Piecewise functions and continuity

A function is discontinous at a particular x value if we need to "lift the pencil" at that point in order to keep drawing the function. Otherwise, the function is said to be continuous.

There are several things that can cause a discontinuity at x = a for a function:

- There is a vertical asymptote at x = a . Typically, (x – a) is a factor of the denominator. (See Example 1).
- A piecewise function abruptly "jumps" at x = a. (See Example 3.)
- There is a "hole" in the graph at x = a. (See Example 5.)

Polynomials are continuous everywhere.

Example 1: Sketch the graph of f(x) (and note the positions of any discontinuities).

$$f(x) = \frac{x}{x^2 + 3x - 18}$$

Example 2: Just by observing the sketch in Example 1, determine the following limits:

$\lim_{x \to -6^-} f(x) = ?$

$\lim_{x \to -6^+} f(x) = ?$

$\lim_{x \to 3^-} f(x) = ?$

$\lim_{x \to 3^+} f(x) = ?$

$\lim_{x \to 3} f(x) = ?$

$f(3) = ?$

Example 3: Sketch this piecewise function.

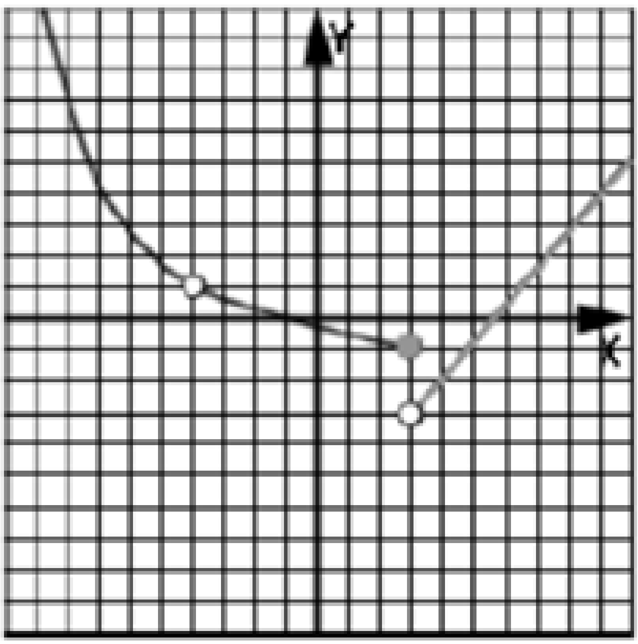
$$f(x) = \begin{Bmatrix} x & when\ x < -3 \\ 5 & when\ x = -3 \\ \sqrt{x+3} + 2 & when\ x > -3 \end{Bmatrix}$$

Example 4: Just by observing the sketch in Example 3, determine the following values:

$\lim_{x \to -3^-} f(x) = ?$ $\lim_{x \to -3^+} f(x) = ?$

$\lim_{x \to -3} f(x) = ?$ $f(-3) = ?$

Example 5: State the *x* positions of discontinuity and identify which are "holes."



$\lim_{x \to -4^-} f(x) = ?$

$\lim_{x \to -4^+} f(x) = ?$

$\lim_{x \to -4} f(x) = ?$

$\lim_{x \to 3^-} f(x) = ?$

$\lim_{x \to 3^+} f(x) = ?$

$\lim_{x \to 3} f(x) = ?$

Example 6. Determine the value of B so as to insure that this function is everywhere continuous.

$$f(x) = \begin{cases} Bx^2 & \text{if } x \le 3 \\ 2 & \text{if } x > 3 \end{cases}$$

Ex 7. Determine the value of *b* so as to insure that the function is everywhere continuous.

$$f(x) = \begin{cases} 3x + b & \text{if } x \le 2 \\ -x - 1 & \text{if } x > 2 \end{cases}$$

Ex 8. Determine the values of m and b so as to insure that the function is everywhere continuous.

$$f(x) = \begin{cases} 4 & \text{if } x \leq 3 \\ mx + b & \text{if } 3 < x < 6 \\ 1 & \text{if } x \geq 6 \end{cases}$$

Intermediate Value Theorem (IVT)

If a function y = f(x) is continuous on a closed interval [a,b] takes on every value between f(a) and f(b).

Ex: If continuous, you cannot go from 1 to 20 without passing through all points in between such as 6, 12, 19.01, …

Example 9:

Show that $f(x) = x^3 + x$ takes on the value of 9 for some x in [1,2].

Example 10:

Show that $g(x) = \frac{x}{x+1}$ take on the value of .429 for some x in [0,1].

Example 11:

. Show that $h(x) = x^2$ takes on the value of ½ for some x in [0,π/4].

Example 12:

Show that $cos(x) = x$ has a solution on the interval [0,1].

Example 13:

. Show that $\sqrt{x} + \sqrt{x+2} = 3$ has a solution.