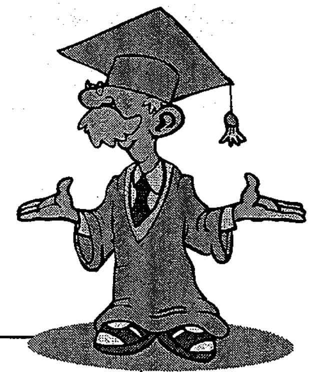
Polynomials

Section 10.1
Introducing Polynomials



You may recall that a term of an algebraic expression is a constant, a variable, or the product of a constant and a variable. Actually, a term can include two or more variables multiplied together. Remember, terms are separated by addition or subtraction.

Terms				
9	$5x$	$7ab$	$2xyz^2$	$3m^2n^4$

Each of these fits our expanded definition of a term. A polynomial is an expression with one or more terms. Polynomials can often be classified by their number of terms.

Number of Terms

- **Monomial** — An expression with one term and positive exponents, if it has them. $2xy$
- **Binomial** — An expression with two terms. $2xy + 1$
- **Trinomial** — An expression with three terms. $2xy + x - 1$
- Expressions with more than three terms do not have special names. $2xy + x - y + 1$

You will need to recognize polynomials by their proper names. The words are important because you may be asked to pick out a trinomial or any of the others for that matter. Remember: it's the number of *terms* and not the exponents or the number of variables that classify polynomials.

Monomial (One Term)	Binomial (Two Terms)	Trinomial (Three Terms)
$2x^2y^2$	$x^2 + y^2$	$x^2 + xy + y^2$

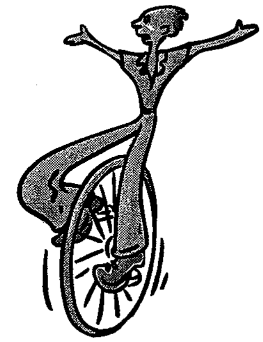
Practice

Label each of the following as a monomial (*m*), a binomial (*b*), or a trinomial (*t*).

______ 1. $2x^2 + x + 1$	______ 2. $a^2 - 2$	______ 3. $2m^2n^3$
______ 4. x^3y^2z	______ 5. $a^3b^2c + a^2bc^3$	______ 6. $3r^5s^2 + r^2s$
______ 7. 15	______ 8. $a - b + c$	______ 9. $mn^2z - m^3n^4z^2 - 1$

Polynomials

Section 10.2
Multiplying Monomials



Multiplying one monomial by another is a three step process:

Steps For Multiplying Monomials

1. Remove the parentheses and multiply like factors together.
2. Multiply the coefficients.
3. Add the exponents of like variables.

Example 1: Multiply $(2x)(3x)$.

Step 1: Group the like factors together.

Step 2: Multiply the coefficients.

Step 3: Add the exponents of like variables.

The result is $6x^2$.

$$(2x)(3x)$$

$$2 \cdot 3 \cdot x \cdot x$$

$$\underbrace{2 \cdot 3}_{2 \times 3 = 6} \cdot \underbrace{x^1 \cdot x^1}_{1 + 1 = 2}$$

Product of coefficients $6x^2$ Sum of Exponents

On simple problems like Example 1, you might not need to write out all the steps, but the process is very important. This is the simplest one you will see!

Example 2: Multiply $(3a^2b^2)(4a^3b^4)$.

Step 1: Group the like factors together.

Step 2: Multiply the coefficients.

Step 3: Add the exponents of like variables.

The result is $12a^5b^6$.

$$(3a^2b^2)(4a^3b^4).$$

$$3 \cdot 4 \cdot a^2 \cdot a^3 \cdot b^2 \cdot b^4$$

$$\underbrace{3 \cdot 4}_{3 \times 4 = 12} \cdot \underbrace{a^2 \cdot a^3}_{2 + 3 = 5} \cdot \underbrace{b^2 \cdot b^4}_{2 + 4 = 6}$$

$$12a^5b^6$$

Example 3: Multiply $(a^2bc^4)(a^3bcd)(b^2d^4)$.

Step 1: Group the like factors together

Step 2: Since all the coefficients are one, add the exponents of like variables.

The result is $a^5b^4c^5d^5$.

$$(a^2bc^4)(a^3bcd)(b^2d^4)$$

$$a^2 \cdot a^3 \cdot b \cdot b \cdot b^2 \cdot c^4 \cdot c \cdot d \cdot d^4$$

$$\underbrace{a^2 \cdot a^3}_{2 + 3 = 5} \cdot \underbrace{b \cdot b \cdot b^2}_{1 + 1 + 2 = 4} \cdot \underbrace{c^4 \cdot c}_{4 + 1 = 5} \cdot \underbrace{d \cdot d^4}_{1 + 4 = 5}$$

$$a^5b^4c^5d^5$$

Multiplying Monomials

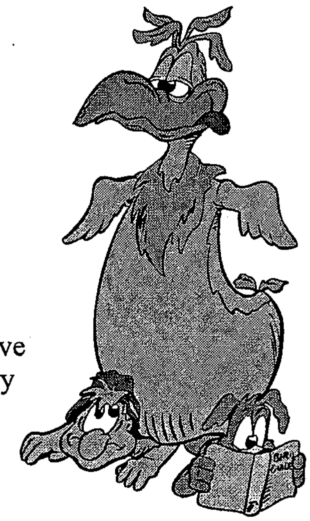
Practice

Multiply the following monomials.

1. $(4m)(3m)$	2. $(a)(3a^2)$
3. $(2x^2y^2)(3x^3)$	4. $(5r^2t)(2r^3s^2t^2)$
5. $(4ab^2c^3)(2a^2b^4c)$	6. $(a^2b^3cd^3)(3a^5bc^3d)$
7. $(3a^2b^2c^2)(2a^4b^4c^4)(3a^5b^4c^3)$	8. $(abcd)(2a^4b^4cd^3)(2a^5b^4c^3d^2)$

Polynomials

Section 10.3
Multiplying a Polynomial by a Monomial



Multiplying a monomial times a polynomial works just like multiplying more than one monomial by the same factor.

$$3m\,(3m^2+m)$$

$$3m \cdot 3m^2 + 3m \cdot m$$

$$\underbrace{3 \cdot 3}_{3\times3=9} \cdot \underbrace{m \cdot m^2}_{1+2=3} + \underbrace{3 \cdot 1}_{3\times1=3} \cdot \underbrace{m \cdot m}_{1+1=2}$$

$$9m^3 + 3m^2$$

The easiest way to do this is to use the distributive property to rearrange the multiplication. Multiply each of the terms by the factor. Now you have two sets of monomials multiplied together, and you know how to do that.

Put the common factors together, multiply the coefficients, and add the exponents of like variables. With a little practice, you will be able to do the mental math rather than writing out all of this stuff.

This would probably be a good time to relate the distributive property to multiplying a monomial times a polynomial. Essentially it says that multiplying a monomial times a polynomial in parentheses is the same as multiplying the monomial times each of the terms in the parentheses. Wasn't that what you did in the example above?

This rule will work no matter how many terms are in the polynomial. The universal formula for the distributive property is the following:

Distributive Property

$$a\,(b + c) = ab + ac$$

Now, let's put the distributive property to work.

Example 1: Multiply: $2a(a^2 - a + 2)$ — $2a(a^2 - a + 2)$

Step 1: Multiply the monomial outside the parentheses times each term of the trinomial. — $2a \cdot a^2 - 2a \cdot a + 2a \cdot 2$

Step 2: Collect terms. (Do the math.) — $2a^3 - 2a^2 + 4a$

Example 2: Multiply: $-x(-2x^2 - 6x + 3)$ — $-x(-2x^2 - 6x + 3)$

Step 1: Multiply the monomial outside the parentheses times each term of the trinomial. Be careful with the signs. — $-x(-2x^2) - (-x)(6x) + (-x)(3)$

$2x^3 - (-6x^2) + (-3x)$

Step 2: Collect terms. (Do the math.) — $2x^3 + 6x^2 - 3x$

Caution:
You may see parentheses with only a negative sign on the outside. That means to multiply every term in the parentheses by negative one. (Or just change the sign of each term and remove the parentheses.)

Section 10.3, continued
Multiplying a Polynomial by a Monomial

Practice

Simplify by using the distributive property.

1. $3x(2x+2)$ ________	2. $4m(3m-3)$ ________
3. $2a(a^2+a+2)$ ________	4. $-2(2x^2-x+1)$ ________
5. $-m(2m^2+m-2)$ ________	6. $-x(-x^2-x-3)$ ________
7. $3n(n^3+2n^2-n+1)$ ________	8. $-(x^2+x-2)$ ________
9. $4a(3a^3-2a^2-a+2)$ ________	10. $-2m(-m^3-2m^2+3m+2)$ ________

Polynomials

Section 10.4
Adding Polynomials

When you add polynomials, there are several things that you should keep in mind. To make addition easier, both polynomials should be in the same form. They can also be arranged one over the other so that like terms match vertically. And finally, watch the signs carefully.

Steps For Adding Polynomials

1. Arrange both polynomials into the same form.
2. Place like terms over each other vertically.
3. Do the math, and watch the signs.

Example 1: **Add these polynomials: $2a^2 - 3a + 5$ and $6a - a^2 - 8$.**

Step 1: The terms of the second expression aren't in the same order as the first expression. Rewrite the second expression to match the form of the first.

$$2a^2 - 3a + 5$$
$$-a^2 + 6a - 8$$

Step 2: Arrange vertically so that like terms match. Draw boxes if you need them.

	$2a^2$	$-\ 3a$	$+5$
+	$-a^2$	$+\ 6a$	-8
	a^2	$+\ 3a$	-3

Step 3: Add like terms vertically. Be careful with the signs.

$$a^2 + 3a - 3$$

Example 2: **What is the sum of these polynomials? $3m^2 - m + 7$ and $2m^2 - 9$**

Step 1: Check the order of terms. The second polynomial is in the same order as the first except it is missing one of the terms.

$$3m^2 - m + 7$$
$$2m^2 - 9$$

Step 2: Arrange vertically so that like terms match. Notice that the middle term is missing. Leave a blank for it, and make sure the other terms match vertically.

	$3m^2$	$-\ m$	$+\ 7$
+	$2m^2$		$-\ 9$
	$5m^2$	$-\ m$	$-\ 2$

Step 3: Add like terms vertically. Be careful with the signs.

$$5m^2 - m - 2$$

Important!

When you add polynomials, only the coefficients of the terms change.
The variables in the terms <u>never</u> change.

Section 10.4, continued
Adding Polynomials

Example 3: **Simplify the following algebraic expression:** $3(x^2y + 3y^2 - x) + 2(xy^2 - y^2 + 2x)$

This one is ugly, but you already have all the skills you need. Just be very careful with like terms. They have to be exactly alike except for the coefficients. ***All the variables in like terms must have the same exponents.***

Step 1: First use the distributive property to remove both sets of parentheses.

Step 2: Find the like terms and line them up vertically as before. CAREFUL — two of these terms look similar but are not alike.

Step 3: Now combine the like terms by adding them. You end up with a polynomial with 4 terms.

$$3(x^2y + 3y^2 - x) + 2(xy^2 - y^2 + 2x)$$

$$3x^2y + 9y^2 - 3x + 2xy^2 - 2y^2 + 4x$$

	$3x^2y$		$+\ 9y^2$	$-\ 3x$
+		$2xy^2$	$-\ 2y^2$	$+\ 4x$
	$3x^2y$	$+\ 2xy^2$	$+\ 7y^2$	$+\ x$

Practice

Add the following polynomials. Show your work and put the sums in the blanks provided.

1. $2x^2 - x + 2$ and $3x - x^2 - 4$

   ____

2. $(x^2 + 2x + 1) + (3x^2 - 4x - 1)$

   ____

3. $(2m^2 + 2) + (m^2 - m + 4)$

   ____

4. $(3a^2 + 2) + (-a^2 - a)$

   ____

5. $2(x^2 + y - 2) + 6(x^2 - y - 1)$

   ____

6. $2(ab^2 + 2ab + 5) + 4(a^2b - ab^2 + ab)$

   ____

Polynomials

Section 10.5
Subtracting Polynomials

Subtracting polynomials uses the same process as adding them. The major difference is that you have to add the opposite of the bottom polynomial. Set it up the same way you do addition and then reverse the signs for *every term* in the bottom polynomial. Just make sure you know which one belongs on the bottom.

Steps For Subtracting Polynomials
1. Arrange both polynomials into the same form.
2. Place like terms over each other vertically.
3. Reverse the signs for *every term* in the bottom polynomial.
4. Do the math, and watch the signs carefully.

Example 1: **Subtract $c + 3$ from $2c + 3$.**

Step 1: Both expression are in the same form.

$$c + 3$$
$$2c + 3$$

Step 2: Arrange vertically so that like terms match. In this case the "from" tells you which one goes on top. The polynomial that comes after the "from" is the top one.

	$2c$	$+3$
$-$	c	$+3$

Step 3: Remember that to subtract, you add the opposite. Reverse the sign of every term in the bottom polynomial.

	$2c$	$+3$
$+$	$-c$	-3
	c	

Step 4: Do the math. The solution is simply c.

Example 2: **Simplify $5n - 3(n + 1)$.**

When the problem is written in this form, the expression after the subtraction sign goes on the bottom.

Step 1: First, distribute the multiplication. Multiply each term inside the parentheses by 3. But keep the parentheses for the result since you are subtracting.

$$5n - 3(n + 1)$$
$$5n - (3n + 3)$$

Step 2: Arrange vertically so that like terms match.

	$5n$	
$-$	$3n$	$+3$

Step 3: Remember that to subtract, you add the opposite. Reverse the sign of every term in the bottom polynomial.

	$5n$	
$+$	$-3n$	-3
	$2n$	-3

Step 4: Do the math. You get $2n - 3$.

Section 10.5, continued
Subtracting Polynomials

Example 3: Simplify $(5a + 3c - 6) - 3(3a - c - 2)$.

Let's try a shortcut this time. Rather than arranging everything in the squares and changing the signs, distribute the sign with the number. That means that everything in the parentheses will be multiplied by –3. Then all you have to do is combine like terms.

$$(5a + 3c - 6)$$
$$-3(3a - c - 2)$$

Step 1: Distribute the multiplication, both the sign and the number.

$$-3(3a) \quad -3(-c) \quad -3(-2)$$

Step 2: Multiply each term by –3. Include the operation as the sign of the term.

$$-9a \quad +3c \quad +6$$

Step 3: Remove the parentheses from the first polynomial and line up like terms.

$$\begin{array}{rrr} 5a & +3c & -6 \\ -9a & +3c & +6 \end{array}$$

Step 4: Combine like terms and you're done.

$$-4a + 6c$$

Example 4: Simplify $(3r^2s - 3r + 11s^2) - (2r^2s + 2r + 7s^2)$.

Don't panic. This one just looks uglier. Distribute the negative sign for the second polynomial and combine like terms. It's just that simple.

$$(3r^2s - 3r + 11s^2)$$
$$-(2r^2s + 2r + 7s^2)$$

Step 1: Distribute the multiplication even though it's only a sign.

$$-(2r^2s) \quad -(2r) \quad -(7s^2)$$

Step 2: Multiply each term by –1. Include the operation as the sign of the term.

$$-2r^2s \quad -2r \quad -7s^2$$

Step 3: Remove the parentheses from the first polynomial and align with the second polynomial.

$$\begin{array}{rrr} 3r^2s & -3r & +11s^2 \\ -2r^2s & -2r & -7s^2 \end{array}$$

Step 4: Combine like terms, and that's all there is to it.

$$r^2s - 5r + 4s^2$$

Example 5: Simplify $(4ab^2 + 3a^2b + 12b^2) - (2a^2b - 2b + 5b^2)$.

This time you don't have a match for every term. Don't worry; just line up the terms where they match. If a term is missing, leave a blank space.

$$4ab^2 + 3a^2b + 12b^2$$
$$-(2a^2b - 2b + 5b^2)$$

Step 1: Distribute the multiplication even though it's only a sign.

Step 2: Line up the terms that match. Leave blanks where there are no matching terms.

$$\begin{array}{rrrr} -2a^2b & & +2b & -5b^2 \\ +3a^2b & +4ab^2 & & +12b^2 \end{array}$$

Step 3: Combine like terms.

$$a^2b + 4ab^2 + 2b + 7b^2$$

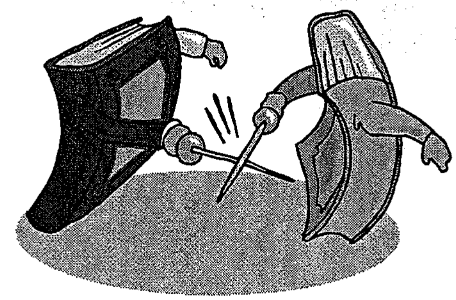
Practice

Subtract the following polynomials. Show your work and write the differences in the blanks.

1. $c + 5$ from $3c + 7$ ____________	2. $3a - 3$ minus $a + 3$ ____________
3. $(n^2 + 3n + 3) - (2n + 3)$ ____________	4. $(a^2 - 2a - 1) - (2a^2 + 2a + 3)$ ____________
5. $(2xy^2 - xy - 1) - (xy^2 - xy - 1)$ ____________	6. $(2x^2y^2 + x^2y + xy^2 + 3) - (x^2y^2 + x^2y - xy - 2y^2)$ ____________
7. $(4y^2 - y - 1) - 3(y^2 - 2y + 2)$ ____________	8. $(2x^2 + x + 3) - (x^2 + 3x - 2)$ ____________

Polynomials

Section 10.6
Multiplying Binomials



FOIL

When two binomials are to be multiplied, each term of the first binomial must be multiplied by each term of the second binomial.

$$(a + b)(c + d)$$

$$ac + ad + bc + bd$$

As you consider this plan, two things become obvious:

1. The number of terms in the product is the same as the number of terms in both binomials.
2. There is a pattern to multiplying the terms. It's called FOIL.

F	O	I	L
First Terms	Outside Terms	Inside Terms	Last Terms

Example 1: Multiply $(a + 1)(a + 2)$.

Step 1: Multiply first terms.

Step 2: Multiply outside terms.

Step 3: Multiply inside terms.

Step 4: Multiply last terms.

Step 5: Arrange according to FOIL and combine like terms.

$$(a + 1)(a + 2)$$

2 $\quad a$

$$(a + 1)(a + 2)$$

a^2 $\quad 2a$

F O I L

$$a^2 + 2a + a + 2$$

$$a^2 + 3a + 2$$

Example 2: Multiply $(x + 3)(x - 2)$.

Step 1: Multiply first terms.

Step 2: Multiply outside terms. Treat the operation as the sign of the term.

Step 3: Multiply inside terms.

Step 4: Multiply last terms. Treat the operation as the sign of the term.

Step 5: Arrange according to FOIL and combine like terms.

$$(x + 3)\ (x - 2)$$

-6 $\quad 3x$

$$(x + 3)(x - 2)$$

x^2 $\quad -2x$

F O I L

$$x^2 - 2x + 3x - 6$$

$$x^2 + x - 6$$

Example 3: Multiply $(2m - 4)(m - 3)$.

Step 1: Multiply according to **FOIL**.

Step 2: Arrange according to FOIL and combine like terms

$$(2m - 4)\ (m - 3)$$

F O I L

$$2m^2 - 6m - 4m + 12$$

$$2m^2 - 10m + 12$$

Section 10.6, continued
Multiplying Binomials

Practice 1
Multiply the following. Show your work and put the product in the space provided.

1. $(x+3)(x+3)$ ________	2. $(x+3)(x-3)$ ________
3. $(x-2)(x-4)$ ________	4. $(x-1)(x+2)$ ________

Special Products
When you multiply two binomials, a special case happens when the factors are the sum and difference of the same terms.

$$(a+b)(a-b)$$
$$a^2-ab+ba-b^2$$
$$a^2-b^2$$

When you multiply the sum and difference of the same terms, the middle term disappears. In fact, it doesn't matter which two terms you use, you get a solution in the same form as this one.

Example 4: Multiply $(a+2)(a-2)$.

$$(a+2)(a-2)$$

Step 1: Multiply according to **FOIL**.

$$a^2-2a+2a-2^2$$

Step 2: Combine like terms and simplify.

$$a^2-4$$

Did you notice that the two solutions have the same form? Since this is consistent with any two terms you choose, you can set up a shortcut for multiplying two binomials like this.

Shortcut for Multiplying Binomials In The Form $(a+b)(a-b)$

1. Square the first term.
2. Put in a minus sign.
3. Square the last term.

$$a^2-b^2$$

Practice 2
Simplify the following by multiplying.

1. $(a+1)(a-1)$ ________	2. $(m+4)(m-4)$ ________

Squaring Binomials

Squaring binomials works the same way as multiplying binomials with one additional step. You have to set up the problem.

$$(a+b)^2$$
$$(a+b)(a+b)$$
$$\overset{F}{a^2} + \overset{O}{ab} + \overset{I}{ba} + \overset{L}{b^2}$$
$$a^2 + 2ab + b^2$$

Think about what squaring an expression means. It's the same as multiplying the expression by itself.

With that in mind, rewrite the problem. In this form, you can use FOIL to multiply it.

Example 5: **Square the binomial $(x+3)^2$.**

Step 1: Write out the multiplication as two binomials.

Step 2: Multiply according to **FOIL.**

Step 3: Combine like terms.

$$(x+3)^2$$
$$(x+3)(x+3)$$
$$\overset{F}{x^2} + \overset{O}{3x} + \overset{I}{3x} + \overset{L}{9}$$
$$x^2 + 6x + 9$$

Again, the products are all in the same form. Here's the shortcut for the square of a sum.

Shortcut for Multiplying Binomials In The Form $(a+b)(a+b)$ or $(a+b)^2$

1. Square the first term.
2. Double the product of the two terms.
3. Square the last term.

$$a^2 + 2ab + b^2$$

A final case of squaring binomials would be a difference of terms. Apply what you know about FOIL to this case.

$$(a-b)^2$$
$$(a-b)(a-b)$$
$$\overset{F}{a^2} - \overset{O}{ab} - \overset{I}{ba} + \overset{L}{b^2}$$
$$a^2 - 2ab + b^2$$

- Set up the problem by multiplying the binomial by itself.
- Multiply according to FOIL.
- Combine like terms.

Example 6: **Square the binomial $(a-2)^2$.**

Step 1: Set up the problem.

Step 2: Multiply according to FOIL.

Step 3: Combine like terms and simplify.

$$(a-2)^2$$
$$(a-2)(a-2)$$
$$a^2 - 2a - 2a + 2^2$$
$$a^2 - 4a + 4$$

Section 10.6, continued
Multiplying Binomials

Notice that the only difference in this case and the last is the negative middle term. Let's write another shortcut.

Shortcut for Multiplying Binomials In The Form $(a-b)(a-b)$ or $(a-b)^2$

1. Square the first term.
2. Multiply the product of the two terms by –2. $a^2 - 2ab + b^2$
3. Square the last term.

Practice 3
Simplify the following binomial squares.

1. $(x+1)^2$ ________	2. $(x+4)^2$ ________
3. $(x+5)^2$ ________	4. $(x+6)^2$ ________
5. $(x-3)^2$ ________	6. $(x-1)^2$ ________
7. $(x-4)^2$ ________	8. $(x-2)^2$ ________
9. $(x+7)^2$ ________	10. $(x-7)^2$ ________

Polynomials
Section 10 Review

Answer each question below. Darken the circle that represents the correct answer.

1. **Which of these would NOT result in a trinomial in its simplified form?**

A $(2r^3s^2)(3rs^3)$

B $3r(r+s)^2$

C $(2r+4)(r-4)$

D $(2s-2)^2$

Ⓐ Ⓑ Ⓒ Ⓓ

2. **Which expression is the result of $(x^2-2)+(2x^2+x-1)$?**

A $2x^2+2x-2$

B $3x^2+x-3$

C x^2+x+1

D $3x^2+x-1$

Ⓐ Ⓑ Ⓒ Ⓓ

3. **Which of the following would be classified as a binomial?**

A a^2+2a-4

B $9y^2-1$

C $x^3y^5z^3$

D $2m^3n$

Ⓐ Ⓑ Ⓒ Ⓓ

4. **Which of the following is equivalent to the expression $(xyz)(xy^2)(y^3z^2)$?**

A $x^2y^6z^3$

B $x^3y^6z^2$

C $x^2y^5z^2$

D $x^3y^5z^3$

Ⓐ Ⓑ Ⓒ Ⓓ

5. **Which is equivalent to the following expression?**

$$-x(3x-1)$$

A $-3x+1$

B $-3x-1$

C $-3x^2+x$

D $-3x^2-x$

Ⓐ Ⓑ Ⓒ Ⓓ

6. **Which polynomial is equivalent to $(2x+3)^2$?**

A $2x^2+9$

B $4x^2+9$

C $4x^2+6x+9$

D $4x^2+12x+9$

Ⓐ Ⓑ Ⓒ Ⓓ

7. **Which polynomial is equivalent to $(x+3)(x-3)$?**

A x^2+9

B x^2-9

C x^2+6x-9

D x^2-6x+9

Ⓐ Ⓑ Ⓒ Ⓓ

8. **Which polynomial is equivalent to $(3y-1)^2$?**

A $9y^2-1$

B $9y^2+1$

C $9y^2+6y-1$

D $9y^2-6y+1$

Ⓐ Ⓑ Ⓒ Ⓓ

9. Which expression is the result of $(2x^2y^2 - x^2y + xy^2 + 3) + (x^2y^2 + 2x^2y - 2xy - 2y^2)$

A $2x^2y^2 + x^2y + xy^2 + 2xy - 2y^2 + 3$

B $3x^2y^2 + x^2y + xy^2 - 2xy - 2y^2 + 3$

C $2x^2y^2 + x^2y + xy^2 - 2xy - 2y^2$

D $3x^2y^2 + 2x^2y - 2xy - 2y^2 + 3$

Ⓐ Ⓑ Ⓒ Ⓓ

10. Which expression is the result of $(3x - 2y + 2) - 2(x + 3y - 3)$?

A $x - 8y + 8$

B $x + 8y + 8$

C $-x - 4y + 8$

D $-x + 4y - 8$

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11. Which expression is the result of $(2mn^2 - 2n - 3m^2n) - 3(2mn^2 - 2m + m^2n)$?

A $-4mn^2 - 2n^2 + 6m$

B $-4mn^2 - 4n + 6m^2n$

C $-4mn^2 + 2n + 6m^2n - 6m$

D $-4mn^2 - 2n - 6m^2n + 6m$

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12. Which expression is equivalent to $(3a^2b^2 + 2ab^2 - 2a^2b + 3ab) - (3ab^2 + ab - 3)$?

A $3a^2b^2 + 2ab + 3$

B $3a^2b^2 + 5ab^2 - 2a^2b + 4ab - 3$

C $3a^2b^2 - 2ab^2 + a^2b + 2ab - 3$

D $3a^2b^2 - ab^2 - 2a^2b + 2ab + 3$

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13. Which expression is the result of $(3a^2c - 4a + 3c^2) - (2a^2c + a - 5c^2)$?

A $a^2c - 5a + 8c^2$

B $a^2c - 3a - 2c^2$

C $5a^2c - 3a - 2c^2$

D $5a^2c - 5a - 2c^2$

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14. Which expression is the result of $2x(x + y - 1) - 3y(2x + 3y + 5)$?

A $2x^2 - 2x + 4xy - 9y^2 - 15y$

B $2x^2 - 2x - 4xy - 9y^2 - 15y$

C $2x^2 - 2x - 8xy - 9y^2 + 15y$

D $2x^2 + 2x + 8xy - 9y^2 + 15y$

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