Reviewing Point Slope Form of a Line

To write an equation of a line, you need a point and a slope.

$$y - y_1 = m(x - x_1)$$

How to find the slope of a tangent line.

The derivative value is the slope of the tangent line. Hence if we wanted the slope of the tangent line for $f(x) = 3x^2 + 4$ at the point (2, 16), then we would take the derivative and substitute in $x = 2$.

$f'(x) = 6x$ so $f'(2) = 12$

Writing the equation for a tangent line.

In the previous slide we determine that f '(2) = 12, and we were given the point (2, 16). Therefore the equation for the tangent line to f(x) at x = 2 is the following:

$$y - 16 = 12(x - 2)$$

Note: You could convert this to slope-intercept form, but this is not required.

A Similar Example:

Write the equation for the tangent line to the curve $g(x) = x^4 - 5x^3 - 7x + 2$ at the point where $x = -1$.

First we calculate the derivative and then substitute in $x = -1$ to find the slope.

$g'(x) = 4x^3 - 15x^2 - 7$ so

$g'(-1) = 4(-1)^3 - 15(-1)^2 - 7 = -26$

Next we need to find the y coordinate of the point on g(x) since it was not initially given.

$g(-1) = (-1)^4 - 5(-1)^3 - 7(-1) + 2 = 15$

Finally write the equation for the tangent line:

$y - 15 = -26(x + 1)$

Lastly, you might be asked to find the equation of the normal line.

A normal line is the line perpendicular to the tangent line. Hence, your goal is to calculate the slope of the tangent line as in the prior examples, and then use the opposite reciprocal to obtain the slope of the normal line. We write the equation for the normal line in point-slope form as well.

LAST EXAMPLE

Write the equation for the normal line to the curve
$p(x) = 4x^2 - 7x + 1$ at the point (2,3)

$p'(x) = 8x-7$ so $p'(2) = 8(2) - 7 = 9$

Hence the slope of the tangent line is 9.

Thus, we now have that the slope of the normal line is -1/9.

Finally, we obtain $y - 3 = -1/9(x - 2)$

Bellringer #1:

Write the equation of the tangent line to the curve $f(x) = 3x^3 - 2x + 5$ at the point where $x = 2$.

Bellringer #2:

Write the equation of the normal line to the curve $g(x) = x^5 - 2x^3 + 5x$ at the point (1, 4).