Co-terminal Angles and Arc Length

Co-terminal angles have a common terminal side.

For example an angle that measures 120° is co-terminal with an angle measuring 480° / 1200°.

We can identify co-terminal angles by adding/subtracting a circle (360° or 2π)

−240°

Name 5 different angles that are co-terminal to 30°.

−330° 390° 750° 1110° −690°

Example 1: Name two co-terminal angles to 35°

$35 + 360 = 395°$

$35 - 360 = -325°$

$360 \cdot 7 + 35$

$2520°$

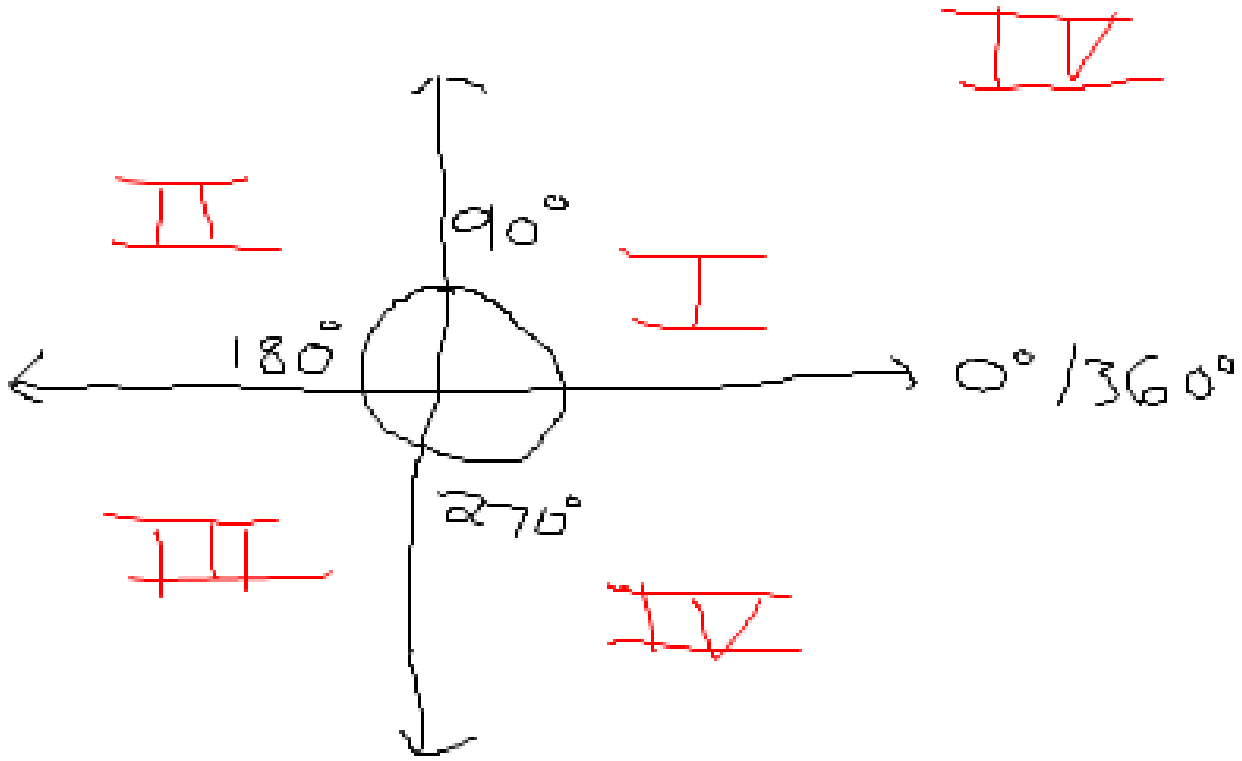
Example 2: Name two co-terminal angles to π/4.

$$\frac{\pi}{4} + \underset{\text{circle}}{2\pi} = \frac{9\pi}{4}$$

$$\frac{9\pi}{4} + 2\pi = \frac{17\pi}{4}$$

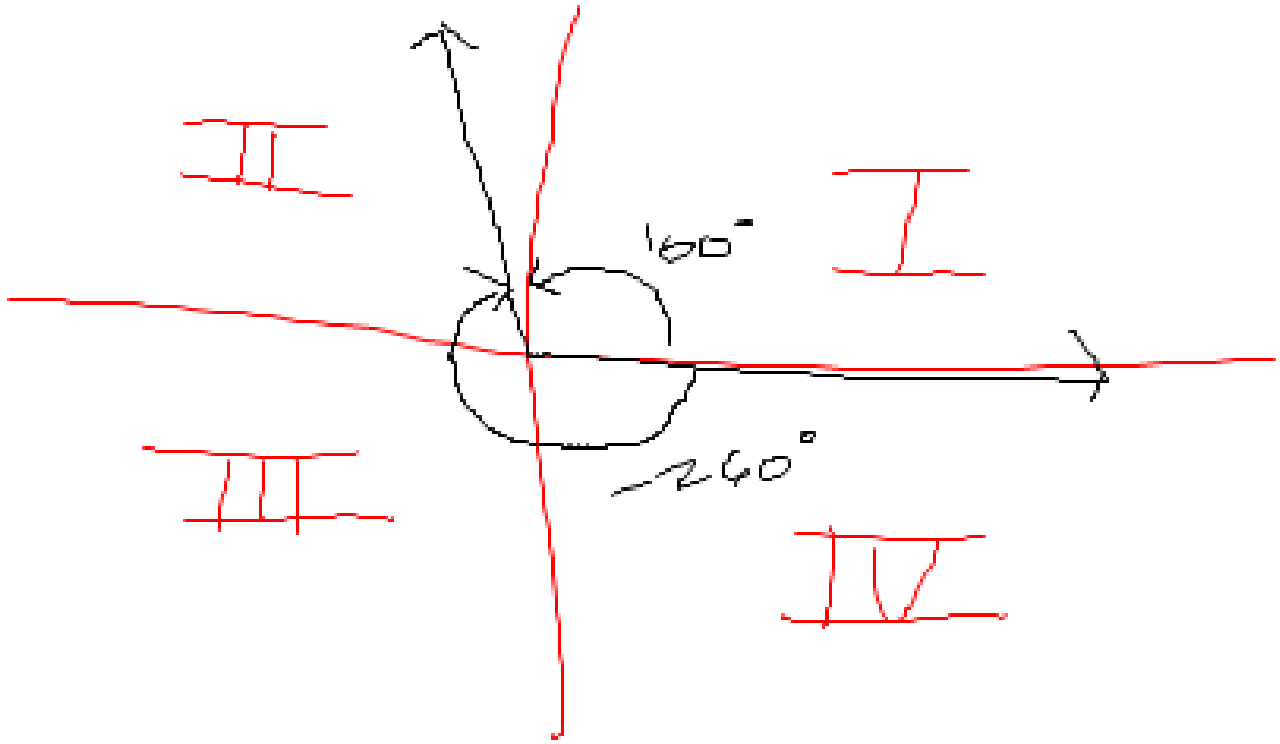
We can determine which quadrant an angle lies in by using it's angle measure. The same holds true for co-terminal angles.

Example 3: Determine which quadrant a 314° angle lies in.

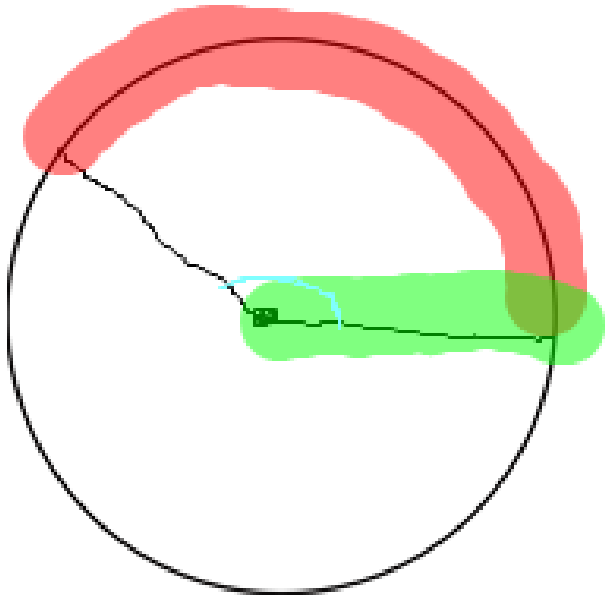


Example 4: Determine which quadrant a -260° angle lies in.

+360
100°



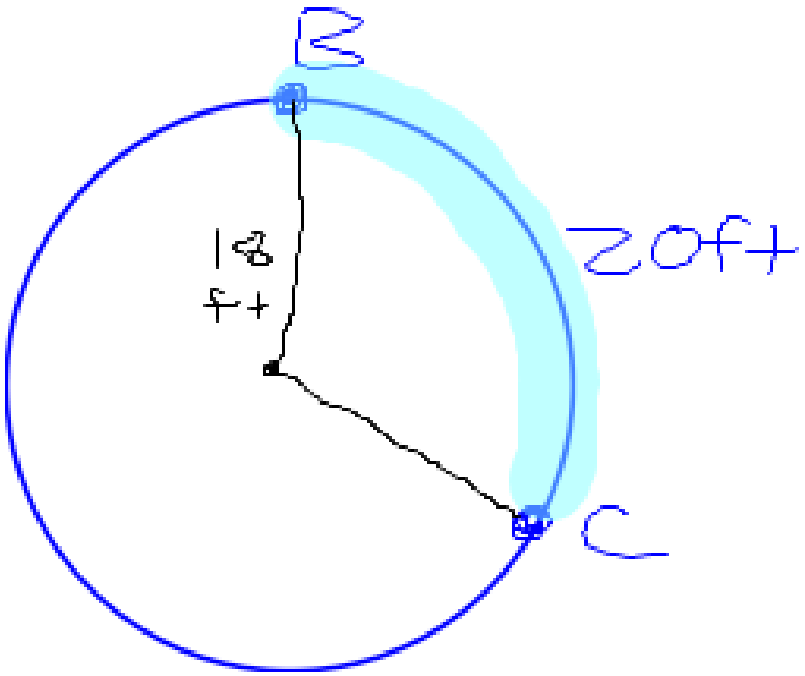
The definition of radians comes from the following drawing and an associated formula: $\Theta = s/r$



$$\text{Angle} = \frac{\text{Arc Length}}{\text{radius}}$$

In the drawing above, *r* is the radius, θ is the angle in radians, and, s is the arc length.

Example 5: In the drawing below the arc length BC is 20 ft and the radius is 18 ft. Find the measure of θ in radians.

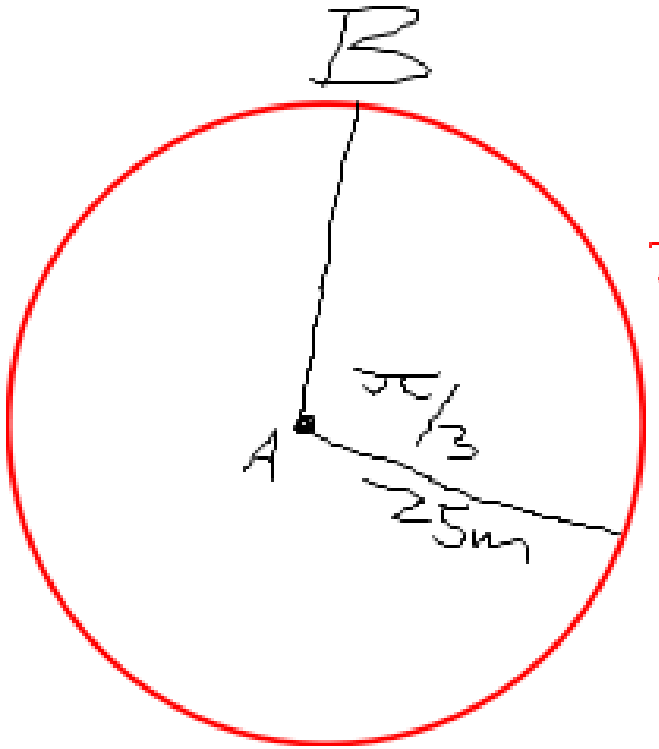


$$\theta = \frac{s}{r}$$

$$\theta = \frac{20}{18}$$

$$\theta = \frac{10}{9} \approx 1.\overline{11}$$

Example 6: In the drawing below find the length of the arc BC when the angle θ is π/3 radians and the line segment AC has length 25 meters.



$$\theta = \frac{s}{r}$$

$$25 \cdot \frac{\pi}{3} = \frac{s}{25} \cdot 25$$

$$\frac{25\pi}{3} = s$$

Arc is 26.18 m

Example 7:

$\theta = \frac{s}{r}$

Find the distance between two cities of the given latitudes. Assume that the earth is a sphere of radius 4000 miles and that the cities are on the same meridian.

Dallas 32°47'9"

Omaha 41°15'42"

$41^\circ 15' 42''$
$-\ 32^\circ 47' 9''$
8.48°

$8.48^\circ \cdot \frac{\pi}{180^\circ}$

$\theta = .148$

$\theta = \frac{s}{4000}$

$.148 = \frac{s}{4000}$

$4000(.148) = s$

$s = 592$ miles

Example 8:

Find the distance between two cities of the given latitudes. Assume that the earth is a sphere of radius 4000 miles and that the cities are on the same meridian.

Erie, NY 42.7500°

Miami, FL 25.7877°

$$16.9623^\circ \cdot \frac{\pi}{180}$$

$$.296 = \frac{s}{4000}$$

$$4000(.296) = s$$

$$s = 1184.2 \text{ miles}$$

Homework

p281 #30-51

(pick 12)

p349 #34-42