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Revision/Draft

Subject/Course: College Prep Statistics
 Grade: 11/12
 Time frame: approx 5-6 weeks

Unit: 6 The Relationship between Two Variables

Stage 1 Desired Results

ESTABLISHED GOALS					
<p><u>CCSS.MATH.CONTENT.HSS.ID.B.6</u> Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <p><u>CCSS.MATH.CONTENT.HSS.ID.B.6.A</u> Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</p> <p><u>CCSS.MATH.CONTENT.HSS.ID.B.6.B</u> Informally assess the fit of a function by plotting and analyzing residuals.</p>	<p style="text-align: center;"><i>Transfer</i></p> <p><i>Students will be able to independently use their learning to...</i></p> <ul style="list-style-type: none"> • apply concepts of graphs and their equations to analysis and predictions related to business and manufacturing models, the sports and gaming industry, weather forecasting and health related services • <u>CCSS.Math.Practice.MP1</u>: Make sense of problems and persevere in solving them • <u>CCSS.Math.Practice.MP2</u>: Reason abstractly and quantitatively. • <u>CCSS.Math.Practice.MP4</u>: Model with mathematics. • <u>CCSS.Math.Practice.MP5</u>: Use appropriate tools strategically • <u>CCSS.Math.Practice.MP6</u>: Attend to precision <p style="text-align: center;"><i>Meaning</i></p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%; text-align: left;">UNDERSTANDINGS <i>Students will understand that...</i></th> <th style="width: 50%; text-align: left;">ESSENTIAL QUESTIONS <i>Students will keep considering...</i></th> </tr> </thead> <tbody> <tr> <td style="vertical-align: top;"> <ul style="list-style-type: none"> • A scatter plot serves two purposes: - to determine if there is a useful relationship between two variables, and - to determine the family of equations that describes the relationship. • Data are collected for a purpose and have meaning in a context. </td> <td style="vertical-align: top;"> <ul style="list-style-type: none"> • How can graphs be used to examine data? • What is the role of outliers in data observations? • What is the strength of an association between two variables? • What is the meaning behind the least squares line? </td> </tr> </tbody> </table>	UNDERSTANDINGS <i>Students will understand that...</i>	ESSENTIAL QUESTIONS <i>Students will keep considering...</i>	<ul style="list-style-type: none"> • A scatter plot serves two purposes: - to determine if there is a useful relationship between two variables, and - to determine the family of equations that describes the relationship. • Data are collected for a purpose and have meaning in a context. 	<ul style="list-style-type: none"> • How can graphs be used to examine data? • What is the role of outliers in data observations? • What is the strength of an association between two variables? • What is the meaning behind the least squares line?
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<p><u>CCSS.MATH.CONTENT.HSS.ID.B.6.C</u> Fit a linear function for a scatter plot that suggests a linear association.</p> <p><u>CCSS.MATH.CONTENT.HSS.ID.C.7</u> Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</p> <p><u>CCSS.MATH.CONTENT.HSS.ID.C.8</u> Compute (using technology) and interpret the correlation coefficient of a linear fit.</p> <p><u>CCSS.MATH.CONTENT.HSS.ID.C.9</u> Distinguish between correlation and causation.</p> <p><u>CCSS.MATH.CONTENT.HSS.IC.B.6</u> Evaluate reports based on data.</p>	<ul style="list-style-type: none"> • The association between two variables considers both the direction and strength of the association • The strength of an association between two variables reflects how accurately the value of one variable can be predicted based on the value of the other variable. • Outliers are observations with large residuals and do not follow the pattern apparent in the other data points. 	<ul style="list-style-type: none"> • What is the meaning of the slope and y-intercept in the line of regression? • What determines a regression equation is an appropriate model?
Assessment		
	<p><i>Students will know...</i></p> <ul style="list-style-type: none"> • How to identify the roles of variables and to place the response variable on the y-axis and the explanatory variable on the x-axis (using proper context). • The conditions for correlation and how to check them. • Correlations are between -1 and +1 (inclusive), and each extreme indicates a perfect linear association. • How the magnitude of the correlation reflects the strength of the linear association. • The correlation has no units. • The correlation coefficient is not changed by changing the center or scale of either variable. • Causation cannot be demonstrated by a scatterplot or correlation. • How a linear equation summarizes the 	<p><i>Students will be skilled at...</i></p> <ul style="list-style-type: none"> • Making a scatter plot by hand (for a small set of data) and with technology. • Computing the correlation of two variables. • Reading a correlation table produced by a statistics program. • Describing the direction, form, and strength of a scatter plot. • Using a correlation as part of the description of the scatterplot. • Being aware of misinterpretations of correlation. • Using a plot of the residuals against predicted values as a check for the appropriateness of the generated line of regression. • Finding a regression equation from the summary statistics for each variable and the correlation between the variables. • Finding a regression equation using a

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relationship between two variables.

- That the least squares slope is easily affected by extreme values.
- Residuals are the differences between data values and the corresponding predicted values.
- How the residuals relate to the least squares linear equation

statistics software output table.

- Using regression to predict a value of y for a given x .
- Computing the residual for each data value and display them.
- Writing a sentence in context showing the meaning of the slope and y -intercept.
- Describing a prediction made from a regression equation, relating the predicted value to the specified x -value.

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Stage 2 – Evidence

Code	Evaluative Criteria	Assessment Evidence
T, M, A	Scoring Rubric used to evaluate successful understanding of data gathering and analysis and a clear presentation on the meaning of the analysis.	<p>PERFORMANCE TASK(S): <i>Students will show that they really understand evidence of...</i></p> <p>Goal: Have students research data for two related variables to determine if there is an existing correlation</p> <p>Role: Statistician</p> <p>Audience: Manager in a field related to the chosen topic</p> <p>Situation: Have students choose 2 variables to research and determine if there is a correlation (i.e.: temperature and ice cream sales, height and foot size, etc..) Students should examine the scatter and residual plots, determine the correlation coefficient and the line of regression. Then they should draw a conclusion as to the nature of the correlation; strength, direction and form and the usefulness of the line of regression as a predictive model for the data.</p> <p>Product or Performance: Presentation on the data gathered and the corresponding conclusion</p> <p>Standards for Success: Accurate calculations and detailed clear explanations of the variables and the correlation</p>

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M, A	Thorough understanding of the properties of a scatter plot and an equation of a line	<p>OTHER EVIDENCE: <i>Students will show they have achieved Stage 1 goals by...</i></p> <ul style="list-style-type: none"> Monitoring class work through board work, group work, questioning, and walk arounds Check for understanding using homework as a checkpoint, white board activities, and reflections and exit tickets Differentiate through purposeful or flexible grouping, use of diagrams and explanations to demonstrate understanding and active lessons involving technology Alternative assessment projects using real world applications Quizzes Unit Test - to include variety of DOK level of problems
M, A	Accurate application of content in graphing a scatter plot by hand, drawing the line of best fit and writing an equation for the regression line	
T, M, A	Accurate application of content by using technology to access the scatter plot and line of regression	
M, A	Thorough understanding of the correlation coefficient, how to make an approximation from a graph and how to calculate the actual value using the graphing calculator	
M, A	Accurate application of the content in determining the nature of the correlation for a set of data	
T, M, A	Accurate application of the content in determining if a regression equation is a good model and then making predictions using the	

T, M, A	equation. Then understanding what the slope and intercept mean in the context of the problem Thorough understanding of residuals and how they relate to the predicted values	
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Stage 3 – Learning Plan

Code	<i>Pre-Assessment</i>	
	<ul style="list-style-type: none"> Teacher checks for prerequisite and prior knowledge via warm-up and questioning activities, such as basic problems on graphing equations in slope intercept form and calculating slope, substitution and evaluation, solving equations Teacher will provide review and assessment on prerequisite correlations, vocabulary knowledge to ensure all students are capable of communicating effectively 	
	<p>Summary of Key Learning Events and Instruction</p> <p><i>Student success at transfer meaning and acquisition depends on...</i></p>	<p>Process Monitoring</p>
<p>A</p> <p>M, A</p> <p>M, A</p> <p>M, A</p> <p>M, A</p> <p>M, A</p> <p>T, M, A</p> <p>T, M, A</p>	<ul style="list-style-type: none"> Teacher will emphasize the importance of the first rule of data analysis: make a picture. Teacher will continue to emphasize the importance of vocabulary and notation. Teacher will introduce formulas and show examples for calculating regression equations by hand. The teacher will then at some point show students how to use technology to get the identical equations in order for students to make connections with what technology can do and be confident they are getting accurate results. Each of these topics can be taught individually or by having students work in small groups verifying results. Teacher supplies visual scatterplots and asks students to describe form, direction, strength, and approximate a correlation coefficient. Students should describe scatter plots verbally indicating direction, form, and strength. Teacher and students will collectively practice finding lines of regressions Students plot unusual values and then determine if they understand if and why they are unusual. Teacher will instruct students on residuals by using a scatter plot and having students record the residual for 	
	<ul style="list-style-type: none"> Warm up questions on graphing equations in slope intercept form and calculating slope, substitution and evaluation, solving equations Class worksheets which practice plotting data and finding regression lines and residuals with direct teacher observation and/or student self assessment Practice on whiteboard with direct teacher observation Kahoot quiz with regression equation review questions Students self-assess homework using answer keys and/or teacher collects homework to assess understanding Projects/performance tasks <ul style="list-style-type: none"> Gathering and graphing real data to determine the equation that can be 	

T, M, A	<p>each point and explain the meaning of the residuals in the context of the problem.</p> <ul style="list-style-type: none"> Teacher will instruct students on the use of the graphing calculator to access a plot of the residuals and how to use it to determine if the regression equation is a good model for the population 	<p>used as a predictive model Work with real data from a variety of areas of study</p> <ul style="list-style-type: none"> Summative assessments
M, A	<ul style="list-style-type: none"> Teacher and students will collectively practice finding residuals for a regression line 	<ul style="list-style-type: none"> quizzes unit test
T, M, A	<ul style="list-style-type: none"> Students practice in small groups working problems by hand and verifying results. They should then do the same exercise using technology in order to see the benefits of using technology with respect to time spent doing calculations and how it relates to productivity and accuracy. 	
T, M, A	<ul style="list-style-type: none"> Students present their work to the class in order for students to observe more instances of good models and models that are not representative of the true nature of the data 	
T, M, A	<ul style="list-style-type: none"> Students work in small groups to find their own bivariate data. For instance each group could gather measurements of height as related to the golden ratio, write their own regression equations, and compare them with the other groups. They should then learn that the larger their sample sizes, the more closely their equations will resemble each other. They can gain a better understanding of the slope and intercept in the context of the problem. (A brief tangent can be taken into an understanding of the significance of the golden ratio) 	
T, M, A	<ul style="list-style-type: none"> Students will be given 4 or 5 ordered pairs to plot, write an equation, and find the correlation coefficient. They would then be instructed to change one of the points and see how the values could have dramatically changed. Using few points gives students a more visual experience. 	
T, M, A	<ul style="list-style-type: none"> Teacher should explain there are three steps that should be followed when answering these types of 	

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T, M	<p>questions. Think (the students should state the question and make a plan), Show (the students should show their calculations), and Tell (interpret your results in the context of the problem)</p> <ul style="list-style-type: none">• Students will explore lines of regression and residual values using the unit's performance task and complete an activity based review in preparation for a unit assessment.	
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ESTABLISHED GOALS					
<p><u>CCSS.MATH.CONTENT.HSS.IC.A1</u></p> <p>Understand statistics as a process for making inferences about population parameters based on a random sample from that population.</p> <p><u>CCSS.MATH.CONTENT.HSS.IC.A2</u></p> <p>Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. <i>For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</i></p> <p><u>CCSS.MATH.CONTENT.HSS.IC.B3</u></p> <p>Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.</p> <p><u>CCSS.MATH.CONTENT.HSS.IC</u></p>	<div style="background-color: #e0e0e0; text-align: center; padding: 5px;">Transfer</div> <p><i>Students will be able to independently use their learning to...</i></p> <ul style="list-style-type: none"> • apply concepts of confidence intervals and hypothesis testing to problems related to business and manufacturing models, the sports and gaming industry, the political arena and health related services. • <u>CCSS.Math.Practice.MP1</u>: Make sense of problems and persevere in solving them • <u>CCSS.Math.Practice.MP2</u>: Reason abstractly and quantitatively. • <u>CCSS.Math.Practice.MP4</u> Model with mathematics. • <u>CCSS.Math.Practice.MP5</u> Use appropriate tools strategically • <u>CCSS.Math.Practice.MP6</u> Attend to precision <div style="background-color: #e0e0e0; text-align: center; padding: 5px;">Meaning</div> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 50%; text-align: left; padding: 5px;">UNDERSTANDINGS <i>Students will understand that...</i></th> <th style="width: 50%; text-align: left; padding: 5px;">ESSENTIAL QUESTIONS <i>Students will keep considering...</i></th> </tr> </thead> <tbody> <tr> <td style="vertical-align: top; padding: 5px;"> <ul style="list-style-type: none"> • A primary goal of sampling is to estimate the value of a parameter based on a statistic. • Confidence intervals use the sample statistic to construct an interval of values that one can be reasonably certain contains the true (unknown) parameter. • Confidence intervals and tests of significance are complementary procedures. </td> <td style="vertical-align: top; padding: 5px;"> <ul style="list-style-type: none"> • Why are confidence intervals and tests of significance important? • How is sampling used and why is it important? • How do you use inferential models to draw statistically significant conclusions from data and make inferences about populations? • How can the language of statistics be used to communicate mathematical ideas coherently and precisely? </td> </tr> </tbody> </table>	UNDERSTANDINGS <i>Students will understand that...</i>	ESSENTIAL QUESTIONS <i>Students will keep considering...</i>	<ul style="list-style-type: none"> • A primary goal of sampling is to estimate the value of a parameter based on a statistic. • Confidence intervals use the sample statistic to construct an interval of values that one can be reasonably certain contains the true (unknown) parameter. • Confidence intervals and tests of significance are complementary procedures. 	<ul style="list-style-type: none"> • Why are confidence intervals and tests of significance important? • How is sampling used and why is it important? • How do you use inferential models to draw statistically significant conclusions from data and make inferences about populations? • How can the language of statistics be used to communicate mathematical ideas coherently and precisely?
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<p><u>.B4</u> Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</p> <p><u>CCSS.MATH.CONTENT.HSS.IC</u></p> <p><u>.B5</u> Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.</p> <p><u>CCSS.MATH.CONTENT.HSS.IC</u></p> <p><u>.B.6</u> Evaluate reports based on data.</p>	<ul style="list-style-type: none"> • Paired comparisons experimental design allows control for possible effects of extraneous variables. • Understand when you use a z test. • Determine if a correlation exists between sets of data. • Find the confidence interval for the mean when the standard deviation is known. • The difference between the alternative hypothesis and null hypothesis in a hypothesis test • How to determine when it is appropriate to reject the null hypothesis 	<ul style="list-style-type: none"> • How can technology be applied to create and interpret models? • How can improperly applied inference procedures lead to bad conclusions? • How do I construct a confidence interval? • What type of information does a confidence interval provide me? • How can hypothesis testing provide the statistical structure to reject or fail to reject the null hypothesis? • When does a person choose to use the z-test type of hypothesis testing
Acquisition		
	<p><i>Students will know...</i></p> <ul style="list-style-type: none"> • That the margin of error of a confidence interval for a proportion change with the sample size and the level of confidence. • How to examine their data for violations of conditions that would make inferences about a population proportion unwise or invalid. • How to find a confidence interval for a population proportion or mean. • The conditions that must be true for a one-proportion z-test to be appropriate and how to check for these conditions. 	<p><i>Students will be skilled at...</i></p> <ul style="list-style-type: none"> • Constructing a one-proportion z-interval. • Interpreting a one-proportion z-interval in a simple sentence or two within the context of the problem. • Stating the null and alternative hypotheses for a one-proportion z-test. • Performing a one-proportion z-test. • Writing a sentence interpreting the results of a one-proportion z-test in context. • Interpreting the meaning of a P-value in nontechnical language. • Explaining the meaning of a confidence interval for a population mean. • Interpreting the result of a test of a

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- How to choose between a one-sided and two-sided alternative hypothesis and be able to explain their choice.
- How the critical value for a test is related to the specified alpha level.
- The close relationship between hypothesis tests and confidence intervals.
- That we do not “accept” a null hypothesis if we cannot reject it, but rather that we can only “fail to reject” the hypothesis for lack of evidence against it.
- Know that the P-value of a test does not give the probability that the null hypothesis is correct.

hypothesis about a population mean..

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Code	Evaluative Criteria	Assessment Evidence
T, M, A	Scoring Rubric used to evaluate successful understanding of data gathering and analysis and a clear presentation on the meaning of the analysis.	<p>PERFORMANCE TASK(S): <i>Students will show that they really understand evidence of...</i></p> <p>Goal: Perform a hypothesis test checking the published proportion of blue M&Ms or red Skittle</p> <p>Role: Marketing Department for Mars Co.</p> <p>Audience: CEO for Mars</p> <p>Situation: Have the marketing department calculate the percentage of blue M&Ms in their bag. Perform a hypothesis test comparing it to the published percentage available at www.mars.com. Decide whether they will reject or fail to reject the null hypothesis.</p> <p>Product or Performance: Board presentation.</p> <p>Standards for Success: Accurate calculations and detailed clear explanations of the testing and the conclusions</p>

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M, A	Thorough understanding of the notation and vocabulary associated with confidence intervals and hypothesis testing	<p>OTHER EVIDENCE: <i>Students will show they have achieved Stage 1 goals by...</i></p> <ul style="list-style-type: none"> Monitoring class work through board work, group work, questioning, and walk-around Check for understanding using homework as a checkpoint, whiteboard activities, and reflections and exit tickets Differentiate through purposeful or flexible grouping, use of diagrams and explanations to demonstrate understanding and active lessons involving technology Alternative assessment projects using real world applications Quizzes Unit Test - to include variety of DOK level of problems
T, M, A	Accurate application of content in computing critical values, probabilities and margins of error	
T, M, A	Accurate application of content by using calculations to determine the confidence interval for certain criteria	
M, A	Thorough understanding of the hypothesis and how to determine the null and alternative hypotheses.	
T, M, A	Appropriate application of content by calculating a test statistic and using it to determine whether or not to reject the null hypothesis	

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Stage 3 – Learning Plan

Code	<i>Pre-Assessment</i>	
	<ul style="list-style-type: none"> Teacher checks for prerequisite and prior knowledge via warm-up and questioning activities, such as basic problems on solving equations, order of operations and substitution and work with the z-score formula Teacher will provide review and assessment on prerequisite inferential statistics vocabulary knowledge to ensure all students are capable of communicating effectively 	
	<p>Summary of Key Learning Events and Instruction <i>Student success at transfer meaning and acquisition depends on...</i></p>	<p>Progress Monitoring</p>
M, A	<ul style="list-style-type: none"> Teacher will continue to emphasize the importance of vocabulary and notation, specifically related to confidence intervals and margins of error 	<p>Warm up questions on solving equations, order of operations and substitution and work with the z-score formula</p>
T, M, A	<ul style="list-style-type: none"> Teacher will introduce formulas and show examples for creating confidence intervals by hand. The teacher will then at some point show students how to use technology to get the identical results in order for students to make connections with what technology can do and be confident they are getting accurate results. 	<ul style="list-style-type: none"> Class worksheets on finding confidence intervals and hypothesis testing with direct teacher observation and/or student self assessment
M, A	<ul style="list-style-type: none"> Each of these topics can be taught individually or by having students work in small groups verifying results. 	<ul style="list-style-type: none"> Practice on whiteboard with direct teacher observation
M, A	<ul style="list-style-type: none"> Teacher may want to supply organized formula/symbol sheets for students as there are extensive formulas and symbols used in this unit. A z-table needs to be provided. 	<ul style="list-style-type: none"> Kahoot quiz with confidence interval and margin of error review questions
M, A	<ul style="list-style-type: none"> Teacher and students will collectively practice calculating confidence intervals and margin of error 	<ul style="list-style-type: none"> Students self-assess homework using answer keys and/or teacher collects homework to assess understanding
T, M, A	<ul style="list-style-type: none"> Students research daily or weekly to find statistics available online or as a hard copy relating to the topics in this unit. They may want to share them individually to the class. 	<ul style="list-style-type: none"> Projects/performance tasks
T, M, A	<ul style="list-style-type: none"> Students individually or in small groups create confidence intervals using published percentiles for M&M colors and their own sample bag. They can 	<ul style="list-style-type: none"> Tasks involving an examination of confidence intervals (i.e. in politics or business) Work with real data to create

<p>T, M, A</p> <p>T, M, A</p> <p>T, M, A</p> <p>M, A</p> <p>M, A</p> <p>M, A</p> <p>T, M</p> <p>T, M</p>	<p>check whether the confidence interval they created captured the true proportion. Hopefully not all of them will if our sample size is large enough.</p> <ul style="list-style-type: none"> • Teacher will introduce hypothesis testing and how to identify the hypothesis, alternative and the null hypothesis for a given claim • Teacher will instruct students on the calculation of the test statistic and its use in determining whether or not to reject the null hypothesis • Teacher will provide students with practice on hypothesis testing and review how to determine whether the data indicates a one or two tailed test • Teacher and students will collectively practice setting up a hypothesis test and determining its relevance • Students will analyze data to determine whether it indicates a one tailed or two tailed test is appropriate in testing a hypothesis • Students will work in small groups working formulas by hand and verify results with each other. • Students will describe confidence intervals and results of their hypothesis test verbally as well as writing complete sentences in context. • Students will explore confidence intervals and hypothesis testing using the unit's performance task and complete an activity based review in preparation for a unit assessment 	<p>confidence intervals such as in medicine or physics</p> <ul style="list-style-type: none"> • Summative assessments quizzes unit test
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