greek letter theta, θ):

First definition (x, y, & r): Begin with an angle in standard position. *** r is always positive***



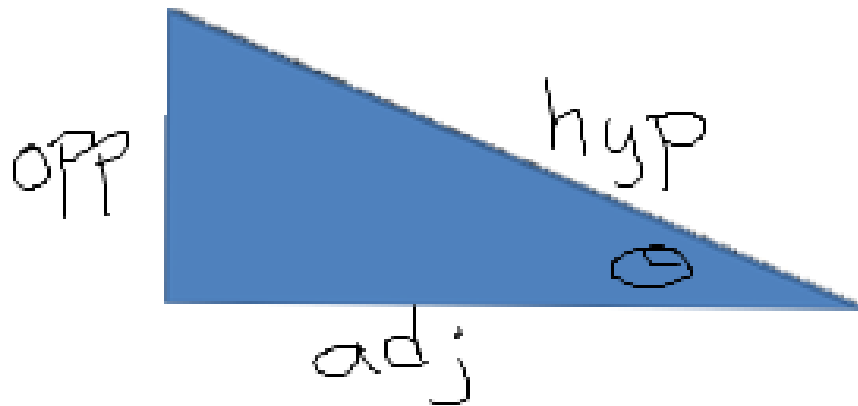
$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

Second definition (opp, adj, hyp): (opposite, adjacent, hypotenuse) Let θ be one of the acute angles of a right triangle



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

Defining the other three trig functions: The basic fact to remember here is that cosecant (csc), secant (sec), and cotangent (cot) are reciprocals respectively of sin, cos, and tan



$$\mathbf{\sin \theta = \frac{opp}{hyp}}$$

$$\mathbf{\cos \theta = \frac{adj}{hyp}}$$

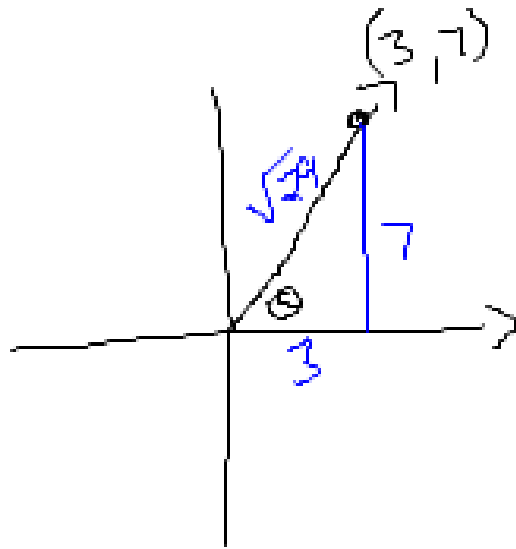
$$\mathbf{\tan \theta = \frac{opp}{adj}}$$

$$\mathbf{\csc \theta = \frac{hyp}{opp}}$$

$$\mathbf{\sec \theta = \frac{hyp}{adj}}$$

$$\mathbf{\cot \theta = \frac{adj}{opp}}$$

Example 1: Draw the angle, θ , in standard position where $(3, 7)$ lies on the terminal side. Find the values of all six trig functions.



$$\begin{aligned}3^2 + 7^2 &= r^2 \\29 &= r^2 \\ \sqrt{29} &= r\end{aligned}$$

$$\sin \theta = \frac{7}{\sqrt{29}} = \frac{7\sqrt{29}}{29}$$

$$\cos \theta = \frac{3}{\sqrt{29}} = \frac{3\sqrt{29}}{29}$$

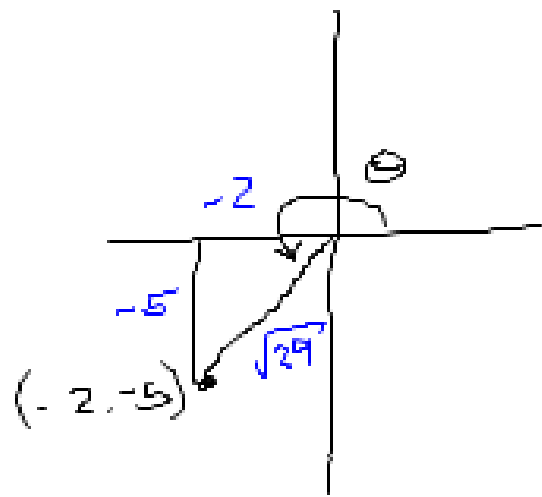
$$\tan \theta = \frac{7}{3}$$

$$\csc \theta = \sqrt{29}/7$$

$$\sec \theta = \sqrt{29}/3$$

$$\cot \theta = 3/7$$

Example 2: Draw the angle, θ , in standard position where $(-2, -5)$ lies on the terminal side. Find the values of all six trig functions.



$$\begin{aligned}(-2)^2 + (-5)^2 &= r^2 \\ 4 + 25 &= r^2 \\ \sqrt{29} &= r\end{aligned}$$

$$\sin \theta = \frac{-5}{\sqrt{29}} = \frac{-5\sqrt{29}}{29}$$

$$\cos \theta = \frac{-2}{\sqrt{29}} = \frac{-2\sqrt{29}}{29}$$

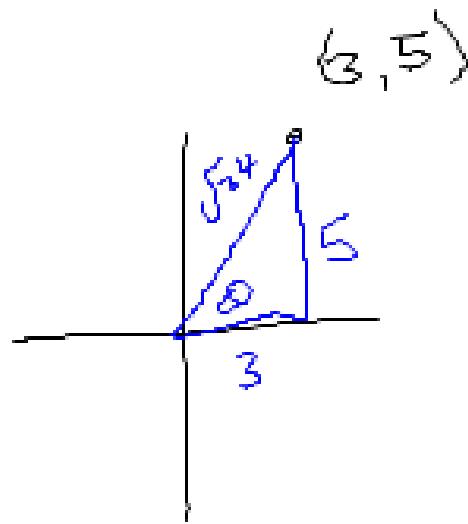
$$\tan \theta = \frac{-5}{-2} = \frac{5}{2}$$

$$\csc \theta = \frac{-\sqrt{29}}{5}$$

$$\sec \theta = \frac{-\sqrt{29}}{2}$$

$$\cot \theta = \frac{2}{5}$$

Example 3: Draw the angle, θ , in standard position where $(3, 5)$ lies on the terminal side. Find the values of all six trig functions.



$$3^2 + 5^2 = r^2$$

$$9 + 25 = r^2$$

$$\sqrt{34} = r$$

$$\sin \theta = \frac{5}{\sqrt{34}} = \frac{5\sqrt{34}}{34}$$

$$\cos \theta = \frac{3}{\sqrt{34}} = \frac{3\sqrt{34}}{34}$$

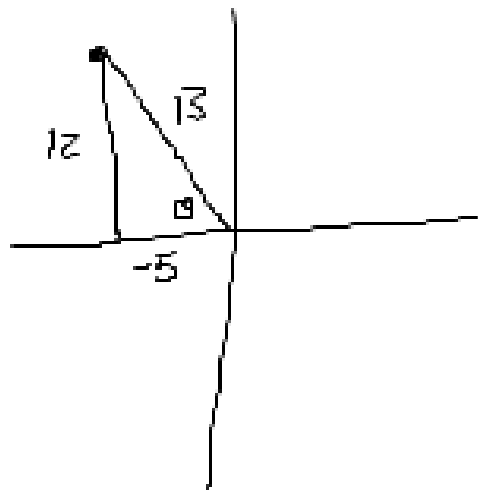
$$\tan \theta = \frac{5}{3}$$

$$\csc \theta = \frac{\sqrt{34}}{5}$$

$$\sec \theta = \frac{\sqrt{34}}{3}$$

$$\cot \theta = \frac{3}{5}$$

Example 4: Draw the angle, θ , in standard position where $\sin\theta = 12/13$ and $\tan\theta$ is negative. Find the values of all six trig functions.



$$x^2 + 12^2 = 13^2$$

$$x^2 + 144 = 169$$

$$x^2 = 25$$

$$x = \pm 5$$

since
 $\tan\theta$ is neg

$$x = -5$$

$$\sin\theta = 12/13$$

$$\cos\theta = -5/13$$

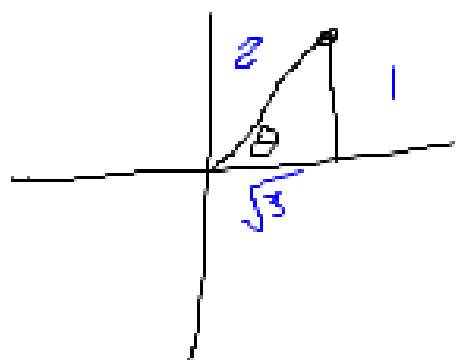
$$\tan\theta = -12/5$$

$$\csc\theta = 13/12$$

$$\sec\theta = -13/5$$

$$\cot\theta = -5/12$$

Example 5: If $\text{Csc}\theta = 2$, and $\text{Tan}\theta > 0$, determine the other 5 trig functions.



$$2^2 = 1^2 + x^2$$

$$4 = 1 + x^2$$

$$3 = x^2$$

$$\sqrt{3} = x$$

since $\tan\theta > 0$

$$x = \sqrt{3}$$

$$\sin\theta = \frac{1}{2}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\csc\theta = \frac{2}{1}$$

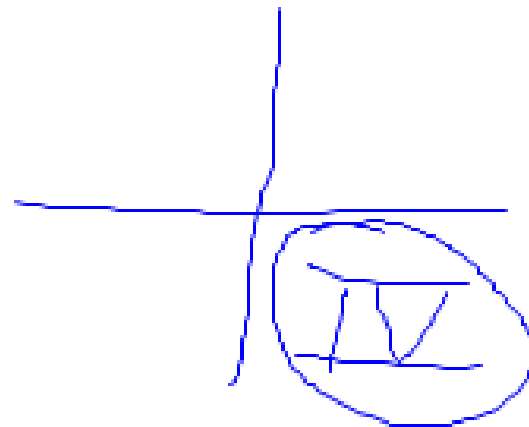
$$\sec\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot\theta = \frac{\sqrt{3}}{1}$$

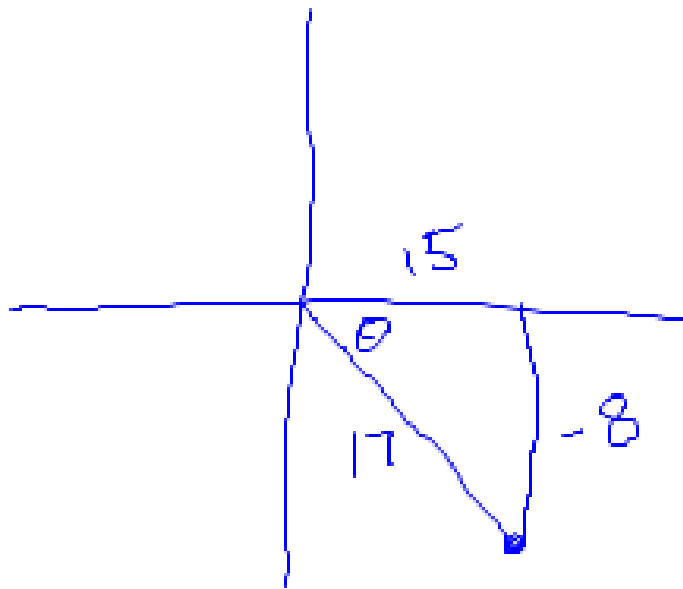
Example 6: If the sine of an angle is negative and the cosine is positive, in what quadrant is the angle?

$$\sin \theta = \frac{y}{r} < 0 \quad \text{so} \quad y < 0$$

$$\cos \theta = \frac{x}{r} > 0 \quad \text{so} \quad x > 0$$



Example 7: Given that $\cos \theta = 15/17$ and the terminal side of the angle is in the 4th quadrant, determine the other 5 trig functions.



$$\sin \theta = -8/17$$

$$\cos \theta = 15/17$$

$$\tan \theta = -8/15$$

$$\csc \theta = -17/8$$

$$\sec \theta = 17/15$$

$$\cot \theta = -15/8$$

Homework:

Complete the handout