DeSoto
COUNTY SCHOOLS

Yearlong Algebra I

Week 5

Name: _____ Date: _____

Factoring Trinomials
Algebra 1

To round out our three main factoring techniques, we now work with factoring trinomials into the product of two binomials. First, we review how to multiply two binomials.

***Exercise* #1:** Write each of the following products without the use of parentheses.

(a) $(x+3)(x+2)=$ (b) $(x-3)(x-5)=$ (c) $(x+7)(x-2)=$

Each of these three problems gave a polynomial of the form x^2+bx+c. These polynomials are called **quadratics** where the x^2 term is called the **quadratic term**, bx the **linear term**, and c the **constant.** Since these equations have one variable, the **degree** of the expression is the largest exponent of the variable. The expression x^2+bx+c has a degree of 2.

***Exercise* #2:** Write each of the following trinomials as the product of two binomials. Hint – look at your results from *Exercise* #1.

(a) $x^2+5x+6=$ (b) $x^2-8x+15=$ (c) $x^2+5x-14=$

We know that our answers to *Exercise* #2 are correct because of *Exercise* #1. This, in fact, serves as the basis for the most general factoring technique – **guess-and-check**. Using this technique, we factor a trinomial by guessing its binomial factors and then checking by multiplication.

***Exercise* #3:** Which of the following expressions is the correct factoring of the trinomial x^2+6x+8? Justify your answer by multiplying each set of binomials.

$(x+1)(x+8)$ $(x+2)(x+4)$

***Exercise* #4:** Consider the trinomial $x^2+8x+12$.

(a) Write all pairs of integral factors that give a product of 12.

(b) Using your answer from part (a), factor the trinomial into the product of two binomials. Make sure to check your answer.

The previous exercise was relatively easy because all coefficients were positive. Still, a student should NEVER get a factoring problem wrong because it should ALWAYS be checked by multiplying the binomials.

Note that if the sign of the constant in the trinomial is "+", the two binomial factors have to have the same sign. If the constant is "−", the signs in the binomials must be different, as noted in exercise #2. In parts 2(a) and 2(b), the trinomial constants have a "+" sign and both binomial factors have like signs. Note you can even determine what those signs will be by looking at the sign of the linear term in that trinomial. Notice in 2(c), the constant in the trinomial is "−" and the two binomial factors have different signs. Even using these rules, you need to check to ensure you have the correct factors.

Exercise #5: Factor each trinomial. Guess as many binomial pairs as necessary and then check each.

(a) $x^2-7x+10$ (b) $x^2+9x+18$ (c) $x^2+6x-16$

(d) $r^2+8r+16$ (e) $m^2+2m-24$ (f) $x^2+2x-15$

(g) w^2-w-42 (h) $x^2-10x+25$ (i) $y^2-18y+32$

Patterns will often emerge that will help make factoring faster in these problems. The last exercise will illustrate the most important of these.

Exercise #6: Consider the following general factoring pattern:

$$x^2+bx+c=(x+m)(x+n)$$

(a) Rewrite the right hand side of this equation by expanding.

(b) Fill in the following:

$m+n=$

$m\cdot n=$

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Factoring Trinomials
Algebra 1 Homework

Skills

1. If the binomial $(x-3)$ is one factor of $x^2-10x+21$, which of the following is the other factor?

   (1) $(x+7)$ (3) $(x+5)$

   (2) $(x-18)$ (4) $(x-7)$

2. The binomials $(x+5)$ and $(x-7)$ are both factors of which of the following trinomials?

   (1) $x^2+12x-35$ (3) $x^2-2x-35$

   (2) $x^2+2x+35$ (4) $x^2-12x+35$

Factor using the guess-and-check method.

3. x^2+x-6

4. $m^2-7m-18$

5. $r^2+9r+20$

6. $x^2-7x+12$

7. x^2+7x+6

8. $t^2+12t+36$

9. $p^2+12p-28$

10. $v^2-9v+18$

11. $b^2 - 11b + 30$

12. $y^2 - 7y - 30$

13. $x^2 + 9x - 36$

14. $n^2 + 13n + 40$

Reasoning

15. Factor: $x^2 + 12xy + 35y^2$ using the guess and check method.

16. Express $x^4 + 5x^2 - 14$ as the product of two binomials. (Hint: $x^2 \cdot x^2 = x^4$)

17. Find all values of k such that $x^2 + kx + 36$ factors into two identical binomials.

18. Consider the trinomial $x^2 + 8x + 6$.

(a) Based on the constant term of 6, state all reasonable guess for how this trinomial could factor.

(b) Show, by expressing the above products as trinomials, that none of these are the correct factors of the given trinomial.

(c) What should we call this trinomial since it cannot be factored?

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Factoring Trinomials Completely
Algebra 1

In the previous lesson, we saw how to factor a trinomial of the form x^2+bx+c by employing the guess-and-check method. In each of those cases, the coefficient of the quadratic (x^2) term was always one, and thus not written. It is also possible to factor trinomials of the form ax^2+bx+c where the coefficient a is a number other than 1 by combining two factoring methods into the same problem.

Exercise #1: Consider the trinomial $3x^2+15x+18$.

(a) What is the GCF of each term in the trinomial?

(b) Write the trinomial as a product involving its GCF.

(c) How does the trinomial inside of the parentheses now factor?

(d) Write $3x^2+15x+18$ in its completely factored form.

We can carry this two-step process out for all trinomials whose three monomial terms have a GCF other than one. In this course, after factoring a GCF out of the trinomial, the quadratic coefficient on the new trinomial will be one.

Exercise #2: Factor each of the following trinomials completely. Remember to mentally check your factors by multiplying.

(a) $2y^2-12y+16$

(b) x^3-4x^2-12x

(c) $5x^2-45x+90$

(d) $9x^2+18x+9$

Additional Classroom Exercises

Write each trinomial in its completely factored form.

1. $2x^2+18x+40$

2. $3x^2-9x-30$

3. $5x^2-35x-90$

4. $3x^2+15x-108$

5. $10p^2+10p-20$

6. $2y^2+8y+6$

7. $7x^3-35x^2+42x$

8. $4m^2-24m+36$

9. $2x^2+2xy-112y^2$

10. $6n^2+18mn-60m^2$

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Factoring Trinomials Completely
Algebra 1 Homework

Skills

1. Which of the following represents the trinomial $4x^2+28x+40$ factored completely?

   (1) $(4x+10)(x+4)$ (3) $4(x+1)(x+10)$

   (2) $4(x+2)(x+5)$ (4) $2(2x+10)(x+2)$

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2. Factored completely, the trinomial $2x^2-10x-12$ is represented by

   (1) $2(x-6)(x+1)$ (3) $2(x-3)(x+2)$

   (2) $(2x+2)(x-6)$ (4) $(2x-12)(x+1)$

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Factor each trinomial expression completely and check.

3. $4w^2-40w+100$

4. $2n^2+4n-16$

5. x^3-17x^2+72x

6. $3x^2-6x-24$

7. $5x^2+40x+60$

8. $2x^2-16x-18$

9. $3x^3+6x^2-24x$

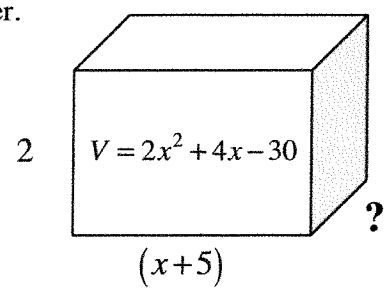
10. $4x^3-36x^2+56x$

11. $9y^2+18xy+9x^2$

12. $x^2y^2z^2-xy^2z^2-20y^2z^2$

Applications

13. Recall that the volume of a rectangular solid (a box) is given by $V=L\cdot W\cdot H$. If a particular rectangular solid has a height of 2, a width of $(x+5)$, and a volume of $2x^2+4x-30$, how would you represent the length of the solid? Justify your answer.



Reasoning

14. Consider the expression: $2p^2(p-3)-12p(p-3)+10(p-3)$.

(a) Rewrite this expression by factoring out the binomial $(p-3)$ from each term.

(b) Completely factor the trinomial from (a).

(c) Write the original expression in its completely factored form.

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The Zero Product Law
Algebra 1

Perhaps the most powerful of all equation solving techniques is what is known as the Zero Product Law. The next two exercises serve to illustrate this law.

Exercise #1: For each of the following, find three sets of rational numbers that give the following products.

(a) 12 (b) -24 (c) 0

From *Exercise* #1 you should notice something that is remarkable about the products you wrote for (c) as opposed to (a) and (b).

Exercise #2: Fill in the box below for the Zero Product Law.

THE ZERO PRODUCT LAW

If $m \cdot n = 0$ then $m =$ _____ or $n =$ _____

We can use the Zero Product Law to solve equations of various types. In this course, we will concentrate on solving quadratic equations using this law.

Exercise #3: Find the values of x that solve each equation.

(a) $(x-5)(x+1)=0$ (b) $5(x+3)(x+4)=0$ (c) $2x(x+6)=0$

(d) In (b), why is it okay to divide by 5 first?

(e) Why, in (c), can we **not** divide by x?

Exercise #4: Which of the following is the solution set of the equation $2(x-4)(x+7)=0$?

(1) $\{-4, 7\}$ (3) $\{-7, 2, 4\}$

(2) $\{-7, 4\}$ (4) $\{-14, 8\}$

Clearly this process is easy enough if our equation is already in factored form. If it is not, which is more common, then we must first use the factoring techniques we have learned.

Exercise #5: Find the values of x that solve each of the following quadratic equations. [illegible] r answers by using **STORE** on your calculator.

(a) $x^2+7x+10=0$ (b) $x^2-25=0$ $3x^2-15x=0$

It is an advantage, but not essential, to factor completely when solving quadratic equations.

Exercise #6: Find the values of x that solve each of the following quadratic equations.

(a) $5x^2-15x-200=0$ (b) $2x^2-14x+20=0$

Additional Classroom Exercises

Solve and check each of the following quadratic equations.

1. $x^2-12x+35=0$

2. $2x^2-3x=0$

3. $x^2-36=0$

4. $4x^2-8x-96=0$

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The Zero Product Law
Algebra 1 Homework

Skills

1. Which of the following is the solution set to the equation $4(x+3)(x-2)=0$?

   (1) $\{-2,3,4\}$ (3) $\{-3,2\}$

   (2) $\{-12,8\}$ (4) $\{-4,-3,2\}$

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2. Which of the following represents all values of x that solve $x^2+6x-16=0$?

   (1) $\{-8,2\}$ (3) $\{-4,4\}$

   (2) $\{-2,8\}$ (4) $\{-1,16\}$

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3. Which of the following quadratic equations has zero as one of its solutions?

   (1) $x^2-25=0$ (3) $x^2-2x-8=0$

   (2) $x^2-16=0$ (4) $2x^2+10x=0$

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Solve each of the following quadratic equations and check using **STORE** on your calculator.

4. $y^2-3y+2=0$

5. $x^2+6x+5=0$

6. $x^2-9=0$

7. $x^2+12x+32=0$

8. $(x+5)(x-3)=0$

9. $7y^2-35y=0$

10. $x^2-7x+6=0$

11. $10p^2+90p-100=0$

12. $2n^2-98=0$

13. $4t^2-81=0$

Reasoning

14. If the solution set of a certain quadratic equation is $\{-2,5\}$ then which of the following could be the quadratic equation?

(1) $(x-2)(x+5)=0$ (3) $2x(x-5)=0$

(2) $(x-5)(x+2)=0$ (4) $5x(x+2)=0$

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15. Illiya believes he has solved the following quadratic equation correctly.

$x^2+2x-3=12$

$(x+3)(x-1)=12$

$x+3=12$ or $x-1=12$

$x=9$ or $x=13$

$\{9,13\}$

(a) Verify using **STORE** on your calculator that neither of Illiya's two solutions are correct.

(b) What error did Illiya make in his reasoning?