Law of Cosines and Heron's Formula

In the previous lessons a problem was never given in which **two sides and their included angle (SAS)** was the initial information given. **To solve such a problem the LAW OF COSINE is needed:**

Writing all 3 possibilities yields the following:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

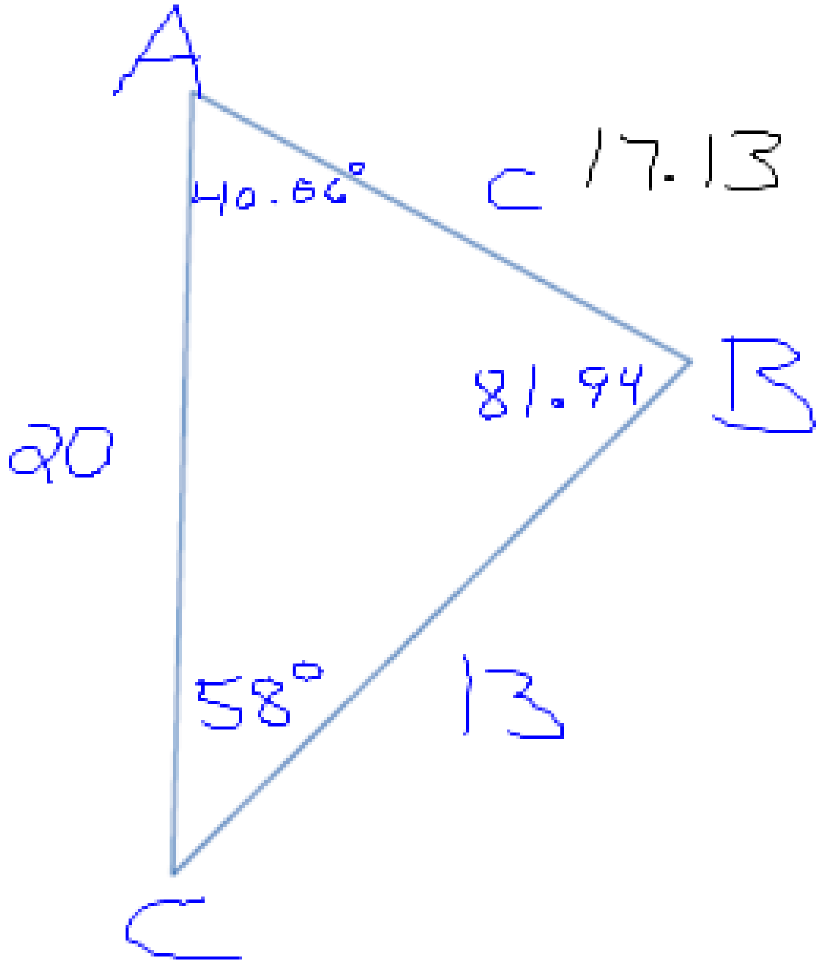
The square of a side equals the sum of the squares of the other two sides minus twice the product of the others sides and the cosine of the angle between them.

EXAMPLE 1:

* Never find angle opp largest side first

$$\frac{17.13}{\sin 58} = \frac{13}{\sin A}$$

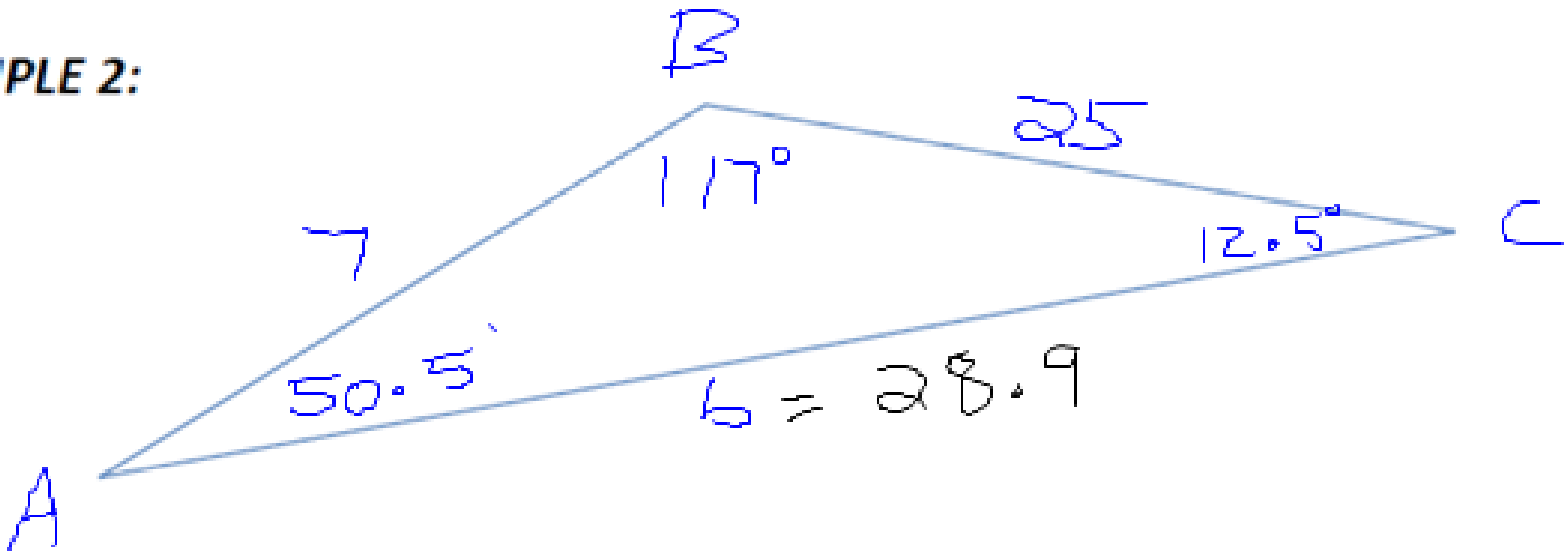
so $A = 40.06°$



$$c = \sqrt{20^2 + 13^2 - 2(20)(13) \cdot \cos 58°}$$

$c = 17.13$

EXAMPLE 2:



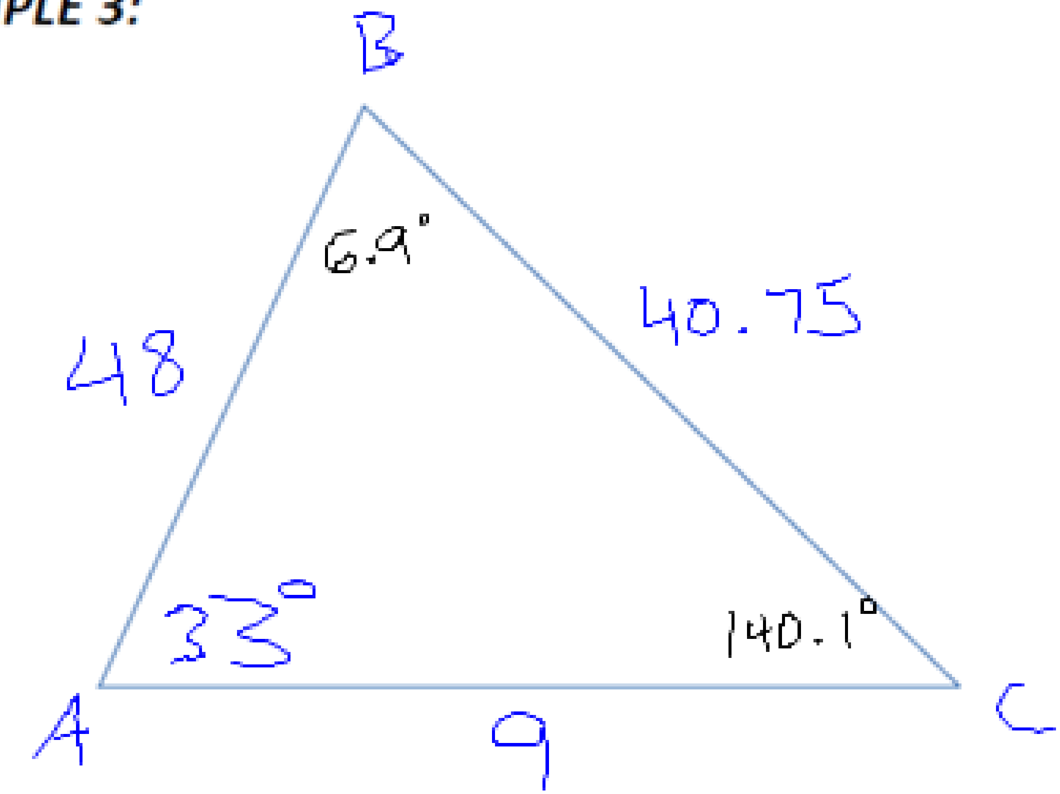
$$\sqrt{7^2 + 25^2 - 2(7)(25)\cos 117°} = b$$

so $b = 28.9$

$$\frac{28.9}{\sin 117°} = \frac{7}{\sin C} \qquad \sin C = 7 \Big/ \left(28.9 / \sin 117° \right)$$

$C = 12.5$

EXAMPLE 3:

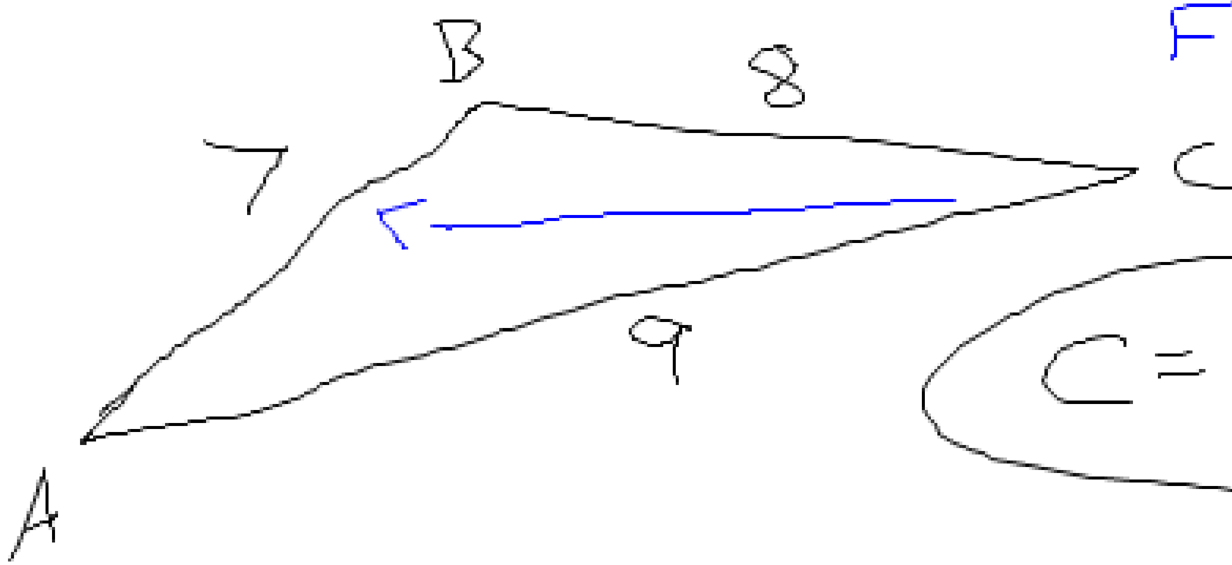


$$a = \sqrt{48^2 + 9^2 - 2(48)(9)\cos 33^\circ} = 40.75$$

$$\frac{40.75}{\sin 33^\circ} = \frac{9}{\sin B} \qquad \sin B = 9 / \left(40.75 / \sin 33^\circ \right) \quad \text{so} \quad B = 6.9$$

Example 4: Find the smallest angle in a triangle with sides 7, 8, and 9.

Find C.



$C = 48.2^\circ$

$$7^2 = 8^2 + 9^2 - 2(8)(9)\cos C$$

$$49 = 64 + 81 - 144\cos C$$

$$49 = 145 - 144\cos C$$

$$-96 = -144 \cdot \cos C$$

so $\cos C = -96/-144$

HERON'S Formula:

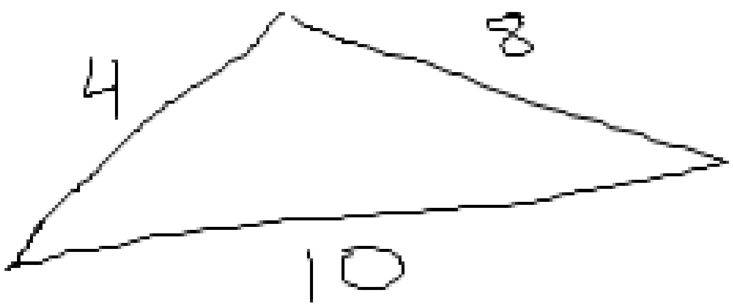
a, b, c are sides

AREA = $\sqrt{s(s-a)(s-b)(s-c)}$

where s = half the perimeter (aka: semi-perimeter, hence s)

$$s = \frac{4+8+10}{2} = 11$$

EXAMPLE 5: Find the area of a triangle with side lengths of 4, 8, & 10.



Area = 15.2

$$\text{Area} = \sqrt{11(11-4)(11-8)(11-10)}$$

EXAMPLE 6: Find the area of a triangle with side lengths of 14, 15, & 21.

Homework: p331 #11, 14-24, 27,28,30