Yellow

$\frac{1}{4}\int 4x^3(x^4+3)^2\,dx$

$u = x^4+3$

$du = 4x^3\,dx$

$\frac{1}{4}\int u^2\,du \quad = \frac{1}{4}\cdot\frac{1}{3}(u)^3 + c$

$\boxed{\frac{1}{12}(x^4+3)^3 + C}$

$-\frac{1}{2}\int -2\cdot 5x\sqrt[3]{1-x^2}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int 5u^{1/3}\,du \quad -\frac{1}{2}\cdot 5\cdot\frac{3}{4}(u)^{4/3} + c$

$\boxed{\frac{-15}{8}(1-x^2)^{4/3} + c}$

$\int 18x^2(2x^3+1)^2\,dx$

$u = 2x^3+1$

$du = 6x^2\,dx$

$\int 3u^2\,du = u^3 + C \quad \boxed{(2x^3+1)^3 + C}$

$\frac{1}{2}\int \frac{2(x-4)}{\sqrt{x^2-8x+1}}\,dx$

$u = x^2-8x+1$

$du = 2x-8\,dx$

$\frac{1}{2}\int u^{-1/2}\,du \quad \frac{1}{2}\cdot 2(u)^{1/2} + c \quad \boxed{(x^2-8x+1)^{1/2} + c}$

$\int x\left(\sqrt{x-3}\right)^3\,dx$

$u = x-3$

$x = x-3+3 \quad du = dx$

$= u+3$

$\int (u+3)(u^{1/2})\,du \quad \boxed{\frac{2}{5}(x-3)^{5/2} + 2(x-3)^{3/2} + c}$

$\int u^{3/2} + 3u^{1/2}\,du$

Blue

$$\int x^2 - 6x + 1\, dx$$

$\frac{1}{3}x^3 - 3x^2 + x + C$

$$\int \cos(4x)\, dx$$

$\frac{1}{4}\sin(4x) + C$

$$\int \frac{x^2 + 2x - 3}{x^4}\, dx$$

$\int x^{-2} + 2x^{-3} - 3x^{-4}\, dx$

$-x^{-1} - x^{-2} + x^{-3} + C$

$$\int \sec^2 x - \sin x\, dx$$

$\tan x + \cos x + C$

$$\int x^{\frac{4}{3}} - \frac{1}{x^3}\, dx = \int x^{4/3} - x^{-3}$$

$\frac{3}{7}x^{7/3} + \frac{1}{2}x^{-2} + C$

Orange

$v(t) = 6t^2 - 4t + 1, \quad s(2) = 7$

$s(t) = \int 6t^2 - 4t + 1 \, dt$

$2t^3 - 2t^2 + t + C$

$s(t) = 2t^3 - 2t^2 + t - 3$

$2(2)^3 - 2(2)^2 + 2 + C = 7$

$C = -3$

$a(t) = 7t - 2, \quad v(1) = 5 \text{ and } s(2) = 7$

$v(t) = \int 7t - 2 \, dt = \frac{7}{2}t^2 - 2t + C$

$7/2(1)^2 - 2t + C = 5$

$C = 3.5$

$s(t) = \int 7/2 t^2 - 2 + 3.5 \, dt = \frac{7}{6}t^3 - t^2 + 3.5t + C$

$s(t) = \frac{7}{6}t^3 - t^2 + 3.5t - 16/3$

$7/6(2)^3 - (2)^2 + 3.5(2) + C = 7$

$C = -16/3$

$a(t) = \sin(t), \quad v(0) = 2 \text{ and } s(0) = 4$

$v(t) = \int \sin t \, dt = -\cos t + C$

$-\cos(0) + C = 2$

$C = 3$

$s(t) = \int -\cos t + 3 \, dt$

$s(t) = -\sin(t) + 3t + 4$

$-\sin t + 3t + C$

$-\sin(0) + 3(0) + C = 4 \quad C = 4$

$f''(x) = e^x, \quad f'(0) = 4 \text{ and } f(1) = 0$

$f'(x) = \int e^x \, dx = e^x + C$

$e^0 + C = 4$

$C = 3$

$f(x) = \int e^x + 3 \, dx = e^x + 3x + C$

$e^1 + 3(1) + C = 0$

$C = -5.718$

$\boxed{f(x) = e^x + 3x - 5.718}$

Red

Approximate the area under $f(x) = \frac{1}{x}$ with a LHS using 4 rectangles of equal width from x = 1 to x =3. Then compare this answer to the actual amount found using FnINT.

1	1.5	2	2.5	3
1	2/3	1/2	2/5	1/3

$$\underset{\text{base}}{.5}\Big[\underset{\text{height}}{1 + 2/3 + 1/2 + 2/5}\Big] = \frac{77}{60} \approx 1.28\overline{3}$$

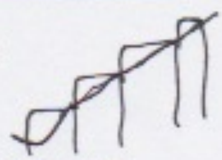
$$\int_1^3 \frac{1}{x}\,dx = 1.0986$$

Approximate the area under the curve using a RHS given the following selected values. Is your approximation an over/under estimate and why? Indicate Units.

Minutes	0	2	5	11	12	15	20
Gallons per Minute	5	8	9	12	18	19	21

$2(8) + 3(9) + 6(12) + 1(18) + 3(19) + 5(21) = 295$ gallons

over   increasing + RHS



Approximate the area under the curve using a TRS given the following selected values. Indicate Units.

Seconds	0	1	2	3	5	7	8
Feet/Second	0	2	6	13	18	25	39

$\frac{1}{2}\Big[1(2) + 1(8) + 1(19) + 2(31) + 2(43) + 1(64)\Big] = 120.5$ ft

Approximate the area under the curve using a MRS given the following selected values.

X	-2	-1	0	5	10	13	16
f(x)	3	1	5	8	9	11	14

$2(1) + 10(8) + 6(11) = 148$

Green

Free Response:

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

a) $a(36) = v'(36) \approx \frac{v(40) - v(32)}{40 - 32} = \frac{7-(-4)}{8} = 11/8 \text{ m/sec}^2$

b) $\int_{20}^{40} v(t)dt$ is the change in position from $t=20$ + $t=40$

$\frac{1}{2}\left[5(-18) + 7(-12) + 8(3)\right] = -75$ meters

c) Must change direction on $8<t<20$ and $32<t<40$ because $v(t) = s'(t)$ changes signs.

$v(8) > 0$ $\quad$ $v(32) < 0$

$v(20) < 0$ $\quad$ $v(40) > 0$

d) Since acceleration + velocity are positive on $0<t<8$, $v(t) \geq 3$ since the particle is speeding up.

So $s(8) \geq 7 + 3 \cdot 8 > 30$