CHAPTER 2

Measurements and Calculations

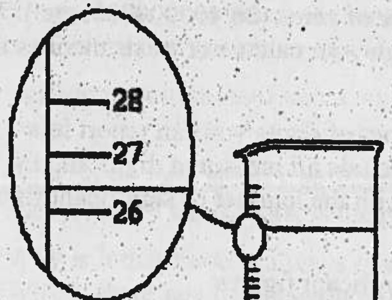
INTRODUCTION

Chemistry is a science that requires observation of the world around us and measurements of the phenomena we observe. In this chapter you will learn how to record your observations and how to perform calculations with measured values. Scientific measurements are usually made using the metric system or the International System. You will need to become familiar with these systems of measurements and know the magnitude of each of the major units.

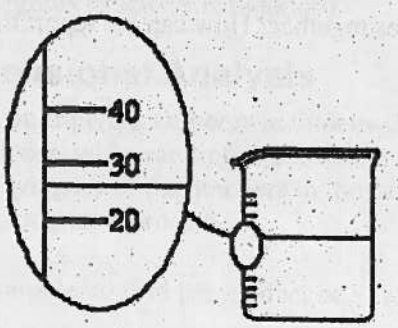
CHAPTER DISCUSSION

Significant Figures

Measurement is an important part of science, and an understanding of uncertainty is an important part of measurement. Science is often thought of (incorrectly) as a body of unchanging absolute truths, which makes the concept of uncertainty seem odd. But you should realize that uncertainty is always a factor in any measurement except for exact counting. For measurements you will be taking in the lab, there is always one (and only one) uncertain digit that we can reasonably estimate. Imagine, for example, measuring water in a beaker as shown below.

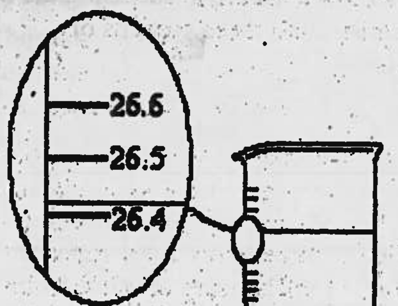


Using this beaker, we know there is more than 26 mL of water and less than 27 mL of water. To report "26 mL" or "27 mL" would be imprecise. Now imagine if we used a beaker as shown below.



In this case we would report an answer to the ones place. In this case, the water appears just over the halfway point between "20" and "30," so "26" is a reasonable estimate. Note that we would not call this an exact measurement. The actual amount of water may be 25 mL or even 27 mL. Unless the glassware is marked, we generally assume our uncertainty is ±1 for the digit that we estimate.

Look back to the first beaker. We can make a reasonable estimate of the tenths place in this case. The water level appears to be just under halfway between the two graduations, so we might report 26.4 mL. In this case, we can assume that the actual amount of water is between 26.3 and 26.5 mL. Therefore we cannot report an answer of 26.42 mL since this would imply we knew the volume was between 26.41 and 26.43 mL (again, this assumes the glassware does not have a precision associated with it). What if we wanted to measure water to the hundredths place? This would require glassware with graduations as shown in the beaker below.



In this case, 26.42 mL is a reasonable estimate for the volume of water although you might think it is 26.41 or 26.43. Again, realize that we never get an exact measurement. Even if the water level seemed to be right on the 26.4 graduation, we would report 26.40 mL, but we cannot report "exactly 26.4 mL." Reporting "exactly 26.4 mL" implies 26.4 with an infinite number of zeros (26.400000000, etc.). Thus, 26.42 is not the same as 26.420 in terms of measurement (although your calculator treats them as the same). The only way to get an exact number is to count it.

Thus the glassware determines the precision that affects the number of digits you can report in a measurement. These digits are the significant figures, and they include all measured digits and the one estimated digit. Our three measurements in this example, along with the number of significant figures, are:

Beaker 1	26.2 mL	three significant figures
Beaker 2	26 mL	two significant figures
Beaker 3	26.42 mL	four significant figures

Now what happens if we add the water from each of these figures together? How can we report the results? Mathematically, we have:

$$\begin{array}{r} 26.4\ \text{mL} \\ 26\ \ \ \ \text{mL} \\ + \\ \underline{26.42\ \text{mL}} \\ 78.82\ \text{mL} \end{array}$$

However, we should realize that we have some uncertain digits. That is, the above procedure implies that the first measurement is 26.40, and the second is 26.00. However, this is simply not true. A better representation for this addition is

$$\begin{array}{r} 26.4?\text{ mL} \\ 26.??\text{ mL} \\ + \\ \underline{26.42\text{ mL}} \\ 78.??\text{ mL} \end{array}$$

Note in the hundredths we are adding a 2 to two unknown digits. What is "2 + ? + ?"? The answer has to be, "We don't know"! In this case, we know the sum only to the ones place, so we can only report it as such. So do we report it as 78? 79? Because the sum of the numbers is 78.82, we round up to 79. We can also justify this by recalling the uncertainty of the numbers. Let's assume two extreme cases. In the first case, assume we estimated too high for all three measurements (that is, assume there was actually less water than we thought). In the second case, assume we estimated too low for all three measurements. Remember that we assume we can be off by ±1 in the last digit. The range for the total amount of water in each case is shown below:

$$\begin{array}{r} 26.3\text{ mL} \\ 25\ \ \ \ \text{ mL} \\ +\underline{26.41\text{ mL}} \\ 77.71\text{ mL} \end{array} \qquad \begin{array}{r} 26.5\text{ mL} \\ 27\ \ \ \ \text{ mL} \\ +\underline{26.43\text{ mL}} \\ 79.93\text{ mL} \end{array}$$

The maximum range of volume should be between 77.71 mL and 79.93 mL. Since we can report the answer only to the ones place, the range should be between 78 mL and 80. mL. Therefore a reported answer of 79 mL (with a range of ±1) is reasonable.

After studying about measurement and significant figures, you should be able to answer the following questions:

1. Why do we care about significant figures? What is the point of determining which figures are significant? That is, what is the practical application?
2. Why is it that there is always one uncertain digit? Why can't we just measure more accurately? Why is there just one uncertain digit in the reported answer?
3. Make sense of the rules for which zeros are significant. Be able to explain them (not just recite) to a classmate or instructor. (One way to understand these is to relate the concept of significant figures to scientific notation).

Dimensional Analysis

When multiplying numbers in dimensional analysis, we are really just multiplying fractions. Remember, when multiplying fractions, multiply all of the numbers in the numerator first, followed by multiplying all of the numbers in the denominator. The last step is to divide the numerator product by the denominator product.

For example, to find the product of $\frac{1}{3}$ and $\frac{2}{5}$ we can write the expression in one of two ways:

$\frac{1}{3} \times \frac{2}{5}$ or $\left(\frac{1}{3}\right)\left(\frac{2}{5}\right)$

We then solve the problem using the following method:

$$\left(\frac{1}{3}\right)\left(\frac{2}{5}\right)=\frac{(1\times 2)}{(3\times 5)}=\frac{2}{15}=0.133$$

Whenever we see the same number in both the numerator and denominator, they cancel out (to equal 1).

$$\left(\frac{1}{\cancel{3}}\right)\left(\frac{\cancel{3}}{5}\right)=\frac{1}{5}=0.20$$

If the number we are analyzing is a whole number, remember that this really means that the number is over 1 (whole number in the numerator, 1 in the denominator). For example, the number 4 really means $\frac{4}{1}$.

When multiplying units, use the same principle that you use for multiplying fractions. If one unit is in the numerator, and the identical unit is in the denominator, they cancel each other out (and ultimately equal 1). Any remaining units are evaluated for the answer.

$$\left(\frac{\cancel{centimeter}}{1}\right)\left(\frac{meter}{\cancel{centimeter}}\right)=\frac{meter}{1}=meter$$

You can also multiply several units together at once using the same principle as for fractions.

$$\left(\frac{\cancel{centimeter}}{second}\right)\left(\frac{\cancel{meter}}{\cancel{centimeter}}\right)\left(\frac{\cancel{kilometer}}{\cancel{meter}}\right)\left(\frac{megameter}{\cancel{kilometer}}\right)=\frac{megameter}{second}$$

It is very important to note that if a unit appears once in the numerator but more than once in the denominator, we can cancel out only one of the unit expressions in the denominator. Think of this concept in terms of fractions. If there were the number 4 in the numerator and two 4's in the denominator of different fractions, we would cancel out only one of the 4's on the bottom, not both.

$$\left(\frac{\cancel{4}}{5}\right)\left(\frac{3}{\cancel{4}}\right)\left(\frac{1}{4}\right)=\frac{(3\times 1)}{(5\times 4)}=\frac{3}{20}=0.15$$

Let's look at an example with units. Consider multiplying the following units.

$$\left(\frac{kilogram}{second}\right)^2\left(\frac{meter}{kilogram}\right)^2\left(\frac{second}{meter}\right)=$$

The squared factor is equivalent to multiplying the fraction by itself.

$$\left(\frac{kilogram}{second}\right)\left(\frac{kilogram}{second}\right)\left(\frac{meter}{kilogram}\right)\left(\frac{meter}{kilogram}\right)\left(\frac{second}{meter}\right)=$$

Now we can evaluate the expression by canceling out units.

$$\left(\frac{\cancel{kilogram}}{second}\right)\left(\frac{\cancel{kilogram}}{\cancel{second}}\right)\left(\frac{meter}{\cancel{kilogram}}\right)\left(\frac{\cancel{meter}}{\cancel{kilogram}}\right)\left(\frac{\cancel{second}}{\cancel{meter}}\right)=\frac{meter}{second}$$

A Warning about Dimensional Analysis

Dimensional analysis is a double-edged sword. It is extremely useful and quite dangerous. It is dangerous because it can allow you to solve problems you do not understand. For example, consider the following problem:

> There are 2 igals in 1 odonku, and 6 odonkus in 4 falgers. If you have 3 igals, how many falgers is this?

We can solve this simply using dimensional analysis:

$$3 \text{ igals} \times \left(\frac{1 \text{ odonku}}{2 \text{ igals}}\right) \times \left(\frac{4 \text{ falgers}}{6 \text{ odonkus}}\right) = 1 \text{ falger}$$

Therefore, the answer is 1 falger. The questions to ask are "What is an igal? ," "What is an odonku?," "What is a falger? ," "What is the point of this problem?". Even though you can solve this problem, it is absolutely meaningless. And this is something you want to avoid in a chemistry course–solving problems without understanding them. Even if you can do this on some occasions, many of the problems in chemistry require an understanding of underlying principles, and it is good practice to start understanding early on. Dimensional analysis is a good tool for unit conversion, but you should never use it to try to replace understanding a problem.

LEARNING REVIEW

1. To express each of the following numbers in scientific notation, would you move the decimal point to the right or to the left? Would the power of 10 be positive or would it be negative (have a minus sign)?

   a. 0.001362

   b. 146,218

   c. 342.016

   d. 0.986

   e. 18.8

2. Complete the table below, and convert the numbers to scientific notation.

		Coefficient		Exponent
a.	0.00602	6.02	×	________
b.	60,000	6	×	________
c.	49	________	×	10^1
d.	1.002	1.002	×	________

3. Convert the numbers below to scientific notation.

   a. 1,999,945

   b. 650,700

   c. 0.1109

   d. 545

   e. 0.0068

   f. 0.042001

g. 1.2

h. 13.921

4. To express the following numbers in decimal notation, would you move the decimal point to the right or to the left? How many places?

a. 1.02×10^{3}

b. 4.1×10^{-6}

c. 5×10^{5}

d. 4.31×10^{2}

e. 9.31×10^{-2}

5. Convert the numbers below to decimal notation.

a. 4.91×10^{10}

b. 5.42×10^{-6}

c. 2.07×10^{3}

d. 1.009×10^{-4}

e. 9.2×10^{1}

f. 4.395×10^{5}

g. 7.03×10^{-2}

6. How can you convert −1235.1 to scientific notation?

7. Which quantity in each pair is larger?

a. 1 meter or ~~1 milliliter~~ 1 millimeter

b. 10 seconds or 1 microsecond

~~c. 1 centimeter or 1 millimeter~~

d. 1 kilogram or 1 decigram

8. Which quantity in each pair is larger?

a. 1 mile or 1 kilometer

b. 1 liter or 1 cubic meter

c. 1 kilogram or 1 pound

d. 1 quart or 1 milliliter

e. 1 micrometer or 12 inches

9. What metric or SI unit would you be likely to use in place of the English units given below?

a. Bathroom scales commonly provide weight in pounds.

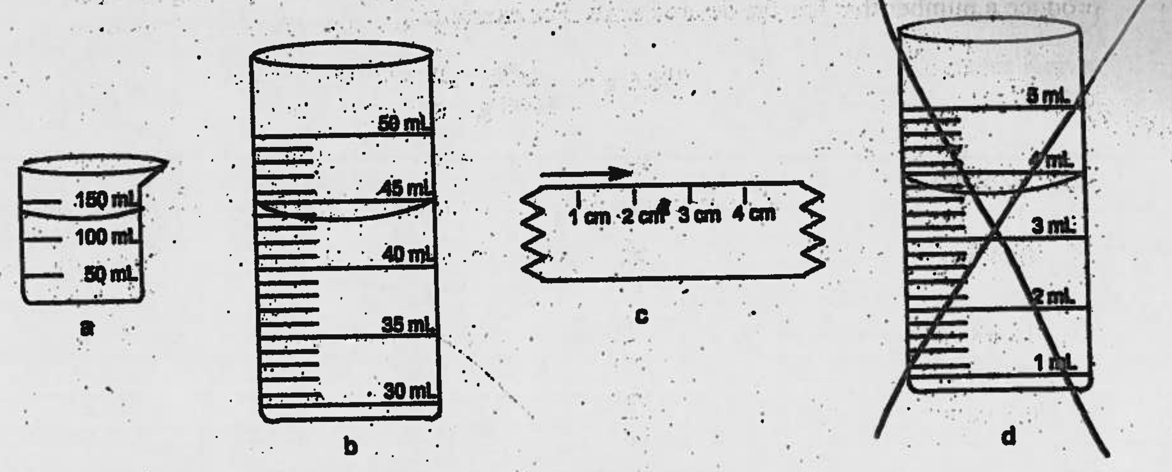
b. A convenient way to purchase small quantities of milk is by the quart.

c. A cheesecake recipe calls for 1 teaspoon of vanilla extract.

d. Carpeting is usually priced by the square yard.

e. "An ounce of prevention is worth a pound of cure."

10. What number would you record for each of the following measurements?



11. How many significant figures are in each of the following numbers?

a. 100

b. 1180.3

c. 0.00198

d. 1.001

e. 67,342

f. 0.0103

g. 4.10×10^4

12. Express the results of each calculation to the correct number of significant figures.

a. 1.8×2.93

b. 0.002/0.041

c. 0.00031×4.030

d. 495.0/390

e. 5024×19.2

f. $91.3 \times 2.10 \times 7.7$

g. $8.003 \times 4.93/61.05$

13. Round off the following numbers to the number of significant figures indicated.

	Number	Number of Significant Figures
a.	0.58333333	four
b.	451.0324	three
c.	942.359	four
d.	0.0090060	two
e.	6.8	one
f.	1346	three
g.	490,000.423	six
h.	0.06295	three

14. For each of the quantities below, give a conversion factor that will cancel the given units and produce a number that has the desired units. For example:

$$8.6\,\cancel{g} \times \frac{1\,kg}{1000\,\cancel{g}} =$$

a. 10.6 m × $\frac{cm}{m}$

b. 0.98 L × $\frac{qt}{L}$

c. 18.98 cm × $\frac{in}{cm}$

d. 0.5 yd × $\frac{m}{yd}$

e. 25.6 kg × $\frac{lb}{kg}$

15. Perform the following conversions:

a. 5.43 kg to g

b. 65.5 in to cm

c. 0.62 L to ft^3

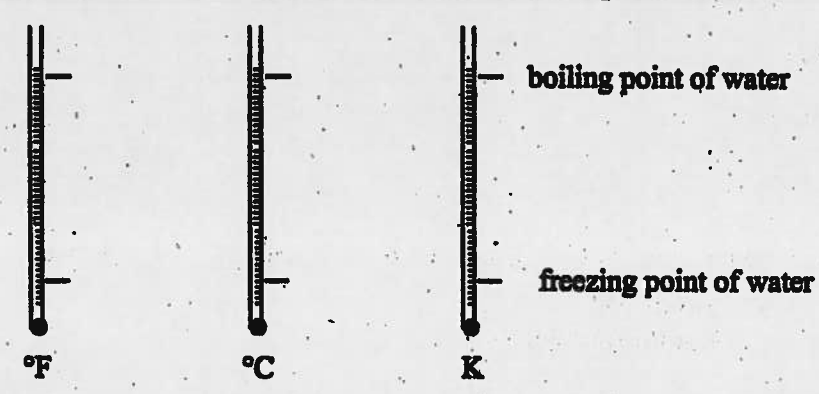
d. 111.3 g to lb

e. 40.0 qt to L

f. 2.83 g to lb

g. 0.21 cm to in

16. Fill in the important reference temperature on each of the temperature scales.



17. How many degrees are there between the freezing point and the boiling point of water on the Fahrenheit and on the Celsius scales?

Also:

a. Calculate the ratio of the number of degrees Fahrenheit to the number of degrees Celsius between the freezing and boiling points of water.

b. Calculate the ratio of the number of degrees Celsius to the number of degrees Kelvin between the freezing and boiling points of water.

c. Calculate the ratio of the number of degrees Fahrenheit to the number of degrees Kelvin between the freezing and boiling points of water.

18. Comfortable room temperature for houses is 75 °F. What is this on the Celsius scale?

19. Ethyl alcohol boils at 78.0 °C. What is this on the Fahrenheit scale?

20. In some parts of the Midwest, temperatures may drop as low as −22 °F in winter. What is this on the Kelvin scale?

21. Perform the temperature conversions below.

   a. 180 °F to °C
   b. −10.8 °C to K
   c. 244 K to °C
   d. 25.1 °F to °C

22. Fill in the missing quantities in the table below.

Substance	Density (g/mL)	Mass	Volume
seawater	1.025	52.6 g	________
diamond	________	2.13 g	0.65 mL
beeswax	0.96	125.5 g	________
oak wood	________	4.63 g	~~6173.3 mL~~ 6.1733 mL

ANSWERS TO LEARNING REVIEW

1. To convert to scientific notation for numbers that are greater than zero but less than one, move the decimal point to the *right*. For numbers that are greater than one, move the decimal point to the *left*. Make sure your final answer has only one number to the left of the decimal point.

   a. right 0.001362 power of ten: negative
   b. left 146218 power of ten: positive
   c. left 342.016 power of ten: positive
   d. right 0.986 power of ten: negative
   e. left 18.8 power of ten: positive

2. Remember that numbers written in scientific notation are divided into two parts. The coefficient on the left is a small number between one and ten, and the exponent on the right is ten raised to some power.

		Coefficient		Exponent
a.	0.00602	6.02	×	10^{-3}
b.	60,000	6	×	10^{4}
c.	49	4.9	×	10^{1}
d.	1.002	1.002	×	10^{0}

3. The answer for g, 1.2×10^0, means that we do not need to move the decimal point of the coefficient. 1.2×10^0 is the same as writing 1.2.

   a. 1.999945×10^6

   b. 6.507×10^5

   c. 1.109×10^{-1}

   d. 5.45×10^2

   e. 6.8×10^{-3}

   f. 4.2001×10^{-2}

   g. 1.2×10^0

   h. 1.3921×10^1

4. When converting from scientific notation to decimal, look first at the exponent. If the exponent is positive (has no negative sign), move the decimal point to the right. If the exponent is negative, move the decimal point to the left.

   a. right 1020

   b. left 0.0000041

   c. right 500,000

   d. right 431

   e. left 0.0931

5. A large number such as 49,100,000,000 has only three significant figures. The trailing zeros are not significant because there is no decimal point at the end.

   a. 49,100,000,000

   b. 0.00000542

   c. 2070

   d. 0.0001009

   e. 92

   f. 439,500

   g. 0.0703

6. This number is different from others we have seen. It is smaller than one and also smaller than zero. You can convert these numbers to scientific notation in much the same way as you convert numbers that are greater than one. First move the decimal point to the left as you normally would.

$$-1235.1$$

Then count the number of times the decimal point was moved, and add the correct exponent.

$$1.2351 \times 10^3$$

Just keep the minus sign in front of the entire number.

$$-1.2351 \times 10^3$$

The minus sign goes in front of 1.235 because this number is less than zero. The exponent is negative only for numbers that are between zero and one.

7. To work this problem you need to have learned the SI prefixes and how they modify the size of the base unit.

   a. A meter is larger than a millimeter.

   b. 10 seconds is larger than 1 microsecond.

   c. ~~1 Mm is larger than 1 cm.~~

   d. 1 kilogram is larger than 1 decigram.

8. This problem asks about the relationship between English units and SI units. You need to know the relative sizes of English and SI units.

   a. 1 mile is larger than 1 kilometer.

   b. 1 cubic meter is larger than 1 liter.

   c. 1 kilogram is larger than 1 pound.

   d. 1 quart is larger than 1 milliliter.

   e. 12 inches is larger than 1 micrometer.

9.

   a. kilograms

   b. liter

   c. milliliter

   d. square meter (m^2)

   e. "A gram of prevention is worth a kilogram of cure."

10.

   a. This measuring device is a beaker. Each division represents 50 mL. The volume of liquid in the beaker is somewhere between 100 mL and 150 mL. We estimate that the volume is 120 mL.

   b. This measuring device is a graduated cylinder. The numbers tell us that each major graduation is 5 mL, so each of the smaller lines must be 1 mL. We can accurately measure the volume to the nearest 1 mL. The volume in this cylinder is between 43 and 44 mL. We estimate the volume to be 43.5 mL.

   c. The length of the arrow lies between 1 cm and 2 cm. We estimate that the arrow lies 0.9 of the way between the two marks. So the reported measurement would be 1.9 cm.

   d. This graduated cylinder has major divisions of 1 mL. The smaller marks represent 0.2 mL. The liquid lies between 3.6 and 3.8 mL. We estimate that the volume is about a quarter (0.05) of the way between the two marks, so the volume would be reported as 3.65 mL.

11. Remember that all nonzero numbers count as significant figures, and zeros in the middle of a number are always significant. Zeros to the right of some nonzero numbers are significant only if they are followed by a decimal point.

   a. 1

   b. 5

c. 3

d. 4

e. 5

f. 3

g. 3

12. For problems involving multiplication and division, your answer should have the same number of decimal points as the measurement with the least number of significant figures. For problems involving addition and subtraction, your answer should have the same number of significant figures as the measurement with the least number of digits to the right of the decimal point.

a. 5.3

b. 0.05

c. 0.0012

d. 1.3

e. 96,500

f. 1500

g. 0.646

13. You can answer problems such as 13.g by putting the decimal point at the end to show that all six digits are significant or use scientific notation with a coefficient that contains six digits.

a. 0.5833

b. 451

c. 942.4

d. 0.0090

e. 7

f. 1350

g. 490,000. or 4.90000×10^5

h. 0.0630

14. To answer this question, you need to know the common equivalencies and how to write them as a unit factor.

a. $10.6\text{ m} \times \dfrac{100\text{ cm}}{1\text{ m}}$

b. $0.98\text{ L} \times \dfrac{1.06\text{ qt}}{1\text{ L}}$

c. $18.98\text{ cm} \times \dfrac{1\text{ in}}{2.54\text{ cm}}$

d. $0.5\text{ yd} \times \dfrac{1\text{ m}}{1.094\text{ yd}}$

e. $25.6 \text{ kg} \times \frac{1000 \text{ g}}{1 \text{ Kg}}$

15.

a. $5.43 \text{ kg} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 5430 \text{ g}$

b. $65.5 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = 166 \text{ cm}$

c. $0.62 \text{ L} \times \frac{1 \text{ ft}^3}{28.32 \text{ L}} = 0.022 \text{ ft}^3$

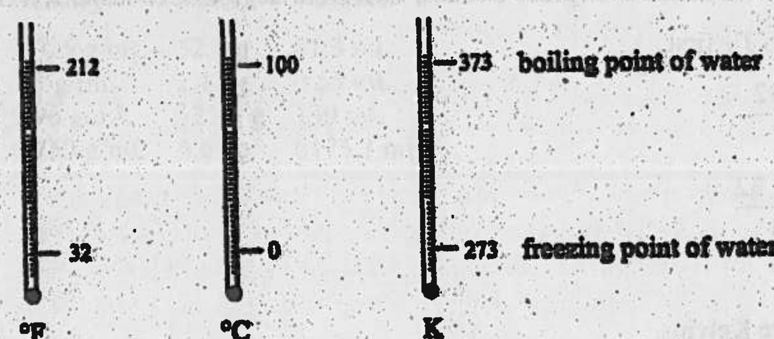
d. $111.3 \text{ g} \times \frac{1 \text{ lb}}{453.6 \text{ g}} = 0.2454 \text{ lb}$

e. $40.0 \text{ qt} \times \frac{1 \text{ L}}{1.06 \text{ qt}} = 38 \text{ L}$ 37.9 L

f. $2.83 \text{ g} \times \frac{1 \text{ lb}}{453.6 \text{ g}} = 6.24 \times 10^{-3} \text{ lb}$

g. $0.21 \text{ cm} \times \frac{1 \text{ in}}{2.54 \text{ cm}} = 0.083 \text{ in}$

16.



17. There are 180 degrees between the freezing and boiling points of water on the Fahrenheit scale and 100 degrees on the Celsius scale.

a. $\frac{°F}{°C} = \frac{180}{100} = 1.80$

b. $\frac{°C}{K} = \frac{100}{100} = 1$

c. $\frac{°F}{K} = \frac{180}{100} = 1.80$

18. We want to convert from degrees Fahrenheit to degrees Celsius.

$T_{°F} = 75$

We can use the formula below to calculate degrees Celsius.

$$T_{°C} = \frac{T_{°F} - 32}{1.80} = \frac{75 - 32}{1.80}$$

$T_{°C} = 24$

75 degrees Fahrenheit is equivalent to 24 degrees Celsius.

19. We want to convert from degrees Celsius to degrees Fahrenheit.

$T_{°C} = 78.0$

We can use the formula below to calculate degrees Fahrenheit.

$T_{°F} = 1.80\,(T_{°C}) + 32$

$T_{°F} = 1.80(78.0) + 32$

$T_{°F} = 172$

78.0 degrees Celsius is equivalent to 172 degrees Fahrenheit.

20. We want to convert from degrees Fahrenheit to Kelvin.

$T_{°F} = -22$

We do not have a formula to directly convert degrees Fahrenheit to Kelvins, but we can convert from degrees Fahrenheit to degrees Celsius, then from degrees Celsius to Kelvins.

Convert $T_{°F}$ to $T_{°C}$ first.

$$T_{°C} = \frac{T_{°F} - 32}{1.80}$$

$$T_{°C} = \frac{-22 - 32}{1.80}$$

$T_{°C} = -30.$

Now, calculate Kelvins.

$T_K = T_{°C} + 273$

$T_K = -30. + 273$

$T_K = 243$

21.

a. $T_{°F} = 180$

$$T_{°C} = \frac{T_{°F} - 32}{1.80}$$

$$T_{°C} = \frac{180 - 32}{1.80}$$

$T_{°C} = 82$

b. $T_{°C} = -10.8$

$T_K = T_{°C} + 273$

$T_K = -10.8 + 273$

$T_K = 262$

c. $T_K = 244$

$T_K = T_{°C} + 273$

Rearrange this equation to isolate $T_{°C}$.

$T_{°C} = T_K - 273$

$T_{°C} = 244 - 273$

$T_{°C} = -29$

d. $T_{°F} = 25.1$

$$T_{°C} = \frac{T_{°F} - 32}{1.80}$$

$$T_{°C} = \frac{25.1 - 32}{1.80}$$

$T_{°C} = -3.8$

22.

Substance	Density	Mass	Volume
seawater	1.025 g/mL	52.6 g	51.3 mL
diamond	3.3 g/mL	2.13 g	0.65 mL
beeswax	0.96 g/mL	125.5 g	130 mL
oak wood	0.750 g/mL	4.63 g	6173.3 mL