Infinite Geometric Series

Consider the infinite geometric series 1/2 + 1/4 + 1/8 + 1/16 + . . .

Does this have a finite sum? Yes!

> Put yourself up against one wall in a room and then walk half the distance to the opposite wall. Now walk half of the remaining distance(1/4), and then, again, half the remaining distance(1/8). Continue this forever and, clearly, the total distance we walk is approaching the width of the room and is the infinite series sum presented above.

The answer to the infinite series above is 1 as shown by our **infinite geometric series formula**:

$$S_{\infty} = a_1/(1 - r) \quad \text{where } |r| < 1$$

If $|r| > 1$ then the infinite series is said to **diverge** and the sum never settles down to a consistent answer as *n* increases. (Typically, the sum diverges to infinity.) When $|r| < 1$ the sum is said to **converge**.

converge

Example 1: $a_n = 7(1/3)^{n-1}$; Find S_∞

$a_1 = 7(1/3)^{1-1} = 7$ $\qquad$ $r = 1/3 < 1$

$$S_\infty = \frac{7}{1 - 1/3}$$

$$S_\infty = \frac{7}{2/3} = 7 \cdot \frac{3}{2} = \frac{21}{2}$$

$$\approx 10.5$$

Example 2: $a_n = -22(5/3)^{n-1}$; Find S_∞

$r = \frac{5}{3} > 1$ diverges

Add all terms from 1 to ∞

Example 3: $\sum_{k=1}^{\infty} 4(\sqrt{3})^k$

$r = \sqrt{3} > 1$ diverge

Example 4: $\sum_{n=1}^{\infty} 4(1/\sqrt{3})^n$

$$r = \frac{1}{\sqrt{3}} < 1$$

converge

$$a_1 = 4\left(\frac{1}{\sqrt{3}}\right)^1$$

$$S = \frac{\frac{4}{\sqrt{3}}}{1 - 1/\sqrt{3}}$$

$$S = 5.46$$

Example 5: $a_n = 7(5/\sqrt{3})^{n-1}$; Find S_∞

$$r = \frac{5}{\sqrt{3}} > 1 \quad \text{diverges}$$

Ex6:

~~13~~. Consider the infinite geometric sum $\sum_{k=1}^{\infty} 17\left(x/5+2\right)^k$. What must be required of x so that this series will converge?

$$\sum_{k=1}^{\infty} 17\left(\frac{x}{5}+2\right)^k$$

$$-1 < \frac{x}{5} \cancel{+2} < 1$$

$$-2 \qquad \cancel{-2} \qquad -2$$

$$5\cdot -3 < \cancel{5}\frac{x}{\cancel{5}} < -1\cdot 5$$

$$-15 < x < -5$$