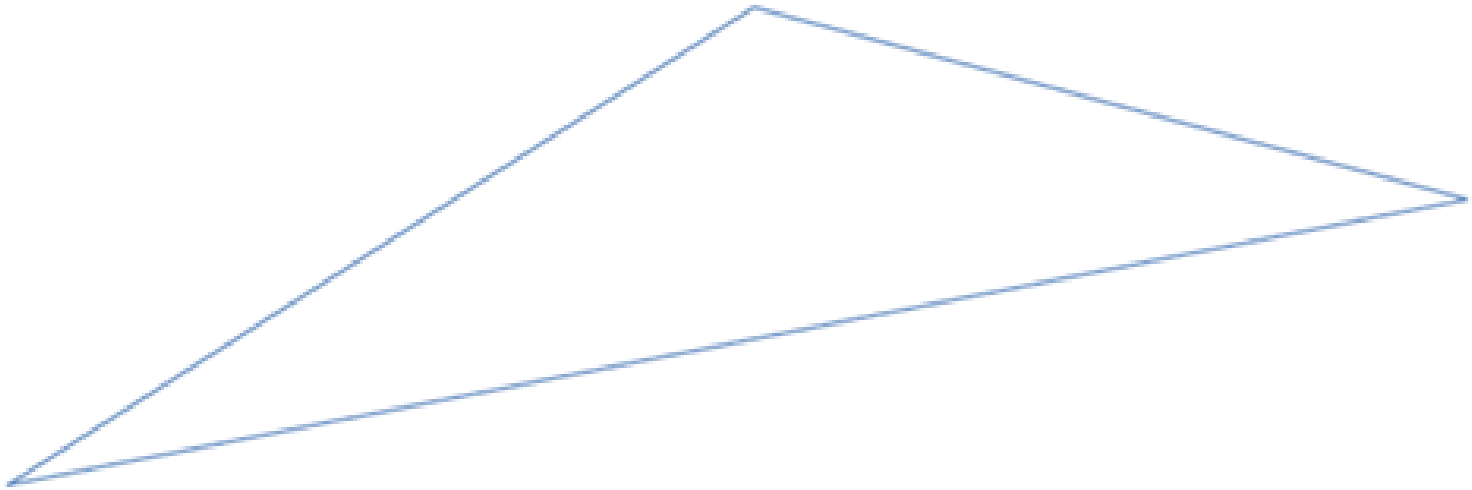
Until this point in our studies we have been restricted to solutions of only right triangles. No longer. The **Law of Sines** applies to **any type of triangle**, not just right triangles.



Each of the following proportions is an expression of the sine law. Notice that each ratio is made of a pair of items that are **opposite each other** in the triangle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

We should remember from Geometry (ha) that a unique triangle can be formed if you know the measures of two angles and the included side length (ASA) or the measures of two angles and the non-included side length (AAS).

We will discuss how to handle triangles when we know two side lengths and the non-included angle in the next section

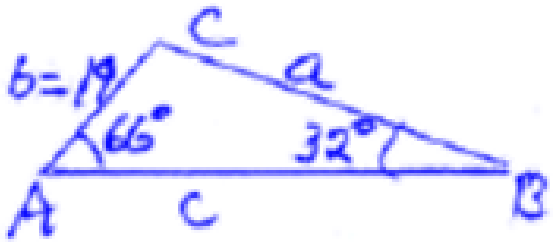
Finding area of non-right triangles:

Remember this formula: $K = \frac{1}{2} a\ b \sin(C)$

Since the names of sides and angles within an oblique triangle are interchangeable, this area formula should be learned as follows:

*************The area of a triangle is equal to one-half the product of any two sides times the sine of their included angle.************

Example 1: A = 66°, B = 32°, b = 19: Determine all the other measures for this triangle and determine the area.



$C + 66 + 32 = 180 \rightarrow C = \boxed{82°}$

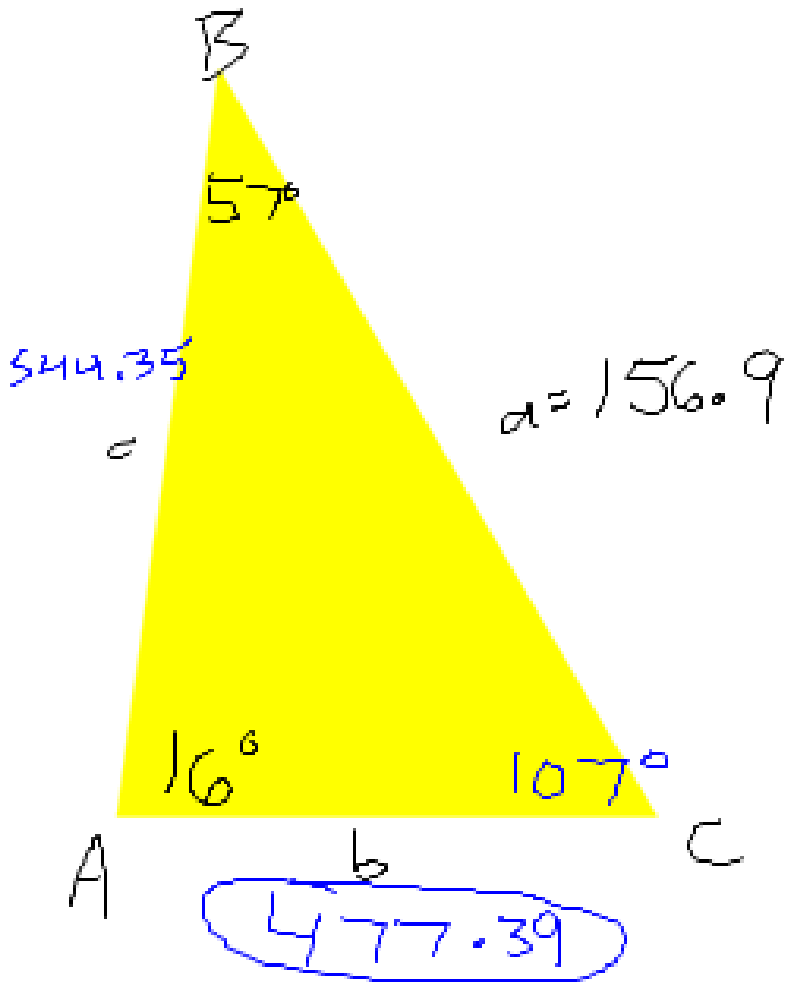
$\frac{\sin 32}{19} = \frac{\sin 66}{a} \rightarrow a = \boxed{32.75}$

$\frac{\sin 32}{19} = \frac{\sin 82}{c} \rightarrow c = \boxed{35.51}$

$K = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(32.75)(19)\sin 82° = \boxed{308.097}$

Example 2: A = 16°, B = 57°, a = 156.9. Determine all the other measures for this triangle and determine the area.



$m\angle C = 107°$

Find b

$\frac{b}{\sin 57°} = \frac{156.9}{\sin 16°}$ · sin57

so $b = 477.39$

Find c

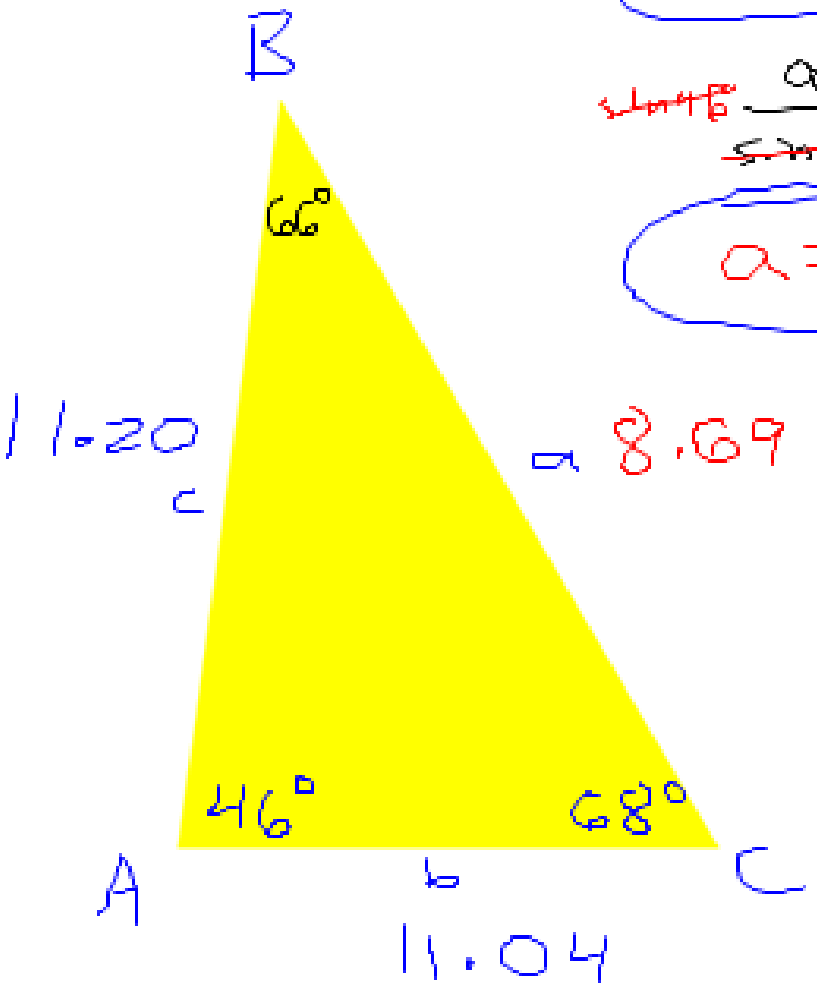
$\frac{\sin 107° \, 156.9}{\sin 16°} = \frac{c}{\sin 107°}$ · sin107

so $c = 544.35$

Area $= \frac{1}{2}(544.35)(156.9)\sin(57°) = 35814.80$

Example 3: A = 46°, C = 68°, b = 11.04. Determine all the other measures for this triangle and determine the area.

$m\angle B = 66^\circ$



$\frac{a}{\sin 46^\circ} = \frac{11.04}{\sin 66^\circ} \cdot \sin 46$

$a = 8.69$

$\frac{c}{\sin 68^\circ} = \frac{11.04}{\sin 66^\circ}$

$c = 11.20$

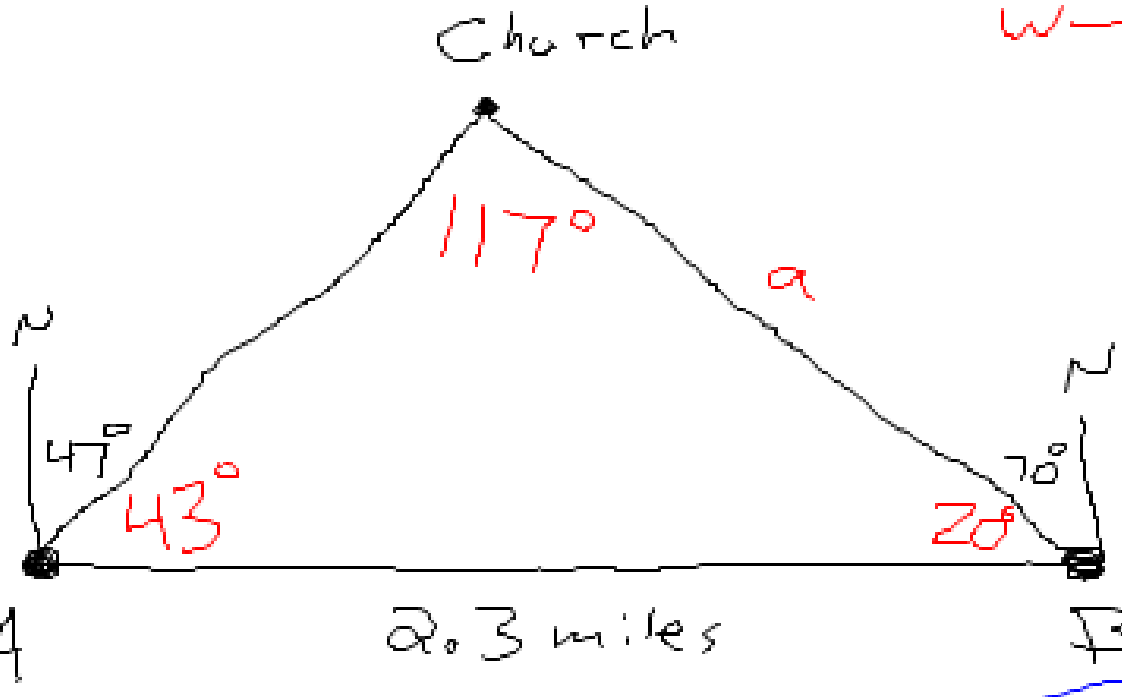
$\text{Area} = \frac{1}{2}(8.69)(11.20)\sin 66^\circ$

$= 44.45$

Example 4: B = 21°, C = 61°, c = 460. Determine all the other measures for this triangle and determine the area.

Let to Reader

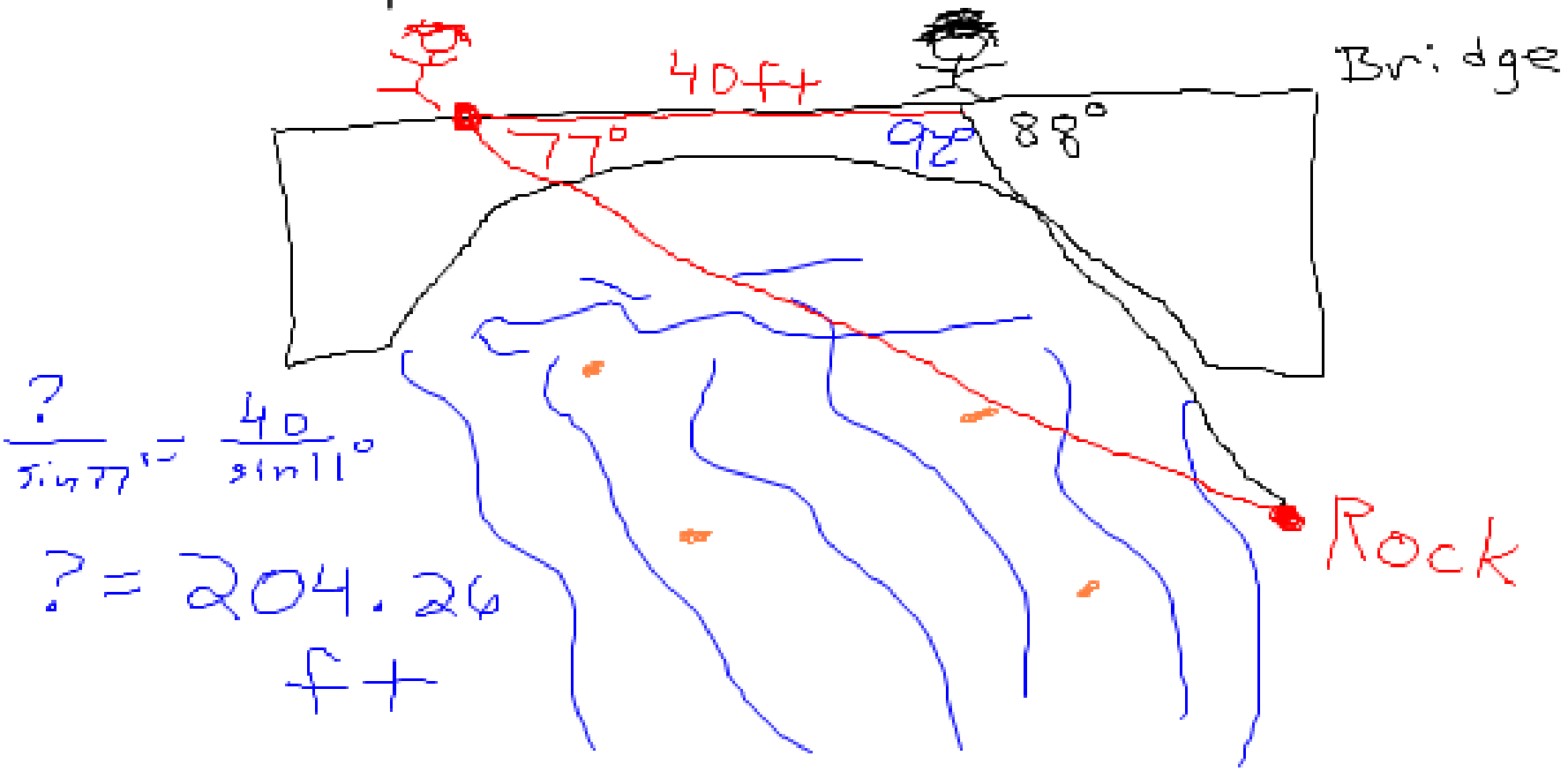
Example 5: Two observers are stationed on an east-west line and are 2.3 miles apart. Observer A sees the steeple of the old church with a bearing of N 47° E. Observer B reports a bearing of N 70° W. How far is the church from B?



$$\frac{\sin 43^\circ}{2.3} = \frac{\sin 117^\circ}{a} \qquad \frac{\sin 43}{} = \frac{\sin 43^\circ}{}$$

$a = 1.76$ miles

Example 6: From a bridge above a river an observer looks along an angle of depression of 77 degrees to a rock on the bank (directly under the bridge). Walking forward 40 ft, she determines a new angle of depression of 88 degrees to the same rock. How far is the rock from her second position?



Homework: p 316 # 11-25, 28, 32, 34