A Trajectory of Learning

A Trajectory of Learning for Numbers and Operations in Base Ten
- Understanding and Reasoning with Whole Numbers -

- Recognizes and reads whole numbers
- Identifies and represents magnitude of each digit in a whole number
- Identifies 0-9 pattern within and across all place value positions
- Works with units of 10 and relates the magnitude of each place value position to the magnitude of place value to the right (e.g., 1 ten is made of 10 ones, 100 is 10x greater than 10)
- Compares and orders whole numbers using reasoning related to place value and/or magnitude
- Performs operations using reasoning and strategies related to place value and/or magnitude
  - Addition
  - Subtraction
  - Multiplication
  - Division

A Trajectory of Learning for Numbers and Operations in Base Ten
- Understanding and Reasoning with Decimal Numbers -

- Recognizes and reads decimal numbers
- Represents the magnitude of a positive number less than 1
- Relates the magnitude of each place value position to the magnitude of place value to the left (e.g., 1 ÷ 10 is .10)
- Compares and orders decimal numbers using reasoning related to place value and/or magnitude
- Performs operations using reasoning and strategies related to place value and/or magnitude
  - Additon
  - Subtraction
  - Multiplication
  - Division
A Trajectory of Learning for Addition and Subtraction

1. Add to and Put Together Situational Problems
   - Commutative Property
   - Represents addition by:
     - using manipulatives
     - in a diagram
     - in a number line model
   - Recognizes the language add to, more, altogether, some more, part
   - States the meaning of the addends and the sum

2. Develop Multiple Reliable Ways of Solving Addition Problems
   - Uses mapping devices to solve problems
   - Understand equivalent representations and expressions

3. Add to and Put Together, Change Unknown and Addend Unknown Situational Problems
   - Develops multiple reliable ways of solving addition and missing addend problems
   - Understands equivalent expressions and representations

4. Develop Multiple Reliable Ways of Solving Addition and Missing Addend Problems
   - Identifies and uses the symbols +, -, =, >, <

5. Commutative Property, Associative Properties, and Fact Strategies
   - Recognizes the relationship between addition and subtraction as "the doing and undoing" of a representation

6. Take From and Take Apart Situational Problems
   - Recognizes two disjoint sets with equal groups can be compared with each other to determine how many times greater, more or fewer one set is than another

7. Compare Situational Word Problems
   - Recognizes the meaning of the addends, the sum, the difference, subtrahend and minuend
A Trajectory of Learning

A Trajectory of Learning for Multiplication and Division Relationships

- Equal Group Multiplication - Unknown Product
  - Represents multiplication by:
    - using manipulatives
    - in a diagram
    - in a number line model
  - Recognizes the language per, for each, for every
  - States the meaning of the factors
  - Identifies and uses the symbol x

- Array/Area Model - Unknown Product
  - Represents multiplication by:
    - using manipulatives
  - Understands equivalent representations and expressions

- The Commutative Property
  - Identifies and uses the symbols ÷ and /

- The Distributive and Associative Properties
  - Recognizes the relationship between multiplication and division as “the doing and undoing” of a representation

- Equal Group and Array/Area Multiplication and Division - Unknown Product - Unknown Factor
  - Represents division by:
    - using manipulatives
    - a grid with square units
    - a number line model
  - Recognizes two disjoint sets with equal groups can be compared with each other to determine how many times greater, more or fewer one set is than another

- Comparison Multiplication
  - How many times more?
  - How many times fewer?

- Equal Group Division
  - How many in the groups?
  - And
  - How many groups?

- Measurement Division
  - How many in the groups?
  - And
  - How many groups?

- Recognizes equal iterations within a whole amount
A Trajectory of Learning for Fractions

1. Recognize fractions as numbers
   - Recognize the meaning of the numerator and the denominator
   - Name fractions
   - Represent fractions

2. Compare fractions
   - Understand equivalence
   - Understand that like-sized pieces can be added and subtracted

3. Add and subtract fractions with like denominators
   - Use repeated addition to solve multiplication problems

4. Multiply a fraction by a whole number
   - Understand taking a portion of an amount
   - Recognize that all pieces need to be of equal size before adding/subtracting

5. Add and subtract fractions with unlike denominators
   - Determine how many of a unit fraction are contained in a whole number

6. Multiply a fraction by a fraction
   - Determine how to partition a fraction into further fractional parts

7. Divide with fractions