

Optimization Problems

Optimization typically involves finding the maximum or minimum of some quantity:

- Decide on what variable is to be optimized.
- Create a function in which that variable is in terms of just one other variable.
- Find the critical points of the function.
- Find the absolute maximum or minimum value (as appropriate).

Example 1 (maximizing): A rectangular sheet of metal 20 in X 30 in will have small squares cut out of each corner and then discarded. The four “sides” are then folded up (and the seams welded) so as to form a lidless rectangular pan for cooking cane syrup. What size squares should be cut out of each corner so as to maximize the volume of the syrup pan? What is the maximum volume?

Example 2 (minimizing): Find two positive numbers whose product is 220 and whose sum is as small as possible.

Example 3 (minimizing): Find two positive numbers whose sum is 10. The sum of their cubes is to be a minimum.

Example 4 (minimizing): What is the shortest distance from $(2, 0)$ to the function $f(x) = x^2$

Example 5 (minimizing): Consider the parabola $f(x) = x^2 - 1$. What is (are) the point(s) on the parabola closest to the point $(0, 3)$?

Example 6 (maximizing): What are the dimensions of a rectangle giving the maximum area when the length must be 3 times the width and the perimeter is 100 feet?

Example 7(minimizing): A construction company is given a contract to build a pipeline from point A on the bank of river to point B on the opposite bank and 2 miles downstream. If the river is .75 miles wide, what is the cheapest construction cost to build the pipeline if it costs 1 million dollars per mile for a pipeline built on dry land and 2 million dollars for each mile in water?