

Properties of Limits

For the following limit theorems, assume:

$$\lim_{x \rightarrow a} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow a} g(x) = M$$

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = L \pm M$$

$$\lim_{x \rightarrow a} [f(x)g(x)] = \left[\lim_{x \rightarrow a} f(x) \right] \cdot \left[\lim_{x \rightarrow a} g(x) \right] = L \cdot M$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{L}{M}$$

$$\lim_{x \rightarrow a} (k \cdot f(x)) = k \lim_{x \rightarrow a} f(x) = k L$$

(where k is a constant)

Example 1: Assume $\lim_{x \rightarrow 3} f(x) = -1$ and $\lim_{x \rightarrow 3} g(x) = 7$

a. $\lim_{x \rightarrow 3} [3g(x) - f(x)] = ?$

b. $\lim_{x \rightarrow 3} \frac{x + f(x)}{g(x) - f(x)}$

Example #2:

$$\lim_{x \rightarrow -4} f(x) = 1 \quad \text{and} \quad \lim_{x \rightarrow -4} g(x) = -2$$

a. $\lim_{x \rightarrow -4} \frac{g(x)}{f(x) + x}$ b. $\lim_{x \rightarrow -4} [xf(x) - g(x)]$

Example 3:

$$\lim_{x \rightarrow -4} f(x) = 1 \quad \text{and} \quad \lim_{x \rightarrow -4} g(x) = -2$$

a. $\lim_{x \rightarrow -4} [f(x)^2 - g(x)^2 - 2]$ b. $\lim_{x \rightarrow -4} [f(x) + g(x)]^2$

The following special trig limits should be memorized

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin(x)} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x} = 0$$

Example 4: $\lim_{x \rightarrow 0} \frac{x}{\sin(7x)} = ?$

Example 5: $\lim_{\theta \rightarrow 0} \frac{\sin(8\theta)}{\theta} = ?$

Example 6:

$$\lim_{\beta \rightarrow 0} \frac{\sin(\beta) \cos(\beta)}{\beta}$$

Example 7:

$$\lim_{x \rightarrow 0} \frac{-\sin(\pi x)}{\pi x} = ?$$

Example 8:

$$\lim_{\alpha \rightarrow 0} \tan(\alpha) = ?$$

