NEW MILFORD PUBLIC SCHOOLS
New Milford, Connecticut

Honors Calculus

June 2011

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New Milford Board of Education

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New Milford's Mission Statement

The mission of the New Milford Public Schools, a collaborative partnership of students, educators, family and community, is to prepare each and every student to compete and excel in an ever-changing world, embrace challenges with vigor, respect and appreciate the worth of every human being, and contribute to society by providing effective instruction and dynamic curriculum, offering a wide range of valuable experiences, and inspiring students to pursue their dreams and aspirations.

Honors Calculus

This course is a study of both differential and integral calculus and some of its applications. This course is for the mathematics students who was not recommended for AP calculus but wants to prepare for mathematics at highly competitive colleges through a rigorous and supportive calculus course. Topics include limits, finding derivatives, applications of derivatives, and an introduction to antiderivatives and definite integrals. A graphing calculator (TI 83+, TI 84+) is required for the course and is used extensively throughout the year.

Common Core State Standards for Mathematics
Mathematics Standards for High School

Standards Key

Number and Quantity

N-RN	The Real Number System
N-Q	Quantities
N-CN	The Complex Number System
N-VM	Vector and Matrix Quantities

Algebra

A-SSE	Seeing Structure in Expressions
A-APR	Arithmetic with Polynomials and Rational Expressions
A-CED	Creating Equations
A-REI	Reasoning with Equations and Inequalities

Functions

F-IF	Interpreting Functions
F-BF	Building Functions
F-LE	Linear, Quadratic, and Exponential Models
F-TF	Trigonometric Functions

Geometry

G-CO	Congruence
G-SRT	Similarity, Right Triangles, and Trigonometry
G-C	Circles
G-GPE	Expressing Geometric Properties with Equations
G-GMD	Geometric Measurement and Dimension
G-MG	Modeling with Geometry

Statistics and Probability

S-ID	Interpreting Categorical and Quantitative Data
S-IC	Making Inferences and Justifying Conclusions
S-CP	Conditional Probability and the Rules of Probability
S-MD	Using Probability to Make Decisions

Pacing Guide

Unit #	Title	Weeks	Pages
1	Functions: Graphs and Limits	8 weeks	7-10
2	Derivatives	15 weeks	11-15
3	Antiderivatives	15 weeks	16-19
	Midterm Review	1 week	
	Final Review	1 week	

New Milford Public Schools

Committee Members: William Knipple & Ellen Tracy Unit 1: Functions, Graphs, and Limits	Course/Subject: Calculus Grade Level: Honors # of Weeks: 8

Identify Desired Results

Common Core State Standards

- **CC 9-12 N-Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **CC 9-12 N-Q.2** Define appropriate quantities for the purpose of descriptive modeling.
- **CC 9-12 A-SSE.1** Interpret expressions that represent a quantity in terms of its context.
- **CC 9-12 F-IF.7**: Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases.
- **CC 9-12 F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Enduring Understandings Generalizations of desired understanding via essential questions (Students will understand that …)	**Essential Questions** Inquiry used to explore generalizations
• Functions can be represented numerically, graphically, and symbolically. • Functions have different properties and characteristics. • Relationships exist between the function graph and the first and the second derivative graphs. • Limits can be found using algebra, graphs, or tables. • Limits can be used to predict the behavior of a function. • Technology helps solve problems and verify answers.	• How can functions be represented numerically, graphically, and symbolically? • What characteristics of the symbolic function tell you the graphical behavior? • How are functions represented numerically, graphically, and symbolically related? • How do first and second derivative graphs relate to the function graph? • How does one calculate a limit using algebra? • How does one determine a limit using graphs or tables? • How does one use technology to understand a function?

Expected Performances
What students should know and be able to do

Students will know the following:

- The differences between functions
- The properties and graphical characteristics of different functions
- The relationship between a function graph, the first derivative, and the second derivative graph
- Local and absolute extremas, points of inflection, concavity, and decreasing/increasing behavior of functions
- Limits of functions algebraically, graphically, and analytically
- Asymptotic behavior graphically and in terms of limits involving infinity
- The definition of continuity or discontinuity of a function at a point of interest
- Left and right side limits
- Limits using substitution, algebra manipulation, graphical and numeric approximations via a graphing calculator
- The relationship between increasing and decreasing behavior of a function and the sign of the first derivative
- The relationship between the concavity of a function and the sign of the second derivative

Students will be able to do the following:

- Recognize different types of functions
- Distinguish between functions via their properties and graphical characteristics
- Analyze a function given the graph of its first derivative or second derivative
- Use the graphs of the first derivative and the second derivative to determine local and absolute extremas, points of inflection, concavity, and decreasing/increasing behavior of functions
- Evaluate limits of functions algebraically, graphically, and analytically
- Describe asymptotic behavior graphically and in terms of limits involving infinity
- Define continuity or discontinuity of a function at a point of interest
- Calculate left and right side limits
- Calculate limits using substitution, algebra manipulation, and graphical and numeric approximations via a graphing calculator
- Understand the relationship between the increasing and decreasing behavior of a function and the sign of its first derivative
- Understand the relationship between the concavity of a function and the sign of its second derivative
- Sketch the function graph given a first and/or second derivative graph(s)
- Appropriately use the graphing calculator

Character Attributes

- Cooperation
- Honesty
- Integrity
- Perseverance
- Respect
- Responsibility

Technology Competency

- Critical Thinking, Problem Solving, and Decision-Making
- Technology Operations and Concepts

Develop Teaching and Learning Plan

Suggested Teaching Strategies:

- Teacher checks for understanding via warm up examples, homework spot checks, and questioning techniques.
- Teacher creates and support cooperative learning groups to enhance learning on topics such as function analysis and curve sketching.
- Teacher models curve sketching using the board and the overhead graphing calculator.
- Teacher provides guided practice during the presentation of limits.
- Teacher leads class discussions on the relationship among functions, graphs, and limits.
- Teacher facilitates small group discussions about moving among the function, first derivative, and second derivative graphs.
- Teacher encourages graphing calculator discovery lessons.
- Teacher uses technology such as a visual presentation to present the idea of a derivative as a slope.

Suggested Learning Activities:

- Students will work independently and as a class analyzing functions.
- Students will use the think-pair-share method of learning to develop a list of rules to use when analyzing graphs.
- Students will work in small groups as peer teachers sketching functions.
- Students will play the Derivative Matching game (Google calculus games).
- Students will play the Limit Wars Card game (Google this title and print the cards).
- Students will verbally explain the process of taking a limit.
- Students will use their calculators to discover characteristics about functions by using the graphing capabilities of the calculator.

Assessments

Performance Task Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	**Other Evidence** Application that is functional in a classroom context to evaluate student achievement of desired results
Goal: Be able to explain a horizontal asymptote in terms of physical constraints **Role**: Park Commissioner **Audience**: The public attending a public hearing **Situation**: Discussion on stocking a lake with fish including what affect that will have on the fish population in the future **Product and Performance**: Oral presentation in which you use graphs to support your mathematical predictions. The predictions should include fish populations in five-year increments as well as a maximum population. **Standards for Success**: Mathematics department scoring rubric	• Monitoring class work • Checking homework • Students working at the board • Cooperative group work • Questioning students • Quizzes (short answer) • Tests (multiple choice and/or short answer)

Suggested Resources

- Textbook: Ostebee, Arnold; Zorn, Paul. *Calculus from Graphical, Numerical, and Symbolic Points of View.* Fort Worth, TX: Harcourt Brace, 1997. Print.
    - Chapter 1 Functions in Calculus
    - Chapter 2 The Derivative
- Supplemental text: Larson, Ron; Hostetler, Robert; Edwards, Bruce. *Calculus of a Single Variable.* 8th ed. Boston, MA: Houghton Mifflin, 2006. Print.
- Supplementary worksheets
- Limit Wars card game (Google calculus games)
- Derivative Matching game (Google this title and print the cards)
- Use Google to search for visual presentations of the derivative
- Graphing calculator

New Milford Public Schools

Committee Members: William Knipple & Ellen Tracy Unit 2: Derivatives	Course/Subject: Calculus Grade Level: Honors # of Weeks: 15

Identify Desired Results

Common Core State Standards

- **CC 9-12 N-RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- **CC 9-12 A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **CC 9-12 A-SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **CC 9-12 F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **CC 9-12 F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **CC 9-12 F-BF.5** (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- **CC 9-12 F-TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **CC 9-12 G-GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Enduring Understandings Generalizations of desired understanding via essential questions (Students will understand that …)	**Essential Questions** Inquiry used to explore generalizations
• Real life situations can be modeled using differential calculus. • The derivative of a function represents the instantaneous rate of change of a function at a given point. • Implicit differentiation can be used to simplify taking the derivative of complicated statements. • Technology helps solve problems and verify answers.	• How is a limit used to define a derivative? • How can a derivative be interpreted as a rate of change both algebraically and geometrically? • How can derivatives help solve differential calculus problems? • When is it appropriate to use implicit differentiation as opposed to explicit differentiation? • How can one use technology to assist in solving differential calculus problems?

Expected Performances
What students should know and be able to do

Students will know the following:

- The definition of a derivative
- The difference between the average rate of change and the instantaneous rate of change
- The relationships between the position, velocity, and acceleration functions
- The basic rules of differentiation including sum, difference, product, quotient, power, and chain rule
- The derivatives of various functions including exponential, logarithmic, and trigonometric
- The slope of the tangent line to a curve at a point is the value of the first derivative at the point
- The first and second derivative tests
- That relative and absolute extrema (optimization) can be used to solve many problems in differential calculus
- That two or more variables can change with respect to time
- The Mean Value Theorem and Rolle's Theorem can be used to solve many problems in differential calculus
- L'Hopital's Rule can be used to take the limits of expressions that yield the indeterminate form
- The difference between explicit and implicit forms of an equation

Students will be able to do the following:

- Calculate the derivative of a function using the limit definition
- Distinguish between the average rate of change and the instantaneous rate of change
- Determine position, velocity, and acceleration given any one of the three functions
- Apply the sum, difference, product, quotient, power, and chain rules to find the derivative of a function
- Determine the derivative of exponential, logarithmic, and trigonometric functions
- Write the equation of a tangent line to a curve at a point
- Use the first and second derivative test to determine relative and absolute extrema, increasing/decreasing behavior, points of inflection, and concavity of a function
- Solve optimizing problems
- Solve related rates problems
- Apply the Mean Value Theorem and Rolle's Theorem to differential calculus problems
- Evaluate limits using L'Hopital's Rule
- Find derivatives implicitly
- Use nDeriv on the graphing calculator

Character Attributes

- Cooperation
- Honesty
- Integrity
- Perseverance
- Respect
- Responsibility

Technology Competencies

- Critical Thinking, Problem Solving, and Decision-Making
- Technology Operations and Concepts

Develop Teaching and Learning Plan

Suggested Teaching Strategies:

- Teacher checks for understanding via warm up examples, homework spot checks, and questioning techniques.
- Teacher creates and supports cooperative learning groups to enhance learning topics such as the Mean Value Theorem and Rolle's Theorem.
- Teacher models average rate of change versus instantaneous rate of change via graphs, tables, and algebraic calculations.
- Teacher provides guided practice on the basic rules of differentiation.
- Teacher leads class discussions on the different derivatives methods.
- Teacher facilitates small group discussions to discover the equation for the tangent line to a curve.
- Teacher encourages graphing calculator discovery lessons.
- Teacher uses technology, such as visual presentations, to demonstrate the relationship among position, velocity, and acceleration.

Suggested Learning Activities:

- Students will work independently and as a class taking derivatives of various functions.
- Students will use the think-pair-share method of learning to discover the first and second derivative tests.
- Students will work in small groups as peer teachers to master the differentiation rules.
- Students will play the Derivative Matching game (Google calculus games).
- Students will play the Limit Wars card game (Google this title and print the cards).
- Students will verbally explain topics L'Hopital's Rule.
- Students will use their calculator to discover characteristics about functions.

Assessments	
Performance Tasks Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	**Other Evidence** Application that is functional in a classroom context to evaluate student achievement of desired results
Performance Task 1 **Goal**: Determine the rate at which an air balloon is losing air **Role**: Engineer **Audience**: Air Balloons Inc. Board of Executives **Situation**: Determine balloon functionality, specifically how long can a balloon safely stay in the air given certain initial conditions? **Product**: Written report that demonstrates through calculus calculations that the balloon is safe to fly **Standards for Success**: Mathematics department scoring rubric **Performance Task 2** **Goal**: Minimize the cost of producing a can **Role**: CEO of the Cans Incorporated Company **Audience**: Stock holders of the Cans Incorporated Company **Situation**: Your charge is to minimize the production cost of a can, thus increasing profits. **Product**: Written report that demonstrates through calculus calculations that the container you designed is of minimum cost **Standards for Success**: Mathematics department scoring rubric	• Monitoring class work • Checking homework • Students working at the board • Cooperative group work • Questioning students • Quizzes (short answer) • Tests (multiple choice and/or short answer)

Suggested Resources

- Textbook: Ostebee, Arnold; Zorn, Paul. *Calculus from Graphical, Numerical, and Symbolic Points of View.* Fort Worth, TX: Harcourt Brace, 1997. Print
    - Chapter 3 Derivatives of Elementary Functions
    - Chapter 4 Applications of the Derivative
- Supplemental text: Larson, Ron; Hostetler, Robert; Edwards, Bruce. *Calculus of a Single Variable.* 8th ed. Boston, MA: Houghton Mifflin, 2006. Print.
- Supplementary worksheets
- Derivative Matching game (Google calculus games)
- Use Google to search for visual presentations of the derivative
- Graphing calculator

New Milford Public Schools

Committee Members: William Knipple & Ellen Tracy Unit 3: Antiderivatives	Course/Subject: Calculus Grade Level: Honors # of Weeks: 15

Identify Desired Results

Common Core State Standards

- **CC 9-12 N-RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- **CC 9-12 A SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **CC 9-12 A-SSE.3a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **CC 9-12 F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **CC 9-12 F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **CC 9-12 F-BF.5** (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- **CC 9-12 F-TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **CC 9-12 G-GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- **CC 9-12 G-GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects.

Enduring Understandings Generalizations of desired understanding via essential questions (Students will understand that ...)	**Essential Questions** Inquiry used to explore generalizations
• The definite integral is a limit of sums. • Antiderivatives and definite integrals are used in a variety of real world applications to model behavior. • Differential and integral calculus together are powerful tools used by mathematicians, scientists, and engineers to solve problems.	• What is the relationship between the integral and the derivative? • How can the definite integral be interpreted as an accumulation function? • What are some of the applications of integral calculus? • How do differential and integral calculus work together to solve problems?

• There can be more than one way to solve a problem. • Technology helps solve problems and verify answers.	• How does one decide what approach to take to solve a problem efficiently? • How can one use technology to assist in solving integral calculus problems?

Expected Performances
What students should know and be able to do

Students will know the following:

- The definition of an antiderivative
- The inverse relationship between exponential and logarithmic functions
- The Trapezoidal Rule and Simpson's Rule
- The different techniques for finding antiderivatives
- The significance of the First Fundamental Theorem of Calculus
- The formula for the average value of a function

Students will be able to do the following:

- Approximate definite integrals by various methods such as Trapezoidal Rule and Simpson's Rule
- Find antiderivatives of various functions using the power rule and u-substitution
- Find the antiderivatives of exponential, logarithmic, and trigonometric functions
- Use the First Fundamental Theorem to evaluate definite integrals and analyze graphical situations
- Distinguish between the average value and the average rate of change of a function
- Find antiderivatives of various functions using the power rule and u-substitution
- Apply integrals to problems involving area under a curve and area between two curves
- Apply the washer, disk, and cylindrical shell methods to find volumes of solids of revolution
- Determine the volumes of solids with known cross sections
- Calculate the arc length of a curve over a specified interval
- Find general and particular solutions to differential equations using integrals
- Apply logarithmic differentiation to functions
- Apply and adapt a variety of appropriate strategies to solve problems with and without graphing calculators
- Use fnInt on the graphing calculator

Character Attributes

- Cooperation
- Honesty
- Integrity
- Perseverance
- Respect
- Responsibility

Technology Competencies

- Critical Thinking, Problem Solving, and Decision-Making
- Technology Operations and Concepts

Develop Teaching and Learning Plan

Suggested Teaching Strategies:

- Teacher checks for understanding via warm up examples, homework spot checks, and questioning techniques.
- Teacher creates and supports cooperative learning groups to enhance learning on topics such as areas and volumes.
- Teacher models the use of a variety of approaches to volumes of revolution problems.
- Teacher provides guided practice when introducing the antiderivative techniques.
- Teacher leads class discussions introducing various methods of taking antiderivatives.
- Teacher facilitates small group discussions about the uses of the First Fundamental Theorem.
- Teacher encourages graphing calculator discovery lessons.
- Teacher uses technology such as visual presentations to demonstrate the area under a curve, the area between two curves, volumes of revolutions, and volumes of cross sections.

Suggested Learning Activities:

- Students will work independently and as a class to take derivatives of various functions.
- Students will use the think-pair-share method of learning to apply the Trapezoidal Rule and Simpson's Rule.
- Students will work in small groups as peer teachers to master antiderivative rules.
- Students will play the game "Who Wants to Be a Millionaire?" calculus version (Google calculus games).
- Students will verbally explain the difference between vertical shells and horizontal shells.
- Students will use their calculator to discover characteristics about functions.

Assessments

Performance Task Authentic application to evaluate student achievement of desired results designed according to GRASPS (one per marking period)	**Other Evidence** Application that is functional in a classroom context to evaluate student achievement of desired results
Goal: Determine whether or not a traffic light is needed at an intersection **Role**: OSHA representative **Audience**: Town officials **Situation**: Given traffic flow at an intersection and the standards for light placement, you must determine if a traffic light is warranted. **Product**: Written report to town officials that uses calculus calculations to support your conclusion as to whether a traffic light is necessary **Standards for Success**: Mathematics department scoring rubric	• Monitoring class work • Checking homework • Students working at the board • Cooperative group work • Questioning students • Quizzes (short answer) • Tests (multiple choice and/or short answer)

Suggested Resources

- Textbook: Ostebee, Arnold; Zorn, Paul. *Calculus from Graphical, Numerical, and Symbolic Points of View.* Fort Worth, TX: Harcourt Brace, 1997. Print
    - Chapter 5 The Integral
    - Chapter 6 Finding Antiderivatives
    - Chapter 7 Numerical Integration
    - Chapter 8 Using the Definite Integral
- Supplemental text: Larson, Ron; Hostetler, Robert; Edwards, Bruce. *Calculus of a Single Variable.* 8th ed. Boston, MA: Houghton Mifflin, 2006. Print.
- Supplementary worksheets
- Use Google to search for visual presentations of the Trapezoidal Rule, Simpson's Rule, area under a curve, area between curves, volumes of rotations, and volumes of cross sections
- Graphing calculator