Applications of Logarithms and Exponentials

We can model logarithmic and exponential equations to model various situations in the real world including growth, decay, and finance.

Compound interest is calculated with:

$$A(t) = P(1 + r/n)^{nt},$$

or

$$A(t) = Pe^{rt} \text{ (continuously compounded)}$$

- A(t) is the amount accumulated after *t* years
- P = principal (initial amount)
- r = interest rate
- n = number of times interest is compounded per year

Example 1: Determine the final balance after 5 years for a $2000 investment that earns interest at the rate of 5.5% compounded monthly.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 2000\left(1 + \frac{.055}{12}\right)^{12 \cdot 5}$$

Final Amount is

$2631.41

Example 2: How much money will accumulate in a savings account after 20 years if \$1000 is initially invested at 6% interest? Interest is compounded semi-annually.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 1000\left(1 + \frac{.06}{2}\right)^{2 \cdot 20}$$

Final Amount is

\$3262.04

Example 3: Use the data from example 2 to determine how much money will be earned by compounding interest continuously.

$$A = Pe^{rt}$$

$$A = 1000e^{.06 \cdot 20}$$

Final Amount is \$3320.12

Example 4: How long will it take for a $1000 investment to turn into $2000 if interest of 5% is compounded monthly?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{2000}{1000} = \frac{1000\left(1 + \frac{.05}{12}\right)^{12t}}{1000}$$

$$2 = \left(1 + \frac{.05}{12}\right)^{12t}$$

$$\frac{\ln 2}{12\ln\left(1+\frac{.05}{12}\right)} = \frac{12t\ln\left(1 + \frac{.05}{12}\right)}{12\ln\left(1+\frac{.05}{12}\right)}$$

so $t = 13.9$ years

Example 5: What is the rate at which an investment of \$1200 will turn a final balance of \$2700 if compounded continuously for 6 years?

$$A = Pe^{rt}$$

$$\frac{2700}{1200} = \frac{\cancel{1200}e^{6r}}{\cancel{1200}}$$

$$\ln 2.25 = \cancel{\ln e}^{6r}$$

$$\ln 2.25 = 6r$$

$$r = \frac{\ln 2.25}{6}$$

$$r \approx .135 \text{ or } 13.5\%$$

Example 6: The decay formula for radium is modeled by $A(t) = A_o e^{-.000428t}$

Initial

Determine the amount of substance left after 500 years with an initial sample of 100 grams.

$$A = 100e^{-.000428 \cdot 500}$$

Final Amount is $\underline{80.78}$ grams

Example 7: The Ebbinghaus model of human memory gives the percent p of acquired knowledge that a person retains after t weeks. The formula is $p = (100 - a)e^{-bt} + a$, where a and b vary from one person to another. If a = 18 and b = 0.6 for a certain student, how much information will the student retain two weeks after learning a topic?

$$p = (100 - a)e^{-bt} + a$$

$$p = (100 - 18)e^{-.6 \cdot 2} + 18$$

$$p = 42.7\%$$

Example 8: The formula is $p = (100 - a)e^{-bt} + a$, where a and b vary from one person to another. If a = 18 and b = 0.6 for a certain student, how much time has past if the student has retained 60% of the material after learning a topic?

$$60 = (100 - 18)e^{-.6t} + 18$$

$$\frac{42}{82} = \frac{\cancel{82}e^{-.6t}}{\cancel{82}}$$

$$.51 = e^{-.6t}$$

$$\ln .51 = \cancel{\ln e}^{-.6t}$$

$$\frac{\ln .51}{-.6} = \frac{\cancel{-.6}t}{\cancel{-.6}}$$

$$t = 1.12$$

weeks

Example 9: An old piece of wood is found to contain only 42% of the radioactive carbon-14 that would have been present when the tree it came from was alive. When did the tree die? The half life of carbon-14 is 5730 years (the time it takes for half of the original amount to decay).