NC Math II Standards

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). These have been indicated throughout the document.

The plus (+) standards are standards that are appropriate for ALL students in this course and not exclusive to Honors level courses. They are foundational standards to prepare students for success in 4th level mathematics courses.

Last revised 7/23/15

NC Math II Standards

Number

N.RN.2

N.Q.1★
N.Q.2★
N.Q.3★

Algebra

A.SSE.1a★
A.SSE.1b★
A.SSE.2
A.SSE.3c★

A.APR.1
A.APR.3

A.CED.1★
A.CED.2★
A.CED.3★
A.CED.4★

A.REI.1
A.REI.2
A.REI.4b
A.REI.7
A.REI.10
A.REI.11★

Function

F.IF.2
F.IF.4★
F.IF.5★
F.IF.7b★
F.IF.7e★
F.IF.8a
F.IF.9

F.BF.1a★
F.BF.1b★
F.BF.3

Geometry

G.CO.2
G-CO.3
G-CO.4
G-CO.5
G-CO.6
G-CO.7
G-CO.8
G.CO.10
G.CO.13

G.SRT.1a
G.SRT.1b
G.SRT.6
G.SRT.7
G.SRT.8★
G-SRT.9 (+)
G-SRT.11 (+)

G.GPE.1
G.GPE.6

G.GMD.4

G.MG.1★
G.MG.2★
G.MG.3★

Statistics & Probability

S.IC.2★
S.IC.6★

S-CP.1★
S-CP.2★
S-CP.3★
S-CP.4★
S-CP.5★
S-CP.6★
S-CP.7★
S-CP.8 (+) ★
S-CP.9 (+) ★

Standard	Cluster: Extend the properties of exponents to rational exponents.
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	This standard is in Math I and II. Math II, students continue to build upon the properties of exponents to include all rational exponents. (In Math I, students focused on exponents with a numerator of one.) **Students rewrite expressions involving rational exponents as expressions involving radicals and simplify those expressions.** ***Example:*** Using the properties of exponents, simplify a. $\left(\sqrt[4]{32^3}\right)^2$ b. $\frac{\sqrt[5]{b^3}}{b^{\frac{4}{3}}}$ ***Example:*** Rewrite the expression $8^{\frac{2}{3}}$ in exponential form. Explain how they are equivalent. ***Solution:*** $8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = \left(8^{\frac{1}{3}}\right)^2$ *In the first expression, the base number is 8 and the exponent is* $2/3$*. This means that the expression represents* 2 *of the* 3 *equal factors whose product is* 8*, thus the value is 4, since* $(2\times2\times2) = 8$*; there are three factors of 2; and two of these factors multiply to be 4. In the second expression, there are 2 equal factors of 8 or 64. The exponent* $1/3$ *represents* 1 *of the* 3 *equal factors of* 64*. Since* $4\times4\times4 = 64$ *then one of the three factors is* 4*. The last expression there is* 1 *of* 3 *equal factors of 8 which is 2 since* $2\times2\times2 = 8$*. Then there are* 2 *of the equal factors of* 2*, which is* 4*.* **Students rewrite expressions involving radicals as expressions using rational exponents and use the properties of exponents to simplify the expressions.** ***Example:*** Given $81^{\frac{3}{4}} = \sqrt[4]{81^3} = \left(\sqrt[4]{81}\right)^3$, which form would be easiest to calculate without using a calculator. Why? ***Example:*** Determine whether each equation is true or false using the properties of exponents. If false, describe at least one way to make the math statement true. Justify a. $\sqrt{32} = 2^{\frac{5}{2}}$ b. $16^{\frac{3}{2}} = 8^2$ c. $4^{\frac{1}{2}} = \sqrt[4]{64}$ d. $2^8 = \left(\sqrt[3]{16}\right)^6$ e. $\left(\sqrt{64}\right)^{\frac{1}{3}} = 8^{\frac{1}{6}}$

Standard	Cluster: Reason quantitatively and use units to solve problems
N.Q.1★ Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included throughout Math I, II and III. Units are a way for students to understand and make sense of problems. **Use units as a way to understand problems and to guide the solution of multi-step problems** • Students use the units of a problem to identify what the problem is asking. They recognize the information units provide about the quantities in context and use units as a tool to help solve multi-step problems. Students analyze units to determine which operations to use when solving a problem. ***For example***, given the speed in *mph* and time traveled in *hours*, what is the distance traveled? From looking at the units, we can determine that we must multiply *mph* times *hours* to get an answer expressed in miles: $\left(\frac{mi}{hr}\right)(hr) = mi$ ***(Note that knowledge of the distance formula, $d = rt$, is not required to determine the need to multiply in this case.)*** ***Another example***, the length of a spring increases 2 cm for every 4 oz. of weight attached. Determine how much the spring will increase if 10 *oz.* are attached: $\left(\frac{2cm}{4oz}\right)(10oz) = 5cm$. This can be extended into a multi-step problem when asked for the length of a 6 cm spring after 10 *oz.* are attached: $\left(\frac{2cm}{4oz}\right)(10oz) + 6cm = 11cm$. **Choose and interpret units consistently in formulas** • Students choose the units that accurately describe what is being measured. Students understand the familiar measurements such as length (unit), area (unit squares) and volume (unit cubes). They use the structure of formulas and the context to interpret units less familiar. ***For example***, if $density = \frac{mass\ in\ grams}{volume\ in\ mL}$ then the unit for density is $\frac{grams}{mL}$. **Choose and interpret the scale and the origin in graphs and data displays** • When given a graph or data display, students read and interpret the scale and origin. When creating a graph or data display, students choose a scale that is appropriate for viewing the features of a graph or data display. Students understand that using larger values for the tick marks on the scale effectively “zooms out” from the graph and choosing smaller values “zooms in.” Students also understand that the viewing window does not necessarily show the *x*- or *y*-axis, but the apparent axes are parallel to the *x*- and *y*-axes. Hence, the intersection of the apparent axes in the viewing window may not be the origin. They are also aware that apparent intercepts may not correspond to the actual *x*- or *y*-intercepts of the graph of a function.

N.Q.2 ★ Define appropriate quantities for the purpose of descriptive modeling.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students define the appropriate variables to describe the model and situation represented. ***Example(s):*** Explain how the units cm, cm^2, and cm^3 are related and how they are different. Describe situations where each would be an appropriate unit of measure.
N.Q.3 ★ Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III throughout all three courses. **Students understand the tool used determines the level of accuracy that can be reported for a measurement.** ***Example(s):*** • When using a ruler, one can legitimately report accuracy to the nearest division. If a ruler has centimeter divisions, then when measuring the length of a pencil the reported length must be to the nearest centimeter, or • In situations where units constant a whole value, as the case with people. An answer of 1.5 people would reflect a level of accuracy to the nearest whole based on the fact that the limitation is based on the context. **Students use the measurements provided within a problem to determine the level of accuracy.** ***Example:*** If lengths of a rectangle are given to the nearest tenth of a centimeter then calculated measurements should be reported to no more than the nearest tenth. **Students recognize the effect of rounding calculations throughout the process of solving problems and complete calculations with the highest degree of accuracy possible, reserving rounding until reporting the final quantity.**

Standard	Cluster: Interpret the structure of expressions
A.SSE.1a, b★ Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included in Math I, II and III. Throughout all three courses, students interpret expressions that represent quantities. In Math II, focus on polynomial expressions, quadratic functions and exponential functions. **Students manipulate the terms, factors, and coefficients in more complicated expressions (for example, expressions including grouping symbols and variable exponents) to explain and interpret the meaning of the individual parts of an expression. They use the manipulated form to make sense of the multiple factors and terms of an expression.** ***Example:*** An astronaut on Planet X that, which a lighter gravity than Earth, throws a ball vertically. The expression – $x^2 + 2x + 3$ represents the height of the ball x seconds after it was thrown. The expression – $(x-3)(x+1)$ represents the equivalent factored form. The zeroes of the function $y = -x^2 + 2x + 3$ would then be $x = 3$ and $x = -1$. The zero of 3 can be interpreted as the number of seconds it took for the ball to hit the ground (where the height, y, is zero). The -1 does not have meaning in the context of this problem since time cannot have a negative value. ***Example:*** The expression $-4.9t^2 + 17t + 0.6$ describes the height in meters of a basketball t seconds after it has been thrown vertically into the air. Interpret the terms and coefficients of the expression in the context of this situation. ***Example:*** An initial investment of $2250 is placed in a bank account with an annual interest rate of 2.5%. Let t represent the time in years. For each of the following, explain how frequently the investment is compounded. a. $2250(1+\frac{0.025}{12})^{\frac{t}{12}}$ b. $2250(1+0.000481)^{\frac{t}{52}}$ c. $2250(1.03333)^{\frac{3t}{4}}$
A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	This standard is included in Math I, II and III. **Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.** • Use factoring techniques such as common factors, grouping, the difference of two squares, or a combination of methods to factor quadratics completely. Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is a factorable quadratic, students should factor the expression further. ***Example:*** Write two expressions that are equivalent forms of expression $m^4 + 5m^2 + 4$ *Solution:* $(m^2)^2 + 5(m^2) + 4$ and $(m^2 + 4)(m^2 + 1)$ • Use the distributive property and combining like terms to simplify an algebraic expression. Connect this to A.APR. 1. ***Example:*** Find a value for a, a value for k, and a value for n, so that $(3x + 2)(2x - 5) = ax^2 + kx + n$

Standard	Cluster: Write expressions in equivalent forms to solve problems
A.SSE.3c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. ***Example:*** Three physicists describe the amount of a radioactive substance, Q in grams, left after t years: 1. $Q = 300\left(\frac{1}{2}\right)^{\frac{t}{12}}$ 2. $Q = 300 \cdot 0.9439^t$ 3. $Q = 252.290 \cdot 0.9439^{t-3}$ a. Show that the expressions describing the radioactive substance are all equivalent (using appropriate rounding). b. What aspect of the decay of the substance does each of the formulas highlight? http://www.illustrativemathematics.org/illustrations/1305 ***Example:*** A recent college grad signed up for a new credit card with a promotional annual interest rate of 7.5% for the first year. What was the monthly interest rate on this credit card during the first year? ***Example:*** A family who lives on Lake Norman is considering the purchase of a jet ski. With some research, the family discovered that a used jet ski depreciates at 8% per year. The family wants to predict the resale value of the jet ski 20 months after purchase. What is the monthly depreciation rate the family should use? Note: The formula $f(y) = a \cdot (1 - .08)^x$ represents annual depreciation. To find the monthly depreciation we must rewrite the expression using the identity property of multiplicationproperties of exponents , knowledge of rational exponents, and the identity property of multiplication. We know that $(1 - .08)^x = (0.92)^{\frac{1}{12} * 12x} = \left(.92^{\frac{1}{12}}\right)^{12x} \approx (0.9931)^{12x}$. To state the depreciation rate, we must compute $1 - 0.9931 = .0069$. This reveals the monthly depreciation rate of .69%. Connect to A.SSE.1 and A.SSE.2

Standard	Cluster: Perform arithmetic operations on polynomials.
A.APR.1 Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	This standard is included In Math I, II and III. Throughout all three courses, students operate with polynomials. Math II, focus on adding and subtracting any polynomial and extending multiplication to as many as three linear expressions. Dividing polynomials is not intended for Math 2. The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, binomial, trinomial, polynomial, factor, and term. **Students recognize that when adding, subtracting, and multiplying polynomials the result is also a polynomial.** ***Example:*** Simplify a. $(x^3 + 3x^2 - 2x + 5)(x - 7)$ b. $4b(cb - zd)$ c. $(4x^2 - 3y^2 + 5xy) - (8xy + 3y^2)$ d. $(x + 4)(x - 2)(3x + 5)$ A set is closed under an operation when any two elements are combined with that operation, the result is always another element of the same set. In order to understand that polynomials are closed under addition, subtraction and multiplication, students can compare these ideas with the analogous claims for integers: The sum, difference or product of any two integers is an integer, but the quotient of two integers is not always an integer.

Standard	Cluster: Understand the relationship between zeroes and factors of polynomials.
A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	This standard is included Math II and III. In both courses, students identify zeroes of polynomials and use them to construct rough graphs. In Math II, focus on factorable quadratics. ***Example:*** Given the function $y = 2x^2 + 6x - 3$, list the zeroes of the function and sketch the graph. ***Example:*** Sketch the graph of the function $f(x) = (x + 5)^2$. How many zeros does this function have? Explain. How does the multiplicity relate to the graph of the function?

Standard	Cluster: Create equations that describe numbers or relationships.
A.CED.1★ Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included In Math I, II and III. Throughout all three courses, students recognize when a problem can be modeled with an equation or inequality and are able to write the equation or inequality. In Math II, focus on quadratic, inverse variation, and exponential contextual situations that students can use to create equations and inequalities in one variable and use them to solve problems. **Students create, select, and use graphical, tabular and/or algebraic representations to solve problems.** ***Example:*** Lava coming from the eruption of a volcano follows a parabolic path. The height h in feet of a piece of lava t seconds after it is ejected from the volcano is given by $h(t) = -16t^2 + 64t + 936$. After how many seconds does the lava reach a height of 1000 feet? ***Example:*** The function $h(x) = 0.04x^2 - 3.5x + 100$ defines the height (in feet) of a major support cable on a suspension bridge from the bridge surface where x is the horizontal distance (in feet) from the left end of the bridge. a. Where is the cable less than 40 feet above the bridge surface? b. Where is the cable at least 60 feet above the bridge surface? ***Example:*** In kickboxing, it is found that the force, *f*, needed to break a board, varies inversely with the length, *l*, of the board. If it takes 5 lbs. of pressure to break a board 2 feet long, how many pounds of pressure will it take to break a board that is 6 feet long? ***Example:*** To be considered a 'fuel efficient' vehicle, a car must get more than 30 miles per gallon. Consider a test run of 200 miles. How many gallons of fuel can a car use and be considered 'fuel-efficient'? Students write exponential equations In Math I and used tables and graphs to solve. In Math II, students should expand their strategy to include the use of common logarithms. Focus on rewriting values into base 10 and reasoning that if the bases are the same for equivalent expressions then the exponents are equivalent. *Note: Further study of logarithms is in Math 3.* ***Example:*** The average rate to rent an apartment is $750 per month and increasing at a rate of 8% per year. If inflation continues at the current rate, when will the rent be $1000 per month? Show how to use common logarithms approximate a solution. ***Solution:*** *The equation is* $750(1.08)^x = 1000$. *Solve using common logarithms* $750(10^{log1.08})^x = 1000$ *Rewrite the factor as base 10 using logarithms* $(10^{0.0334})^x = 1.3333$ *Divide both sides by 750 and rewrite the exponent approximately* $(10^{0.0334})^x = 10^{log1.3333}$ *Rewrite 1.3333 as base 10 using logarithms* $0.0334x = log1.333$ *If bases are the same for equivalent expressions then the exponents must also be equivalent* $x = \frac{log1.333}{0.0334} \approx 3.74$ *Divide both sides by 0.0334* *The apartment rent will be $1000 about ¾ of the way through the 3rd year. If it only increases at the end of a year then the cost will be slightly more than a $1000 in 4 years.*

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A.CED.2 ★

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

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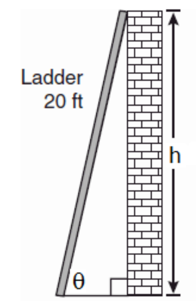
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is included In Math I, II and III. Throughout all three courses, students create equations in two variables and graph them on coordinate axes. In Math II, focus on quadratics, power, and inverse variation and simple trig (sine, cosine and tangent) equations.

Example: The area of a rectangle is 40 in^2. Write an equation for the length of the rectangle related to the width. Graph the length as it relates to the width of the rectangle. Interpret the meaning of the graph.

Example: The formula for the surface area of a cylinder is given by $A = \pi r^2 h$, where *r* represents the radius of the circular cross-section of the cylinder and h represents the height. Select a fixed value for *h* and graph the area as it relates to the radius. Select a fixed value for *r* and graph the area as it relates to the height. Compare the graphs. What is the appropriate domain for *r* and *h*? Be sure to label your graphs and use an appropriate scale.

Example: John has a 20-foot ladder leaning against a wall. Create an equation that represents the relationship between the angle the ladder makes with the ground and the maximum height of ladder can reach against the wall.



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A.CED.3 ★

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

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This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is included In Math I, II and III. Throughout all three courses, students recognize when a constraint can be modeled with an equation, inequality or system of equations/inequalities. Students create, select, and use graphical, tabular and/or algebraic representations to solve the problem. This is an optimization standard where students will progress to study linear programming. In Math II, the standard is extended to linear with quadratic relationships and linear with inverse variation relationships.

Example: In making a business plan for a pizza sale fundraiser, students determined that both the income and the expenses would depend on the number of pizzas sold. They predicted that $I(n) = -0.05n^2 + 20n$ and $E(n) = 5n + 250$. Determine values for which $I(n) = E(n)$ and explain what the solution(s) reveal about the prospects of the pizza sale fundraiser.

Example: The FFA has $2400 in a fund to raise money for a new tractor. They are selling trees and have determined that the number of trees they can buy to sell depends on the price of the tree *p*, according to the function $n(p) = \frac{2400}{p}$. Also, after allowing for profit, the number of trees that customers will purchase depends on the price which the group purchased the trees with function $c(p) = 300 - 6p$. For what price per tree will the number of trees that can be bought

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	be greater than the number of trees that will be sold? ***Example:*** Four artists and three writers create two types of greeting cards. Each art card requires four hours of art and two hours of writing. Each sonnet card takes two hours of art and four hours of writing. Each employee can work up to 40 hours per week. The company makes a profit of \$2 on each art card and \$1 on each sonnet card. How many of each type of card should be created and sold to maximize profits?
A.CED.4★ Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included In Math I, II and III. Throughout all three courses, students solve multi-variable formulas or literal equations, for a specific variable. Math II, extend to compound variation. ***Example:*** Solve $V = \frac{4}{3}\pi r^3$ for radius *r*. ***Example:*** The "condition of average" is the insurance term used when calculating a payout against a claim where the policy undervalues the sum insured. In the event of a partial loss, the amount paid against the claim will be in the same proportion as the value of the underinsurance. The formula used is $Payout = Claim \times \frac{sum\ insured}{current\ value}$. Solve the formula for the current value.

Standard	Cluster: Understand solving equations as a process of reasoning and explain the reasoning
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	This standard is included in Math I, II and III. In Math II, students should focus on solving factorable quadratic equations and be able to extend and apply their reasoning to other types of equations in future courses. **Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution.** • When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method. ***Example:*** Explain why the equation $x^2 + 14 = 9x$ can be solved by determining values of x such that $x - 7 = 0$ and $x - 2 = 0$. ***Example:*** Below are two methods for solving the equation $5x^2 + 10 = 90$. Select one of the solution methods and construct a viable argument for the use of the method. ***Method A*** $5x^2 + 10 = 90$ $-10 = -10$ $5x^2 = 80$ $\frac{5x^2}{5} = \frac{80}{5}$ $x^2 = 16$ $x = \pm\sqrt{16}$ $x = 4 \text{ or } x = -4$ ***Method B*** $5x^2 + 10 = 90$ $-90 = -90$ $5x^2 - 80 = 0$ $5(x^2 - 16) = 0$ $5(x+4)(x-4) = 0$ $x + 4 = 0 \text{ or } x - 4 = 0$ $x = 4 \text{ or } x = -4$
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	This standard is included Math II and III. In Math II, focus on solving inverse variation in one variable. ***Example:*** Solve $y = \frac{70}{x}$, when $y = 5$. ***Example:*** Solve $\sqrt{x-1} = x - 7$. Note: The process of solving a simple rational equation will be used again in solving trigonometric equations for right triangles. (Connect to G.SRT.8)

Standard	Cluster: Solve equations and inequalities in one variable
A.REI.4b Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	This standard is included Math II and III. Math II, students should focus on solving quadratic equations by inspection, taking square roots, quadratic formula, and factoring when lead coefficient is one. Completing the square and writing complex solutions is not expected Math II; however recognizing when there are non-real solutions is expected. ***Examples:*** a. Inspection $x^2 = 49$ b. Square root $3x^2 + 9 = 72$ c. Quadratic formula $4x^2 + 13x - 7 = 0$ d. Factoring ($a = 1$) $x^2 + 8x + 12 = 0$ ***Example:*** Ryan used the quadratic formula to solve an equation and his result was $x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-2)}}{2(1)}$. a. Write the quadratic equation Ryan started with. b. Simplify the expression to find the solutions. c. What are the x-intercepts of the graph of the corresponding quadratic function? ***Example:*** Solve $x^2 + 8x = -17$ for x.

Standard	Cluster: Solve systems of equations.
A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.	Students solve a system containing a linear equation and a quadratic equation in two-variables. Students solve graphically and algebraically. *Note: Quadratics may include conic sections such as a circle.* ***Example:*** Solve $\begin{cases} y = x^2 - x - 6 \\ 2x - y = 2 \end{cases}$ both graphically and algebraically ***Example:*** Describe the possible number of solutions of a linear and quadratic system. Illustrate the possible number of solutions with graphs.

Standard	Cluster: Represent and solve equations and inequalities graphically.

A.REI.10

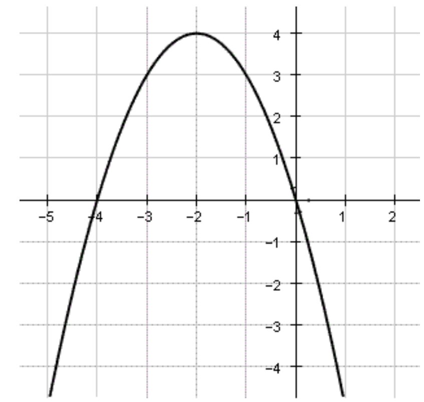
Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

This standard is included In Math I, II and III.

In Math I, students focused on linear and exponential equations. In Math II, extend this same work to focus on quadratics, inverse variation and power equations.

Students can explain and verify that every point (x, y) on the graph of an equation represent values for x and y that make the equation true.

Example: Given the graph of $g(x)$, provide at least three solutions to $g(x) = y$.



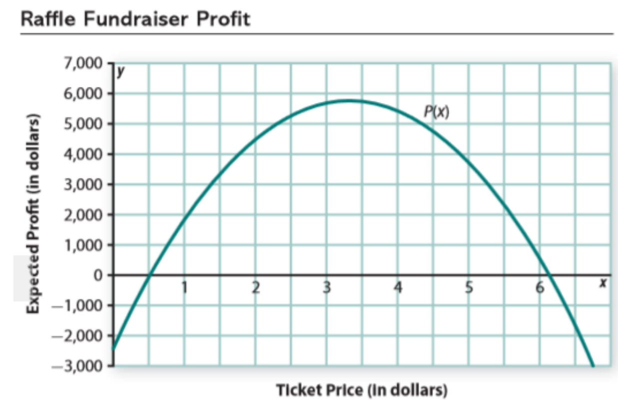
A-REI.11★

Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

This standard is included In Math I, II and III. Throughout all three courses, students recognize that a graph is the set of all solutions (A.REI.10) to an equation in two variables; and thus, when two equations are graphed on the same coordinate axes, the point(s) of intersection are solutions to both equations. Students find solutions using graphs, tables, or finding successive approximations.

Math II, extend to quadratics.

Example: The graph at the right represents $P(x)$, the profit from a fundraiser as a function of ticket price. For what ticket price(s) will $P(x) = \$2,500$?



Example: Given the following equations determine the x-value that results in an equal output for both functions.

$$f(x) = 3x - 2$$
$$g(x) = (x + 3)^2 - 1$$

Connect to A.REI.7

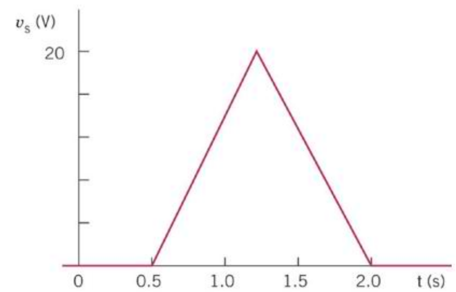
Standard	Cluster: Understand the concept of a function and use function notation
F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	This standard is included in Math I, II and III. Throughout all three courses, students use function notation, evaluate functions, and interpret statements that use function notation. In Math II, focus on quadratic, simple power, and inverse variation. **Use function notation** ***Example:*** Find the value of k for $f(x) = 5x^2 + kx + 2$ if $f(3) = 23$. **Evaluate functions for inputs in their domains** ***Example:*** Let $g(x) = 2(x+3)^2$. Find $g(3)$, $g\left(-\frac{1}{2}\right)$, and $g(a)$. **Interpret statements that use function notation in terms of a context** ***Example:*** If $h(t) = -16t^2 + 5t + 7$ models the path of an object projected into the air where t is time in seconds and $h(t)$ is the vertical height in feet . a. Find $h(5)$ and explain the meaning of the solution. b. Find t, when $h(t) = 0$. Explain your reasoning. ***Example:*** The light intensity of a flashlight is inversely related to the square of the distance from the light and can be represented by the function $I(d) = \frac{160}{\pi d^2}$. Determine each of the following and interpret what it means in context. a. $I(2)$ b. $I(d) = \frac{10}{\pi}$

Standard	Cluster: Interpret functions that arise in applications in terms of the context.
F.IF.4★ For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included In Math I, II and III. Throughout all three courses, students interpret the key features of graphs and tables for a variety of different functions. In Math II, focus on interpreting key features of functions* given a graph, table, or verbal description. For trigonometric functions, limit to sine, cosine and tangent in standard position with angle measures of 180° or less. When given a table or graph of a function that models a real-life situation, explain the meaning of the characteristics of the table or graph in the context of the problem. Key features include intercepts, intervals of increase/decrease, positive/negative, relative maximum/minimums, symmetries, and end behavior. Note: periodicity is not addressed at this level.

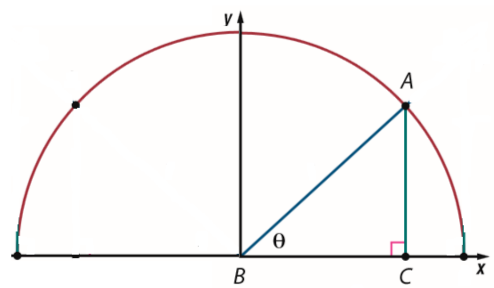
and minimums; symmetries; end behavior; and periodicity.[*]

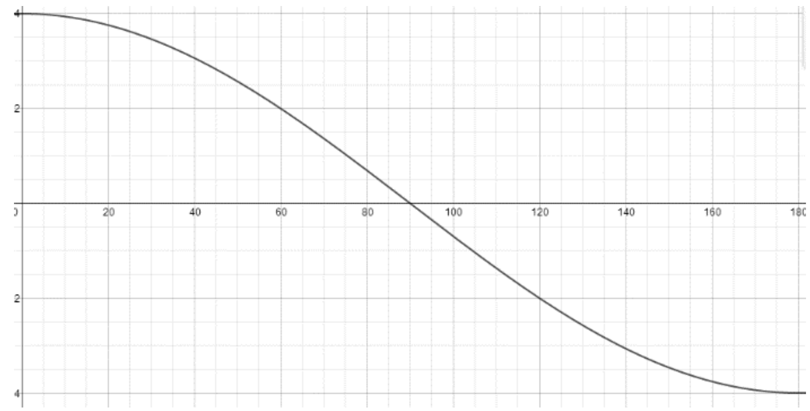
Example: It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.

Example: The graph at the right is the voltage, v, in a given circuit as a function of the the time (in seconds). What was the maximum voltage and for how long did it take to complete the circuit?



Example: The graph at the right represents the horizontal distance point A is from the y axis as A travels at a constant rate counterclockwise around the semicircle.





a. What is the length of $\overline{BA}$?

b. For what angle(s) is the length of $\overline{BC}$ the longest? The shortest? Explain.

****Note:*** The note on this standard addresses trigonometric functions; however it should be intepreted that the standard applies to any function standard up to and including those addressed in Math I and II.

F.IF.5★

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*[*]

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is included In Math I, II and III. Throughout all three courses, students relate the domain of a function to its graph and in an application situation to the quantitative relationship it describes.

In Math II, extend to quadratics, right triangle trigonometry, and inverse variation functions.

Example: A hotel has 10 stories above ground and 2 levels in its parking garage below ground. What is an appropriate domain for a function, $T(n)$, that gives the average number of times an elevator in the hotel stops at the nth floor each day? *Connect this example to F.IF.4 by having students sketch a possible graph, provide justification for key features and then compare graphs. Also, recognize that $n = 0$ is not in the domain. Buildings don't have a 0 floor, so the domain is strictly* $\{-2, -1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Standard

F.IF.7 (b, e)★

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

e. Graph exponential and logarithmic functions, showing intercept and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Cluster: Analyze functions using different representations.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Math II, students graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Example: Describe key characteristics of the graph of $f(x) = |x - 3| + 5$.

Example: Graph $f(x)) = \begin{cases} |x+4| & x < 0 \\ 4 - 3x & 0 \le x < 1 \\ \sqrt{x} & x \ge 1 \end{cases}$ and describe the key features of $f(x)$.

Example: The Oriental Trading Company charges shipping based on cost of the order before taxes.

Order Total	Shipping Price
Up to $30	$7.00
$30.01 to $60	$9.00
$60.01 to $90	$11.00
Above $90	Free

a. Explain *why* the shipping cost is a function of the order total.
b. Write and graph the function $s(t)$.

Partial Solution: $s(t)) = \begin{cases} 7 & 0 < t \le 30 \\ 9 & 30 < t \le 60 \\ 11 & 60 < t \le 90 \\ 0 & t > 90 \end{cases}$

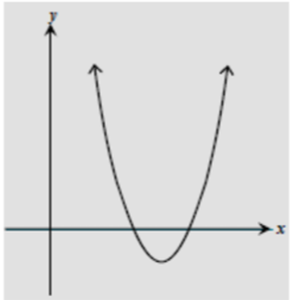
c. Explain *how* the piecewise function *s(t)* represents the shipping price as a function of the order total.

Math II, students graph simple trigonometric functions.

Example: Graph each of the following for a domain $0° \le x \le 180°$. Identify and explain the period, midline and amplitude.

a. $y = sin\ x$
b. $y = cos\ x$
c. $y = tan\ x$

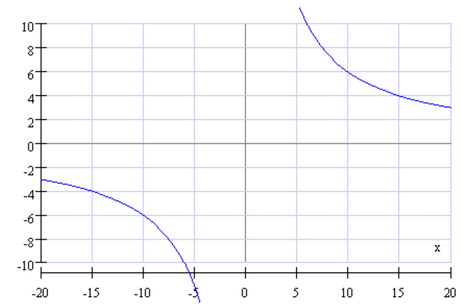
Connect and extend from F.IF.4.

Standard	Cluster: Analyze functions using different representations
F.IF.8a Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	This standard is included in Math I, II and III. Throughout all three courses, students rewrite expressions in different ways to reveal properties. In Math II, focus on factoring as a process to show zeroes, extreme values, and symmetry of the graph. Students should also interpret these characteristics in terms of a context. At this level, completing the square is still NOT expected. An exemplar formative assessment lesson plan of F.IF.8 and F.IF.9 is available from the Shell Center for Mathematics Education titled "L20: Forming Quadratics" at http://map.mathshell.org/materials/lessons.php?taskid=224 ***Example:*** Coyote was chasing roadrunner, seeing no easy escape, Roadrunner jumped off a cliff towering above the roaring river below. Molly Mathematician was observing the chase and obtained a digital picture of this fall. Using her mathematical knowledge, Molly modeled the Road Runner's fall with the following quadratic functions: $h(t) = -16t^2 + 32t + 48$ $h(t) = -16(t+1)(t-3)$ a. How can Molly have two equations? b. Which of the rules would be most helpful in answering each of these questions? Explain. i. What is the maximum height the Road Runner reaches and when will it occur? ii. When would the Road Runner splash into the river? iii. At what height was the Road Runner when he jumped off the cliff? ***Example:*** Which of the following equations could describe the function of the given graph to the right? Explain.  $f_1(x) = (x+12)^2 + 4$ $f_5(x) = -4(x+2)(x+3)$ $f_2(x) = -(x-2)^2 - 1$ $f_6(x) = (x+4)(x-6)$ $f_3(x) = (x+18)^2 - 40$ $f_7(x) = (x-12)(-x+18)$ $f_4(x) = (x+12)^2 + 4$ $f_8(x) = (20-x)(30-x)$ Connect this standard to A.SSE.2 in terms of the types of factoring problems expected.
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*	This standard is included In Math I, II and III. Throughout all three courses, students compare characteristics of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description. In Math II, students focus on extending quadratics from Math I and include simple power and inverse variation. ***Example:*** A science class collected data and created models for the height and time of the bottle rockets. Group A's model: $a(t) = (-8t + 11)(2t + 1)$. Group B's model: $b(t) = -16t^2 + 20t + 5$. Time was measured in seconds and height was measured in feet. Determine which rocket went the highest and explain your reasoning.

Example: Compare the constant of proportionality for each of the following inverse variation models and list them in order from least to greatest.

x	y
5	36
10	18
15	12
20	9
25	7.2

$$y = \frac{90}{x}$$



Standard	Cluster: Build a function that models a relationship between two quantities

F.BF.1a, b★

Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is included In Math I, II and III. Throughout all three courses, students write functions that describe relationship between two quantities.

In Math II, extend to building quadratic and inverse variation functions.

Write a function that describes a relationship between two quantities.

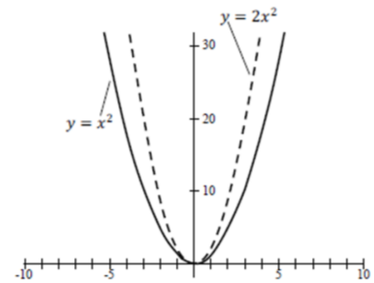
Determine an explicit expression, a recursive process, or steps for calculation from a context

Example: Skeleton Tower from http://www.illustrativemathematics.org/illustrations/75

Example: In kickboxing, it is found that the force, *f*, needed to break a board, varies inversely with the length, *l*, of the board. If it takes 5 lbs. of pressure to break a board 2 feet long, how many pounds of pressure will it take to break a board that is 6 feet long?

Combine standard function types using arithmetic operations

Example: The Sleek Bike Company repairs bicycles. The function $I(p) = -0.8p^2 - 1360p - 240$ represents the income as a function of the average price charged for repairs. The function $C(p) = 0.5p^2 - 6p + 200$ represents the company's monthly costs based on the average price charged for repairs. Build a function to represent the profit as a function of the average price charged for repairs.

Standard	Cluster: Build new functions from existing functions.
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	This standard is included In Math I, II and III. The standard addresses specific functions and their transformations throughout each course. In Math I, the focus was on vertical and horizontal translations of linear and exponential functions. In Math II, extend to quadratics and vertical stretch/compression including reflections and vertical and horizontal translations. Focus on quadratic functions; however, working with other functions, such as absolute value functions, will assist students in developing a deeper understanding of the transformations and prevent the common misconception that the effect of transformations are specific to a function type. ***Example:*** Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$ and explain the differences in terms of the algebraic expressions for the functions.  ***Example:*** Describe the effect of varying the parameters a, h, and k on the shape and position of the graph of the equation $f(x) = a(x - h)^2 + k$ *(Note: This example is NOT suggesting that students Math II need to know vertex form or how to complete the square to write a quadratic into vertex form. It is an exercise to explore the transformation of a quadratic.)* Recognize even and odd functions from their graphs and algebraic expressions. Students recognize that a function is even if $f(-x) = f(x)$ and is odd if $f(-x) = -f(x)$. Visual approaches to identifying the graphs of even and odd functions can be used as well; especially since $f(kx)$ is not intended for Math II.

<table>
<tr><th>Standard</th><th>Cluster: Experiment with transformations in the plane</th></tr>
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G.CO.2

Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

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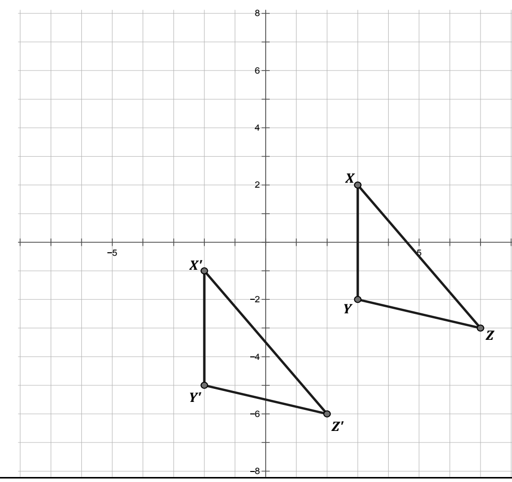
Students describe and compare function transformations on a set of points as inputs to produce another set of points as outputs. They distinguish between transformations that are rigid (preserve distance and angle measure: reflections, rotations, translations, or combinations of these) and those that are not (dilations or rigid motions followed by dilations). Transformations produce congruent figures while dilations produce similar figures.

Example: A plane figure is translated 3 units right and 2 units down. The translated figure is then dilated with a scale factor of 4, centered at the origin.

a. Draw a plane figure and represent the described transformation of the figure in the plane.
b. Explain how the transformation is a function with inputs and outputs.
c. Determine if the relationship between the pre-image and the image after a series of transformations. Provide evidence to support your answer.

Example: Transform ΔABC with vertices A (1,1), B (6,3) and C (2,13) using the function rule $(x, y) \rightarrow (-y, x)$ and describe the transformation as completely as possible. *(This is not intended for students to memorize transformation rules and thus be able to identify the transformation from the rule. Students should understand the structure of the rule and how to use it as a function to generate outputs from the provided inputs.)*

Example: Complete the rule for the transformation at the right: $(x, y) \rightarrow$ (____, ____) and determine if the transformations preserve distance and angle. Provide justification for your answer.



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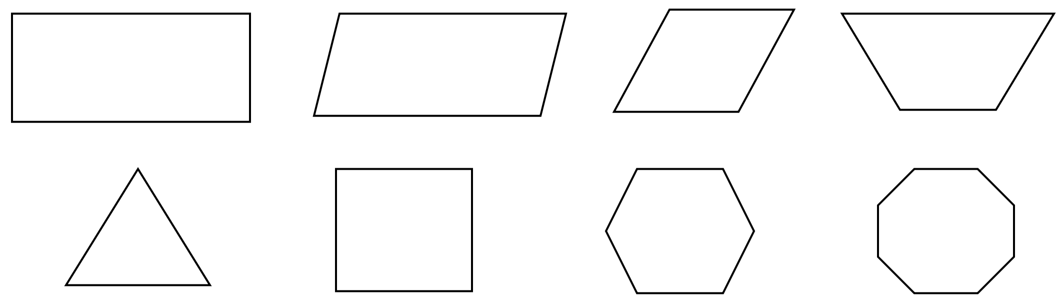
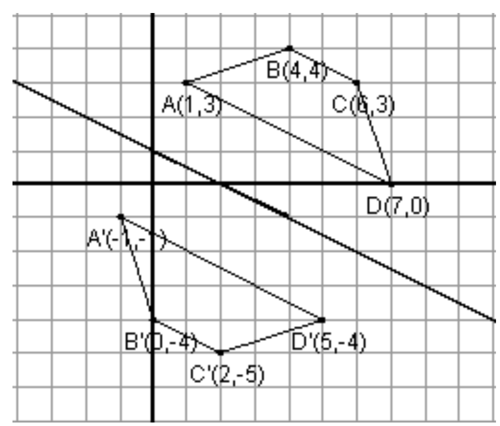
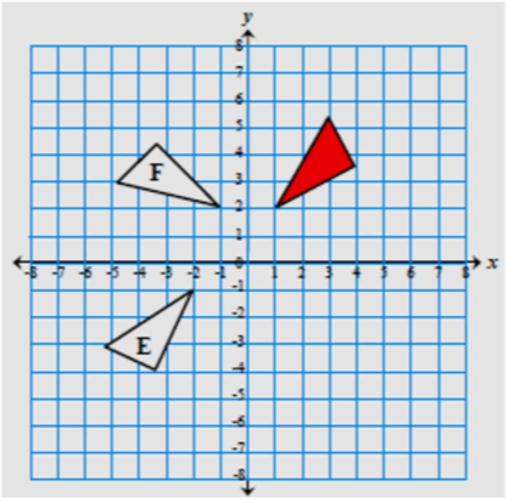
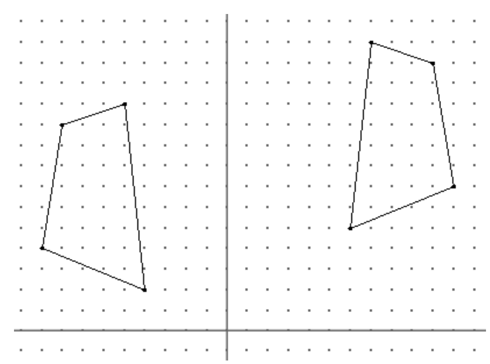
G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	Students describe and illustrate how a rectangle, parallelogram, isosceles trapezoid or regular polygon are mapped onto themselves using transformations. Students determine the number of lines of reflection symmetry and the degree of rotational symmetry of any regular polygon. ***Example:*** For each of the following shapes, describe the rotations and reflections that carry it onto itself. 
G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	Students develop the definition of each transformation in regards to the characteristics between pre-image and image points. • For a *translation*: connecting any point on the pre-image to its corresponding point on the translated image, and connecting a second point on the pre-image to its corresponding point on the translated image, the two segments are equal in length, translate in the same direction, and are parallel. • For a *reflection*: connecting any point on the pre-image to its corresponding point on the reflected image, the line of reflection is a perpendicular bisector of the line segment. • For a *rotation*: connecting the center of rotation to any point on the pre-image and to its corresponding point on the rotated image, the line segments are equal in length and the measure of the angle formed is the angle of rotation. ***Example:*** Is quadrilateral A'B'C'D' a reflection of quadrilateral ABCD across the given line? Justify your reasoning. 

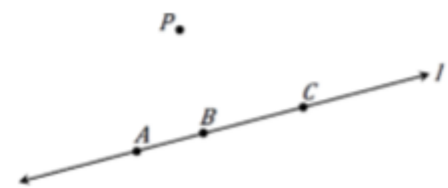
G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	Students transform a geometric figure given a rotation, reflection, or translation. They create sequences of transformations that map a geometric figure onto itself and another geometric figure. Students predict and verify the sequence of transformations (a composition) that will map a figure onto another. ***Example:*** ***Part 1*** Draw the shaded triangle after: a. It has been translated −7 units horizontally and +1 units vertically. Label your answer *A*. b. It has been reflected over the *x*-axis. Label your answer *B*. c. It has been rotated 90° clockwise about the origin. Label your answer *C*. d. It has been reflected over the line *y* = *x*. Label your answer *D*. ***Part 2*** Describe fully the single transformation that: a. Takes the shaded triangle onto the triangle labeled *E*. b. Takes the shaded triangle onto the triangle labeled *F*. 

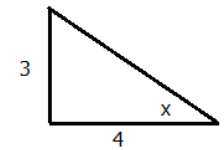
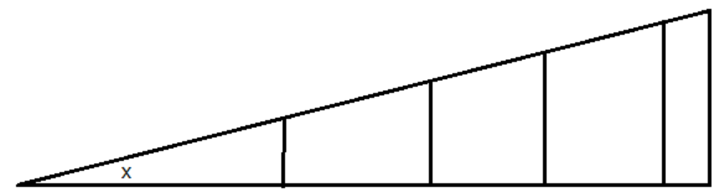
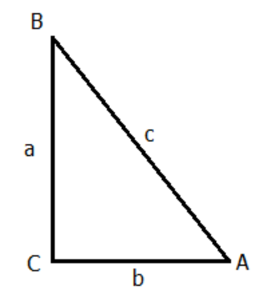
Standard	Cluster: Understand congruence in terms of rigid motions.
G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	Students use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane. Students recognize rigid transformations preserve size and shape or distance and angle and develop the definition of congruent. Students determine if two figures are congruent by determining if rigid motions will turn one figure into the other. ***Example***: Consider parallelogram ABCD with coordinates A(2,-2), B(4,4), C(12,4) and D(10,-2). Perform the following transformations. Make predictions about how the lengths, perimeter, area and angle measures will change under each transformation. a. A reflection over the x-axis. b. A rotation of 270° counter clockwise about the origin. c. A dilation of scale factor 3 about the origin. d. A translation to the right 5 and down 3. Verify your predictions. Compare and contrast which transformations preserved the size and/or shape with those that did not preserve size and/or shape. Generalize, how could you determine if a transformation would maintain congruency from the pre-image to the image? ***Example***: Determine if the figures below are congruent. If so, tell what rigid motions were used. 

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to *preserve distances and angle measures*. Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur. Students identify corresponding sides and corresponding angles of congruent triangles. Explain that in a pair of congruent triangles, corresponding sides are congruent (distance is preserved) and corresponding angles are congruent (angles measure is preserved). They demonstrate that when distance is preserved (corresponding sides are congruent) and angle measure is preserved (corresponding angles are congruent) the triangles must also be congruent. ***Example:*** Properties of Congruent Triangles (http://www.illustrativemathematics.org/illustrations/1637)
G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	Students list the sufficient conditions to prove triangles are congruent: ASA, SAS, and SSS. They map a triangle with one of the sufficient conditions (e.g., SSS) onto the original triangle and show that corresponding sides and corresponding angles are congruent. ***Examples:*** Why Does SAS Work? from Illustrative Mathematics (http://www.illustrativemathematics.org/illustrations/109) Why Does ASA Work? from Illustrative Mathematics (http://www.illustrativemathematics.org/illustrations/339) Why Does SSS Work? From Illustrative Mathematics (http://www.illustrativemathematics.org/illustrations/110) ***Example:*** Josh is told that two triangles ΔABC and ΔDEF share two sets of congruent sides and one set of congruent angles: $\overline{AB}$ is congruent to $\overline{DE}$, $\overline{BC}$ is congruent to $\overline{EF}$, and $\angle B$ is congruent to $\angle E$. He is asked if these two triangles must be congruent. Josh draws the two triangles below and says, "They are definitely congruent because two pairs of sides are congruent and the angle between them is congruent!" a. Explain Josh's reasoning using one of the triangle congruence criteria: ASA, SSS, SAS. b. Given two triangles ΔABC and ΔDEF, what is an example of three congruent parts that will not guarentee the two triangles are congruent.

Standard	Cluster: Prove geometric theorems
G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	This standard is included Math II and III. In both courses students prove theorems about triangles. Math II, students construct proofs that are focused on characteristics within a certain triangle. Encourage multiple ways of writing proofs, such as *narrative paragraphs*, using *flow diagrams*, and *two-column format* Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between and within geometric objects should be central to any geometric study and certainly to proof. The use of dynamic geometry software can be important tools for helping students conceptually understand important geometric concepts. Although it is impossible to provide an exhaustive list of all the proofs possible for Math II, here are some theorems about triangles to consider: • Measures of interior angles of a triangle sum to 180° • Midpoint Connector Theorem • Exterior Angle Theorem • Triangle Inequality Theorem • Converse of Pythagorean Theorem • Centroid • Orthocenter • Circumcenter • Incenter

Standard	Cluster: Make geometric constructions
G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	Drawing geometric shapes with rulers, protractors and technology is developed in middle grades. In high school, students perform formal geometry constructions using a variety of tools. Students will utilize proofs to justify validity of their constructions. **Students complete three specific constructions:** • Equilateral triangle inscribed by a circle • Square inscribed by a circle • Regular hexagon inscribed by a circle ***Examples:*** Illustrative Mathematics (https://www.illustrativemathematics.org/HSG-CO) *Students may use geometric software to make geometric constructions.*

Standard	Cluster: Understand similarity in terms of similarity transformations
G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	Students should understand that a dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. **Students perform a dilation with a given center and scale factor on a figure in the coordinate plane.** ***Example:*** Given ΔABC with A $(-2,-4)$, B $(1,2)$ and C $(4,-3)$ apply the rule (x,y)→$(3x,3y)$ **Students verify that when a side passes through the center of dilation, the side and its image lie on the same line and the remaining corresponding sides of the pre-image and images are parallel.** ***Example:*** Using ΔABC and its image $\Delta A'B'C'$ from the previous example, connect the corresponding pre-image and image points. Describe how the corresponding sides are related. Determine the center of dilation. **Students verify that a side length of the image is equal to the scale factor multiplied by the corresponding side length of the pre-image.** ***Example:*** Calculate the side length of each side of the triangle. How do the side lengths compare? How does the perimeters compare? ***Example:*** Suppose we apply a dilation by a factor of 2, centered at the point P to the figure below. a. In the picture, locate the images A', B', and C' of the points A, B, C under this dilation. b. Based on you picture in part a., what do you think happens to the line *l* when we perform the dilation? c. Based on your picture in part a., what appears to be the relationship between the distance of A'B' and the distance of AB? d. Prove your observations in part c.  ***Example:*** Given two similar figures that are related by dilation, determine the center of dilation and scale factor. Connect to G.CO.2 ***Example:*** Given two similar figures that are related by a dilation followed by a sequence of rigid motions, determine the parameters of the dilation and rigid motions that will map one onto the other

Standard	Cluster: Define trigonometric ratios and solve problems involving right triangles
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles	Students establish that the side ratios of a right triangle are equivalent to the corresponding side ratios of *similar* right triangles and are a function of the acute angle(s). ***Example:*** Find the sine, cosine, and tangent of x.  ***Example:*** Explain why the sine of x is the same regardless of which triangle is used to find it in the figure below. 
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.	**Students can explain why the sine of an acute angle in a right triangle is the cosine of complementary angle in the same right triangle.**  ***Example:*** Using the diagram at the right, provide an argument justifying why $sin\ A = cos\ B$. **Students use the relationship between the sine and cosine of complementary angles.** ***Example:*** Complete the following statement: If $sin\ 30° =$ ½, then cos _____ $=$ ½ ***Example:*** Given that angle F and angle G are complementary. As the measure of angle F varies from a value of x to a value of y, $sin\ (F)$ increases by 0.2. How does $cos\ (G)$ change as F varies from x to y?
G.SRT.8★ Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Solve application problems involving right triangles, including angle of elevation and depression, navigation, and surveying.** ***Example:*** Find the height of a flagpole to the nearest tenth if the angle of elevation of the sun is 28° and the shadow of the flagpole is 50 feet. ***Example:*** A new house is 32 feet wide. The rafters will rise at a 36° angle and meet above the centerline of the house. Each rafter also needs to overhang the side of the house by 2 feet. How long should the carpenter make each rafter?

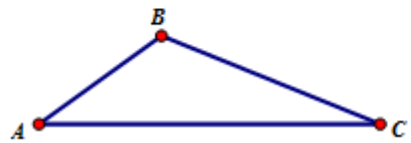
Standard	Cluster: Apply trigonometry to general triangles

G.SRT.9 (+)

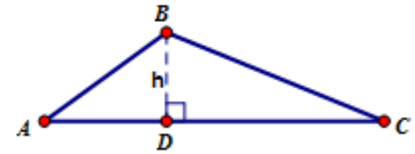
Derive the formula $A = 1/2\, ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

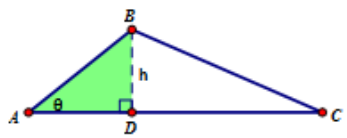
Students derive the formula for the area of a triangle when only two sides and the angle in between is known.

Oblique ΔABC



Drop an altitude to a side





$$\sin A = \frac{h}{AB}$$

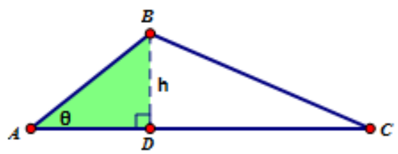
$$h = (\sin A)(AB)$$

$$Area = \frac{1}{2}bh$$

Now we substitute the values for b and h.

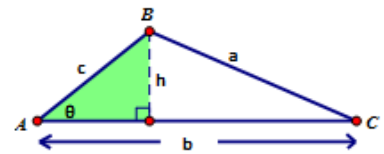
$$Area = \frac{1}{2}(AC)(\sin A)(AB)$$

$$Area = \frac{1}{2}(AC)(AB)(\sin A)$$



$$Area = \frac{1}{2}(AC)(AB)(\sin A)$$

Notice that this formula requires no provided value for the height, the height is being calculated using the sine ratio. This is a very handy formula for area. **Calculating area then requires two sides of a triangle and the INCLUDED ANGLE (SAS).**



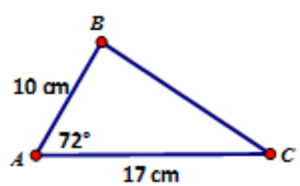
$$Area = \frac{1}{2}bc(\sin A)$$

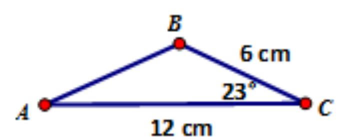
"SPECIAL LABELLING"

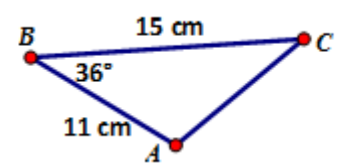
Sometimes to simplify a formula we refer to the opposite sides of an angle as the lowercase letter of the vertex that is opposite it. So c is the opposite sides of ∠C, b is the opposite side of ∠B and of course a is the opposite sides of ∠A.

Students recognize the structure of the formula and do not rely on the labels. The area of a triangle is half the product of the two known sides and the sine of the angle in between.

Example: Find the area of each triangle.







G.SRT.11 (+)

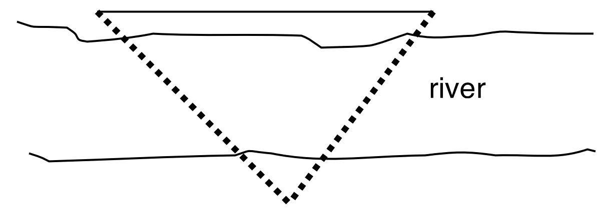
Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Students know where the Law of Sines and the Law of Cosines originates and when they should be utilized in solving problems.

Law of Sines	Law of Cosine
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	$c^2 = a^2 + b^2 - 2ab(cosC)$

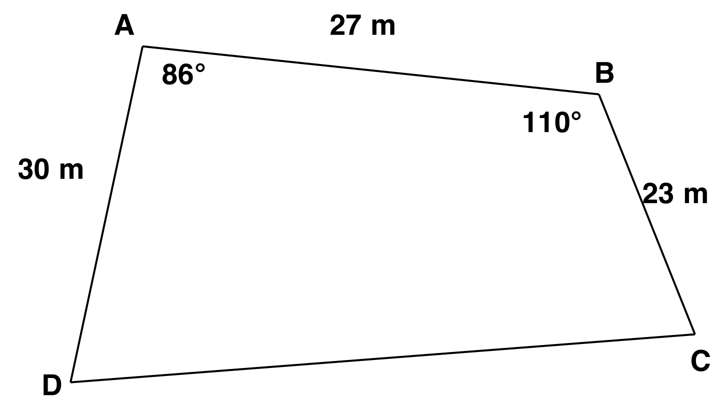
Students will use the *Law of Sines* and the *Law of Cosines* to solve problems.

Example: Two surveyors are 180 meters apart on the same side of a river. They measure their respective angles to a point on the other side of the river and obtain 54° and 68°. How far from the point (line-of-sight) is each surveyor?



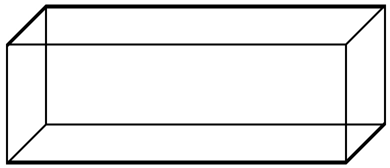
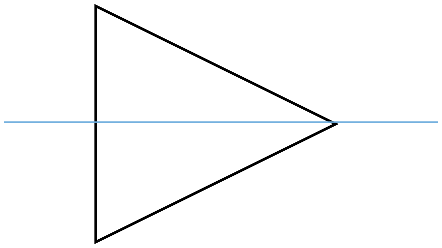
Example: A park in the shape of a quadrilateral ABCD as shown below.

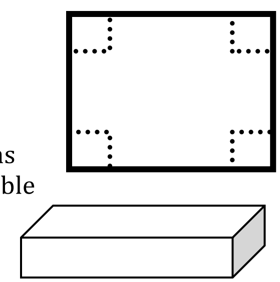
a. Find BD. Show or explain your work.
b. Find the remaining angle measures of the field. Show or explain your work.



Standard	Cluster: Translate between geometric description and the equation for a conic section
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	This standard is included in Math II and Math III. In both courses students prove theorems about triangles. In Math II, students use the Pythagorean Theorem to derive the equation of a circle. *Completing the square is not intended at this level.* **Students define a circle as the set of points whose distance from a fixed point is constant.** Given a point on the circle and the fixed point, they identify that the difference in the x-coordinates represents the horizontal distance and the difference in the y-coordinates represents the vertical distance. Students apply the Pythagorean Theorem to calculate the distance between the two points. Generalizing this process, students derive the equation of a circle. Students connect the derivation of the equation of a circle to the distance formula. ***Example:*** Write the equation of a circle that is centered at $(-1, 3)$ with a radius of 5 units. ***Example:*** Write an equation for a circle given that the endpoints of the diameter are $(-2, 7)$ and $(4, -8)$

Standard	Cluster: Use coordinates to prove simple geometric theorems algebraically
G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	This standard is included in Math I and II. In Math I, students found the midpoint. Math II students focus on ratios other than 1:1. Understanding the process to find the point on a directed line segment requires students to: • Interpret the ratio a:b as *part*:*part* and recognize that there are $(a + b)$ parts. Thus a point is $\frac{a}{(a+b)}$ from the starting endpoint. • Describe the difference between a directed line segment AB and directed line segment BA. The first starts at A and goes to B while the latter starts at B and goes to A. • Calculate the vertical change Δx and horizontal change Δy in regards to the direction of the line segment Thus the point is located at $\left(\left(\frac{a}{(a+b)}\right)\Delta x, \left(\frac{a}{(a+b)}\right)\Delta y\right)$. ***Example:*** Given directed line segment AB with $A(-1,2)$ and $B(7,14)$, find point P that partitions the segment into a ratio of 1:3. ***Example:*** Scaling a Triangle in the Coordinate Plane https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/6/tasks/1867

Standard	Cluster: Visualize relationships between two-dimensional and three dimensional objects
G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	**Students identify shapes of two-dimensional cross-sections of three-dimensional objects.** The Cross Section Flyer at http://www.shodor.org/interactivate/activities/CrossSectionFlyer/ can be used to allow students to predict and verify the cross section of different three-dimensional objects. ***Example:*** Identify two-dimensional cross sections of a rectangular prism.  **Students identify three-dimensional objects generated by rotations of two-dimensional objects.** The 3D Transmographer at http://www.shodor.org/interactivate/activities/3DTransmographer/ can be used to allow students to predict and verify three-dimensional objects generated by rotations of two-dimensional objects. ***Example:*** Identify the object generated when the following object is rotated about the indicated line. 

Standard	Cluster: Use geometric concepts in modeling situations
G.MG.1 ★ Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students recognize situations that require relating two- and three- dimensional objects.** They estimate measures (circumference, area, perimeter, volume) of real-world objects using comparable geometric shapes or three-dimensional objects. Students apply the properties of geometric figures to comparable real-world objects (e.g., The spokes of a wheel of a bicycle are equal lengths because they represent the radii of a circle). ***Example:*** Describe each of the following as a simple geometric shape or combination of shapes. Illustrate with a sketch and label dimensions important to describing the shape. a. Soup can label b. A bale of hay c. Paperclip d. Strawberry
G.MG.2 ★ Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. ***Example:*** A King Size waterbed has the following dimensions 72 in. x 84 in. x 9.5in. It takes 240.7 gallons of water to fill it, which would weigh 2071 pounds. What is the weight of a cubic foot of water? ***Example:*** Wichita, Kansas has 344,234 people within 165.9 square miles. What is Wichita's population density?
G.MG.3 ★ Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included Math II and Math III. ***Example:*** You are the manager of a packing company responsible for manufacturing identical rectangular boxes from rectangular sheet of cardboard, each sheet having the same dimensions (18" X 24"). To save money, you want to manufacture boxes that will have the maximum possible volume. Determine the maximum volume possible.  ***Example:*** The Bolero Chocolate Company makes square prisms to package their famous chocolate almond balls. The package holds 5 of the chocolate almond balls that are 1.5" in diameter. They are considering changing packaging to a triangular prism. What would be the difference in material cost if the cardboard used is currently purchased at $1.25 per square foot? (Consider both the top and bottom of the box.)

Standard	Cluster: Understand and evaluate random processes underlying statistical experiments
S.IC.2★ Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students explain how well and why a sample represents the variable of interest from a population.** ***Example:*** Multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group's results will most likely approach the theoretical probability? ***Example:*** Illustrative Mathematics – *Block Scheduling* at http://www.illustrativemathematics.org/illustrations/125

Standard	Cluster: Make inferences and justify conclusions from sample surveys, experiments, and observational studies
S.IC.6★ Evaluate reports based on data.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard is included Math II and Math III. **Students read and explain, in context, data from outside reports.** They identify the variables as quantitative or categorical. Students describe how the data was collected, indicate any potential biases or flaws and identify inferences the author of the report made from sample data. ***Example:*** A reporter used the two data sets below to calculate the mean housing price in Kansas as $629,000. Why is this calculation not representative of the typical housing price in Kansas?

Wichita Area
1.2 million
242,000
265,500
140,000
281,000
265,000
211,000

Overland Park Homes
5 million
154,000
250,000
250,000
200,000
160,000
190,000

Standard	Cluster: Understand independence and conditional probability and use them to interpret data

S.CP.1★

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Students define a sample space and events within the sample space. The sample space is the set of all possible outcomes of an experiment. Students describe sample spaces using a variety of different representations.

Example: Describe the sample space for rolling two number cubes. *Note: This may be modeled well with a 6x6 table with the rows labeled for the first event and the columns labeled for the second event.*

Example: Describe the sample space for picking a colored marble from a bag with red and black marbles. *Note: This may be modeled with set notation.*

Example: Andrea is shopping for a new cellphone. She is either going to contract with Verizon (60% chance) or with Sprint (40% chance). She must choose between an Android phone (25% chance) or an IPhone (75% chance). Describe the sample space. *Note: This may be modeled well with an area model.*

Example: The 4 aces are removed from a deck of cards. A coin is tossed and one of the aces is chosen. Describe the sample space. *Note: This may be modeled well with a tree diagram.*

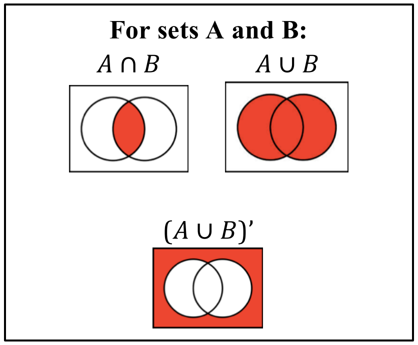
Students establish events as subsets of a sample space. An event is a subset of a sample space.

Example: Describe the event of rolling two number cubes and getting evens.

Example: Describe the event of pulling two marbles from a bag of red/black marbles.

Students define union, intersection, and complement with and without the use of notation.

- The **intersection** of two sets A and B is the set of elements that *are common to both* set A and set B.
  It is denoted by $A \cap B$ and is read "A intersection B"
- The **union** of two sets A and B is the set of elements, which are *in A or in B, or in both*.
  It is denoted by $A \cup B$, and is read "A union B"
- The **complement** of the set $A \cup B$ is the set of elements that are members of the universal set U but *are not in* $A \cup B$. It is denoted by $(A \cup B)'$



Example: Describe the event that the summing of two rolled number cubes is larger than 7 and even, and contrast it with the event that the sum is larger than 7 or even.

	Example: If the subset of outcomes for choosing one card from a standard deck of cards is the intersection of two events: {queen of hearts, queen of diamonds}. a. Describe the sample space for the experiment. b. Describe the subset of outcomes for the union of two events.
S.CP.2★ Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students calculate the probability of events.** ***Example:*** When rolling two number cubes… a. What is the probability of rolling a sum that is greater than 7? b. What is the probability of rolling a sum that is odd? c. Are the events, rolling a sum greater than 7, and rolling a sum that is odd, independent? Justify your response. ***Example:*** You are picking a number out of a hat, which contains the numbers 1 through 100. What are the following events and their probabilities? a. The number has a single digit b. The number has two digits c. The number is a multiple of 4 d. The number is not a multiple of 4 e. The sum of the number's digits is 5 **Students understand that two events A and B are independent when the probability that one event occurs in no way affects the probability of the other event occurring. In other words, the probability of A is the same even of event B has occurred.** If events are independent then the $P(A \cap B) = P\ (A)\ \cdot P\ (B)$ ***Example:*** Determine if the events are independent or not. Explain your reasoning. a. Flipping a coin and getting heads and rolling a number cube and getting a 4 b. When rolling a pair of number cubes consider the events: getting a sum of 7 and getting doubles c. From a standard deck of cards consider the events: draw a diamond and draw an ace

S.CP.3 ★

Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Students understand conditional probability as the probability of A occurring given B has occurred.

Example: What is the probability that the sum of two rolled number cubes is 6 given that you rolled doubles?

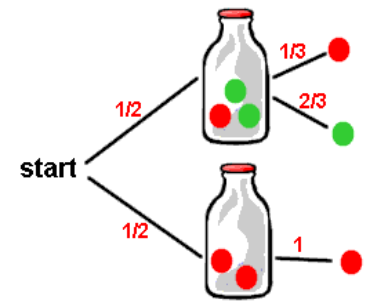
Example: Each student in the Junior class was asked if they had to complete chores at home and if they had a curfew. The table represents the data.

a. What is the probability that a student who has chores also has a curfew?
b. What is the probability that a student who has a curfew also has chores?
c. Are the two events have chores and have a curfew independent? Explain.

		Curfew		
		Yes	**No**	**Total**
Chores	**Yes**	51	24	75
	No	30	12	42
	Total	81	36	117

Example: There are two identical bottles. A bottle is selected at random and a single ball is drawn. Use the tree diagram at the right to determine each of the following:

a. $P\ (red|bottle\ 1)$
b. $P\ (red|bottle\ 2)$



S.CP.4 ★

Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities

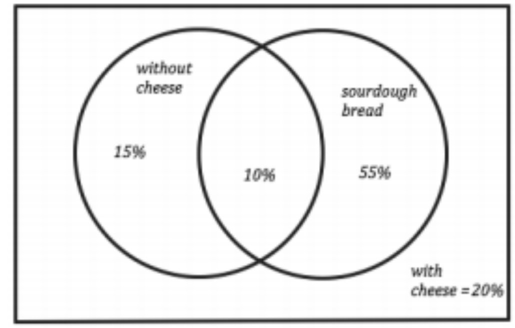
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Students

- Determine when a two-way frequency table is an appropriate display for a set of data
- Collect data from a random sample
- Construct a two-way frequency table for the data using the appropriate categories for each variable
- Calculate probabilities from the table
- Use probabilities from the table to evaluate independence of two variables.

Example: The Venn diagram to the right shows the data collected at a sandwich shop for the last six months with respect to the type of bread people ordered (sourdough or wheat) and whether or not they got cheese on their sandwich. Use the diagram to construct a two-way frequency table and then answer the following questions.

a. *P* (sourdough)
b. *P* (cheese | wheat)
c. *P* (without cheese or sourdough)
d. Are the events "sourdough" and "with cheese" independent events? Justify your reasoning.



Example: Complete the two-way frequency table at the right and develop three conditional statements regarding the data. Determine if there are any set of events that independent. Justify your conclusion.

	Ice Cream	Cake	Total
Male		20	
Female	10		60
Total	85		

Example: Collect data from a random sample of students in your school on their favorite subject among math, science, history, and English. Estimate the probability that a randomly selected student from your school will favor science give that the student is in tenth grade. Do the same for other subjects and compare the results.

S.CP.5 ★

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Example: Felix is a good chess player and a good math student. Do you think that the events "being good at playing chess" and "being a good math student" are independent or dependent? Justify your answer.

Example: Juanita flipped a coin 10 times and got the following results: T, H, T, T, H, H, H, H, H, H. Her math partner Harold thinks that the next flip is going to result in tails because there have been so many heads in a row. Do you agree? Explain why or why not.

Example: At your high school the probability that a student takes a Business class and Spanish is 0.062. The probability that a student takes a Business class is 0.43. What is the probability that a student takes Spanish given that the student is taking a Business class?

<table>
<tr><th>Standard</th><th>Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model</th></tr>
<tr><td>

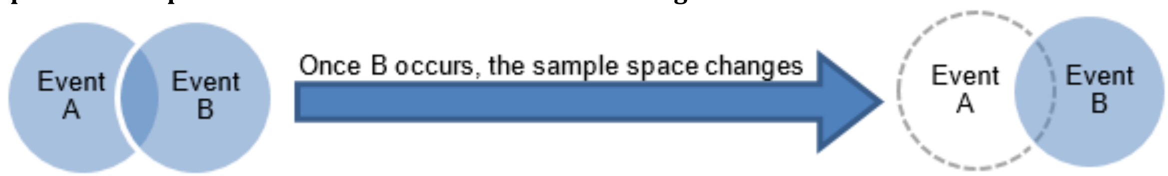
S.CP.6★

Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

</td><td>

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

The sample space of an experiment can be modeled with a Venn diagram such as:



So, the $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

Example: Peter has a bag of marbles. In the bag are 4 white marbles, 2 blue marbles, and 6 green marbles. Peter randomly draws one marble, sets it aside, and then randomly draws another marble. What is the probability of Peter drawing out two green marbles? *Note: Students must recognize that this a conditional probability P(green | green).*

Example: A teacher gave her class two quizzes. 30% of the class passed both quizzes and 60% of the class passed the first quiz. What percent of those who passed the first quiz also passed the second quiz?

Example: If a balanced tetrahedron with faces 1, 2, 3, 4 is rolled twice, what is the probability that the sum is prime (A) of those that show a 3 on at least one roll (B)?

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S.CP.7★

Apply the Addition Rule, $P\ (A \text{ or } B) = P\ (A) + P\ (B) - P\ (A \text{ and } B)$, and interpret the answer in terms of the model.

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This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Students understand that the $\boldsymbol{P\ (A \text{ or } B) = P\ (A) + P\ (B) - P\ (A \text{ and } B)}$. Students may recognize that if two events A and B are mutually exclusive, also called **disjoint,** the rule can be simplified to $P\ (A \text{ or } B) = P\ (A) + P\ (B)$ since for mutually exclusive events $P\ (A \text{ and } B) = 0$.

Example: Given the situation of drawing a card from a standard deck of cards, calculate the probability of the following:

a. Drawing a red card or a king
b. Drawing a ten or a spade
c. Drawing a four or a queen

Example: In a math class of 32 students, 18 boys and 14 are girls. On a unit test, 5 boys and 7 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?

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S.CP.8 (+) ★ Apply the general Multiplication Rule in a uniform probability model, $P\ (A\ and\ B)\ =$ $P\ (A)\ P\ (B\|A)\ =$ $P\ (B)\ P\ (A\|B)$, and interpret the answer in terms of the model.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **A uniform probability model is one in which all events in the sample space have an equal opportunity of occurring.** ***Example:*** You have a box with 3 blue marbles, 2 red marbles, and 4 yellow marbles. You are going to pull out one marble, record its color, put it back in the box and draw another marble. What is the probability of pulling out a red marble followed by a blue marble? ***Example:*** Consider the same box of marbles as in the previous example. However in this case, we are going to pull out the first marble, leave it out, and then pull out another marble. What is the probability of pulling out a red marble followed by a blue marble? ***Example:*** Suppose you are going to draw two cards from a standard deck. What is the probability that the first card is an ace and the second card is a jack (just one of several ways to get "blackjack" or 21)?
S.CP.9 (+) ★ Use permutations and combinations to compute probabilities of compound events and solve problems.	This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. **Students determine if the sample space is a permutation or combination.** They define the sample space and calculate probabilities using permutations and combinations. ***Example:*** There are seven children to be lined up in a straight line for a photograph. a. How many different ways are possible? b. How many different ways are possible if Sally must be in the middle? c. What is the probability that Sally is in the middle of the picture? ***Example:*** North Carolina has proposed a state lottery (pick 6) of 55 numbers. a. What is the probability of winning this lottery? b. A person buys 5000 different tickets? What is their chance of winning?

References

This document includes examples, illustrations and references from the following websites:

Illustrative Mathematics: www.illustrativemathematics.org
Math Assessment Project: www.map.mathshell.org
Shodor: www.shodor.org