Completing the Square

Completing the Square is a method for solving quadratic equations whenever factoring is not possible. The idea is to force a perfect square trinomial by adding some constant to each side of the equation.

Example 1: Determine the value of c that completes the square:

$x^2 + 8x + C$

Find c by taking $\left(\frac{b}{2}\right)^2$

$x^2 + 8x + 16$

so

$(x+4)^2$

$\left(\frac{8}{2}\right)^2$ so $4^2 = 16$

Determine the value of c that completes the square:

Example 2: $x^2 + 10x + C$ $\quad C = \left(\frac{10}{2}\right)^2 = 25$

Example 3: $x^2 + 22x + C$ $\quad C = \left(\frac{22}{2}\right)^2 = 121$

Example 4: $\underline{3x^2 + 6x} + C$

Make an adjustment by factoring

$3(x^2 + 2x + 1)$

Now we will actually solve these equations for x.

Example 5: $x^2 + 6x + 3 = 0$

$$x^2 + 6x + 9 = -3 + 9$$

$$\sqrt{(x+3)^2} = \sqrt{6}$$

$$x + 3 = \pm\sqrt{6}$$

$$x = -3 \pm \sqrt{6}$$

1) Move 3

2) Is there a # in front of x^2

3) Take $\left(\frac{6}{2}\right)^2$

4) Add 9 to each side

5) Make PSr & solve for x

Example 6: $x^2 + 4x - 8 = 0$

$x^2 + 4x + 4 = 8 + 4$ $\qquad \left(\frac{4}{2}\right)^2$

$\sqrt{(x+2)^2} = \sqrt{12}$ $\qquad 2^2 = 4$

$x + 2 = \pm\sqrt{12}$

$x = -2 \pm \sqrt{12}$

Example 7: $3x^2 + 12x - 10 = 8$

$$3x^2 + 12x = 18$$

$$3(x^2 + 4x + 4) = 18 + 12$$

$$3(x + 2)^2 = 30$$

$$\sqrt{(x+2)^2} = \sqrt{10}$$

$$x + 2 = \pm\sqrt{10}$$

$$x = -2 \pm \sqrt{10}$$

The final question type involving completing the square desires you to get a polynomial in VERTEX FORM. This is useful when graphing with transformations as in the past chapter.

Example 8: $y = x^2 + 12x - 7$

$36 + y + 7 = x^2 + 12x + 36$

$y + 43 = (x + 6)^2$

$y = (x + 6)^2 - 43$

Example 9: $y = 2x^2 + 6x + 6$

$$y - 6 = 2x^2 + 6x$$

$$4.5 + y - 6 = 2(x^2 + 3x + 2.25)$$

$$y - 1.5 = 2(x + 1.5)^2$$

so

$$y = 2(x + 1.5)^2 + 1.5$$