Synthetic Divison

Synthetic division is a shorthand, or shortcut, method of polynomial division in the special case of dividing by a linear factor -- and it only works in this case. Synthetic division is generally used, however, not for dividing out factors but for finding zeroes (or roots) of polynomials.

Synthetic Division

Use synthetic division to divide $x^3 + 13x^2 + 46x + 48$ by $x + 3$. What is the quotient and remainder?

Step 1 Set up your polynomial division.

$$(x^3 + 13x^2 + 46x + 48) \div (x + 3)$$

Step 2 Reverse the sign of the constant, 3, in the divisor. Write the coefficients of the dividend: 1 13 46 48.

Step 3 Bring the first coefficient, 1, down to the bottom line.

Step 4 Multiply the coefficient, 1, by the divisor, –3. Put this product, –3, underneath the second coefficient –13, and add those two numbers: 13 + (–3) = 10.

Step 5 Continue multiplying and adding through the last coefficient. The final sum is the remainder.

Here is the final product.

$$
\begin{array}{r|rrrr}
-3 & 1 & 13 & 46 & 48 \\
 & & & & \\
\hline
 & & & &
\end{array}
$$

$$
\begin{array}{r|rrrr}
-3 & 1 & 13 & 46 & 48 \\
 & & & & \\
\hline
 & 1 & & &
\end{array}
$$

$$
\begin{array}{r|rrrr}
-3 & 1 & 13 & 46 & 48 \\
 & & -3 & & \\
\hline
 & 1 & 10 & &
\end{array}
$$

$$
\begin{array}{r|rrrr}
-3 & 1 & 13 & 46 & 48 \\
 & & -3 & -30 & -48 \\
\hline
 & 1 & 10 & 16 & 0
\end{array}
$$

The quotient is $x^2 + 10x + 16$ and since the remainder is zero, then (x-3) is a factor of the polynomial.

What is the quotient and remainder of the following polynomials?

Example 1. $(x^3 - 2x + 8) \div (x + 2)$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -2 & 8 \\ & & -2 & 4 & -4 \\ \hline & 1 & -2 & 2 & 4 \end{array}$$

$x^2 - 2x + 2$

Remainder
4

$(x+2)$ is not a factor

Example 2. $(12x^3 - 71x^2 + 57x - 10) \div (x - 5)$

so $x = 5$

$$\begin{array}{r|rrrr} 5 & 12 & -71 & 57 & -10 \\ & & 60 & -55 & 10 \\ \hline & 12 & -11 & 2 & 0 \end{array}$$

$12x^2 - 11x + 2$

Remainder 0

$(x-5)$ is a factor

Example 3. $(3x^4 + x^3 - 6x^2 - 9x + 12) \div (x + 1)$

$x = -1$

$$\begin{array}{r|rrrrr} -1 & 3 & 1 & -6 & -9 & 12 \\ & & -3 & 2 & 4 & 5 \\ \hline & 3 & -2 & -4 & -5 & 17 \end{array}$$

$3x^3 - 2x^2 - 4x - 5$ Remainder 17

$(x+1)$ is not a factor

Example 4. $(2x^3 - 15x + 23) \div (x - 2)$

Left to reader

Example 5. $(x^3 + x + 10) \div (x + 2)$

Left to reader

Use synthetic division to completely factor the polynomial to a product of linear terms. Also state the zeros.

Example 6. $f(x) = x^3 - 3x^2 - 15x + 125$ given $(x + 5)$ is a factor.

$$\begin{array}{r|rrrr} -5 & 1 & -3 & -15 & 125 \\ & & -5 & 40 & -125 \\ \hline & 1 & -8 & 25 & 0 \end{array}$$

$x^2 - 8x + 25$

$y = (x+5)(x^2 - 8x + 25)$

Example 7. $f(x) = 3x^3 - 2x^2 - 15x + 10$ given (3x - 2) is a factor.

$3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$

$$\begin{array}{c|cccc} \frac{2}{3} & 3 & -2 & -15 & 10 \\ & & 2 & 0 & -10 \\ \hline & 3 & 0 & -15 & 0 \end{array}$$

$3x^2 - 15$

$x^2 - 5$

$y = (3x - 2)(x^2 - 5)$

$x = 2/3$

$x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

Example 8. $f(x) = 5x^3 + 4x^2 - 20x - 16$ given

(x-2) is a factor.

$$\begin{array}{r|rrrr} 2 & 5 & 4 & -20 & -16 \\ & & 10 & 28 & 16 \\ \hline & 5 & 14 & 8 & 0 \end{array}$$

$$y = (x-2)(5x^2 + 14x + 8)$$

$$y = (x-2)(5x+4)(x+2)$$

$x = 2$ $\quad$ $x = -4/5$ $\quad$ $x = -2$