Paulsboro Schools

Curriculum

Precalculus

October 2012

*** For adoption by all regular education programs as specified and for adoption or adaptation by all Special Education Programs in accordance with Board of Education Policy.**

Board Approved: Nov 2012

AP Calculus AB

Course Overview

Our course covers a review of functions as well as both Differential and Integral Calculus. This is a college level course to prepare students to take the Advance Placement Test and for all higher level mathematics courses. We have aligned our objectives with the AP Calculus Course description. Students are asked to complete an extensive summer assignment prior to the start of school. The first week of school we review their summer assignment. All students are given a graphing calculator (TI-83) to use during the year.

Course Outline

I. Preparation for Calculus Chapter P

1 week

Students complete this review of Precalculus as a summer assignment.

II. Limits and Their Properties

3 weeks

1. Finding limits numerically and graphically
2. Evaluate limits analytically
3. Determine one-sided limits and continuity
4. Determine infinite limits and vertical asymptotes

III. Differentiation

6 weeks

1. Use the definition of derivative to find the derivative of a function
2. Use the basic differentiation rules to find the derivative of a function
3. Use derivatives to find rate of change
4. Use the Product and Quotient rules to find the derivative of a function
5. Find higher-order derivatives
6. Use the Chain Rule to find the derivative of a function
7. Use Implicit Differentiation to find the derivative of a function
8. Use Related Rates to solve real-life problems

IV. Applications of Differentiation

6 weeks

1. Finding extrema on an interval
2. Use Rolle’s Theorem and Mean Value Theorem
3. Determine the intervals on which a function is increasing or decreasing
4. Apply the first derivative test to find relative extrema of a function
5. Determine concavity and find points of inflection
6. Apply the Second derivative test to find relative extrema
7. Determine limits at infinity and find horizontal asymptotes
8. Use L’Hopital’s rule to evaluate limits of indeterminate form
9. Analyze and sketch the graph of a function
10. Solve optimization problems
11. Approximate a zero of a function using Newton’s Method
12. Linear Approximations

V. Integration

6 weeks

1. Use basic integration rules to find the antiderivatives
2. Approximate the area of a plane figure using Riemann sums
3. Evaluate definite integrals
4. Find the average value of a function
5. Evaluate a definite integral using the Fundamental Theorem of Calculus
6. Use the Second Fundamental Theorem of Calculus
7. Use substitution to evaluate an integral
8. Approximate a definite integral using the Trapezoidal and Simpson’s Rule

VI. Logarithmic and Exponential Functions

5 weeks

1. Find the derivatives of logarithmic and exponential functions
2. Integrate rational functions
3. Find the derivative and integral of trigonometric functions

4. Solve growth and decay problems
5. Use separation of variables to solve differential equations
6. Slope Fields

VII. Applications of Integration

4 weeks

1. Find area under a curve
2. Find the area between two curves
3. Find the volume of a solid of revolution using the disk and washer method
4. Find the volume of a solid with known cross sections
5. Find the volume of a solid of revolution using shell methods

Teaching Strategies

Each student is given a TI-83 Plus calculator for their use for the entire year. The first few weeks of school we review the calculator's capabilities. Calculators are used on a daily basis and calculator use is restricted on some assessments. Before each chapter students are given a syllabus containing assignments to be covered. Students are taught to be able to complete work analytically, graphically, and numerically. In addition they are expected to verbally relate this information using mathematical terms. Throughout the year students are required and encouraged to work together in groups both in and out of class. We review for the AP Calculus Exam by completing previous AP Exams.

Student evaluation

Students are given quarter grades based on homework, class work, quizzes and tests. Tests and quizzes are made up of multiple choice, short answer and free-response questions. Students are assigned free-repsonse questions, usually from previous AP Exam, to be completed as a group project or in some instances to be completed individually. They are graded in accordance with the AP Exam rubric.

Student Activities

1. To introduce evaluating limits graphically and numerically and analytically, I use the table, trace and zoom feature of the calculator. We not only evaluate limits, but review basic features of the TI – 83 as well. We explore different types of functions and review the concepts of removable and non-removable discontinuities as well as asymptotes which lead into when limits exist or fail to exist... Students are reminded discontinuities on a calculator are not always visible. We complete the activity by evaluating limits by direct substitution.

2. I use discovery to introduce basic differential formulas. Students are asked to use the definition of the derivative to find f'(x) on several polynomial functions. They are asked to find and describe a pattern. As a class we discuss the pattern and develop the basic differential formulas.

3. Students are placed in groups for their mid term project. Each group is given a different set of free-response questions from previous AP Exams to discuss and solve. Each group is given a set of different questions. They are given approximately one week to complete the project outside of class. A portion of their project is to share their solutions with the class; their solutions must be checked for accuracy before sharing. In addition, they must verbally explain their process for solving each free-response question. They are graded on accuracy of their work (AP Exam Rubrics) and their presentation.

Primary Textbook

Larson, Hostsetler, Edwards, and Heyd. *Calculus of a Single Variable*. 7th ed... Houghton Mifflin, 2002

Precalculus

Scope and Sequence

Quarter I	
Big Idea: Functions I. Transformations of functions a. Reflection in the x-axis, y-axis, and line y=x (inverse functions) b. Symmetry in the x-axis, y-axis, and origin c. Periodic Functions d. Translations of y=f(x) to y-k=f(x-h) e. Vertical (y=cf(x)) and horizontal (y=f(cx)) stretching or shrinking of y=f(x).	**Big Idea: Trigonometric Functions** II. Introduction to Trigonometric Functions a. Degree and radian measures of angles b. Arc length and area of a sector c. Evaluating trigonometric expressions d. Graphs of trigonometric functions
Big Idea: Trigonometric Equations and Applications III. Trigonometric Equations and Applications a. Translation of sine and cosine graphs b. Vertical and horizontal stretching and shrinking of sine and cosine functions c. Simplifying trigonometric expressions and proving trigonometric identities d. Trigonometric equations	
Quarter II	
Big Idea: Triangle Trigonometry IV. Triangle Trigonometry a. Measurements in right triangles b. Area of a triangle c. Law of Sines d. Law of Cosines	**Big Idea: Trigonometric Addition Formulas** V. Trigonometric Addition Formulas a. Sum and difference formulas for sine, cosine, and tangent b. Double angle formulas c. Trigonometric equations

<table>
<tr><th colspan="2">Quarter III</th></tr>
<tr><td>Big Idea: Polar Coordinates and Complex Numbers
VI. Polar Coordinates and Complex Numbers
a. Graphing polar coordinates
b. Conversions of rectangular and polar coordinates
c. Graphs of polar functions
d. Conversions of complex numbers between rectangular and polar form
e. Product of two complex numbers in polar form
f. De Moivre’s theorem
g. Roots of complex numbers</td><td>Big Idea: Sequences and Series
VII. Sequences and series
a. nth term of an arithmetic sequence
b. nth term of a geometric sequence
c. Recursive definitions
d. Sum of finite arithmetic and geometric series
e. Sum of infinite geometric series
f. Sigma notation</td></tr>
<tr><td></td><td></td></tr>
<tr><th colspan="2">Quarter IV</th></tr>
<tr><td>Big Idea: Limits
VIII. Limits
a. Limits of functions that approach infinity or negative infinity
b. Limits of functions that approach a real number
c. Graphs of rational functions</td><td>Big Idea: Exponents and Logarithms
IX. Exponents and Logarithms
a. Simplify numeric and algebraic expressions with integral and rational exponents
b. Compound interest formula and formula for interest compounded continually
c. Evaluate logarithmic expressions (change of base formula)
d. Expand and condense logarithmic expressions
e. Logarithmic and exponential equations</td></tr>
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Suggested blocks of Instruction	**Curriculum Management System** **<u>Grade Level/Subject</u>:** **Grade 11/Precalculus**	**Big Idea: Functions** **<u>Goal 1:</u> The student will be able to stretch, shrink, reflect, or translate the graph of a function, and determine the inverse of a function, if it exists.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	1.1. To sketch the reflection of a graph in the x-axis and y-axis. (4.2.12B.1; 4.2.12B.3) 1.2. To write the inverse of an equation and sketch the graph of the reflection in the line $y \quad x$. (4.2.12B.1; 4.3.12B.3) 1.3. To determine if the graph of an equation has symmetry in the x-axis, the y-axis, the line $y \quad x$, and the origin. (4.2.12B.1; 4.3.12B.4) 1.4. To determine if a function is periodic. (4.3.12B.1) 1.5. To evaluate a function using the fundamental period. (4.3.12B.2) 1.6. To determine the period and amplitude of a periodic function. (4.3.12B.2) 1.7. To understand the effect of c and sketch the graph of $y \quad cf(x)$ by vertically stretching or shrinking the graph of $y \quad f(x)$. (4.2.12B.1; 4.3.12B.3) 1.8. To understand the effect of c and sketch the graph of	**Essential Questions:** What changes in an equation produces the reflection of its graph in the x-axis, the y-axis, and the line $y \quad x$? How do you tell whether the graph of an equation has symmetry in the the x-axis, the y-axis, the line $y \quad x$, and the origin? Given the graph of $y \quad f(x)$, what effect does c have on the graph of $y \quad cf(x)$ and $y \quad f(cx)$? Given the graph of $y \quad f(x)$, what effect does h and k have on the graph of $y \quad k \quad f(x \quad h)$? How can the vertical-line test be used to justify the horizontal-line test? **Enduring Understandings:** If the equation $y \quad f(x)$ is changed to: a. $y \quad f(x)$, then the graph of $y \quad f(x)$ is reflected in the x-axis. b. $y \quad \|f(x)\|$, then the graph of $y \quad f(x)$ is unchanged when $f(x) \quad 0$ and reflected in the x-axis when $f(x) \quad 0$. c. $y \quad f(\quad x)$, then the graph of $y \quad f(x)$ is reflected in the y-axis. d. $x \quad f(y)$, then the graph of $y \quad f(x)$ is reflected	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Resources:** *Precalculus with Limits A Graphing Approach,* Fifth Edition, Larson et al; Houghton Mifflin, 2008 **Learning Activity:** In the following activity, how does a change in the equation result in the reflection of its graph in some line? (analysis) 1. Graph $y \quad x^2$ and $y \quad x^2$. Graph $y \quad x^3 \quad 2x^2$ and $y \quad (x^3 \quad 2x^2)$. How are the graphs of $y \quad f(x)$ and $y \quad f(x)$ related? 2. Graph $y \quad x^2 \quad 1$ and $y \quad \|x^2 \quad 1\|$. Graph $y \quad x(x \quad 1)(x \quad 3)$ and $y \quad \|x(x \quad 1)(x \quad 3)\|$.

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Functions** **Goal 1:** **The student will be able to stretch, shrink, reflect, or translate the graph of a function, and determine the inverse of a function, if it exists.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	$y \quad f(cx)$ by horizontally stretching or shrinking the graph of $y \quad f(x)$. (4.2.12B.1; 4.3.12B.3) 1.9. To understand the effect of h and k and sketch the graph of the equation $y \quad k \quad f(x \quad h)$ by translating the graph of $y \quad f\ (x)$ horizontally h units and vertically k units. 1.10. To determine if two functions are inverse functions by applying the definition. 1.11. To apply the Horizontal Line Test to determine if a function has an inverse.	in the line $y \quad x$. e. $y \quad cf(x), c \quad 1$, then the graph of $y \quad f(x)$ is stretched vertically. f. $y \quad cf(x), 0 \quad c \quad 1$, then the graph of $y \quad f(x)$ is shrunk vertically. g. $y \quad f(cx), c \quad 1$, then the graph of $y \quad f(x)$ is shrunk horizontally. h. $y \quad f(cx), 0 \quad c \quad 1$, then the graph of $y \quad f(x)$ is stretched horizontally. i. $y \quad k \quad f(x \quad h)$, then the graph of $y \quad f(x)$ is translated h units horizontally and k units vertically. A graph is symmetric in the x-axis if $(x, \quad y)$ is on the graph whenever (x, y) is. An equation of a graph is symmetric in the x-axis if an equivalent equation results after substituting y for y. A graph is symmetric in the y-axis if $(\quad x, y)$ is on the graph whenever (x, y) is. An equation of a graph is symmetric in the x-axis if an equivalent equation results after substituting x for x. A graph is symmetric in the line $y \quad x$ if (y, x) is on the graph whenever (x, y) is. An equation of a graph is symmetric in the line $y \quad x$ if an equivalent equation results after interchanging x and y. A graph is symmetric in the origin if $(\quad x, \quad y)$ is on the graph whenever (x, y) is. An equation of a graph is symmetric in the origin if an equivalent equation results	How are the graphs of $y \quad f(x)$ and $y \quad \|f(x)\|$ related? 3. Graph $y \quad 2x \quad 1$ and $y \quad 2(\quad x) \quad 1$. Graph $y \quad \sqrt{x}$ and $y \quad \sqrt{\quad x}$. How are the graphs of $y \quad f(x)$ and $y \quad f(\quad x)$ related? 4. Graph $y \quad 2x \quad 1$ and $x \quad 2y \quad 1$. Graph $y \quad x^2$ and $x \quad y^2$. How is the graph of an equation affected when you interchange the variables in the equation? Given the graph of $f(x)$, sketch the graphs of $y \quad f(x)$, $y \quad f(\quad x)$, and $y \quad \|f(x)\|$ using different colored crayons.

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Functions** **Goal 1:** **The student will be able to stretch, shrink, reflect, or translate the graph of a function, and determine the inverse of a function, if it exists.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		after substituting x for x and y for y. Two functions f and g are inverse functions if: 1. $g(f(x))$ x for all x in the domain of f, and 2. $f(g(x))$ x for all x in the domain of g. The Horizontal Line Test: If the graph of the function y $f(x)$ is such that no horizontal line intersects the graph in more than one point, then f is one-to-one and has an inverse.	

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Trigonometric Functions** **Goal 2:** **The student will be able to evaluate and graph trigonometric functions.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	2.1. To convert degree measures of angles to radians. 2.2. To convert radian measures of angles to degrees. 2.3. To determine coterminal angles. 2.4. To determine the arc length and area of a sector of a circle with central angles in either degrees or radians. 2.5. To use the definitions of *sine* and *cosine* to evaluate these functions. (4.3.12D.1) 2.6. To use reference angles, calculators or tables, and special angles to evaluate *sine* and *cosine* functions. (4.3.12D.1) 2.7. To sketch the graph of *sine* and *cosine* functions. (4.3.12B.2) 2.8. To use reference angles, calculators or tables, and special angles to evaluate *tangent, cotangent, secant,* and *cosecant* functions. (4.3.12D.1) 2.9. To sketch the graphs of *tangent, cotangent, secant,* and *cosecant* functions. (4.3.12B.2)	**Essential Questions:** What are radians and how are they related to degrees? Explain the process of evaluating a trigonometric function using reference angles and the unit circle. How do the values on the unit circle correlate to the rectangular graph of a trigonometric function? Why is it necessary to restrict the domain in order to discuss inverse trigonometric functions? **Enduring Understandings:** $\frac{s}{r}$, where is the measure of the central angle, in radians, s is the arc length, and r is the length of the radius. To convert each degree measure to radians, multiply by $\frac{\ }{180}$. To convert each radian measure to degrees, multiply by $\frac{180}{\ }$. The following formulas are used for the arc length s and area K of a sector with central angle : a. If is in degrees, then $s \quad \frac{\ }{360} \ 2 \quad r$ and $K \quad \frac{\ }{360} \quad r^2$.	**Sample Assessment Questions:** Convert each angle to radians in terms of . a) 60° b) 140° c) 180° d) 315° Convert each angle to degrees. e) 2 f) $\frac{2}{3}$ g) $\frac{11}{6}$ h) $\frac{5}{4}$ Give one positive and one negative coterminal angle for each angle below. Use the given form of the angle. i) 125° j) $\frac{3}{2}$. A sector of a circle has central angle 1.2 radians and radius 6cm. k) Find its arc length. l) Find its area. Find the value of each expression leave answers in simplest radical form. Show reference angle statement when necessary.

Suggested blocks of Instruction	Curriculum Management System Grade Level/Subject: **Grade 11/Precalculus**	**Big Idea:** Trigonometric Functions **Goal 2:** The student will be able to evaluate and graph trigonometric functions.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	2.10. To sketch the graph of the inverse of the *sine, cosine,* and *tangent* functions, and determine the domain and range. (4.3.12B.2) 2.11. To evaluate the inverse of *sine, cosine,* and *tangent* functions with and without a calculator or table. (4.3.12D.1)	b. If is in radians, then $s \quad r$ and $K \quad \frac{1}{2}r^2 \quad \frac{1}{2}rs$. $\sin \quad \frac{y}{r}$ $\cos \quad \frac{x}{r}$ $\tan \quad \frac{y}{x},\ x \quad 0$ $\cot \quad \frac{x}{y},\ y \quad 0$ $\sec \quad \frac{r}{x},\ x \quad 0$ $\csc \quad \frac{r}{y},\ y \quad 0$ The signs of the trigonometric functions (**s**ine and cosecant, **c**osine and secant, and **t**angent and cotangent) in the four quadrants can be summarized by the following phrase: **A**ll **S**tudents **T**ake **C**alculus.	m) sin 135° n) cos 270° o) tan 240° p) sec 420° q) cos (120°) r) sin 3 s) csc $\frac{5}{3}$ t) cot $\frac{11}{6}$ u) sec $\frac{7}{6}$ 2) **Graphing Project (See Addendum)** Students will complete the following tasks and present their graphs in a neat and accurate presentation. a) Complete a table of exact values for all special angles and quadrantal angles $2 \quad 2$. b) Graph each of the 6 trigonometric functions on a separate graph. Include an accurate scale and asymptotes where appropriate.

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Trigonometric Functions** **Goal 2:** **The student will be able to evaluate and graph trigonometric functions.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		In order to evaluate a trigonometric expression, 1. Determine the quadrant of the terminal ray of the angle. 2. Determine if it is positive, negative, or zero. 3. Determine the reference angle. 4. Determine the exact value, if possible, in simplest radical form.	

Suggested blocks of Instruction	Curriculum Management System Grade Level/Subject: **Grade 11/Precalculus**	**Big Idea:** **Trigonometric Equations and Applications** Goal 3: **The student will be able to stretch, shrink, and translate *sine* and *cosine* functions, simplify trigonometric expressions, and solve trigonometric equations.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	3.1. To solve and apply simple trigonometric equations. (4.3.12D.2) 3.2. To determine the slope and equation of a line given the angle of inclination and the coordinates of a point on the line, and determine the angle of inclination given the equation of a line or information about the line. (4.2.12C.1) 3.3. To stretch and shrink the graphs of *sine* and *cosine* functions. (4.2.12B.1) 3.4. To determine the period and amplitude of $y \quad A\sin Bx$ and $y \quad A\cos Bx$. (4.3.12B.2) 3.5. To determine the amplitude and period, and write the equation of *sine* and *cosine* curves. (4.3.12B.2) 3.6. To solve equations of the form $A\sin Bx \quad \#$ and $A\cos Bx \quad \#$. (4.3.12D.2) 3.7. To determine the amplitude, period, axis of wave, and	**Essential Questions:** How does a change in amplitude or period affect the graph of a Sine or Cosine curve? Explain the effect of A, B, h, and k on the graph of a sine or Cosine curve using the equations $y \quad A\sin B(x \quad h) \quad k$ $y \quad A\cos B(x \quad h) \quad k$ Explain how to find all possible solutions to simple trigonometric equations over a given domain. How can the graph of a trigonometric function be used to anticipate the number of solutions to a trigonometric equation? Could a single curve be described using both a Sine function and a Cosine function? Why? **Enduring Understandings:** For any line with slope *m* and inclination , $m \quad \tan$ if 90^0. If 90^0, then the line has no slope. (The line is vertical.) For functions $y \quad A\sin Bx$ and $y \quad A\cos Bx$ $(A \quad 0$ and $B \quad 0)$: amplitude = $\|A\|$ and period = $\frac{2}{B}$. If the graphs of $y \quad A\sin Bx$ and $y \quad A\cos Bx$ are translated horizontally *h* units and vertically *k* units	**Learning Activities:** Translating Graphs of Trigonometric Functions **(See Addendum)** Students will work in pairs to complete the graphing calculator activity on translating graphs of trigonometric functions. Deriving Basic Trigonometric Identities **(See Addendum)** Students will learn about negative angle relationships, Pythagorean relationships, and reciprocal relationships among trigonometric functions. **Sample Assessment Questions:** Solve $\sin x \quad 0.6$ for $0 \quad x \quad 2$. Solve $3\cos \quad 9 \quad 7$ for $0^0 \quad\quad 360^0$. Solve $\sec x \quad \frac{1}{2}$ for $0 \quad x \quad 2$ without using tables or a calculator. To the nearest degree, find the inclination of the line $2x \quad 5y \quad 15$. Find the slope and equation of a line with an inclination of 45^0 and contains (2,3). Give the amplitude and period of the function

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Trigonometric Equations and Applications** **Goal 3:** **The student will be able to stretch, shrink, and translate *sine* and *cosine* functions, simplify trigonometric expressions, and solve trigonometric equations.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	sketch the graph of translated *sine* and *cosine* functions. (4.3.12B.3; 4.3.12B.4) 3.8. To determine the amplitude, period, axis of wave, and write the equation of the graph of translated *sine* and *cosine* functions. (4.2.12B.1; 4.3.12B.2) 3.9. To simplify trigonometric expressions using the reciprocal relationships, relationships with negatives, Pythagorean relationships, and cofunction relationships of trigonometric functions. (4.3.12D.1) 3.10. To prove trigonometric identities using the reciprocal relationships, relationships with negatives, Pythagorean relationships, and cofunction relationships of trigonometric functions. (4.2.12A.4) 3.11. To use trigonometric identities or graphing calculator to solve more difficult trigonometric equations. (4.3.12D.2)	then the resulting graphs have equations $y \quad k \quad A\sin B(x \quad h)$ and $y \quad k \quad A\cos B(x \quad h)$. The amplitude is $A \quad \frac{Max \quad \min}{2}$. To find the period, *p* = horizontal distance between successive maximums. Use the formula $B \quad \frac{2}{period}$. Relationships with negatives: $\sin(\quad) \quad \sin$ and $\cos(\quad) \quad \cos$ $\csc(\quad) \quad \csc$ and $\sec(\quad) \quad \sec$ $\tan(\quad) \quad \tan$ and $\cot(\quad) \quad \cot$ Pythagorean Relationships: $\sin^2 \quad \cos^2 \quad 1$ $1 \quad \tan^2 \quad \sec^2$ $1 \quad \cot^2 \quad \csc^2$ Cofunction Relationships $\sin \quad \cos(90^0 \quad)$ and $\cos \quad \sin(90^0 \quad)$ $\tan \quad \cot(90^0 \quad)$ and $\cot \quad \tan(90^0 \quad)$ $\sec \quad \csc(90^0 \quad)$ and $\csc \quad \sec(90^0 \quad)$	$y \quad 4\sin 3x$. Then sketch at least one cycle of its graph. Solve $6\sin 2x \quad 5$ for $0 \quad x \quad 2$. For each function: a) State the amplitude of the curve. b) State the period of the curve. c) Describe any vertical or horizontal translations of the curve. d) Sketch the graph by hand. e) Confirm your sketch using a graphing calculator. i) $y \quad \frac{1}{2}\sin 3x$ ii) $y \quad \cos x \quad 1$ iii) $y \quad \sin(x \quad \frac{}{4})$ iv) $y \quad 2 \quad 2\cos\frac{1}{2}x$ Simplify $\sec x \quad \sin x \tan x$. Prove: $\frac{\cot A(1 \quad \tan^2 A)}{\tan A} \quad \csc^2 A$. Solve $2\sin^2 \quad 1 \quad 0$ for $0^0 \quad 360^0$. Solve $\sin^2 x \quad \sin x \quad \cos^2 x$ for $0^0 \quad x \quad 2$. Solve $\sin x \tan x \quad 3$ for $0^0 \quad x \quad 2$. Solve $2\sin \quad \cos$ for $0^0 \quad 360^0$.

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Triangle Trigonometry** **Goal 4:** **The student will be able to apply the trigonometric definitions, Law of Sines, and Law of Cosines to determine the lengths of unknown sides or measures of unknown angles in triangles.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	4.1. To use trigonometry to find the lengths of unknown sides or measures of unknown angles of a right triangle. (4.2.12A.1; 4.2.12E.1) 4.2. To determine the area of a triangle given the lengths of two sides of a triangle and the measure of the included angle. (4.2.12E.2) 4.3. To use the Law of Sines to find unknown parts of a triangle. (4.2.12E.1) 4.4. To use the Law of Cosines to find unknown parts of a triangle. (4.2.12E.1) 4.5. To use trigonometry to solve navigation and surveying problems. (4.2.12E.1)	**Essential Questions:** What does the acronym SOH-CAH-TOA stand for? Given the lengths of two sides of a right triangle, or the length of one side and the measure of one acute angle, how can you find the measures of the remaining sides and angles using the trigonometric functions? How can the area of a triangle be determined given the lengths of two sides and the measure of the included angle? When given the lengths of two sides and the measure of a non-included angle of a triangle, how many measurements are possible for the unknown angles and why? For which of the following situation is the Law of Cosines used? SAS, SSS, ASA, AAS, or SSA For which of the following situations is the Law of Sines used? SAS, SSS, ASA, AAS, or SSA How is measuring an angle in standard form different from measuring an angle from a compass bearing? **Enduring Understandings:** In $\triangle ABC$ with right angle C $\sin A = \frac{opposite}{hypotenuse} = \frac{a}{c}$ $\cos A = \frac{adjacent}{hypotenuse} = \frac{b}{c}$ $\tan A = \frac{opposite}{adjacent} = \frac{a}{b}$	**Learning Activities:** Perform the following activity to illustrate that when given the lengths of two sides of a triangle and the measure of a non-included angle (SSA), it may be possible to construct no triangle, one triangle, or two triangles. Draw $\angle A$ with measure 30^0. Along one ray of $\angle A$, locate point C 10 cm from point A. For each of the following compass settings, draw a large arc. Then determine if the arc intersects the other ray of $\angle A$ and, if so, in how many points. a. Compass at C and opened to 4 cm. (0) b. Compass at C and opened to 5 cm. (1) c. Compass at C and opened to 6 cm. (2) **Sample Assessment Questions:** For right triangle $\triangle ABC$ with right angle C, $m\angle A = 28^0$ and $BC = 40$. Find AC. The safety instructions for a 20 ft. ladder indicate that the ladder should not be inclined at more than a 70^0 angle with the ground. Suppose the ladder is leaned against a house with a 70^0 angle with the ground. Find (a) the distance *x* from the base of the house to the foot of the ladder and (b) the height *y* reached by the ladder. The highest tower in the world is in Toronto, Canada, and is 553 m high. An observer at point A, 100 m from the center of the tower's

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea: Triangle Trigonometry** **Goal 4: The student will be able to apply the trigonometric definitions, Law of Sines, and Law of Cosines to determine the lengths of unknown sides or measures of unknown angles in triangles.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		$\csc A = \frac{1}{\sin A} = \frac{hypotenuse}{opposite} = \frac{c}{a}$ $\sec A = \frac{1}{\cos A} = \frac{hypotenuse}{adjacent} = \frac{c}{b}$ $\cot A = \frac{1}{\tan A} = \frac{adjacent}{opposite} = \frac{b}{a}$ The area K of $\triangle ABC$ is given by: $K = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$. In other words, the area of any triangle is $\frac{1}{2}$ (one side) (another side) (sine of the included angle). Derive the formula for the area of a triangle by rewriting the height of the triangle in terms of an angle and a side (that doesn't intersect the height of the triangle). Derive the Law of Sines using the three different ways of writing the area of a triangle (using each of the three angles). The Law of Sines: In $\triangle ABC$, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. The Law of Cosines: in $\triangle ABC$, $c^2 = a^2 + b^2 - 2ab\cos C$. In other words, the square of the side opposite an angle is equal to the square of one side of the angle plus the square of the other side of the angle minus twice the product of the two sides of the angle times the cosine of the angle. The Law of Cosines (alternate form):	base, sights the top of the tower. The *angle of elevation* is $\angle A$. Find the measure of this angle. A triangle has sides of length 8, 8, and 4. Find the measures of the angles of the triangle. Two sides of a triangle have lengths 7cm and 4cm. The angle between the sides measures 73^o. Find the area of the triangle. If the angle between the sides is changed to 107^o, what is the area of the new triangle? Why are the two areas the same? The area of $\triangle PQR$ is 15. If $p = 5$ and $q = 10$, find all possible measures of $\angle R$. Why are there two answers? In $\triangle ABC$, $AB = 25$, $m\angle A = 110^0$, and $m\angle B = 20^0$. Find AC and BC. In $\triangle ABC$, $m\angle B = 126^0$, $b = 12$, and $c = 7$. Determine whether $\angle B$ exists, and, if so, find all possible measures of $\angle B$. Two sides of a triangle have lengths 3 cm and 7 cm, and the included angle has a measure of 130^0. Find the length of the third side. The lengths of the sides of a triangle are 5, 6, and 7. Solve the triangle.

Suggested blocks of Instruction	Curriculum Management System Grade Level/Subject: **Grade 11/Precalculus**	**Big Idea:** Triangle Trigonometry Goal 4: The student will be able to apply the trigonometric definitions, Law of Sines, and Law of Cosines to determine the lengths of unknown sides or measures of unknown angles in triangles.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		$\cos C \quad \frac{a^2 \quad b^2 \quad c^2}{2ab}$. Given SAS, use the Law of Cosines to find the measure of the third side and then one of the remaining angles. Given SSS, use the Law of Cosines to find the measures of any two angles. Given ASA or AAS, use the Law of Sines to find the measures of the remaining sides. Given SSA, use the Law of Sines to find an angle opposite a given side and then the third side. (Note that 0, 1, or 2 triangles are possible.)	

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Trigonometric Addition Formulas** **Goal 5: The student will be able to derive and apply the sum and difference formulas and the double-angle formulas for *sine, cosine,* and *tangent*, and apply these formulas to solve trigonometric equations.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**

5.1. To derive and apply the formulas for $\cos(\quad)$ and for $\sin(\quad)$. (4.3.12D.1)

5.2. To derive and apply the formulas for $\tan(\quad)$. (4.3.12D.1)

5.3. To derive and apply the double-angle formulas. (4.3.12D.1)

5.4. To use the double-angle formulas to solve trigonometric equations. (4.3.12D.2)

Essential Questions:

For what angle measurements can the sum and difference formulas be used?

How can the double-angle formulas be derived from the sum and difference formulas for sine, cosine, and tangent?

Enduring Understandings:

Use the Law of Cosines or the distance formula to derive the formulas for the difference (and then the sum) of two angles for cosines. Then use the cofunction relationship to derive the formula for the sum (and then the difference) of two angles for sine. (See pages 369-370.)

The *sum* and *difference formulas* for sine, cosine, and tangent are as follows:

$$\sin(\quad) \quad \sin \quad \cos \quad \cos \quad \sin$$

$$\sin(\quad) \quad \sin \quad \cos \quad \cos \quad \sin$$

$$\cos(\quad) \quad \cos \quad \cos \quad \sin \quad \sin$$

$$\cos(\quad) \quad \cos \quad \cos \quad \sin \quad \sin$$

$$\tan(\quad) \quad \frac{\tan \quad \tan}{1 \quad \tan \quad \tan}$$

$$\tan(\quad) \quad \frac{\tan \quad \tan}{1 \quad \tan \quad \tan}$$

Use the sum and difference formulas for sine, cosine, and tangent, and the Pythagorean relationships to derive the double angle formulas.

Sample Assessment Questions:

Find the exact value of $\sin 15^0$.

Find the exact value of :

a. $\cos 50^0 \cos 10^0 \quad \sin 50^0 \sin 10^0$ [redacted]

b. $\sin \frac{5}{12} \cos \frac{}{12} \quad \cos \frac{5}{12} \sin \frac{}{12}$.

If $\sin \quad \frac{4}{5}$ and $\sin \quad \frac{5}{13}$, where

$0 \quad \frac{}{2}$ and $\frac{}{2}$, find $\cos(\quad)$.

If $\tan A \quad \frac{1}{2}$, find $\tan 2A$.

Solve $\cos 2x \quad 1 \quad \sin x$ for $0 \quad x \quad 2$ using a graphing calculator and algebraically.

Solve $3\cos 2x \quad \cos x \quad 2$ for $0 \quad x \quad 2$.

Solve $2\sin 2x \quad 1$ for $0^0 \quad x \quad 360^0$.

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Trigonometric Addition Formulas** **Goal 5:** **The student will be able to derive and apply the sum and difference formulas and the double-angle formulas for *sine, cosine,* and *tangent*, and apply these formulas to solve trigonometric equations.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		The *double angle formulas* are as follows: $\sin 2 \quad 2\sin \quad \cos$ $\cos 2 \quad \cos^2 \quad \sin^2 \quad 1 \quad 2\sin^2 \quad 2\cos^2 \quad 1$ $\tan 2 \quad \frac{2\tan}{1 \quad \tan^2}$ A quick review of factoring polynomials may be helpful before solving trigonometric equations using the double-angle formulas.	

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Polar Coordinates and Complex Numbers** **Goal 6:** **The student will be able to represent points in rectangular and polar coordinates, and multiply and find powers of complex numbers.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	6.1. To graph points given polar coordinates. (4.2.12C.1) 6.2. To state two additional polar coordinates for the same point. (4.2.12C.1) 6.3. To convert from rectangular (Cartesian) to polar coordinates. 6.4. To convert from polar to rectangular coordinates. 6.5. To graph polar equations. (4.3.12B.1) 6.6. Explore special polar graphs including Cardiod, Limacon, and Rose curves. (4.3.12B.1; 4.3.12B.4) 6.7. To convert complex numbers in rectangular form to polar form. 6.8. To express complex numbers in polar form to rectangular form. 6.9. To determine the product of two complex numbers in polar form. (4.3.12D.3) 6.10. To use De Moivre's theorem to determine powers of complex numbers. (4.3.12D.3) 6.11. To determine roots of complex numbers. (4.3.12D.2)	**Essential Questions:** How many ways can a point be represented using polar coordinates? How are rectangular coordinates converted to polar coordinates? How are polar coordinates converted to rectangular coordinates? What is the advantage of using *De Moivre's* theorem? **Enduring Understandings:** The polar coordinates of a point P are $(r, \ )$, where r is the directed distance from the *pole* to P and is the *polar angle* measured from the polar axis to the ray OP . Although a point has only one pair of rectangles coordinates, it has many pairs of polar coordinates. For example, $(2, 20^0) \quad (2, \quad 340^0) \quad (\quad 2, 200^0) \quad (\quad 2, \quad 160^0)$ all represent the same point. Formulas for converting from polar to rectangular coordinates: $x \quad r\cos \quad , y \quad r\sin \quad$. Formulas for converting from rectangular to polar coordinates: $r \quad \sqrt{x^2 \quad y^2}, \tan \quad \frac{y}{x}$. The *complex plane* can be represented by an *Argand diagram*. In this diagram, the complex number $a \quad bi$ is represented by the point (a, b) or by an arrow from the origin to (a, b) . The *absolute value* of a complex number $z \quad a \quad bi$ is	**Graphing Project:** **(See Addendum)** Polar vs. Rectangular Graphs In this activity may help students see the connections between rectangular and polar graphs. **Sample Assessment Questions:** Express $2cis50^0$ in rectangular form. Express $1 \quad 2i$ in polar form. If $z_1 \quad 2 \quad 2i\sqrt{3}, z_2 \quad \sqrt{3} \quad i$: a. find $z_1 z_2$ in rectangular form by multiplying z_1 and z_2 . b. find z_1 , z_2 , and $z_1 z_2$ in polar form. c. show that $z_1 z_2$ in polar form agree with $z_1 z_2$ in rectangular form. If $z \quad \frac{1}{2} \quad \frac{\sqrt{3}}{2}i$, find z^2, z^3, z^4, z^5, and z^6. Plot these on an Argand diagram.

Suggested blocks of Instruction	**Curriculum Management System** **<u>Grade Level/Subject</u>:** **Grade 11/Precalculus**	**Big Idea:** **Polar Coordinates and Complex Numbers** **<u>Goal 6:</u> The student will be able to represent points in rectangular and polar coordinates, and multiply and find powers of complex numbers.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		$\lvert z\rvert \quad \sqrt{a^2 \quad b^2}$. Thus, $\lvert z\rvert \quad r$. To multiply two complex numbers in polar form: 1. Multiply their absolute values. 2. Add their polar angles. In other words, if $z_1 \quad rcis$ and $z_2 \quad scis$ [redacted]en $z_1 z_2 \quad (rcis \quad)(scsi \quad) \quad rscis(\qquad)$. *De Moivre's Theorem*: If $z \quad rcis$, then $z^n \quad r^n cisn$. The n nth roots of $z \quad rcis$ are: $\sqrt[n]{z} \quad z^{\frac{1}{n}} \quad r^{\frac{1}{n}} cis \quad \frac{}{n} \quad \frac{k \; 360^0}{n}$ for $k \quad 0,1,2,...,n \quad 1$.	Find the cube roots of $8i$. Find the four fourth roots of -16.

Suggested blocks of Instruction	Curriculum Management System Grade Level/Subject: **Grade 11/Precalculus**	**Big Idea:** Sequences and Series **Goal 7:** **The student will be able to identify arithmetic and geometric sequences and series, write a formula for the *n*th term of sequences and series using an explicit or recursive definition, apply sigma notation, and find the sum of finite arithmetic, and finite and infinite geometric series.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	7.1. To identify an arithmetic and geometric sequence. 7.2. To write a formula for the *n*th term of an arithmetic sequence. (4.3.12A.1) 7.3. To write a formula for the *n*th term of a geometric sequence. (4.3.12A.1) 7.4. To write the formula for the *n*th term of a sequence that is neither arithmetic nor geometric. (4.3 12A.1; 4.3.12A.3) 7.5. To use and write recursive definitions of sequences. (4.3.12A.1) 7.6. To determine the sum of the first *n* terms of arithmetic and geometric series. (4.3 12A.1) 7.7. To find or estimate the limit of an infinite sequence or to determine that the limit does not exist. (4.2.12B.4; 4.3.12A.1; 4.3.12A.2) 7.8. To determine the sum of an infinite geometric series. (4.2.12B.4; 4.3.12A.1) 7.9. To expand series written in sigma notation. (4.3.12A.1) 7.10. To condense series into sigma notation. (4.3.12A.1)	**Essential Questions:** What makes a sequence arithmetic? What is the graph of an arithmetic sequence? What makes a sequence geometric? What is the graph of a geometric sequence? What is the difference between an explicit definition and a recursive definition of a sequence? What is the difference between a sequence and a series? When does the sum of a series converge or diverge? **Enduring Understandings:** A sequence is *arithmetic* if the difference *d* of any two consecutive terms is constant. The formula for the *nth* term in an arithmetic sequence is: $a_n \quad a_1 \quad d(n \quad 1)$. A sequence is *geometric* if the ratio *r* of any two consecutive terms is constant. The formula for the *nth* term in a geometric sequence is: $a_n \quad a_1 \quad r^{n \quad 1}$. A *recursive definition* consists of two parts: 1. An *initial condition* that states the first term of the sequence. 2. A *recursive equation* (or *recursive formula*) that states how any term in the sequence is related to the preceding term. A series is an indicated sum of terms of a sequence. The sum of the first *n* terms in an arithmetic series is	**Sample Assessment Questions:** State the next term in each sequence. Then write a rule for the nth term. Identify is the sequence is arithmetic, geometric, or neither. 1. $6, 9, 12, 15, \ldots$ 2. $3, 9, 27, 81, \ldots$ 3. $0, 3, 8, 15, 24, \ldots$ 4. $1, 8, 27, 64, 125, \ldots$ 5. $3, 4, 7, 11, 18, \ldots$ 6. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ (discuss $\lim_{x} \frac{(\ 1)^x}{x} \quad 0$) Write a recursive rule for the sequences #1, 2, and 5 above. Write a rule for the ***nth*** term of the arithmetic sequence. Then find a_{30}. 1. $1, 3, 7, 11, 15, \ldots$ 2. $\frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3, \frac{13}{4}, \ldots$ 3. $a_6 \quad 27.2, a_{13} \quad 44$ Write a rule for the ***nth*** term of the geometric sequence. Then find a_8. $8, 12, 18, 27, \ldots$ One term of a geometric sequence is $a_3 \quad 5$.

Suggested blocks of Instruction	Curriculum Management System Grade Level/Subject: **Grade 11/Precalculus**	**Big Idea:** Sequences and Series **Goal 7:** The student will be able to identify arithmetic and geometric sequences and series, write a formula for the *n*th term of sequences and series using an explicit or recursive definition, apply sigma notation, and find the sum of finite arithmetic, and finite and infinite geometric series.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		$S_n \quad \frac{n}{2} \; a_1 \quad a_n$. The sum of the first *n* terms of a geometric series is $S_n \quad \frac{a_1(1 \quad r^n)}{1 \quad r}$, where *r* is the common ratio and $r \quad 1$. The sum of an infinite geometric series when $\lvert r \rvert \quad 1$ is: $S \quad \frac{a_1}{1 \quad r}$. If $\lvert r \rvert \quad 1$ and $a_1 \quad 0$, then the series diverges.	The common ratio is $r \quad 2$. Write a rule for the ***nth*** term. Two terms of a geometric sequence are $a_2 \quad 45$ and $a_5 \quad 1215$. Find a rule for the ***nth*** term. State the first five terms of the sequence. 1. $a_1 \quad 10$ $a_n \quad a_{n \; 1} \quad 2$ 2. $a_1 \quad 10$ $a_n \quad \frac{1}{2} a_{n \; 1}$ 3. $a_1 \quad 10, a_2 \quad 2$ $a_n \quad a_{n \; 1} \; a_{n \; 2}$ 4. $a_n \quad 10, a_2 \quad 2$ $a_n \quad a_{n \; 2} \quad a_{n \; 1}$ For the arithmetic series $4 \quad 7 \quad 10 \quad 13 \quad 16 \quad 19 \quad ...$ 1. Determine the sum of the first 30 terms. 2. Find ***n*** such that $S_n \quad 175$. For the geometric series $1 \quad 5 \quad 25 \quad 125 \quad 625 \quad ...$ 1. Determine the sum of the first 10 terms. 2. Find ***n*** such that $S_n \quad 3906$.

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Sequences and Series** **Goal 7: The student will be able to identify arithmetic and geometric sequences and series, write a formula for the *n*th term of sequences and series using an explicit or recursive definition, apply sigma notation, and find the sum of finite arithmetic, and finite and infinite geometric series.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
			Find the sum of $1 \quad \frac{1}{4} \quad \frac{1}{16} \quad \frac{1}{64} \quad \ldots$ A ball is dropped from a height of 5 feet. Each time it hits the ground, it bounces one half of its previous height. Find the total distance traveled by the ball. Find the sum of the series. 1. $\overset{30}{\underset{n\ 1}{}}(\ 3 \quad 5n)$ 2. $\overset{40}{\underset{n\ 1}{}}4 \quad \frac{1}{2}n$ 3. $\overset{10}{\underset{n\ 1}{}}4 \quad \frac{1}{2}^{n\ 1}$ 4. $\overset{10}{\underset{n\ 1}{}}n^2$ Write in sigma notation. 1. $\frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{4}{5} \quad \ldots$ 2. $1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \ldots$

Suggested blocks of Instruction	Curriculum Management System **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** Limits **Goal 8:** The student will be able to determine the limit of a function or the quotient of two functions, and sketch the graph of a rational function using limits.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	8.1. To determine the limit of a function or the quotient of two functions as *x* approaches or . (4.3.12A.2) 8.2. To determine the limit of a function or the quotient of two functions as *x* approaches a real number. (4.3.12A.2) 8.3. To determine if a function is continuous. (4.3.12B.1; 4.3.12B.2) 8.4. To sketch the graph of a rational function by determining the *x*-intercepts, vertical and horizontal asymptotes, performing a sign analysis, and use limits. (4.3.12B.1; 4.3.12B.2)	**Essential Questions:** How do we find the limit of a function as x approaches or ? How do we find the limit of a function as x approaches a real number c? How do we find the limit of a quotient of two functions? What is an inverse variation and how does it compare to other rational functions? What is significant about the graph of a rational function? How can the vertical and horizontal asymptotes of a rational function be identified analytically? Explain the procedure to sketch a possible graph of a rational function. **Enduring Understandings:** If $\|r\| \quad 1$, then $\lim_{x} r^n \quad 0$.. The Quotient Theorem for Limits: If $\lim n(x)$ and $\lim d(x)$ both exist, and $\lim d(x) \quad 0$, then $\lim \frac{n(x)}{d(x)} \quad \frac{\lim n(x)}{\lim d(x)}$. Techniques for evaluating $\lim \frac{n(x)}{d(x)}$: 1. If possible, use the quotient theorem for limits. 2. If $\lim n(x) \quad 0$ and $\lim d(x) \quad 0$, try the following	**NOTE: The assessment models provided in this document are suggestions for the teacher. If the teacher chooses to develop his/her own model, *it must be of equal or better quality and at the same or higher cognitive levels (as noted in parentheses*).** **Depending upon the needs of the class, the assessment questions may be answered in the form of essays, quizzes, mobiles, PowerPoint, oral reports, booklets, or other formats of measurement used by the teacher.** **Sample Assessment Questions:** Evaluate: a) $\lim_{x} (0.99)^n$ b) $\lim_{x} \cos \frac{1}{n}$ c) $\lim_{x} \frac{n^2 \quad 1}{2n^2 \quad 3n}$ d) $\lim_{x} \frac{5n^2 \quad \sqrt{n}}{3n^3 \quad 7}$ e) $\lim_{x} (4n \quad 1)$ f) $\lim_{x} (\quad 10^n)$

Suggested blocks of Instruction	Curriculum Management System Grade Level/Subject: **Grade 11/Precalculus**	**Big Idea:** Limits **Goal 8:** The student will be able to determine the limit of a function or the quotient of two functions, and sketch the graph of a rational function using limits.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		techniques: a) Factor $d(x)$ and $n(x)$, and reduce $\frac{n(x)}{d(x)}$ to lowest terms. b) If $d(x)$ or $n(x)$ involves a square root, try multiplying both $d(x)$ and $n(x)$ by the conjugate of the square root expression. 3. If $\lim n(x) \quad 0$ and $\lim d(x) \quad 0$, then either statement (a) or (b) is true: a) $\lim \frac{n(x)}{d(x)}$ does not exist b) $\lim \frac{n(x)}{d(x)}$ or $\lim \frac{n(x)}{d(x)}$ 4. If x is approaching infinity or negative infinity, divide the numerator and denominator by the highest power of x in the denominator. 5. Evaluate $\lim_{x} \frac{n(x)}{d(x)}$ by evaluating $\frac{n(x)}{d(x)}$ for very large values of x, and evaluate $\lim_{x \quad c} \frac{n(x)}{d(x)}$ by evaluating $\frac{n(x)}{d(x)}$ for x-values very near $x \quad c$. These limits can also be guessed by using a graphing calculator to examine the graph of $y \quad \frac{n(x)}{d(x)}$ for very	g) $\lim_{x} \frac{7n^3}{4n^2 \quad 5}$ h) $\lim_{x} \frac{(\quad 1)^n \quad n}{n \quad 1}$ i) $\lim_{x} x^{\frac{1}{3}}$ j) $\lim_{x} x^{\frac{1}{3}}$ k) $\lim_{x \quad 2} \frac{x^2 \quad 4}{x \quad 2}$ l) $\lim_{x \quad 3} \frac{\sqrt{x \quad 1}}{x^2 \quad 1}$ m) $\lim_{x \quad 1} \frac{x^2 \quad 2x \quad 3}{x^2 \quad 1}$ n) $\lim_{x \quad 0} \frac{1 \quad \sqrt{1 \quad x}}{x}$ o) $\lim_{x \quad 1} \frac{1}{(x \quad 1)^2}$ p) $\lim_{x \quad 1} \frac{1}{x \quad 1}$ For each rational function: a) Find all horizontal and vertical asymptotes

Suggested blocks of Instruction	Curriculum Management System Grade Level/Subject: **Grade 11/Precalculus**	**Big Idea:** Limits **Goal 8:** The student will be able to determine the limit of a function or the quotient of two functions, and sketch the graph of a rational function using limits.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		large values of x, or for x-values very near $x \quad c$. (A graphing calculator might not show points of discontinuity.) Given a rational function of the form $f(x) \quad \frac{n(x)}{d(x)}$, find the horizontal asymptotes by the following rules: a. If the degree of $n(x)$ and $d(x)$ are the same, then the horizontal asymptote is the ratio of the leading coefficients of $n(x)$ and $d(x)$. b. If the degree of $n(x)$ is greater then the degree of $d(x)$, then the horizontal asymptote is the x axis. c. If the degree of $n(x)$ is less than the degree of $d(x)$, there is no horizontal asymptote. Given a rational function of the form $f(x) \quad \frac{n(x)}{d(x)}$, a discontinuity (vertical asymptote or hole) will occur whenever $d(x) \quad 0$. Given a rational function of the form $f(x) \quad \frac{n(x)}{d(x)}$, the x-intercepts will occur when $n(x) \quad 0$ (unless it is also a zero of $d(x)$).	of the graph. *(synthesis)* b) Identify any holes in the graph. *(application)* c) Sketch a possible graph of the function using asymptotes, intercepts and sign analysis. *(synthesis)* d) Confirm your graph using a graphing calculator. *(application)* e) State the domain and range of the function. *(analysis)* i) $y \quad \frac{2}{(x \quad 1)}$ ii) $y \quad \frac{x \quad 2}{x^2 \quad 5x \quad 6}$ iii) $y \quad \frac{x^2 \quad 6x \quad 9}{x^2 \quad 4x \quad 3}$ iv) $y \quad \frac{x \quad 3}{x^2 \quad 1}$ v) $y \quad \frac{x^2 \quad 2x \quad 3}{x^2 \quad 4}$ The function $p \quad \frac{69.1}{a \quad 2.3}$ relates atmospheric pressure, p, in inches of mercury, to altitude, a, in miles. a) Graph the function. *(application)* b) Find the atmospheric pressure at Mt. Kilimanjaro with altitude 19,340 ft. *(application)*

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea: Limits** **Goal 8: The student will be able to determine the limit of a function or the quotient of two functions, and sketch the graph of a rational function using limits.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
			c) Is there an altitude at which the atmospheric pressure is 0 inches of mercury? Use your graph to justify your answer. *(synthesis/evaluation*

Suggested blocks of Instruction	Curriculum Management System Grade Level/Subject: **Grade 11/Precalculus**	**Big Idea:** Exponents and Logarithms **Goal 9:** The student will be able to simplify expressions and solve equations with exponents and logarithms.	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**

The student will be able to:

9.1. To apply the laws of exponents to simplify numeric and algebraic expressions with integral exponents. (4.1.12B.4; 4.3.12D.1)

9.2. To apply the laws of exponents to simplify numeric and algebraic expressions with rational exponents. (4.1.12B.4; 4.3.12D.1)

9.3. To solve real-world problems involving exponential growth and decay. (4.3.12C.1)

9.4. To estimate the doubling time using the Rule of 72. (4.3.12C.1)

9.5. To apply the formula for compounded interest. (4.3.12C.1)

9.6. To define and apply the natural exponential function.

9.7. To derive and apply the formula for interest compounded continuously. (4.3.12C.1)

9.8. To define and evaluate logarithms. (4.3.12D.1)

9.9. To solve logarithmic equations without and with a

Essential Questions:

For what values of b does $y \quad b^x$ represent exponential growth? Exponential decay?

Enduring Understandings:

Properties of Exponents:

1. $x^0 \quad 1$
2. $x^{\ n} \quad \frac{1}{x^n}$
3. When multiplying expressions with the same base, add the exponents. $x^m \quad x^n \quad x^{m \ \ n}$
4. When dividing expressions with the same base, subtract the exponents. $\frac{x^m}{x^n} \quad x^{m \ \ n}$
5. When raising a power to a power, multiply the exponents. $x^m \ {}^n \quad x^{mn}$
6. $(xy)^n \quad x^n \ y^n$
7. $\frac{x}{y} \ ^n \quad \frac{x^n}{y^n}$
8. If $b \quad 0,1, \quad 1$, then $b^x \quad b^y$ if and only if $x \quad y$.

An *exponential function* has the form $f(x) \quad ab^x$, where $a \quad 0, b \quad 0,$ and $b \quad 1$. If $b \quad 1$, it is an exponential growth function, and if $0 \quad b \quad 1$, then it is

Sample Assessment Questions:

Simplify $\frac{b^2}{a} \ ^2 \ \frac{a^2}{b} \ ^3$

Simplify $a^{\ 2} \ b^{\ 2} \ ^1$, where $a \quad 0$ and $b \quad 0$.

Simplify $\frac{x^5 \ x^{\ 2}}{x^{\ 3}}$ and $\frac{x^5 \ x^{\ 2}}{x^{\ 3}}$, where $x \quad 0$.

A radioactive isotope decays so that the radioactivity present decreases by 15% per day. If 40 kg are present now, find the amount present (a) 6 days from now, and (b) 6 days ago.

Find the balance after 10 years if $5000 is invested in a bank account that pays 4.5% interest compounded

1. annually.
2. quarterly.
3. monthly.
4. daily.

Find the balance after 10 years if $5000 is invested in a bank account that pays 4.5% annual interest compounded continuously.

In about how many years will it take for $1000 to double in value with a 6% annual interest rate compounded continuously? (Discuss the rule of 72)

Evaluate without a calculator:

Suggested blocks of Instruction	Curriculum Management System Grade Level/Subject: **Grade 11/Precalculus**	**Big Idea:** **Exponents and Logarithms** **Goal 9:** **The student will be able to simplify expressions and solve equations with exponents and logarithms.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
	calculator. (4.3.12D.2) 9.10. To apply the laws of logarithms to expand and condense logarithmic expressions. (4.3.12D.1) 9.11. To solve exponential equations by rewriting both sides of the equation with the same base. (4.3.12D.2) 9.12. To solve exponential equations by using the definition of logarithms to rewrite as a logarithmic equation. (4.3.12D.2) 9.13. To evaluate logarithmic expressions using the Change of Base formula. (4.3.12D.1)	an exponential growth function. Compound Interest Formula: The amount A in an account earning interest compounded n times per year for t years is: $A \quad P \quad 1 \quad \frac{r}{n}^{nt}$, where P is the principal and r is the annual interest rate expressed as a decimal. The *rule of 72* provides an approximation of the doubling time for exponential growth. If a quantity is growing at *r%* per year, then the doubling time $72 \quad r$. The number e is defined as: $\lim_{x} \; 1 \quad \frac{1}{n}^{n}$. Continuously Compounded Interest Formula: $A \quad Pe^{rt}$. The *logarithm* of *x* to the base *b* $(b \quad 0, b \quad 1)$ is the exponent *a* such that $x \quad b^x$. Thus, $\log_b x \quad a$ if and only if $x \quad b^a$. Laws of Logarithms: 1. $\log_b MN \quad \log_b M \quad \log_b N$ 2. $\log_b \frac{M}{N} \quad \log_b M \quad \log_b N$ 3. $\log_b M^k \quad k\log_b M$ 4. $\log_b M \quad \log_b N$ if and only if $M \quad N$ The *change of base formula* enables you to write	1. $\log_5 125$ 2. $\log_8 2$ 3. $\log_3 \frac{1}{27}$ 4. $\log_7 7$ 5. $\log_9 1$ 6. $\log_{16} 64$ Expand the expression. 1. $\log a^2bc^4$ 2. $\log \frac{x^5 y^{\;2}}{2y}$ Condense the expression. 1. $\frac{1}{2}\log x \quad 3\log y$ 2. $3(\ln 3 \quad \ln x) \quad (\ln x \quad \ln 9)$ Use the change of base formula to evaluate 1. $\log_6 9$ 2. $\log_9 6$ Solve. Check for extraneous solutions. 1. $4^{3x} \quad 8^{x \; 1}$ 2. $3^{2x} \quad 5$

Suggested blocks of Instruction	**Curriculum Management System** **Grade Level/Subject:** **Grade 11/Precalculus**	**Big Idea:** **Exponents and Logarithms** **Goal 9:** **The student will be able to simplify expressions and solve equations with exponents and logarithms.**	
	Objectives / Cluster Concepts / Cumulative Progress Indicators (CPI's) **The student will be able to:**	**Essential Questions** **Sample Conceptual Understandings**	**Instructional Tools / Materials / Technology / Resources / Learning Activities / Interdisciplinary Activities / Assessment Model**
		logarithms in any given base in terms in any other base. $\log_b a \quad \frac{\log_c a}{\log_c b}$	3. $10^{2x \quad 3} \quad 4 \quad 21$ 4. $\log_3(5x \quad 1) \quad \log_3(x \quad 7)$ 5. $\log_5(3x \quad 1) \quad 2$ 6. $\log 5x \quad \log(x \quad 1) \quad 2$

Precalculus

COURSE BENCHMARKS

1. The student will be able to stretch, shrink, reflect, or translate the graph of a function, and determine the inverse of a function, if it exists.

2. The student will be able to evaluate and graph trigonometric functions.

3. The student will be able to stretch, shrink, and translate *sine* and *cosine* functions, simplify trigonometric expressions, and solve trigonometric equations.

4. The student will be able to apply the trigonometric definitions, Law of Sines, and Law of Cosines to determine the lengths of unknown sides or measures of unknown angles in triangles.

5. The student will be able to derive and apply the sum and difference formulas and the double-angle formulas for *sine, cosine,* and *tangent*, and apply these formulas to solve trigonometric equations.

6. The student will be able to represent points in rectangular and polar coordinates, and multiply and find powers of complex numbers.

7. The student will be able to identify arithmetic and geometric sequences and series, write a formula for the *n*th term of sequences and series using an explicit or recursive definition, apply sigma notation, and find the sum of finite arithmetic, and finite and infinite geometric series.

8. The student will be able to determine the limit of a function or the quotient of two functions, and sketch the graph of a rational function using limits.

9. The student will be able to simplify expressions and solve equations with exponents and logarithms.

Addendum

Goal 2: Graphing Project

GRAPHING TRIGONOMETRIC FUNCTIONS

DIRECTIONS: Fill in all values on the table below. Use the values to graph each trig function on a separate sheet of graph paper. You will be graded on the following:

A complete and correct table of values in simplest radical form.

Six separate graphs including : **title, scale, points plotted accurately using decimal approximations, asymptotes where necessary and smooth continuous curves**

in degrees	in radians	sin	cos	tan	csc	sec	cot
0°							
30°							
45°							
60°							
90°							
120°							
135°							
150°							
180°							
210°							
225°							
240°							
270°							
300°							
315°							
330°							
360°							

in degrees	in radians	sin	cos	tan	csc	sec	cot
-360°							
-330°							
-315°							
-300°							
-270°							
-240°							
-225°							
-210°							
-180°							
-150°							
-135°							
-120°							
-90°							
-60°							
-45°							
-30°							
0°							

(The grid below can be used instead of traditional graph paper to help students create a reasonable scale on the horizontal axis)

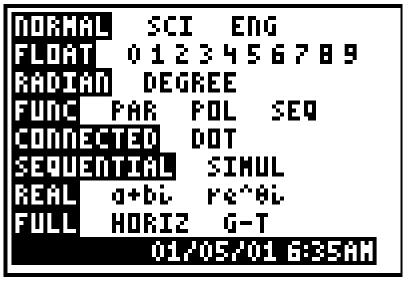
TITLE OF GRAPH:

Goal 3 Learning Activity-Translating Graphs of Trigonometric Functions
Students may work in pairs or alone to complete the exercises and discover the relationship between translations of trigonometric functions and their equations.

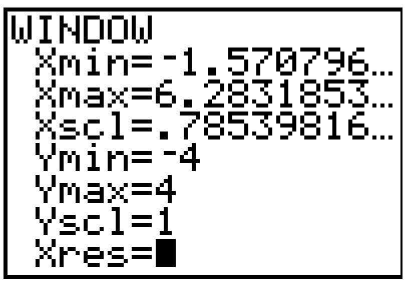
TRANSLATING GRAPHS OF TRIGONOMETRIC FUNCTIONS

Complete each task in the space provided using a graphing calculator.

1. Set the MODE on your calculator to agree with the window below.

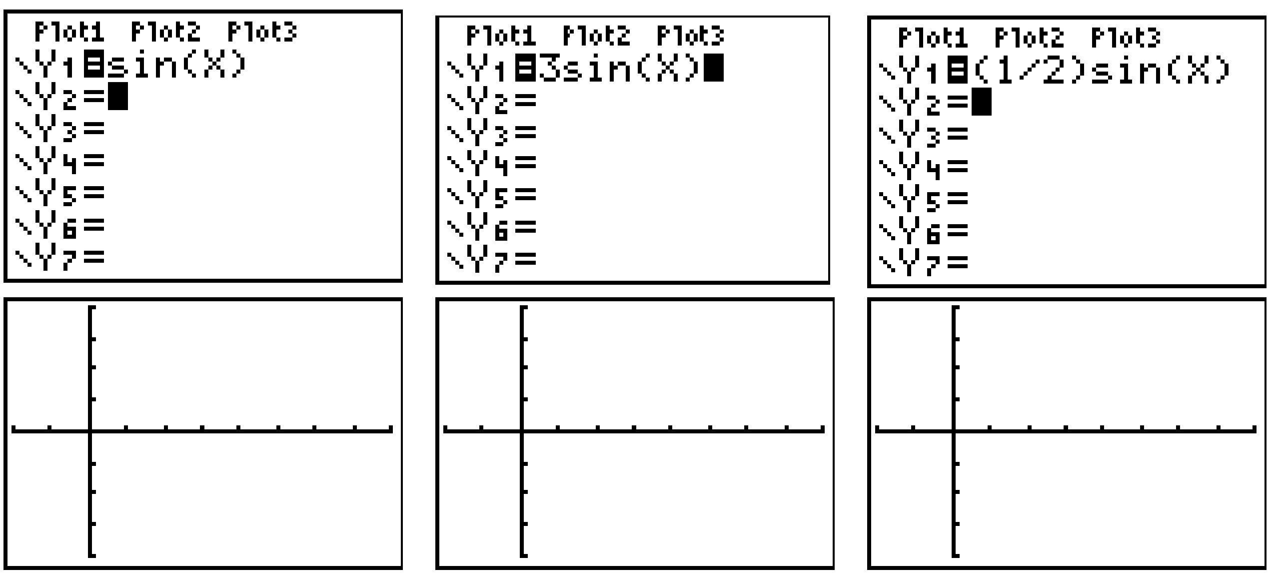


2. Set you WINDOW to agree with the window below.



/2
type in for the first three values.
/4

3. Graph the following equations in the window provided.



4. Using the graphs above and your class notes, describe the effect that a change in A has on the graph of y = A sin(x). Use the proper vocabulary word for A in your explanation.

Will the effect be the same for y = A cos(x)? Test your conclusion using the calculator.

5. Sketch each graph in the space provided.

6. Using the graphs above, describe the effects that a change in B has on the graph of y = sin(Bx) or y = cos(Bx).

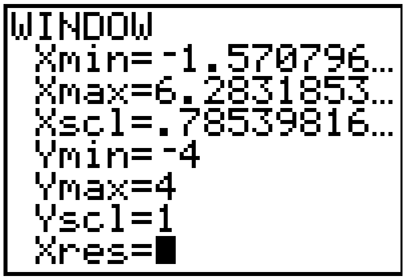
7. Use the information below to answer questions about the exercises that follow.
   Given y = Asin(Bx) or y = Acos(Bx)

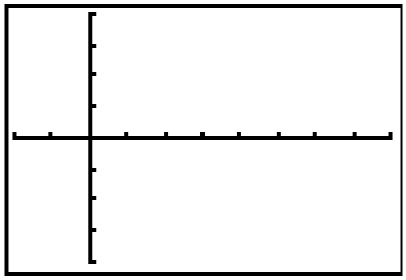
   A represents **amplitude** $A \quad \frac{\text{max} \quad \text{min}}{2}$.

   B represents the number of cycles in 2 units. B is related to the period P as follows:

   $B \quad \frac{2}{P}$ or $P \quad \frac{2}{B}$.

Try the following exercises without the calculator. You may check your work with the calculator when you are done.
NOTE: use the same window that you set up in the beginning of this packet.

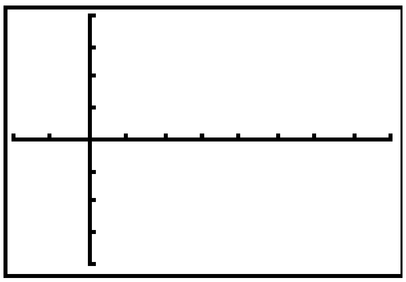


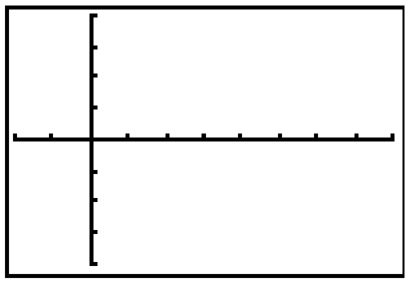


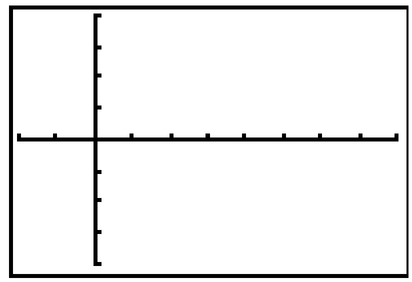
Remember: each mark on the x-axis represents $/4$ which means that 4 marks = .

8. Sketch each graph and state the amplitude and period.

$y \quad 3\sin 2x$ $\qquad$ $y \quad 2\cos x$ $\qquad$ $y \quad 4\sin(1/\ 2x)$

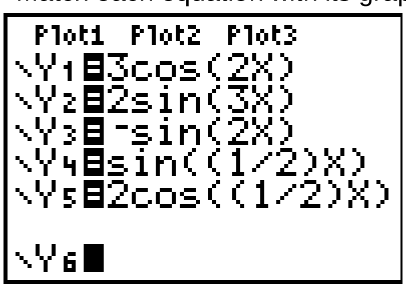




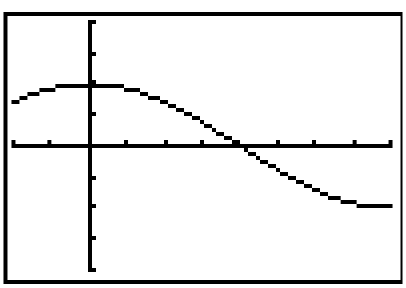


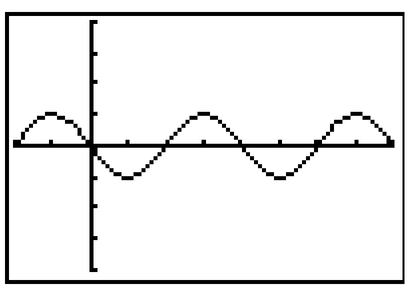
A=_______ P=_______ $\qquad$ A=_______ P=_______ $\qquad$ A=________P=________

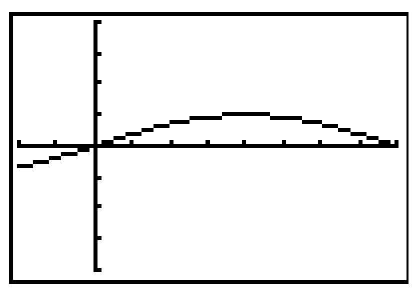
9. Match each equation with its graph.

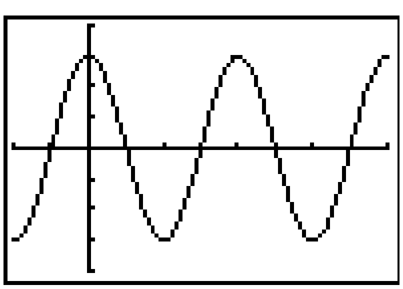


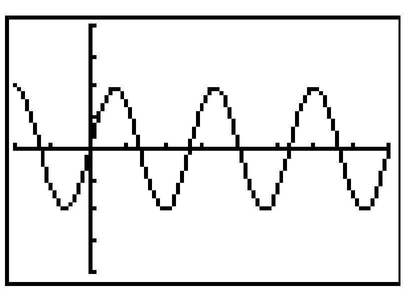
Label each graph $Y_1, Y_2, Y_3, Y_4,$ or Y_5.



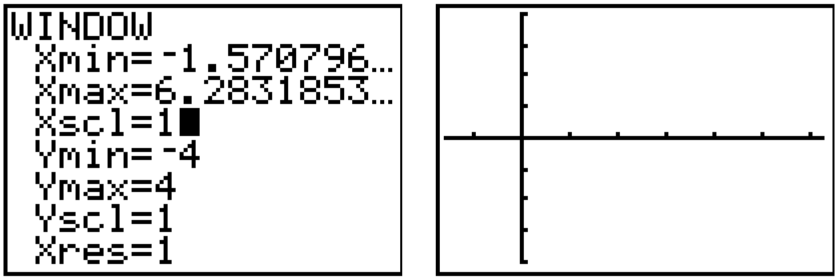






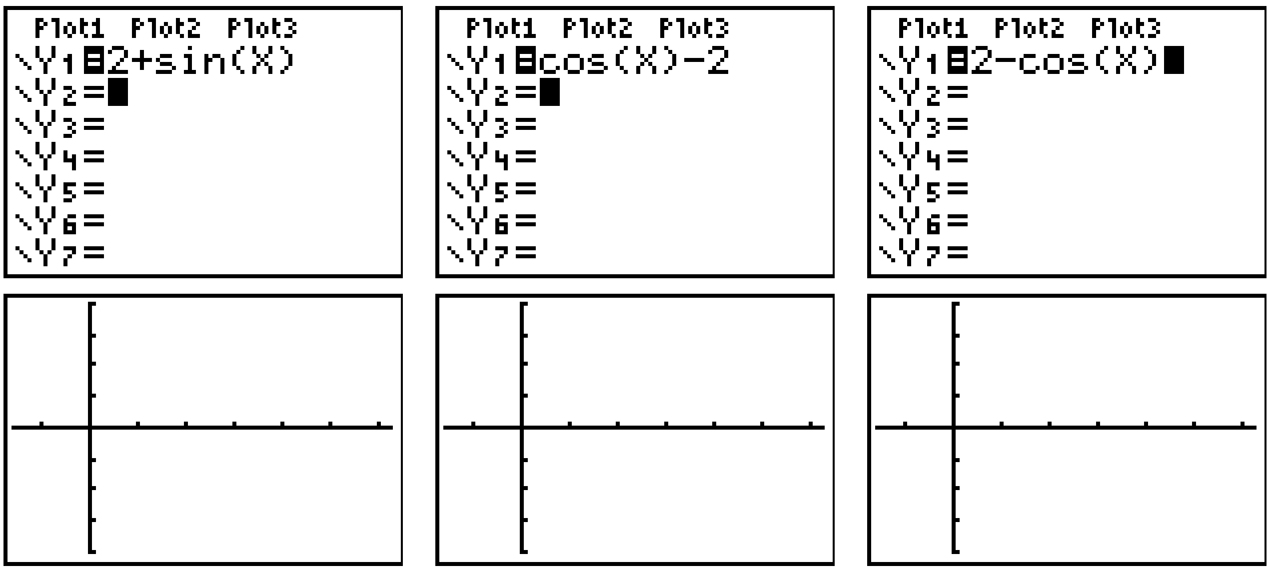


10. Reset your window using the sample below:



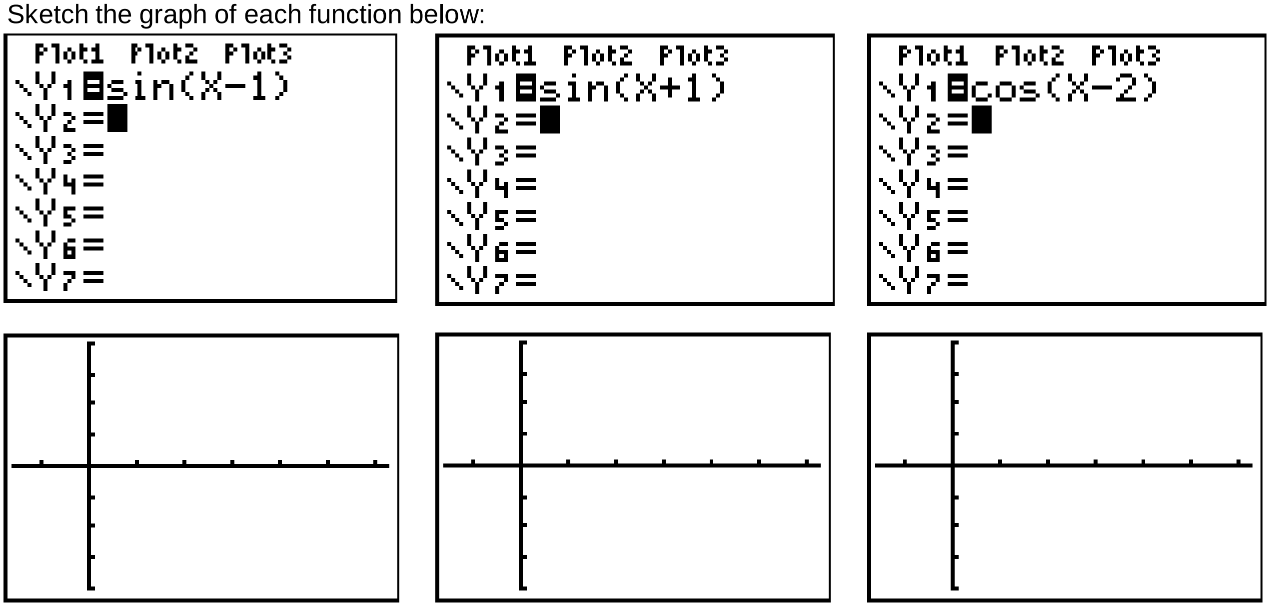
NOTE: Now each mark on the x-axis is equal to 1.0 decimal radians.

11. Sketch each graph in the window provided.



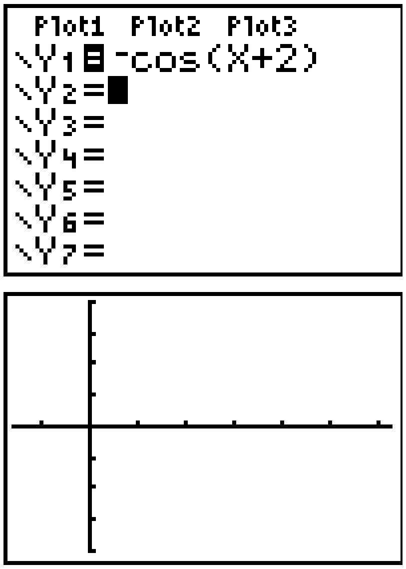
12. Using the graphs above, describe the effect of adding or subtracting a number on the outside of the function had on the graph.

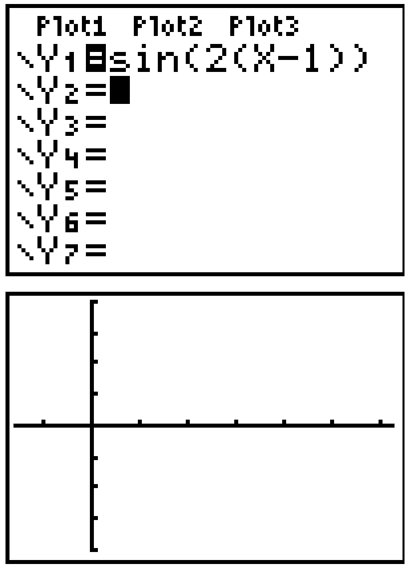
13. Sketch the graph of each function below:

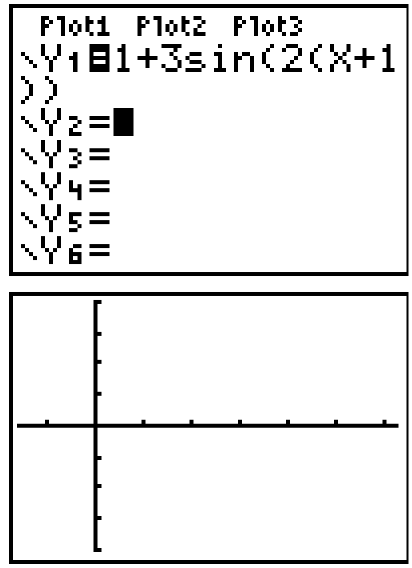


14. Using the graphs above, describe the effect that adding or subtracting a number on the inside of the function has on its graph. (You may want to graph the y = sin(x) or y = cos(x) on the same axis to compare.

15. Sketch each graph below:

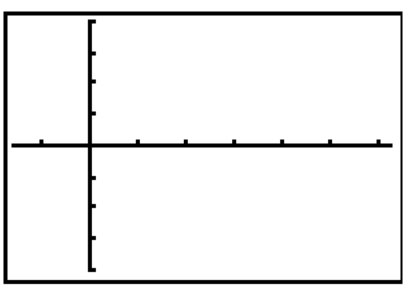




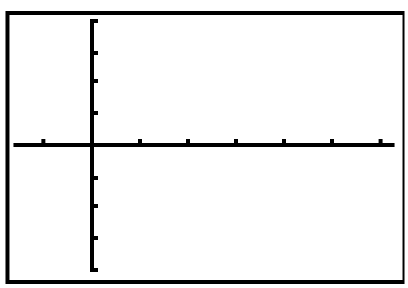


16. Try to sketch each graph without the calculator, then check your answer.

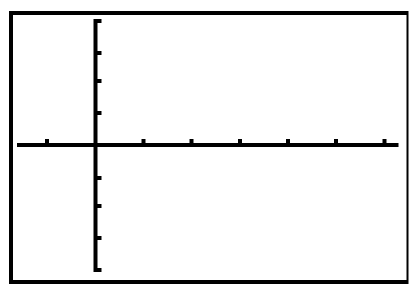
$y \quad 1 \quad \sin(x \quad 2)$



$y \quad 2\cos(2x) \quad 1$

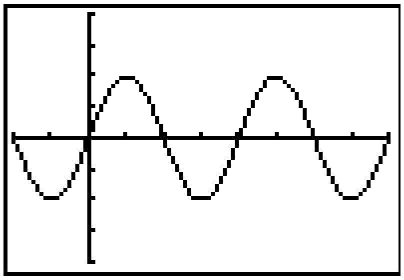


$y \quad 2 \quad \cos(x \quad 2)$

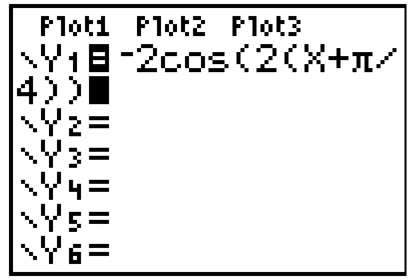


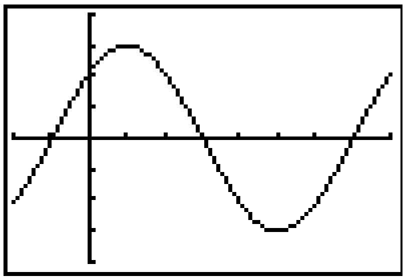
17. Use the descriptions in #7 to find the amplitude and period for each graph in #16.

18. Match each equation with is graph. Note the scale on each graph.

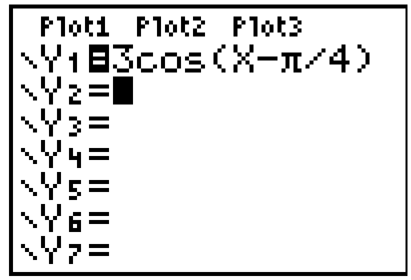


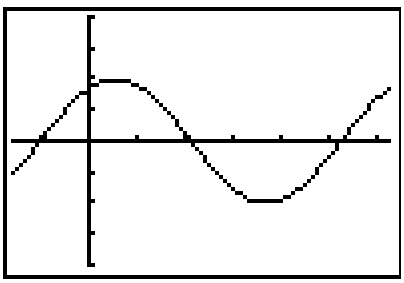
x-scale = / 4



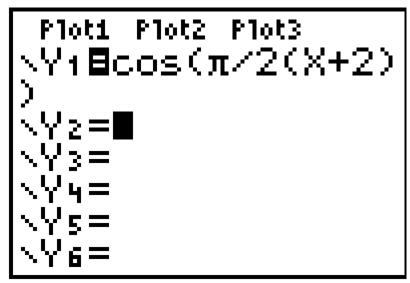


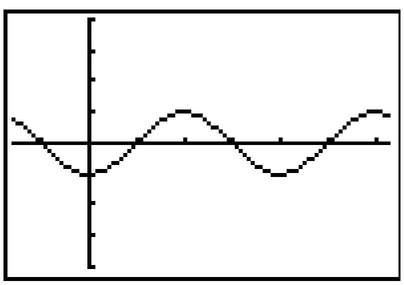
x-scale = / 4



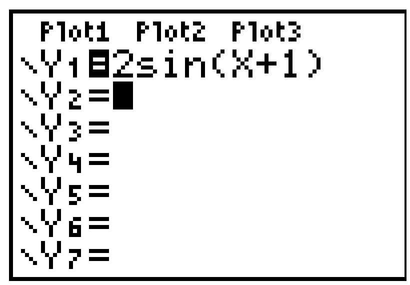


x-scale = 1.0





x-scale = 1.0



19. Summarize the effects of A,B,H, and K on the graphs of sine and cosine using the equations
$y \quad A\sin B(x \quad H) \quad K \qquad y \quad A\cos B(x \quad H) \quad K$.

Goal 3: Learning Activity-Deriving Basic Trigonometric Identities.
The following exercises can be used to help students discover the basic trigonometric identities. After students have completed the exercises, a formal discussion of trigonometric identities should follow.

DERIVING TRIGONOMETRIC IDENTITIES

COMPETE EACH SET OF EXERCISES AND DRAW CONCLUSIONS IN THE SPACE PROVIDED.

1. Give the exact value for each expression in simplest radical form.

a) $\sin(30^\circ)$ b) $\cos(120^\circ)$ c) $\tan(45^\circ)$

$\sin(\ \ 30^\circ)$ $\cos(\ \ 120^\circ)$ $\tan(\ \ 45^\circ)$

d) $\csc(150^\circ)$ e) $\sec(45^\circ)$ f) $\cot(60^\circ)$

$\csc(\ \ 150^\circ)$ $\sec(\ \ 45^\circ)$ $\cot(\ \ 60^\circ)$

2. Which functions changed signs when the angle was changed from a positive to a negative? Will that happen every time? Explain.

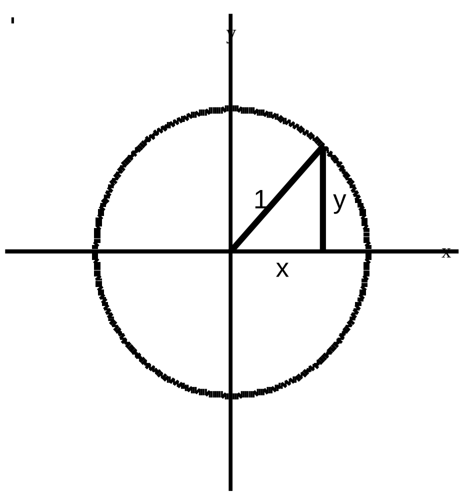
3. Give the value of each expression to the nearest hundredth.

a) $\sin(30^\circ)$ b) $\tan(20^\circ)$ c) $\sec(40^\circ)$

$\cos(60^\circ)$ $\cot(70^\circ)$ $\csc(50^\circ)$

4. What is the relationship between the angles in each pair of functions? What can you conclude about the “cofunctions”?

5. '



For the given unit circle, we know that

$y \quad \sin$ and $x \quad \cos$.

Use the Pythagorean Theorem to state the relationship between x, y and 1.

Substitute $\sin$ and $\cos$ into the Pythagorean expression above.

Goal 3: Graphing Assessment
The following activity may be used as an alternative assessment for translating graphs of trigonometric functions.

GRAPHING PROJECT: Translating Graphs of Trigonometric Functions

For each situation listed below, write an original equation and use The Geometer's Sketchpad to illustrate the translation.
For 4-6, show the original sine or cosine curve and the translation on the same set of axes.
Each graph should contain a text window with an explanation, in complete sentences, of the effects of the equation on the graph of sine or cosine.
Include a text window on each graph with your name and date.

1. Show a change in amplitude of a sine or cosine curve.
2. Show a change in period of sine or cosine function. Be sure to state the period on this graph.
3. Show a reflection of sine or cosine over the horizontal axis.
4. Show a horizontal translation of sine or cosine. Make notations on the graph to illustrate the translation of a particular point on the curve.
5. Show a vertical translation of sine or cosine. Make notations on the graph to illustrate the translation of a particular point on the curve.
6. Show all of the above in a single graph. State this equation in terms of sine and cosine.

NOTE: The equations to be used are your choice.
Be sure to use sine and cosine equally throughout this project.

Goal 4: Group Assessment-Surveying Poster
Each group of students will be assigned a surveying problem. They should work together to solve the problem and then present a poster of the property that was surveyed.

DIRECTIONS:
Read the problem below. Working with the members of your group, make a rough diagram and find the area of the irregular quadrilateral. Document all work to show how you got the area. When all members of the group agree on the solution, draw a large scale diagram on the paper provided. Provide a **NEAT** copy of all work necessary to get the area.
You will be graded on neatness and accuracy. All members of the group must present the solutions in their own notebook and be able to explain how they got their answers.

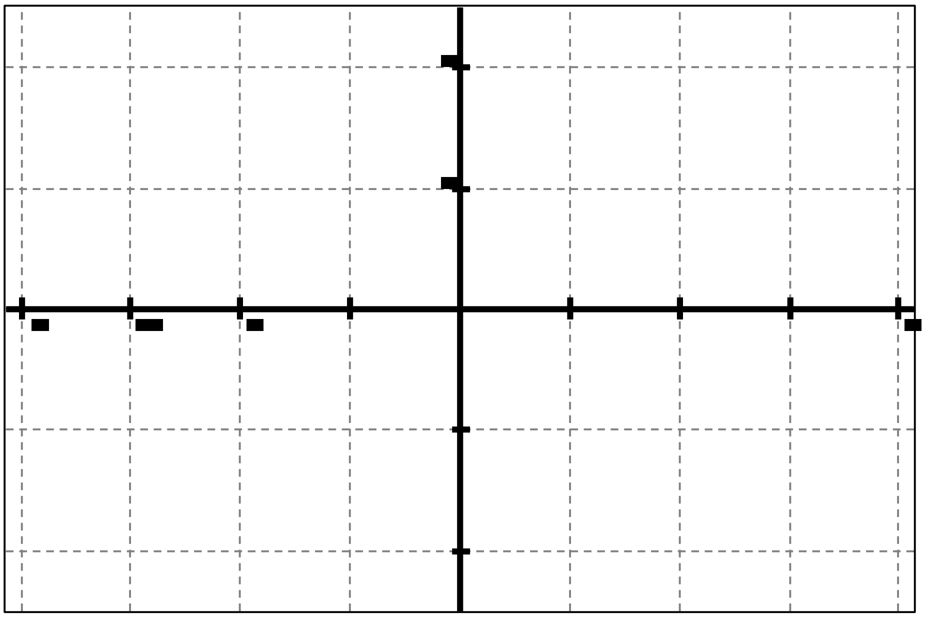
Sample problem: Find the area of the plot of land described below. From a granite post, proceed 195 feet East along Tasker Hill Blvd, then along a bearing of S32°E for 260 feet, then along a bearing of S68°W for 385 feet, and finally along a line back to the granite post.

Goal 6 Learning Activity-Polar vs. Rectangular Graphs
The following activity may help students see the connections between rectangular and polar graphs.

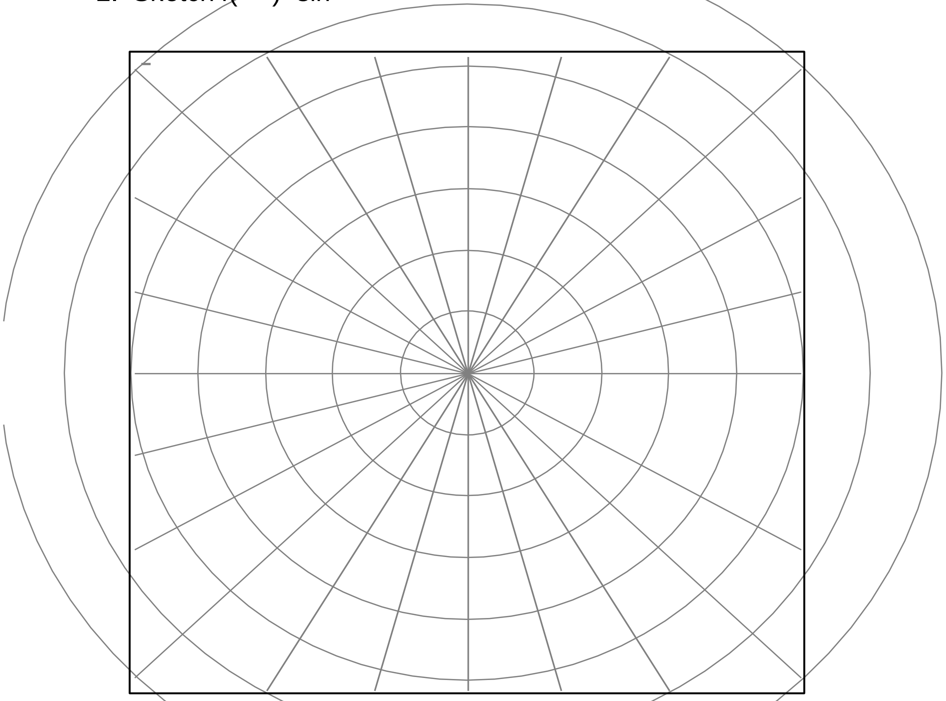
GRAPHING POLAR EQUATIONS

Using Geometer's Sketchpad or a graphing calculator, graph each of the equations below using rectangular coordinates and then polar coordinates. Sketch your graphs in the space provided.

1. Sketch f(x)=sin x



2. Sketch f( )=sin



Describe the characteristics that the two graphs have in common

Goal 11: Polar Graphing Project

DIRECTIONS:

For each equation below, complete the table of values for $0^\circ \quad x \quad 360^\circ$, then graph.

You will be graded on the following:

- a complete and correct table of values
- reasonable scales, clearly labeled
- neat and accurate curves (hint: there are no line segments on these graphs!)
- all points must be plotted!
- **BE CREATIVE!**

Sample problems:

1. $r \quad \sin 2$
2. $r \quad 1 \quad 2\cos$

Sample graph and table:

EQUATION:________________________________

	r