

First and Second Derivative Test

Determining Increasing/Decreasing Intervals,
Maximum/Minimum Values
Concavity Intervals
Points of Inflection

Critical values:

- If $f(x)$ is defined at c , and
- if either
 - $f'(c) = 0$, or
 - $f'(c)$ does not exist, then

c is said to be a critical x -value of $f(x)$.

If a function $f(x)$ is continuous over an interval and $f'(x) > 0$ then $f(x)$ is **increasing** in that interval.

If a function $f(x)$ is continuous over an interval and $f'(x) < 0$ then $f(x)$ is **decreasing** in that interval.

A **local maximum** occurs when the derivative switches from positive to negative.

A **local minimum** occurs when the derivative switches from negative to positive.

Absolute maximum

In an interval the function $f(x)$ has an absolute maximum at $x = c$ if $f(c)$ is greater than or equal to **all** other $f(x)$ in that interval.

Absolute minimum

In an interval the function $f(x)$ has an absolute minimum at $x = c$ if $f(c)$ is less than or equal to **all** other $f(x)$ in that interval.

Concave upward:

If the graph of $f(x)$ is everywhere **above** all of the tangents to the curve in an interval, then the curve is said to be concave upward.

$$f''(x) > 0 \text{ everywhere in the interval.}$$

Concave downward:

If the graph of $f(x)$ is everywhere **below** all of the tangents to the curve in an interval, then the curve is said to be concave downward.

$$f''(x) < 0 \text{ everywhere in the interval.}$$

Point of inflection:

The point at which a curve **changes its concavity** is called a point of inflection.

$f''(x)$ changes sign.

Example 1: Find the coordinates of any local maximum/minimums, increasing/decreasing intervals, points of inflection, and concavity intervals for the following function:

Example 2: Find the coordinates of any local maximum/minimums, increasing/decreasing intervals, points of inflection, and concavity intervals for the following function:

Example 3: Find the coordinates of any local maximum/minimums, increasing/decreasing intervals, points of inflection, and concavity intervals for the following function:

Example 4: Find the coordinates of any local maximum/minimums, increasing/decreasing intervals, points of inflection, and concavity intervals for the following function:

Example 5: Would the tangent line to the function below be above or below the curve at the given point:

Example 6: Would the tangent line to the function below be above or below the curve at the given point: