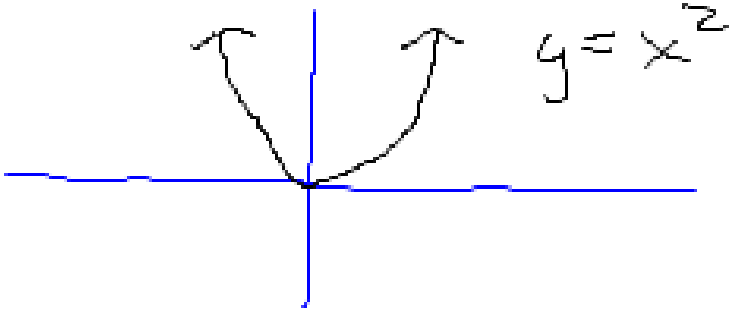
Unit 6: Lesson 05 — Even and odd functions

Functions can fall into the category of **even**, **odd**, or **neither**.

Even function

Graphical definition: A relation that has symmetry with respect to (w.r.t) the vertical axis.



Algebraic definition: A relation that satisfies:

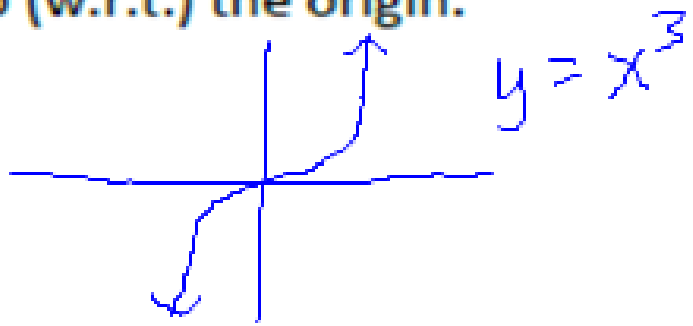
$$f(x) = f(-x)$$

$$y = (-x)^2$$

$$y = x^2$$

Odd function

Graphical definition: A relation that has symmetry with respect to (w.r.t.) the origin.



Algebraic definition: A relation that satisfies:

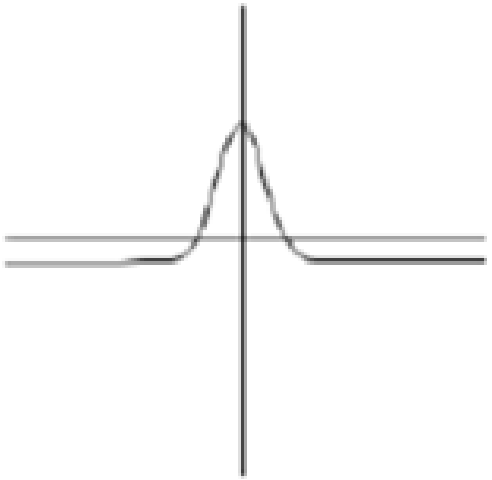
$$f(x) = -f(-x)$$

$$y = (-x)^3$$

$$y = -x^3$$

In the following examples, is the relation an even or an odd function (or neither)?

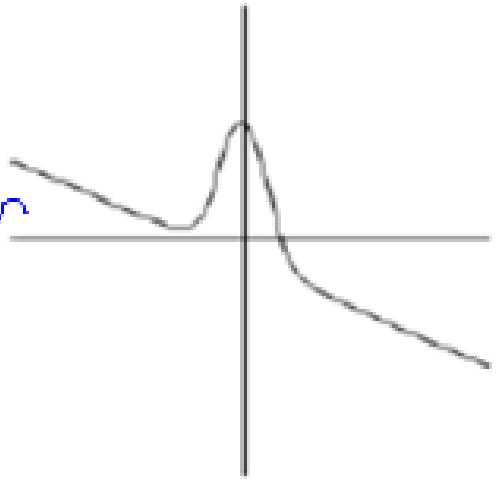
Example 1:



even

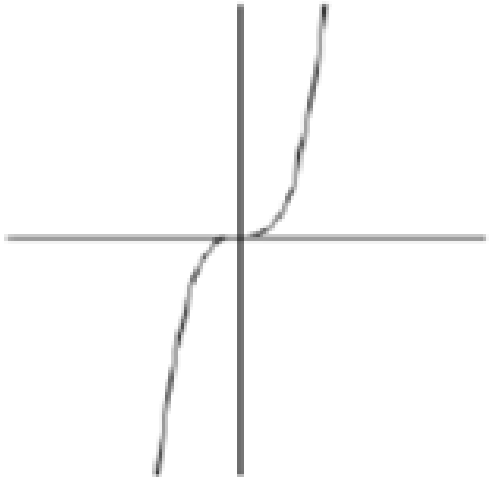
Example 2:

neither



Example 3:

odd



Example 4: $f(x) = x^2 + 3$

$$f(-x) = (-x)^2 + 3$$

$$= x^2 + 3$$

even

Example 5: $f(x) = 4x^3 - x$

$$f(-x) = 4(-x)^3 - (-x)$$

$$= -4x^3 + x$$

odd

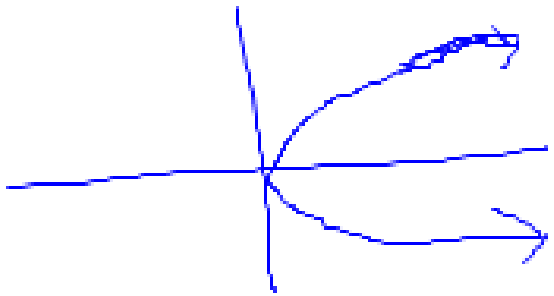
Example 6: $f(x) = x^2 + x^3 - 1$

$$f(-x) = (-x)^2 + (-x)^3 - 1$$

$$= x^2 - x^3 - 1$$

neither

It is possible for a relation to have **symmetry w.r.t. the horizontal** (x) axis. It is a moot point to ask if this is an even or odd function because it is **not a function**...notice that it fails the vertical line test.



See **Calculator Appendix G** for a look at even and odd functions on the graphing calculator.

Homework: Complete problems 1-14 on the worksheet